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Dynamics of a particle moving in a straight line
Exercise A, Question 1

Question:

Remember that g should be taken as 9.8 m s $^{-2}$.

Find the weight in newtons of a particle of mass 4 kg.

Solution:

$W = m\ g = 4 \times 9.8 = 39.2$

The weight of the particle is 39.2 N.

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Dynamics of a particle moving in a straight line
Exercise A, Question 2

Question:

Find the mass of a particle whose weight is 490 N.

Solution:

$$W = mg \text{ so } 490 = m \times 9.8$$
$$\Rightarrow m = \frac{490}{9.8} = 50$$

The mass of the particle is 50 kg.

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Dynamics of a particle moving in a straight line
Exercise A, Question 3

Question:

The weight of an astronaut on the Earth is 686 N. The acceleration due to gravity on the Moon is approximately 1.6 m s^{-2}. Find the weight of the astronaut when he is on the Moon.

Solution:

$$
\begin{aligned}
W_{\text{EARTH}} &= m\ g_{\text{EARTH}} \\
\text{so,}\ 686 &= m \times 9.8 \\
\Rightarrow m &= 70 \text{ i.e. the mass of the astronaut is 70 kg} \\
W_{\text{MOON}} &= m\ g_{\text{MOON}} \\
&= 70 \times 1.6 \\
&= 112
\end{aligned}
$$

The weight of the astronaut on the Moon is 112 N.

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Dynamics of a particle moving in a straight line
Exercise A, Question 4

Question:

Find the force required to accelerate a 1.2 kg mass at a rate of 3.5 m s^{-2}.

Solution:

$$
\begin{aligned}
F &= m\ a \\
&= 1.2 \times 3.5 \\
&= 4.2
\end{aligned}
$$

So the force required is 4.2 N.

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Dynamics of a particle moving in a straight line

Exercise A, Question 5

Question:

Find the acceleration when a particle of mass 400 kg is acted on by a resultant force of 120 N.

Solution:

$$\begin{aligned} F &= m\ a \\ 120 &= 400a \\ a &= 0.3 \end{aligned}$$

The acceleration is 0.3 $\mathrm{m\ s^{-2}}$.

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Dynamics of a particle moving in a straight line
Exercise A, Question 6

Question:

An object moving on a rough surface experiences a constant frictional force of 30 N which decelerates it at a rate of 1.2 m s $^{-2}$. Find the mass of the object.

Solution:

$$\begin{aligned} F &= m\ a \\ 30 &= 1.2m \\ m &= 25 \end{aligned}$$

The mass of the object is 25 kg.

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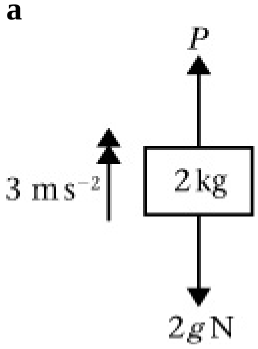
Dynamics of a particle moving in a straight line
Exercise A, Question 7

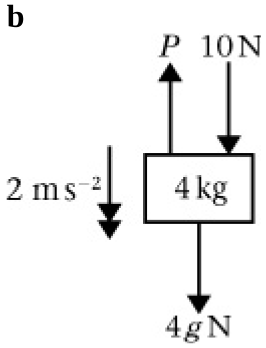
Question:

In each of the following scenarios, the forces acting on the body cause it to accelerate as shown. Find the magnitude of the unknown forces *P* and *Q*.

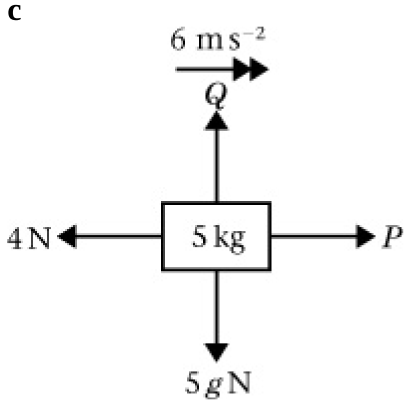
a



b



c



Solution:

a

$$\begin{aligned} R(\uparrow), \quad P - 2g &= 2 \times 3 \\ P &= 25.6 \end{aligned}$$

The magnitude of *P* is 25.6 N.

Always resolve in the direction of the acceleration.

b

$$\begin{aligned} R(\downarrow), \quad 4g + 10 - P &= 4 \times 2 \\ 49.2 - P &= 8 \\ P &= 41.2 \end{aligned}$$

The magnitude of *P* is 41.2 N.

c

$$\begin{aligned} R(\rightarrow), \quad P - 4 &= 5 \times 6 \\ P &= 34 \end{aligned}$$

The magnitude of *P* is 34 N.

$$\begin{aligned} R(\uparrow), \quad Q - 5g &= 5 \times 0 \\ Q &= 49 \end{aligned}$$

No acceleration vertically.

The magnitude of *Q* is 49 N.

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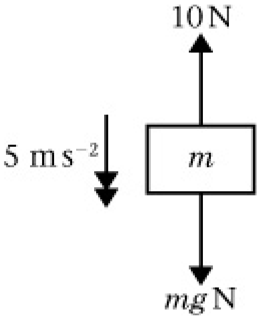
Dynamics of a particle moving in a straight line
Exercise A, Question 8

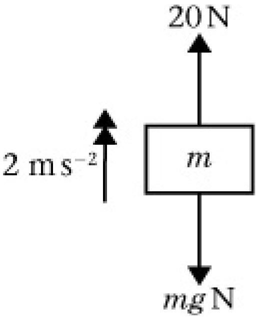
Question:

In each of the following situations, the forces acting on the body cause it to accelerate as shown. In each case find the mass of the body, *m*.

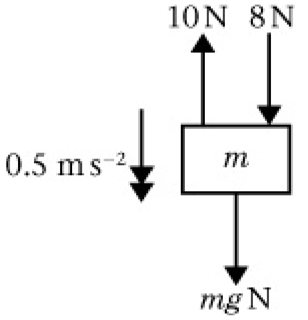
a



b



c



Solution:

a

$R(\downarrow), \quad mg - 10 = m \times 5$

$9.8m - 10 = 5m$

$4.8m = 10$

$m = 2.1 \quad (2 \text{ s.f.})$

The mass of the body is 2.1 kg (2 s.f.).

b

$R(\uparrow), \quad 20 - mg = m \times 2$

$20 - 9.8m = 2m$

$20 = 11.8m$

$m = 1.7 \quad (2 \text{ s.f.})$

The mass of the body is 1.7 kg (2 s.f.).

c

$R(\downarrow), \quad mg + 8 - 10 = m \times 0.5$

$9.8m - 2 = 0.5m$

$9.3m = 2$

$m = 0.22 \quad (2 \text{ s.f.})$

The mass of the body is 0.22 kg (2 s.f.).

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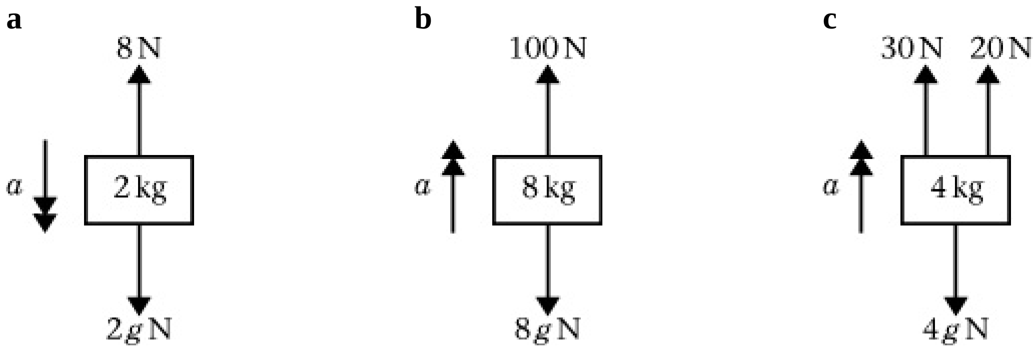
Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line

Exercise A, Question 9

Question:

In each of the following situations, the forces acting on the body cause it to accelerate as shown with magnitude a m s $^{-2}$. In each case find the value of a.



Solution:

a

R (↓) , $2g - 8 = 2a$

$19.6 - 8 = 2a$

$11.6 = 2a$

$5.8 = a$

The acceleration of the body is 5.8 m s $^{-2}$.

b

R (↑) , $100 - 8g = 8a$

$100 - 78.4 = 8a$

$21.6 = 8a$

$2.7 = a$

The acceleration of the body is 2.7 m s $^{-2}$.

c

R (↑) , $30 + 20 - 4g = 4a$

$50 - 39.2 = 4a$

$10.8 = 4a$

$2.7 = a$

The acceleration of the body is 2.7 m s $^{-2}$.

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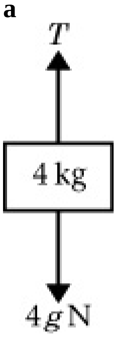
Dynamics of a particle moving in a straight line
Exercise A, Question 10

Question:

The diagram shows a block of mass 4 kg being lowered vertically by a rope.

a

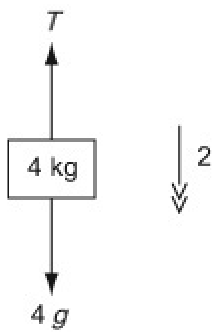


Find the tension in the rope when the block is lowered **a** with an acceleration of 2 m s^{-2}, **b** at a constant speed of 4 m s^{-1},
c with a deceleration of 0.5 m s^{-2}.

Solution:

a

$$\begin{array}{lll} R(\downarrow), & 4g - T & = 4 \times 2 \\ 39.2 - T & & = 8 \\ T & & = 31.2 \end{array}$$

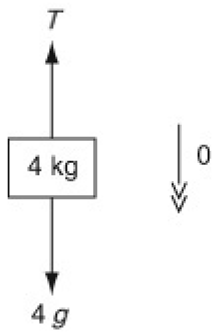


The tension in the rope is 31.2 N.

b

$$\begin{array}{lll} R(\downarrow), & 4g - T & = 4 \times 0 \\ 39.2 & & = T \end{array}$$

$\boxed{\text{Constant speed} \Rightarrow \text{zero acceleration.}}$

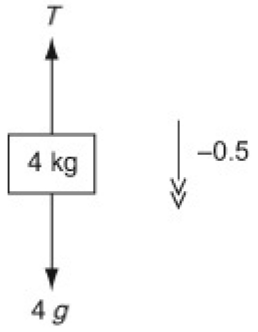


The tension in the rope is 39.2 N.

c

R (↓) , $4g - T = 4 \times -0.5$
$39.2 - T = -2$
$T = 41.2$

Deceleration is a negative (downward) acceleration



The tension in the rope is 41.2 N.

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Dynamics of a particle moving in a straight line
Exercise B, Question 1

Question:

A ball of mass 200 g is attached to the upper end of a vertical light rod. Find the thrust in the rod when it raises the ball vertically with an acceleration of 1.5 m s $^{-2}$.

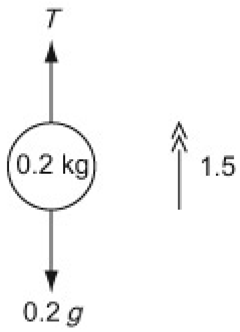
Solution:

R (↑), $T - 0.2g = 0.2 \times 1.5$

$T - 1.96 = 0.3$

$T = 2.26 = 2.3$ (2 s.f.)

The thrust in the rod is 2.3 N (2 s.f.).



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Dynamics of a particle moving in a straight line
Exercise B, Question 2

Question:

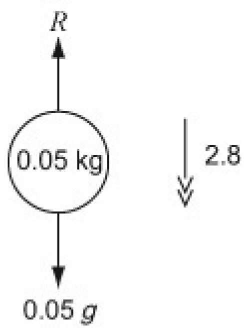
A small pebble of mass 50 g is dropped into a pond and falls vertically through it with an acceleration of 2.8 m s $^{-2}$. Assuming that the water produces a constant resistance, find its magnitude.

Solution:

R (↓) , $0.05g - R = 0.05 \times 2.8$

$R = 0.35$

The resistance of the water is 0.35 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 3

Question:

A lift of mass 500 kg is lowered or raised by means of a metal cable attached to its top. The lift contains passengers whose total mass is 300 kg. The lift starts from rest and accelerates at a constant rate, reaching a speed of 3 m s^{-1}after moving a distance of 5 m. Find

a the acceleration of the lift,

b the tension in the cable if the lift is moving vertically downwards,

c the tension in the cable if the lift is moving vertically upwards.

Solution:

a $u = 0$, $v = 3$, $s = 5$, $a =$?

$v^2 = u^2 + 2as$

$3^2 = 0^2 + 2a \times 5$

$9 = 10a$

$a = 0.9$

The acceleration of the lift is 0.9 m s^{-2}.

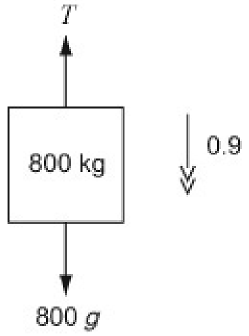
b

R (↓) , $800g - T = 800 \times 0.9$

$7840 - T = 720$

$T = 7120$

The tension in the cable is 7120 N.



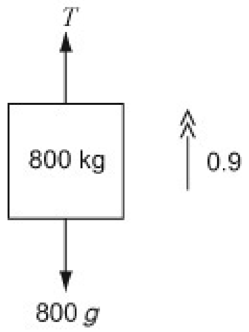
c

R (↑) , $T - 800g = 800 \times 0.9$

$T - 7840 = 720$

$T = 8560$

The tension in the cable is 8560 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 4

Question:

A block of mass 1.5 kg falls vertically from rest and hits the ground 16.6 m below after falling for 2 s. Assuming that the air resistance experienced by the block as it falls is constant, find its magnitude.

Solution:

$u = 0$, $s = 16.6$, $t = 2$, $a =$?

$s = ut + \frac{1}{2}at^2 \quad (\downarrow)$

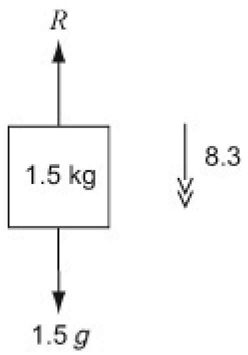
$16.6 = 0 + \frac{1}{2}a \times 2^2$

$a = 8.3$

R (↓) , $1.5g - R = 1.5 \times 8.3$

$R = 2.25$

The magnitude of the air resistance is 2.25 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 5

Question:

A trolley of mass 50 kg is pulled from rest in a straight line along a horizontal path by means of a horizontal rope attached to its front end. The trolley accelerates at a constant rate and after 2 s its speed is 1 m s^{-1}. As it moves, the trolley experiences a resistance to motion of magnitude 20 N. Find

a the acceleration of the trolley,

b the tension in the rope.

Solution:

a $u = 0$, $v = 1$, $t = 2$, $a =$?

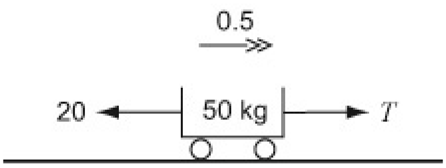
$$v = u + at$$
$$1 = 0 + a \times 2$$
$$\Rightarrow a = 0.5$$

The acceleration of the trolley is 0.5 m s^{-2}.

b

$$R(\rightarrow), \quad T - 20 = 50 \times 0.5$$
$$T = 45$$

The tension in the rope is 45 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 6

Question:

A trailer of mass 200 kg is attached to a car by a light tow-bar. The trailer is moving along a straight horizontal road and decelerates at a constant rate from a speed of 15 m s^{-1} to a speed of 5 m s^{-1} in a distance of 25 m. Assuming there is no resistance to the motion, find

a the deceleration of the trailer,

b the thrust in the tow-bar.

Solution:

a $u = 15$, $v = 5$, $s = 25$, $a = ?$

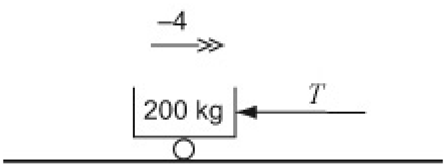
$$\begin{aligned} v^2 &= u^2 + 2as \quad (\rightarrow) \\ 5^2 &= 15^2 + 2a \times 25 \\ 25 &= 225 + 50a \\ -200 &= 50a \\ a &= -4 \end{aligned}$$

The deceleration of the trailer is 4 m s^{-2}.

b

$$\begin{aligned} \text{R}(\rightarrow), \quad -T &= 200 \times -4 \\ T &= 800 \end{aligned}$$

The thrust in the tow-bar is 800 N.



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Dynamics of a particle moving in a straight line
Exercise B, Question 7

Question:

A woman of mass 60 kg is in a lift which is accelerating upwards at a rate of 2 m s^{-2}.

a Find the magnitude of the normal reaction of the floor of the lift on the woman.
The lift then moves at a constant speed and then finally decelerates to rest at 1.5 m s^{-2}.

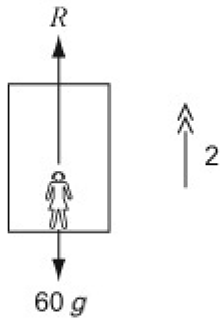
b Find the magnitude of the normal reaction of the floor of the lift on the woman during the period of deceleration.

c Hence explain why the woman will feel heavier during the period of acceleration and lighter during the period of deceleration.

Solution:

a

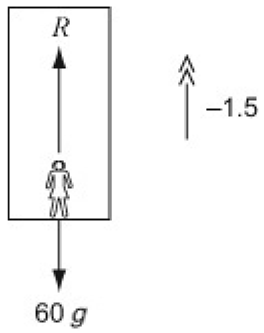
$$\begin{aligned} &R(\uparrow), \quad R - 60g &&= 60 \times 2 \\ &R - 588 &&= 120 \\ &R &&= 708 \end{aligned}$$



The normal reaction on the woman has magnitude 708 N.

b

$$\begin{aligned} &R(\uparrow), \quad R - 60g &&= 60 \times -1.5 \\ &R - 588 &&= -90 \\ &R &&= 498 \end{aligned}$$



The normal reaction on the woman has magnitude 498 N.

c The woman's sense of her own weight is the magnitude of the force that she feels from the floor i.e. the normal reaction. This is ‘usually’ 60g i.e. 588 N but when the lift is accelerating upwards it increases to 708 N i.e. she feels heavier and when the lift is decelerating upwards it decreases to 498 N i.e. she feels lighter.

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise B, Question 8

Question:

The engine of a van of mass 400 kg cuts out when it is moving along a straight horizontal road with speed 16 m s^{-1}. The van comes to rest without the brakes being applied.

In a model of the situation it is assumed that the van is subject to a resistive force which has constant magnitude of 200 N.

a Find how long it takes the van to stop.

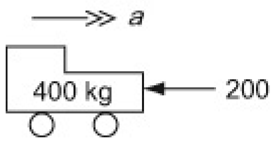
b Find how far the van travels before it stops.

c Comment on the suitability of the modelling assumption.

Solution:

a

$$R(\rightarrow), \quad -200 = 400a$$
$$\Rightarrow a = -0.5$$



$u = 16$, $v = 0$, $a = -0.5$, t ?

$$v = u + at \quad (\rightarrow)$$
$$0 = 16 - 0.5t$$
$$0.5t = 16$$
$$t = 32$$

It takes 32 s for the van to stop.

b $u = 16$, $v = 0$, $a = -0.5$, s ?

$$v^2 = u^2 + 2as \quad (\rightarrow)$$
$$0^2 = 16^2 + 2(-0.5)s$$
$$0 = 256 - s$$
$$s = 256$$

The van travels 256 m before it stops.

c Air resistance is unlikely to be of constant magnitude. (It is usually a function of speed.)

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Dynamics of a particle moving in a straight line
Exercise B, Question 9

Question:

Albert and Bella are both standing in a lift. The mass of the lift is 250 kg. As the lift moves upward with constant acceleration, the floor of the lift exerts forces of magnitude 678 N and 452 N respectively on Albert and Bella. The tension in the cable which is pulling the lift upwards is 3955 N.

a Find the acceleration of the lift.

b Find the mass of Albert.

c Find the mass of Bella.

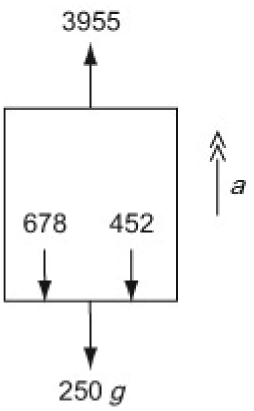
Solution:

a

R (↑) ,	$3955 - 678 - 452 - 250g$	$= 250a$
375		$= 250a$
a		$= 1.5$

The acceleration of the lift upwards is 1.5 m s^{-2}.

Draw a diagram showing the forces acting an the LIFT only.

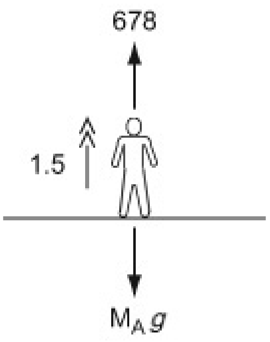


b

R (↑) , $678 - M_A g$	$= M_A \times 1.5$
678	$= 11.3 M_A$
M_A	$= 60$

The mass of Albert is 60 kg.

Draw a diagram showing the forces acting on ALBERT only.

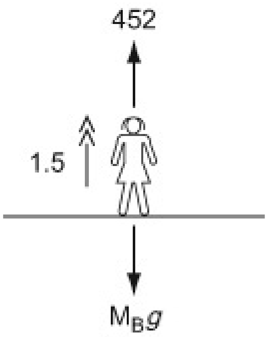


c

Draw a diagram showing the forces acting an

R (↑) , $452 - M_B g$	$= M_B \times 1.5$	BELLA only.
452	$= 11.3M_B$	
M_B	$= 40$	

The mass of Bella is 40 kg.



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Dynamics of a particle moving in a straight line

Exercise B, Question 10

Question:

A small stone of mass 400 g is projected vertically upwards from the bottom of a pond full of water with speed 10 m s^{-1}. As the stone moves through the water it experiences a constant resistance of magnitude 3 N. Assuming that the stone does not reach the surface of the pond, find

a the greatest height above the bottom of the pond that the stone reaches,

b the speed of the stone as it hits the bottom of the pond on its return,

c the total time taken for the stone to return to its initial position on the bottom of the pond.

Solution:

a

$R(\uparrow),\quad -3 - 0.4g = 0.4a$

$a = -17.3$

$u = 10,\ v = 0,\ a = -17.3,\ s = ?$

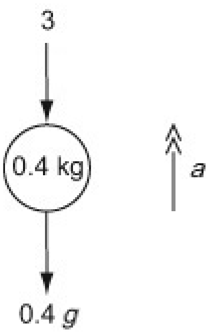
$v^2 = u^2 + 2as \quad (\uparrow)$

$0 = 10^2 + 2(-17.3)s$

$0 = 100 - 34.6s$

$s = 2.89\ldots = 2.9 \quad (2\text{ s.f.})$

The stone rises to a height of 2.9 m (2 s.f.) above the bottom of the pond.



b

$R(\downarrow),\quad 0.4g - 3 = 0.4a$

$0.92 = 0.4a$

$a = 2.3$

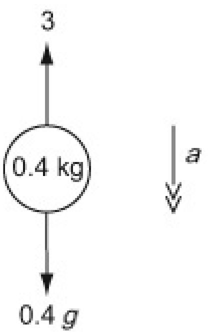
$u = 0,\ s = \left(\frac{100}{34.6}\right),\ a = 2.3,\ v = ?$

$v^2 = u^2 + 2as \quad (\downarrow)$

$v^2 = 0^2 + 2 \times 2.3 \times \left(\frac{100}{34.6}\right)$

$v = 3.646.. = 3.6 \quad (2\text{ s.f.})$

The stone hits the bottom of the pond with speed 3.6 m s^{-1} (2 s.f.).



c $u = 10,\ v = 0,\ a = -17.3,\ t = ?$

$v = u + at \quad (\uparrow)$

$0 = 10 - 17.3t$,

$t_1 = \frac{10}{17.3} = 0.57803 \ldots$

$u = 0$, $a = 2.3$, $s = \left(\frac{100}{34.6}\right)$, $t = ?$

$s = ut + \frac{1}{2}at^2 \quad (\downarrow)$

$\frac{100}{34.6} = 0 + \frac{1}{2} \times 2.3t_2^{\,2}$

$t_2^{\,2} = \cdot \frac{2 \times 100}{2.3 \times 34.6} = 2.51319$

$t_2 = 1.585$

$t_1 + t_2 = 0.57803 + 1.585 = 2.16$

The total time is 2.16 s (3 s.f.).

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Edexcel AS and A Level Modular Mathematics

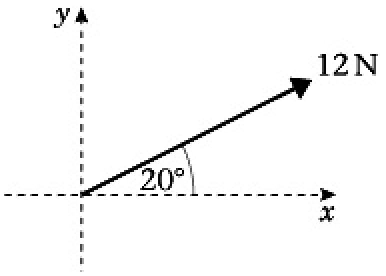
Dynamics of a particle moving in a straight line
Exercise C, Question 1

Question:

Find the component of each force that acts in

i the *x*-direction, **ii** the *y*-direction.

a



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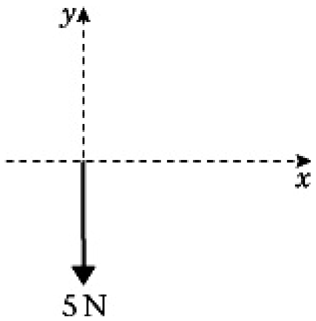

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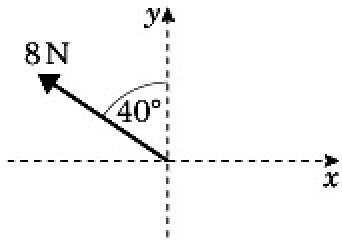

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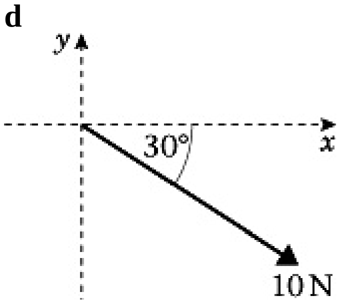

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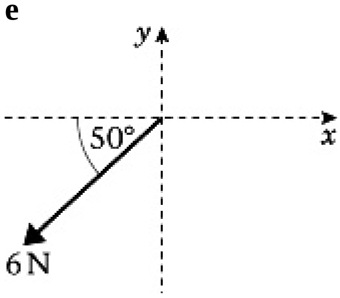
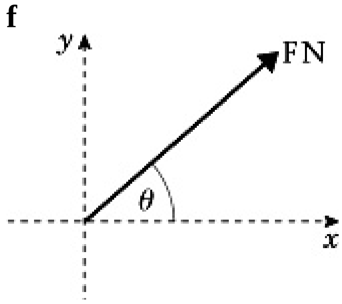


f



Solution:

a

i $12 \cos 20° = 11.3$ N (3 s.f.)

ii $12 \cos 70° = 12 \sin 20° = 4.10$ N (3 s.f.)

b

i $5 \cos 90° = 0$ N

ii $-5 \cos 0° = -5$ N

(or $5 \cos 180°$)

c

i $-8 \cos 50° = -5.14$ N (3 s.f.)

(or $8 \cos 130°$)

ii $8 \cos 40° = 6.13$ N (3 s.f.)

d

i $10 \cos 30° = 8.66 \text{ N} \ (3 \text{ s.f.})$

ii $-10 \cos 60° = -5 \text{ N}$

(or $10 \cos 120°$)

e

i $-6 \cos 50° = -3.86 \text{ N} \ (3 \text{ s.f.})$

(or $6 \cos 130°$)

ii $-6 \cos 40° = -4.60 \text{ N} \ (3 \text{ s.f.})$

(or $6 \cos 140°$)

f

i $F \cos \theta$ N

ii $F \cos (90° - \theta) = F \sin \theta$ N

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

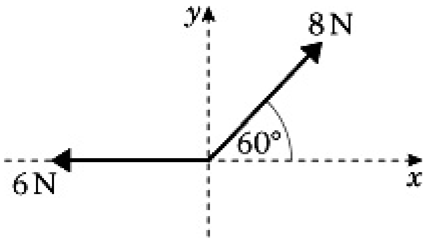
Dynamics of a particle moving in a straight line
Exercise C, Question 2

Question:

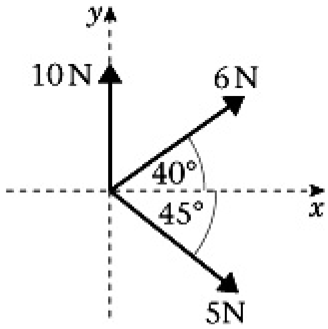
For each of the following systems of forces, find the sum of the components in

i the x-direction, **ii** the y-direction.

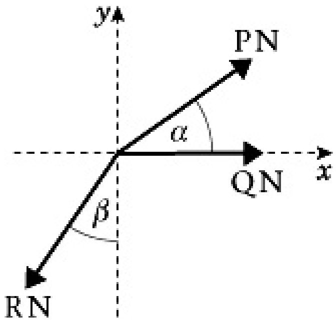
a



b



c



Solution:

a

i $8\cos 60° - 6 = -2$ N

ii $8\cos 30° = 6.93$ N (3 s.f.)

b

i $6\cos 40° + 5\cos 45° + (10\cos 90°) = 8.13$ N (3 s.f.)

ii $10(\cos 0°) + 6\cos 50° - 5\cos 45° = 10.3$ N (3 s.f.)

c

i $P\cos\alpha + Q(\cos 0°) - R\cos(90° - \beta)$

$= P\cos\alpha + Q - R\sin\beta$

ii $P\cos(90° - \alpha) - R\cos\beta = P\sin\alpha - R\cos\beta$

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Dynamics of a particle moving in a straight line
Exercise D, Question 1

Question:

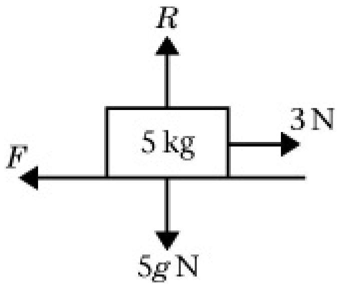
Each of the following diagrams shows a body of mass 5 kg lying initially at rest on rough horizontal ground. The coefficient of friction between the body and the ground is $\frac{1}{7}$. In each diagram R is the normal reaction from the ground on the body and F is the friction force exerted on the body by the ground. Any other forces applied to the body are as shown on the diagram. In each case

i find the magnitude of F,

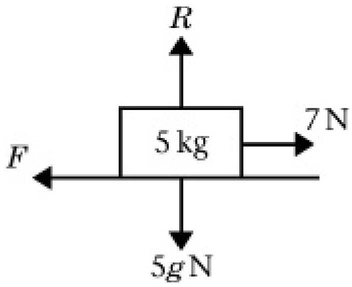
ii state whether the body will remain at rest or accelerate from rest along the ground,

iii find, when appropriate, the magnitude of this acceleration.

a



b



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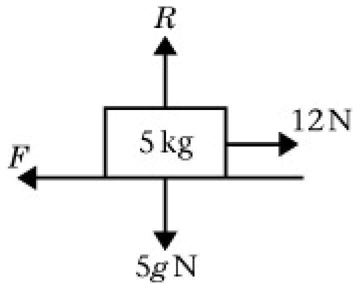

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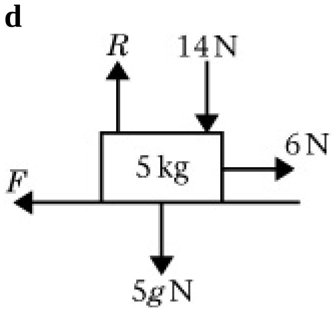

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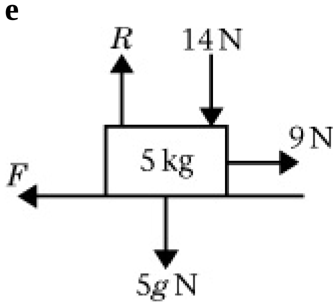
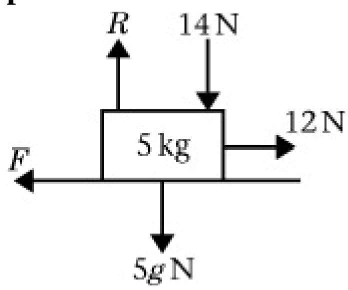
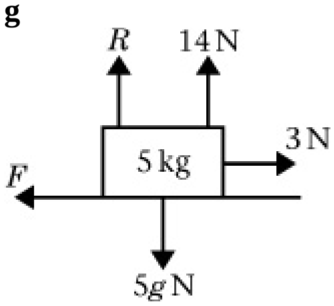


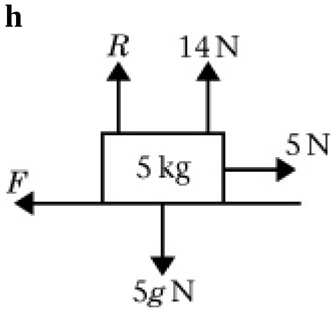
f



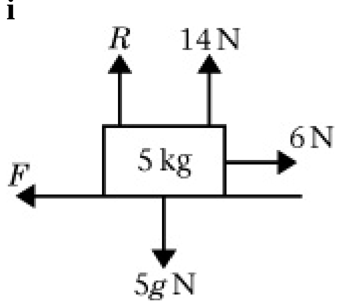
g



h



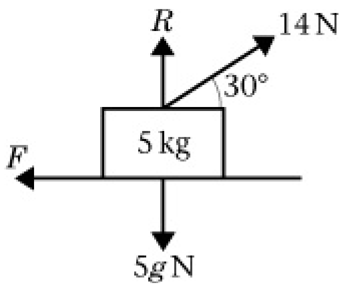
i

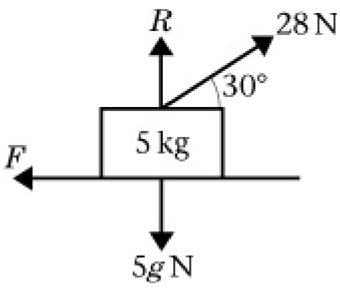


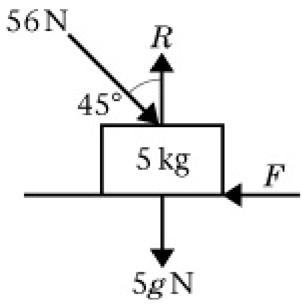
j

k

l







Solution:

a

i

$$\begin{aligned} \mathrm{R}(\uparrow)\ R - 5g &= 0 \\ R &= 5g = 49\text{ N} \end{aligned}$$

$\therefore F_{\text{MAX}} = \frac{1}{7} \times 49 = 7\text{ N} \;\; \therefore F = 3\text{ N}$

ii $\therefore F = 3$ N and body remains at rest

b

i $F_{\text{MAX}} = 7\text{ N} \;\; \therefore F = 7\text{ N}$

ii $F = 7$ N and body remains at rest (in limiting equilibrium)

c

i $F_{\text{MAX}} = 7\text{ N} \;\; \therefore F = 7\text{ N}$

ii $F = 7$ N and body accelerates

iii

$$\begin{aligned} \mathrm{R}(\rightarrow),12 - 7 &= 5a \\ a &= 1\text{ m s}^{-2} \end{aligned}$$

Body accelerates at 1 m s^{-2}

d

i

$$\begin{aligned} \mathrm{R}(\uparrow),R - 14 - 5g &= 0 \\ R &= 63\text{ N} \end{aligned}$$

$\therefore F_{\text{MAX}} = \frac{1}{7} \times 63 = 9\text{ N} \;\; \therefore F = 6\text{ N}$

ii $F = 6$ N and body remains at rest

e

i $F = 9$ N

ii $F = 9$ N and body remains at rest in limiting equilibrium

f

i $F = 9$ N

ii $F = 9$ N and body accelerates

iii

$$\begin{aligned} \mathrm{R}(\rightarrow),\ 12 - 9 &= 5a \\ a &= 0.6\ \mathrm{m\ s^{-2}} \end{aligned}$$

Body accelerates at 0.6 m s^{-2}

g

i

$$\begin{aligned} \mathrm{R}(\uparrow),\ R + 14 - 5g &= 0 \\ R &= 35\ \mathrm{N} \end{aligned}$$

$\therefore F_{\mathrm{MAX}} = \frac{1}{7} \times 35 = 5\ \mathrm{N}\ \therefore F = 3\ \mathrm{N}$

ii $F = 3$ N and body remains at rest

h

i $F = 5$ N

ii $F = 5$ N and body remains at rest in limiting equilibrium

i

i $F = 5$ N

ii $F = 5$ N and body accelerates

iii

$$\begin{aligned} \mathrm{R}(\rightarrow),\ 6 - 5 &= 5a \\ a &= 0.2\ \mathrm{m\ s^{-2}} \end{aligned}$$

Body accelerates at 0.2 m s^{-2}

j

i

$$\begin{aligned} \mathrm{R}(\uparrow),\ R + 14\cos 60^{\circ} - 5g &= 0 \\ R &= 42\ \mathrm{N} \end{aligned}$$

$\therefore F_{\mathrm{MAX}} = \frac{1}{7} \times 42 = 6\ \mathrm{N}$

R (→) ,$14\cos 30° - F = 5a$

Since $14\cos 30° > 6$

$\therefore F = 6$ N

ii $F = 6$ N and body accelerates

iii

$$\begin{aligned} R(\rightarrow),\ 14\cos 30° - 6 &= 5a \\ a &= 1.22 \text{ m s}^{-2} \ (3 \text{ s.f.}) \end{aligned}$$

Body accelerates at 1.22 m s^{-2} (3 s.f.)

k

i

$$\begin{aligned} R(\uparrow),\ R + 28\cos 60° - 5g &= 0 \\ R &= 35 \text{ N} \\ \therefore F_{MAX} = \tfrac{1}{7} \times 35 &= 5 \text{ N} \end{aligned}$$

R (→) ,$28\cos 30° - F = 5a$

Since $28\cos 30° > 5$

$\therefore F = 5$ N

ii $F = 5$ N and body accelerates

iii

$$\begin{aligned} R(\rightarrow),\ 28\cos 30° - 5 &= 5a \\ a &= 3.85 \text{ m s}^{-2} \ (3 \text{ s.f.}) \end{aligned}$$

Body accelerates at 3.85 m s^{-2} (3 s.f.)

l

i R (↑) ,$R - 56\cos 45° - 5g = 0$

$\therefore R = 88.6$ N (3 s.f.)

$\therefore F_{MAX} = \frac{1}{7} \times 88.6 = 12.657$ N

R (→) ,$56\cos 45° - F$

Since $56\cos 45° > 12.657$ N

$F = F_{MAX} = 12.657$ or 12.7 N (3 s.f.)

ii body accelerates

iii $56 \cos(45°) = 39.5979$

$5a = 39.598 - 12.657 = 26.941$

$a = 5.3882$

So the acceleration is 5.39 m s^{-2} (3 s.f.).

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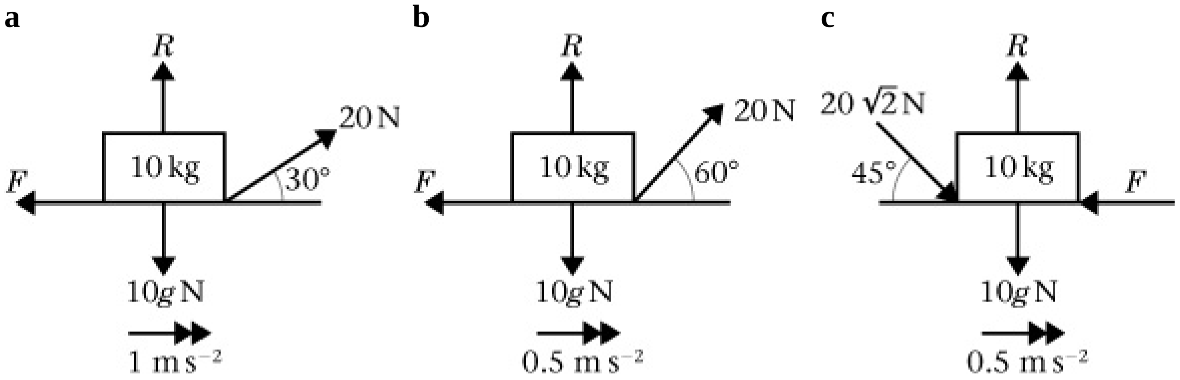
Dynamics of a particle moving in a straight line
Exercise D, Question 2

Question:

In each of the following diagrams, the forces shown cause the body of mass 10 kg to accelerate as shown along the rough horizontal plane. R is the normal reaction and F is the friction force. Find the coefficient of friction in each case.



Solution:

a

$R(\uparrow), R + 20\cos 60° - 10g = 0$

$R = 88$ N

$R(\rightarrow), 20\cos 30° - \mu \times 88 = 10 \times 1$

$\mu = 0.083$ (2 s.f.)

b

$R(\uparrow), R + 20\cos 30° - 10g = 0$

$R = 80.679...$ N

$R(\rightarrow), 20\cos 60° - \mu \times 80.679 = 10 \times 0.5$

$\mu = 0.062$ (2 s.f.)

c

$R(\uparrow), R - 20\sqrt{2}\cos 45° - 10g = 0$

$R = 118$ N

$R(\rightarrow), 20\sqrt{2}\cos 45° - \mu \times 118 = 10 \times 0.5$

$\mu = 0.13$ (2 s.f.)

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise E, Question 1

Question:

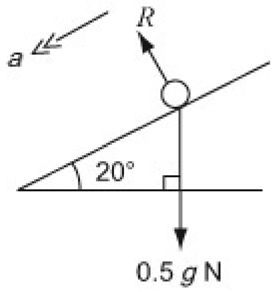
A particle of mass 0.5 kg is placed on a smooth inclined plane. Given that the plane makes an angle of 20° with the horizontal, find the acceleration of the particle.

Solution:

a

$R(\swarrow)$, $0.5g\cos 70^\circ = 0.5a$

$a = 3.35$ (3 s.f.)



The acceleration of the particle is 3.35 m s^{-2} (3 s.f.) .

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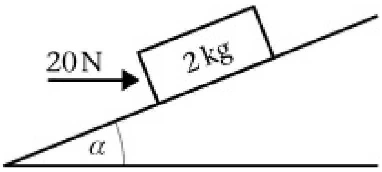
Dynamics of a particle moving in a straight line
Exercise E, Question 2

Question:

The diagram shows a box of mass 2 kg being pushed up a smooth plane by a horizontal force of magnitude 20 N. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.



Find

a the normal reaction between the box and the plane,

b the acceleration of the box up the plane.

Solution:

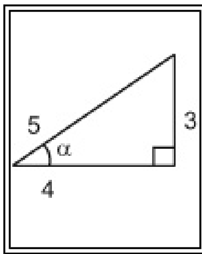
a

R (↖) , $R - 20\cos(90° - \alpha) - 2g\cos\alpha = 0$

$R = 20\sin\alpha + 19.6\cos\alpha$

$= 12 + 15.68$

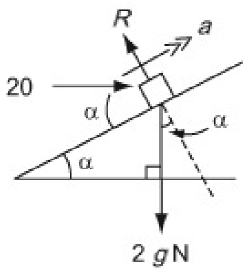
$= 27.7$ N (3 s.f.)



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$



The normal reaction is 27.7 N (3 s.f.).

b

R (↗) , $20\cos\alpha - 2g\cos(90° - \alpha) = 2a$

$20\cos\alpha - 2g\sin\alpha = 2a$

$a = 2.12 \text{ m s}^{-2}$

The acceleration of the box is 2.12 m s^{-2}.

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise E, Question 3

Question:

A boy of mass 40 kg slides from rest down a straight slide of length 5 m. The slide is inclined to the horizontal at an angle of 20°. The coefficient of friction between the boy and the slide is 0.1. By modelling the boy as a particle, find

a the acceleration of the boy,

b the speed of the boy at the bottom of the slide.

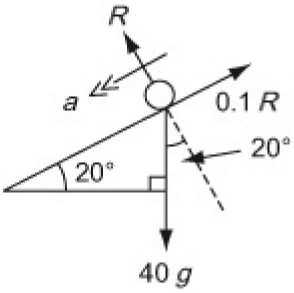
Solution:

a

$R(\swarrow)$, $40g\cos 70° - 0.1R = 40a$

$R(\nwarrow)$, $R - 40g\cos 20° = 0$

$R = 368.36$



Substituting,

$392\cos 70° - 36.836 = 40a$

$a = 2.43$ (3 s.f.)

The acceleration of the boy is 2.43 m s^{-2} (3 s.f.).

b $u = 0$, $a = 2.43$, $s = 5$, $u = ?$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times 2.43 \times 5 = 24.3$

$u = 4.93$ m s^{-1} (3 s.f.)

The speed of the boy is 4.93 m s^{-1} (3 s.f.).

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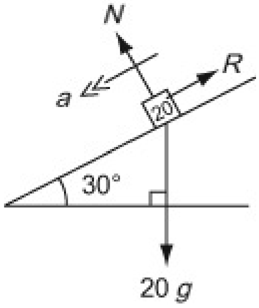
Dynamics of a particle moving in a straight line
Exercise E, Question 4

Question:

A block of mass 20 kg is released from rest at the top of a rough slope. The slope is inclined to the horizontal at an angle of 30°. After 6 s the speed of the block is 21 m s^{-1}. As the block slides down the slope it is subject to a constant resistance of magnitude *R* N. Find the value of *R*.

Solution:



$u = 0$, $v = 21$, $t = 6$, $a =$?

$v = u + at$

$21 = 0 + 6a$

$a = 3.5 \text{ m s}^{-2}$

R (↙) , $20g \cos 60° - R = 20 \times 3.5$

$98 - R = 70$

$R = 28 \text{ N}$

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Dynamics of a particle moving in a straight line
Exercise E, Question 5

Question:

A book of mass 2 kg slides down a rough plane inclined at 20° to the horizontal. The acceleration of the book is 1.5 m s $^{-2}$. Find the coefficient of friction between the book and the plane.

Solution:

R (↖) , $R - 2g \cos 20° = 0$

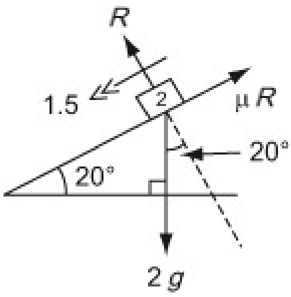
$R = 2g \cos 20°$

R (↙) ,

$2g \cos 70° - \mu \times 2g \cos 20° = 2 \times 1.5$

$\mu = \dfrac{2g \cos 70° - 3}{2g \cos 20°} = 0.201$ (3 s.f.)

The coefficient of friction is 0.20 (2 s.f.).



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Dynamics of a particle moving in a straight line
Exercise E, Question 6

Question:

A block of mass 4 kg is pulled up a rough slope, inclined at 25° to the horizontal, by means of a rope. The rope lies along the line of greatest slope. The tension in the rope is 30 N. Given that the acceleration of the block is 2 m s $^{-2}$ find the coefficient of friction between the block and the plane.

Solution:

R (↖) , $R - 4g \cos 25° = 0$

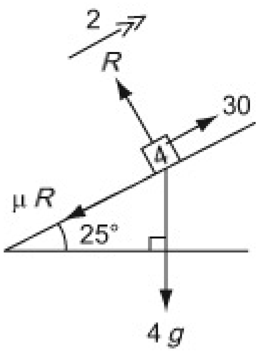
$R = 4g \cos 25°$

R (↗) , $30 - 4g \cos 65° - \mu 4g \cos 25° = 4 \times 2$

$\frac{22 - 4g \cos 65°}{4g \cos 25°} = \mu$

$0.15 \ (2 \text{ s.f.}) = \mu$

The coefficient of friction is 0.15 (2 s.f.).



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Dynamics of a particle moving in a straight line
Exercise E, Question 7

Question:

A parcel of mass 10 kg is released from rest on a rough plane which is inclined at 25° to the horizontal.

a Find the normal reaction between the parcel and the plane.

Two seconds after being released the parcel has moved 4 m down the plane.

b Find the coefficient of friction between the parcel and the plane.

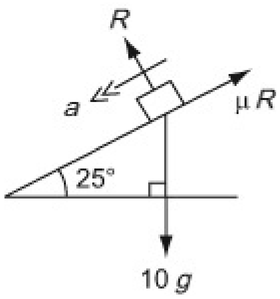
Solution:

a

$$\text{R}(\nwarrow), \quad R - 10g\cos 25° = 0$$

$$R = 98\cos 25° = 88.8\text{ N } (3\text{ s.f.})$$

The normal reaction is 88.8 N (3 s.f.).



b

$$u = 0,\ s = 4,\ t = 2,\ a = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$4 = 0 + \frac{1}{2}a \times 2^2$$

$$a = 2\text{ m s}^{-2}$$

$$\text{R}(\swarrow), \quad 10g\cos 65° - \mu R = 10 \times 2$$

$$\mu \times 98\cos 25° = 10g\cos 65° - 20$$

$$\mu = \frac{98\cos 65° - 20}{98\cos 25°}$$

$$= 0.241\ (3\text{ s.f.})$$

The coefficient of friction is 0.24 (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise E, Question 8

Question:

A particle P is projected up a rough plane which is inclined at an angle α to the horizontal, where tan $\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{3}$. The particle is projected from the point A with speed 20 m $s^{-1}$and comes to instantaneous rest at the point B.

a Show that while P is moving up the plane its deceleration is $\frac{13g}{15}$.

b Find, to three significant figures, the distance AB.

c Find, to three significant figures, the time taken for P to move from A to B.

d Find the speed of P when it returns to A.

Solution:

a

Let mass of particle be m.

$R(\nwarrow)$, $\quad R - mg\cos\alpha = 0$

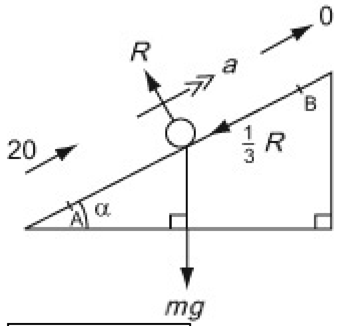
$R = \frac{4mg}{5}$

$R(\nearrow)$, $\quad -mg\sin\alpha - \frac{1}{3}R = ma$

$-\frac{3\cancel{m}g}{5} - \frac{1}{3} \times \frac{4\cancel{m}g}{5} = \cancel{m}a$

$-\frac{13g}{15} = a$

The deceleration is $\frac{13g}{15}$.



$\tan\alpha$	$= \frac{3}{4}$
$\sin\alpha$	$= \frac{3}{5}$
$\cos\alpha$	$= \frac{4}{5}$

b

$u = 20$, $v = 0$, $a = -\frac{13g}{15}$, $s = ?$

$v^2 = u^2 + 2as$

$0 = 20^2 - \frac{26g}{15}s \Rightarrow s = \frac{6000}{26g} = 23.5$ m (3 s.f.)

$AB = 23.5$ m (3 s.f.)

c

$u = 20$, $v = 0$, $a = \frac{-13g}{15}$, $t = $?

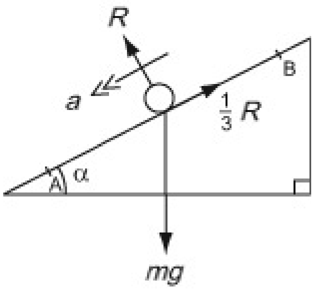
$v = u + a\ t$

$0 = 20 - \frac{13g\ t}{15}$

$t = \frac{300}{13g} = 2.35$ s (3 s.f.)

d

$R = \frac{4\,m\ g}{5}$ as before



R (↙) , $m\ g\ \sin\ \alpha - \frac{1}{3}R = m\ a$

$\frac{3\,m\ g}{5} - \frac{1}{3} - \frac{4\,m\ g}{5} = m\ a$

$\frac{g}{3} = a$

$u = 0$, $a = \frac{g}{3}$, $s = \frac{6000}{26g}$, $v = $?

$v^2 = u^2 + 2a\ s$

$v^2 = 0 + \frac{2g}{3} \times \frac{6000}{26g} = \frac{4000}{26}$

$u = 12.4\ \text{m s}^{-1}$ (3 s.f.)

The speed of the particle as it passes A on the way down is 12.4 m s^{-1} (3 s.f.).

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Dynamics of a particle moving in a straight line
Exercise F, Question 1

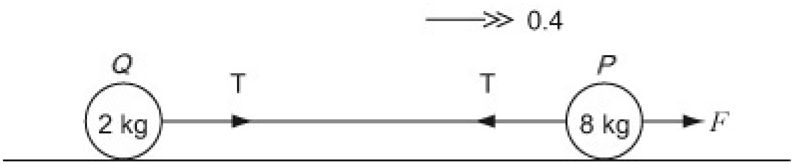
Question:

Two particles P and Q of mass 8 kg and 2 kg respectively, are connected by a light inextensible string. The particles are on a smooth horizontal plane. A horizontal force of magnitude F is applied to P in a direction away from Q and when the string is taut the particles move with acceleration 0.4 m s^{-2}.

a Find the value of F.

b Find the tension in the string.

Solution:



For whole system:

a

$$\begin{aligned} R(\rightarrow): \quad F &= (2+8) \times 0.4 \\ &= 4 \end{aligned}$$

Hence F is 4 N.

b For Q:

$$\begin{aligned} R(\rightarrow): \quad T &= 2 \times 0.4 \\ &= 0.8 \end{aligned}$$

The tension in the string is 0.8 N.

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Dynamics of a particle moving in a straight line
Exercise F, Question 2

Question:

Two bricks P and Q, each of mass 5 kg, are connected by a light inextensible string. Brick P is held at rest and Q hangs freely, vertically below P. A force of 200 N is then applied vertically upwards to P causing it to accelerate at 1.2 m s^{-2}. Assuming there is a resistance to the motion of each of the bricks of magnitude R N, find

a the value of R,

b the tension in the string connecting the bricks.

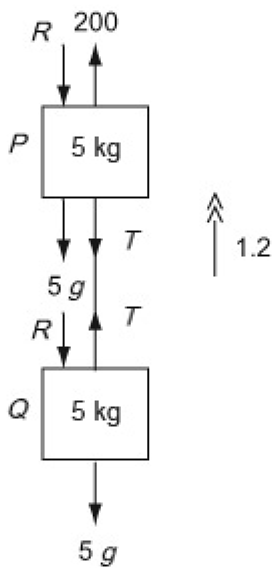
Solution:

a

For whole system

R (↑) ,

$$\begin{aligned} 200 - 2R - 10g &= 10 \times 1.2 \\ 200 - 2R - 98 &= 12 \\ 2R &= 90 \\ R &= 45 \end{aligned}$$



b For Q only:

$$\begin{aligned} \text{R}(\uparrow),\quad T - R - 5g &= 5 \times 1.2 \\ T - 45 - 49 &= 6 \\ T &= 100 \end{aligned}$$

∴ The tension in the string is 100 N.

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Dynamics of a particle moving in a straight line
Exercise F, Question 3

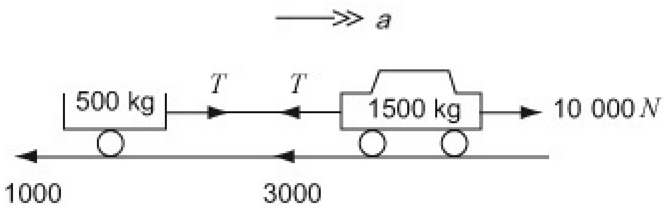
Question:

A car of mass 1500 kg is towing a trailer of mass 500 kg along a straight horizontal road. The car and the trailer are connected by a light inextensible tow-bar. The engine of the car exerts a driving force of magnitude 10 000 N and the car and the trailer experience resistances of magnitudes 3000 N and 1000 N respectively.

a Find the acceleration of the car.

b Find the tension in the tow-bar.

Solution:



a For whole system:

$$\begin{aligned} R(\rightarrow), \quad 10\,000 - 1000 - 3000 &= 2000a \\ a &= 3 \end{aligned}$$

The acceleration of the car is 3 m s^{-2}.

b For trailer:

$$\begin{aligned} R(\rightarrow), \quad T - 1000 &= 500 \times 3 \\ T &= 2500 \end{aligned}$$

The tension in the tow-bar is 2500 N.

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Dynamics of a particle moving in a straight line
Exercise F, Question 4

Question:

Two particles *A* and *B* of mass 4 kg and 3 kg respectively are connected by a light inextensible string which passes over a small smooth fixed pulley. The particles are released from rest with the string taut.

a Find the tension in the string.

When *A* has travelled a distance of 2 m it strikes the ground and immediately comes to rest.

b Assuming that *B* does not hit the pulley find the greatest height that *B* reaches above its initial position.

Solution:

a

For A : R ($\downarrow$) , $4g - T = 4a$ (1)

For B : R ($\uparrow$) , $T - 3g = 3a$ (2)

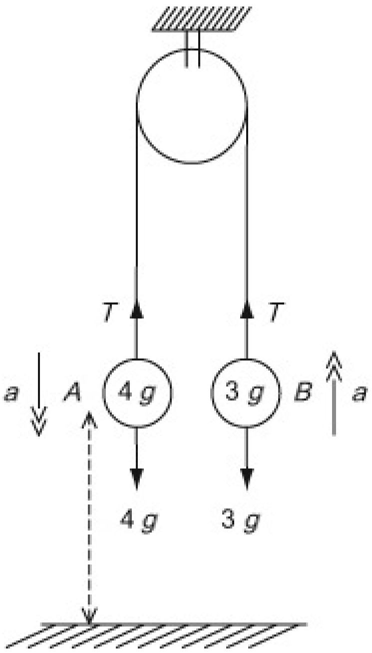
(1) + (2) : $4g - 3g = 7a$

$\frac{1}{7}g = a$

i.e. $a = 1.4 \text{ m s}^{-2}$

$\therefore T = 3g + \frac{3g}{7}$

$= 33.6$ N (3 s.f.)



The tension in the string is 33.6 N (3 s.f.).

b For *A*: ($\downarrow$) $u = 0$, $s = 2$, $a = \frac{5}{7}$, $v = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + \left(2 \times \frac{5}{7} \times 2 \right)$$

$$= \frac{4g}{7}$$

For *B*: ($\uparrow$) $u^2 = \frac{4g}{7}$, $v = 0$, $a = -g$, $s = ?$

$v^2 = u^2 + 2as$

$0 = \frac{4g}{7} - 2gs \Rightarrow s = \frac{2}{7}$

$\therefore$ Height above initial position is $2\frac{2}{7}$ m.

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Dynamics of a particle moving in a straight line
Exercise F, Question 5

Question:

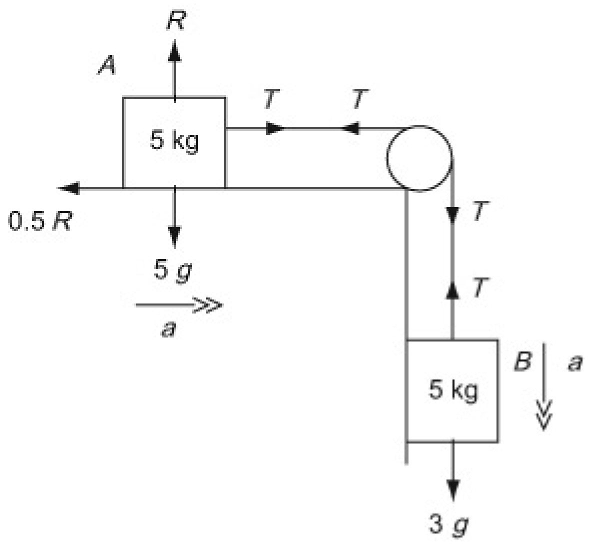
Two particles *A* and *B* of mass 5 kg and 3 kg respectively are connected by a light inextensible string. Particle *A* lies on a rough horizontal table and the string passes over a small smooth pulley which is fixed at the edge of the table. Particle *B* hangs freely. The coefficient of friction between *A* and the table is 0.5. The system is released from rest. Find

a the acceleration of the system,

b the tension in the string,

c the magnitude of the force exerted on the pulley by the string.

Solution:



a For *A*:

$$\text{R}(\uparrow),\ R-5g=0$$
$$R=49$$

$$\text{R}(\rightarrow),\quad T-0.5R=5a$$
$$\text{i.e. } T-24.5=5a \qquad (1)$$

$$\text{For } B:\quad \text{R}(\downarrow),\quad 3g-T=3a$$
$$29.4-T=3a \qquad (2)$$

$$(1)+(2):\quad 29.4-24.5=8a$$
$$4.9=8a$$
$$0.6125=a$$

The acceleration of the system is 0.613 m s^{-2} (3 s.f.) or 0.61 m s^{-2} (2 s.f.) .

b

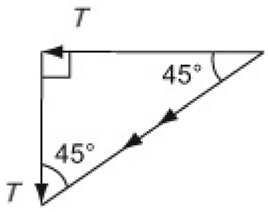
$$T - 24.5 = 5 \times 0.6125$$
$$T = 27.5625$$

The tension in the string is 27.6 N (3 s.f.) or 28 N (2 s.f.).

c

By Pythagoras,

$$F^2 = T^2 + T^2 = 2T^2$$
$$F = T\sqrt{2}$$
$$= 38.979 \ldots$$



The magnitude of the force exerted on the pulley is 39 N (2 s.f.) or 39.0 N (3 s.f.).

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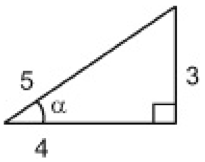
Dynamics of a particle moving in a straight line
Exercise F, Question 6

Question:

Two particles P and Q of equal mass are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a smooth inclined plane. The plane is inclined to the horizontal at an angle α where tan $\alpha = 0.75$. Particle P is held at rest on the inclined plane at a distance of 2 m from the pulley and Q hangs freely on the edge of the plane at a distance of 3 m above the ground with the string vertical and taut. Particle P is released. Find the speed with which it hits the pulley.

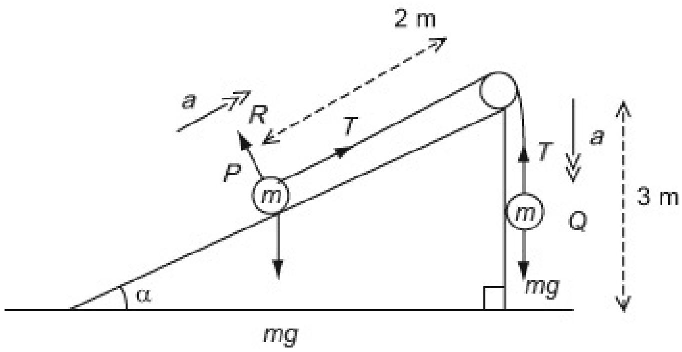
Solution:

$\tan \alpha = \frac{3}{4}$



$\sin \alpha = \frac{3}{5}$

$\cos \alpha = \frac{4}{5}$



For P : R (↗) , $T - mg\sin\alpha = ma$

$$T - \frac{3mg}{5} = ma \qquad (1)$$

For Q : R (↓) , $mg - T = ma \qquad (2)$

$(1) + (2)$: $\cancel{m}g - \frac{3\cancel{m}g}{5} = 2\cancel{m}a$

$$\frac{g}{5} = a$$

For P: $u = 0$, $a = \frac{g}{5}$, $s = 2$, $v = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + \frac{2g}{5} \times 2$$

$$v = \sqrt{\frac{4g}{5}} = 2.8 \text{ m s}^{-1}$$

P hits the pulley with speed 2.8 m s^{-1}.

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Dynamics of a particle moving in a straight line
Exercise F, Question 7

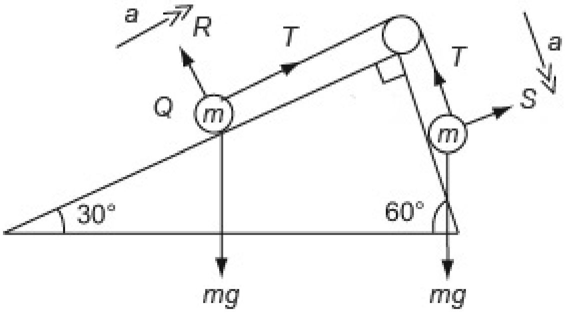
Question:

Two particles P and Q of equal mass are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed wedge. One face of the wedge is smooth and inclined to the horizontal at an angle of 30° and the other face of the wedge is rough and inclined to the horizontal at an angle of 60°. Particle P lies on the rough face and particle Q lies on the smooth face with the string connecting them taut. The coefficient of friction between P and the rough face is 0.5.

a Find the acceleration of the system.

b Find the tension in the string.

Solution:



a

For Q : R ($\rightarrow$) , $T - mg\cos 60° = ma$ (1)

For P : R ($\nearrow$) , $S = mg\cos 60°$

R ($\searrow$) , $mg\cos 30° - \frac{1}{2}S - T = ma$

$mg\cos 30° - \frac{1}{2}mg\cos 60° - T = ma$ (2)

$(1) + (2)$: $\cancel{m}g\frac{\sqrt{3}}{2} - \frac{3\cancel{m}g}{4} = 2\cancel{m}a$

$\boxed{\cos 30° = \frac{\sqrt{3}}{2} \quad \cos 60° = \frac{1}{2}}$

$\frac{g}{8}(2\sqrt{3} - 3) = a$

$\therefore$ The acceleration of the system is 0.569 m s^{-2} (3 s.f.) or 0.57 m s^{-2} (2 s.f.)

b From (1),

$T = \frac{1}{2}mg + \frac{mg}{8}(2\sqrt{3} - 3)$

$= \frac{mg}{8}(1 + 2\sqrt{3})$

The tension in the string is $\frac{mg}{8}\left(1 + 2\sqrt{3}\right) - 0.56mg$.

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Dynamics of a particle moving in a straight line
Exercise F, Question 8

Question:

A van of mass 900 kg is towing a trailer of mass 500 kg up a straight road which is inclined to the horizontal at an angle α where tan α = 0.75. The van and the trailer are connected by a light inextensible tow-bar. The engine of the van exerts a driving force of magnitude 12 kN and the van and the trailer experience resistances to motion of magnitudes 1600 N and 600 N respectively.

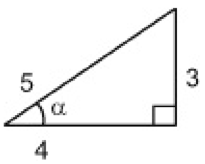
a Find the acceleration of the van.

b Find the tension in the tow-bar.

Solution:

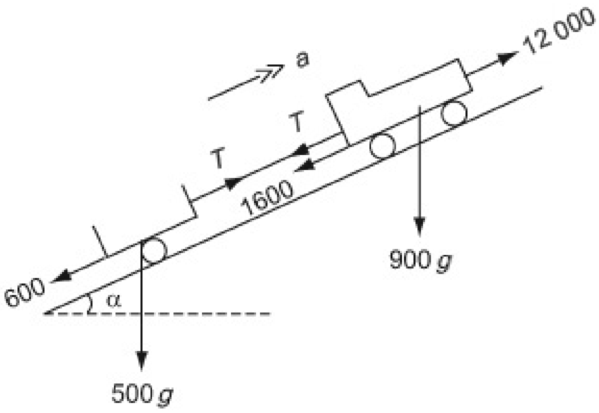
a

$\tan \alpha = 0.75 = \frac{3}{4}$



$\sin \alpha = \frac{3}{5}$

$\cos \alpha = \frac{4}{5}$



For whole system:

$$\begin{aligned} R(\nearrow),\ 12\,000 - 1600 - 600 - 1400g\sin \alpha &= 1400a \\ 9800 - 1400g\sin \alpha &= 1400a \\ 7 - 5.88 &= a \\ 1.12 &= a \end{aligned}$$

The acceleration of the van is 1.12 m s^{-2}.

b For trailer:

$$\begin{aligned} R(\nearrow),\quad T - 600 - 500g\sin \alpha &= 500 \times 1.12 \\ T - 600 - 2940 &= 560 \\ T &= 4100 \end{aligned}$$

The tension in the tow-bar is 4100 N.

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Dynamics of a particle moving in a straight line
Exercise F, Question 9

Question:

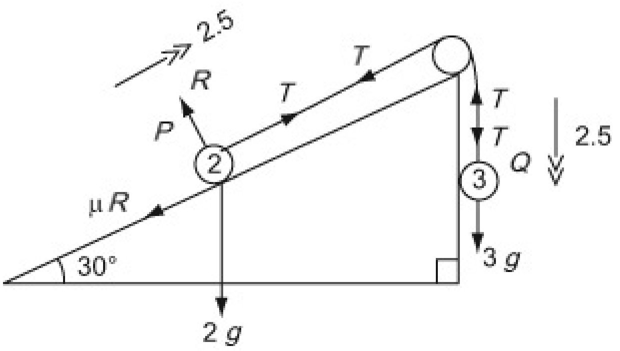
Two particles *P* and *Q* of mass 2 kg and 3 kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle of 30°. Particle *P* is held at rest on the inclined plane and *Q* hangs freely on the edge of the plane with the string vertical and taut. Particle *P* is released and it accelerates up the plane at 2.5 m s^{-2}. Find

a the tension in the string,

b the coefficient of friction between *P* and the plane,

c the force exerted by the string on the pulley.

Solution:



a For *P*:

$$R(\nwarrow),\quad R - 2g\cos 30° = 0 \qquad \boxed{\cos 30° = \frac{\sqrt{3}}{2}}$$
$$R = g\sqrt{3}$$
$$R(\nearrow),\quad T - \mu g\sqrt{3} - 2g\cos 60° = 2 \times 2.5 \qquad \boxed{\cos 60° = \frac{1}{2}}$$
$$T - \mu g\sqrt{3} - g = 5 \qquad (1)$$

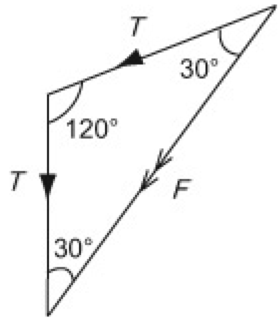
$$\text{For } Q: R(\downarrow),\ 3g - T = 3 \times 2.5$$
$$3g - T = 7.5 \qquad (2)$$

$\therefore T = 21.9$
The tension is 21.9 N.

b
$$(1) + (2) \quad 2g - \mu g\sqrt{3} = 12.5$$
$$\mu g\sqrt{3} = 7.1$$
$$\mu = \frac{7.1}{g\sqrt{3}} = 0.418\ (3\text{ s.f.})\ \text{or}\ 0.42\ (2\text{ s.f.})$$

The coefficient of friction is 0.42 (2 s.f.).

c



$$F = 2T \cos 30°$$
$$= 43.8 \cos 30°$$
$$= 37.9 \text{ N } (3 \text{ s.f.}) \text{ or } 38 \text{ N } (2 \text{ s.f.})$$

The force exerted by the string on the pulley is 38 N (2 s.f.).

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Dynamics of a particle moving in a straight line
Exercise F, Question 10

Question:

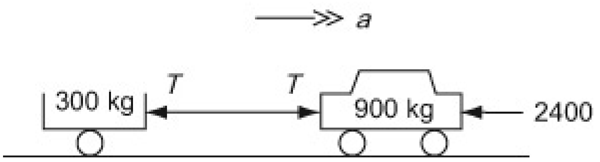
A car of mass 900 kg is towing a trailer of mass 300 kg along a straight horizontal road. The car and the trailer are connected by a light inextensible tow-bar and when the speed of the car is 20 m s^{-1} the brakes are applied. This produces a braking force of 2400 N. Find

a the deceleration of the car,

b the magnitude of the force in the tow-bar,

c the distance travelled by the car before it stops.

Solution:



a For whole system:

$$\begin{aligned} \mathrm{R}(\rightarrow), \quad -2400 &= 1200a \\ a &= -2 \end{aligned}$$

The deceleration of the car is 2 m s^{-2}.

b For trailer:

$$\begin{aligned} \mathrm{R}(\rightarrow), \quad -T &= 300 \times -2 \\ \Rightarrow T &= 600 \end{aligned}$$

The thrust in the tow-bar is 600 N.

c $u = 20$, $a = -2$, $v = 0$, $s = ?$

$$\begin{aligned} v^2 &= u^2 + 2as \ (\rightarrow), \\ 0^2 &= 20^2 + 2(-2)s \\ 0 &= 400 - 4s \\ s &= 100 \end{aligned}$$

The car stops in 100 m.

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Dynamics of a particle moving in a straight line
Exercise G, Question 1

Question:

A ball of mass 0.5 kg is at rest when it is struck by a bat and receives an impulse of 15 N s. Find its speed immediately after it is struck.

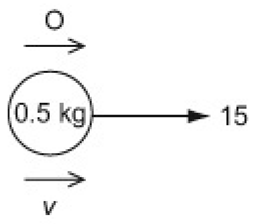
Solution:

$(\rightarrow)$:

$15 = 0.5v$

$30 = v$

Its initial speed is 30 m s $^{-1}$.



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Dynamics of a particle moving in a straight line
Exercise G, Question 2

Question:

A ball of mass 0.3 kg moving along a horizontal surface hits a fixed vertical wall at right angles with speed 3.5 m s $^{-1}$. The ball rebounds at right angles to the wall. Given that the magnitude of the impulse exerted on the ball by the wall is 1.8 N s, find the speed of the ball just after it has hit the wall.

Solution:

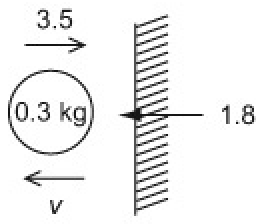
(←) :

$1.8 = 0.3 (v - - 3.5)$

$6 = v + 3.5$

$2.5 = v$

The ball rebounds with speed 2.5 m s $^{-1}$.



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Dynamics of a particle moving in a straight line
Exercise G, Question 3

Question:

A ball of mass 0.2 kg is dropped from a height of 2.5 m above horizontal ground. After hitting the ground it rises to a height of 1.8 m above the ground. Find the magnitude of the impulse received by the ball from the ground.

Solution:

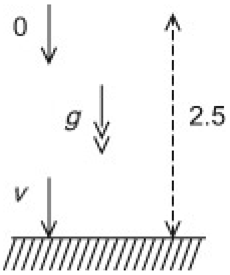
$u = 0$, $a = g$, $s = 2.5$, $v =$?

$v^2 = u^2 + 2as \quad (\downarrow)$

$v^2 = 0^2 + 2 \times 9.8 \times 2.5$

$= 49$

$v = 7 \ (\text{not} - 7)$



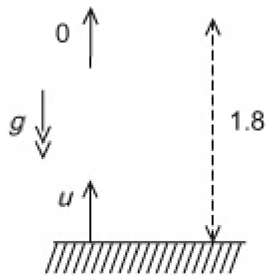
$(\uparrow)$: $v = 0$, $a = -g$, $s = 1.8$, $u =$?

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2(-9.8) \times 1.8$

$u^2 = 35.28$

$u = 5.94 \ (\text{not} - 5.94)$

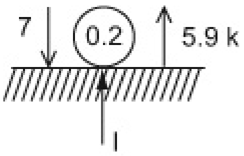


Take downwards as positive.

$(\uparrow)$: $I = 0.2(5.94 - 7)$

$= -2.588$

The magnitude of the impulse on the ball is 2.59 N s (3 s.f.).



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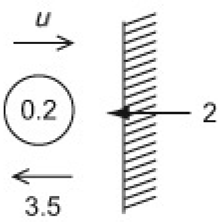
Dynamics of a particle moving in a straight line
Exercise G, Question 4

Question:

A ball of mass 0.2 kg, moving along a horizontal surface, hits a fixed vertical wall at right angles. The ball rebounds at right angles to the wall with speed 3.5 m s $^{-1}$. Given that the magnitude of the impulse exerted on the ball by the wall is 2 N s, find the speed of the ball just before it hit the wall.

Solution:



$(\leftarrow)$:

$2 = 0.2 (3.5 - - u)$

$10 = 3.5 + u$

$u = 6.5$

The speed of the ball before it hits the wall is 6.5 m s $^{-1}$.

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise G, Question 5

Question:

A toy car of mass 0.2 kg is pushed from rest along a smooth horizontal floor by a horizontal force of magnitude 0.4 N for 1.5 s. Find its speed at the end of the 1.5 s.

Solution:

$$\begin{aligned} Ft &= mv - mu \\ 0.4 \times 1.5 &= 0.2(v - 0) \\ 0.6 &= 0.2v \\ 3 &= v \end{aligned}$$

The speed of the toy car is 3 m s $^{-1}$.

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Dynamics of a particle moving in a straight line
Exercise H, Question 1

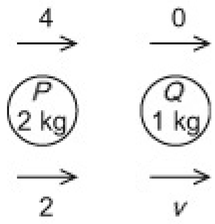
Question:

A particle P of mass 2 kg is moving on a smooth horizontal plane with speed 4 m s^{-1}. It collides with a second particle Q of mass 1 kg which is at rest. After the collision P has speed 2 m s^{-1} and it continues to move in the same direction. Find the speed of Q after the collision.

Solution:

Conservation of Momentum ($\rightarrow$)

$$\begin{aligned} (2 \times 4) + (1 \times 0) &= (2 \times 2) + (1 \times v) \\ 8 &= 4 + v \\ 4 &= v \end{aligned}$$



The speed of Q is 4 m s^{-1}.

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Dynamics of a particle moving in a straight line
Exercise H, Question 2

Question:

A railway truck of mass 25 tonnes moving at 4 m s^{-1} collides with a stationary truck of mass 20 tonnes. As a result of the collision the trucks couple together. Find the common speed of the trucks after the collision.

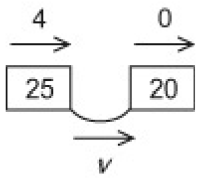
Solution:

Conservation of Momentum $(\rightarrow)$

$$(4 \times 25) + (20 \times 0) = 45v$$

$$100 = 45v$$

$$\frac{20}{9} = v$$



The common speed of the trucks is $2\frac{2}{9}$ m s^{-1}.

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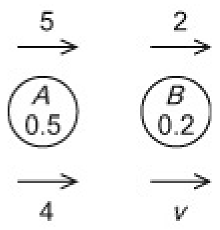
Dynamics of a particle moving in a straight line
Exercise H, Question 3

Question:

Particles A and B have mass 0.5 kg and 0.2 kg respectively. They are moving with speeds 5 m s^{-1} and 2 m s^{-1} respectively in the same direction along the same straight line on a smooth horizontal surface when they collide. After the collision A continues to move in the same direction with speed 4 m s^{-1}. Find the speed of B after the collision.

Solution:

Conservation of Momentum $(\rightarrow)$



$(0.5 \times 5) + (0.2 \times 2)$	$= (0.5 \times 4) + (0.2 \times v)$
$2.5 + 0.4$	$= 2.0 + 0.2v$
0.9	$= 0.2v$
4.5	$= v$

The speed of B after the collision is 4.5 m s^{-1}.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise H, Question 4

Question:

A particle of mass 2 kg is moving on a smooth horizontal plane with speed 4 m s^{-1}. It collides with a second particle of mass 1 kg which is at rest. After the collision the particles join together.

a Find the common speed of the particles after the collision.

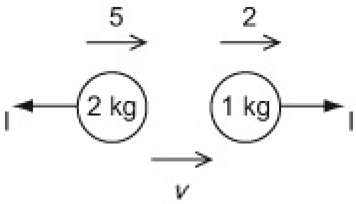
b Find the magnitude of the impulse in the collision.

Solution:

a

Conservation of Momentum $(\rightarrow)$

$$(2 \times 4) + (1 \times 0) = 3v$$
$$8 = 3v$$
$$\frac{8}{3} = v$$



The common speed of the particles is $2\frac{2}{3}$ m s^{-1}.

b For 1 kg: $(\rightarrow)$ $I = 1 \times v = 2\frac{2}{3}$ m s^{-1}

The impulse in the collision is $2\frac{2}{3}$ m s^{-1} N s.

$$\left[\text{For 2 kg: } (\leftarrow) \quad I = 2(-v - -4) = 2\left(-2\frac{2}{3} + 4\right) = 2 \times 1\frac{1}{3} = 2\frac{2}{3} \right]$$

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise H, Question 5

Question:

Two particles *A* and *B* of mass 2 kg and 5 kg respectively are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speeds of *A* and *B* are 6 m s^{-1} and 4 m s^{-1} respectively. After the collision the direction of motion of *A* is reversed and its speed is 1.5 m s^{-1}. Find

a the speed and direction of *B* after the collision,

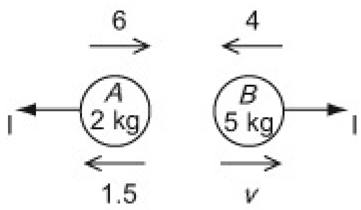
b the magnitude of the impulse given by *A* to *B* in the collision.

Solution:

a

Conservation of Momentum ($\rightarrow$)

$$
\begin{aligned}
(2 \times 6) + (5 \times -4) &= (2 \times -1.5) + 5v \\
12 - 20 &= -3 + 5v \\
-5 &= 5v \\
v &= -1
\end{aligned}
$$



The speed of *B* is 1 m s^{-1} and its direction of motion is unchanged by the collision.

b

$$
\begin{aligned}
\text{For } A \ (\leftarrow): \ I &= 2(1.5 - -6) \\
&= 2 \times 7.5 \\
&= 15
\end{aligned}
$$

$$
\begin{aligned}
[\text{ or for } B \ (\rightarrow): \ I &= 5(v - -4) \\
&= 5(-1 + 4) \\
&= 15 \ ]
\end{aligned}
$$

The magnitude of the impulse given to *B* is 15 N s.

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Dynamics of a particle moving in a straight line
Exercise H, Question 6

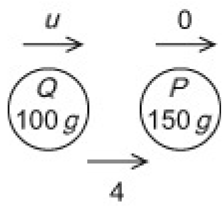
Question:

A particle *P* of mass 150 g is at rest on a smooth horizontal plane. A second particle *Q* of mass 100 g is projected along the plane with speed u m s^{-1} and collides directly with *P*. On impact the particles join together and move on with speed 4 m s^{-1}. Find the value of u.

Solution:

Conservation of Momentum ($\rightarrow$)

$$100u + (150 \times 0) = 250 \times 4$$
$$100u = 1000$$
$$u = 10$$



The value of u is 10.

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Dynamics of a particle moving in a straight line
Exercise H, Question 7

Question:

A particle A of mass $4m$ is moving along a smooth horizontal surface with speed $2u$. It collides with another particle B of mass $3m$ which is moving with the same speed along the same straight line but in the opposite direction. Given that A is brought to rest by the collision, find

a the speed of B after the collision and state its direction of motion,

b the magnitude of the impulse given by A to B in the collision.

Solution:

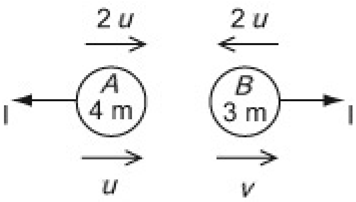
a

Conservation of Momentum ($\rightarrow$)

$$(4m \times 2u) + (3m \times -2u) = (4m \times 0) + 3m\ v$$

$$8\cancel{m}\ u - 6\cancel{m}\ u = 3\cancel{m}\ v$$

$$\frac{2u}{3} = v$$



The speed of B after the collision is $\frac{2u}{3}$ and its direction of motion is reversed by the collision.

b

For A ($\leftarrow$): $I = 4m(0 - -2u)$

$= 8m\ u$

[or For B ($\rightarrow$): $I = 3m(v - -2u)$

$= 3m\left(\frac{2u}{3} + 2u\right)$

$= 2m\ u + 6m\ u = 8m\ u$]

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Dynamics of a particle moving in a straight line
Exercise H, Question 8

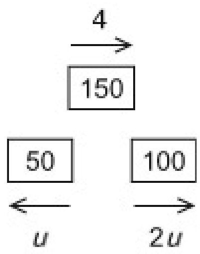
Question:

An explosive charge of mass 150 g is designed to split into two parts, one with mass 100 g and the other with mass 50 g. When the charge is moving at 4 m s^{-1} it splits and the larger part continues to move in the same direction whilst the smaller part moves in the opposite direction. Given that the speed of the larger part is twice the speed of the smaller part, find the speeds of the two parts.

Solution:

Conservation of Momentum ($\rightarrow$)

(150×4)	$= (100 \times 2u) + (50 \times -u)$
600	$= 200u - 50u$
600	$= 150u$
4	$= u$



The larger has speed 8 m s^{-1} and the smaller part has speed 4 m s^{-1}.

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Dynamics of a particle moving in a straight line
Exercise H, Question 9

Question:

Two particles *P* and *Q* of mass *m* and *km* respectively are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speeds of *P* and *Q* are 3*u* and *u* respectively. After the collision the direction of motion of both particles is reversed and the speed of each particle is halved.

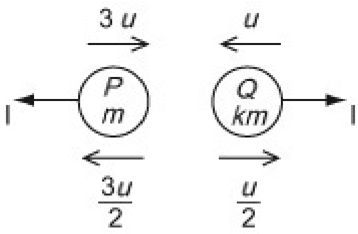
a Find the value of *k*.

b Find, in terms of *m* and *u*, the magnitude of the impulse given by *P* to *Q* in the collision.

Solution:

a

Conservation of Momentum $(\rightarrow)$



$$(m \times 3u) + (km \times -u) = \left(m \times -\frac{3u}{2}\right) + \left(km \times \frac{u}{2}\right)$$

$$3mu - kmu = -3\frac{mu}{2} + \frac{kmu}{2}$$

$$6 - 2k = -3 + k$$

$$9 = 3k$$

$$3 = k$$

The value of *k* is 3.

b

For P $(\leftarrow)$: $I = m\left(\frac{3u}{2} - -3u\right) = \frac{9mu}{2}$

[or For Q: $(\rightarrow)$ $I = km\left(\frac{u}{2} - -u\right) = 3m \times \frac{3u}{2} = \frac{9mu}{2}$]

The magnitude of the impulse is $\frac{9mu}{2}$.

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Dynamics of a particle moving in a straight line
Exercise H, Question 10

Question:

Two particles A and B of mass 4 kg and 2 kg respectively are connected by a light inextensible string. The particles are at rest on a smooth horizontal plane with the string slack. Particle A is projected directly away from B with speed u m s^{-1}. When the string goes taut the impulse transmitted through the string has magnitude 6 N s. Find

a the common speed of the particles just after the string goes taut,

b the value of u.

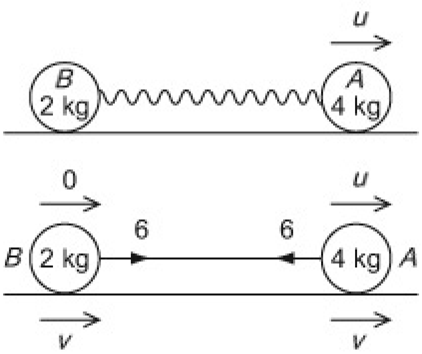
Solution:

a

For B $(\rightarrow)$:

$6 = 2(v - 0)$

$3 = v$



The common speed is 3 m s^{-1}.

b Conservation of Momentum $(\rightarrow)$

$4u = 2v + 4v = 6 \times 3 = 18$

$u = 4.5$

[or For A : $(\rightarrow)$ $-6 = 4(3 - u)$

$-1.5 = 3 - u$

$u = 4.5$]

The value of u is 4.5.

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Dynamics of a particle moving in a straight line
Exercise H, Question 11

Question:

Two particles P and Q of mass 3 kg and 2 kg respectively are moving along the same straight line on a smooth horizontal surface. The particles collide. After the collision both the particles are moving in the same direction, the speed of P is 1 m s^{-1}and the speed of Q is 1.5 m s^{-1}. The magnitude of the impulse of P on Q is 9 N s. Find

a the speed and direction of P before the collision,

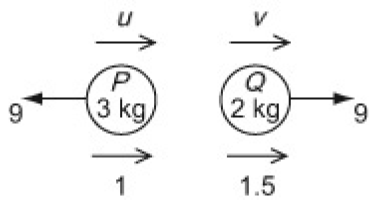
b the speed and direction of Q before the collision.

Solution:

a

For P : ($\leftarrow$)

$$\begin{aligned} 9 &= 3(-1 - -u) \\ 9 &= 3(-1 + u) \\ 3 &= -1 + u \\ 4 &= u \end{aligned}$$



The speed of P before the collision is 4 m s^{-1} and it was moving in the same direction as it was after the collision.

b For Q : ($\rightarrow$)

$$\begin{aligned} 9 &= 2(1.5 - v) \\ 9 &= 3 - 2v \\ 2v &= -6 \\ v &= -3 \end{aligned}$$

[or Conservation of Momentum ($\rightarrow$)

$$\begin{aligned} 3u + 2v &= (3 \times 1) + (2 \times 1.5) \\ 12 + 2v &= 3 + 3 = 6 \\ 2v &= -6 \\ v &= -3 \,] \end{aligned}$$

The speed of Q before the collision was 3 m s^{-1} and it was moving in the opposite direction to its direction after the collision.

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Dynamics of a particle moving in a straight line
Exercise H, Question 12

Question:

Two particles A and B are moving in the same direction along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speed of B is 1.5 m s^{-1}. After the collision the direction of motion of both particles is unchanged, the speed of A is 2.5 m s^{-1}and the speed of B is 3 m s^{-1}. Given that the mass of A is three times the mass of B,

a find the speed of A before the collision.

Given that the magnitude of the impulse on A in the collision is 3 N s.

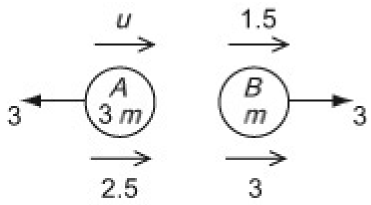
b find the mass of A.

Solution:

a

Conservation of Momentum $(\rightarrow)$

$$3m\ u + 1.5m = (3m \times 2.5) + (m \times 3)$$
$$3\cancel{m}\ u + 1.5\cancel{m} = 7.5\cancel{m} + 3\cancel{m}$$
$$3u = 9$$
$$u = 3$$



The speed of A before the collision is 3 m s^{-1}.

b For B: $(\rightarrow)$

$$3 = m(3 - 1.5)$$
$$2 = m$$

[or For A: $\leftarrow$

$$3 = 3m(-2.5 - -u)$$
$$3 = 3m(-2.5 + 3)$$
$$1 = 0.5m$$
$$2 = m\ ]$$

The mass of A is 6 kg.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise I, Question 1

Question:

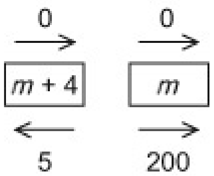
A bullet is fired by a gun which is 4 kg heavier than the bullet. Immediately after the bullet is fired, it is moving with speed 200 m s^{-1} and the gun recoils in the opposite direction with speed 5 m s^{-1}. Find

a the mass of the bullet,

b the mass of the gun.

Solution:

a



Draw a diagram showing velocities, before and after, with arrows.

$$\begin{aligned} \text{CLM} (\rightarrow) : \quad O &= 200m - 5(m+4) \\ 20 &= 195m \\ 0.103 &= m \end{aligned}$$

Momentum is conserved, solving for m.

Mass of bullet is 0.103 kg.

b Mass of gun is 4.103 kg.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise I, Question 2

Question:

A child of mass 25 kg moves from rest down a slide which is inclined to the horizontal at an angle of 35°. When the child has moved a distance of 4 m, her speed is 6 m s^{-1}. By modelling the child as a particle, find the coefficient of friction between the child and the slide.

Solution:

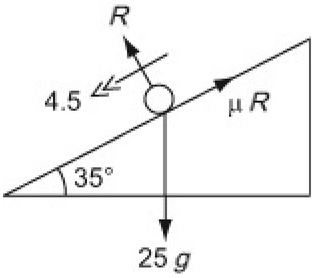
$v^2 = u^2 + 2as$

$6^2 = 0^2 + 2a \times 4$

$a = 4.5$

Choose the appropriate equation.

Solving for a



R (↖) $R - 25g \cos 35° = 0$

$R = 25g \cos 35°$

We need R to find the friction.

R (↙) $25g \cos 55° - \mu R = 25 \times 4.5$

$25g \cos 55° - \mu \times 25g \cos 35° = 112.5$

Resolve in the direction of the acceleration. Solve for μ.

$$\mu = \frac{25g \cos 55° - 112.5}{25g \cos 35°}$$

$= 0.14$ (2 s.f.)

Answer to 2 s.f. since $g = 9.8$ has been used.

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Dynamics of a particle moving in a straight line
Exercise I, Question 3

Question:

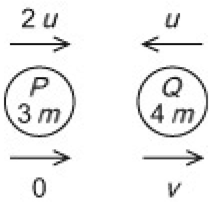
A particle P of mass $3m$ is moving along a straight line with constant speed $2u$. It collides with another particle Q of mass $4m$ which is moving with speed u along the same line but in the opposite direction. As a result of the collision P is brought to rest.

a Find the speed of Q after the collision and state its direction of motion.

b Find the magnitude of the impulse exerted by Q on P in the collision.

Solution:

a



A diagram showing all the velocities, before and after, with arrows.

CLM $(\rightarrow)$:

$$6mu - 4mu = 4mv$$

$$\frac{1}{2}u = v$$

Conserving momentum

b I = $(3m \times 2u - 0) = 6mu$ impulse *6 mu*

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Edexcel AS and A Level Modular Mathematics

Dynamics of a particle moving in a straight line
Exercise I, Question 4

Question:

A small box of mass 2 kg is projected with speed 2 m s^{-1} up a line of greatest slope of a rough plane. The plane is inclined to the horizontal at an angle α where tan α = 0.75. The coefficient of friction between the box and the plane is 0.4. The box is projected from the point *P* on the plane.

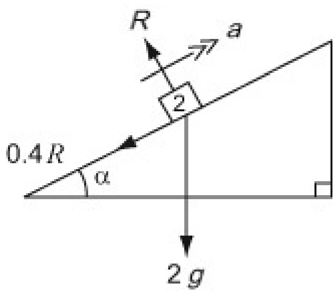
a Find the distance that the box travels up the plane before coming to rest.

b Show that the box will slide back down the plane.

c Find the speed of the box when it reaches the point *P*.

Solution:

a



R (↖) $R = 2g\cos\alpha = \frac{8g}{5}$

R is needed to find the friction.

R (↗)

$$-\frac{2R}{5} - 2g\sin\alpha = 2a$$

Note the signs.

$$-\frac{16g}{25} - \frac{6s}{5} = 2a$$

$$-\frac{23s}{25} = a$$

Solving for *a*.

$$v^2 = u^2 + 2as\ (\nearrow)$$

$$0 = 2^2 - 2 \times \frac{23s}{25} \times s$$

$$s = \frac{50}{23g} = 0.22\ m\ \ (2\text{ s.f.})$$

2 s.f. since *g* = 9 has been used.

b Wt component down plane $= 2g \times \frac{3}{5} = \frac{6g}{5}$

Limiting friction $= \frac{2}{5} \times R = \frac{16g}{25}$

Hence, net force down the plane $= \frac{6g}{5} - \frac{16g}{25}$

$= \frac{14s}{25}$ so it will slide back down

c

R (↙) $\frac{14g}{25} = 2a \Rightarrow a =$

$\frac{7g}{25}$

$v^2 = u^2 + 2as$ (↙) :

$v^2 = 2 \times \frac{7g}{25} \times \frac{50}{23g} = \frac{28}{23}$

2 s.f since g = 9.8 has been used.

v $= 1.1 \text{ m s}^{-1}$ (2 s.f.)

Finding the acceleration. 2 s.f since g = 9.8 has been used.

Solutionbank M1

Edexcel AS and A Level Modular Mathematics

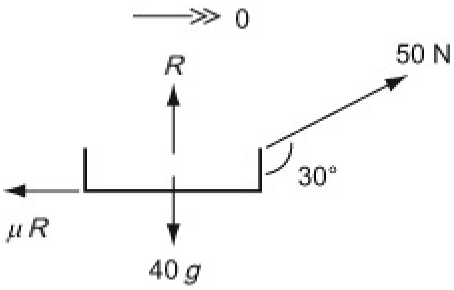
Dynamics of a particle moving in a straight line

Exercise I, Question 5

Question:

Peter is pulling Paul, who is on a toboggan, along a rough horizontal snow surface using a rope which makes an angle of 30° with the horizontal. Paul and the toboggan have a total mass of 40 kg and the toboggan is moving in a straight line with constant speed. The rope is modelled as a light inextensible string. Given that the tension in the rope is 50 N, find the coefficient of friction between the toboggan and the snow.

Solution:



R (↑) , $R + 50\cos 60° - 40g = 0$

$R = 367$ N

No vertical acceleration.

R (→) , $50\cos 30° - \mu \times 367 = 0$

$\Rightarrow \mu = 0.12$ (2 s.f.)

No horizontal acceleration as moving with constant speed.

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Dynamics of a particle moving in a straight line
Exercise I, Question 6

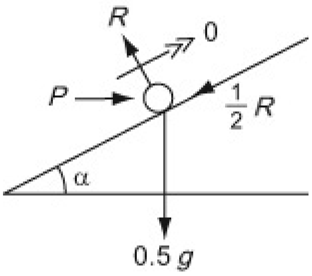
Question:

A particle of mass 0.5 kg is pushed up a line of greatest slope of a rough plane by a horizontal force of magnitude *P* N. The plane is inclined to the horizontal at an angle α where tan α = 0.75 and the coefficient of friction between *P* and the plane is 0.5. The particle moves with constant speed. Find

a the magnitude of the normal reaction between the particle and the plane

b the value of *P*.

Solution:



a

$$R(\uparrow),\quad R\cos\alpha - \frac{1}{2}R\sin\alpha - 0.5g = 0$$

$$R = g = 9.8\text{ N}$$

$\boxed{\text{Resolving at right angles to } P \text{ gives a quick solution.}}$

b

$$R(\rightarrow),\quad P - R\sin\alpha - \frac{1}{2}R\cos\alpha = 0$$

$\boxed{\text{Alternatively, resolve up the plane.}}$

$$\Rightarrow P = \frac{3}{5}R + \frac{2R}{5} = R = 9.8\text{ N}$$

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Dynamics of a particle moving in a straight line
Exercise I, Question 7

Question:

A pile driver consists of a pile of mass 200 kg which is knocked into the ground by dropping a driver of mass 1000 kg onto it. The driver is released from rest at a point which is 10 m vertically above the pile. Immediately after the driver impacts with the pile it can be assumed that they both move off with the same speed. By modelling the pile and the driver as particles,

a find the speed of the driver immediately before it hits the pile,

b find the common speed of the pile and driver immediately after the impact. The ground provides a constant resistance to the motion of the pile driver of magnitude 120,000 N.

c Find the distance that the pile driver is driven into the ground before coming to rest.

Solution:

a

$v^2 = u^2 + 2as$

$v^2 = 2 \times 10 \times 9.8$

$v = 14 \text{ m s}^{-1}$

Motion under gravity.

b

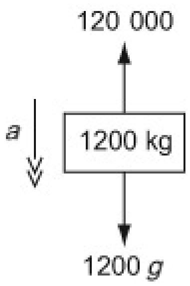
$CLM \ (\downarrow)$

$1000 \times 14 = 1200v$

$v = \frac{35}{3} \text{ m s}^{-1}$

Momentum is conserved.

c



R (↓) , first find the deceleration.

$1200g - 120\,000 = 1200a$

$a = -90.2$

$v^2 = u^2 + 20g \ (\downarrow)$

Using u = answer from part **b**.

$0^2 = \left(\frac{35}{3} \right)^2 - 2 \times 90.2 \times s$

$s = 0.75 \ m \ \ (2 \text{ s.f.})$

2 s.f. as $g = 9.8$ has been used.

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Dynamics of a particle moving in a straight line
Exercise I, Question 8

Question:

Particles P and Q of masses $2m$ and m respectively are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. They both hang at a distance of 2 m above horizontal ground. The system is released from rest.

a Find the magnitude of the acceleration of the system.

b Find the speed of P as it hits the ground.

Given that particle Q does not reach the pulley,

c find the greatest height that Q reaches above the ground.

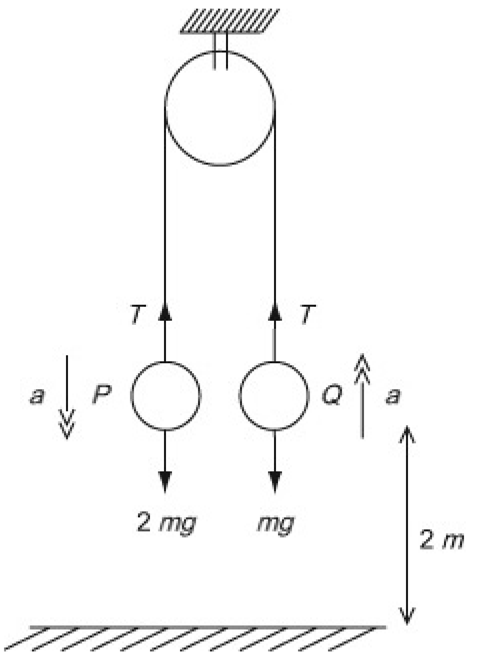
d State how you have used in your calculation,

(i) the fact that the string is inextensible,

(ii) the fact that the pulley is smooth.

Solution:

a



The diagram should show all the forces and the accelerations.

For P : R (↓) , $2m\,g - T = 2m\,a$

For Q : R (↑) , $T - m\,g = m\,a$

ADD , $mg = 3m\,a$

$a = \frac{1}{3}g$

Resolve in the direction of the acceleration for each mass.

b

For P $v^2 = u^2 + 2a\ g$ | The acceleration is constant.

$v^2 = 2 \times \frac{5}{3} \times 2$

$v = \sqrt{\frac{4g}{3}}$

$= 3.6 \text{ m s}^{-1}$ (2 s.f.)

c

For Q: R (↑) , $-\boxed{m}\ g = \boxed{m}\ a$ | Q now moves under gravity as the string is now slack.

$v^2 = u^2 + 2a\ s$ (↑) ,

$0 = \frac{4g}{3} - 2gs$

$s = \frac{2}{3}$m | Note that g cancels.

∴ Height above $= 2\frac{2}{3}$m

d

(i) In extensible string ⇒ acceleration of both masses in equal. | Learn this.

(ii) Smooth pulley ⇒ same tension in string either side of the pulley. | Learn this.

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Dynamics of a particle moving in a straight line
Exercise I, Question 9

Question:

The diagram shows two blocks *A* and *B*, of masses 20 kg and 30 kg respectively, inside a cage of mass 10 kg. Block *A* is on top of block *B*. The blocks are being lowered to the ground using a rope which is attached to the cage. The rope is modelled as a light inextensible string. Given that the blocks are moving vertically downwards with acceleration 0.8 m s^{-2}, find

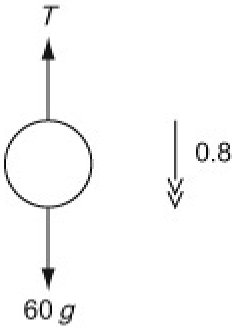
a the tension in the rope,

b the magnitude of the force that block *B* exerts on block *A*,

c the normal reaction between block *B* and the floor of the cage.

Solution:

a

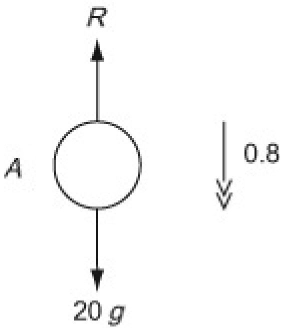


R (↓) , for whole system

$$
\begin{aligned}
60g - T &= 60 \times 0.8 \\
T &= 588 - 48 \\
&= 540 \text{ N}
\end{aligned}
$$

Note that the reactions between *A* and *B*, and between *B* and the cage cancel.

b
R (↓) , for *A*



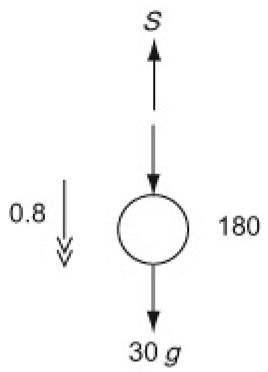
$$
\begin{aligned}
20g - R &= 20 \times 0.8 \\
R &= 196 - 16 \\
&= 180 \text{ N}
\end{aligned}
$$

Include only the forces acting on *A*.

c

R (↓) for *B*

Include only the forces acting on *B*.



$$30g + 180 - S = 30 \times 0.8$$
$$474 - S = 24$$
$$S = 450 \text{ N}$$

An alternative would be to consider the cage only.

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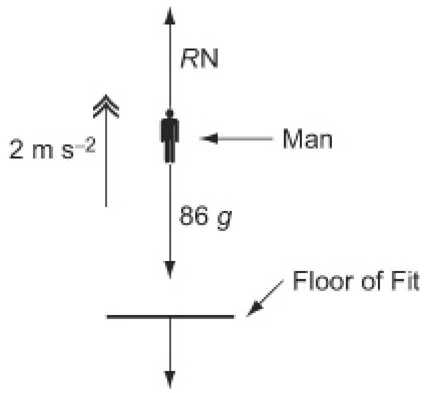
Dynamics of a particle moving in a straight line
Exercise I, Question 10

Question:

A man, of mass 86 kg, is standing in a lift which is moving upwards with constant acceleration 2 m s^{-2}. Find the magnitude and direction of the force that the man is exerting on the floor of the lift.

Solution:



By Newton's Third Law the action of the man on the floor and the reaction of the floor on the man are equal in magnitude, here labelled *R*, and in opposite directions.

$F = ma$

For the man

R (↑) $R - 86g = 86 \times 2$

$R = 86 \times 9.8 + 86 \times 2$

$= 1014.8 \approx 1000$

As the numerical value 9.8 has been used for *g* you should give your answer to 2 significant figures. You should not give any answer to an accuracy greater than the data you have used to calculate that answer.

The reaction on the man on the floor is of equal magnitude to the action of the floor on the man and in the opposite direction.
The force that the man exerts on the floor of the lift is of magnitude 1000 N (2 s.f.) and acts vertically downwards.

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Dynamics of a particle moving in a straight line
Exercise I, Question 11

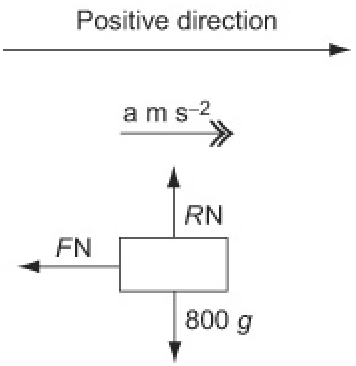
Question:

A car, of mass 800 kg and travelling along a straight horizontal road. A constant retarding force of F N reduces the speed of the car from 18 m s^{-1} to 12 m s^{-1} in 2.4 s. Calculate

a the value of F,

b the distance moved by the car in these 2.4 s.

Solution:



a

$u = 18$, $v = 12$, $t = 2.4$, $a =$?

$v = u + at$

$12 = 18 + 2.4a$

$a = \frac{12 - 18}{2.4} = -2.5$

$F = ma$

$-F = 800 \times -2.5 = -2000 \Rightarrow F = 2000$

You are going to have to use $F = ma$ to find F. So the first step of your solution must be to find a.

The retarding force is slowing the car down and is in the negative direction. So, in the positive direction, the force is $-F$.

b

$u = 18$, $v = 12$, $t = 2.4$, $s =$?

$s = \left(\frac{u + v}{2} \right) t$

$= \left(\frac{18 + 12}{2} \right) \times 2.4 = 15 \times 2.5 = 36$

You could use the value of a you found in part a and another formula. Unless it causes you extra work, it is safer to use the data in the question.

The distance moved by the car is 36 m.

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Dynamics of a particle moving in a straight line
Exercise I, Question 12

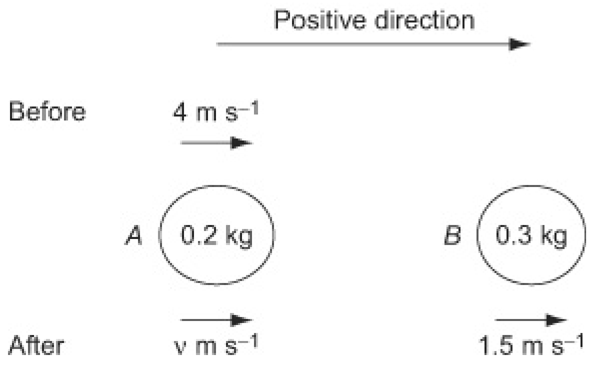
Question:

Two particles *A* and *B*, of mass 0.2 kg and 0.3 kg respectively, are free to move in a smooth horizontal groove. Initially *B* is at rest and *A* is moving toward *B* with a speed of 4 m s^{-1}. After the impact the speed of *B* is 1.5 m s^{-1}. Find

a the speed of *A* after the impact,

b the magnitude of the impulse of *B* on *A* during the impact.

Solution:



a

Conservation of linear momentum

$0.2 \times 4 = 0.2 \times v + 0.3 \times 1.5$

$0.8 = 0.2v + 0.45$

$v = \frac{0.8 - 0.45}{0.2} = 1.75$

A full formula for the conservation of momentum is $m_A u_A + m_B u_B = m_A v_A + m_B v_B$. In this case the velocity of *B* is zero.

The speed of *A* after the impact is 1.75 m s^{-1}.

b

Consider the impulse of *A*

$I = mv - mu$

$= 0.2 \times 1.75 - 0.2 \times 4$

$= 0.35 - 0.8 = -0.45$

The magnitude of the impulse of *B* on *A* during the impact is 0.45 N s.

It is a common mistake to mix up the particles. The impulses on the two particles are equal and opposite. Finding the magnitude of the impulse, you can consider either particle – either would give the same magnitude. However, you must work on only one single particle. Here you can work on *A* or *B* but not both.

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Dynamics of a particle moving in a straight line
Exercise I, Question 13

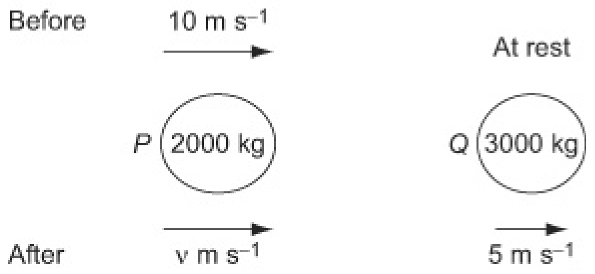
Question:

A railway truck *P* of mass 2000 kg is moving along a straight horizontal track with speed 10 m s^{-1}. The truck *P* collides with a truck *Q* of mass 3000 kg, which is at rest on the same track. Immediately after the collision *Q* moves with speed 5 m s^{-1}. Calculate

a the speed of *P* immediately after the collision

b the magnitude of the impulse exerted by *P* on *Q* during the collision.

Solution:



a Conservation of linear momentum

$$2000 \times 10 = 2000 \times v + 3000 \times 5$$
$$20\,000 = 2000v + 15\,000$$
$$v = \frac{20\,000 - 15\,000}{2000} = 2.5$$

The speed of *P* immediately after the collision is 2.5 m s^{-1}.

b

For *Q*,
$$I = mv - mu$$
$$I = 3000 \times 5 - 3000 \times 0 = 15\,000$$

To find the magnitude of the impulse you could consider **either** the change in momentum of *P* **or** the change of momentum of *Q*. You must not mix them up.

The magnitude of the impulse of *P* on *Q* is 15 000 N s.

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Dynamics of a particle moving in a straight line

Exercise I, Question 14

Question:

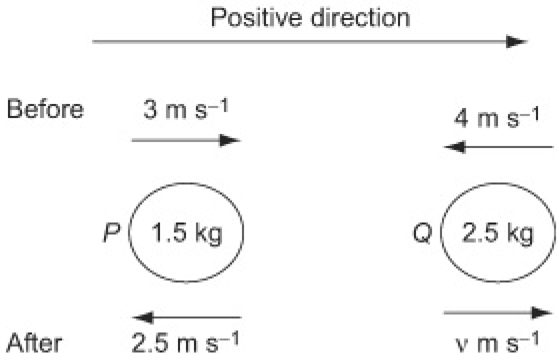
A particle P of mass 1.5 kg is moving along a straight horizontal line with speed 3 m s^{-1}. Another particle Q of mass 2.5 kg is moving, in the opposite direction, along the same straight line with speed 4 m s^{-1}. The particles collide. Immediately after the collision the direction of motion of P is reversed and its speed is 2.5 m s^{-1}.

a Calculate the speed of Q immediately after the impact.

b State whether or not the direction of motion of Q is changed by the collision.

c Calculate the magnitude of the impulse exerted by Q on P, giving the units of your answer.

Solution:



You do not know which direction Q will be moving in after the impact. Mark the unknown velocity as v m s^{-1} in the positive direction. After you have worked out v, the sign of v will tell you the direction Q is moving in.

a Conservation of linear momentum

$1.5 \times 3 + 2.5 \times (-4) = 1.5 \times (-2.5) + 2.5 \times v$

$$4.5 - 10 = -3.75 + 2.5v$$

$$2.5v = 4.5 - 10 + 3.75 = -1.75$$

$$v = -\frac{1.75}{2.5} = -0.7$$

The sign of v is negative, so Q is moving in the negative direction. It was moving in the negative direction before the impact and so its direction has not changed.

The speed of Q immediately after the impact is 0.7 m s^{-1}.

b The direction of Q is unchanged.

c For P, $I = mv - mu$

$I = 1.5 \times (-2.5) - 1.5 \times 3 = 8.25$

The magnitude of the impulse exerted by Q on P is 8.25 N s.

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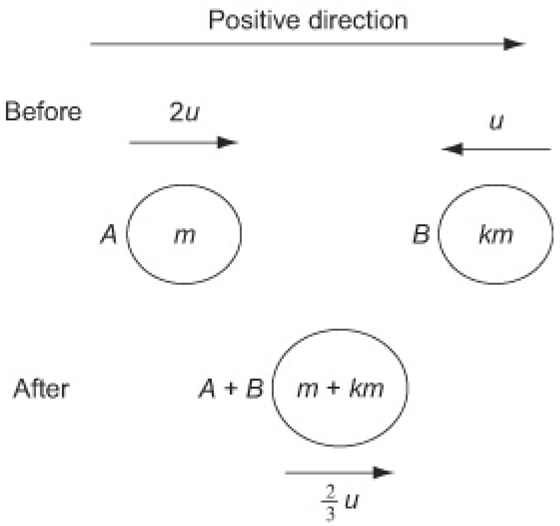
Dynamics of a particle moving in a straight line
Exercise I, Question 15

Question:

A particle A of mass m is moving with speed $2u$ in a straight line on a smooth horizontal table. It collides with another particle B of mass km which is moving in the same straight line on the table with speed u in the opposite direction to A. In the collision, the particles form a single particle which moves with speed $\frac{2}{3}u$ in the original direction of A's motion.

Find the value of k.

Solution:



After the collision A (of mass m) and B (of mass km) combine to form a single particle. That particle will have the mass which is the sum of the two individual masses, $m + km$.

Conservation of linear momentum

$$m \times 2u + km \times (-u) = (m + km) \times \frac{2}{3}u$$

The total linear momentum before impact must equal the total linear momentum after impact. Particle B is moving in the negative direction before the collision and so it has a negative linear momentum.

m and u are common factors on both sides of the equation and can be cancelled.

$$2mu - kmu = \frac{2}{3}mu + \frac{2}{3}kmu$$

$$2mu - \frac{2}{3}mu = kmu + \frac{2}{3}kmu$$

$$\frac{4}{3}\cancel{m}\cancel{u} = \frac{5}{3}k\cancel{m}\cancel{u}$$

$$k = \frac{4}{3} \times \frac{3}{5} = \frac{4}{5}$$

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Dynamics of a particle moving in a straight line
Exercise I, Question 16

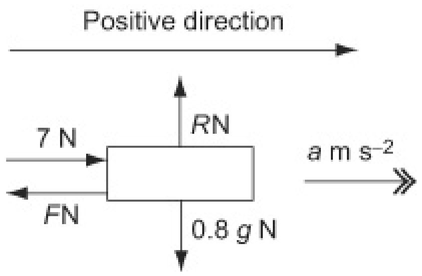
Question:

A block of mass 0.8 kg is pushed along a rough horizontal floor by a constant horizontal force of magnitude 7 N. The speed of the block increases from 2 m s $^{-1}$ to 4 m s $^{-1}$ in a distance of 4.8 m. Calculate

a the magnitude of the acceleration of the block,

b the coefficient of friction between the block and the floor.

Solution:



You should begin by drawing a diagram which shows all of the forces acting on the block and the acceleration of the block.

a

$u = 2,\ v = 4,\ s = 4.8,\ a = ?$

$v^2 = u^2 + 2as$

$4^2 = 2^2 + 9.6a$

$a = \frac{16-4}{9.6} = 1.25$

The magnitude of the acceleration of the block is 1.25 m s $^{-1}$.

b

R ($\uparrow$) $R - 0.8g = 0$

$R = 0.8g$

$F = \mu R = \mu 0.8g$

$F = ma$

R ($\rightarrow$) $7 - F = 0.8 \times 1.5$

$F = 7 - 0.8 \times 1.5 = 7 - 1 = 6$

Newton's Second Law is a relation between vector quantities and so you must be careful with the directions of forces. In this case, the force of 7 N pushing the block is in the positive direction and the friction force of F N is in the negative direction.

From $*$

$\mu 0.8g = 6$

$\mu = \frac{6}{0.8g} = \frac{6}{0.8 \times 9.8} = 0.765 \ldots$

As a numerical approximation for g has been used, you should correct your final answer to 2 significant figures.

The coefficient of friction is 0.77 (2 s.f.).

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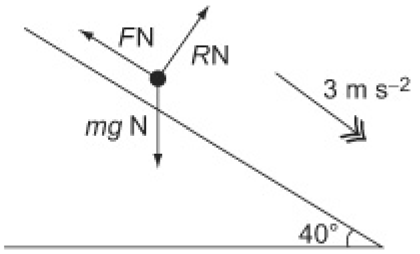
Dynamics of a particle moving in a straight line
Exercise I, Question 17

Question:

A particle is sliding with acceleration 3 m s^{-2} down a line of greatest slope of a fixed plane. The plane is inclined at $40°$ to the horizontal.

Calculate the coefficient of friction between the particle and the plane.

Solution:



You are given no value for mass of the particle and you will need to have an expression for the weight of the particle. Let the mass of the particle be *m* kg, then the weight of the stone is *mg* N.

R (↗) $\quad R - mg\cos 40° = 0$

$$R = mg\cos 40°$$

You could multiply 9.8 by $\cos 40°$ using your calculator here but, in general, it is advisable to do all of the calculation, in one go, at the end of the question. This avoids rounding errors.

Friction is limiting

$F = \mu R \qquad = \mu mg\cos 40°$

R (↘) $\quad mg\sin 40° - F \qquad = ma$

$\cancel{m}g\sin 40° - \mu\cancel{m}g\cos 40° \qquad = \cancel{m}3$

$$\mu = \frac{g\sin 40° - 3}{g\cos 40°} = \frac{9.8\sin 40° - 3}{9.8\cos 40°} = 0.4394\ldots$$

The coefficient of friction between the particle and the plane is 0.44, (2 s.f.).

As a numerical value of *g* has been given, you should give your answer to 2 significant figures unless the question instructs you otherwise.

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Dynamics of a particle moving in a straight line

Exercise I, Question 18

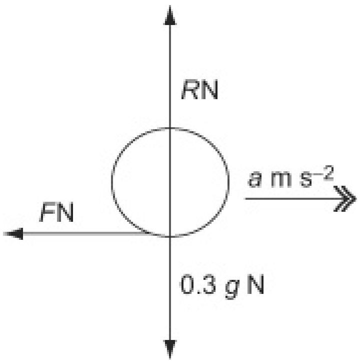
Question:

A pebble of mass 0.3 kg slides in a straight line on the surface of a rough horizontal concrete path. Its initial speed is 12.6 m s^{-1}. The coefficient of friction between the pebble and the path is $\frac{3}{7}$.

a Find the frictional force retarding the pebble.

b Find the total distance covered by the pebble before it comes to rest.

Solution:



a

R (↑) $R - 0.3g = 0 \Rightarrow R = 0.3g$

Friction is limiting

$F = \mu R = \frac{3}{7} \times 0.3 \times 9.8 = 1.26$

The frictional force retarding the pebble is 1:3 N (2 s.f.).

Although the answer was correctly given to 2 significant figures in part **a**, in working out the deceleration in part **b**, you should use the value of 1.26 for the friction force. All working should be carried out to at least 3 figures.

b

$$F = ma$$

$$\text{R} (\rightarrow) \; -1.26 = 0.3a$$

$$a = -\frac{1.26}{0.3} = -4.2$$

The question has not asked you to work out the deceleration but you could not find out *s* without working out *a* first. You have to see this kind of step for yourself.

$u = 12.6$, $v = 0$, $a = -4.2$, $s = ?$

$$v^2 = u^2 + 2as$$

$$0^2 = 12.6^2 - 8.4s$$

$$s = \frac{12.6^2}{8.4} = 18.9$$

The total distance covered by the pebble before it comes to rest is 19 m (2 s.f.).

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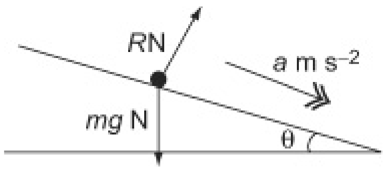
Dynamics of a particle moving in a straight line
Exercise I, Question 19

Question:

A particle moves down a line of greatest slope of a smooth plane inclined at an angle θ to the horizontal. The particle starts from rest and travels 3.5 m in time 2 s. Find the value of $\sin\theta$.

Solution:



$u = 0$, $s = 3.5$, $t = 2$, $a = ?$

$s = ut + \frac{1}{2}at^2$

$3.5 = 0 + \frac{1}{2}a \times 2^2 = 2a$

$a = \frac{3.5}{2} = 1.75$

R (↗) $\cancel{m}g \sin\theta = \cancel{m}a$

$\sin\theta = \frac{1.75}{9.8} = \frac{5}{28}$

You are given no value for mass of the particle and you will need to have an expression for the weight of the particle. Let the mass of the particle be *m* kg, then the weight of the stone is *mg* N. As often happens, the *m* can later be removed from the working, using the usual processes of algebra.

There is no friction in this question and the only force acting on the particle parallel to the plane is the component of the weight of the particle. There is an exact answer here which can be left. However a decimal answer, $\sin\theta \approx 0.18$, would be acceptable.

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Dynamics of a particle moving in a straight line
Exercise I, Question 20

Question:

A man of mass 80 kg stands in a lift. The lift has mass 60 kg and is being raised vertically by a cable attached to the top of the lift. Given that the lift with the man inside is rising with a constant acceleration of 0.6 m s^{-2}, find, to two significant figures,

a the magnitude of the force exerted by the lift on the man,

b the magnitude of the force exerted by the cable on the lift.

The lift starts from rest and, 5 s after starting to rise, the coupling between the cable and the lift suddenly snaps. There is an emergency cable attached to the lift but this only becomes taut when the lift is at the level of its initial position. After the coupling snaps, the lift moves freely under gravity until it is suddenly brought to rest in its initial position by the emergency cable. By modelling the lift with the man inside as a particle moving freely under gravity,

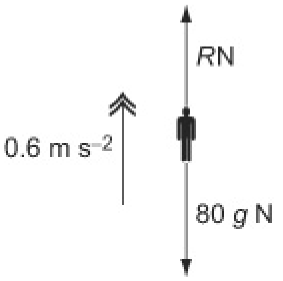
c find, to two significant figures, the magnitude of the impulse exerted by the emergency cable on the lift when it brings the lift to rest.

The model used in calculating the value required in part **c** ignores any effect of air resistance.

d State, with a reason, whether the answer obtained in **c** is higher or lower than the answer which would be obtained using a model which did incorporate the effect of air resistance.

Solution:

a



R N is the normal reaction of the lift on the man. The only forces on the man are this reaction and his weight. The cable has no direct contact with the man and you should not include this in your equation.

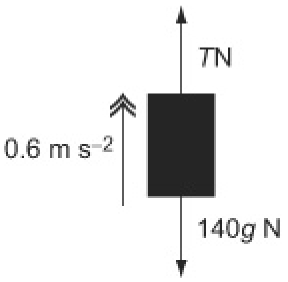
R (↑) $R - 80g = 80 \times 0.6$

$R = 80 \times 9.8 + 80 \times 0.6 = 832$

The magnitude of the force exerted by the lift on the man is 830 N (2 s.f.).

b

Combining the man and the lift, and modelling them as a single particle of mass 140 kg



There is a reaction of the lift on the man and an equal and opposite reaction of the man on the lift. When the lift and the man are combined, these cancel out and you need not consider them when writing down the equation of motion of the man and lift combined.

$\boldsymbol{F} = m\boldsymbol{a}$

R (↑) $T - 140g = 140 \times 0.6$

$T = 140 \times 9.8 + 140 \times 0.6 = 1456$

The magnitude of the force exerted by the cable on the lift is 1500 N (2 s.f.)

c

To find the speed of the lift when the cable breaks, take the **upward** direction as positive.

$u = 0$, $t = 5$, $a = 0.6$, $v = ?$

$v = u + at = 0 + 0.6 \times 5 = 3$

To find the distance travelled before the cable breaks, take the **upward** direction as positive.

$u = 0$, $t = 5$, $a = 0.6$, $s = ?$

After the cable breaks, take **downwards** as positive.

$u = -3$, $a = 9.8$, $s = 75$, $v = ?$

$v^2 = u^2 + 2as = 3^2 + 2 \times 9.8 \times 7.5 = 156$

$v = \sqrt{156}$

$I = mv - mu$

$I = 0 - 140\sqrt{156} = -1748 \ldots$

The magnitude of the impulse exerted by the emergency cable on the lift when it brings the lift to rest is 1700 N s (2 s.f.).

The emergency cable reduces the lift to rest. To find the impulse, you need to find the speed with which the lift returns to its original position. After the lift breaks, the lift falls freely under gravity. Its acceleration is no longer 0.6 m s^{-2} upwards but 9.8 m s^{-2} downwards.

d

Air resistance would reduce the speed of the lift as it falls and so the impulse would be reduced.

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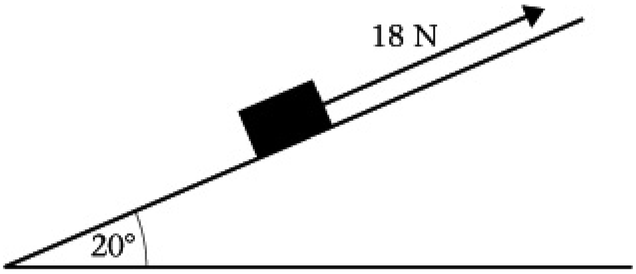
Dynamics of a particle moving in a straight line
Exercise I, Question 21

Question:

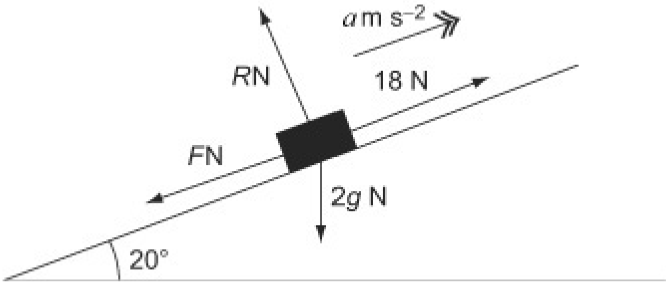
A box of mass 2 kg is pulled up a rough plane face by means of a light rope. The plane is inclined at an angle of 20 ° to the horizontal, as shown in the figure. The rope is parallel to a line of greatest slope of the plane. The tension in the rope is 18 N. The coefficient of friction between the box and the plane is 0.6. By modelling the box as a particle, find



a the normal reaction of the plane on the box,

b the acceleration of the box.

Solution:



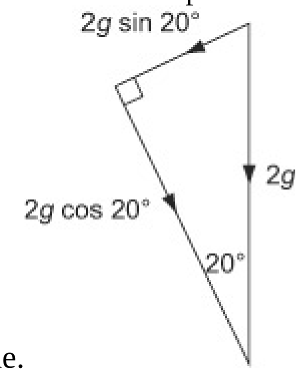
a

R (↗) $R - 2g\cos 20° = 0$

$R = 2g\cos 20° = 18.417\,97\ \ldots$

The normal reaction of the plane on the box is 18 N (2 s.f.).

The weight 2g N has components perpendicular and parallel to the plane.



b

Friction is limiting

There are three forces acting on the box in the direction parallel to the plane. The

$F = \mu R$	$= 0.6 \times 18.417\,97 \ldots$ $= 11.0507 \ldots$	tension in the rope acting up the plane, and the friction force and the component of the weight acting down the plane. The **F** in $F = ma$ is the vector sum of these three forces.
R (↘) 18	$- F - 2g \sin 20° = 2a$	
18	$- 11.0507 \ldots - 6.70359 \ldots = 2a$	
$2a$	$= 0.2456 \ldots \Rightarrow a = 0.1228 \ldots$	

The acceleration of the box is 0.12 m s $^{-1}$ (2 s.f.).

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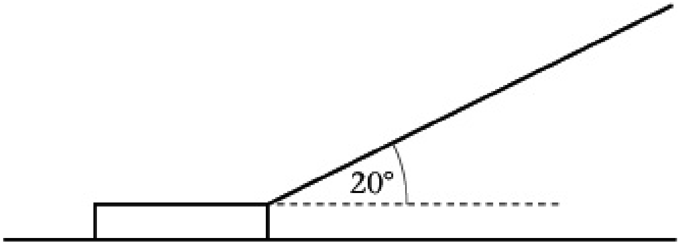
Dynamics of a particle moving in a straight line
Exercise I, Question 22

Question:

A sledge has mass 30 kg. The sledge is pulled in a straight line along horizontal ground by means of a rope. The rope makes an angle 20 ° with the horizontal, as shown in the figure. The coefficient of friction between the sledge and the ground is 0.2. The sledge is modelled as a particle and the rope as a light inextensible string. The tension in the rope is 150 N. Find, to three significant figures,



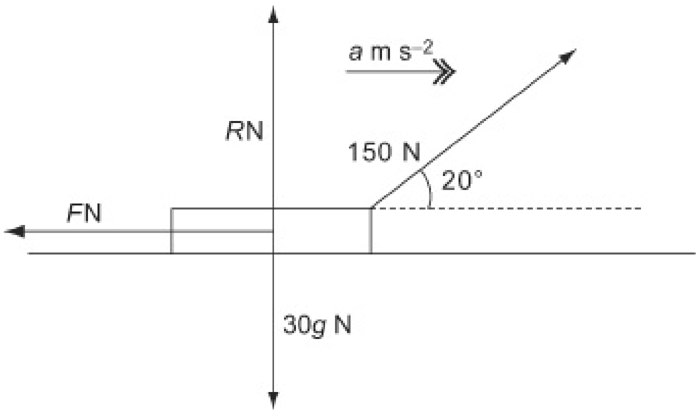
a the normal reaction of the ground on the sledge,

b the acceleration of the sledge.

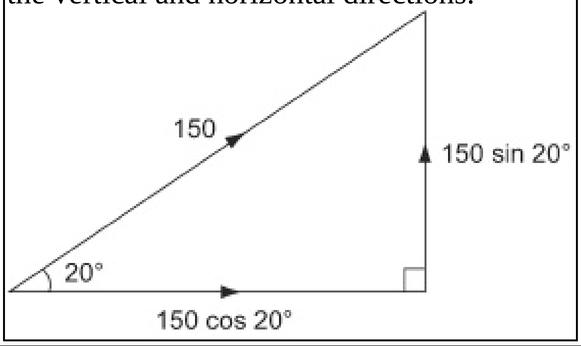
When the sledge is moving at 12 m s^{-1}, the rope is released from the sledge.

c Find, to three significant figures, the distance travelled by the sledge from the moment when the rope is released to the moment when the sledge comes to rest.

Solution:



The tension in the rope has components in the vertical and horizontal directions.



a

R (↑) $R + 150 \sin 20^\circ - 30g = 0$

$R = 30g - 150 \sin 20^\circ$

$= 30 \times 9.8 - 150 \sin 20^\circ = 242.69 \ldots$

The normal reaction of the ground on the sledge is 243 N (3 s.f.).

This question asks you to give answers to 3 significant figures. Usually when a numerical value of *g* is used you correct to 2 significant figures, but in all A level papers, you must follow the instructions given in a particular question. From time to time the conditions in a question may vary.

b

Friction is limiting

$F = \mu R = 0.2 \times 242.69 \ldots = 48.539 \ldots$

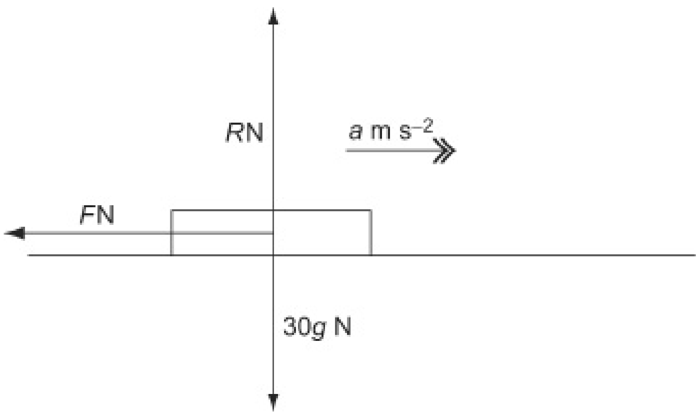
R ($\rightarrow$)

$150 \cos 20^\circ - 48.539 \ldots = 30a$

$a = \dfrac{150 \cos 20^\circ - 48.539 \ldots}{30} = 3.080 \ldots$

The acceleration of the sledge is 3.08 m s^{-1} (3 s.f.).

c



R ($\uparrow$) $R - 30g = 0 \Rightarrow R = 30g$

Friction is limiting

$F = \mu R = 0.2 \times 30g = 6g$

R ($\rightarrow$) $-F = 30a$

$a = \dfrac{-6g}{30} = -1.96$

$u = 12$, $v = 0$, $a = -1.96$, $s = ?$

$v^2 = u^2 + 2as$

$0^2 = 12^2 - 2 \times 1.96 \times s$

$s = \dfrac{12^2}{2 \times 1.96} = 36.734 \ldots$

The distance travelled by the sledge is 36.7 m (3 s.f.).

The only force acting in a horizontal direction is the friction force. However, with the removal of the rope, this has changed. The friction force depends on the normal reaction and that is now $30g \approx 294$ N. It was about 243 N in part **a**. It has increased with the removal of the rope. Assuming the friction and the normal reaction are unchanged is a common error. You must start again, draw a fresh diagram and work through the question again, resolving in both directions.

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Dynamics of a particle moving in a straight line
Exercise I, Question 23

Question:

A metal stake of mass 2 kg is driven vertically into the ground by a blow from a sledgehammer of mass 10 kg. The sledgehammer falls vertically on to the stake, its speed just before impact being 9 m s $^{-1}$. In a model of the situation it is assumed that, after impact, the stake and the sledgehammer stay in contact and move together before coming to rest.

a Find the speed of the stake immediately after impact.

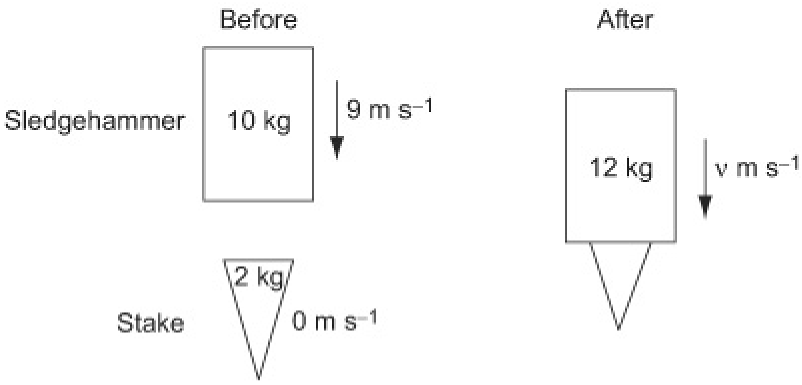
The stake moves 3 cm into the ground before coming to rest. Assuming in this model that the ground exerts a constant resistive force of magnitude *R* newtons as the stake is driven down,

b find the value of *R*.

c State one way in which this model might be refined to be more realistic.

Solution:

a



After impact, the sledgehammer and the metal stake move together. You model the sledgehammer and stake as a single particle of mass 12 kg.

Conservation of linear momentum

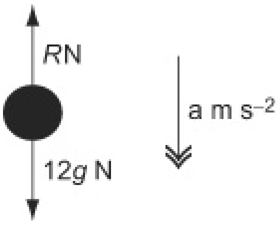
$10 \times 9 + 2 \times 0 = 12 \times v$

$v = \frac{90}{12} = 7.5$

The speed of the stake immediately after impact is 7.5 m s $^{-1}$.

The model given in the question assumes that the stake and sledgehammer stay in contact and move together after impact, before coming to rest. Although the question only refers to the stake, you must consider the stake and the sledgehammer as moving together, with the same velocity and the same acceleration, throughout the motion after the impact.

b



$u = 7.5,\ v = 0,\ s = 0.03,\ a = ?$

$v^2 = u^2 + 2as$

$0^2 = 7.5^2 + 2 \times a \times 0.03$

$a = -\frac{7.5^2}{0.06} = -937.5$

$F = ma$

$12g - R = 12 \times (-937.5)$

$R = 12 \times 9.8 + 12 \times 937.5 = 11367.6$

$= 11\ 000\ (2\text{ s.f.})$.

c The resistance (R) could be modelled as varying with speed.

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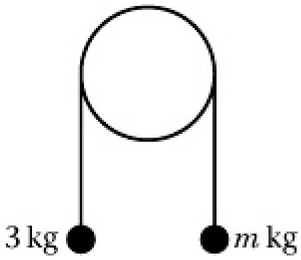
Dynamics of a particle moving in a straight line
Exercise I, Question 24

Question:

The particles have mass 3 kg and m kg, where $m < 3$. They are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The particles are held in position with the string taut and the hanging parts of the string vertical, as shown in the figure. The particles are then released from rest. The initial acceleration of each particle has magnitude $\frac{3}{7}g$.



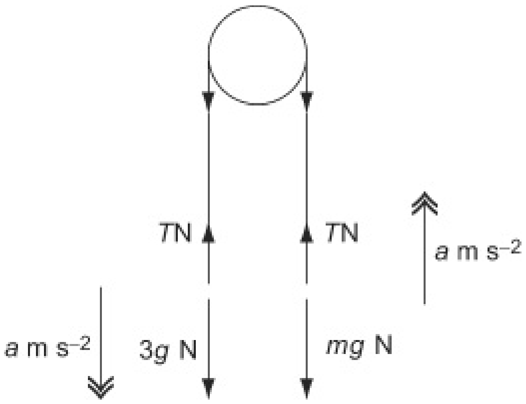
Find

a the tension in the string immediately after the particles are released,

b the value of m.

Solution:

a



As $m < 3$, the 3 kg mass will move downwards and the m kg mass will move downwards.

For the 3 kg mass

R (↓) $F = ma$

$3g - T = 3 \times \frac{3}{7}g$

$T = 3g - \frac{9}{7}g = \frac{12}{7}g$

Newton's second law is a relation between vectors and the forces must be given their correct sign. For the 3 kg mass, the weight is in the same direction as the acceleration and the tension in the string is in the opposite direction.

The tension in the string is $\frac{12}{7}$ g N.

The answer can be left in this exact form and, in this question, leaving the tension in this form leads to g cancelling in part **b**.

b

For the m kg mass

For the m kg mass, the tension is in the

R (↑) $F = ma$ $T - mg = m \times \frac{3}{7}g$	same direction as the acceleration and the weight is in the opposite direction.

Using the answer to part **a**

$$\frac{12}{7}g - mg = \frac{3}{7}mg$$

$$\frac{12}{7} = \frac{10}{7}m \Rightarrow m = 1.2$$

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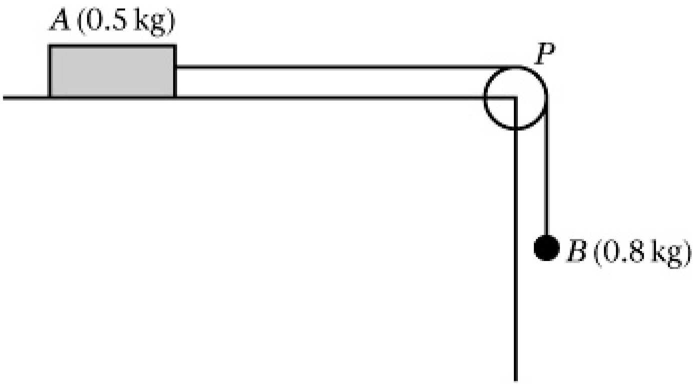
Dynamics of a particle moving in a straight line
Exercise I, Question 25

Question:



A block of wood A of mass 0.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a ball B of mass 0.8 kg which hangs freely below the pulley, as shown in the figure. The coefficient of friction between A and the table is μ. The system is released from rest with the string taut. After release, B descends a distance of 0.4 m in 0.5 s. Modelling A and B as particles, calculate

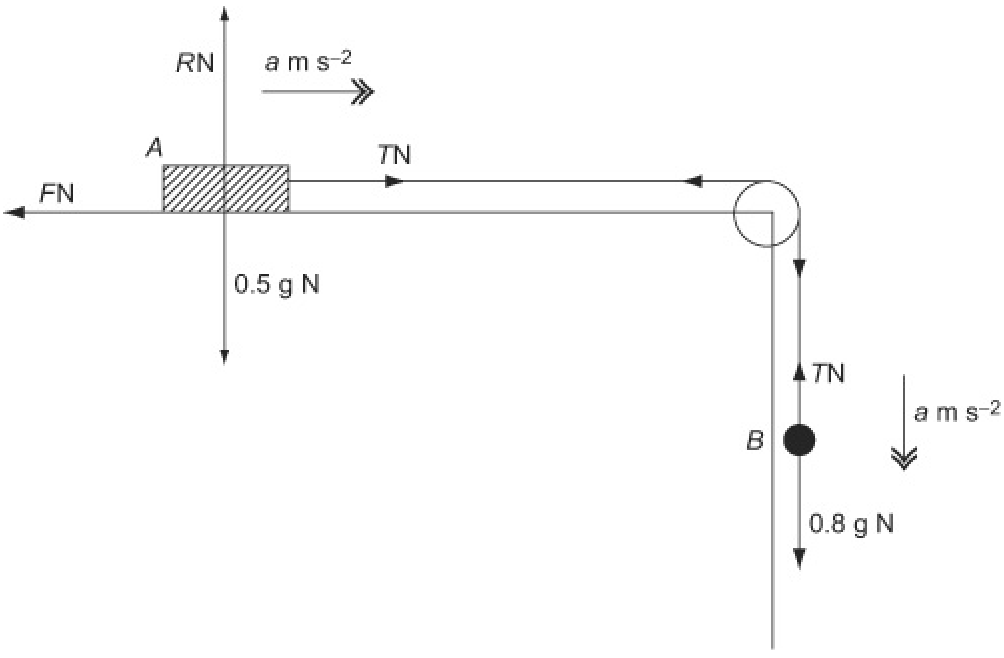
a the acceleration of B,

b the tension in the string,

c the value of μ.

d State how in your calculations you have used the information that the string is inextensible.

Solution:



a For B

$u = 0$, $s = 0.4$, $t = 0.5$, $a =$?

$s = ut + \frac{1}{2}at^2$

$0.4 = 0 + \frac{1}{2}a \times 0.5^2 = \frac{1}{8}a$

$a = 8 \times 0.4 = 3.2$

The acceleration of B is 3.2 ms^{-2}.

b For B

$F = ma$

$0.8g - T = 0.8 \times 3.2$

$T = 0.8 \times 9.8 - 0.8 \times 3.2 = 5.28$

As the numerical value $g = 9.8$ has been used, you should correct your answer to 2 significant figures.

The tension in the string is 5.3 N (2 s.f.).

c For A

R (↑) $R - 0.5g = 0 \Rightarrow R = 0.5g$

Friction is limiting

$F = \mu R = \mu 0.5g$

R (→) $T - F = ma$

$5.28 - \mu 0.5g = 0.5 \times 3.2$

$\mu = \frac{5.28 - 0.5 \times 3.2}{0.5 \times 9.8} = 0.751 \ldots$

$= 0.75$ (2 s.f.)

It is a common error to include the weight $0.5g$ in this equation. The weight acts vertically downwards and has no component in the horizontal direction, which is the direction you are resolving in. The weight does, however, affect the friction force.

Although you, correctly, gave the answer for the tension to two significant figures in part **b**, all working should be given to at least 3 significant figures, so you should use $T = 5.28$ here.

d The information that the string is inextensible has been used when, in part **c**, the acceleration of A has been taken equal to the acceleration of B obtained in part **a**.

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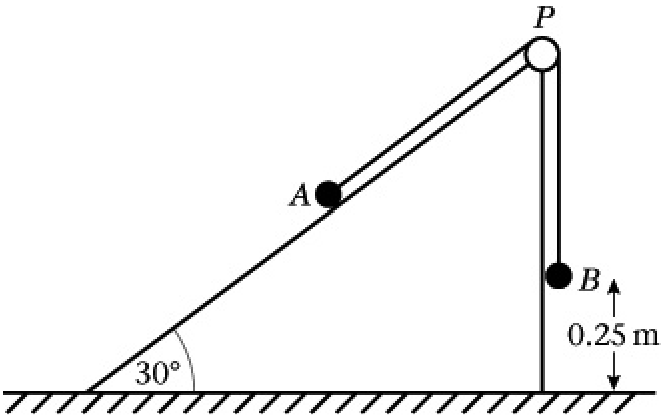
Dynamics of a particle moving in a straight line
Exercise I, Question 26

Question:



Two particles *A* and *B*, of mass *m*kg and 3 kg respectively, are connected by a light inextensible string. The particle *A* is held resting on a smooth fixed plane inclined at 30 ° to the horizontal. The string passes over a smooth pulley *P* fixed at the top of the plane. The portion *AP* of the string lies along a line of greatest slope of the plane and *B* hangs freely from the pulley, as shown in the figure. The system is released from rest with *B* at a height of 0.25 m above horizontal ground. Immediately after release, *B* descends with an acceleration of $\frac{2}{5}g$. Given that *A* does not reach *P*, calculate

a the tension in the string while *B* is descending,

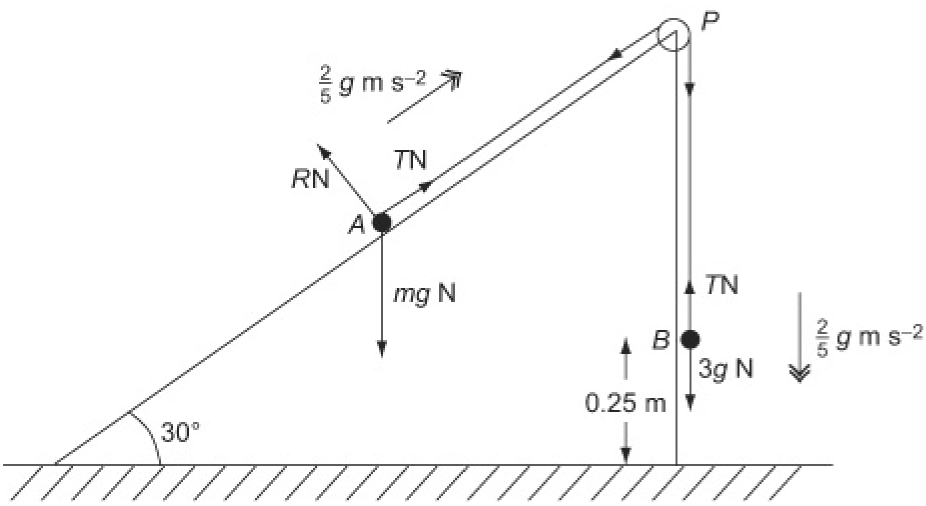
b the value of *m*.

The particle *B* strikes the ground and does not rebound. Find

c the magnitude of the impulse exerted by *B* on the ground,

d the time between the instant when *B* strikes the ground and the instant when *A* reaches its highest point.

Solution:



a

For B $\quad F = ma$

$$3g - T = 3 \times \frac{2}{5}g$$

$$T = 3g - \frac{6}{5}g = \frac{9}{5}g = 17.64$$

The tension in the string while B is descending is 18 N, to 2 significant figures.

b

For A $\quad F = ma$

$$R(\nearrow) \quad T - mg\sin 30° = m \times \frac{2}{5}g$$

$$\frac{9}{5}\cancel{g} - \frac{1}{2}m\cancel{g} = \frac{2}{5}m\cancel{g}$$

$$\left(\frac{1}{2} + \frac{2}{5}\right)m = \frac{9}{10}m = \frac{9}{5} \Rightarrow m = 2$$

$\sin 30° = \frac{1}{2}$ is the exact value.

c To find the speed of B immediately before it strikes the ground

$$u = 0, \; a = \frac{2}{5}g, \; s = 0.25, \; v = ?$$

$$v^2 = u^2 + 2as = 0^2 + 2 \times \frac{2}{5}g \times 0.25 = 1.96$$

$$v = \sqrt{1.96} = 1.4$$

$$I = mv - mu$$

$$I = 3 \times 0 - 3 \times 1.4 = -4.2$$

As the particle does not rebound, the velocity of B, after it strikes the ground, is zero.

The magnitude of the impulse exerted by B on the ground is 4.2 N s.

The impulse I calculated here is the impulse exerted on B by the ground – it is upwards.The impulse asked for is equal to I in magnitude but is in the opposite direction.

d

After B strikes the ground, for A

$$R(\searrow) \quad -\cancel{m}g\sin 30° = \cancel{m}a$$

$$a = -\frac{1}{2}g$$

$$u = 1.4, \; v = 0, \; a = -\frac{1}{2}g, \; t = ?$$

$$v = u + at$$

$$0 = 1.4 - \frac{1}{2}gt \Rightarrow t = \frac{2.8}{9.8} = \frac{28}{98} = \frac{2}{7}$$

After B strikes the ground, there is no tension in the string and the only force acting on B parallel to the plane is the component of its weight acting down the plane.

The approximate answer, 0.28 s, would also be acceptable.

The time between the instants is $\frac{2}{7}$ s.