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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Find the position of the centre of mass of four particles of masses 1 kg, 4 kg, 3 kg and 2 kg placed on the x-axis at the points (6, 0), (3, 0) (2, 0) and (4, 0) respectively.

Solution:

$$(1\times 6)+(4\times 3)+(3\times 2)+(2\times 4)=\bar{x}(1+4+3+2)$$

Use $\Sigma m_i x_i = \bar{x}\Sigma m_i$

$$6+12+6+8=10\bar{x}$$

$$32=10\bar{x}$$

Simplify.

$$3.2=\bar{x}$$

Solve for $\bar{x}$.

Centre of mass is at (3.2, 0).

Give both coordinates.

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Exercise A, Question 2

Question:

Three masses 1 kg, 2 kg and 3 kg, are placed at the points with coordinates (0, 2), (0, 5) and (0, 1) respectively. Find the coordinates of G, the centre of mass of the three masses.

Solution:

$$(1\times 2)+(2\times 5)+(3\times 1)=\bar{y}(1+2+3)$$ ← Use $\Sigma m_i y_i = \bar{y}\Sigma m_i$.

$$2+10+3=6\bar{y}$$ ← Simplify.

$$15=6\bar{y}$$

$$2.5=\bar{y}$$ ← Solve for $\bar{y}$.

Centre of mass is at (0, 2.5). ← Give both coordinates.

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Exercise A, Question 3

Question:

Three particles of mass 2 kg, 3 kg and 5 kg, are placed at the points $(-1, 0)$, $(-4, 0)$ and $(5, 0)$ respectively. Find the coordinates of the centre of mass of the three particles.

Solution:

$$(2\times -1)+(3\times -4)+(5\times 5)=\bar{x}(2+3+5)$$ ← Use $\Sigma m_i x_i = \bar{x}\Sigma m_i$.

$$-2+-12+25=10\bar{x}$$

$$11=10\bar{x}$$ ← Simplify.

$$1.1=\bar{x}$$ ← Solve for $\bar{x}$.

Centre of mass is at $(1.1, 0)$. ← Give both coordinates.

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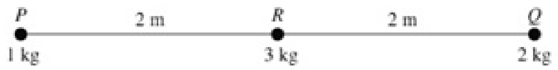
Exercise A, Question 4

Question:

A light rod PQ of length 4 m has particles of mass 1 kg, 2 kg and 3 kg attached to it at the points P, Q and R respectively, where $PR = 2\text{ m}$. The centre of mass of the loaded rod is at the point G. Find the distance PG.

Solution:



Draw a diagram. The rod has no mass.

$$(1\times 0)+(3\times 2)+(2\times 4)=\bar{x}(1+3+2)$$

Take P as the origin and use $\Sigma m_i x_i = \bar{x}\Sigma m_i$.

$$0+6+8=6\bar{x}$$

Simplify.

$$\frac{7}{3}=\bar{x}$$

Solve for $\bar{x}$.

$$PG = 2\frac{1}{3}\text{ m}$$

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Exercise A, Question 5

Question:

Three particles of mass 5 kg, 3 kg and m kg lie on the y-axis at the points (0, 4), (0, 2) and (0, 5) respectively. The centre of mass of the system is at the point (0, 4). Find the value of m.

Solution:

$$(5\times 4)+(3\times 2)+(m\times 5)=4(5+3+m)$$ — Use $\Sigma m_i y_i = \bar{y}\Sigma m_i$.

$$20+6+5m=32+4m$$ — Simplify

$$m=6$$ — Solve for m.

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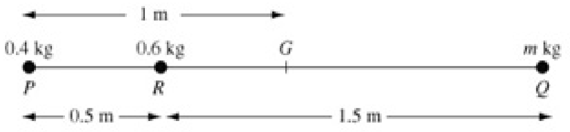
Exercise A, Question 6

Question:

A light rod PQ of length 2 m has particles of masses 0.4 kg and 0.6 kg fixed to it at the points P and R respectively, where $PR = 0.5$ m . Find the mass of the particle which must be fixed at Q so that the centre of mass of the loaded rod is at its mid-point.

Solution:



The rod, being light, has no mass.

Draw a diagram showing all the information.
G is the centre of mass.
Assume the mass of the particle required is m kg.

Take P as the origin.

$$(0.4 \times 0) + (0.6 \times 0.5) + (m \times 2) = 1 \times (0.4 + 0.6 + m)$$

$$0.3 + 2m = 1.0 + m$$

$$m = 0.7$$

Use $\Sigma m_i x_i = \bar{x} \Sigma m_i$.

Simplify.

Solve for m.

The mass of the particle is 0.7 kg.

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Exercise A, Question 7

Question:

The centre of mass of four particles of masses $2m$, $3m$, $7m$ and $8m$, which are positioned at the points $(0, a)$, $(0, 2)$, $(0, -1)$ and $(0, 1)$ respectively, is the point G. Given that the coordinates of G are $(0, 1)$, find the value of a.

Solution:

$(2m \times a) + (3m \times 2) + (7m \times -1) + (8m \times 1) = 1 \times (2m + 3m + 7m + 8m)$ ← Use $\Sigma m_i y_i = \bar{y} \Sigma m_i$.

$2ma + 6m - 7m + 8m = 20m$ ← Divide by m.

$2a + 7 = 20$

$a = 6\frac{1}{2}$ ← Solve for a.

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Exercise A, Question 8

Question:

Particles of mass 3 kg, 2 kg and 1 kg lie on the y-axis at the points with coordinates $(0, -2)$, $(0, 7)$ and $(0, 4)$ respectively. Another particle of mass 6 kg is added to the system so that the centre of mass of all four particles is at the origin. Find the position of this particle.

Solution:

Suppose the particle is placed at $(0, y)$.

$$(3\times -2)+(2\times 7)+(1\times 4)+(6\times y)=0\times(3+2+1+6)$$

Use $\Sigma m_i y_i = \bar{y}\Sigma m_i$.

$$-6+14+4+6y=0$$

Simplify.

$$6y=-12$$

$$y=-2$$

Solve for y.

The particle must be placed at $(0, -2)$.

Give both coordinates.

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Exercise A, Question 9

Question:

Three particles A, B and C are placed along the x-axis. Particle A has mass 5 kg and is at the point (2, 0). Particle B has mass m_1 kg and is at the point (3, 0) and particle C has mass m_2 kg and is at the point $(-2, 0)$. The centre of mass of the three particles is at the point $G(1, 0)$. Given that the total mass of the three particles is 10 kg, find the values of m_1 and m_2.

Solution:

$5 + m_1 + m_2 = 10$ — Use the total mass.

$m_1 + m_2 = 5 \quad (1)$

$(5 \times 2) + (m_1 \times 3) + (m_2 \times -2) = 1 \times 10$ — Use $\Sigma m_i x_i = \bar{x} \Sigma m_i$, and $m_i = 10$.

$10 + 3m_1 - 2m_2 = 10$ — Simplify.

$3m_1 - 2m_2 = 0 \quad (2)$

Adding $(2) + 2 \times (1)$ $2m_1 + 2m_2 = 10$

$5m_1 = 10$ — Eliminate m_2.

$m_1 = 2$ — Solve for m_1.

$m_2 = 3$ — Use (1).

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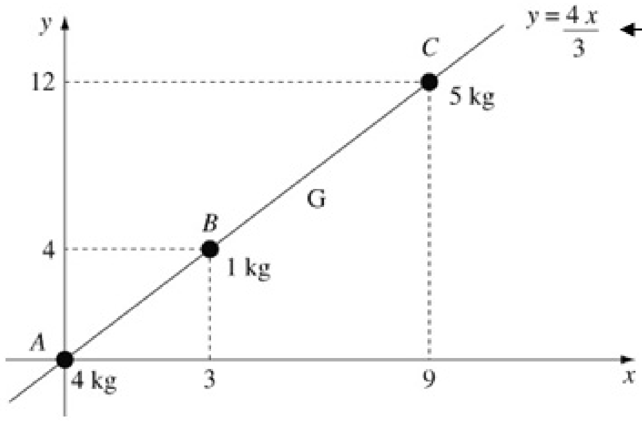
Exercise A, Question 10

Question:

Three particles A, B and C have masses 4 kg, 1 kg and 5 kg respectively. The particles are placed on the line with equation $3y - 4x = 0$. Particle A is at the origin, particle B is at the point (3, 4) and particle C is at the point (9, 12). Find the coordinates of the centre of mass of the three particles.

Solution:



Draw a diagram showing the positions of the three particles.

$AB = \sqrt{3^2 + 4^2} = 5$

$AC = \sqrt{9^2 + 12^2} = 15$

We need the distances along the line.

$(4 \times 0) + (1 \times 5) + (5 \times 15) = \bar{x}(4 + 1 + 5)$

Taking our axis as the line $3y - 4x = 0$ and A as the origin.

$0 + 5 + 75 = 10\bar{x}$

Use $\Sigma m_i x_i = \bar{x}\Sigma m_i$.

$8 = \bar{x}$

Solve for $\bar{x}$.

i.e. $AG = 8$

Since $AB = 5$,

$\overrightarrow{AG} = \frac{8}{5} \times \overrightarrow{AB} = \frac{8}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

The vectors $\overrightarrow{AB}$ and $\overrightarrow{AG}$ are parallel.

$= \begin{pmatrix} 4.8 \\ 6.4 \end{pmatrix}$

Multiply out.

The coordinates of G are (4.8, 6.4).

Section 2.2 will show you a neater and easier way to solve this problem.

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Exercise B, Question 1

Question:

Two particles of equal mass are placed at the points $(1,-3)$ and $(5, 7)$. Find the centre of mass of the particles.

Solution:

$$m\begin{pmatrix}1\\-3\end{pmatrix}+m\begin{pmatrix}5\\7\end{pmatrix}=(m+m)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i\mathbf{r}_i=\bar{\mathbf{r}}\,\Sigma m_i$

$$\begin{pmatrix}m\\-3m\end{pmatrix}+\begin{pmatrix}5m\\7m\end{pmatrix}=2m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}6m\\4m\end{pmatrix}=2m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Simplify by dividing both sides by m.

$$\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Centre of mass is (3, 2).

This is the mid-point of the line joining the two points.

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Exercise B, Question 2

Question:

Four particles of equal mass are situated at the points (2, 0), (−1,3),(2,−4) and (−1,−2). Find the coordinates of the centre of mass of the particles.

Solution:

$$m\begin{pmatrix}2\\0\end{pmatrix}+m\begin{pmatrix}-1\\3\end{pmatrix}+m\begin{pmatrix}2\\-4\end{pmatrix}+m\begin{pmatrix}-1\\-2\end{pmatrix}=4m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i \mathbf{r}_i = \bar{\mathbf{r}}\,\Sigma m_i$ as before.

$$\begin{pmatrix}2\\-3\end{pmatrix}=4\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Divide both sides by m.

$$\begin{pmatrix}\frac{1}{2}\\-\frac{3}{4}\end{pmatrix}=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Solve.

Centre of mass is $\left(\frac{1}{2},-\frac{3}{4}\right)$.

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Exercise B, Question 3

Question:

A system of three particles consists of 10 kg placed at (2, 3), 15 kg placed at (4, 2) and 25 kg placed at (6, 6). Find the coordinates of the centre of mass of the system.

Solution:

$$10\begin{pmatrix}2\\3\end{pmatrix}+15\begin{pmatrix}4\\2\end{pmatrix}+5\begin{pmatrix}6\\6\end{pmatrix}=50\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}4\\6\end{pmatrix}+\begin{pmatrix}12\\6\end{pmatrix}+\begin{pmatrix}30\\30\end{pmatrix}=10\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$ ← Divide both sides by 5.

$$\begin{pmatrix}46\\42\end{pmatrix}=10\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$ ← Simplify.

$$\begin{pmatrix}4.6\\4.2\end{pmatrix}=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$ ← Solve.

Centre of mass is (4.6, 4.2).

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Exercise B, Question 4

Question:

Find the position vector of the centre of mass of three particles of masses 0.5 kg, 1.5 kg and 2 kg which are situated at the points with position vectors $(6\mathbf{i}-3\mathbf{j}), (2\mathbf{i}+5\mathbf{j})$ and $(3\mathbf{i}+2\mathbf{j})$ respectively.

Solution:

$$0.5\begin{pmatrix}6\\-3\end{pmatrix}+1.5\begin{pmatrix}2\\5\end{pmatrix}+2\begin{pmatrix}3\\2\end{pmatrix}=4\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

4 is the total mass.

$$\begin{pmatrix}3\\-1.5\end{pmatrix}+\begin{pmatrix}3\\7.5\end{pmatrix}+\begin{pmatrix}6\\4\end{pmatrix}=4\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

It's easier to use column vectors.

$$\begin{pmatrix}12\\10\end{pmatrix}=4\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}3\\2.5\end{pmatrix}=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

You could leave your answer as a column vector.

The position vector is $(3\mathbf{i}+2.5\mathbf{j})$.

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Exercise B, Question 5

Question:

Particles of masses m, $2m$, $5m$ and $2m$ are situated at $(-1,-1),(3,2),(4,-2)$ and $(-2,5)$ respectively. Find the coordinates of the centre of mass of the particles.

Solution:

$$m\begin{pmatrix}-1\\-1\end{pmatrix}+2m\begin{pmatrix}3\\2\end{pmatrix}+5m\begin{pmatrix}4\\-2\end{pmatrix}+2m\begin{pmatrix}-2\\5\end{pmatrix}=10m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}-1\\-1\end{pmatrix}+\begin{pmatrix}6\\4\end{pmatrix}+\begin{pmatrix}20\\-10\end{pmatrix}+\begin{pmatrix}-4\\10\end{pmatrix}=10\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Divide both sides by m.

$$\begin{pmatrix}21\\3\end{pmatrix}=10\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Simplify.

$$\begin{pmatrix}2.1\\0.3\end{pmatrix}=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Solve.

Centre of mass is at (2.1, 0.3).

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 6

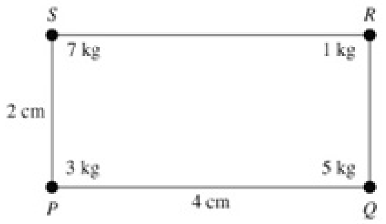
Question:

A light rectangular metal plate $PQRS$ has $PQ = 4$ cm and $PS = 2$ cm. Particles of masses 3 kg, 5 kg, 1 kg and 7 kg are attached respectively to the corners P, Q, R and S of the plate.
Find the distance of the centre of mass of the loaded plate from

a the side PQ,

b the side PS.

Solution:



Draw a diagram. (Note that the plate is light.)

Draw the rectangle with the 2 'axes' (PQ and PS) in the bottom L. H. corner.

Taking P as the origin, and axes, PQ and PS, P is (0, 0); Q is $(4, 0)$; R is $(4, 2)$; S is $(0, 2)$.

$$3\begin{pmatrix}0\\0\end{pmatrix}+5\begin{pmatrix}4\\0\end{pmatrix}+1\begin{pmatrix}4\\2\end{pmatrix}+7\begin{pmatrix}0\\2\end{pmatrix}=16\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Total mass is 16 kg.

$$\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}20\\0\end{pmatrix}+\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}0\\14\end{pmatrix}=16\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}24\\16\end{pmatrix}=16\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Simplify.

$$\begin{pmatrix}1.5\\1\end{pmatrix}=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Solve.

a Distance from PQ is 1 $(\bar{y})$.

b Distance from PS is 1.5 $(\bar{x})$.

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Exercise B, Question 7

Question:

Three particles of masses 1 kg, 2 kg and 3 kg are positioned at the points (1, 0), (4, 3) and (p, q) respectively. Given that the centre of mass of the particles is at the point (2, 0), find the values of p and q.

Solution:

$$1\begin{pmatrix}1\\0\end{pmatrix}+2\begin{pmatrix}4\\3\end{pmatrix}+3\begin{pmatrix}p\\q\end{pmatrix}=(1+2+3)\begin{pmatrix}2\\0\end{pmatrix}$$

Use $\Sigma m_i\mathbf{r}_i = \bar{\mathbf{r}}\,\Sigma m_i$.

$$\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}8\\6\end{pmatrix}+\begin{pmatrix}3p\\3q\end{pmatrix}=\begin{pmatrix}12\\0\end{pmatrix}$$

Simplify.

$$\begin{pmatrix}9+3p\\6+3q\end{pmatrix}=\begin{pmatrix}12\\0\end{pmatrix}$$

Collect terms.

$$9+3p=12$$
$$6+3q=0$$

Equate **i** and **j** components.

$$\Rightarrow p=1$$
$$q=-2$$

Solve for p and q.

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Exercise B, Question 8

Question:

A system consists of three particles with masses $3m$, $4m$ and $5m$. The particles are situated at the points with coordinates $(-3,-4)$, $(0.5, 4)$ and $(0,-5)$ respectively. Find the coordinates of the position of a fourth particle of mass $7m$, given that the centre of mass of all four particles is at the origin.

Solution:

$$3m\begin{pmatrix}-3\\-4\end{pmatrix}+4m\begin{pmatrix}0.5\\4\end{pmatrix}+5m\begin{pmatrix}0\\-5\end{pmatrix}+7m\begin{pmatrix}x\\y\end{pmatrix}=19m\begin{pmatrix}0\\0\end{pmatrix}$$

Use $\Sigma m_i \mathbf{r}_i = \bar{\mathbf{r}}\,\Sigma m_i$.

$$\begin{pmatrix}-9\\-12\end{pmatrix}+\begin{pmatrix}2\\16\end{pmatrix}+\begin{pmatrix}0\\-25\end{pmatrix}+\begin{pmatrix}7x\\7y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

Divide by m and simplify.

$$\begin{pmatrix}-7+7x\\-21+7y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

Collect terms.

$$-7+7x=0$$
$$-21-7y=0$$

Equate $\mathbf{i}$ and $\mathbf{j}$ components.

$$x=1$$
$$y=3$$

Solve for x and y.

Coordinates of particle are $(1, 3)$.

State answer.

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Exercise B, Question 9

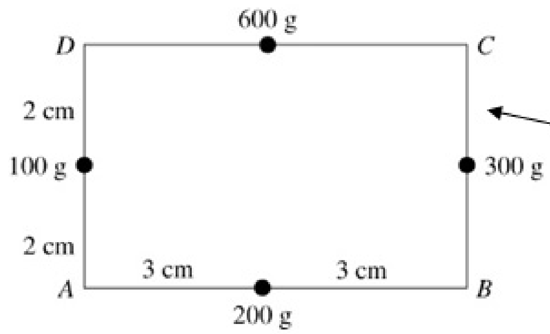
Question:

A light rectangular piece of card $ABCD$ has $AB = 6$ cm and $AD = 4$ m. Four particles of mass 200 g, 300 g, 600 g and 100 g are fixed to the rectangle at the mid-points of the sides AB, BC, CA and AD respectively. Find the distance of the centre of mass of the loaded rectangle from

a the side AB,

b the side AD.

Solution:



The card is light so has no mass.

Draw a diagram showing all the information.

Taking axes through A, the coordinates of the masses are (3, 0), (6, 2), (3, 4) and (0, 2).

Here you have to set up your own axes.

So,

$$200\begin{pmatrix}3\\0\end{pmatrix}+300\begin{pmatrix}6\\2\end{pmatrix}+600\begin{pmatrix}3\\4\end{pmatrix}+100\begin{pmatrix}0\\2\end{pmatrix}=1200\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i \mathbf{r}_i = \bar{\mathbf{r}}\,\Sigma m_i$.

$$\begin{pmatrix}600\\0\end{pmatrix}+\begin{pmatrix}1800\\600\end{pmatrix}+\begin{pmatrix}1800\\2400\end{pmatrix}+\begin{pmatrix}0\\200\end{pmatrix}=1200\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Simplify and divide by 100.

$$\begin{pmatrix}42\\32\end{pmatrix}=\begin{pmatrix}12\bar{x}\\12\bar{y}\end{pmatrix}$$

Collect terms.

$$42 = 12\bar{x}$$
$$32 = 12\bar{y}$$

Equate **i** and **j** components.

$$3.5 = \bar{x}$$
$$2.\dot{6} = \bar{y}$$

Solve for $\bar{x}$ and $\bar{y}$.

a Distance from AB is $2\frac{2}{3}$ cm ($\bar{y}$)

b Distance from AD is $3\frac{1}{2}$ cm ($\bar{x}$)

State answer.

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Exercise B, Question 10

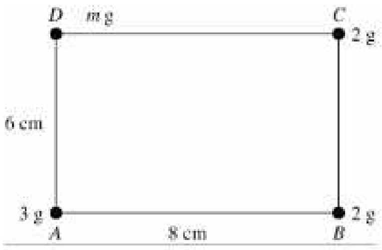
Question:

A light rectangular piece of card $ABCD$ has $AB = 8$ cm and $AD = 6$ cm. Three particles of mass 3 g, 2 g and 2 g are attached to the rectangle at the points A, B and C respectively.

a Find the mass of a particle which must be placed at the point D for the centre of mass of the whole system of four particles to lie 3 cm from the line AB.

b With this fourth particle in place, find the distance of the centre of mass of the system from the side AD.

Solution:



Draw a diagram.

The card has no mass. Here 'g' is grams!

Let mass of particle of D be m g.

Taking axes through A, the coordinates of the particles are (0, 0), (8, 0), (8, 6) and (0, 6).

Here we have to set up our own axes.

$$3\begin{pmatrix}0\\0\end{pmatrix}+2\begin{pmatrix}8\\0\end{pmatrix}+2\begin{pmatrix}8\\6\end{pmatrix}+m\begin{pmatrix}0\\6\end{pmatrix}=(3+2+2+m)\begin{pmatrix}\bar{x}\\3\end{pmatrix}$$

'3 cm from A' means $\bar{y}=3$.

$$\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}16\\0\end{pmatrix}+\begin{pmatrix}16\\12\end{pmatrix}+\begin{pmatrix}0\\6m\end{pmatrix}=(7+m)\begin{pmatrix}\bar{x}\\3\end{pmatrix}$$

Simplify.

$$\begin{pmatrix}32\\12+6m\end{pmatrix}=\begin{pmatrix}(7+m)\bar{x}\\21+3m\end{pmatrix}$$

Collect terms.

$$12+6m=21+3m$$

Equate $\mathbf{j}$ components.

$$3m=9$$
$$m=3$$

Solve for m.

$$32=10\bar{x}$$

Equate $\mathbf{i}$ component and substitute for m.

$$3.2=\bar{x}$$

Solve for $\bar{x}$.

a $m=3$

b 3.2 cm

State your answer.

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Exercise C, Question 1

Question:

Find the centre of mass of a uniform triangular lamina whose vertices are

a (1, 2), (2, 6) and (3, 1),

b $(-1, 4)$, (3, 5) and (7, 3)

c $(-3, 2)$, (4, 0) and (0,1)

d (a, a), $(3a, 2a)$ and $(4a, 6a)$.

Solution:

a G is $\left(\frac{1+2+3}{3}, \frac{2+6+1}{3}\right)$
i.e. (2, 3)

← Find the mean of the vertices of the triangle.

b G is $\left(\frac{-1+3+7}{3}, \frac{4+5+3}{3}\right)$
i.e. (3, 4)

← Find the mean of the vertices of the triangle.

c G is $\left(\frac{-3+4+0}{3}, \frac{2+0+1}{3}\right)$
i.e. $\left(\frac{1}{3}, 1\right)$

← Find the mean of the vertices of the triangle.

d G is $\left(\frac{a+3a+4a}{3}, \frac{a+2a+6a}{3}\right)$
i.e. $\left(\frac{8a}{3}, 3a\right)$

← Find the mean of the vertices of the triangle.

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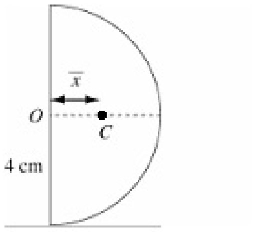
Exercise C, Question 2

Question:

Find the position of the centre of mass of a uniform semi-circular lamina of radius 4 cm and centre O.

Solution:

For a semi-circle,



$$2\alpha = \pi$$

$$\alpha = \frac{\pi}{2}$$

$$\bar{x} = \frac{2r\sin\alpha}{3\alpha} = \frac{8\sin\frac{\pi}{2}}{3\frac{\pi}{2}}$$

$$= \frac{16}{3\pi}$$

This result is in the formula booklet provided by Edexcel.

Centre of mass is on the axis of symmetry at a distance $\frac{16}{3\pi}$ cm from the centre.

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Exercise C, Question 3

Question:

The centre of mass of a uniform triangular lamina ABC is at the point $(2, a)$. Given that A is the point $(4, 3)$, B is the point $(b, 1)$ and C is the point $(-1, 5)$, find the values of a and b.

Solution:

$$(2, a) = \left(\frac{4+b-1}{3}, \frac{3+1+5}{3}\right)$$ ← Use the fact that the centre of mass is at the 'mean point'.

i.e. $(2, a) = \left(\frac{3+b}{3}, 3\right)$ ← Simplify.

So,

$2 = \frac{3+b}{3}$ and $a = 3$ ← Compare x and y coordinates.

$6 = 3 + b$

$3 = b$ ← Solve for b.

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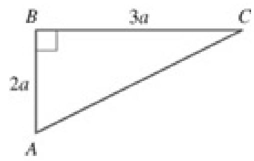
Exercise C, Question 4

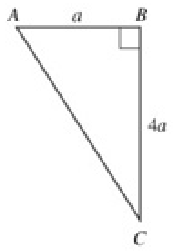
Question:

Find the position of the centre of mass of the following uniform triangular laminas:

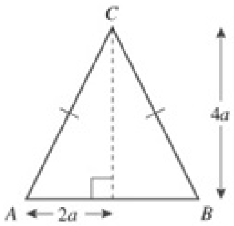
a



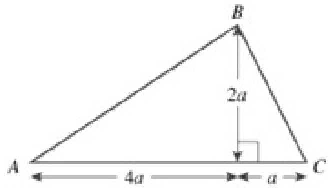
b



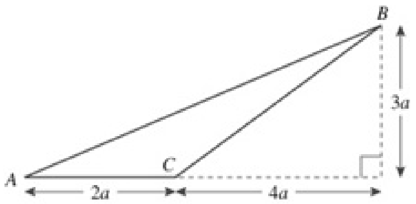
c



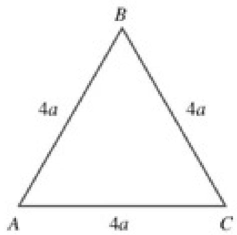
d



e

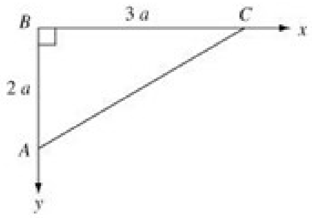


f



Solution:

a



We need to set up our own axes here.

Using the axes shown, B is (0, 0)
C is $(3a,\ 0)$ and A is $(0,\ 2a)$.

Centre of mass G is $\left(\frac{0+0+3a}{3}, \frac{2a+0+0}{3}\right)$

Use the fact that the centre of mass is at the 'mean point'.

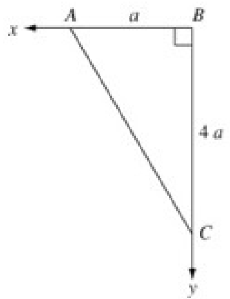
i.e. $\left(a, \frac{2a}{3}\right)$

Centre of mass is a distance a from AB and a distance $\frac{2a}{3}$ from BC.

State your answer carefully.

B is $(0,0)$
A is $(a,0)$
C is $(0,4a)$

b



Use the axes chosen (see the diagram).

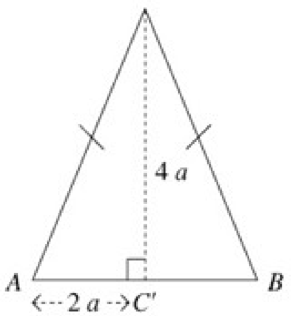
Centre of mass G is $\left(\frac{0+a+0}{3}, \frac{0+0+4a}{3}\right)$

i.e. $\left(\frac{a}{3}, \frac{4a}{3}\right)$

Centre of mass is a distance $\frac{a}{3}$ from BC and a distance $\frac{4a}{3}$ from AB.

Note that $\bar{x}$ gives the distance from the y-axis and $\bar{y}$ gives the distance from the x-axis.

c



Since $AC = BC$ i.e. the Δ is isosceles so $AB = 4a$.

Taking A as the origin with AB as the x-axis, the coordinates of A, B and C are (0, 0), (4a, 0) and (2a, 4a) respectively.

We need to set up our own axes here.

G is $\left(\frac{0+4a+2a}{3}, \frac{0+0+4a}{3}\right)$

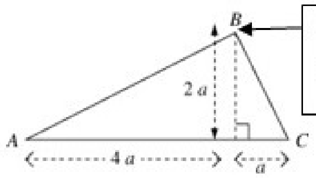
i.e. $\left(2a, \frac{4a}{3}\right)$

Take the mean of the vertices.

Note that we could have found G by using the *symmetry* of the Δ. G must lie on the axis of symmetry and since this line is also a median, G divides CC' in the ratio 2:1 i.e. it is $\frac{2}{3}$ of the way down the median from C.

This type of argument is perfectly acceptable when answering examination questions.

d



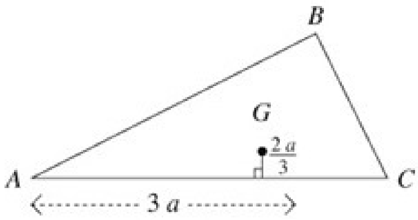
Again we need to set up some axes.

A is (0, 0); B is (4a, 2a); C is (5a, 0).

Then G is $\left(\frac{0+4a+5a}{3}, \frac{0+2a+0}{3}\right)$

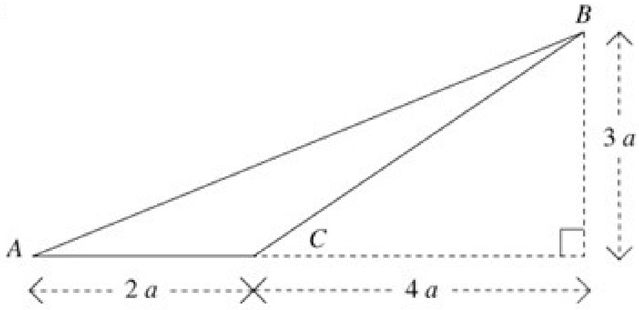
i.e. $\left(3a, \frac{2a}{3}\right)$

Take the mean of the vertices.



The diagram shows the position of G.

e



Take A as the origin with AC as the x-axis.

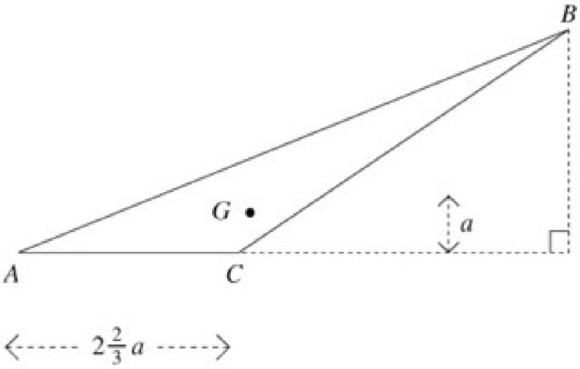
We need to set up axes.

The coordinates of A, B and C are (0, 0), (6a, 3a) and (2a, 0) respectively.

Then G is $\left(\frac{0+6a+2a}{3}, \frac{0+3a+0}{3}\right)$

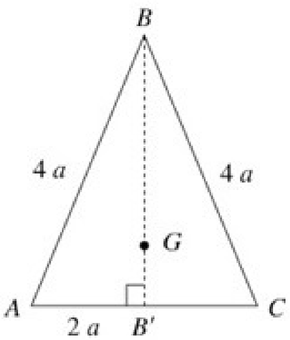
Take the mean point.

i.e. $\left(\frac{8a}{3}, a\right)$



The diagram shows the position of G, the centre of mass.

f



The ΔABC is equilateral, so G is on a line of symmetry, and divides it (BB') in ratio 2:1,

This type of argument is acceptable in an examination.

So,

$$BG = \frac{2}{3} \times 4a \sin 60°$$
$$= \frac{2}{3} \times 4a \times \frac{\sqrt{3}}{2}$$
$$= \frac{4a\sqrt{3}}{3}$$

$\left(GB' = \frac{2a\sqrt{3}}{3}\right)$

OR

If we take A as our origin, and the line AC as the x-axis, then A is (0, 0), C is (4a, 0) and B is $(2a, 4a \sin 60°)$

Set up axes.

so, G is $\left(\frac{0+4a+2a}{3}, \frac{0+0+4a\sin 60°}{3}\right)$

Take the mean point.

i.e. $(2a, \frac{2a\sqrt{3}}{3})$

Simplify.

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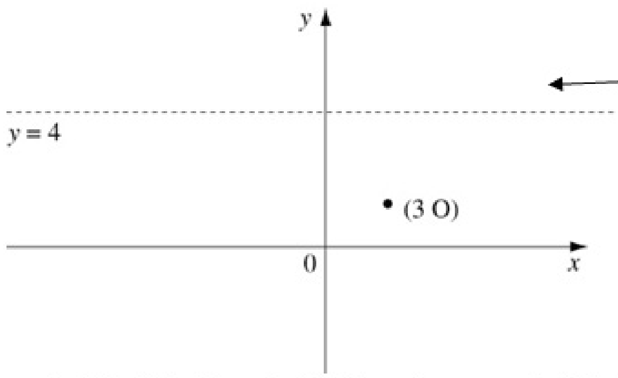
Exercise C, Question 5

Question:

A uniform triangular lamina is isosceles and has the line $y = 4$ as its axis of symmetry. One of the vertices of the triangle is the point (2, 1). Given that the x-coordinate of the centre of mass of the lamina is -3, find the coordinates of the other two vertices.

Solution:



Axis of symmetry $y = 4$

Draw a diagram showing all the information.

Let (2, 1) be the point A. Then since $y = 4$ is the line of symmetry, the point (2, 7) must be another vertex.

A is 3 units below the axis of symmetry, so the other after vertex must be 3 units above the axis of symmetry.

The third vertex must be on the line of symmetry, $(x, 4)$ say.
Then,

$$-3 = \frac{2+2+x}{3}$$

$$-13 = x$$

The mean of the x-coordinates must be -3.

The other two vertices are (2, 7) and $(-13, 4)$.

State the answer.

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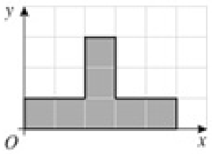
Exercise D, Question 1

Question:

<JN note: I've amended the question stem, as I'm assuming that each of these questions will appear separately in Solution bank.>

The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

1



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

$$5\begin{pmatrix}2\frac{1}{2}\\ \frac{1}{2}\end{pmatrix}+2\begin{pmatrix}2\frac{1}{2}\\ 2\end{pmatrix}=(5+2)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$$

Use $\Sigma m_i\mathbf{r}_i=\bar{\mathbf{r}}\,\Sigma m_i$.

Clearly, $\bar{x}=2\frac{1}{2}$

OR By symmetry, $\bar{x}=2\frac{1}{2}$

This is acceptable in an examination.

$$\frac{5}{2}+4=7\bar{y}$$

$$\frac{13}{14}=\bar{y}$$

Equate the y-components.

Centre of mass is $\left(2\frac{1}{2},\frac{13}{14}\right)$.

State your answer, using both coordinaters.

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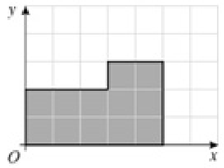
Exercise D, Question 2

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

$$6\begin{pmatrix}1\frac{1}{2}\\1\end{pmatrix}+6\begin{pmatrix}4\\1\frac{1}{2}\end{pmatrix}=(6+6)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

There are other ways of splitting the lamina up (*see below*).

$$\begin{pmatrix}9\\6\end{pmatrix}+\begin{pmatrix}24\\9\end{pmatrix}=12\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}33\\15\end{pmatrix}=12\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\frac{33}{12}=\frac{11}{4}=\bar{x}$$

$$\frac{15}{12}=\frac{5}{4}=\bar{y}$$

Equate **i** and **j** components.

Centre of mass is $\left(\frac{11}{4},\frac{5}{4}\right)$.

OR

$$10\begin{pmatrix}2\frac{1}{2}\\1\end{pmatrix}+2\begin{pmatrix}4\\2\frac{1}{2}\end{pmatrix}=(10+2)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

This splits the lamina differently.

$$\begin{pmatrix}25\\10\end{pmatrix}+\begin{pmatrix}8\\5\end{pmatrix}=12\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

As above, etc.

$$\begin{pmatrix}33\\15\end{pmatrix}=12\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

OR

$$15\begin{pmatrix}2\frac{1}{2}\\1\frac{1}{2}\end{pmatrix}-3\begin{pmatrix}1\frac{1}{2}\\2\frac{1}{2}\end{pmatrix}=(15-3)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

This uses 'rectangle – rectangle'.

$$\begin{pmatrix}37\frac{1}{2}\\22\frac{1}{2}\end{pmatrix}-\begin{pmatrix}4\frac{1}{2}\\7\frac{1}{2}\end{pmatrix}=12\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}33\\5\end{pmatrix}=12\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

As above.

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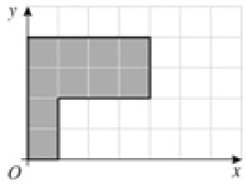
Exercise D, Question 3

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

$$16\begin{pmatrix}2\\2\end{pmatrix}-6\begin{pmatrix}2\frac{1}{2}\\1\end{pmatrix}=(16-6)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$ ← Use 'square – rectangle'.

$$\begin{pmatrix}32\\32\end{pmatrix}-\begin{pmatrix}15\\6\end{pmatrix}=10\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$ ← Simplify.

$$\begin{pmatrix}17\\26\end{pmatrix}=10\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$ ← Equate **i** and **j** components.

$$\begin{pmatrix}1.7\\2.6\end{pmatrix}=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Centre of mass is (1.7, 2.6). ← State your answer.

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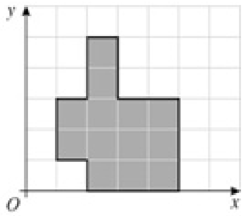
Exercise D, Question 4

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

$$2\begin{pmatrix}1\frac{1}{2}\\2\end{pmatrix}+2\begin{pmatrix}2\frac{1}{2}\\4\end{pmatrix}+9\begin{pmatrix}3\frac{1}{2}\\1\frac{1}{2}\end{pmatrix}=(2+2+9)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i\mathbf{r}_i=\bar{\mathbf{r}}\Sigma m_i$.

$$\begin{pmatrix}3\\4\end{pmatrix}+\begin{pmatrix}5\\8\end{pmatrix}+\begin{pmatrix}31\frac{1}{2}\\13\frac{1}{2}\end{pmatrix}=13\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Simplify.

$$\begin{pmatrix}39\frac{1}{2}\\25\frac{1}{2}\end{pmatrix}=13$$

Collect terms.

$$\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}\frac{79}{26}=\bar{x};\frac{51}{26}=\bar{y}$$

Centre of mass is $\left(\frac{79}{26},\frac{51}{26}\right)$.

You could use decimals.

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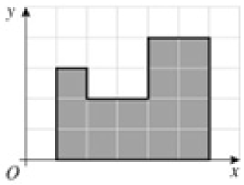
Exercise D, Question 5

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

$$1\begin{pmatrix}1\frac{1}{2}\\2\frac{1}{2}\end{pmatrix}+10\begin{pmatrix}3\frac{1}{2}\\1\end{pmatrix}+4\begin{pmatrix}5\\3\end{pmatrix}=(1+10+4)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i \mathbf{r}_i = \bar{\mathbf{r}}\,\Sigma m_i$.

$$\begin{pmatrix}1\frac{1}{2}\\2\frac{1}{2}\end{pmatrix}+\begin{pmatrix}35\\10\end{pmatrix}+\begin{pmatrix}20\\12\end{pmatrix}=15\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Multiply out.

$$\begin{pmatrix}56\frac{1}{2}\\24\frac{1}{2}\end{pmatrix}=15\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Collect terms.

$$\frac{113}{30}=\bar{x};\ \frac{49}{30}=\bar{y}$$

Centre of mass is $\left(\frac{113}{30},\frac{49}{30}\right)$.

Check that your answer looks reasonable.

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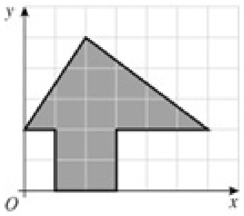
Exercise D, Question 6

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

$$\frac{1}{3}\left\{\begin{pmatrix}0\\2\end{pmatrix}+\begin{pmatrix}6\\2\end{pmatrix}+\begin{pmatrix}2\\5\end{pmatrix}\right\}=\frac{1}{3}\begin{pmatrix}8\\9\end{pmatrix}=\begin{pmatrix}\frac{8}{3}\\3\end{pmatrix}$$

The centre of mass of the triangle can be found by taking the average (mean) of its vertices.

Area of the triangle $=\frac{1}{2}\times 6\times 3=9$

$$4\begin{pmatrix}2\\1\end{pmatrix}+9\begin{pmatrix}\frac{8}{3}\\3\end{pmatrix}=13\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i\mathbf{r}_i=\bar{\mathbf{r}}\Sigma m_i$.

$$\begin{pmatrix}8\\4\end{pmatrix}+\begin{pmatrix}24\\27\end{pmatrix}=13\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Multiply out.

$$\begin{pmatrix}32\\31\end{pmatrix}=13\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Centre of mass is $\left(\frac{32}{13},\frac{31}{13}\right)$.

Improper fractions are acceptable.

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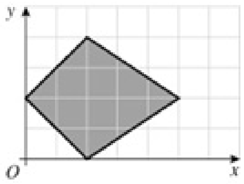
Exercise D, Question 7

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

By symmetry, $\bar{y} = 2$

Area of L.H. triangle $= \frac{1}{2} \times 4 \times 2 = 4$

Area of R.H. triangle $= \frac{1}{2} \times 4 \times 3 = 6$

$\frac{1}{2} \times \text{base} \times \text{height}$

x-coordinate of centre of mass of L.H. triangle $= \frac{1}{3}(0+2+2) = \frac{4}{3}$

Take the mean of the x-coordinates of its vertices.

x-coordinate of centre of mass of R.H. triangle $= \frac{1}{3}(2+2+5) = 3$

As before.

So,

$$\left(4 \times \frac{4}{3}\right) + (6 \times 3) = 10\bar{x}$$

Use the x-coordinates only.

$$\frac{16}{3} + 18 = 10\bar{x}$$

$$\frac{70}{3} = 10\bar{x}$$

$$\frac{7}{3} = \bar{x}$$

Centre of mass is $\left(\frac{7}{3}, 2\right)$.

Check that your answer looks sensible.

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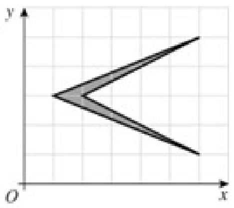
Exercise D, Question 8

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

By symmetry, $\bar{y} = 3$

Area of each triangle $= \frac{1}{2} \times 1 \times 2 = 1$ ← Split the shape in to two triangles.

x-coordinate of the centre of mass of each triangle is $\frac{1}{3}(1+2+6) = \frac{9}{3} = 3$ ← Find the mean of the x-coordinates of the vertices.

So,

$$(1\times3)+(1\times3) = (1+1)\times\bar{x}$$
$$3+3 = 2\bar{x}$$
$$3 = \bar{x}$$

← Use $\Sigma m_i x_i = \bar{x}\Sigma m_i$.

Centre of mass is (3, 3).

OR Area of large triangle $= \frac{1}{2} \times 4 \times 5 = 10$ ← Treat the lamina as a large triangle — a small triangle.

Area of small triangle $= \frac{1}{2} \times 4 \times 4 = 8$

x-coordinate of centre of mass of large triangle ← Take the mean of the corners.

$= \frac{1}{3}(1+6+6) = \frac{13}{3}$

x-coordinate of centre of mass of small triangle

$= \frac{1}{3}(2+6+6) = \frac{14}{3}$

So,

$$\left(10\times\frac{13}{3}\right)-\left(8\times\frac{14}{3}\right) = (10-8)\quad\bar{x}$$
$$\frac{130}{3}-\frac{112}{3} = 2\bar{x}$$
$$\frac{18}{3} = 2\bar{x}$$
$$3 = \bar{x}$$

← As before.

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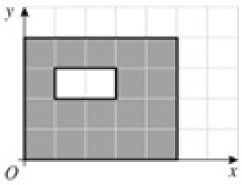
Exercise D, Question 9

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

$$20\begin{pmatrix}2\frac{1}{2}\\2\end{pmatrix}-2\begin{pmatrix}2\\2\frac{1}{2}\end{pmatrix}=(20-2)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}50\\40\end{pmatrix}-\begin{pmatrix}4\\5\end{pmatrix}=18\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}46\\35\end{pmatrix}=18\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Centre of mass is $\left(\frac{23}{9},\frac{35}{18}\right)$.

It's much easier to treat this lamina as a rectangle with a rectangle removed.

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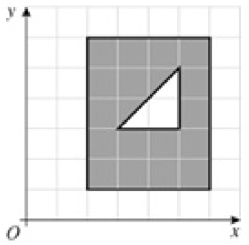
Exercise D, Question 10

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

$$\text{Centre of mass of triangle} = \frac{1}{3}\left\{\begin{pmatrix}3\\3\end{pmatrix}+\begin{pmatrix}5\\3\end{pmatrix}+\begin{pmatrix}5\\5\end{pmatrix}\right\}$$

$$= \frac{1}{3}\begin{pmatrix}13\\11\end{pmatrix}$$

$$= \begin{pmatrix}\frac{13}{3}\\\frac{11}{3}\end{pmatrix}$$

Take the mean of the vertices of the triangle.

$$20\begin{pmatrix}4\\3\frac{1}{2}\end{pmatrix} - 2\begin{pmatrix}\frac{13}{3}\\\frac{11}{3}\end{pmatrix} = (20-2)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

It's much easier to treat the lamina as (a rectangle – a triangle).

$$\begin{pmatrix}80\\70\end{pmatrix} - \begin{pmatrix}\frac{26}{3}\\\frac{22}{3}\end{pmatrix} = 18\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}\frac{214}{3}\\\frac{188}{3}\end{pmatrix} = 18\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Simplify and collect terms.

So, centre of mass is $\left(\frac{107}{27}, \frac{94}{27}\right)$.

State the answer.

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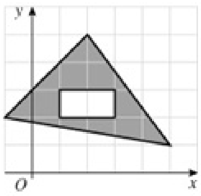
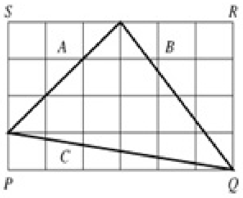
Exercise D, Question 11

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:



$$\text{Area of triangle} = \text{Area of } PQRS - \text{Area of } (A+B+C)$$
$$= (6\times4)-(\tfrac{1}{2}\times3\times3+\tfrac{1}{2}\times3\times4+\tfrac{1}{2}\times1\times6)$$
$$= 24-\tfrac{9}{2}-6-3$$
$$= \tfrac{21}{2}$$

This is the easiest way of finding the area of the triangle.

$$\text{Centre of mass of triangle} = \frac{1}{3}\left\{\begin{pmatrix}-1\\3\end{pmatrix}+\begin{pmatrix}5\\2\end{pmatrix}+\begin{pmatrix}2\\6\end{pmatrix}\right\}$$
$$= \frac{1}{3}\begin{pmatrix}6\\11\end{pmatrix}$$
$$= \begin{pmatrix}2\\ \frac{11}{3}\end{pmatrix}$$

Take the mean of the vertices of the triangle.

So,

$$\frac{21}{2}\begin{pmatrix}2\\ \frac{11}{3}\end{pmatrix}-2\begin{pmatrix}2\\ 3\frac{1}{2}\end{pmatrix}=\left(\frac{21}{2}-2\right)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$$
$$\begin{pmatrix}21\\ \frac{77}{2}\end{pmatrix}-\begin{pmatrix}4\\7\end{pmatrix}=\frac{17}{2}\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$$
$$\begin{pmatrix}17\\ \frac{63}{2}\end{pmatrix}=\frac{17}{2}\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}\Rightarrow\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}2\\ \frac{63}{17}\end{pmatrix}$$

This is the only viable method here.

Centre of mass is $(2, \frac{63}{17})$.

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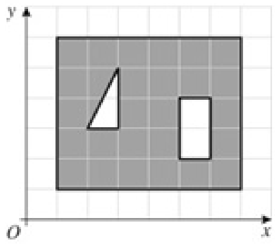
Exercise D, Question 12

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

Centre of mass of triangle $= \frac{1}{3}\left\{\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}3\\3\end{pmatrix}+\begin{pmatrix}3\\5\end{pmatrix}\right\}$

$= \begin{pmatrix}\frac{8}{3}\\\frac{11}{3}\end{pmatrix}$

Find the 'average' of the vertices.

$$30\begin{pmatrix}4\\3\frac{1}{2}\end{pmatrix}-1\begin{pmatrix}\frac{8}{3}\\\frac{11}{3}\end{pmatrix}-2\begin{pmatrix}5\frac{1}{2}\\3\end{pmatrix}=(30-1-2)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

This is the only sensible method here.

$$\begin{pmatrix}120\\105\end{pmatrix}-\begin{pmatrix}\frac{8}{3}\\\frac{11}{3}\end{pmatrix}-\begin{pmatrix}11\\6\end{pmatrix}=27\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}\frac{319}{3}\\\frac{286}{3}\end{pmatrix}=27\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Centre of mass is $\left(\frac{319}{81},\frac{286}{81}\right)$.

Check that your answer looks reasonable for the lamina in question.

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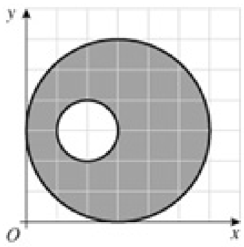
Exercise D, Question 13

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

$$\pi \times 3^2 \begin{pmatrix} 3 \\ 3 \end{pmatrix} - \pi \times 1^2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} = (\pi \times 3^2 - \pi \times 1^2) \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

Cancel the π s.

$$\begin{pmatrix} 27 \\ 27 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 8 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

This is the only method possible for this lamina.

$$\begin{pmatrix} 25 \\ 24 \end{pmatrix} = 8 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\begin{pmatrix} \frac{25}{8} \\ 3 \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

Note that we could have said $\bar{y} = 3$, by symmetry.

This can be used in the examination.

Centre of mass is $\left(\frac{25}{8}, 3\right)$.

State your answer.

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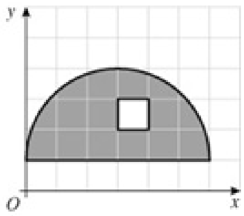
Exercise D, Question 14

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

Centre of mass of semi-circle $= \begin{pmatrix} 3 \\ 1+\frac{4\times 3}{3\pi} \end{pmatrix}$

$= \begin{pmatrix} 3 \\ \frac{\pi+4}{\pi} \end{pmatrix}$

The position of the centre of mass of a semi-circular lamina is given in the formulae booklet.

$$\frac{\pi\times 3^2}{2}\begin{pmatrix} 3 \\ \frac{\pi+4}{\pi} \end{pmatrix} - 1\begin{pmatrix} 3\frac{1}{2} \\ 2\frac{1}{2} \end{pmatrix} = \left(\frac{\pi\times 3^2}{2} - 1\right)\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

Simplify.

$$\begin{pmatrix} \frac{27\pi}{2} \\ \frac{9}{2}(\pi+4) \end{pmatrix} - \begin{pmatrix} 3\frac{1}{2} \\ 2\frac{1}{2} \end{pmatrix} = \left(\frac{9\pi}{2} - 1\right)\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\left(\begin{pmatrix} \frac{27\pi-7}{2} \\ \frac{9\pi+31}{2} \end{pmatrix}\right) = \frac{9\pi-2}{2}\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

So, $\bar{x} = \frac{27\pi-7}{9\pi-2}$; $\bar{y} = \frac{9\pi+31}{9\pi-2}$

Decimal answers would, of course, be acceptable.

Centre of mass is $\left(\frac{27\pi-7}{9\pi-2}, \frac{9\pi+31}{9\pi-2}\right)$.

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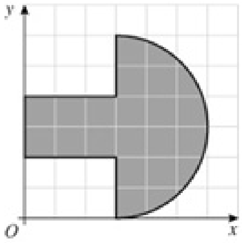
Exercise D, Question 15

Question:



The diagram shows a uniform plane figure drawn on a grid of unit squares. Find the coordinates of the centre of mass.

Solution:

By symmetry, $\bar{y} = 3$

x-coordinate of centre of mass of semi-circle is $3 + \frac{4 \times 3}{3\pi} = \frac{3\pi + 4}{\pi}$

$$(6 \times 1\tfrac{1}{2}) + \frac{\pi \times 3^2}{2}\left(\frac{3\pi + 4}{\pi}\right) = (6 + \frac{\pi \times 3^2}{2})\bar{x}$$

Use $\Sigma m_i x_i = \bar{x}\Sigma m_i$.

$$9 + \frac{9}{2}(3\pi + 4) = (6 + \frac{9\pi}{2})\bar{x}$$

$$18 + 27\pi + 36 = (12 + 9\pi)\bar{x}$$

Multiply through by 2 to clear the fractions.

$$\frac{18 + 9\pi}{4 + 3\pi} = \bar{x}$$

Centre of mass is $\left(\frac{18+9\pi}{4+3\pi}, 3\right)$.

Divide by 3.

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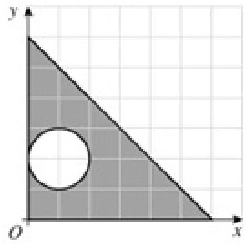
Exercise D, Question 16

Question:



Solution:

Centre of mass of triangle

$$= \frac{1}{3}\left\{\begin{pmatrix}0\\0\end{pmatrix} + \begin{pmatrix}6\\0\end{pmatrix} + \begin{pmatrix}0\\6\end{pmatrix}\right\} = \begin{pmatrix}2\\2\end{pmatrix}$$

This is two thirds of the way along the median through O.

$$\left(\frac{1}{2}\times 6\times 6\right)\begin{pmatrix}2\\2\end{pmatrix} - \pi\times 1^2\begin{pmatrix}1\\2\end{pmatrix} = (18-\pi)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i\mathbf{r}_i = \bar{\mathbf{r}}\,\Sigma m_i$.

$$\begin{pmatrix}36\\36\end{pmatrix} - \begin{pmatrix}\pi\\2\pi\end{pmatrix} = (18-\pi)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\bar{x} = \frac{36-\pi}{18-\pi};\ \bar{y} = \frac{36-2\pi}{18-\pi}$$

Centre of mass is $\left(\frac{36-\pi}{18-\pi}, \frac{36-2\pi}{18-\pi}\right)$.

Check that your answer is reasonable for the lamina in question.

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Exercise E, Question 1

Question:

Find the coordinates of the centres of mass of the shapes shown in Exercise 2D questions **1** to **5**, regarding them as uniform plane wire frameworks.

Solution:

By symmetry, $\bar{x} = 2\frac{1}{2}$

Always use the symmetry if possible.

$$(5\times 0) + (1\times\frac{1}{2}) + (2\times 1) + (2\times 2) + (1\times 3) + (2\times 2) + (2\times 1) + (1\times\frac{1}{2})$$
$$= (5+1+2+2+1+2+2+1)\bar{y}$$

Use $\Sigma m_i y_i = \bar{y}\Sigma m_i$.

$$\frac{1}{2} + 2 + 4 + 3 + 4 + 2 + \frac{1}{2} = 16\bar{y}$$

The framework has 8 sides.

$$16 = 16\bar{y}$$
$$1 = \bar{y}$$

Simplify.

$\therefore$ Centre of mass is $\left(2\frac{1}{2}, 1\right)$.

Note that it was $(2\frac{1}{2}, \frac{13}{14})$ when regarded as a lamina.

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Exercise E, Question 2

Question:

Find the coordinates of the centres of mass of the shapes shown in Exercise 2D questions **1** to **5**, regarding them as uniform plane wire frameworks.

Solution:

$$5\begin{pmatrix}2\frac{1}{2}\\0\end{pmatrix}+3\begin{pmatrix}5\\1\frac{1}{2}\end{pmatrix}+2\begin{pmatrix}4\\3\end{pmatrix}+1\begin{pmatrix}3\\2\frac{1}{2}\end{pmatrix}+3\begin{pmatrix}1\frac{1}{2}\\2\end{pmatrix}+2\begin{pmatrix}0\\1\end{pmatrix}$$

Use $\Sigma m_i \mathbf{r}_i = \bar{\mathbf{r}}\,\Sigma m_i$.

$$=(5+3+2+1+3+2)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

The framework has 6 sides.

$$\begin{pmatrix}12\frac{1}{2}\\0\end{pmatrix}+\begin{pmatrix}15\\4\frac{1}{2}\end{pmatrix}+\begin{pmatrix}8\\6\end{pmatrix}+\begin{pmatrix}3\\2\frac{1}{2}\end{pmatrix}+\begin{pmatrix}4\frac{1}{2}\\6\end{pmatrix}+\begin{pmatrix}0\\2\end{pmatrix}=16\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}43\\21\end{pmatrix}=16\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Simplify and collect the terms.

$\therefore$ Centre of mass is $\left(\frac{43}{16}, \frac{21}{16}\right)$.

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Exercise E, Question 3

Question:

Find the coordinates of the centres of mass of the shapes shown in Exercise 2D questions **1** to **5**, regarding them as uniform plane wire frameworks.

Solution:

$$1\begin{pmatrix}\frac{1}{2}\\0\end{pmatrix}+2\begin{pmatrix}1\\1\end{pmatrix}+3\begin{pmatrix}2\frac{1}{2}\\2\end{pmatrix}+2\begin{pmatrix}4\\3\end{pmatrix}+4\begin{pmatrix}2\\4\end{pmatrix}+4\begin{pmatrix}0\\2\end{pmatrix}$$

$$=(1+2+3+2+4+4)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i \mathbf{r}_i = \bar{\mathbf{r}}\,\Sigma m_i$.

The framework has 6 sides.

$$\begin{pmatrix}\frac{1}{2}\\0\end{pmatrix}+\begin{pmatrix}2\\2\end{pmatrix}+\begin{pmatrix}7\frac{1}{2}\\6\end{pmatrix}+\begin{pmatrix}8\\6\end{pmatrix}+\begin{pmatrix}8\\16\end{pmatrix}+\begin{pmatrix}0\\8\end{pmatrix}=16\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}26\\38\end{pmatrix}=16\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$\therefore$ Centre of mass is $\left(\frac{13}{8},\frac{19}{8}\right)$.

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Exercise E, Question 4

Question:

Find the coordinates of the centres of mass of the shapes shown in Exercise 2D questions **1** to **5**, regarding them as uniform plane wire frameworks.

Solution:

$$3\begin{pmatrix}3\frac{1}{2}\\0\end{pmatrix}+3\begin{pmatrix}5\\1\frac{1}{2}\end{pmatrix}+2\begin{pmatrix}4\\3\end{pmatrix}+2\begin{pmatrix}3\\4\end{pmatrix}+1\begin{pmatrix}2\frac{1}{2}\\5\end{pmatrix}+2\begin{pmatrix}2\\4\end{pmatrix}+1\begin{pmatrix}1\frac{1}{2}\\3\end{pmatrix}$$

The shape has 10 sides.

$$+2\begin{pmatrix}1\\2\end{pmatrix}+1\begin{pmatrix}1\frac{1}{2}\\1\end{pmatrix}+1\begin{pmatrix}2\\\frac{1}{2}\end{pmatrix}=(3+3+2+2+1+2+1+2+1+1)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}10\frac{1}{2}\\0\end{pmatrix}+\begin{pmatrix}15\\4\frac{1}{2}\end{pmatrix}+\begin{pmatrix}8\\6\end{pmatrix}+\begin{pmatrix}6\\8\end{pmatrix}+\begin{pmatrix}2\frac{1}{2}\\5\end{pmatrix}+\begin{pmatrix}4\\8\end{pmatrix}+\begin{pmatrix}1\frac{1}{2}\\3\end{pmatrix}+\begin{pmatrix}2\\4\end{pmatrix}+\begin{pmatrix}1\frac{1}{2}\\1\end{pmatrix}+\begin{pmatrix}2\\\frac{1}{2}\end{pmatrix}$$

Simplify.

$$=18\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}53\\40\end{pmatrix}=18\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Centre of mass is $\left(\frac{53}{18},\frac{20}{9}\right)$.

Check the sense of your answer.

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Exercise E, Question 5

Question:

Find the coordinates of the centres of mass of the shapes shown in Exercise 2D questions **1** to **5**, regarding them as uniform plane wire frameworks.

Solution:

$$5\begin{pmatrix}3\frac{1}{2}\\0\end{pmatrix}+4\begin{pmatrix}6\\2\end{pmatrix}+2\begin{pmatrix}5\\4\end{pmatrix}+2\begin{pmatrix}4\\3\end{pmatrix}+2\begin{pmatrix}3\\2\end{pmatrix}+1\begin{pmatrix}2\\2\frac{1}{2}\end{pmatrix}+1\begin{pmatrix}1\frac{1}{2}\\3\end{pmatrix}+3\begin{pmatrix}1\\1\frac{1}{2}\end{pmatrix}$$

The framework has 8 sides.

$$=(5+4+2+2+2+1+1+3)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i\mathbf{r}_i=\bar{\mathbf{r}}\Sigma m_i$.

$$\begin{pmatrix}17\frac{1}{2}\\0\end{pmatrix}+\begin{pmatrix}24\\8\end{pmatrix}+\begin{pmatrix}10\\8\end{pmatrix}+\begin{pmatrix}8\\6\end{pmatrix}+\begin{pmatrix}6\\4\end{pmatrix}+\begin{pmatrix}2\\2\frac{1}{2}\end{pmatrix}+\begin{pmatrix}1\frac{1}{2}\\3\end{pmatrix}+\begin{pmatrix}3\\4\frac{1}{2}\end{pmatrix}=20\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}72\\36\end{pmatrix}=20\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Centre of mass is $\left(\frac{18}{5},\frac{9}{5}\right)$.

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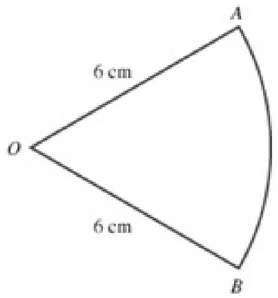
Exercise E, Question 6

Question:

Find the position of the centre of mass of the framework shown in the diagram which is formed by bending a uniform piece of wire of total length $(12+2\pi)$ cm to form a sector of a circle, centre O, radius 6 cm.



Solution:

$AB=(12+2\pi)-12=2\pi$

Let $A\hat{O}B=\theta$ (radians) $(=2\alpha)$

Then

$6\theta=2\pi$ — Use $S=r\theta$.

$\theta=\frac{2\pi}{6}=\frac{\pi}{3}=60^\circ$

Distance of G from O $=\bar{x}$ say.

> $\frac{r\sin\alpha}{\alpha}$; α in RADIANS. from the formula booklet.

Then,

$$\left(6\times3\cos30^\circ\right)\times2+2\pi\times\frac{6\sin\frac{\pi}{6}}{\frac{\pi}{6}}=\bar{x}(12+2\pi)$$

> Use $\Sigma m_i x_i=\bar{x}\Sigma m_i$.

$18\sqrt{3}+36=\bar{x}(12+2\pi)$

> $\cos30^\circ=\frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6}=\frac{1}{2}$

$\therefore\ \bar{x}=\frac{18\sqrt{3}+36}{12+2\pi}$

$=\frac{9(\sqrt{3}+2)}{6+\pi}$

> Simplify by dividing top and bottom by 2.

Centre of mass is on line of symmetry through O, and a distance of $\frac{9(\sqrt{3}+2)}{6+\pi}$ from O.

> State your answer.

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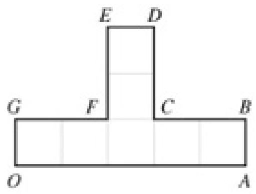
Exercise F, Question 1

Question:

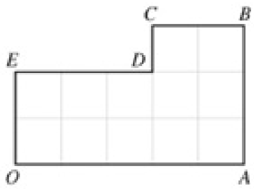
a The lamina from question 1 in Exercise 2D is shown.

The lamina is freely suspended from the point O and hangs in equilibrium.

Find the angle between OA and the downward vertical.

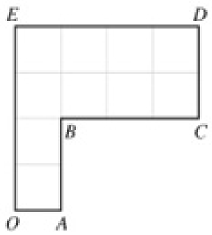


b The lamina from question 2 in Exercise 2D is shown below.



The lamina is freely suspended from the point O and hangs in equilibrium. Find the angle between OA and the downward vertical.

c The lamina from question 3 in Exercise 2D is shown below.



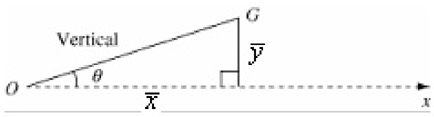
The lamina is freely suspended from the point O and hangs in equilibrium. Find the angle between OA and the downward vertical.

Solution:

a From question 1**a** in Exercise 2D,

$\bar{x} = 2\frac{1}{2}; \bar{y} = \frac{13}{14}$

Vertical



In equilibrium, G will be vertically below O i.e. OG is the vertical.

$$\tan\theta = \frac{\bar{y}}{\bar{x}} = \frac{\frac{13}{14}}{\frac{5}{2}}$$

$$= \frac{13}{14} \times \frac{2}{5} = \frac{13}{35}$$

$$\theta = \tan^{-1}\left(\frac{13}{35}\right) = 20.4^\circ \text{ (3 s.f.)}$$

b From question 1**b** in Exercise 2D,

$\bar{x} = \frac{11}{4}; \bar{y} = \frac{5}{4}$

As above, $\tan\theta = \frac{\bar{y}}{\bar{x}} = \frac{\frac{5}{4}}{\frac{11}{4}}$

i.e. $\tan\theta = \frac{5}{11}$

$$\theta = \tan^{-1}\left(\frac{5}{11}\right) = 24.4^\circ \text{ (3 s.f.)}$$

c From question 1**c** in Exercise 2D,

$\bar{x} = 1.7; \bar{y} = 2.6$

$$\tan\theta = \frac{2.6}{1.7} = \frac{26}{17}$$

$$\theta = \tan^{-1}\left(\frac{26}{17}\right) = 56.8^\circ \text{ (3 s.f.)}$$

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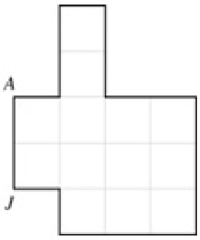
Exercise F, Question 2

Question:

The lamina from question 4 in Exercise 2D is shown below.

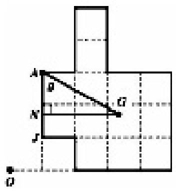


The lamina is freely suspended from the point A and hangs in equilibrium. Find the angle between AJ and the downward vertical.

Solution:

From question 1**a** in Exercise 2D,
$\bar{x} = \frac{79}{26}; \bar{y} = \frac{51}{26}$

These are the coordinates of the centre of mass, G, referred to O as origin.



A is the point of suspension.

G is $\left(\frac{79}{26}, \frac{51}{26}\right)$.

When the lamina hangs in equilibrium from A, AG will be the downward vertical.
Let N be the point on AJ such that GN is perpendicular to AJ.
Then $N\hat{A}G = \theta$ is the required angle.

See diagram.

$$\tan\theta = \frac{GN}{AN} = \frac{\bar{x}-1}{3-\bar{y}}$$

Since A is the point (1, 3).

$$= \frac{\frac{79}{26}-1}{3-\frac{51}{26}} = \frac{79-26}{78-51}$$

Multiply top and bottom by 26.

$$= \frac{53}{27} \Rightarrow \theta = 63.0° \text{ (3 s.f.)}$$

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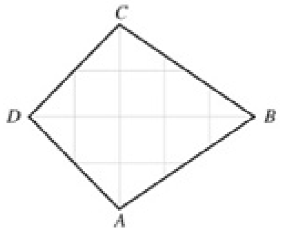
Exercise F, Question 3

Question:

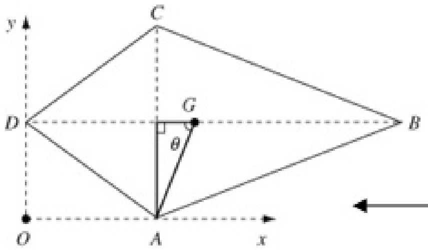
The lamina from question 7 in Exercise 2D is shown below.



The lamina is free to rotate about a fixed smooth horizontal axis, perpendicular to the plane of the lamina, passing through the point A.

Find the angle between AC and the horizontal.

Solution:



G, the centre of mass has coordinates $\left(\frac{7}{3}, 2\right)$ taking O as origin.

Since AG will be *vertical* in equilibrium, the angle between AC and the *horizontal* will be θ.

θ is the required angle.

$$\tan\theta = \frac{2}{\frac{7}{3}-2} = \frac{6}{7-6} = 6$$

Multiply top and bottom by 3 to clear fractions.

$$\theta = 80.5^\circ \text{ (3 s.f.)}$$

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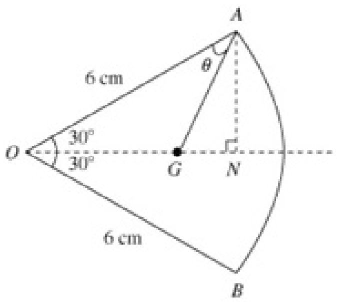
Exercise F, Question 4

Question:

The framework in question 6, Exercise 2E is freely suspended from the point A and allowed to hang in equilibrium. Find the angle between OA and the downward vertical.

Solution:



G is the centre of mass if the framework

$$OG = \bar{x} = \frac{9(\sqrt{3}+2)}{(6+\pi)}$$

G is on the line of symmetry.

(see question 2 in Exercise 2E)

AG will be vertical, when the framework hangs in equilibrium.

θ (see diagram) is the required angle.

$\theta = 60° - G\hat{A}N$

$$\tan G\hat{A}N = \frac{GN}{AN} = \frac{6\cos 30° - \bar{x}}{6\sin 30°}$$

$$= \frac{3\sqrt{3} - \bar{x}}{3}$$

$$= \sqrt{3} - \frac{3(\sqrt{3}+2)}{6+\pi}$$

So, $G\hat{A}N = 26.898°\ldots$

So, $\theta = 33.1°$ (3 s.f.)

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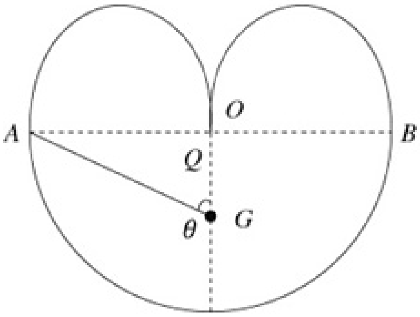
Exercise F, Question 5

Question:

The shape in question 7, Exercise 2E is freely suspended from the point A and allowed to hang in equilibrium. Find the angle between OA and the horizontal.

Solution:



$OG = \frac{3}{2\pi}$, where G is the centre of mass so, AG will be vertical in equilibrium.

See question 3 in Exercise 2E.

Required angle is $A\hat{G}O = \theta$

Since the angle with the horizontal will be $90° -$ angle with the vertical.

$$\tan\theta = \frac{AO}{OG} = \frac{3}{\frac{3}{2\pi}} = 2\pi$$

$$\theta = \tan^{-1}(2\pi) = 81.0° \text{ (3 s.f.)}$$

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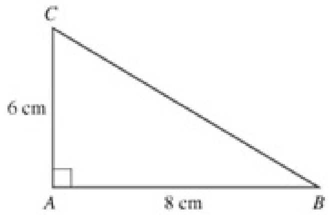
Exercise F, Question 6

Question:

The uniform triangular lamina ABC shown below is placed on a rough plane inclined at an angle α to the horizontal.



The edge AB is in contact with the plane, with A below B.

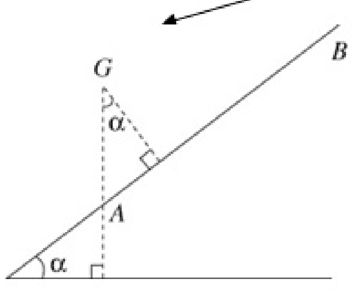
Given that the lamina is on the point of toppling about A, find the value of α.

Solution:

G, the centre of mass of the lamina, has position vector $\frac{1}{3}\left\{\begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}8\\0\end{pmatrix}+\begin{pmatrix}0\\6\end{pmatrix}\right\}$ referred to axes, AB

and AC respectively. i.e.: $\begin{pmatrix}\frac{8}{3}\\2\end{pmatrix}$

G will be vertically above A when the lamina is about to topple.



$$\tan\alpha = \frac{\frac{8}{3}}{2} = \frac{4}{3}$$

$$\alpha = 53.1°$$

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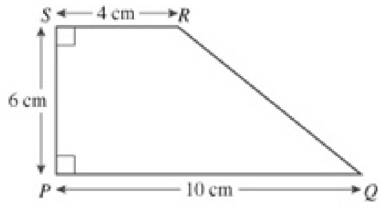
Exercise F, Question 7

Question:

$PQRS$ is a uniform lamina.

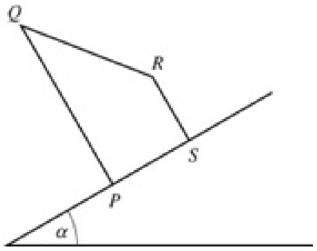


a Find the distance of the centre of mass of the lamina from

i PS

ii PQ.

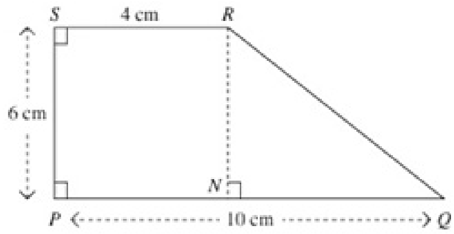
b The diagram shows the lamina on a rough inclined plane of angle α.



Given that the lamina is about to topple about the point P, find the value of α, giving your answer to 3 s.f.

Solution:

a



Centre of mass of ΔRNQ has position vector

$$\frac{1}{3}\left\{\begin{pmatrix}4\\0\end{pmatrix}+\begin{pmatrix}10\\0\end{pmatrix}+\begin{pmatrix}4\\6\end{pmatrix}\right\}=\begin{pmatrix}6\\2\end{pmatrix}$$

Taking PQ and PS as axes.

$$24\begin{pmatrix}2\\3\end{pmatrix}+18\begin{pmatrix}6\\2\end{pmatrix}=42\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i\mathbf{r}_i = \bar{\mathbf{r}}\Sigma m_i$.

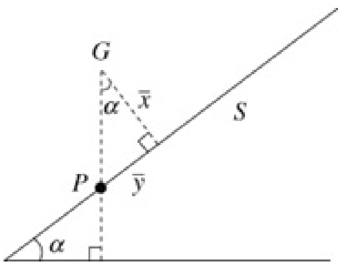
$$\begin{pmatrix}48\\72\end{pmatrix}+\begin{pmatrix}108\\36\end{pmatrix}=42\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}156\\108\end{pmatrix}=42\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Simplify.

$$\begin{pmatrix}\frac{26}{7}\\\frac{18}{7}\end{pmatrix}=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

b



G will be above P, on the point of toppling.

$$\tan\alpha = \frac{\bar{y}}{\bar{x}}$$

$$= \frac{\frac{18}{7}}{\frac{26}{7}}$$

$$= \frac{9}{13} = 0.6924$$

$$\alpha = 34.7^\circ$$

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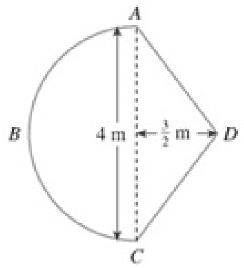
Exercise G, Question 1

Question:

The diagram shows a uniform lamina consisting of a semi-circle joined to a triangle ADC.



The sides AD and DC are equal.

a Find the distance of the centre of mass of the lamina from AC.

The lamina is freely suspended from A and hangs at rest.

b Find, to the nearest degree, the angle between AC and the vertical.

The mass of the lamina is M. A particle P of mass kM is attached to the lamina at D. When suspended from A, the lamina now hangs with its axis of symmetry, BD, horizontal.

c Find, to 3 s.f., the value of k.

Solution:

a

$$\frac{\pi\times2^2}{2}\times\left(\frac{4\times2}{3\pi}\right)+2\times\frac{3}{2}\times\left(-\frac{1}{2}\right)=\left(\frac{\pi\times2^2}{2}+2\times\frac{3}{2}\right)\bar{x}$$

$$\frac{16}{3}-\frac{3}{2}=(2\pi+3)\bar{x}$$

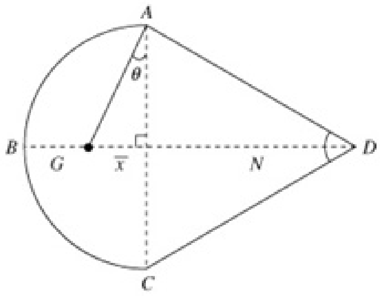
$$\frac{23}{6(2\pi+3)}=\bar{x}$$

Use $\Sigma m_i x_i = \bar{x}\Sigma m_i$ taking AC as the y-axis.

0.413m (3 s.f.)

A decimal answer is acceptable.

b



G is the centre of mass

G will be on the line of symmetry.

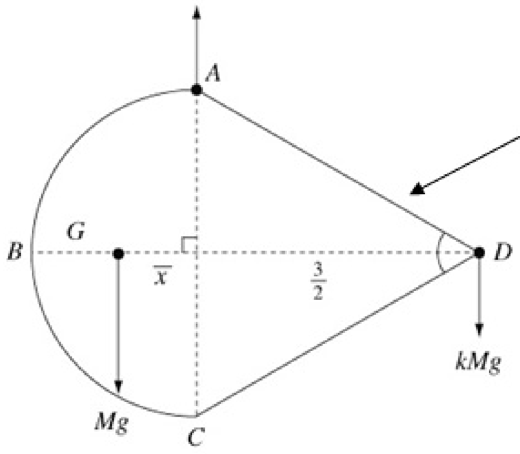
θ is the required angle

In equilibrium, AG will be vertical.

$$\tan\theta = \frac{\bar{x}}{2}=\frac{23}{12(2\pi+3)}$$

$\theta = 13°$ (nearest degree)

c



Draw a diagram showing all the forces.

M(A),

$$Mg\bar{x} = kMg\times\frac{3}{2}$$

$$\Rightarrow k = \frac{2}{3}\times\frac{23}{6(2\pi+3)}$$

$$= \frac{23}{9(2\pi+3)} = 0.275 \text{ (3 s.f.)}$$

Taking moments about A means we don't need to know the force A.

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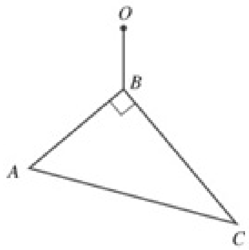
Exercise G, Question 2

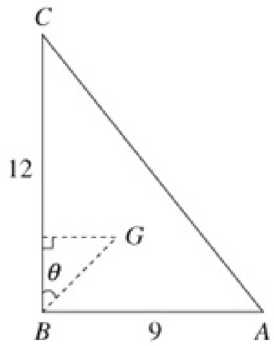
Question:

A uniform triangular lamina ABC is in equilibrium, suspended from a fixed point O by a light inextensible string attached to the point B of the lamina, as shown in the diagram.



Given that $AB = 9$ cm, $BC = 12$ cm and $A\hat{B}C = 90°$, find the angle between BC and the downward vertical.

Solution:



A is (9, 0)
B is (0, 0)
C is (0, 12)
then G is (3, 4)

← Take BA and BC as axes.

← Take the mean of the 3 points.

← G will be vertically below B.

In equilibrium, BG will be vertical.

Hence required angle is $G\hat{B}C = \theta$.

$\tan\theta = \frac{3}{4} \Rightarrow \theta = 36.9°$.

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Exercise G, Question 3

Question:

Four particles P, Q, R and S of masses 3 kg, 5 kg, 2 kg and 4 kg are placed at the points (1, 6), (−1,5),(2,−3) and (−1,−4) respectively. Find the coordinates of the centre of mass of the particles.

Solution:

$$3\begin{pmatrix}1\\6\end{pmatrix}+5\begin{pmatrix}-1\\5\end{pmatrix}+2\begin{pmatrix}2\\-3\end{pmatrix}+4\begin{pmatrix}-1\\-4\end{pmatrix}=(3+5+2+4)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Use $\Sigma m_i \mathbf{r}_i = \bar{\mathbf{r}}\Sigma m_i$.

$$\begin{pmatrix}3\\18\end{pmatrix}+\begin{pmatrix}-5\\25\end{pmatrix}+\begin{pmatrix}4\\-6\end{pmatrix}+\begin{pmatrix}-4\\-16\end{pmatrix}=14\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Simplify.

$$\begin{pmatrix}-2\\21\end{pmatrix}=14\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Hence, coordinates of the centre of mass are $\left(-\frac{1}{7},\frac{3}{2}\right)$.

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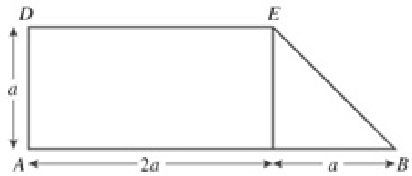
Exercise G, Question 4

Question:

A uniform rectangular piece of card $ABCD$ has $AB = 3a$ and $BC = a$. One corner of the rectangle is folded over to form a trapezium $ABED$ as shown in the diagram:



Find the distance of the centre of mass of the trapezium from

a AD,

b AB.

The lamina $ABED$ is freely suspended from E and hangs at rest.

c Find the angle between DE and the horizontal.

The mass of the lamina is M. A particle of mass m is attached to the lamina at the point B. The lamina is freely suspended from E and it hangs at rest with AB horizontal.

d Find m in terms of M.

Solution:

Taking AB and AD as axes:

$$2a^2\begin{pmatrix} a \\ \frac{1}{2}a \end{pmatrix} + 2\times\frac{1}{2}a^2\begin{pmatrix} \frac{7a}{3} \\ \frac{a}{3} \end{pmatrix} = 3a^2\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

Centre of mass of the *two* triangles.

$$\begin{pmatrix} 2a \\ a \end{pmatrix} + \begin{pmatrix} \frac{7a}{3} \\ \frac{a}{3} \end{pmatrix} = 3\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

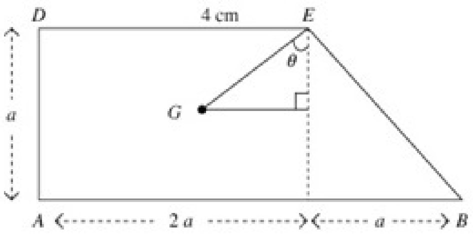
$$\frac{1}{3}\left\{\begin{pmatrix} 2a \\ 0 \end{pmatrix} + \begin{pmatrix} 3a \\ 0 \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix}\right\} = \begin{pmatrix} \frac{7a}{3} \\ \frac{a}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{13a}{9} \\ \frac{4a}{9} \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

a Distance from AD is $\frac{13a}{9}$.

b Distance from AB is $\frac{4a}{9}$.

c



EG will be vertical in equilibrium.

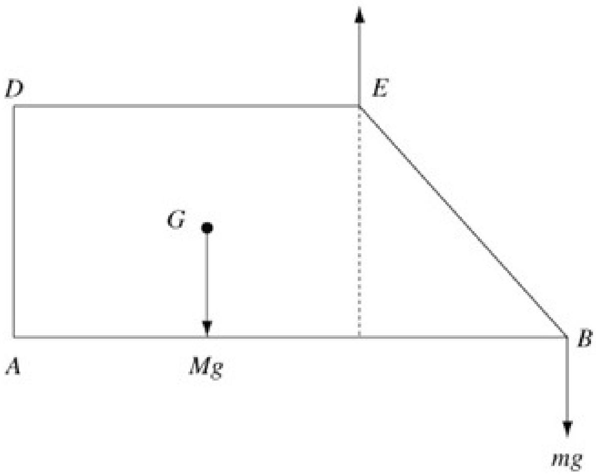
θ is the required angle

$D\hat{E}G$ is the angle between DE and the vertical so $(90^\circ - D\hat{E}G)$ will be the angle between DE and the horizontal.

$$\tan\theta = \frac{2a-\bar{x}}{a-\bar{y}} = \frac{2a-\frac{13a}{9}}{a-\frac{4a}{9}} = \frac{18-13}{9-4} = 1$$

So, θ is 45°

d



$$M(\mathrm{E}),\ Mg(2a - \bar{x}) = mga$$

$$M\frac{5a}{9} = ma$$

$$\text{i.e. } m = \frac{5M}{9}$$

Take moments about E to give an equation relating M and m.

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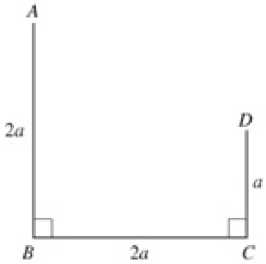
Exercise G, Question 5

Question:

A thin uniform wire of length $5a$ is bent to form the shape $ABCD$, where $AB = 2a, BC = 2a, CD = a$ and BC is perpendicular to both AB and CD, as shown in the diagram:



a Find the distance of the centre of mass of the wire from

i AB,

ii BC.

The wire is freely suspended from B and hangs at rest.

b Find, to the nearest degree, the angle between AB and the vertical.

Solution:

a Taking axes BC and BA:

$$2a\begin{pmatrix}0\\a\end{pmatrix}+2a\begin{pmatrix}a\\0\end{pmatrix}+a\begin{pmatrix}2a\\\frac{1}{2}a\end{pmatrix}=5a\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}0\\2a\end{pmatrix}+\begin{pmatrix}2a\\0\end{pmatrix}+\begin{pmatrix}2a\\\frac{1}{2}a\end{pmatrix}=5\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

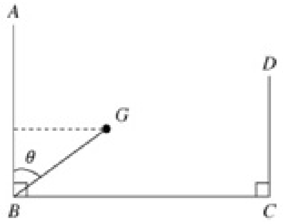
$$\begin{pmatrix}4a\\\frac{5a}{2}\end{pmatrix}=5\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}\frac{4a}{5}\\\frac{a}{2}\end{pmatrix}=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

Take axes through the point B, and use $\Sigma m_i \mathbf{r}_i = \bar{\mathbf{r}}\Sigma m_i$.

i $\frac{4a}{5}$ **ii** $\frac{a}{2}$

b



BG will be vertical when the wire hangs in equilibrium.

θ is the required angle.

$\tan\theta = \frac{\bar{x}}{\bar{y}}$

$= \frac{4a}{5}\times\frac{2}{a}=\frac{8}{5}$

$\Rightarrow \theta = 58°$ (nearest degree)

Solutionbank M2

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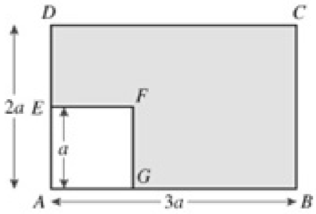
Exercise G, Question 6

Question:

A uniform lamina consists of a rectangle $ABCD$, where $AB = 3a$ and $AD = 2a$, with a square hole $EFGA$, where $EF = a$, as shown in the diagram:

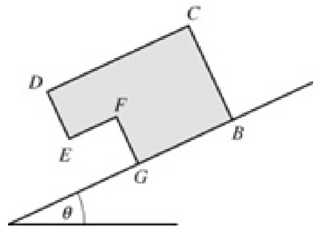


a Find the distance of the centre of mass of the lamina from

i AD,

ii AB.

The lamina is balanced on a rough plane inclined to the horizontal at an angle θ. The plane of the lamina is vertical and the inclined plane is sufficiently rough to prevent the lamina from slipping. The side GB is in contact with the plane with G lower than B, as shown in the diagram:



b Find, in degrees to 1 decimal place, the greatest value of θ for which the lamina can rest in equilibrium without toppling.

Solution:

a Taking AB and AD as axes:

$$6a^2\begin{pmatrix}\frac{3a}{2}\\ a\end{pmatrix} - a^2\begin{pmatrix}\frac{1}{2}a\\ \frac{1}{2}a\end{pmatrix} = 5a^2\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$$

Treat the lamina as a rectangle with a square removed.

$$\begin{pmatrix}9a\\ 6a\end{pmatrix} - \begin{pmatrix}\frac{1}{2}a\\ \frac{1}{2}a\end{pmatrix} = 5\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$$

$$\begin{pmatrix}1.7a\\ 1.1a\end{pmatrix} = \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$$

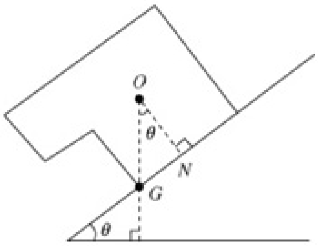
Simplify.

i $1.7a$

ii $1.1a$

State your answers.

b



At critical point, the centre of mass, O, will be vertically above the point G.

$$\tan\theta = \frac{GN}{ON}$$

$$= \frac{\bar{x}-a}{\bar{y}}$$

Since $\bar{x}$ was measured from the point A.

$$= \frac{1.7a-a}{1.1a}$$

Substitute for $\bar{x}$.

$$= \frac{0.7}{1.1}$$

$$= \frac{7}{11}$$

Simplify.

So, $\theta = 32.5°$ (1d.p.)