Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Motion in a circle
Exercise A, Question 1

Question:

Express

a an angular speed of 5 revolutions per minute in rad s^{-1},

b an angular speed of 120 revolutions per minute in rad s^{-1},

c an angular speed of 4 rad s^{-1} in revolutions per minute,

d an angular speed of 3 rad s^{-1} in revolutions per hour.

Solution:

a $5 \text{ rev min}^{-1} = 5 \times 2\pi \text{ rad min}^{-1} = \frac{5 \times 2\pi}{60} \text{ rad s}^{-1} \approx 0.524 \text{ rad s}^{-1}$.

b $120 \text{ rev min}^{-1} = 120 \times 2\pi \text{ rad min}^{-1} = \frac{120 \times 2\pi}{60} \text{ rad s}^{-1} \approx 12.6 \text{ rad s}^{-1}$.

c $4 \text{ rad s}^{-1} = 4 \times 60 \text{ rad min}^{-1} = \frac{4 \times 60}{2\pi} \text{ rev min}^{-1} \approx 38.2 \text{ rev min}^{-1}$.

d $3 \text{ rad s}^{-1} = 3 \times 60 \times 60 \text{ rad h}^{-1} = \frac{3 \times 60 \times 60}{2\pi} \text{ rev h}^{-1} \approx 1720 \text{ rev h}^{-1}$.

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Edexcel AS and A Level Modular Mathematics

Motion in a circle
Exercise A, Question 2

Question:

Find the speed in m s^{-1} of a particle moving on a circular path of radius 20 m at

a 4 rad s^{-1},

b 40 rev min^{-1}.

Solution:

a $v = r\omega$: $v = 20 \times 4 = 80 \text{ m s}^{-1}$.

b Distance per minute $= 40 \times 2\pi \times 20 = 1600\pi$ m

Distance per second $= \dfrac{1600\pi}{60} \approx 83.8 \text{ m}, v = 83.8 \text{ m s}^{-1}$.

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Motion in a circle
Exercise A, Question 3

Question:

A particle moves on a circular path of radius 25 cm at a constant speed of 2 m s^{-1}.
Find the angular speed of the particle

a in rad s^{-1},

b in rev min^{-1} .

Solution:

a $\omega = \frac{v}{r} = \frac{2}{0.25} = 8 \text{ rad s}^{-1}$ ← Need to convert cm to m.

b $8 \text{ rad s}^{-1} = 8 \times 60 \text{ rad min}^{-1} = \frac{8 \times 60}{2\pi} \approx 76.4 \text{ rev min}^{-1}$.

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Edexcel AS and A Level Modular Mathematics

Motion in a circle

Exercise A, Question 4

Question:

Find the speed in m s^{-1} of a particle moving on a circular path of radius 80 cm at

a 2.5 rad s^{-1},

b 25 rev min^{-1} .

Solution:

a $v = r\omega : v = 0.8 \times 2.5 = 2 \text{ m s}^{-1}$.

b $25 \text{ rev min}^{-1} = 25 \times 2\pi \text{ rad min}^{-1} = \frac{25 \times 2\pi}{60} \text{ rad s}^{-1} = 2.617\ldots \text{ rad s}^{-1}$.

$v = r\omega : v = 0.8 \times 2.617\ldots \simeq 2.09 \text{ m s}^{-1}$

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Edexcel AS and A Level Modular Mathematics

Motion in a circle
Exercise A, Question 5

Question:

An athlete is running round a circular track of radius 50 m at 7 m s^{-1}.

a How long does it take the athlete to complete one circuit of the track?

b Find the angular speed of the athlete in rad s^{-1}.

Solution:

a $\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{2\pi \times 50}{7} \approx 44.9 \text{ s}$

b $\omega = \frac{v}{r} = \frac{7}{50} = 0.14 \text{ rad s}^{-1}$

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Edexcel AS and A Level Modular Mathematics

Motion in a circle

Exercise A, Question 6

Question:

A disc of radius 12 cm rotates at a constant angular speed, completing one revolution every 10 seconds. Find

a the angular speed of the disc in rad s^{-1},

b the speed of a particle on the outer rim of the disc in m s^{-1},

c the speed of a particle at a point 8 cm from the centre of the disc in m s^{-1}.

Solution:

a $1 \text{ rev in } 10 \text{ s} = 0.1 \text{ rev s}^{-1} = 0.1 \times 2\pi \text{ rad s}^{-1} \approx 0.628 \text{ rad s}^{-1}$.

b $v = r\omega : v = 0.12 \times 0.628\ldots \approx 0.0754 \text{ m s}^{-1}$.

c $v = r\omega : v = 0.08 \times 0.628\ldots \approx 0.0503 \text{ m s}^{-1}$

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Motion in a circle
Exercise A, Question 7

Question:

A cyclist completes two circuits of a circular track in 45 seconds. Calculate

a his angular speed in rad s^{-1},

b the radius of the track given that his speed is 40 km h^{-1}.

Solution:

a 2 circuits $= 2 \times 2\pi$ radians in 45 seconds $= \frac{4\pi}{45} \text{ rad s}^{-1} \approx 0.279 \text{ rad s}^{-1}$.

b $40 \text{ km h}^{-1} = \frac{40 \times 1000}{3600} = 11.1\ldots \text{ m s}^{-1}$.

$r = \frac{v}{\omega} = \frac{11.111}{0.279} \approx 39.8 \text{ m}$

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Motion in a circle
Exercise A, Question 8

Question:

Anish and Bethany are on a fairground roundabout. Anish is 3 m from the centre and Bethany is 5 m from the centre. If the roundabout completes 10 revolutions per minute, calculate the speeds with which Anish and Bethany are moving.

Solution:

$10 \text{ rev min}^{-1} = 10 \times 2\pi \text{ rad min}^{-1} = \dfrac{10 \times 2\pi}{60} \text{ rad s}^{-1} \approx 1.05 \text{ rad s}^{-1}.$

$v = r\omega$: Anish's speed $= 3 \times 1.047\ldots \approx 3.14 \text{ m s}^{-1}$,

Bethany's speed $= 5 \times 1.047\ldots \approx 5.24 \text{m s}^{-1}$.

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Motion in a circle
Exercise A, Question 9

Question:

A model train completes one circuit of a circular track of radius 1.5 m in 26 seconds.
Calculate

a the angular speed of the train in rad s^{-1},

b the linear speed of the train in m s^{-1}.

Solution:

a 1 circuit in 26 seconds $= \frac{2\pi}{26} \text{ rad s}^{-1} \approx 0.242 \text{ rad s}^{-1}$.

b $v = r\omega : v = 1.5 \times 0.24166\ldots = 0.362 \text{ m s}^{-1}$

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Motion in a circle
Exercise A, Question 10

Question:

A train is moving at 150 km h^{-1} round a circular bend of radius 750 m. Calculate the angular speed of the train in rad s^{-1}.

Solution:

$$150\text{ km h}^{-1} = \frac{150 \times 1000}{3600}\text{ m s}^{-1} \approx 41.7\text{ m s}^{-1}$$

$$\omega = \frac{v}{r} = \frac{41.66666}{750} \approx 0.056\text{ rad s}^{-1}$$

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Motion in a circle
Exercise A, Question 11

Question:

The hour hand on a clock has radius 10 cm, and the minute hand has radius 15 cm.
Calculate
a the angular speed of the end of each hand,
b the linear speed of the end of each hand.

Solution:

a Hour hand: 2π radians in 12 hours $= \dfrac{2\pi}{12 \times 3600}$ rad s^{-1} ≈ 0.000145 rad s^{-1}.

Minute hand: 2π radians in 1 hour $= \dfrac{2\pi}{3600}$ rad s^{-1} ≈ 0.00175 rad s^{-1}.

b $v = r\omega$: End of hour hand moves at $0.1 \times 0.000145 = 1.45 \times 10^{-5}$ m s^{-1}.

End of minute hand moves at $0.15 \times 0.00175 = 2.62 \times 10^{-4}$ m s^{-1}.

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Motion in a circle
Exercise A, Question 12

Question:

The drum of a washing machine has diameter 50 cm. The drum spins at 1200 rev min^{-1}. Find the linear speed of a point on the drum.

Solution:

$1200 \text{ rev min}^{-1} = 1200 \times 2\pi \text{ rad min}^{-1} = \dfrac{1200 \times 2\pi}{60} \text{ rad s}^{-1} \approx 126 \text{ rad s}^{-1}.$

$v = r\omega : v = 125.66.. \times 0.5 = 62.8 \text{ m s}^{-1}.$

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Motion in a circle
Exercise A, Question 13

Question:

A gramophone record rotates at 45 rev min^{-1}.

a Find the angular speed of the record in rad s^{-1}.

b Find the distance from the centre of a point moving at 12 cm s^{-1}.

Solution:

a $45 \text{ rev min}^{-1} = 45 \times 2\pi \text{ rad min}^{-1} = \frac{45 \times 2\pi}{60} \text{ rad s}^{-1} \approx 4.71 \text{ rad s}^{-1}$.

b $r = \frac{v}{\omega} = \frac{12}{4.712} \approx 2.55 \text{ cm}$ ← Working is in cm, not m here.

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Motion in a circle

Exercise A, Question 14

Question:

The Earth completes one orbit of the sun in a year. Taking the orbit to be a circle of radius 1.5×10^{11} m, and a year to be 365 days, calculate the speed at which the Earth is moving.

Solution:

Distance travelled in one year $= 2\pi \times 1.5 \times 10^{11}$ m,

So speed $= \dfrac{2\pi \times 1.5 \times 10^{11}}{365 \times 24 \times 3600} \approx 30\,000 \text{ m s}^{-1}$.

We can not achieve more than 2 s.f. accuracy in the answer because one of the original values only contained 2 s.f.

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Motion in a circle
Exercise B, Question 1

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle is moving on a horizontal circular path of radius 16 cm with a constant angular speed of 5 rad s^{-1}. Calculate the acceleration of the particle.

Solution:

$a = r\omega^2 : a = 0.16 \times 25 = 4 \text{ m s}^{-2}$.

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Motion in a circle
Exercise B, Question 2

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle is moving on a horizontal circular path of radius 0.3 m at a constant speed of 2.5 m s^{-1}. Calculate the acceleration of the particle.

Solution:

$$a = \frac{v^2}{r} : a = \frac{2.5^2}{0.3} \approx 20.8 \text{ m s}^{-2}.$$

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Motion in a circle
Exercise B, Question 3

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle is moving on a horizontal circular path of radius 3 m. Given that the acceleration of the particle is 75 m s^{-2} towards the centre of the circle, find

a the angular speed of the particle,
b the linear speed of the particle.

Solution:

a $a = r\omega^2 : 75 = 3\omega^2, \omega^2 = 25, \omega = 5 \text{ rad s}^{-1}$.

b $a = \frac{v^2}{r} : 75 = \frac{v^2}{3}, v^2 = 3 \times 75 = 225, v = 15 \text{ m s}^{-1}$.

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Motion in a circle
Exercise B, Question 4

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle is moving on a horizontal circular path of diameter 1.2 m. Given that the acceleration of the particle is 100 m s^{-2} towards the centre of the circle, find

a the angular speed of the particle,
b the linear speed of the particle.

Solution:

a $a = r\omega^2 : 100 = 0.6\omega^2, \omega^2 \approx 166.7, \omega \approx 12.9 \text{ rad s}^{-1}$.

b $a = \dfrac{v^2}{r} : 100 = \dfrac{v^2}{0.6}, v^2 = 100 \times 0.6 = 60, v \approx 7.75 \text{ m s}^{-1}$.

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Motion in a circle
Exercise B, Question 5

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A car is travelling round a bend which is an arc of a circle of radius 90 m. The speed of the car is 50 km h^{-1}. Calculate its acceleration.

Solution:

$$50 \text{ km h}^{-1} = \frac{50 \times 1000}{3600} \approx 13.89 \text{ m s}^{-1}.$$
$$a = \frac{v^2}{r} : a = \frac{13.89^2}{90} \approx 2.14 \text{ m s}^{-2}.$$

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Motion in a circle
Exercise B, Question 6

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A car moving along a horizontal road which follows an arc of a circle of radius 75 m has an acceleration of 6 m s^{-2} directed towards the centre of the circle. Calculate the angular speed of the car.

Solution:

$a = r\omega^2 : 6 = 75\omega^2, \omega^2 = 0.08, \omega \approx 0.283 \text{ rad s}^{-1}$.

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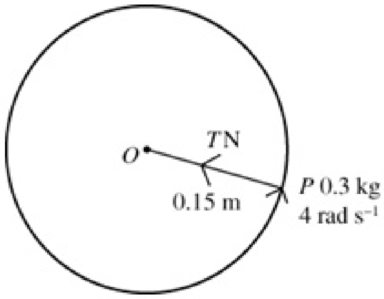
Motion in a circle
Exercise B, Question 7

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

One end of a light inextensible string of length 0.15 m is attached to a particle P of mass 300 g. The other end of the string is attached to a fixed point O on a smooth horizontal table. P moves in a horizontal circle centre O at constant angular speed 4 rad s^{-1}. Find the tension in the string.

Solution:



Suppose that the tension in the string is T.
Using $F = ma$
$T = 0.3 \times 0.15 \times 4^2 = 0.72 \text{ N}$

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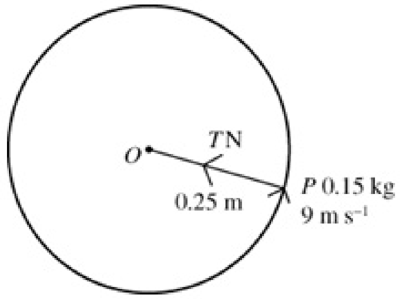
Motion in a circle
Exercise B, Question 8

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
One end of a light inextensible string of length 25 cm is attached to a particle P of mass 150 g. The other end of the string is attached to a fixed point O on a smooth horizontal table. P moves in a horizontal circle centre O at constant speed 9 m s^{-1}. Find the tension in the string.

Solution:



Suppose that the tension in the string is T.
Using $F = ma$

$$T = \frac{0.15 \times 9^2}{0.25} = 48.6 \text{ N}$$

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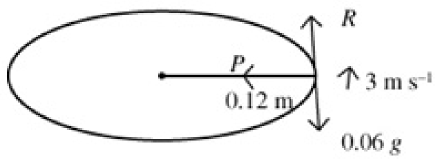
Motion in a circle
Exercise B, Question 9

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A smooth wire is formed into a circle of radius 0.12 m. A bead of mass 60 g is threaded onto the wire. The wire is horizontal and the bead is made to move along it with a constant speed of 3 m s^{-1}. Find

a the vertical component of the force on the bead due to the wire,
b the horizontal component of the force on the bead due to the wire.

Solution:



a Suppose the vertical component of the force is R:
$R(\updownarrow) R = 0.06g = 0.59 \text{ N}$

b Suppose the horizontal component of the force is F:
Using $F = ma$,

$$F = \frac{0.06 \times 3^2}{0.12} = 4.5 \text{ N}$$

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Motion in a circle
Exercise B, Question 10

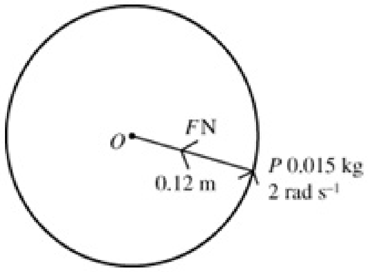
Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle P of mass 15 g rests on a rough horizontal disc at a distance 12 cm from the centre. The disc rotates at a constant angular speed of 2 rad s^{-1}, and the particle does not slip. Calculate

a the linear speed of the particle,

b the force due to the friction acting on the particle.

Solution:



a $v = r\omega$: $v = 0.12 \times 2 = 0.24 \text{ m s}^{-1}$

b Using $F = ma$

$F = 0.015 \times 0.12 \times 2^2 = 0.0072 \text{ N}$

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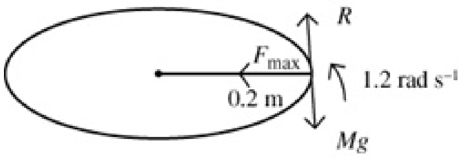
Edexcel AS and A Level Modular Mathematics

Motion in a circle
Exercise B, Question 11

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle P rests on a rough horizontal disc at a distance 20 cm from the centre. When the disc rotates at constant angular speed of 1.2 rad s^{-1}, the particle is just about to slip. Calculate the value of the coefficient of friction between the particle and the disc.

Solution:



Let R be the normal reaction between the particle and the disc, F the frictional force, M the mass of the particle, and μ be the coefficient of friction between the particle and the disc.

R(↕): $R = Mg$

The particle is about to slip, so $F = F_{max} = \mu R = \mu Mg$.

Using $F = ma$, $\mu Mg = M \times 0.2 \times 1.2^2 = M \times 0.288$,

$$\mu = \frac{0.288}{g} \approx 0.029$$

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Edexcel AS and A Level Modular Mathematics

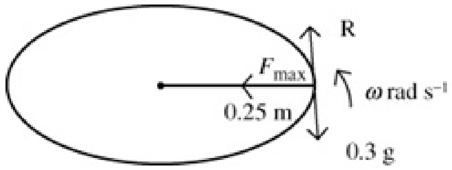
Motion in a circle
Exercise B, Question 12

Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.

A particle P of mass 0.3 kg rests on a rought horizontal disc at a distance 0.25 m from the centre of the disc. The coefficient of friction between the particle and the disc is 0.25. Given that P is on the point of slipping, find the angular speed of the disc.

Solution:



Let R be the normal reaction between the particle and the disc, F the frictional force.

Given $\mu = 0.25$ and $F = F_{max} = 0.25R$.

$R(\updownarrow)$: $R = 0.3g$, so $F_{max} = 0.25 \times 0.3g$.

Using $F = ma$, $0.25 \times 0.3\,g = 0.3 \times 0.25 \times \omega^2$,

$g = \omega^2$

$\omega \approx 3.1\text{ rad s}^{-1}$.

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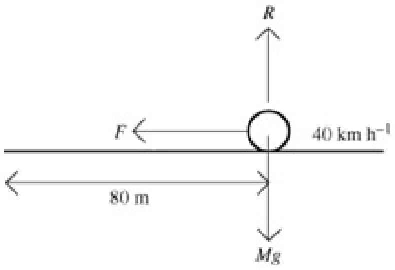
Edexcel AS and A Level Modular Mathematics

Motion in a circle
Exercise B, Question 13

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A car is travelling round a bend in the road which is an arc of a circle of radius 80 m. The greatest speed at which the car can travel round the bend without slipping is 40 km h^{-1}. Find the coefficient of friction between the tyres of the car and the road.

Solution:



$40 \text{ km h}^{-1} = \frac{40 \times 1000}{3600} \approx 11.11 \text{ m s}^{-1}$.

Let the mass of the car be M. Let F be the force due to friction between the car tyres and the road, μ the coefficient of friction, and R the normal reaction between the car and the road.

At maximum speed the car is about to slip, so $F = F_{max}$

R($\updownarrow$) $R = Mg$, so $F = F_{max} = \mu R = \mu Mg$

R($\leftrightarrow$) Using $F = ma$, $\mu Mg = \frac{M \times 11.11^2}{80}$, $\mu = \frac{11.11^2}{80 \times 9.8} \approx 0.16$

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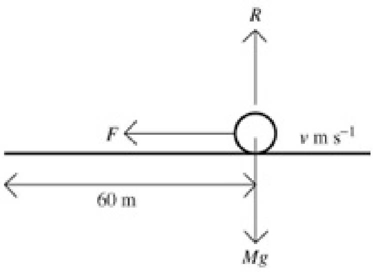
Edexcel AS and A Level Modular Mathematics

Motion in a circle
Exercise B, Question 14

Question:

Whenever a numerical value of g is required take $g = 9.8$ m s^{-2}.
A car is travelling round a bend in the road which is an arc of a circle of radius 60 m.
The coefficient of friction between the tyres of the car and the road is $\frac{1}{3}$. Find the greatest angular speed at which the car can travel round the bend without slipping.

Solution:



Let the mass of the car be M.
Let F be the force due to friction between the car tyres and the road, and R the normal reaction between the car and the road.
Max speed $\Rightarrow F = F_{max}$.

R(↕) $R = ma$, so $F = F_{max} = \mu R = \frac{1}{3}Mg$.

R(↔) Using $F = ma$, $\frac{1}{3}Mg = M \times 60 \times \omega^2$, $\omega^2 = \frac{g}{180} \approx 0.054$, $\omega \approx 0.23$ rad s^{-1}.

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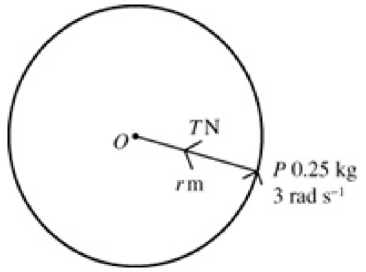
Motion in a circle
Exercise B, Question 15

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
One end of a light extensible string of natural length 0.3 m and modulus of elasticity 10 N is attached to a particle P of mass 250 g. The other end of the string is attached to a fixed point O on a smooth horizontal table. P moves in a horizontal circle centre O at constant angular speed 3 rad s^{-1}. Find the radius of the circle.

Solution:



If the extension in the string is x m, then the radius of the circle is $(0.3 + x)$ m, and the tension in the string is given by

$$T = \frac{\lambda x}{a} = \frac{10x}{0.3}$$

Using $F = ma$, $\frac{10x}{0.3} = 0.25 \times (0.3 + x) \times 3^2$, $10x = \frac{9}{4} \times \frac{3}{10}(0.3 + x)$

$400x = 8.1 + 27x$, $373x = 8.1$, $x \approx 0.0217 \text{ m}$

$\Rightarrow$ the radius of the circle ≈ 0.322 m

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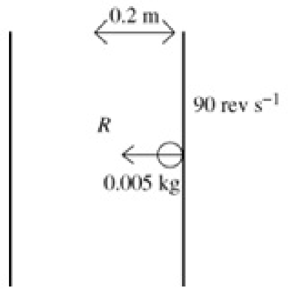
Motion in a circle
Exercise B, Question 16

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A centrifuge consists of a vertical hollow cylinder of radius 20 cm rotating about a vertical axis through its centre at 90 rev s^{-1}. Calculate the magnitude of the normal reaction between the cylinder and a particle of mass 5 g on the inner surface of the cylinder.

Solution:



$90 \text{ rev s}^{-1} = 90 \times 2\pi \text{ rad s}^{-1}$
$= 180\pi \text{ rad s}^{-1}$.

Let the normal reaction between the particle and the cylinder be R.

$(\leftrightarrow)$ Using $F = ma$, $R = 0.005 \times 0.2 \times (180\pi)^2 \approx 320 \text{ N}$

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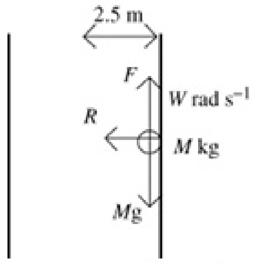
Motion in a circle
Exercise B, Question 17

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A fairground ride consists of a vertical hollow cylinder of diameter 5 m which rotates about a vertical axis through its centre. When the ride is rotating at $W \text{ rad s}^{-1}$ the floor of the cylinder opens. The people on the ride remain, without slipping, in contact with the inner surface of the cylinder. Given that the coefficient of friction between a person and the inner surface of the cylinder is $\frac{2}{3}$, find the minimum value for W.

Solution:



Suppose that the person has mass M. Let the normal reaction between the person and the cylinder be R. F is the frictional force between the person and the wall of the cylinder.

Minimum $W \Rightarrow$ the person is about to slip $\Rightarrow F = F_{max} = \frac{2}{3}R$

Also, R(↕) $\Rightarrow F_{max} = Mg$, so $Mg = \frac{2}{3}R, R = \frac{3Mg}{2}$.

↔Using $F = ma$,

$R = \frac{3Mg}{2} = M \times 5 \times W^2, W^2 = \frac{3g}{5}, W = \sqrt{\frac{3g}{5}} \approx 2.42 \text{ rad s}^{-1}$.

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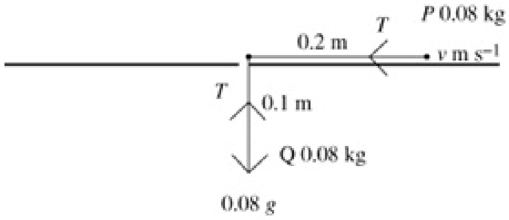
Motion in a circle
Exercise B, Question 18

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
Two particles P and Q, both of mass 80 g, are attached to the ends of a light inextensible string of length 30 cm. Particle P is on a smooth horizontal table, the string passes through a small smooth hole in the centre of the table, and particle Q hangs freely below the table at the other end of the string. P is moving on a circular path about the centre of the table at constant linear speed. Find the linear speed at which P must move if Q is in equilibrium 10 cm below the table.

Solution:



Let the tension in the string be T N, and the speed of P be $v \text{ m s}^{-1}$.
Q is in equilibrium, so
R(↕) at $Q \Rightarrow T = 0.08\,g$

For P, Using $F = ma$,
$T = 0.08\,g = \dfrac{0.08v^2}{0.2}, v^2 = 0.2 \times g \approx 1.96, v \approx 1.4 \text{ m s}^{-1}$.

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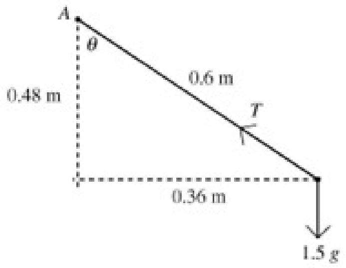
Motion in a circle
Exercise C, Question 1

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle of mass 1.5 kg is attached to one end of a light inextensible string of length 60 cm. The other end of the string is attached to a fixed point A. The particle moves with constant angular speed in a horizontal circle of radius 36 cm. The centre of the circle is vertically below A. Calculate the tension in the string and the angular speed of the particle.

Solution:



Let the tension in the string be T, and the angular speed be ω. The angle between the string and the vertical is θ. Since the triangle is right angled, the third side will have length 0.48 m (3, 4, 5 triangle).

$R(\updownarrow): T\cos\theta = 1.5\,g$

$$\Rightarrow \frac{4}{5}T = \frac{3g}{2}, T = \frac{15\,g}{8} \approx 18 \text{ N}$$

$R(\leftrightarrow): T\sin\theta = mr\omega^2$

$$\Rightarrow \frac{3}{5}T = \frac{3}{2} \times 0.36 \times \omega^2, T = 0.9\omega^2$$

Equating the two expressions for T:

$$\frac{15g}{8} = 0.9\omega^2$$

$$\omega^2 = \frac{15g}{0.9 \times 8} \approx 20.41, \omega \approx 4.5 \text{ rad s}^{-1}.$$

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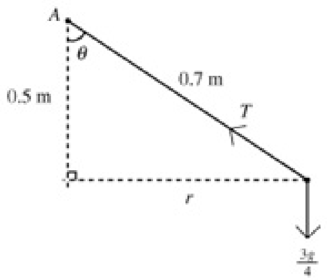
Motion in a circle
Exercise C, Question 2

Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.

A particle of mass 750 g is attached to one end of a light inextensible string of length 0.7 m. The other end of the string is attached to a fixed point A. The particle moves with constant angular speed in a horizontal circle whose centre is 0.5 m vertically below A. Calculate the tension in the string and the angular speed of the particle.

Solution:



Let the tension in the string be T, and the angular speed be ω. The angle between the string and the vertical is θ, and the radius of the circle is r.

$R(\updownarrow): T\cos\theta = \frac{3g}{4}$

$\Rightarrow \frac{5}{7}T = \frac{3g}{4}, T = \frac{21g}{20} \approx 10\text{ N}$

$R(\leftrightarrow): T\sin\theta = mr\omega^2$

$\Rightarrow \frac{r}{0.7}T = \frac{3}{4} \times r \times \omega^2, T = \frac{3}{4} \times 0.7\omega^2$

Equating the two expressions for T:

$\frac{21g}{20} = \frac{3}{4} \times 0.7\omega^2$

$\omega^2 = \frac{7g}{5 \times 0.7} \approx 19.6, \omega \approx 4.4\text{ rad s}^{-1}$.

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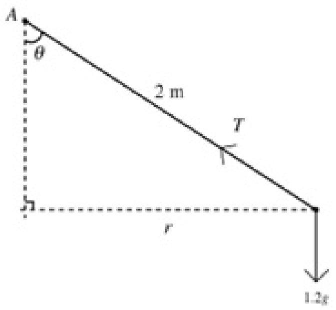
Motion in a circle
Exercise C, Question 3

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

A particle of mass 1.2 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point A. The particle moves in a horizontal circle with constant angular speed. The centre of the circle is vertically below A. The particle takes 2 seconds to complete one revolution. Calculate the tension in the string and the angle between the string and the vertical.

Solution:



Let the tension in the string be T, and the angular speed be ω. The angle between the string and the vertical is θ, and the radius of the circle is r.

2 seconds to complete 2π radians $\Rightarrow$ angular speed is π rad s^{-1}.

$R(\updownarrow): T\cos\theta = 1.2g$

$R(\leftrightarrow): T\sin\theta = mr\omega^2$

$$\Rightarrow T \times \frac{r}{2} = 1.2 \times r \times \pi^2, T = 2.4\pi^2$$

$$= 23.7 \text{ N}$$

and using this value in the first equation gives

$$\theta = \cos^{-1}\left(\frac{1.2g}{T}\right) \approx \cos^{-1} 0.496 \approx 60^\circ.$$

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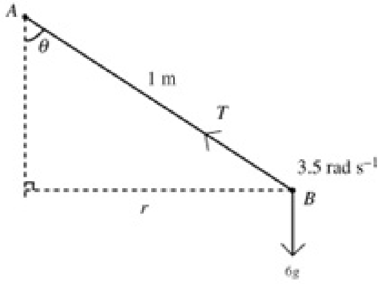
Motion in a circle
Exercise C, Question 4

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A conical pendulum consists of a light inextensible string AB of length 1 m, fixed at A and carrying a small ball of mass 6 kg at B. The particle moves in a horizontal circle, with centre vertically below A, at constant angular speed 3.5 rad s^{-1}. Find the tension in the string and the radius of the circle.

Solution:



Let the tension in the string be T. The angle between the string and the vertical is θ, and the radius of the circle is r.

$R(\updownarrow): T\cos\theta = 6g$

$R(\leftrightarrow): T\sin\theta = 6 \times r \times 3.5^2$

$$T \times \frac{r}{1} = 73.5r, \; T = 73.5 \text{ N}$$

and using this value in the first equation gives

$73.5\cos\theta = 6g$, $\cos\theta = 0.8$ radius $= \sin\theta = 0.6$ m

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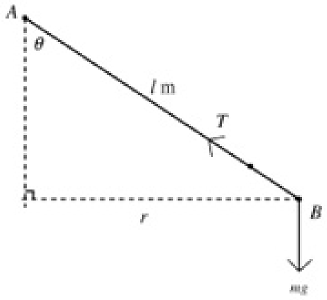
Motion in a circle
Exercise C, Question 5

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

A conical pendulum consists of a light inextensible string AB of length l, fixed at A and carrying a small ball of mass m at B. The particle moves in a horizontal circle, with centre vertically below A, at constant angular speed ω. Find, in terms of m, l and ω, the tension in the string.

Solution:



Let the tension in the string be T. The angle between the string and the vertical is θ, and the radius of the circle is r.

$$R(\leftrightarrow): T\sin\theta = m \times r \times \omega^2$$

$$T\frac{r}{l} = m \times r \times \omega^2$$

$$T = ml\omega^2$$

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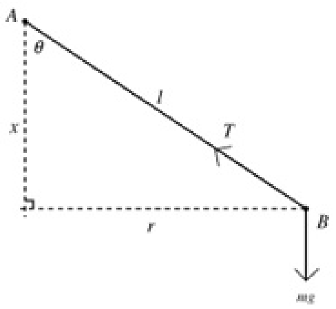
Motion in a circle
Exercise C, Question 6

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A conical pendulum consists of a light inextensible string AB fixed at A and carrying a small ball of mass m at B. With the string taut the particle moves in a horizontal circle at constant angular speed ω. The centre of the circle is at distance x vertically below A. Show that $\omega^2 x = g$.

Solution:



Let the tension in the string be T. The angle between the string and the vertical is θ, and the radius of the circle is r.

$R(\leftrightarrow): T\sin\theta = m \times r \times \omega^2$

$R(\updownarrow): T\cos\theta = mg$

Dividing the first equation by the second

$\Rightarrow \tan\theta = \frac{mr\omega^2}{mg}$

$\frac{r}{x} = \frac{r\omega^2}{g}, \ \omega^2 x = g$

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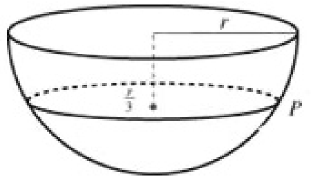
Motion in a circle
Exercise C, Question 7

Question:

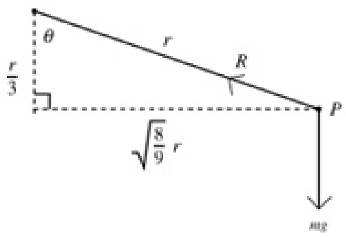
Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A hemispherical bowl of radius r is resting in a fixed position with its rim horizontal.
A particle P of mass m is moving in a horizontal circle around the smooth inside surface of the bowl.

The centre of the circle is $\frac{r}{3}$ below the centre of the bowl.

Find the angular speed of the particle and the magnitude of the reaction between the bowl and the particle.



Solution:



R is the normal reaction at P.
Using geometry, we know that the radius at P is perpendicular to the tangent at P, so R acts along this radius.
θ is the angle between the radius and the vertical.
Using Pythagoras' theorem we know that the radius of the circle is $\sqrt{\frac{8}{9}}r$.

$\text{R}(\uparrow): R\cos\theta = mg$

$$\frac{R}{3} = mg, R = 3mg$$

$\text{R}(\leftarrow): R\sin\theta = m\sqrt{\frac{8}{9}}r\omega^2$

$$R\frac{\sqrt{\frac{8}{9}}r}{r} = m\sqrt{\frac{8}{9}}r\omega^2$$

Substituting for R and simplifying:

$$3mg = mr\omega^2, \omega = \sqrt{\frac{3g}{r}}$$

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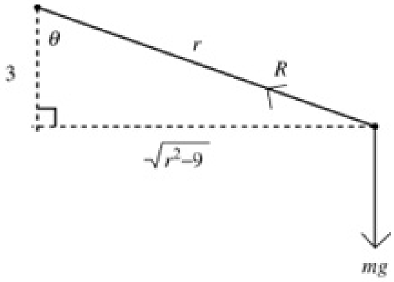
Motion in a circle
Exercise C, Question 8

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A hemispherical bowl of radius r cm is resting in a fixed position with its rim horizontal. A small marble of mass m is moving in a horizontal circle around the smooth inside surface of the bowl. The plane of the circle is 3 cm below the plane of the rim of the bowl. Find the angular speed of the marble.

Solution:



R is the normal reaction at the marble.
Using geometry, we know that the radius at the marble is perpendicular to the tangent at that point, so R acts along this radius.
θ is the angle between the radius and the vertical.
Using Pythagoras' theorem we know that the radius of the circle is $\sqrt{r^2 - 9}$.

$$\text{R}(\leftrightarrow): R\sin\theta = m\sqrt{r^2-9}\,\omega^2$$

$$R\frac{\sqrt{r^2-9}}{r} = m\sqrt{r^2-9}\,\omega^2$$

$$\omega^2 = \frac{R}{mr}$$

$$\text{R}(\updownarrow): R\cos\theta = mg = R\times\frac{3}{r},\ R = \frac{mgr}{3}$$

Substituting this expression for R in the first equation:

$$\omega^2 = \frac{mgr}{3mr} = \frac{g}{3},\ \omega = \sqrt{\frac{g}{3}}$$

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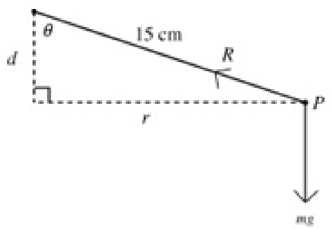
Motion in a circle
Exercise C, Question 9

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A hemispherical bowl of radius 15 cm is resting in a fixed position with its rim horizontal. A particle P of mass m is moving at 14 rad s^{-1} in a horizontal circle around the smooth inside surface of the bowl. Find the distance of the plane of the circle below the plane of the rim of the bowl.

Solution:



R is the normal reaction at the marble. Using geometry, we know that the radius at P is perpendicular to the tangent at that point, so R acts along this radius.
θ is the angle between the radius of the bowl and the vertical.
The particle moves on a circle of radius r m, depth d m below the rim of the bowl.

$\text{R}(\updownarrow): R\cos\theta = mg$ ①

$\text{R}(\leftrightarrow): R\sin\theta = mr\omega^2$ ②

Dividing ② ÷ ① to eliminate R,

$$\tan\theta = \frac{r\omega^2}{g} = \frac{r}{d}$$

$$\Rightarrow d = \frac{g}{\omega^2} \approx \frac{9.8}{196} = 0.05 \text{ m} = 5 \text{ cm}$$

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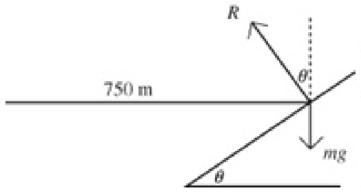
Motion in a circle
Exercise C, Question 10

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A car travels round a bend of radius 750 m on a road which is banked at angle θ to the horizontal. The car is assumed to be moving at constant speed in a horizontal circle. If there is no frictional force acting on the car when it is travelling at 126 km h^{-1}, find the value of θ.

Solution:



No friction, so just the normal reaction, R, between the car and the road with a horizontal component.

$$126 \text{ km h}^{-1} = \frac{126 \times 1000}{3600} = 35 \text{ m s}^{-1}$$

$R(\updownarrow): R\cos\theta = mg$ ①

$R(\leftrightarrow): R\sin\theta = m\frac{v^2}{r} = m \times \frac{35^2}{750}$ ②

Dividing ② ÷ ① to eliminate R and m

$$\Rightarrow \tan\theta = \frac{35^2}{750g} \approx 0.167, \theta \approx 9.5°$$

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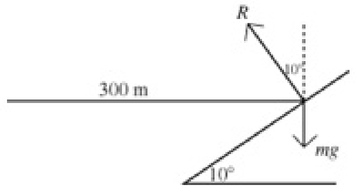
Motion in a circle
Exercise C, Question 11

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A car travels round a bend of radius 300 m on a road which is banked at an angle of $10°$ to the horizontal. The car is assumed to be moving at constant speed in a horizontal circle. At what speed does the car move if there is no frictional force?

Solution:



No friction, so just the normal reaction, R, between the car and the road with a horizontal component.

$R(\updownarrow): R\cos 10° = mg$

$R(\leftrightarrow): R\sin 10° = \frac{mv^2}{r} = \frac{mv^2}{300}$

Dividing to eliminate R

$\Rightarrow \tan 10° = \frac{mv^2}{300\,mg}$

$v^2 = 300\,g\tan 10° = 518.4\ldots$

$v \approx 23 \text{ m s}^{-1}$

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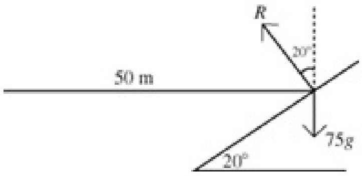
Motion in a circle
Exercise C, Question 12

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A boy rides his cycle round a circular track of diameter 50 m. The track is banked at $20°$ to the horizontal. There is no force due to friction. By modelling the boy and his cycle as a particle of mass 75 kg, find the speed at which the cycle is moving.

Solution:



No friction, so just the normal reaction, R, between the cycle and the road with a horizontal component.

$$R(\updownarrow): R\cos 20° = 75g$$

$$R(\leftrightarrow): R\sin 20° = \frac{75 \times v^2}{50}$$

Dividing to eliminate R

$$\Rightarrow \tan 20° = \frac{75v^2}{50 \times 75g} = \frac{v^2}{50g}$$

$$v^2 = 50g\tan 20° = 178.3\ldots$$

$$v \approx 13 \text{ m s}^{-1}.$$

It was not necessary to know the value of the mass because it cancels out at the stage when the two equations are combined to find $\tan 20°$.

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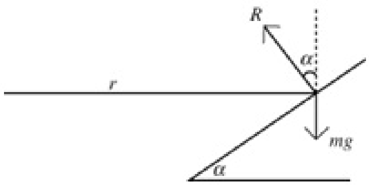
Motion in a circle
Exercise C, Question 13

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A bend in the road is a horizontal circular arc of radius r. The surface of the bend is banked at an angle α to the horizontal. When a vehicle is driven round the bend there is no tendency to slip. Show that the speed of the vehicle is $\sqrt{rg \tan \alpha}$.

Solution:



No friction, so just the normal reaction, R, between the vehicle and the road with a horizontal component.

$R(\updownarrow): R\cos\alpha = mg$

$R(\leftrightarrow): R\sin\alpha = \frac{mv^2}{r}$

Dividing to eliminate R

$\Rightarrow \tan\alpha = \frac{mv^2}{rmg} = \frac{v^2}{rg}$

$v^2 = rg\tan\alpha, v = \sqrt{rg\tan\alpha}$

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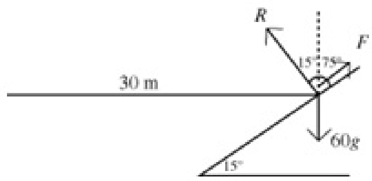
Motion in a circle
Exercise C, Question 14

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A girl rides her cycle round a circular track of diameter 60 m. The track is banked at $15°$ to the horizontal. The coefficient of friction between the track and the tyres of the cycle is 0.25. Modelling the girl and her cycle as a particle of mass 60 kg moving in a horizontal circle, find the minimum speed at which she can travel without slipping.

Solution:



R is the normal reaction between the cycle and the track. F is the force due to friction. At minimum speed the force due to friction is acting up the slope to stop the cycle from sliding down. (At maximum speed the friction will act down the slope to prevent sliding up the slope.)
As slipping is about to occur, $F = \mu R$.

$$\text{R}(\updownarrow): R\cos 15° + F\cos 75° = 60\,g$$

$$R\left(\cos 15° + \frac{\cos 75°}{4}\right) = 60\,g$$

$$\text{R}(\leftrightarrow): R\cos 75° - F\cos 15° = 60 \times \frac{v^2}{30}$$

$$R\left(\cos 75° - \frac{\cos 15°}{4}\right) = 2v^2$$

Dividing to eliminate R

$$\Rightarrow \frac{\cos 75° - \frac{\cos 15°}{4}}{\cos 15° + \frac{\cos 75°}{4}} = \frac{2v^2}{60\,g}$$

$$v^2 = \frac{\cos 75° - 0.25 \times \cos 15°}{\cos 15° + 0.25 \times \cos 75°} \times 30\,g$$

$$v^2 = 4.94\ldots, \quad v \approx 2.2 \text{ m s}^{-1}.$$

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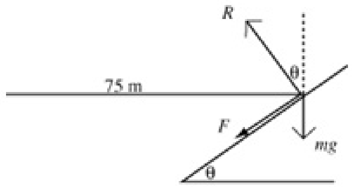
Motion in a circle
Exercise C, Question 15

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A van is moving on a horizontal circular bend in the road of radius 75 m. The bend is banked at $\tan^{-1}\frac{1}{3}$ to the horizontal. The maximum speed at which the van can be driven round the bend without slipping is 90 km h^{-1}. Calculate the coefficient of friction between the road surface and the tyres of the van.

Solution:



R is the normal reaction between the van and the track. F is the force due to friction. At maximum speed the force due to friction is acting down the slope to stop the van from sliding up.
μ is the coefficient of friction between the tyres and the road.
As slipping is about to occur, $F = \mu R$.

$$90 \text{ km h}^{-1} = \frac{90 \times 1000}{3600} = 25 \text{ m s}^{-1}.$$

$$R(\updownarrow): F\sin\theta + mg = R\cos\theta$$

$$R(\leftrightarrow): F\cos\theta + R\sin\theta = m \times \frac{25^2}{75}$$

Substituting $F = \mu R$

$$\Rightarrow mg = R(\cos\theta - \mu\sin\theta)$$

$$\frac{25m}{3} = R(\mu\cos\theta + \sin\theta)$$

Dividing to eliminate m

$$\Rightarrow \frac{25}{3g} = \frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta}$$

(on dividing top and bottom by $\cos\theta$)

$$= \frac{\mu + \frac{1}{3}}{1 - \frac{\mu}{3}}\left(\text{using } \tan\theta = \frac{1}{3}\right)$$

$$= \frac{3\mu + 1}{3 - \mu}$$

$$\Rightarrow 25(3 - \mu) = 3g(3\mu + 1)$$

Rearranging this equation gives

$$\mu(9g + 25) = 75 - 3g$$

$$\mu = \frac{75 - 3g}{9g + 25} \approx 0.40$$

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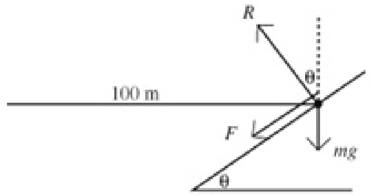
Motion in a circle
Exercise C, Question 16

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A car moves on a horizontal circular path round a banked bend in a race track. The radius of the path is 100 m. The coefficient of friction between the car tyres and the track is 0.3. The maximum speed at which the car can be driven round the bend without slipping is 144 km h^{-1}. Find the angle at which the track is banked.

Solution:



R is the normal reaction between the car and the track. F is the force due to friction. At maximum speed the force due to friction is acting down the slope to stop the car from sliding up.
The track is banked at θ to the horizontal.
As slipping is about to occur, $F = \mu R$.

$$144 \text{ km h}^{-1} = \frac{144 \times 1000}{3600} = 40 \text{ m s}^{-1}.$$

$$\text{R}(\updownarrow): mg = R\cos\theta - F\sin\theta$$

$$\text{R}(\leftrightarrow): R\sin\theta + F\cos\theta = m \times \frac{40^2}{100}$$

Substituting $F = \mu R$ and dividing to eliminate m

$$\Rightarrow \frac{\sin\theta + 0.3\cos\theta}{\cos\theta - 0.3\sin\theta} = \frac{40^2}{100g}$$

Dividing top and bottom of the left hand side by $\cos\theta$

$$\Rightarrow \frac{\tan\theta + 0.3}{1 - 0.3\tan\theta} = \frac{1600}{100g} = \frac{16}{g}$$

$$g(\tan\theta + 0.3) = 16(1 - 0.3\tan\theta)$$

$$\tan\theta(g + 4.8) = 16 - 0.3g$$

$$\tan\theta = \frac{16 - 0.3g}{g + 4.8} = 0.894\ldots$$

$$\theta \approx 42°$$

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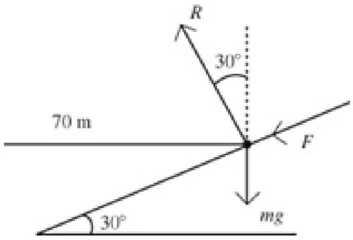
Motion in a circle
Exercise C, Question 17

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

A bend in a race track is banked at $30°$. A car will follow a horizontal circular path of radius 70 m round the bend. The coefficient of friction between the car tyres and the track surface is 0.4. Find the maximum and minimum speeds at which the car can be driven round the bend without slipping.

Solution:



R is the normal reaction between the car and the track, F is force due to friction. At maximum speed F acts down the slope and is equal to $\mu R = 0.4R$

$R(\updownarrow) R\cos 30° - F\sin 30° = mg$

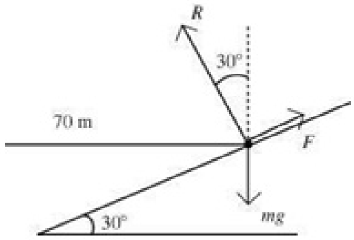
$R(\leftrightarrow) F\cos 30° + R\sin 30° = \frac{mv}{70}$

Substituting $F = 0.4R$ and dividing

$\Rightarrow \frac{v^2}{70g} = \frac{0.4\cos 30° + \sin 30°}{\cos 30° - 0.4\sin 30°}$

$\Rightarrow v^2 = 871.7\ldots$

$\Rightarrow v \simeq 29.5 \text{ m s}^{-1}$



At minimum speed, F acts up the slope

$R(\updownarrow) R\cos 30° + F\sin 30° = mg$

$R(\leftrightarrow) R\sin 30° - F\cos 30° = \frac{mv^2}{70}$

which leads to $\frac{v^2}{70\,g} = \frac{\sin 30° - 0.4\cos 30°}{\cos 30° + 0.4\sin 30°}$

giving $v^2 = 98.83\ldots \Rightarrow v \approx 9.9 \text{ m s}^{-1}$

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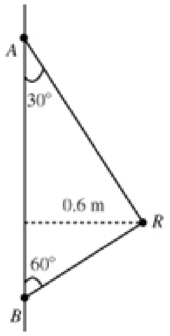
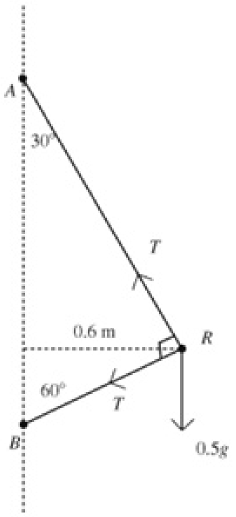
Motion in a circle
Exercise C, Question 18

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
The diagram shows a small smooth ring R of mass 500 g threaded on a light inextensible string. The ends of the string are attached to fixed points A and B, where A is vertically above B. The string is taut and the system rotates about AB. The ring moves with constant angular speed on a horizontal circle of radius 0.6 m.
$\angle ABR = 60°$ and $\angle BAR = 30°$. Modelling the ring as a particle, calculate the tension in the string and the angular speed of the particle.



Solution:



Because the ring is smooth and the string is continuous, we have only one value for tension, T.

$$R(\updownarrow): T\cos 30° - T\cos 60° = 0.5\,g$$
$$T(\cos 30° - \cos 60°) = 0.5\,g$$
$$T = \frac{0.5\,g}{\cos 30° - \cos 60°} = \frac{g}{\sqrt{3}-1}$$
$$\approx 13.4 \text{ N}$$
$$R(\leftrightarrow): T\cos 60° + T\cos 30° = 0.5 \times 0.6 \times \omega^2$$
$$T(\cos 60° + \cos 30°) = \frac{0.6\omega^2}{2}$$

Substituting for T and the trigonometric ratios:

$$\frac{g}{\sqrt{3}-1} \times \left(\frac{1+\sqrt{3}}{2}\right) = \frac{0.6\omega^2}{2}$$

Rearranging:

$$\omega^2 = \frac{g(1+\sqrt{3})}{0.6(\sqrt{3}-1)} = 60.9\ldots$$
$$\omega \approx 7.8 \text{ rad s}^{-1}.$$

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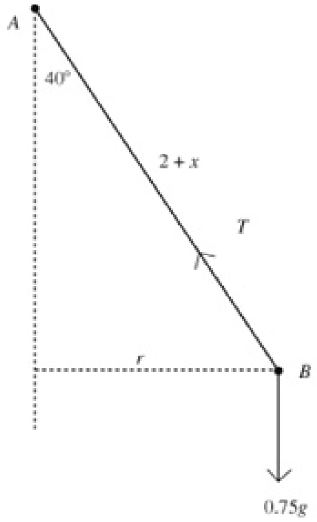
Motion in a circle
Exercise C, Question 19

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

A light elastic string AB has natural length 2 m and modulus of elasticity 30 N. The end A is attached to a fixed point. A particle of mass 750 g is attached to the end B. The particle is moving in a horizontal circle below A with the string inclined at $40°$ to the vertical. Find the angular speed of the particle.

Solution:



T is the tension in AB. AB is an elastic string, if the extension in the string is x then $T = \frac{\lambda x}{l} = \frac{30x}{2}$

$R(\updownarrow): T\cos 40° = 0.75\,g$

$R(\leftrightarrow): T\sin 40° = 0.75 \times r \times \omega^2$

The radius of the circle is $(2 + x)\sin 40°$, so substituting for r and T gives

$15x \sin 40° = 0.75 \times (2 + x)\sin 40° \times \omega^2$

$$\Rightarrow \omega^2 = \frac{15x}{0.75(2 + x)} = \frac{20x}{2 + x}$$

From the first equation:

$15x \cos 40° = 0.75\,g$

$$x = \frac{0.75g}{15\cos 40°} = 0.639\ldots$$

$$\Rightarrow \omega^2 = 4.846, \omega \approx 2.2 \text{ rad s}^{-1}.$$

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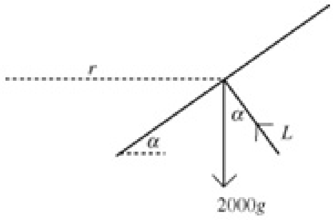
Motion in a circle
Exercise C, Question 20

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
An aircraft of mass 2 tonnes flies at 400 km h^{-1} on a path which follows a horizontal circular arc in order to change course from a bearing of $060°$ to a bearing of $015°$. It takes 25 seconds to change course, with the aircraft banked at $\alpha°$ to the horizontal. Calculate the two possible values of α and the corresponding values of the magnitude of the lift force perpendicular to the surface of the aircraft's wings.

Solution:



L is the lift force, and r is the radius of the circular arc.

$400 \text{ km h}^{-1} = \frac{400 \times 1000}{3600} = \frac{1000}{9} \text{ m s}^{-1}$.

In 25 seconds the aircraft travels $\frac{25\,000}{9}$ m .

The direction changes by $45°$ or $315°$ depending on the direction of turning.

For $45°$, $\frac{25\,000}{9} = \frac{1}{8} \times 2\pi r, r = 3536.7.. \text{ m}$

For $315°$, $\frac{25\,000}{9} = \frac{7}{8} \times 2\pi r, r = 505.25 \text{ m}$

$\text{R}(\updownarrow): L\cos\alpha = 2000\text{ g}$

$\text{R}(\leftrightarrow): L\sin\alpha = \frac{2000v^2}{r}$

Dividing the second equation by the first

$\Rightarrow \tan\alpha = \frac{v^2}{rg}$

When $r = 3536.7\ldots$

$\tan\alpha = \frac{1000^2}{9^2 \times 3537 \times 9.8} \approx 0.356$

$\alpha \approx 20°$, and $L \approx 21\,000$ N

When $r = 505.2\ldots$

$\tan\alpha = \frac{1000^2}{9^2 \times 505.3 \times 9.8} \approx 2.493$

$\alpha \approx 68°$, and $L \approx 53\,000$ N

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Motion in a circle
Exercise D, Question 1

Question:

At time t seconds the position vector, relative to the centre of the circle, of a particle moving in a horizontal circle, centre O, at constant angular speed ω rad s^{-1} is given by

$\mathbf{r} = r\cos\omega t\mathbf{i} + r\sin\omega t\mathbf{j}$

a Differentiate $\mathbf{r}$ with respect to t to obtain the velocity, $\mathbf{v}$, of the particle.

b Hence calculate the linear speed of the particle and deduce that $\mathbf{v} = r\omega$.

Solution:

a

$$\mathbf{r} = \mathbf{r}\cos\omega t\mathbf{i} + r\sin\omega t\mathbf{j}$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r} = \frac{\mathrm{d}}{\mathrm{d}t}(r\cos\omega t\mathbf{i} - r\sin\omega t\mathbf{j}) = \mathbf{v}$$

$$= \frac{\mathrm{d}}{\mathrm{d}t}(r\cos\omega t)\mathbf{i} + \frac{\mathrm{d}}{\mathrm{d}t}(r\sin\omega t)\mathbf{j}$$

$$= -r\omega\sin\omega t\mathbf{i} + r\omega\cos\omega t\mathbf{j}$$

b

$$\text{speed} = v = |\mathbf{v}| = \sqrt{(-\omega r\sin\omega t)^2 + (\omega r\cos\omega t)^2}$$

$$= \omega r\sqrt{\sin^2\omega t + \cos^2\omega t} = \omega r, \text{ as required.}$$

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Motion in a circle
Exercise D, Question 2

Question:

a By considering the gradients of the vectors **r** and **v**, or by taking the scalar product of **r** and **v**, find the angle between these two vectors.
b What does this tell you about the velocity of the particle?

Solution:

a $r.\mathbf{v} = (r\cos\omega t\mathbf{i} + r\sin\omega t\mathbf{j}).(-r\omega\sin\omega t\mathbf{i} + r\omega\cos\omega t\mathbf{j})$
$= (r\cos\omega t)\times(-r\omega\sin\omega t) + (r\sin\omega t)\times(r\omega\cos\omega t)$
$= -r^2\omega\cos\omega t.\sin\omega t + r^2\omega\sin\omega t.\cos\omega t = 0$, so **r** is perpendicular to **v**.

b The direction of **v** is perpendicular to the direction of **r**. Since **r** is always directed from the origin to the particle, the direction of the velocity at any instant is along the tangent to the circular path.

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Motion in a circle
Exercise D, Question 3

Question:

a Differentiate $\mathbf{v}$ with respect to t to obtain the acceleration, $\mathbf{a}$, of the particle.
b Express $\mathbf{a}$ in terms of $\mathbf{r}$. What does this tell you about the direction of the acceleration?
c Calculate the magnitude of $\mathbf{a}$.

Solution:

a
$$\mathbf{v} = -r\omega \sin \omega t\mathbf{i} + r\omega \cos \omega t\mathbf{j}$$
$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v} = \frac{\mathrm{d}}{\mathrm{d}t}(-r\omega \sin \omega t\mathbf{i} + r\omega \cos \omega t\mathbf{j}) = \mathbf{a}$$
$$= \frac{\mathrm{d}}{\mathrm{d}t}(-r\omega \sin \omega t)\mathbf{i} + \frac{\mathrm{d}}{\mathrm{d}t}(r\omega \cos \omega t)\mathbf{j}$$
$$= -r\omega^2 \cos \omega t\mathbf{i} - r\omega^2 \sin \omega t\mathbf{j}$$
$$= -\omega^2(r \cos \omega t\mathbf{i} + r \sin \omega t\mathbf{j})$$

b $\mathbf{a} = -\omega^2(r \cos \omega t\mathbf{i} + r \sin \omega t\mathbf{j}) = -\omega^2\mathbf{r}$

$\mathbf{a}$ is parallel to $\mathbf{r}$ but in the opposite direction, so the direction of the acceleration is towards the centre of the circle.

c $|\mathbf{a}| = |-\omega^2\mathbf{r}|$, so $|\mathbf{a}| = \omega^2 \times |\mathbf{r}|$, $a = r\omega^2$.

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Motion in a circle
Exercise E, Question 1

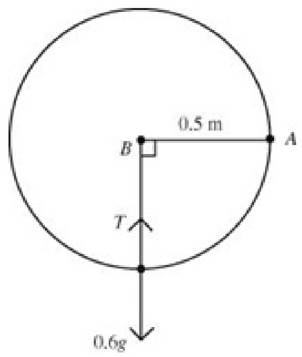
Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.
A particle of mass 0.6 kg is attached to end A of a light rod AB of length 0.5 m. The rod is free to rotate in a vertical plane about B. The particle is held at rest with AB horizontal. The particle is released. Calculate

a the speed of the particle as it passes through the lowest point of the path,
b the tension in the rod at this point.

Solution:

a



Let the speed of the particle at the lowest point be $v\text{ m s}^{-1}$, and the tension in the rod be T N.
At the lowest point the particle has fallen a distance 0.5 m, so the P.E. lost $= 0.6 \times g \times 0.5$

and the K.E. gained $= \frac{1}{2} \times 0.6 \times v^2$

$$\therefore 0.6 \times g \times 0.5 = \frac{1}{2} \times 0.6 \times v^2$$

$$v^2 = g, v \approx 3.1\text{ m s}^{-1}$$

b At the lowest point, the force towards the centre of the circle

$$= T - 0.6\,g = \frac{0.6v^2}{0.5}$$

$$\Rightarrow T = 0.6\,g + \frac{0.6\,g}{0.5} = 1.8\,g \approx 17.6\text{ N}$$

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Motion in a circle
Exercise E, Question 2

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

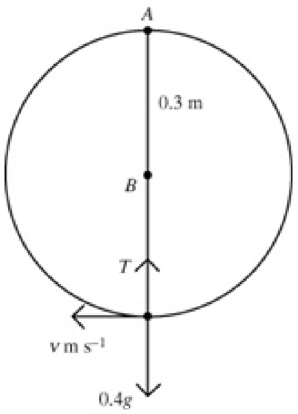
A particle of mass 0.4 kg is attached to one end A of a light rod AB of length 0.3 m. The rod is free to rotate in a vertical plane about B. The particle is held at rest with A vertically above B. The rod is slightly displaced so that the particle moves in a vertical circle. Calculate

a the speed of the particle as it passes through the lowest point of the path,

b the tension in the rod at this point.

Solution:

a



Let the speed of the particle at the lowest point be $v \text{ m s}^{-1}$, and the tension in the rod be T N.

At the lowest point the particle has fallen a distance 0.6 m, so the P.E. lost $= 0.4 \times g \times 0.6$,

and the K.E. gained $= \frac{1}{2} \times 0.4 \times v^2$.

$$\therefore 0.4 \times g \times 0.6 = \frac{1}{2} \times 0.4 \times v^2$$

$$v^2 = 2 \times g \times 0.6 = 1.2g, v \approx 3.4 \text{ m s}^{-1}$$

b At the lowest point, the force towards the centre of the circle

$$= T - 0.4\,g = \frac{0.4v^2}{0.3}$$

$$\Rightarrow T = 0.4g + \frac{0.4 \times 1.2\,g}{0.3} = 2\,g \approx 19.6 \text{ N}$$

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Motion in a circle
Exercise E, Question 3

Question:

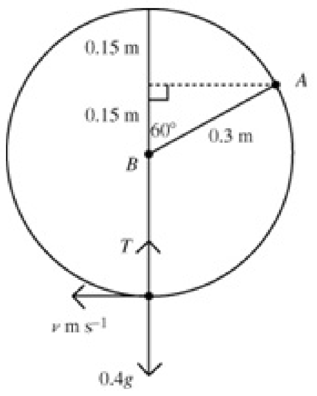
Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.
A particle of mass 0.4 kg is attached to end A of a light rod AB of length 0.3 m. The rod is free to rotate in a vertical plane about B. The particle is held at rest with AB at $60°$ to the upward vertical. The particle is released. Calculate

a the speed of the particle as it passes through the lowest point of the path,

b the tension in the rod at this point.

Solution:

a



Let the speed of the particle at the lowest point be $v\text{ m s}^{-1}$, and the tension in the rod be T N.
At the lowest point the particle has fallen a distance $0.3\cos 60° + 0.3 = 0.45\text{ m}$, so the
P.E. lost $= 0.4 \times g \times 0.45$, and the
K.E. gained $= \frac{1}{2} \times 0.4 \times v^2$.

$$\therefore 0.4 \times g \times 0.45 = \frac{1}{2} \times 0.4 \times v^2$$

$$v^2 = 2 \times g \times 0.45 = 0.9\,g, v \approx 3.0\text{ m s}^{-1}$$

b At the lowest point, the force towards the centre of the circle

$$= T - 0.4\,g = \frac{0.4v^2}{0.3}$$

$$\Rightarrow T = 0.4\,g + \frac{0.4 \times 0.9g}{0.3} = 1.6\,g \approx 15.7\text{ N}$$

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Motion in a circle
Exercise E, Question 4

Question:

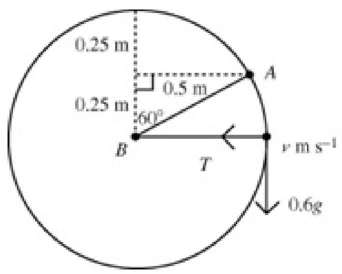
Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle of mass 0.4 kg is attached to end A of a light rod AB of length 0.3 m. The rod is free to rotate in a vertical plane about B. The particle is held at rest with AB at $60°$ to the upward vertical. The particle is released. Calculate

a the speed of the particle as it passes through the lowest point of the path,

b the tension in the rod at this point.

Solution:

a



Let the speed of the particle where AB is horizontal be $v \text{ m s}^{-1}$, and the tension in the rod be T N.
At the point where AB is horizontal, the particle has fallen a distance $0.5\cos 60° = 0.25 \text{ m}$, so the
P.E. lost $= 0.6 \times g \times 0.25$,
and the K.E. gained $= \frac{1}{2} \times 0.6 \times v^2$.

$\therefore 0.6 \times g \times 0.25 = \frac{1}{2} \times 0.6 \times v^2$

$v^2 = 2 \times g \times 0.25 = 0.5g, v \approx 2.2 \text{ m s}^{-1}$

b When AB is horizontal, the force towards the centre of the circle

$= T = \frac{0.6v^2}{0.5}$

$\Rightarrow T = \frac{0.6 \times 0.5\,g}{0.5} = 0.6\,g \approx 5.9 \text{ N}$

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Motion in a circle
Exercise E, Question 5

Question:

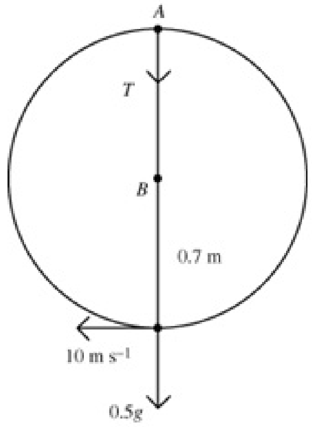
Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle of mass 0.5 kg is attached to end A of a light rod AB of length 0.7 m. The rod is free to rotate in a vertical plane about B. The particle is hanging with A vertically below B when it is projected horizontally with speed 10 m s^{-1}. Calculate

a the speed of the particle when it is vertically above B,

b the tension in the rod at this point.

Solution:

a



Let the speed of the particle at the highest point be $v \text{ m s}^{-1}$, and the tension in the rod be T N.
At the highest point the particle has risen a distance 1.4 m, so the
P.E. gained $= 0.5 \times g \times 1.4$, and the
K.E. lost $= \frac{1}{2} \times 0.5 \times 10^2 - \frac{1}{2} \times 0.5 \times v^2$.

$$\therefore 0.5 \times g \times 1.4 = \frac{1}{2} \times 0.5 \times (100 - v^2)$$

$$100 - v^2 = 2 \times g \times 1.4 = 2.8g, v \approx 8.5 \text{ m s}^{-1}$$

b At the highest point, the force towards the centre of the circle

$$= T + 0.5\,g = \frac{0.5v^2}{0.7}$$

$$\Rightarrow T = \frac{0.5 \times (100 - 2.8g)}{0.7} - 0.5g = 46.9 \text{ N}$$

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Motion in a circle
Exercise E, Question 6

Question:

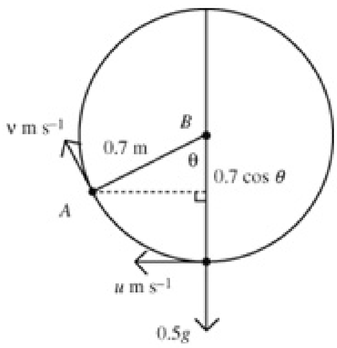
Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.
A particle of mass 0.5 kg is attached to end A of a light rod AB of length 0.7 m. The rod is free to rotate in a vertical plane about B. The particle is hanging with A vertically below B when it is projected horizontally with speed $u\text{ ms}^{-1}$. Find

a an expression in terms of u and θ for the speed of the particle when AB makes an angle of θ with the downward vertical through B,

b the restriction on u if the particle is to reach the highest point of the circle.

Solution:

a



When the angle between AB and the vertical is θ particle has speed $v\text{ m s}^{-1}$.

P.E. gained $= mgh = 0.5 \times g \times 0.7(1 - \cos\theta)$

Loss in K.E. $=$

$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{0.5}{2}(u^2 - v^2)$

Energy is conserved

$\therefore 0.5\,g \times 0.7(1 - \cos\theta) = \frac{0.5}{2}(u^2 - v^2)$

$v^2 = u^2 - 1.4g(1 - \cos\theta)$

$\Rightarrow v = \sqrt{u^2 - 1.4g(1 - \cos\theta)}$

b If the particle is to reach to top of the circle then we require $v > 0$ when $\theta = 180°$.

$\Rightarrow u^2 - 1.4g(1 - \cos 180°) > 0$

But $\cos 180° = -1$, so $u^2 > 1.4g \times 2$, $u > \sqrt{2.8\,g}$

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Motion in a circle
Exercise E, Question 7

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

A particle A of mass 1.5 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point B. The particle is hanging in equilibrium when it is set in motion with a horizontal speed of $u \text{ ms}^{-1}$.

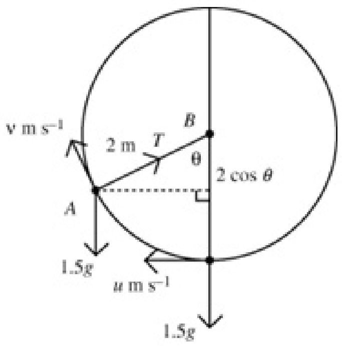
Find

a an expression for the tension in the string when it is at an angle θ to the downward vertical through B,

b the minimum value of u for which the particle will perform a complete circle.

Solution:

a



Take the lowest point of the circle as the zero level for potential energy.

At the lowest level the particle has

$\text{K.E.} = \frac{1}{2} \times 1.5 \times u^2 = 0.75u^2 \text{ J}$ and $\text{P.E.} = 0 \text{ J}$

When the rod is at angle θ to the vertical the particle has

$\text{K.E.} = \frac{1}{2} \times 1.5 \times v^2 = 0.75v^2 \text{ J}$ and

$\text{P.E.} = 1.5 \times g \times 2(1 - \cos\theta) \text{ J}.$

Energy is conserved

$\therefore 0.75u^2 = 0.75v^2 + 3g(1 - \cos\theta)$

Resolving towards the centre of the circle:

$T - 1.5g\cos\theta = \frac{mv^2}{r} = \frac{1.5v^2}{2}$, so substituting for v^2 gives

$T = 1.5g\cos\theta + \frac{3}{4}(u^2 - 4g + 4g\cos\theta)$

$= 4.5g\cos\theta + \frac{3u^2}{4} - 3g$

b If the particle is to reach to top of the circle then we require $T > 0$ when $\theta = 180°$.

$\Rightarrow -4.5g + \frac{3u^2}{4} - 3g > 0, \frac{3u^2}{4} > 7.5g, u^2 > 10g, u > \sqrt{10g}$

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Motion in a circle
Exercise E, Question 8

Question:

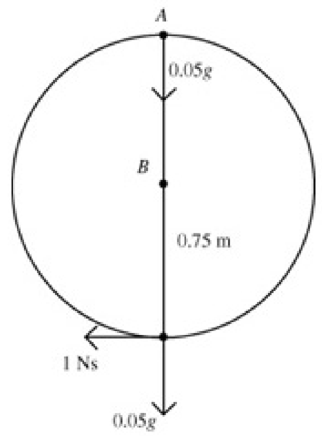
Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A small bead of mass 50 g is threaded on a smooth circular wire of radius 75 cm which is fixed in a vertical plane. The bead is at rest at the lowest point of the wire when it is hit with an impulse of I Ns horizontally causing it to start to move round the wire. Find the value of I if

a the bead just reaches the top of the circle,

b the bead just reaches the point where the radius from the bead to the centre of the circle makes an angle of $\tan^{-1}\frac{3}{4}$ with the upward vertical and then starts to slide back to its original position.

Solution:

a



Impulse = change in momentum, so if the initial speed of the bead is u m s^{-1} then

$I = 0.05u, u = 20I$.

Take the lowest point of the circle as the zero level for potential energy.

At the lowest level the particle has

$\text{K.E.} = \frac{1}{2} \times 0.05 \times u^2 = 0.025 \times 400I^2 = 10I^2$ J and

$\text{P.E.} = 0$ J

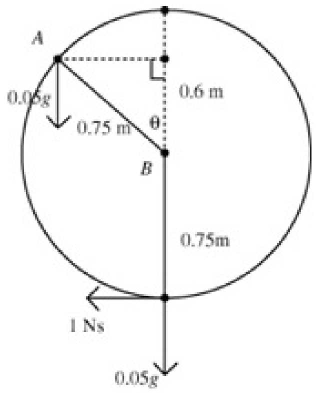
At the highest level the particle has $\text{K.E.} = 0$ (since we are told that the bead if just reaches the top) and it has risen 1.5 m so it has

$\text{P.E.} = 0.05 \times g \times 1.5 = 0.075\,g$

Energy is conserved, $\therefore 10I^2 = 0.075g$,

$\Rightarrow I^2 = 0.0075\,g, I \approx 0.27$

b



Take the lowest point of the circle as the zero level for potential energy.

At the lowest level the particle has

$\text{K.E.} = \frac{1}{2} \times 0.05 \times u^2 = 0.025 \times 400I^2 = 10I^2$ J and

$\text{P.E.} = 0$ J

When the rod is at angle $\tan^{-1}\frac{3}{4}$ to the vertical the particle has $\text{K.E.} = 0$ J (we are told that the particle reaches this point and then starts to slide back, so we can deduce that the speed is zero here) and

$\text{P.E.} = 0.05 \times g \times 0.75 \left\{1 + \cos\left(\tan^{-1}\frac{3}{4}\right)\right\}$

$= 0.0675\,g$ J

Energy is conserved

$10I^2 = 0.0675\,g$

$I = 0.26$

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Motion in a circle
Exercise E, Question 9

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

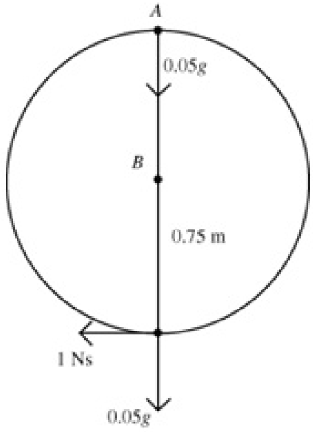
A particle of mass 50 g is attached to one end of a light inextensible string of length 75 cm. The other end of the string is attached to a fixed point. The particle is hanging at rest when it is hit with an impulse of I Ns horizontally causing it to start to move in a vertical circle. Find the value of I if

a the particle just reaches the top of the circle,

b the string goes slack at the instant when the particle reaches the point where the string makes an angle of $\tan^{-1}\frac{3}{4}$ with the upward vertical.

Solution:

a



Impulse = change in momentum, so if the initial speed of the bead is u m s^{-1} then

$I = 0.05u, u = 20I.$

Take the lowest point of the circle as the zero level for potential energy.

At the lowest level the particle has

$$\text{K.E.} = \frac{1}{2}\times 0.05\times u^2 = 0.025\times 400I^2 = 10I^2 \text{ J}$$

and P.E. = 0 J

If the bead just reaches the top of the circle then this is the point at which the tension in the string becomes zero. If the speed of the bead at this point is v m s^{-1} then

$$0.05g = \frac{mv^2}{r} = \frac{0.05v^2}{0.75}, v^2 = 0.75\,g$$

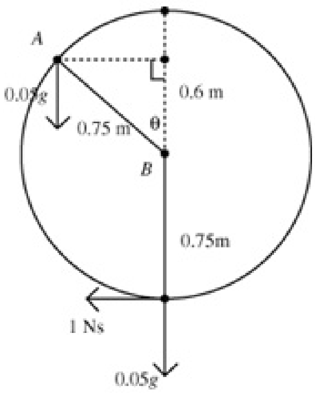
The bead has risen 1.5 m so it has

P.E. $= 0.05\times g\times 1.5 = 0.075\,g$

Energy is conserved, so

$$10I^2 = 0.075\,g + \frac{1}{2}\times 0.05\times 0.75\,g = 0.09375\,g, I \approx 0.30$$

b



Take the lowest point of the circle as the zero level for potential energy.

At the lowest level the particle has

$$\text{K.E.} = \frac{1}{2}\times 0.05\times u^2 = 0.025\times 400I^2 = 10I^2 \text{ J}$$

and P.E. = 0 J

When the bead just reaches the point where AB is at $\tan^{-1}\frac{3}{4}$ to the vertical the tension in the string becomes zero. If the speed of the bead at this point is v m s^{-1} then

$$0.05g\cos\left(\tan^{-1}\frac{3}{4}\right) = \frac{mv^2}{r} = \frac{0.05v^2}{0.75}$$

$$v^2 = 0.75\times g\times\frac{4}{5} = 0.6\,g$$

The bead has risen $0.75 + 0.6 = 1.35$ m,

so gain in P.E. $= 0.05\times g\times 1.35 = 0.0675\,g$.

Energy is conserved $\Rightarrow 10I^2 = 0.0675\,g + \frac{1}{2}\times 0.05\times 0.6\,g = 0.0825\,g$

$$I \approx 0.28$$

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Motion in a circle
Exercise E, Question 10

Question:

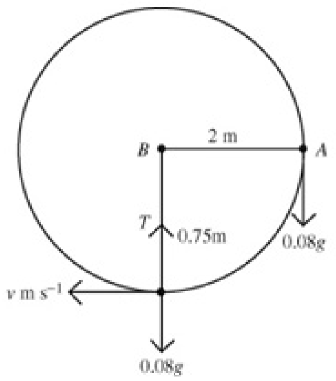
Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.
A particle of mass 0.8 kg is attached to end A of a light rod AB of length 2 m. The end B is attached to a fixed point so that the rod is free to rotate in a vertical circle with its centre at B. The rod is held in a horizontal position and then released. Calculate the speed of the particle and the tension in the rod when

a the particle is at the lowest point of the circle,

b the rod makes an angle of $\tan^{-1}\frac{3}{4}$ with the downward vertical through B.

Solution:

a



Let the speed of the particle at the lowest point be v m s^{-1}, and the tension in the rod be T N.

Take the starting level as the zero level for potential energy, the particle starts with

P.E. $= 0$ and K.E. $= 0$.

At the lowest level,

$$\text{P.E.} = -0.8 \times g \times 2 = -1.6\,g$$

$$\text{K.E.} = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times 0.8v^2 = 0.4v^2$$

Conservation of energy

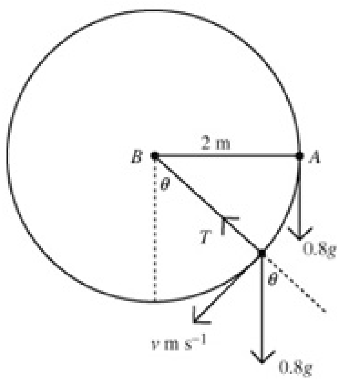
$$\Rightarrow -1.6g + 0.4v^2 = 0, \quad v^2 = 4g$$

$$v \approx 6.3 \text{ m s}^{-1}.$$

Force towards the centre of the circle $= T - 0.8g = \dfrac{mv^2}{r} = \dfrac{0.8 \times 4g}{2} = 1.6g$

$$T = 2.4\,g \approx 24 \text{ N}$$

b



Let the speed of the particle at the point when $\theta = \tan^{-1}\dfrac{3}{4}$ be v m s^{-1}, and the tension in the rod be T N.

Take the starting level as the zero level for potential energy, the particle starts with

P.E. $= 0$ and K.E. $= 0$.

When AB is at θ to the vertical the particle has fallen $2\cos\theta = 1.6$ m.

$$\text{P.E.} = -0.8 \times g \times 1.6 = -1.28\,g$$

$$\text{K.E.} = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times 0.8v^2 = 0.4v^2$$

Conservation of energy

$$\Rightarrow -1.28g + 0.4v^2 = 0, \quad v^2 = 3.2g$$

$$v \approx 5.6 \text{ m s}^{-1}.$$

Force towards the centre of the circle $= T - 0.8g\cos\theta = \dfrac{mv^2}{r} = \dfrac{0.8 \times 3.2g}{2} = 1.28g$

$$T = 1.28\,g + 0.8\,g \times \frac{4}{5} = 1.92g \approx 19 \text{ N}$$

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Motion in a circle
Exercise E, Question 11

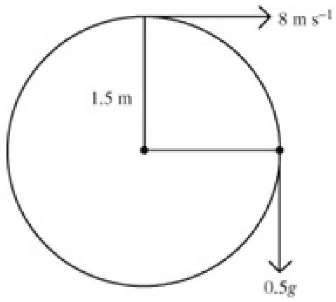
Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.
A particle of mass 500 g describes complete vertical circles on the end of a light inextensible string of length 1.5 m. Given that the speed of the particle is 8 m s^{-1} at the highest point, find

a the speed of the particle when the string is horizontal,
b the magnitude of the tangential acceleration when the string is horizontal,
c the tension in the string when the particle is at the lowest point of the circle.

Solution:

a



Let the speed of the particle when the string is horizontal be $v\text{ m s}^{-1}$.
Take the lowest point as the zero level for potential energy, the particle starts with

$\text{P.E.} = 0.5 \times g \times 3$ and $\text{K.E.} = \frac{1}{2} \times 0.5 \times 8^2$.

When the string is horizontal,

$\text{P.E.} = 0.5 \times g \times 1.5$ and $\text{K.E.} = \frac{1}{2} \times 0.5 \times v^2$.

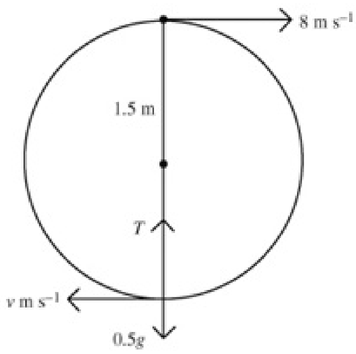
Energy is conserved

$$\Rightarrow 1.5g + 16 = 0.75g + \frac{v^2}{4}$$

$$v^2 = 4(0.75g + 16),\ v \approx 9.7\text{ m s}^{-1}.$$

b The only force with a vertical component is the weight.
Acceleration $= g\text{ m s}^{-2}$.

c



Let the speed of the particle at the lowest point be $v\text{ m s}^{-1}$, and the tension in the rod be T N.
Take the lowest point as the zero level for potential energy, the particle starts with

$\text{P.E.} = 0.5 \times g \times 3$ and $\text{K.E.} = \frac{1}{2} \times 0.5 \times 8^2$.

When the string is vertical,

$\text{P.E.} = 0$ and $\text{K.E.} = \frac{1}{2} \times 0.5 \times v^2$.

Energy is conserved

$$\Rightarrow 1.5g + 16 = \frac{v^2}{4},\quad v \approx 11.1\text{ m s}^{-1}$$

$$T - 0.5g = \frac{0.5v^2}{1.5},\quad T \approx 45.8\text{ N}$$

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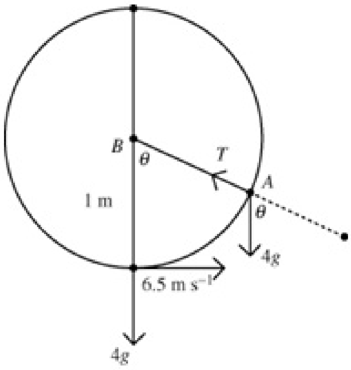
Motion in a circle
Exercise E, Question 12

Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.
A light rod AB of length 1 m has a particle of mass 4 kg attached at A. End B is pivoted to a fixed point so that AB is free to rotate in a vertical plane. When the rod is vertical with A below B the speed of the particle rod is 6.5 m s^{-1}. Find the angle between AB and the vertical at the instant when the tension in the rod is zero, and calculate the speed of the particle at that instant.

Solution:



At the lowest point let P.E. $= 0$ J.
The K.E. at the lowest point is
$\frac{1}{2} \times 4 \times 6.5^2 = 84.5$ J.
When AB is at angle θ to the vertical, the tension in the rod is T, and the particle has speed $v\text{ m s}^{-1}$.
Particle has risen $(1 - \cos\theta)$, so
P.E. $= 4 \times g \times (1 - \cos\theta)$ J and
K.E. $= \frac{1}{2} \times 4 \times v^2 = 2v^2$

Energy is conserved $\Rightarrow 84.5 = 2v^2 + 4g(1 - \cos\theta)$,
$v^2 = 42.25 - 2g(1 - \cos\theta)$
Force towards the centre of the circle
$= T - 4g\cos\theta = \frac{mv^2}{r} = \frac{4(42.25 - 2g(1 - \cos\theta))}{1}$.
$\therefore T = 4g\cos\theta + 169 - 8g(1 - \cos\theta) = 169 + 12g\cos\theta - 8g = 0$ when
$\cos\theta = \frac{8g - 169}{12g} = -0.77\ldots$ giving $\theta \approx 140°$
i.e. $\theta \approx 40°$ to the upward vertical and

$v^2 = 42.25 - 2g(1 - \cos\theta) = 7.5\ldots, v \approx 2.7\text{ m s}^{-1}$

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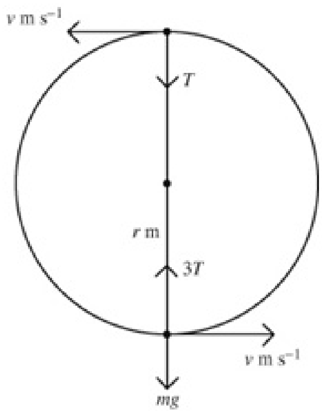
Motion in a circle
Exercise E, Question 13

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A particle P of mass m kg is attached to one end of a light rod of length r m which is free to rotate in a vertical plane about its other end. The particle describes complete vertical circles. Given that the tension at the lowest point of P's path is three times the tension at the highest point, find the speed of P at the lowest point on its path.

Solution:



Let the speed at the lowest point be $u \text{ m s}^{-1}$, and the speed at the highest point be $v \text{ m s}^{-1}$.
The gain in P.E. in moving from the lowest point to the highest is $2mgr$.
The loss in K.E. is $\frac{1}{2}mu^2 - \frac{1}{2}mv^2$
Energy is conserved
$\therefore 2mgr = \frac{1}{2}mu^2 - \frac{1}{2}mv^2, \ v^2 = u^2 - 4gr$
At the lowest point $3T - mg = \frac{mu^2}{r}$
At the highest point $T + mg = \frac{mv^2}{r}$

Substituting for T and v^2 in the first of these two equations:

$$3\left(\frac{m(u^2-4gr)}{r} - mg\right) - mg = \frac{mu^2}{r}, 3\frac{(u^2-4gr)}{r} - 4g = \frac{u^2}{r}$$

$$\frac{2u^2}{r} = 16g, u^2 = 8gr, u = \sqrt{8gr}$$

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Motion in a circle
Exercise E, Question 14

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

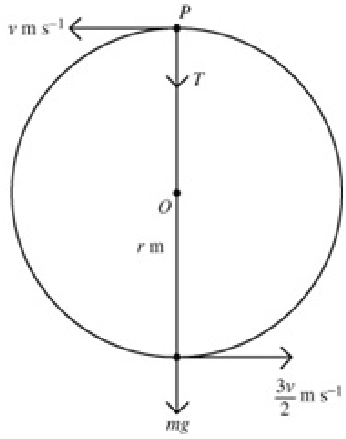
A particle P of mass m kg is attached to one end of a light inextensible string of length r m. The other end of the string is attached to a fixed point O, and P describes complete vertical circles about O. Given that the speed of the particle at the lowest point is one-and-a-half times the speed of the particle at the highest point, find

a the speed of the particle at the highest point,

b the tension in the string when the particle is at the highest point.

Solution:

a



Let the speed at the lowest point be $\frac{3v}{2} \text{ m s}^{-1}$, and the speed at the highest point be $v \text{ m s}^{-1}$.

The gain in P.E. in moving from the lowest point to the highest is $2mgr$.

The loss in K.E. is $\frac{1}{2}m\left(\frac{3v}{2}\right)^2 - \frac{1}{2}mv^2$

Energy is conserved

$$\therefore 2mgr = \frac{1}{2}m \times \frac{9v^2}{4} - \frac{1}{2}mv^2 = \frac{5}{8}mv^2$$

$$v^2 = \frac{16gr}{5}, v = \sqrt{\frac{16\,gr}{5}}$$

b At the highest point, $T + mg = \frac{mv^2}{r} = \frac{m\frac{16gr}{5}}{r} = \frac{16mg}{5}, T = \frac{11mg}{5}$

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Motion in a circle
Exercise E, Question 15

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

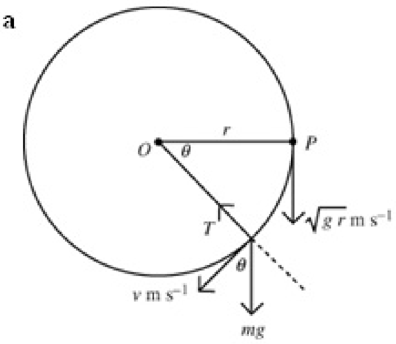
A light inelastic string of length r has one end attached to a fixed point O. A particle P of mass m kg is attached to the other end. P is held with OP horizontal and the string taut. P is then projected vertically downwards with speed $\sqrt{gr}$.

a Find, in terms of θ, m and g, the tension in the string when OP makes an angle θ with the horizontal.

b Given that the string will break when the tension in the string is $2mg$ N, find the angle between the string and the horizontal when the string breaks.

Solution:

a



With OP horizontal, the particle has

P.E. $= 0$ and K.E. $= \frac{1}{2}mv^2 = \frac{1}{2}mgr$.

When OP is $\theta°$ below the horizontal, the tension in the string is T and the speed of the particle is v.

The particle has

P.E. $= -mgr\sin\theta$ and K.E. $= \frac{1}{2}mv^2$

Energy is conserved

$$\therefore \frac{1}{2}mgr = -mgr\sin\theta + \frac{1}{2}mv^2$$

$$v^2 = gr(1+2\sin\theta)$$

Resolving towards O:

$$T - mg\sin\theta = \frac{mv^2}{r} = \frac{mgr(1+2\sin\theta)}{r}$$

$$T = mg(1+3\sin\theta)\text{N}$$

b When $T = 2mg$ N, $2mg = mg(1+3\sin\theta)$, $\sin\theta = \frac{1}{3}$, $\theta \approx 19.5°$

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Motion in a circle
Exercise F, Question 1

Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.

A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is hanging in equilibrium at a point A, directly below O, when it is set in motion with a horizontal speed $\sqrt{3gl}$.

When OP has turned through an angle θ and the string is still taut, the tension in the string is T.

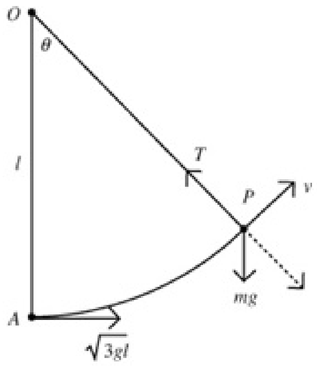
a Find an expression for T.

b Find the height of P above A at the instant when the string goes slack.

c Find the maximum height above A reached by P before it starts to fall to the ground again.

Solution:

a



Conservation of energy.

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgl(1-\cos\theta)$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mgl(1-\cos\theta)$$

$$v^2 = 3gl - 2gl(1-\cos\theta) = gl(1+2\cos\theta)$$

Resolving towards the centre of the circle:

$$T - mg\cos\theta = \frac{mv^2}{l}$$

$$T = mg\cos\theta + \frac{mgl}{l}(1+2\cos\theta) = mg + 3mg\cos\theta$$

b String slack $\Rightarrow T = 0 \Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \text{height} = l + \frac{l}{3} = \frac{4l}{3}$

c When the string goes slack, $v^2 = gl\left(1 + 2\times\left(-\frac{1}{3}\right)\right) = \frac{gl}{3}$

So horizontal component of velocity $= \frac{1}{3}\sqrt{\frac{gl}{3}}$

Using energy, if the maximum additional height is h, then

$$mgh + \frac{1}{2}m\times\left(\frac{1}{3}\sqrt{\frac{gl}{3}}\right)^2 = \frac{1}{2}m\left(\frac{gl}{3}\right)$$

$$h = \frac{l}{6} - \frac{l}{6\times 9} = \frac{8l}{54} = \frac{4l}{27},\ \text{height above } A = \frac{4l}{3} + \frac{4l}{27} = \frac{40l}{27}$$

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Motion in a circle
Exercise F, Question 2

Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.
A smooth solid hemisphere with radius 6 m and centre O is resting in a fixed position on a horizontal plane with its flat face in contact with the plane. A particle P of mass 3 kg is slightly disturbed from rest at the highest point of the hemisphere.
When OP has turned through an angle θ and the particle is still on the surface of the hemisphere the normal reaction of the sphere on the particle is R.

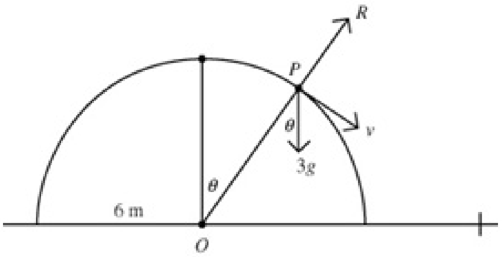
a Find an expression for R.

b Find the angle between OP and the upward vertical when the particle leaves the surface of the hemisphere.

c Find the distance of the particle from the centre of the hemisphere when it hits the ground.

Solution:

a



Conservation of energy from top to P:

$$mg \times 6 = mg \times 6\cos\theta + \frac{1}{2}mv^2$$

$$v^2 = 12g(1 - \cos\theta)$$

Resolving towards O:

$$3g\cos\theta - R = \frac{mv^2}{r} = \frac{12 \times 3g}{6}(1 - \cos\theta)$$

$$9g\cos\theta - 6g = R$$

b $R = 0 \Rightarrow \cos\theta = \frac{2}{3}, \theta \approx 48°$

c $v^2 = 12g \times \frac{1}{3} = 4g$

Speed $\rightarrow v\cos\theta$, $\quad \downarrow v\sin\theta + gt$

Distance $\rightarrow v\cos\theta t$, $\quad \downarrow v\sin\theta t + \frac{1}{2}gt^2 = 6 \times \frac{2}{3} = 4$

$$2\sqrt{g}\,\frac{\sqrt{5}}{3}t + \frac{g}{2}t^2 = 4,\ 4.9t^2 + \frac{14}{3}t - 4 = 0$$

$t \approx 0.545\ldots$

Total horizontal distance from $O = 6\sin\theta + \sqrt{4g}\cos\theta \times t \approx 6.7$ m

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Motion in a circle
Exercise F, Question 3

Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.
A smooth solid hemisphere is fixed with its plane face on a horizontal table and its curved surface uppermost. The plane face of the hemisphere has centre O and radius r. The point A is the highest point on the hemisphere. A particle P is placed on the hemisphere at A. It is then given an initial horizontal speed u, where $u^2 = \frac{rg}{4}$. When OP makes an angle θ with OA, and while P remains on the hemisphere, the speed of P is v.

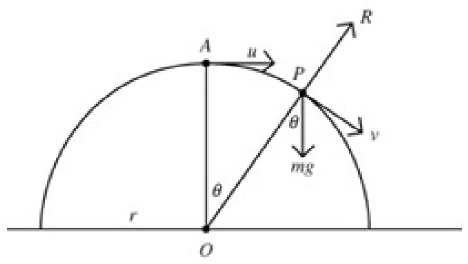
a Find an expression for v^2.
b Find the value of $\cos\theta$ when P leaves the hemisphere.
c Find the value of v when P leaves the hemisphere.

After leaving the hemisphere P strikes the table at B.

d Find the speed of P at B.
e Find the angle at which P strikes the table.

Solution:

a



Conservation of energy:

$$\frac{1}{2}mu^2 + mgr = \frac{1}{2}mv^2 + mgr\cos\theta$$

$$\frac{rg}{8} + rg = \frac{9rg}{8} = \frac{1}{2}v^2 + rg\cos\theta$$

$$v^2 = \frac{9rg}{4} - 2rg\cos\theta$$

b Resolving towards O: $mg\cos\theta - R = \frac{mv^2}{r} = mg\left(\frac{9}{4} - 2\cos\theta\right)$

$$R = 0 \Rightarrow 3mg\cos\theta = mg \times \frac{9}{4}, \cos\theta = \frac{3}{4}$$

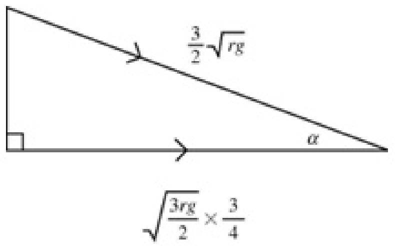
c $v^2 = \frac{9rg}{4} - 2rg \times \frac{3}{4} = \frac{3rg}{4}, \quad v = \sqrt{\frac{3rg}{4}}$

d Conservation of energy from A to the table:

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgr$$

$$v^2 = u^2 + 2gr = \frac{rg}{4} + 2gr = \frac{9rg}{4}, v = \frac{3}{2}\sqrt{rg}$$

e



After leaving the hemisphere the horizontal component of the velocity remains constant.

Direction is angle α to the ground,

$$\cos\alpha = \frac{\sqrt{\frac{3rg}{4}} \times \frac{3}{4}}{\frac{3}{2} \times \sqrt{rg}} = \frac{\sqrt{3}}{4}$$

$$\alpha = 64^\circ$$

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Motion in a circle
Exercise F, Question 4

Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.

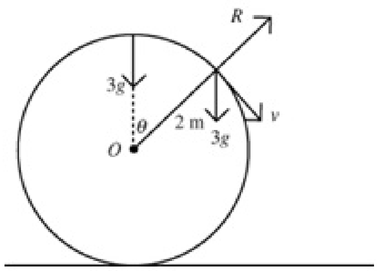
A smooth sphere with centre O and radius 2 m is fixed to a horizontal surface. A particle P of mass 3 kg is slightly disturbed from rest at the highest point of the sphere and starts to slide down the surface of the sphere.

a Find the angle between OP and the upward vertical at the instant when P leaves the surface of the sphere.

b Find the magnitude and direction of the velocity of the particle as it hits the horizontal surface.

Solution:

a



Conservation of energy:

$$mgr = \frac{1}{2}mv^2 + mgr\cos\theta$$

$$v^2 = 2mgr(1-\cos\theta) = 4mg(1-\cos\theta)$$

Resolving towards O:

$$3g\cos\theta - R = \frac{3v^2}{2} = \frac{3\times 4g(1-\cos\theta)}{2}$$

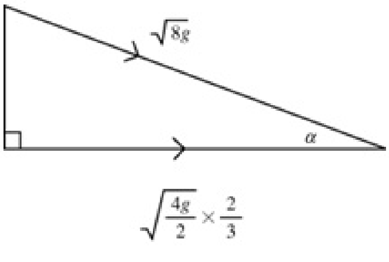
$$R = 0 \Rightarrow 9g\cos\theta = 6g, \quad \cos\theta = \frac{2}{3}$$

$$\theta \approx 48°$$

b Using conservation of energy from the highest point to the ground:

$\frac{1}{2}mv^2 = mgh = mg\times 4$, $v = \sqrt{8g}$ when P hits the ground.

When P leaves the sphere $v^2 = 4mg(1-\cos\theta) = 4mg\times\frac{1}{3}, v = \sqrt{\frac{4mg}{3}}$



After leaving the hemisphere the horizontal component of the velocity remains constant.
Direction is angle α to the ground,

$$\cos\alpha = \frac{\sqrt{\frac{4g}{3}}\times\frac{2}{3}}{\sqrt{8g}} = \frac{2}{3}\times\sqrt{\frac{1}{6}}$$

$$\alpha = 74°$$

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Motion in a circle
Exercise F, Question 5

Question:

Whenever a numerical value of g is required take $g = 9.8\text{ m s}^{-2}$.

A particle of mass m is projected with speed v from the top of the outside of a smooth sphere of radius a. In the subsequent motion the particle slides down the surface of the sphere and leaves the surface of the sphere with speed $\frac{\sqrt{3ga}}{2}$.

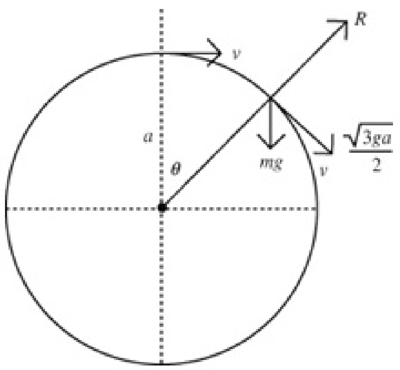
a Find the vertical distance travelled by the particle before it loses contact with the surface of the sphere.

b Find v.

c Find the magnitude and direction of the velocity of the particle when it is at the same horizontal level as the centre of the sphere.

Solution:

a



Forces acting along the radius:

$$mg\cos\theta - R = \frac{mv^2}{r} = \frac{m \times 3ga}{4a} = \frac{3mg}{4}$$

$$R = 0 \Rightarrow \cos\theta = \frac{3}{4}$$

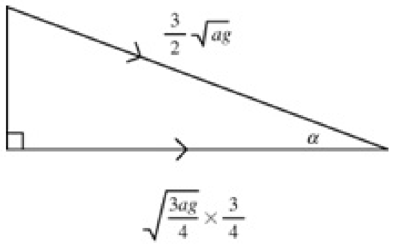
$$\text{Distance fallen} = a - a\cos\theta = \frac{a}{4}$$

b Conservation of energy from the top to the point where the particle leaves the sphere:

$$mg\frac{a}{4} = \frac{1}{2}m \times \frac{3ga}{4} - \frac{1}{2}mv^2, \quad \frac{1}{2}v^2 = \frac{3ga}{8} - \frac{ga}{4} = \frac{ga}{8}, v^2 = \frac{ga}{4}, v = \sqrt{\frac{ga}{4}}$$

c Looking at the energy at the top and level with the centre:

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga = \frac{1}{2}m\frac{ga}{4} + mga, v^2 = \frac{9ga}{4}, v = \frac{3}{2}\sqrt{ga}$$



After leaving the hemisphere the horizontal component of the velocity remains constant.

$$\text{Direction is } \alpha, \ \cos\alpha = \frac{\sqrt{\frac{3ag}{4}} \times \frac{3}{4}}{\frac{3}{2} \times \sqrt{ag}} = \frac{\sqrt{3}}{4}$$

$$\alpha = 64^\circ \text{ to the horizontal}$$

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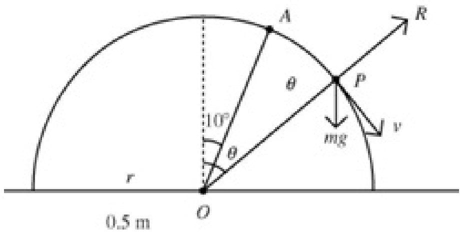
Edexcel AS and A Level Modular Mathematics

Motion in a circle
Exercise F, Question 6

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
A smooth hemisphere with centre O and radius 50 cm is fixed with its plane face in contact with a horizontal surface. A particle P is released from rest at point A on the sphere, where OA is inclined at $10°$ to the upward vertical. The particle leaves the sphere at point B. Find the angle between OB and the upward vertical.

Solution:



Conservation of energy:

$$\frac{1}{2}mv^2 + mg\frac{1}{2}\cos\theta = mg\frac{1}{2}\cos 10°$$

$$v^2 = g(\cos 10° - \cos\theta)$$

Forces acting towards O:

$$mg\cos\theta - R = \frac{mv^2}{0.5} = 2mv^2$$

$$R = 0 \Rightarrow g\cos\theta = 2v^2 = 2g(\cos 10° - \cos\theta) \Rightarrow 3g\cos\theta = 2g\cos 10°$$

$$\cos\theta = \frac{2}{3}\cos 10°, \theta \approx 49°$$

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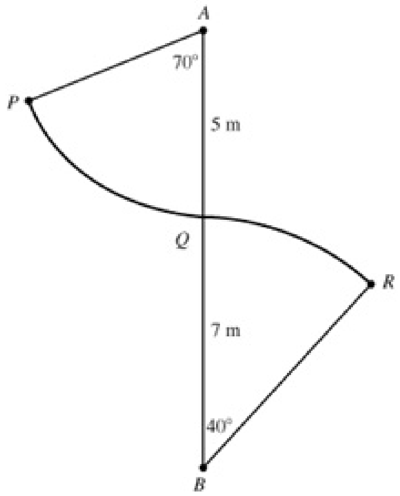
Motion in a circle
Exercise F, Question 7

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.

A smooth laundry chute is built in two sections, PQ and QR. Each section is in the shape of an arc of a circle. PQ has radius 5 m and subtends an angle of $70°$ at its centre, A. QR has radius 7 m and subtends an angle of $40°$ at its centre, B. The points A, Q and B are in a vertical straight line. The laundry bags are collected in a large bin $\frac{1}{2}$ m below R. To test the chute, a small particle of mass 2 kg is released from rest at P.



a Calculate the speed with which the particle reaches the bin at the bottom of the chute.

b Determine whether or not the particle loses contact with the chute before it reaches R.

Solution:

a Total height lost $= 5(1-\cos 70°) + 7(1-\cos 40°) + 0.5 = 5.427\ldots \text{ m}$

Conservation of energy:

$\frac{1}{2} \times 2 \times v^2 = 2 \times g \times 5.427\ldots \quad v \approx 10.3 \text{ m s}^{-1}$

b At R: $\frac{1}{2} \times 2 \times v^2 = 2g\left(12 - 5\cos 70° - 7\cos 40°\right), \quad v^2 = 96.58$

Resolving ↗ towards B: $mg\cos\theta - R = \frac{mv^2}{7}$

$R = 2g\cos 40° - \frac{2v^2}{7} \approx -2.6 < 0$, which is impossible, so the particle has lost contact with the chute before this point.

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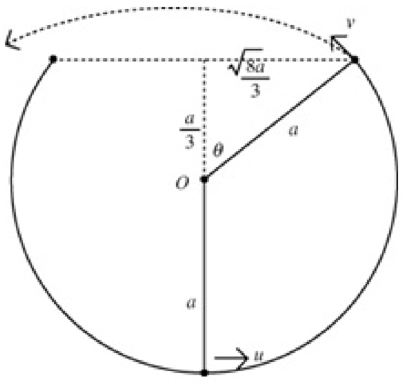
Motion in a circle
Exercise F, Question 8

Question:

Whenever a numerical value of g is required take $g = 9.8 \text{ m s}^{-2}$.
Part of a hollow spherical shell, centre O and radius a, is removed to form a bowl with a plane circular rim. The bowl is fixed with the rim uppermost and horizontal. The centre of the circular rim is $\frac{4a}{3}$ vertically above the lowest point of the bowl. A marble is projected from the lowest point of the bowl with speed u. Find the minimum value of u for which the marble will leave the bowl and not fall back in to it.

Solution:



K.E. + P.E. at lowest point $= \frac{1}{2}mu^2$

K.E. + P.E. at rim $= \frac{1}{2}mv^2 + mg \times \frac{4a}{3}$

$$\Rightarrow u^2 = v^2 + \frac{8ga}{3}$$

After the particle leaves the bowl:

The vertical speed when the particle returns to the level of the rim of the bowl is $v \sin\theta$ downwards, so using $v = u + at, -v\sin\theta = v\sin\theta - gt, t = \frac{2v\sin\theta}{g}$

The horizontal distance covered in this time is $v\cos\theta \times \frac{2v\sin\theta}{g}$

The width of the top of the bowl $= 2 \times \frac{\sqrt{8}}{3}a = \frac{4\sqrt{2}a}{3}$

$\Rightarrow 2\frac{v^2}{g}\sin\theta\cos\theta > \frac{4\sqrt{2}a}{3}, v^2 \times \frac{\sqrt{8}}{3} \times \frac{1}{3} > \frac{2\sqrt{2}ag}{3}, \quad v^2 > 3ag$

$\Rightarrow u^2 > 3ag + \frac{8ga}{3} = \frac{17ga}{3}$

so minimum value of u is $\sqrt{\frac{17ag}{3}}$

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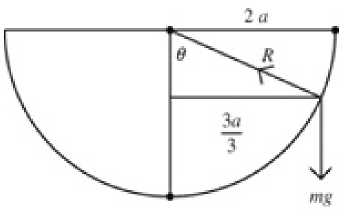
Motion in a circle
Exercise G, Question 1

Question:

A particle of mass m moves with constant speed u in a horizontal circle of radius $\frac{3a}{2}$ on the inside of a fixed smooth hollow sphere of radius $2a$. Show that $9ag = 2\sqrt{7}u^2$.

Solution:



$$R(\updownarrow)\ R\cos\theta = mg$$

$$R(\leftrightarrow)\ R\sin\theta = \frac{mv^2}{r} = \frac{2mv^2}{3a}$$

$$\text{Dividing} \Rightarrow \tan\theta = \frac{2u^2}{3ag}, \text{ but}$$

$$\tan\theta = \frac{\frac{3a}{2}}{\frac{\sqrt{7}a}{2}} = \frac{3}{\sqrt{7}}, \text{ so}$$

$$\frac{2u^2}{3ag} = \frac{3}{\sqrt{7}},\ 9ag = 2\sqrt{7}u^2$$

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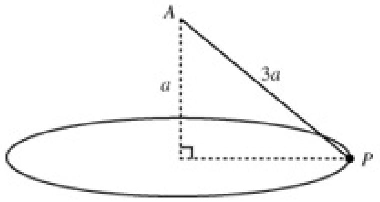
Motion in a circle
Exercise G, Question 2

Question:



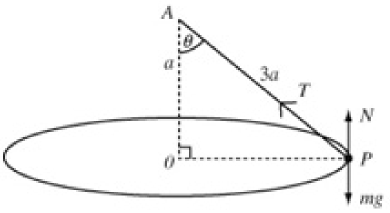
A particle P of mass m is attached to one end of a light inextensible string of length $3a$. The other end of the string is attached to a fixed point A which is a vertical distance a above a smooth horizontal table. The particle moves on the table in a circle whose centre O is vertically below A, as shown in the diagram. The string is taut and the speed of P is $2\sqrt{ag}$. Find

a the tension in the string,

b the normal reaction of the table on P.

Solution:

a



N is the normal reaction of the table on P, T is the tension in the string, and θ is the angle between the string and the vertical.

Right-angled triangle so

$OP = a\sqrt{8}$

$$R(\leftarrow): T\sin\theta = \frac{mv^2}{a\sqrt{8}}$$

$$T\frac{\sqrt{8}a}{3a} = \frac{m \times 4ga}{a\sqrt{8}}$$

$$\Rightarrow T = \frac{3mg}{2}$$

b $R(\uparrow): T\cos\theta + N = mg \Rightarrow N = mg - \frac{3}{2}mg \times \frac{1}{3} = \frac{1}{2}mg$

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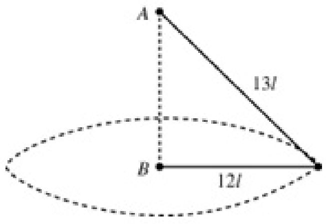
Motion in a circle
Exercise G, Question 3

Question:



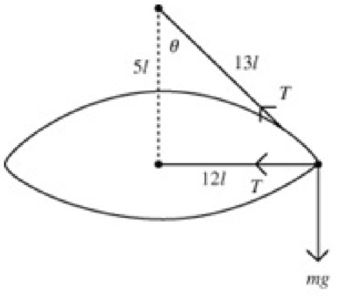
A light inextensible string of length $25l$ has its ends fixed to two points A and B, where A is vertically above B. A small smooth ring of mass m is threaded on the string. The ring is moving with constant speed in a horizontal circle with centre B and radius $12l$, as shown in the diagram. Find

a the tension in the string,

b the speed of the ring.

Solution:

a



Let θ be the angle between the string and the vertical.

We have a 5, 12, 13 triangle.

$R(\updownarrow) T\cos\theta = mg$

$$T = \frac{mg}{\cos\theta} = \frac{13mg}{5}$$

b $R(\leftrightarrow) T + T\sin\theta = \frac{mv^2}{r} \Rightarrow T\left(1+\frac{12}{13}\right) = \frac{mv^2}{12l}, \frac{25}{13}\times\frac{13mg}{5} = 5mg = \frac{mv^2}{12l}$

$$\Rightarrow v^2 = 60gl, \quad v = \sqrt{60gl}$$

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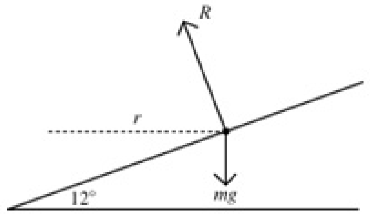
Motion in a circle
Exercise G, Question 4

Question:

A car moves round a bend which is banked at a constant angle of $12°$ to the horizontal. When the car is travelling at a constant speed of 15 m s^{-1} there is no sideways frictional force on the car. The car is modelled as a particle moving in a horizontal circle of radius r metres. Calculate the value of r.

Solution:



R is the normal reaction of the surface on the car.

No friction.

$$\text{R}(\updownarrow)\ R\cos 12° = mg$$

$$\text{R}(\leftrightarrow)\ R\sin 12° = \frac{mv^2}{r} = \frac{m \times 15^2}{r}$$

$$\text{Dividing}: \tan 12° = \frac{225}{gr}$$

$$r = \frac{225}{g\tan 12°} \approx 108 \text{ m}$$

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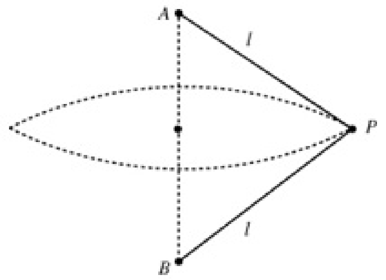
Motion in a circle
Exercise G, Question 5

Question:



A particle P of mass m is attached to the ends of two light inextensible strings AP and BP each of length l. The ends A and B are attached to fixed points, with A vertically above B and $AB = l$, as shown in the diagram. The particle P moves in a horizontal circle with constant angular speed ω. The centre of the circle is the mid-point of AB and both strings remain taut.

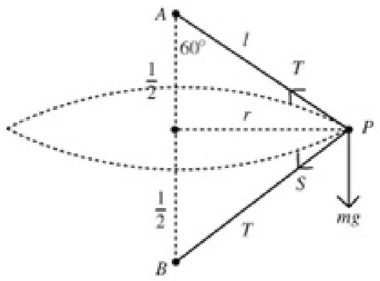
a Show that the tension in AP is $\frac{m}{2}\left(2g + l\omega^2\right)$.

b Find, in terms of m, l, ω and g, an expression for the tension in BP.

c Deduce that $\omega^2 \geq \frac{2g}{l}$.

Solution:

a



T is the tension in AP and S is the tension in BP.
The triangle is equilateral (3 equal sides).

$$R(\updownarrow): T\cos 60° = mg + S\cos 60°$$
$$T - S = 2mg$$
$$R(\leftrightarrow): T\cos 30° + S\cos 30° = mr\omega^2$$
$$(T+S)\cos 30° = ml\cos 30° \times \omega^2$$
$$T + S = ml\omega^2$$

Adding these two equations gives

$$2T = 2mg + ml\omega^2, T = \frac{m}{2}(2g + l\omega^2).$$

b $S = T - 2mg = \frac{m}{2}(l\omega^2 - 2g)$

c Both strings taut $\Rightarrow l\omega^2 - 2g > 0, \quad \omega^2 > \frac{2g}{l}$

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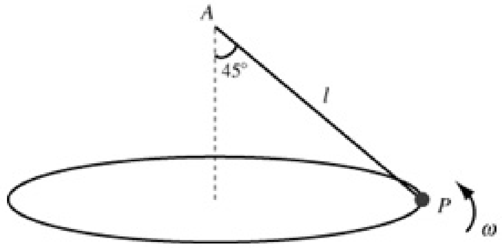
Motion in a circle
Exercise G, Question 6

Question:



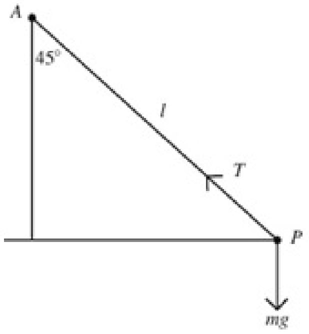
A particle P of mass m is attached to one end of a light string of length l. The other end of the string is attached to a fixed point A. The particle moves in a horizontal circle with constant angular speed ω and with the string inclined at an angle of 45° to the vertical, as shown in the diagram.

a Show that the tension in the string is $\sqrt{2}mg$.

b Find ω in terms of g and l.

Solution:

a



T is the tension in the string.

R(↕): $T\cos 45° = mg, T = \sqrt{2}mg$

b R(↔): $T\cos 45° = mr\omega^2 = ml\cos 45°\omega^2, T = ml\omega^2, \omega = \sqrt{\frac{T}{ml}} = \sqrt{\frac{\sqrt{2}g}{l}}$

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Motion in a circle
Exercise G, Question 7

Question:

A rough disc rotates in a horizontal plane with constant angular velocity ω about a fixed vertical axis. A particle P of mass m lies on the disc at a distance $\frac{3}{5}a$ from the axis. The coefficient of friction between P and the disc is $\frac{3}{7}$. Given that P remains at rest relative to the disc,

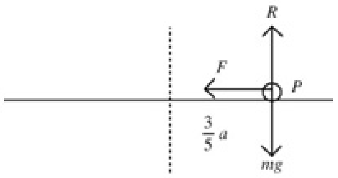
a prove that $\omega^2 \le \frac{5g}{7a}$.

The particle is now connected to the axis by a horizontal light elastic string of natural length $\frac{a}{2}$ and modulus of elasticity $\frac{5mg}{2}$. The disc again rotates with constant angular velocity ω about the axis and P remains at rest relative to the disc at a distance $\frac{3}{5}a$ from the axis.

b Find the range of possible values of ω^2.

Solution:

a



F is the force due to friction,
R is the normal reaction.

$R(\updownarrow): R = mg$

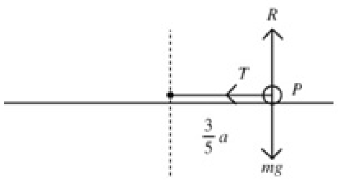
$R(\leftrightarrow): F = mr\omega^2$

If P is not to slip then

$$\frac{3}{7}mg \ge m\frac{3}{5}a\omega^2$$

$$\therefore \omega^2 \le \frac{5g}{7a}.$$

b



T is the tension in the elastic string.

$$T = \frac{\lambda x}{l} = \frac{\frac{5mg}{2}\times\left(\frac{3}{5}a - \frac{a}{2}\right)}{\frac{a}{2}} = \frac{5mg}{10} = \frac{mg}{2}$$

The limits for ω^2 depend on whether the friction is acting with the tension or against it.

$$R(\leftrightarrow): \frac{3}{7}mg + \frac{mg}{2} \ge m\frac{3}{5}a\omega^2, \omega^2 \le \frac{5}{3a}\times\frac{13g}{14} = \frac{65g}{42a}$$

$$\text{or } R(\leftrightarrow): -\frac{3}{7}mg + \frac{mg}{2} \le m\frac{3}{5}a\omega^2, \omega^2 \ge \frac{5}{3a}\times\frac{g}{14} = \frac{5g}{42a}$$

$$\frac{5g}{42a} \le \omega^2 \le \frac{65g}{42a}$$

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Motion in a circle
Exercise G, Question 8

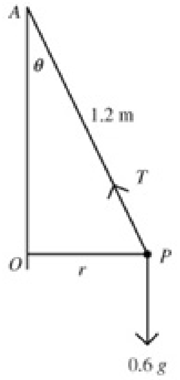
Question:

A particle P of mass 0.6 kg is attached to one end of a light inextensible string of length 1.2 m. The other end of the string is attached to a fixed point A. The particle is moving, with the string taut, in a horizontal circle with centre O vertically below A. The particle is moving with constant angular speed 3 rad s^{-1}. Find

a the tension in the string,
b the angle between AP and the downward vertical.

Solution:

a



r is the radius of the circle,
T is the tension in the string and $\angle OAP$ is θ.
From the triangle, $r = 1.2 \sin\theta$.

$R(\leftrightarrow): T\sin\theta = mr\omega^2 = 0.6 \times 1.2 \sin\theta \times 9$

$T = 0.6 \times 1.2 \times 9 = 6.48 \text{ N}$

b $R(\updownarrow) T\cos\theta = mg, 6.48\cos\theta = 0.6g, \cos\theta = \dfrac{0.6g}{6.48} \approx 0.907, \theta \approx 25°.$

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Motion in a circle
Exercise G, Question 9

Question:

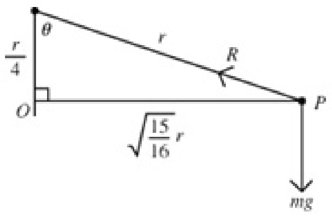
A particle P of mass m moves on the smooth inner surface of a spherical bowl of internal radius r. The particle moves with constant angular speed in a horizontal circle, which is at a depth $\frac{r}{4}$ below the centre of the bowl. Find

a the normal reaction of the bowl on P,

b the time it takes P to complete three revolutions of its circular path.

Solution:

a



The angle between the radius through P and the vertical is θ. P has angular speed ω rad s^{-1}.

R is the reaction of the bowl on P.

$\text{R}(\updownarrow): R\cos\theta = mg, R = 4mg.$

b $\text{R}(\leftrightarrow): R\sin\theta = mr\omega^2 = m \times r\sin\theta \times \omega^2,\ \omega = \sqrt{\frac{4mg}{mr}} = \sqrt{\frac{4g}{r}}$

Three revolutions is 6π radians, time taken $= \frac{6\pi}{\sqrt{\frac{4g}{r}}} = 3\pi\sqrt{\frac{r}{g}}$

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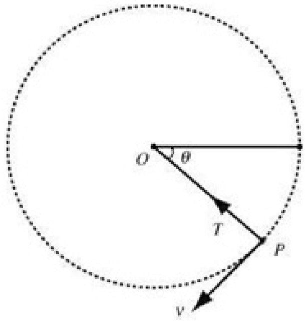
Motion in a circle
Exercise G, Question 10

Question:

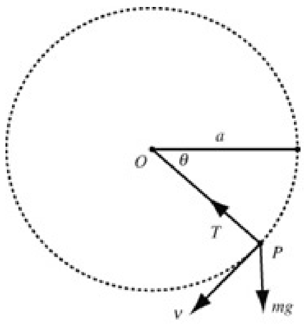


A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is fixed at a point O. The particle is held with the string taut and OP horizontal. It is then projected vertically downwards with speed u, where $u^2 = \frac{4}{3}ga$. When OP has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in the diagram.

a Find an expression for v^2 in terms of a, g and θ.
b Find an expression for T in terms of m, g and θ.
c Find, to the nearest degree, the value of θ when the string becomes slack.
d Explain why P would not complete a vertical circle if the string were replaced by a light rod.

Solution:

a



Loss in P.E. = gain in K.E. so

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga\sin\theta$$

$$\Rightarrow v^2 = \frac{4}{3}ga + 2ga\sin\theta$$

b Resolving towards O: $T - mg\sin\theta = \frac{mv^2}{a}$

$$T = \frac{4}{3}mg + 2mg\sin\theta + mg\sin\theta = mg\left(\frac{4}{3} + 3\sin\theta\right)$$

c $T = 0$ when $\sin\theta = -\frac{4}{9}, \theta = 206°$

d When $v = 0, \sin\theta = -\frac{4}{6} = -\frac{2}{3}, (\theta \approx 222°)$ so the particle can not complete the circle.

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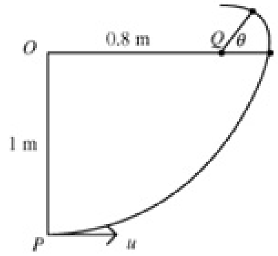
Motion in a circle
Exercise G, Question 11

Question:

A particle P of mass 0.4 kg is attached to one end of a light inelastic string of length 1 m. The other end of the string is fixed at point O. P is hanging in equilibrium below O when it is projected horizontally with speed u ms^{-1}. When OP is horizontal it meets a small smooth peg at Q, where $OQ = 0.8$ m. Calculate the minimum value of u if P is to describe a complete circle about Q.

Solution:



Consider the circle centre Q, radius 0.2 m.
When QP is at θ above the horizontal:

Energy: $\frac{1}{2}mw^2 + mg \times 0.2\sin\theta = \frac{1}{2}mv^2$,

$w^2 = v^2 - 0.4g\sin\theta$

where v is the speed when $\theta = 0$, and w the speed at angle θ.

Circular motion: $T + mg\sin\theta = \frac{mw^2}{r} = \frac{m(v^2 - 0.4g\sin\theta)}{0.2}$

$T = \frac{m(v^2 - 0.4g\sin\theta)}{0.2} - mg\sin\theta = \frac{m(v^2 - 0.6\,g\sin\theta)}{0.2} \geq 0$

Looking at the larger circle, conservation of energy

$\Rightarrow \frac{1}{2}mv^2 + mg \times 1 = \frac{1}{2}mu^2, \quad v^2 = u^2 - 2g$

At the top of the small circle, $\sin\theta = 1$,

$\Rightarrow u^2 - 2g - 0.6g \geq 0, u^2 \geq 2.6g, u \geq \sqrt{2.6\,g}$

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Motion in a circle
Exercise G, Question 12

Question:

A smooth solid hemisphere is fixed with its plane face on a horizontal table and its curved surface uppermost. The plane face of the hemisphere has centre O and radius a. The point A is the highest point on the hemisphere. A particle P is placed on the hemisphere at A.

It is then given an initial horizontal speed u, where $u^2 = \frac{ag}{2}$. When OP makes an angle θ with OA, and while P remains on the hemisphere, the speed of P is v.

a Find an expression for v^2.

b Show that P is still on the hemisphere when $\theta = \cos^{-1} 0.9$

c Find the value of

i $\cos\theta$ when P leaves the hemisphere,

ii v when P leaves the hemisphere.

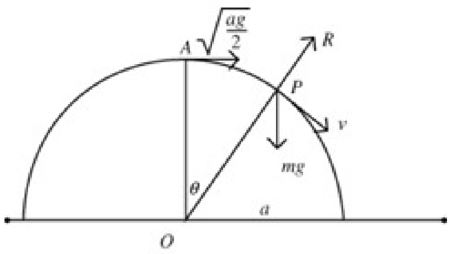
After leaving the hemisphere P strikes the table at B.

d Find the speed of P at B.

e Find the angle at which P strikes the table.

Solution:

a



R is the reaction between the particle and the surface.
If the level of P is the level of zero P.E., conservation of energy

$$\Rightarrow \frac{1}{2}m\frac{ag}{2} + mga(1-\cos\theta) = \frac{1}{2}mv^2,$$

$$v^2 = \frac{ga}{2} + 2ga(1-\cos\theta)$$

$$= \frac{ga}{2}(5-4\cos\theta)$$

b Resolving towards O: $mg\cos\theta - R = \frac{mv^2}{r} = \frac{mg}{2}(5-4\cos\theta)$

Substituting $\cos\theta = 0.9$: $R = mg\times 0.9 - \frac{mg}{2}(5-3.6) = 0.2mg > 0$

so P is still on the hemisphere.

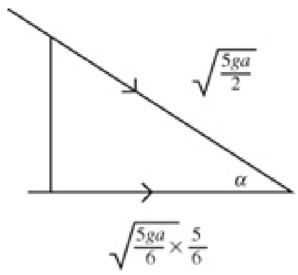
c **i** $R = 0 \Rightarrow \cos\theta = \frac{1}{2}(5-4\cos\theta),\ 3\cos\theta = \frac{5}{2}, \cos\theta = \frac{5}{6}.$

ii $v^2 = \frac{ga}{2}(5-4\cos\theta) = \frac{ga}{2}(5-\frac{10}{3}) = \frac{5ga}{6}, v = \sqrt{\frac{5ga}{6}}$

d By considering K.E. + P.E. at A and B, if v is the speed at B,

$$\frac{1}{2}mv^2 = \frac{1}{2}m\frac{ag}{2} + mga, v^2 = \frac{5ga}{2}, v = \sqrt{\frac{5ga}{2}}$$

e



After the particle leaves the sphere the horizontal velocity remains constant $= \sqrt{\frac{5ga}{6}}\times\frac{5}{6}$.

If α is the angle at which the particle strikes the table then $\cos\alpha = \frac{\sqrt{\frac{5ga}{6}}\times\frac{5}{6}}{\sqrt{\frac{5ga}{2}}} = \frac{5}{6\sqrt{3}}$

$$\alpha \approx 61°$$

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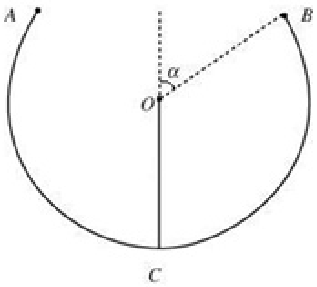
Motion in a circle
Exercise G, Question 13

Question:



Part of a hollow spherical shell, centre O and radius r, is removed to form a bowl with a plane circular rim. The bowl is fixed with the circular rim uppermost and horizontal. The point C is the lowest point of the bowl. The point B is on the rim of the bowl and OB is at an angle α to the upward vertical as shown in the diagram. Angle α satisfies $\tan\alpha = \frac{4}{3}$. A smooth small marble of mass m is placed inside the bowl at C and given an initial horizontal speed u. The direction of motion of the marble lies in the vertical plane COB. The marble stays in contact with the bowl until it reaches B.

When the marble reaches B it has speed v.

a Find an expression for v^2.

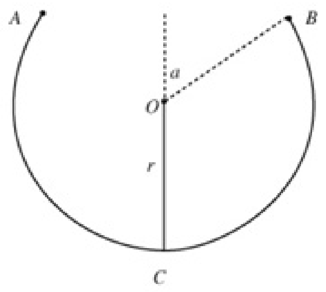
b If $u^2 = 4gr$, find the normal reaction of the bowl on the marble as the marble reaches B.

c Find the least possible value of u for the marble to reach B.

The point A is the other point of the rim of the bowl lying in the vertical plane COB.

d Find the value of u which will enable the marble to leave the bowl at B and meet it again at A.

Solution:

a 

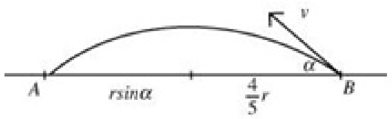
K.E. + P.E. at C = K.E. + P.E. at B.
If P.E. = 0 at C then

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(r + r\cos\alpha) = \frac{1}{2}mv^2 + \frac{8}{5}mgr$$

$$v^2 = u^2 - \frac{16}{5}gr$$

b $u^2 = 4gr \Rightarrow v^2 = \frac{4}{5}gr$. Resolving towards O: $R + \frac{3}{5}mg = \frac{mv^2}{r} = \frac{4mg}{5}, R = \frac{mg}{5}$

c $R = 0$ at $B \Rightarrow \frac{3mg}{5} = \frac{mv^2}{r} = \frac{m\left(u^2 - \frac{16gr}{5}\right)}{r}, \frac{mu^2}{r} = \frac{3mg}{5} + \frac{16mg}{5}, u = \sqrt{\frac{19gr}{5}}$

d 

The particle is now moving freely under gravity.
Horizontal distance

$$= 2r\sin\alpha = \frac{8r}{5} = v\cos\alpha \times t$$

so $t = \frac{8r}{3v}$.

Vertical distance $= 0 = \frac{4v}{5}t - \frac{1}{2}gt^2 \Rightarrow t = \frac{8v}{5g} = \frac{8r}{3v}, \Rightarrow v = \sqrt{\frac{5rg}{3}}$

$\Rightarrow u^2 = \frac{5rg}{3} + \frac{16gr}{5} = \frac{73}{15}gr; u = \sqrt{\frac{73gr}{15}}$

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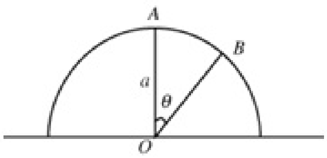
Motion in a circle
Exercise G, Question 14

Question:



A particle is at the highest point A on the outer surface of a fixed smooth hemisphere of radius a and centre O. The hemisphere is fixed to a horizontal surface with the plane face in contact with the surface. The particle is projected horizontally from A with speed u, where $u < \sqrt{ag}$. The particle leaves the sphere at the point B, where OB makes an angle θ with the upward vertical, as shown in the diagram.

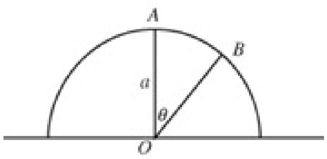
a Find an expression for $\cos\theta$ in terms of u, g and a.

The particle strikes the horizontal surface with speed $\sqrt{\frac{5ag}{2}}$.

b Find the value of θ.

Solution:

a



Equating the K.E. + P.E. at A and B:

$$\frac{1}{2}mu^2 + mga = \frac{1}{2}mv^2 + mga\cos\theta$$

$$\Rightarrow v^2 = u^2 + 2ga(1-\cos\theta)$$

Resolving towards O: $mg\cos\theta - R = \frac{mv^2}{a}$

$$R = 0 \Rightarrow ag\cos\theta = u^2 + 2ga(1-\cos\theta)$$

$$3ag\cos\theta = u^2 + 2ag$$

$$\cos\theta = \frac{u^2 + 2ag}{3ag}$$

b Conservation of energy from A to surface:

$$\frac{1}{2}mu^2 + mga = \frac{1}{2}m\times\frac{5ag}{2}, u^2 = \frac{ag}{2}, \cos\theta = \frac{5}{6}, \theta \approx 34°$$