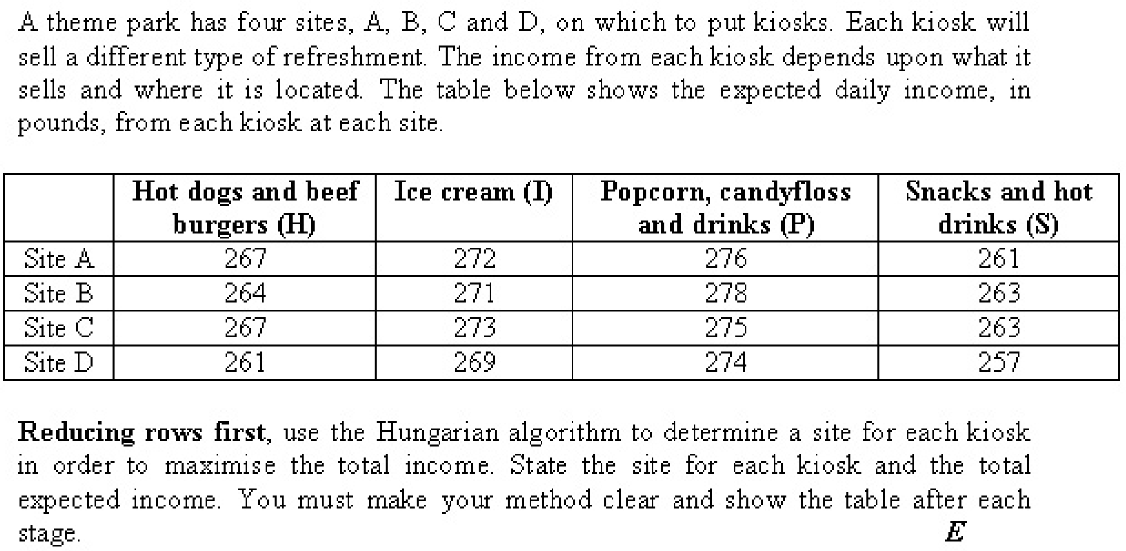
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 1

Question:



A theme park has four sites, A, B, C and D, on which to put kiosks. Each kiosk will sell a different type of refreshment. The income from each kiosk depends upon what it sells and where it is located. The table below shows the expected daily income, in pounds, from each kiosk at each site.

	Hot dogs and beef burgers (H)	**Ice cream (I)**	**Popcorn, candyfloss and drinks (P)**	**Snacks and hot drinks (S)**
Site A	267	272	276	261
Site B	264	271	278	263
Site C	267	273	275	263
Site D	261	269	274	257

Reducing rows first, use the Hungarian algorithm to determine a site for each kiosk in order to maximise the total income. State the site for each kiosk and the total expected income. You must make your method clear and show the table after each stage. ***E***

Solution:

To maximise, subtract all entries from $n \geq 278$

e.g. $\begin{bmatrix} 11 & 6 & 2 & 17 \\ 14 & 7 & 0 & 15 \\ 11 & 5 & 3 & 15 \\ 17 & 9 & 4 & 21 \end{bmatrix}$

Reduce rows $\begin{bmatrix} 9 & 4 & 0 & 15 \\ 14 & 7 & 0 & 15 \\ 8 & 2 & 0 & 12 \\ 13 & 5 & 0 & 17 \end{bmatrix}$ then columns $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 6 & 5 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 5 \end{bmatrix}$

Minimum element is 1

$\begin{bmatrix} 0 & 1 & 0 & 2 \\ 5 & 4 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 4 & 2 & 0 & 4 \end{bmatrix}$

Minimum element is 1

$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 5 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 4 & 1 & 0 & 3 \end{bmatrix}$

Then Minimum element is 1

$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 4 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$

optimal

$\begin{bmatrix} 0 & 1 & 2 & 2 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$

Minimum element is 2
optimal

So $\begin{matrix} A-H \\ B-P \\ C-S \\ D-I \end{matrix}$ or $\begin{matrix} H \\ S \\ I \\ P \end{matrix}$

(both £1077)

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Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 2

Question:

A coach company has 20 coaches. At the end of a given week, 8 coaches are at depot A, 5 coaches are at depot B and 7 coaches are at depot C. At the beginning of the next week, 4 of these coaches are required at depot D, 10 of them at depot E and 6 of them at depot F. The following table shows the distances, in miles, between the relevant depots.

	D	E	F
A	40	70	25
B	20	40	10
C	35	85	15

The company needs to move the coaches between depots at the weekend. The total mileage covered is to be a minimum.
Formulate this information as a linear programming problem.

a State clearly your decision variables.
b Write down the objective function in terms of your decision variables.
c Write down the constraints, explaining what each constraint represents. ***E***

Solution:

a x_{11} number of coaches from A to D
x_{12} number of coaches from A to E
x_{13} number of coaches from A to F
x_{21} number of coaches from B to D
x_{22} number of coaches from B to E
x_{23} number of coaches from B to F
x_{31} number of coaches from C to D
x_{32} number of coaches from C to E
x_{33} number of coaches from C to F

b Minimise
$$\begin{aligned} z = 40x_{11} + 70x_{12} + 25x_{13} \\ +20x_{21} + 40x_{22} + 10x_{23} \\ +35x_{31} + 85x_{32} + 15x_{33} \end{aligned}$$

c Depot A $x_{11} + x_{12} + x_{13} = 8$ (number of coaches at A)
Depot B $x_{21} + x_{22} + x_{23} = 5$ (number of coaches at B)
Depot C $x_{31} + x_{32} + x_{33} = 7$ (number of coaches at C)
Depot D $x_{11} + x_{21} + x_{31} = 4$ (number required at D)
Depot E $x_{12} + x_{22} + x_{32} = 10$ (number required at E)
Depot F $x_{13} + x_{32} + x_{33} = 6$ (number required at F)
Reference to number of coaches at A, B and C = number of coaches at D, E and F

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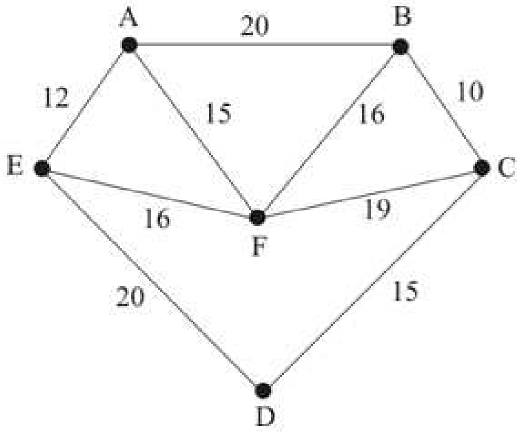
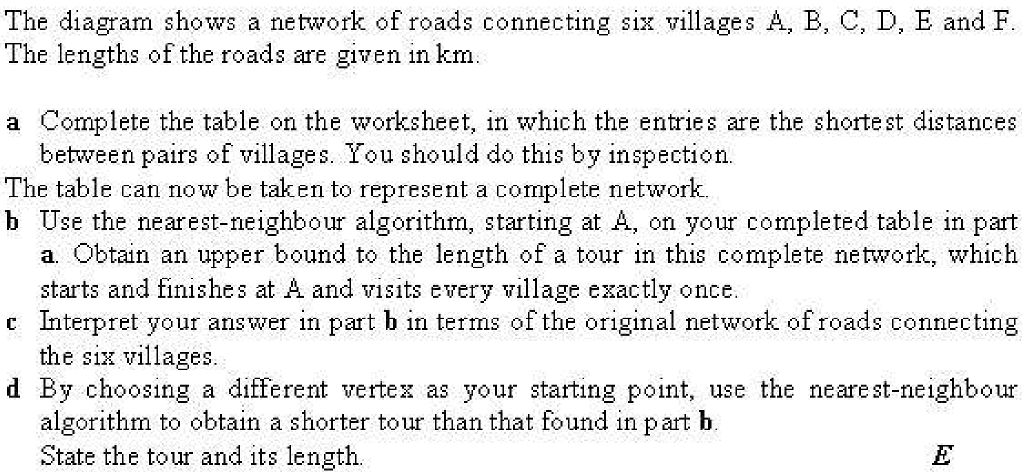
Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 3

Question:





The diagram shows a network of roads connecting six villages A, B, C, D, E and F. The lengths of the roads are given in km.

a Complete the table on the worksheet, in which the entries are the shortest distances between pairs of villages. You should do this by inspection.

The table can now be taken to represent a complete network.

b Use the nearest-neighbour algorithm, starting at A, on your completed table in part **a**. Obtain an upper bound to the length of a tour in this complete network, which starts and finishes at A and visits every village exactly once.

c Interpret your answer in part **b** in terms of the original network of roads connecting the six villages.

d By choosing a different vertex as your starting point, use the nearest-neighbour algorithm to obtain a shorter tour than that found in part **b**.
State the tour and its length. ***E***

Solution:

a

	A	B	C	D	E	F
A	0	20	30	32	12	15
B	20	0	10	25	32	16
C	30	10	0	15	35	19
D	32	25	15	0	20	34
E	12	32	35	20	0	16
F	15	16	19	34	16	0

b AE(12), EF(16), FB(16) BC(10), CD(15), DA(32)

i.e.

A E F B C D A upper bound = 101 km

c In the original network AD is not a direct path. The tour becomes A E F B C D E A

d e.g.

$$\left.\begin{array}{l} \text{B C D E A F B} \\ \text{C B F A E D C} \\ \text{D C B F A E D} \\ \text{E A F B C D E} \\ \text{F A E D C B F} \end{array}\right\} \quad \text{length 88}$$

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Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 4

Question:

A manufacturing company makes 3 products X, Y and Z. The numbers of each product made are x, y and z respectively and £P is the profit. There are two machines which are available for a limited time. These time limitations produce two constraints. In the process of using the simplex algorithm, the following tableau is obtained, where r and s are slack variables.

Basic variable	x	y	z	r	s	**Value**
y	0	1	$3\frac{1}{3}$	1	$-\frac{1}{3}$	1
x	1	0	−3	−1	$\frac{1}{2}$	3
P	0	0	1	1	1	33

a State how you know that this tableau is optimal (final).

b By writing out the profit equation, or otherwise, explain why a further increase in profit is not possible under these constraints.

c From this tableau, deduce

i the maximum profit,

ii the optimum number of X, Y and Z that should be produced to maximise the profit. ***E***

Solution:

a There are no negative entries in the objective row

b Profit equation

$P+z+r+s=33$

$P\ =33-(z+r+s)$

At present z, r and s are all zero. If they increase P will decrease. Hence P is maximal

c i $P=33$

ii $x=3\quad y=1, z=0$

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Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 5

Question:

Freezy Co. has three factories A, B and C. It supplies freezers to three shops D, E and F. The table shows the transportation cost in pounds of moving one freezer from each factory to each outlet. It also shows the number of freezers available for delivery at each factory and the number of freezers required at each shop. The total number of freezers required is equal to the total number of freezers available.

	D	**E**	**F**	**Available**
A	21	24	16	24
B	18	23	17	32
C	15	19	25	14
Required	20	30	20	

a Use the north-west corner rule to find an initial solution.
b Obtain improvement indices for each unused route.
c Use the stepping-stone method **once** to obtain a better solution and state its cost.

E

Solution:

a

	D	E	F
A	20	4	
B		26	6
C			14

b $S_A = 0$ $S_B = 1$ $S_C = 7$

$D_D = 21$ $D_E = 24$ $D_F = 18$

$$I_{AF} = 16 - 0 - 18 = -2$$
$$I_{BD} = 18 + 1 - 21 = -2$$
$$I_{CD} = 15 - 7 - 21 = -13$$
$$I_{CE} = 19 - 7 - 24 = -12$$

	D	E	F
A	$20-\theta$	$4+\theta$	
B		$26-\theta$	$6+\theta$
C	θ		$14-\theta$

entering cell
CD
$\theta = 14$
exiting cell CF

	D	E	F
A	6	18	
B		12	20
C	14		

cost £1384

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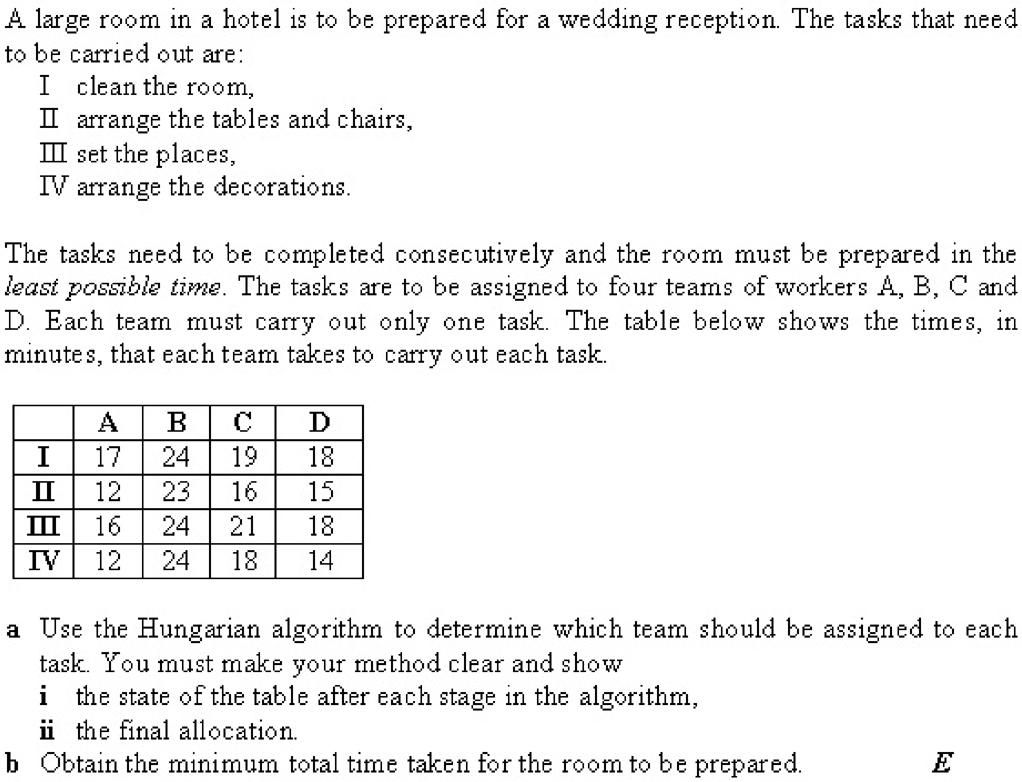
Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 6

Question:



A large room in a hotel is to be prepared for a wedding reception. The tasks that need to be carried out are:

I clean the room,
II arrange the tables and chairs,
III set the places,
IV arrange the decorations.

The tasks need to be completed consecutively and the room must be prepared in the *least possible time*. The tasks are to be assigned to four teams of workers A, B, C and D. Each team must carry out only one task. The table below shows the times, in minutes, that each team takes to carry out each task.

	A	B	C	D
I	17	24	19	18
II	12	23	16	15
III	16	24	21	18
IV	12	24	18	14

a Use the Hungarian algorithm to determine which team should be assigned to each task. You must make your method clear and show
  i the state of the table after each stage in the algorithm,
  ii the final allocation.
b Obtain the minimum total time taken for the room to be prepared. *E*

Solution:

a

17	24	19	18
12	23	16	15
16	24	21	18
12	24	18	14

Reducing rows gives:

0	7	2	1
0	11	4	3
0	8	5	2
0	12	6	2

Reducing columns gives:

0	0	0	0
0	4	2	2
0	1	3	1
0	5	4	1

No assignment possible as zeroes can all be covered by 2 lines (2 < 4)
Minimum uncovered element is 1
Applying algorithm gives:

1	0	0*	0
0*	3	1	1
0	0*	2	0
0	4	3	0*

Now requires 4 lines to cover all zeroes so assignment now possible
(1, 3) – only zero in column 3
(3, 2) – row 1 already used and now only zero in C2
(4, 4) – only remaining possibility in C4
(2, 1) – must then be used
I – C, II – A, III – B, IV – D

b Time of this assignment
$19 + 12 + 24 + 14 = 69$ minutes

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Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 7

Question:

A three-variable linear programming problem in x, y and z is to be solved. The objective is to maximise the profit P. The following tableau was obtained.

Basic variable	x	y	z	r	s	t	Value
s	3	0	2	0	1	$-\frac{2}{3}$	$\frac{2}{3}$
r	4	0	$\frac{7}{2}$	1	0	8	$\frac{9}{2}$
y	5	1	7	0	0	3	7
P	3	0	2	0	0	8	63

a State, giving your reason, whether this tableau represents the optimal solution.

b State the values of every variable.

c Calculate the profit made on each unit of y. ***E***

Solution:

a *Yes*. there are *no negative* values in the *profit row*

b $P = 63, x = 0, y = 7, z = 0, r = \frac{9}{2}, s = \frac{2}{3}, t = 0$

c $\frac{63}{7} = 9$

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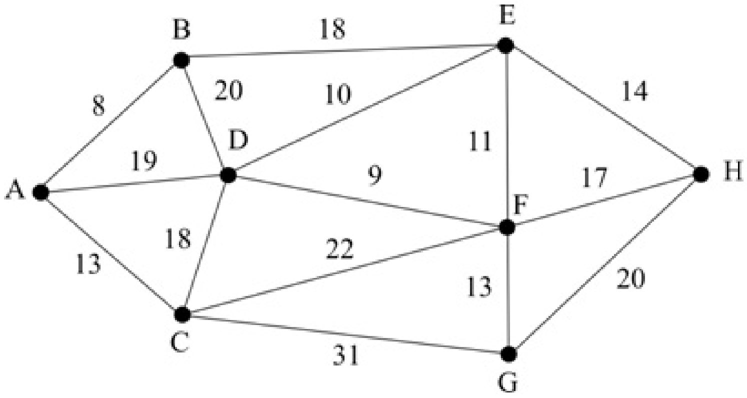
Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 8

Question:

a Explain the difference between the classical and practical travelling salesman problems.



The network above shows the distances, in kilometres, between eight McBurger restaurants. An inspector from head office wishes to visit each restaurant. His route should start and finish at A, visit each restaurant at least once and cover a minimum distance.

b Obtain a minimum spanning tree for the network using Kruskal's algorithm. You should draw your tree and state the order in which the arcs were added.

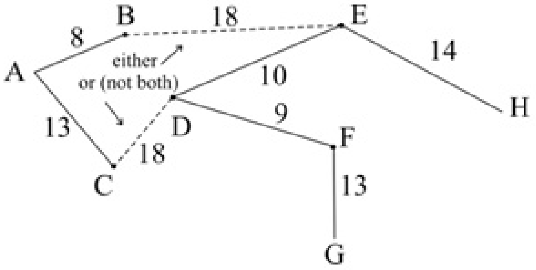
c Use your answer to part **b** to determine an initial upper bound for the length of the route.

d Starting from your initial upper bound and using an appropriate method, find an upper bound which is less than 135 km. State your tour. ***E***

Solution:

a In the *practical* T.S.P each vertex must be visited *at least once*
In the *classical* T.S.P. each vertex must be visited *exactly once*

b AB, DF, DE, (reject EF) $\begin{Bmatrix} \text{FG} \\ \text{AC} \end{Bmatrix}$, EH $\begin{Bmatrix} \text{DC} \\ \text{or} \\ \text{BE} \end{Bmatrix}$



c Initial upper bound $= 2 \times 85 = 170$ km

d When CD is part of tree
Use GH (saving 26) and BD (saving 19) giving a new upper bound of 125 km
Tour A B D E H G F D C A
e.g. when BE is part of tree
Use CG (saving 40) giving a new upper bound of 130 km
Tour A B E H E D F G C A

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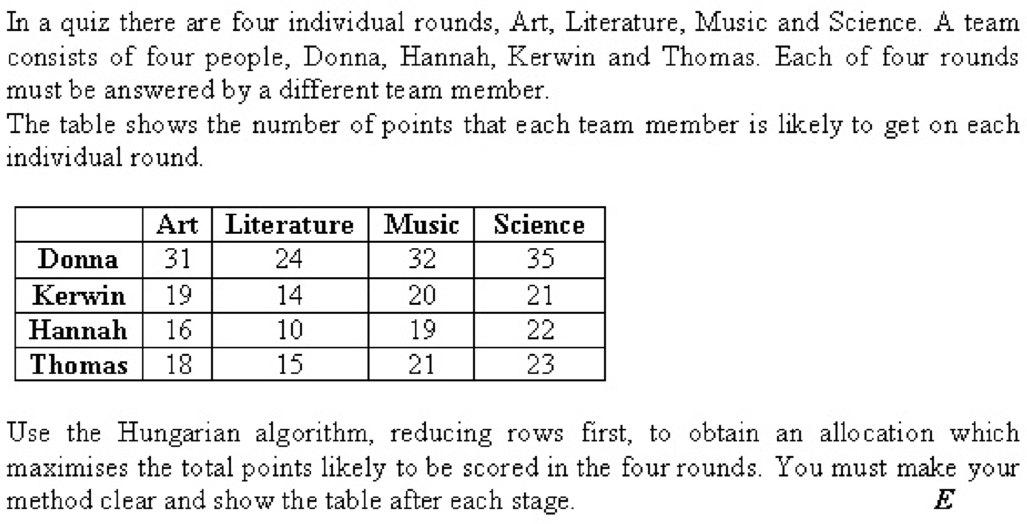
Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 9

Question:



In a quiz there are four individual rounds, Art, Literature, Music and Science. A team consists of four people, Donna, Hannah, Kerwin and Thomas. Each of four rounds must be answered by a different team member.

The table shows the number of points that each team member is likely to get on each individual round.

	Art	Literature	Music	Science
Donna	31	24	32	35
Kerwin	19	14	20	21
Hannah	16	10	19	22
Thomas	18	15	21	23

Use the Hungarian algorithm, reducing rows first, to obtain an allocation which maximises the total points likely to be scored in the four rounds. You must make your method clear and show the table after each stage. ***E***

Solution:

Subtract all terms from some $n \geq 35$, eg 35

$$\begin{array}{cccc} 4 & 11 & 3 & 0 \\ 16 & 21 & 15 & 14 \\ 19 & 25 & 16 & 13 \\ 17 & 20 & 14 & 12 \end{array}$$

Reducing rows then column $\begin{array}{cccc} 2 & 4 & 2 & 0 \\ 4 & 5 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 \end{array}$

minimum uncovered 1 $\begin{array}{cccc} 1 & 3 & 1 & 0 \\ 3 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \end{array}$

minimum uncovered 1 $\begin{array}{cccc} 0 & 2 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 1 \end{array}$

e.g. matching $\begin{array}{l} D-A \\ H-S \\ K-M \\ T-L \end{array}$ or $\begin{array}{c} A \\ S \\ L \\ M \end{array}$ or $\begin{array}{c} M \\ S \\ A \\ L \end{array}$ or $\begin{array}{c} S \\ M \\ A \\ L \end{array}$

Total 88 points

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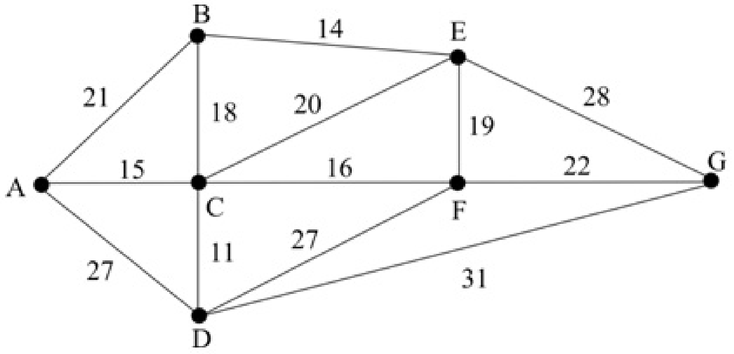
Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 10

Question:



The network above shows the distances, in km, of the cables between seven electricity relay stations A, B, C, D, E, F and G. An inspector needs to visit each relay station. He wishes to travel a minimum distance, and his route must start and finish at the same station.

By deleting C, a lower bound for the length of the route is found to be 129 km.

a Find another lower bound for the length of the route by deleting F. State which is the best lower bound of the two.

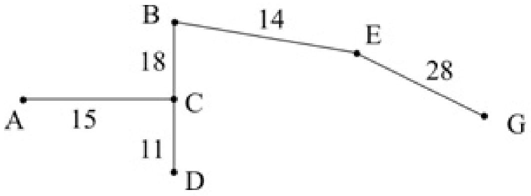
b By inspection, complete the table of least distances.

The table can now be taken to represent a complete network.

c Using the nearest-neighbour algorithm, starting at F, obtain an upper bound to the length of the route. State your route. ***E***

Solution:

a Deleting F leaves residual spanning tree



r.s.t. length $= 86$

So lower bound $= 86 + 16 + 19 = 121$

$\therefore$ better lower bound is 129 by deleting C

b Add 33 to BF and FB
Add 31 to DE and ED

c Tour visits each vertex, order correct using table of least distances.
e.g. F C D A B E G F (actual route F C D C A B E G F) upper bound of 138 km

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Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 11

Question:

Three warehouses W, X and Y supply televisions to three supermarkets J, K and L. The table gives the cost, in pounds, of transporting a television from each warehouse to each supermarket. The warehouses have stocks of 34, 57 and 25 televisions respectively, and the supermarkets require 20, 56 and 40 televisions respectively. The total cost of transporting the televisions is to be minimised.

	J	**K**	**L**
W	3	6	3
X	5	8	4
Y	2	5	7

Formulate this transportation problem as a linear programming problem. Make clear your decision variables, objective function and constraints. ***E***

Solution:

Let x_{ij} be *number* of unit transported from i to j

when $i \in \{\text{W}, \text{X}, \text{Y}\}$ and $j \in \{\text{J}, \text{K}, \text{L}\}$

Objective minimise
$$C = 3x_{\text{WJ}} + 6x_{\text{WK}} + 3x_{\text{WL}} + 5x_{\text{XJ}} + 8x_{\text{XK}} + 4x_{\text{XL}} + 2x_{\text{YJ}} + 5x_{\text{YK}} + 7x_{\text{YL}}$$

Subject to
$$x_{\text{WJ}} + x_{\text{WK}} + x_{\text{WL}} = 34$$
$$x_{\text{XJ}} + x_{\text{XK}} + x_{\text{XL}} = 57$$
$$x_{\text{YJ}} + x_{\text{YK}} + x_{\text{YL}} = 25$$
$$x_{\text{WJ}} + x_{\text{XJ}} + x_{\text{YJ}} = 20$$
$$x_{\text{WK}} + x_{\text{XK}} + x_{\text{YK}} = 56$$
$$x_{\text{WL}} + x_{\text{XL}} + x_{\text{YL}} = 40$$

$x_{ij} \geq 0 \quad i \in \{\text{W}, \text{X}, \text{Y}\}$ and $j \in \{\text{J}, \text{K}, \text{L}\}$

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Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 12

Question:

A manager wishes to purchase seats for a new cinema. He wishes to buy three types of seat: standard, deluxe and majestic. Let the number of standard, deluxe and majestic seats to be bought be x, y and z respectively.

He decides that the total number of deluxe and majestic seats should be at most half of the number of standard seats.

The number of deluxe seats should be at least 10% and at most 20% of the total number of seats.

The number of majestic seats should be at least half of the number of deluxe seats.

The total number of seats should be at least 250.

Standard, deluxe and majestic seats each cost £20, £26 and £36, respectively.

The manager wishes to minimise the total cost, £C, of the seats.

Formulate this situation as a linear programming problem, simplifying your inequalities so that all coefficients are integers. ***E***

Solution:

$$y+z \le \frac{1}{2}x \Rightarrow 2(y+z) \le x$$

$$y \ge \frac{10}{100}(x+y+z) \Rightarrow x+z \le 9y$$

$$y \ge \frac{20}{100}(x-y+z) \Rightarrow x+z \ge 4y$$

$$z \ge \frac{1}{2}y \qquad \Rightarrow 2z \ge y$$

$x \ge 0, y \ge 0, z \ge 0$.

$x+y+z \ge 250$

objective function: minimise $C = 20x+26y+36z$

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Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 13

Question:

Talkalot College holds an induction meeting for new students. The meeting consists of four talks: I (Welcome), II (Options and Facilities), III (Study Tips) and IV (Planning for Success). The four department heads, Clive, Julie, Nicky and Steve, deliver one of these talks each. The talks are delivered consecutively and there are no breaks between talks. The meeting starts at 10 a.m. and ends when all four talks have been delivered. The time, in minutes, each department head takes to deliver each talk is given in the table below.

	Talk I	**Talk II**	**Talk III**	**Talk IV**
Clive	12	34	28	16
Julie	13	32	36	12
Nicky	15	32	32	14
Steve	11	33	36	10

a Use the Hungarian algorithm to find the earliest time that the meeting could end. You must make your method clear and show

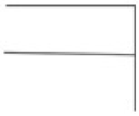
- **i** the state of the table after each stage in the algorithm.
- **ii** the final allocation.

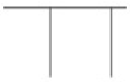
b Modify the table so it could be used to find the latest time that the meeting could end. (You do not have to find this latest time.) ***E***

Solution:

a i Reduce *rows then columns* giving

$$\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{C} & 0 & 22 & 16 & 4 \\ \text{J} & 1 & 20 & 24 & 0 \\ \text{N} & 1 & 18 & 18 & 0 \\ \text{S} & 1 & 23 & 26 & 0 \end{array} \quad \text{then} \quad \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{C} & 0 & 4 & 0 & 4 \\ \text{J} & 1 & 2 & 8 & 0 \\ \text{N} & 1 & 0 & 2 & 0 \\ \text{S} & 1 & 5 & 10 & 0 \end{array}$$

3 lines only needed  least element 1

or least element 

$$\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{C} & 0 & 4 & 0 & 5 \\ \text{J} & 0 & 1 & 7 & 0 \\ \text{N} & 1 & 0 & 2 & 1 \\ \text{S} & 0 & 4 & 9 & 0 \end{array} \quad \text{or} \quad \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{C} & 0 & 5 & 0 & 5 \\ \text{J} & 0 & 2 & 7 & 0 \\ \text{N} & 0 & 0 & 1 & 0 \\ \text{S} & 0 & 5 & 9 & 0 \end{array}$$

or *columns then rows* giving

$$\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{C} & 1 & 2 & 0 & 6 \\ \text{J} & 2 & 0 & 8 & 2 \\ \text{N} & 4 & 0 & 4 & 4 \\ \text{S} & 0 & 1 & 8 & 0 \end{array}$$

(then no change)

3 lines only needed ⊥ and *either* row 1 *or* column 3

if row 1 least uncovered element is 2

$$\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{C} & 1 & 4 & 0 & 6 \\ \text{J} & 0 & 0 & 6 & 0 \\ \text{N} & 2 & 0 & 2 & 2 \\ \text{S} & 0 & 3 & 8 & 0 \end{array}$$

if column 3 least uncovered element is 1

$$\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{C} & 0 & 2 & 0 & 5 \\ \text{J} & 1 & 0 & 8 & 1 \\ \text{N} & 3 & 0 & 4 & 3 \\ \text{S} & 0 & 2 & 9 & 0 \end{array}$$

then I least uncovered element 1 $\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{C} & 0 & 3 & 0 & 5 \\ \text{J} & 0 & 0 & 7 & 0 \\ \text{N} & 2 & 0 & 3 & 2 \\ \text{S} & 0 & 3 & 9 & 0 \end{array}$

ii C - III, J - I or IV, N - II, S - IV or I
83 minutes $\therefore$ 11:23 am

b Subtracting all entries from some $n \geq 36$
e.g. subtracting from 36

	I	II	III	IV
C	24	2	8	20
J	23	4	0	24
N	21	4	4	22
S	25	3	0	26

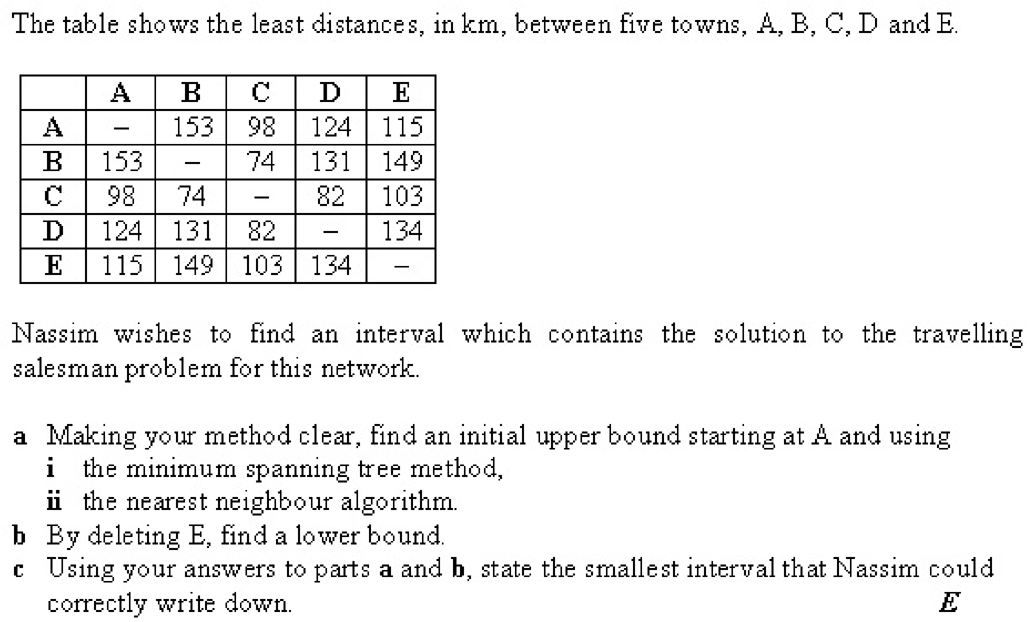
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 14

Question:



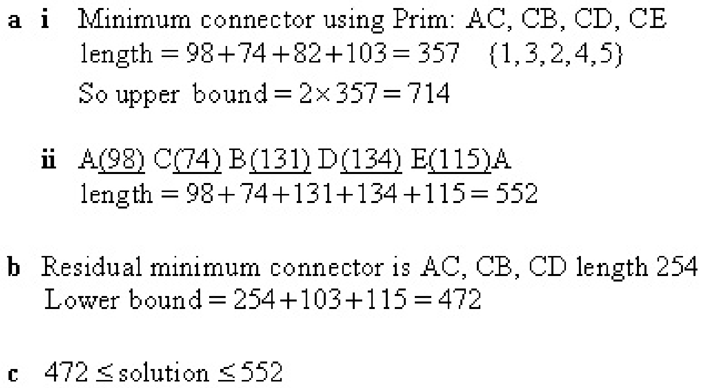
The table shows the least distances, in km, between five towns, A, B, C, D and E.

	A	B	C	D	E
A	–	153	98	124	115
B	153	–	74	131	149
C	98	74	–	82	103
D	124	131	82	–	134
E	115	149	103	134	–

Nassim wishes to find an interval which contains the solution to the travelling salesman problem for this network.

a Making your method clear, find an initial upper bound starting at A and using
  i the minimum spanning tree method,
  ii the nearest neighbour algorithm.
b By deleting E, find a lower bound.
c Using your answers to parts **a** and **b**, state the smallest interval that Nassim could correctly write down. ***E***

Solution:



a **i** Minimum connector using Prim: AC, CB, CD, CE
length $= 98+74+82+103 = 357 \quad \{1,3,2,4,5\}$
So upper bound $= 2\times 357 = 714$

ii A$\underline{(98)}$ C$\underline{(74)}$ B$\underline{(131)}$ D$\underline{(134)}$ E$\underline{(115)}$A
length $= 98+74+131+134+115 = 552$

b Residual minimum connector is AC, CB, CD length 254
Lower bound $= 254+103+115 = 472$

c $472 \le \text{solution} \le 552$

Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 15

Question:

Three depots, F, G and H, supply petrol to three service stations, S, T and U.
The table gives the cost, in pounds, of transporting 1000 litres of petrol from each depot to each service station.

	S	**T**	**U**
F	23	31	46
G	35	38	51
H	41	50	63

F, G and H have stocks of 540 000, 789 000 and 673 000 litres respectively. S, T and U require 257 000, 348 000 and 410 000 litres respectively. The total cost of transporting the petrol is to be minimised.

Formulate this problem as a linear programming problem. Make clear your decision variables, objective function and constraints. ***E***

Solution:

Let x_{ij} be the *number* of units transported from i to j, in 1000 litres where $i \in \{\mathrm{F}, \mathrm{G}, \mathrm{H}\}$ and $j \in \{\mathrm{S}, \mathrm{T}, \mathrm{U}\}$

$$\begin{aligned} \text{minimise } C = {} & 23x_{fs} + 31x_{ft} + 46x_{fu} + \\ & 35x_{gs} + 38x_{gt} + 51x_{gu} + \\ & 41x_{hs} + 50x_{ht} + 63x_{hu} \end{aligned} \quad \text{unbalanced}$$

$$\begin{aligned} \text{subject to } & x_{fs} + x_{ft} + x_{fu} \le 540 \\ & x_{gs} + x_{gt} + x_{gu} \le 789 \\ & x_{hs} + x_{ht} + x_{hu} \le 673 \\ & \left. \begin{aligned} x_{fs} + x_{gs} + x_{hs} &\le 257 \\ x_{ft} + x_{gt} + x_{ht} &\le 348 \\ x_{fu} + x_{gu} + x_{hu} &\le 410 \end{aligned} \right\} \text{accept} = \text{here} \end{aligned}$$

$$x_{ij} \ge 0$$

Accept introduction of a dummy demand methods.

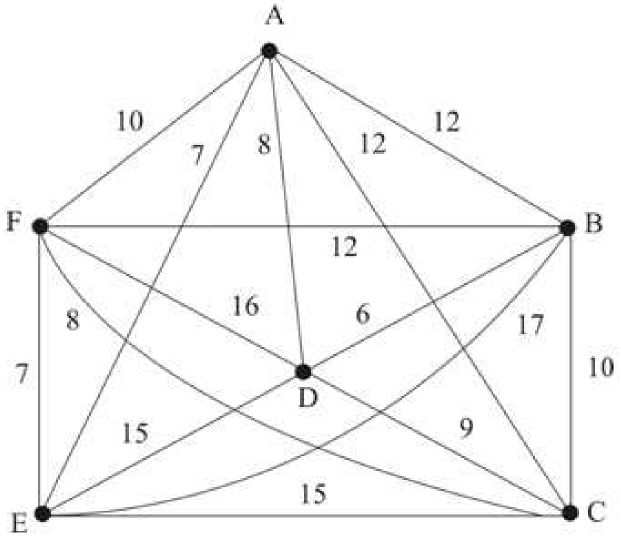
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 16

Question:



The diagram shows six towns A, B, C, D, E and F and the roads joining them. The number on each arc gives the length of that road in miles.

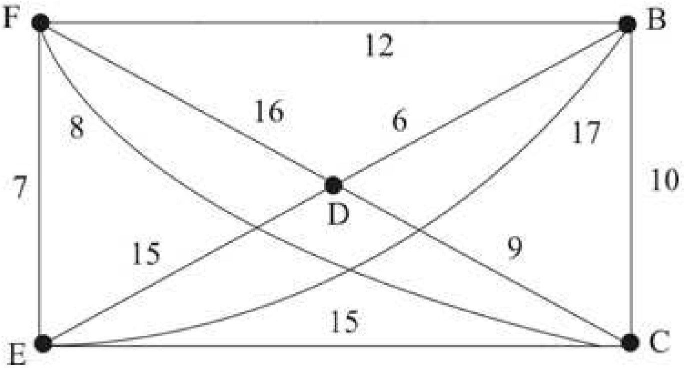
a By deleting vertex A, obtain a lower bound for the solution to the travelling salesman problem.

The nearest neighbour algorithm for finding a possible salesman tour is as follows:
Step 1: Let V be the current vertex.
Step 2: Find the nearest unvisited vertex to the current vertex, move directly to that vertex and call it the current vertex.
Step 3: Repeat step 2 until all vertices have been visited and then return directly to the start vertex.

b **i** Use this algorithm to find a tour starting at the vertex A. State clearly the tour and give its length.
ii Starting at an appropriate vertex, use the algorithm to find a tour of shorter length. ***E***

Solution:

a Deleting vertex A we obtain



By Kruskal's algorithm an MST is DB(6), EF(7), CF(8), DC(9) of weight 30

The two edges of least weight at A are AE(7) and AD(8)

$\therefore$ A lower bound is $30+8+7=45$

b **i** A – nearest neighbour E(7)
E – nearest neighbour F(7)
F – nearest neighbour C(8)
C – nearest neighbour D(9)
D – nearest neighbour B(6)

Complete tour with BA(12)
A E F C D B A – length 49

ii Choose a tour that does not use AB
e.g. DB(6) BC(10), CF(8), FE(4), EA(4)
Complete with AD(8), D B C F E A D.
Total weight 46

Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 17

Question:

Warehouse / Factory	W_1	W_2	W_3	Availabilities
F_1	7	8	6	4
F_2	9	2	4	3
F_3	5	6	3	8
Requirements	2	9	4	

A manufacturer has 3 factories F_1, F_2, F_3 and 3 warehouses W_1, W_2, W_3. The table shows the cost C_{ij}, in appropriate units, of sending one unit of product from factory F_i to warehouse W_j. Also shown in the table are the number of units available at each factory F_i and the number of units required at each warehouse W_j. The total number of units available is equal to the number of units required.

a Use the north-west corner rule to obtain a possible pattern of distribution and find its cost.

b Calculate shadow costs R_i and K_j for this pattern and hence obtain improvement indices I_{ij} for each route.

c Using your answer to part **b**, explain why the pattern is optimal. ***E***

Solution:

a

	W_1	W_2	W_3	Available
F_1	2	2		4
F_2		3		3
F_3		4	4	8
Require	2	9	4	

Cost $2\times7+2\times8+3\times2+4\times6+4\times3=14+16+6+24+12=72$

b For occupied cells $R_1+K_2=C_3$ gives

$(1,1)R_1+K_1 = 7j(1.2)R_1+K_2=8:(2.2)R_2-K_2=2$

$(3,2)R_3+K_2 = 6j(3,3)R_3+K_3=3$

Taking $R_1=0$ we obtain $K_1=7, K_2=8, R_2=-6, R_3=-2, K_3=5$

Shadow costs		7	8	5	
		W_1	W_2	W_3	
0	F_1	7	8		4
−6	F_2		2		3
−2	F_3		6	3	8
		2	9	4	

$F_1=0$ $W_1=7$

$F_2=-6$ $W_2=8$

$F_3=-2$ $W_3=5$

Improvement indices $I_{ij}=C_{ij}-R_i-K_j$

$I_{13}=6-5-0=1$

$I_{21}=9-7-(-6)=8$

$I_{23}=4-5-(-6)=5$

$I_{31}=5-7-(-2)=0$

c No negative improvement indices and so given solution is optimal and gives minimum cost. If there was a negative I_{ij} then using this route would reduce cost.

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Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 18

Question:

a State the circumstances under which it is necessary to use the simplex algorithm, rather than a graphical method.

The tableau given below arose after one complete iteration of the simplex algorithm.

Basic variable	x	y	z	r	s	Value
y	$\frac{4}{5}$	1	$\frac{2}{5}$	$\frac{1}{5}$	0	$429\frac{2}{5}$
s	$2\frac{1}{5}$	0	$5\frac{3}{5}$	$-\frac{1}{5}$	1	$1243\frac{3}{5}$
P	$-\frac{3}{5}$	0	$-\frac{4}{5}$	$1\frac{3}{5}$	0	$3435\frac{1}{5}$

b State the column that was used as the pivotal column for the first iteration.

c Perform one further complete iteration to obtain the next complete tableau.

d State the values of P, x, y and z displayed by your tableau in part **c**.

e State, giving a reason, whether your values in part **d** give the optimal solution. ***E***

Solution:

a If the number of variables ≥ 3 use simplex

b Column y

c

b.v.	x	y	z	r	s	numbers	
y	$\frac{9}{14}$	1	0	$\frac{2}{7}$	$\frac{-1}{14}$	$340\frac{4}{7}$	$R1-\frac{2}{5}R3$
z	$\frac{11}{28}$	0	1	$\frac{-3}{14}$	$\frac{5}{28}$	$222\frac{1}{14}$	$R2+5\frac{3}{5}$
P	$-\frac{2}{7}$	0	0	$1\frac{3}{7}$	$\frac{1}{7}$	$3612\frac{6}{7}$	$R3+\frac{4}{5}R3$

d $P=3612\frac{6}{7}$ $x=0$ $y=340\frac{4}{7}$ $z=222\frac{1}{14}$

e No. bottom row still contains a negative, x can be increased.

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Edexcel AS and A Level Modular Mathematics

Review Exercise 1
Exercise A, Question 19

Question:

An engineering company has 4 machines available and 4 jobs to be completed. Each machine is to be assigned to one job. The time, in hours, required by each machine to complete each job is shown in the table below.

	Job 1	Job 2	Job 3	Job 4
Machine 1	14	5	8	7
Machine 2	2	12	6	5
Machine 3	7	8	3	9
Machine 4	2	4	6	10

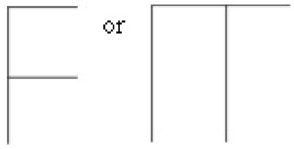
Use the Hungarian algorithm, *reducing rows first*, to obtain the allocation of machines to jobs which minimises the total time required. State this minimum time. ***E***

Solution:

a Reducing rows

$$\begin{matrix} 9 & 0 & 3 & 2 \\ 0 & 10 & 4 & 3 \\ 4 & 5 & 0 & 6 \\ 0 & 2 & 4 & 8 \end{matrix} \xrightarrow[\text{columns}]{\text{reducing}} \begin{matrix} 9 & 0 & 3 & 0 \\ 0 & 10 & 4 & 1 \\ 4 & 5 & 0 & 4 \\ 0 & 2 & 4 & 6 \end{matrix}$$

b Testing for optimality – 3 lines are enough



Minimum uncovered element is 1

$$\begin{matrix} 10 & 0 & 3 & 0 \\ 0 & 9 & 3 & 0 \\ 5 & 5 & 0 & 4 \\ 0 & 1 & 3 & 5 \end{matrix} \quad \text{or} \quad \begin{matrix} 10 & 0 & 4 & 0 \\ 0 & 9 & 4 & 0 \\ 4 & 4 & 0 & 3 \\ 0 & 1 & 4 & 5 \end{matrix}$$

4 lines now needed

c Final matching
Machine 1 - Job 2 (5)
Machine 2 - Job 4 (5)
Machine 3 - Job 3 (3)
Machine 4 - Job 1 (2)
Minimum time: 15 hours

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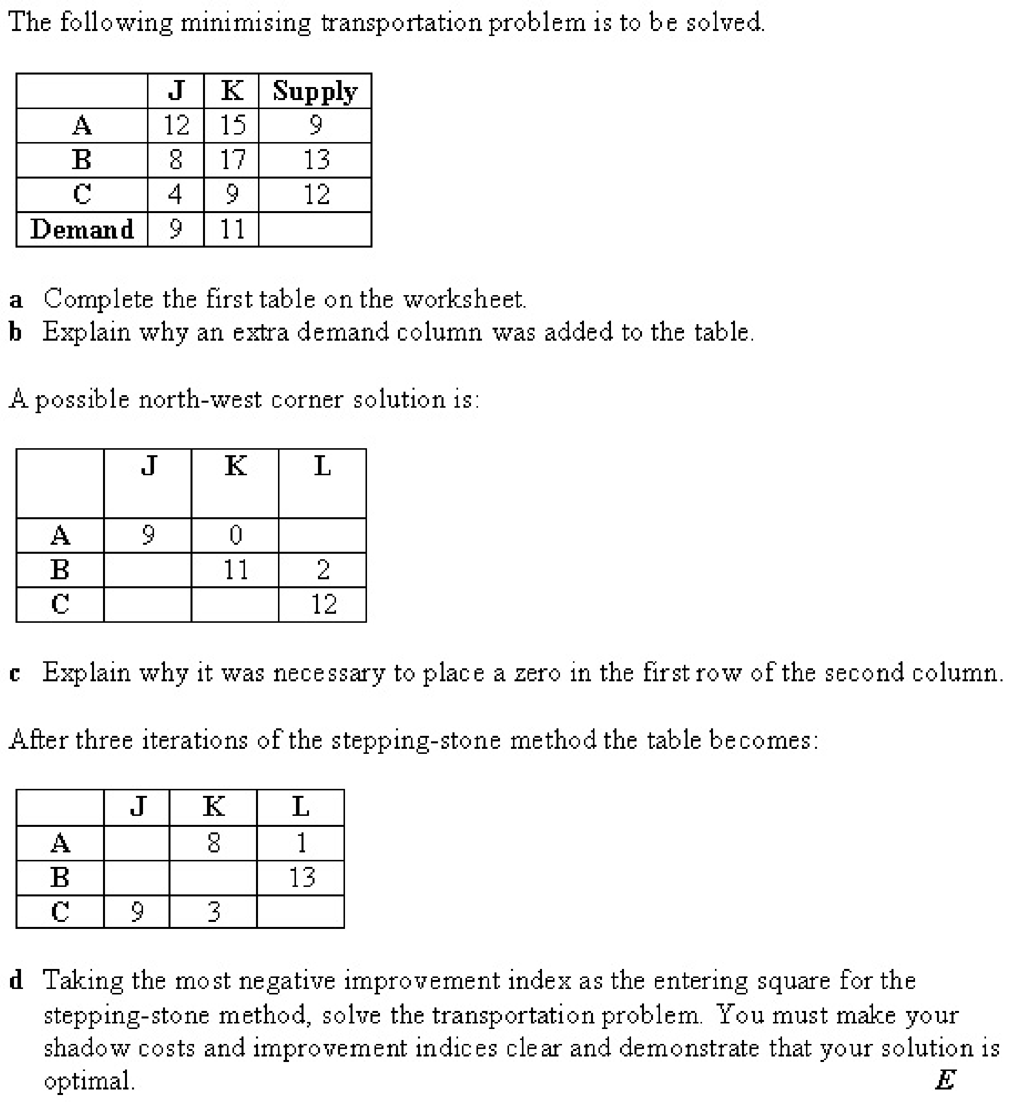
Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 20

Question:



The following minimising transportation problem is to be solved.

	J	K	Supply
A	12	15	9
B	8	17	13
C	4	9	12
Demand	9	11	

a Complete the first table on the worksheet.
b Explain why an extra demand column was added to the table.

A possible north-west corner solution is:

	J	K	L
A	9	0	
B		11	2
C			12

c Explain why it was necessary to place a zero in the first row of the second column.

After three iterations of the stepping-stone method the table becomes:

	J	K	L
A		8	1
B			13
C	9	3	

d Taking the most negative improvement index as the entering square for the stepping-stone method, solve the transportation problem. You must make your shadow costs and improvement indices clear and demonstrate that your solution is optimal. ***E***

Solution:

a Adds zero for cost in third column
Adds 14 as the demand value

b The total supply is greater than the total demand

c The solution would otherwise be degenerate

d

		10	15	0
		J	K	L
0	A		8	1
0	B			13
−6	C	9	3	

$I_{AJ} = 12 - 0 - 10 = 2$
$I_{BJ} = 8 - 0 - 10 = -2*$
$I_{BC} = 17 - 0 - 15 = 2$
$I_{CL} = 0 + 6 - 0 = 6$

	J	K	L
A		$8-\theta$	$1+\theta$
B	θ		$13-\theta$
C	$9-\theta$	$3+\theta$	

$\theta = 8$
Entering square BJ
Exiting square AK

		8	13	0
		J	K	L
0	A			9
0	B	8		5
−4	C	1	11	

$I_{AJ} = 12 - 0 - 3 = 4$
$I_{AK} = 15 - 0 - 13 = 2$
$I_{BK} = 17 - 0 - 13 = 4$
$I_{CL} = 0 + 4 - 0 = 4$
No negatives, so optimal

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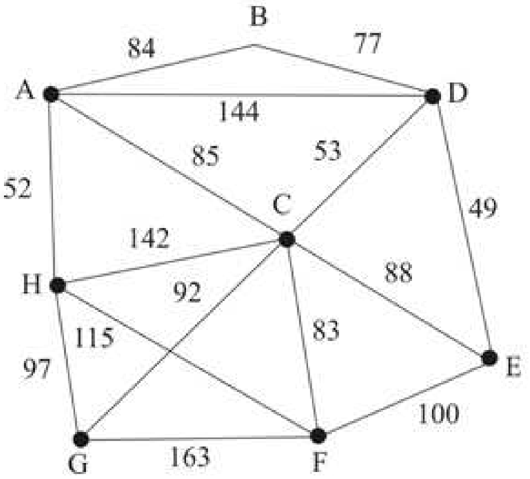
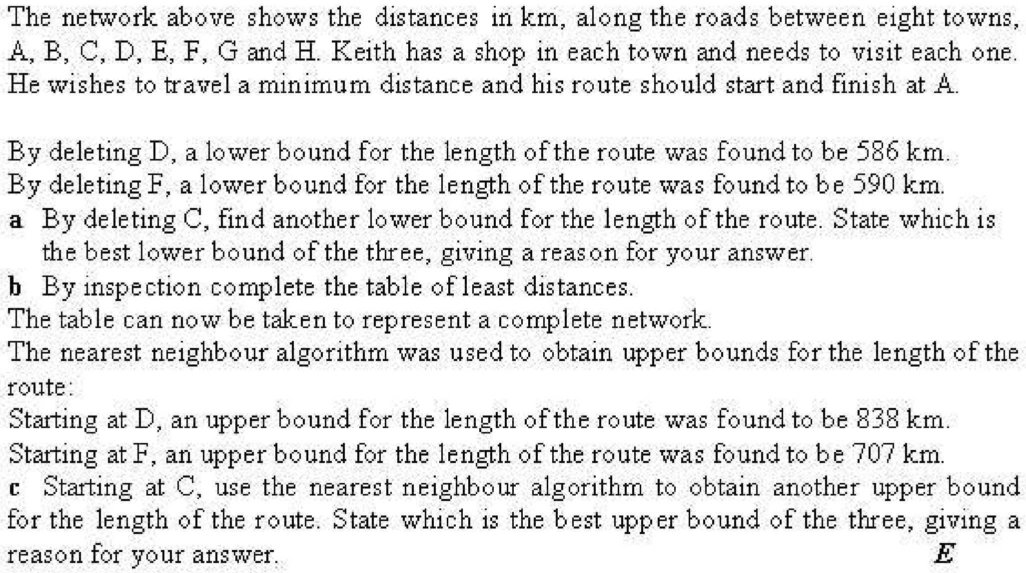
Edexcel AS and A Level Modular Mathematics

Review Exercise 1
Exercise A, Question 21

Question:





The network above shows the distances in km, along the roads between eight towns, A, B, C, D, E, F, G and H. Keith has a shop in each town and needs to visit each one. He wishes to travel a minimum distance and his route should start and finish at A.

By deleting D, a lower bound for the length of the route was found to be 586 km.
By deleting F, a lower bound for the length of the route was found to be 590 km.

a By deleting C, find another lower bound for the length of the route. State which is the best lower bound of the three, giving a reason for your answer.

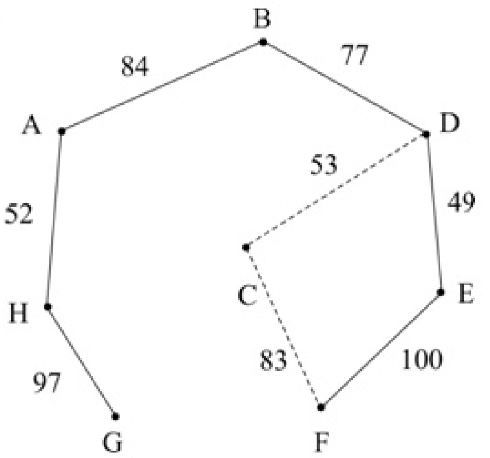
b By inspection complete the table of least distances.

The table can now be taken to represent a complete network.
The nearest neighbour algorithm was used to obtain upper bounds for the length of the route:
Starting at D, an upper bound for the length of the route was found to be 838 km.
Starting at F, an upper bound for the length of the route was found to be 707 km.

c Starting at C, use the nearest neighbour algorithm to obtain another upper bound for the length of the route. State which is the best upper bound of the three, giving a reason for your answer. ***E***

Solution:

a



R.M.S.T
e.g. AH, AB, BD, DE, HG, EF
using Prim's

length of R M S T = 459
$\therefore$ lower bound = 459 + 53 + 83 = 595 km (deleting C)
Best lower bound is 595 km, by deleting C as it is the highest lower bound found.

b Adds 167 to AF and FA
137 to CH and HC
136 to DF and FD
145 to DG and GD

c $C_{53}\ D_{49}\ E_{120}\ F_{115}\ H_{52}\ A_{84}\ B_{222}\ G_{92}\ C$
Upper bound, starting at C = 767 km
$\therefore$ Best upper bound is 707 starting at F as it is the lowest upper bound found.

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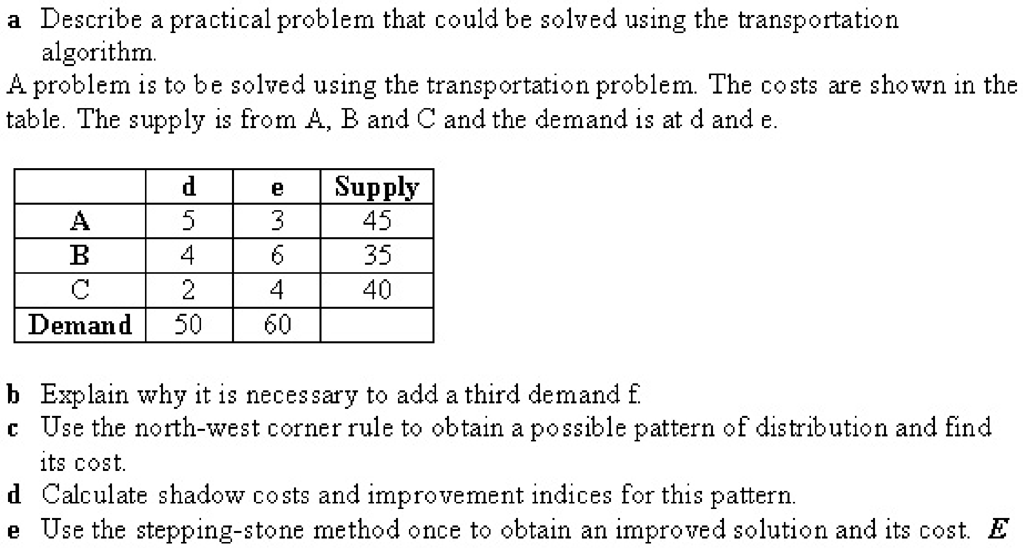
Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 22

Question:



a Describe a practical problem that could be solved using the transportation algorithm.

A problem is to be solved using the transportation problem. The costs are shown in the table. The supply is from A, B and C and the demand is at d and e.

	d	e	Supply
A	5	3	45
B	4	6	35
C	2	4	40
Demand	50	60	

b Explain why it is necessary to add a third demand f.

c Use the north-west corner rule to obtain a possible pattern of distribution and find its cost.

d Calculate shadow costs and improvement indices for this pattern.

e Use the stepping-stone method once to obtain an improved solution and its cost. ***E***

Solution:

a Idea of many supply and demand points and many units to be moved. Costs are variable and dependent upon the supply and demand points, need to minimise costs. *Practical*

b Supply $= 120$ Demand $= 110$ so not balanced

c Adds 0, 0, 0, 10 to column f

	d	e	f
A	45		
B	5	30	
C		30	10

Cost 545

d $R_1 = 0 \quad R_2 = -1 \quad R_3 = -3$

$K_1 = 5 \quad K_2 = 7 \quad K_3 = 3$

Shadow costs

$Ae = 3 - 0 - 7 = -4 \leftarrow$

$Af = 0 - 0 - 3 = -3$

$Bf = 0 + 1 - 3 = -2$

$Cd = 2 + 3 - 5 = 0$

Improvement indices

e $Ae^+ \to Be^- \to Bd^+ \to Ad^-$ so $\theta = 30$

	d	e	f
A	15	30	
B	35		
C		30	10

Cost 425

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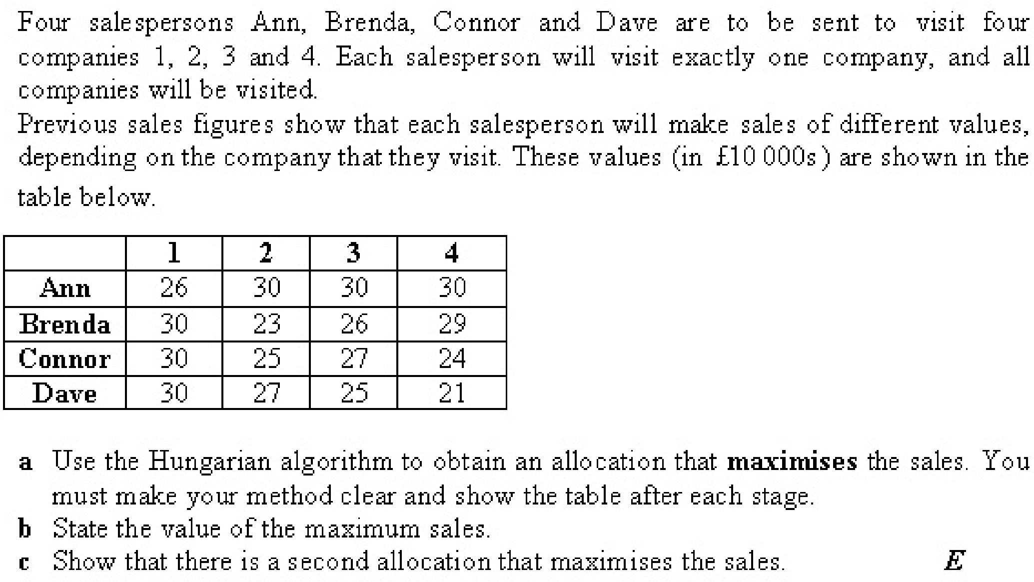
Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 23

Question:



Four salespersons Ann, Brenda, Connor and Dave are to be sent to visit four companies 1, 2, 3 and 4. Each salesperson will visit exactly one company, and all companies will be visited.

Previous sales figures show that each salesperson will make sales of different values, depending on the company that they visit. These values (in £10 000s) are shown in the table below.

	1	**2**	**3**	**4**
Ann	26	30	30	30
Brenda	30	23	26	29
Connor	30	25	27	24
Dave	30	27	25	21

a Use the Hungarian algorithm to obtain an allocation that **maximises** the sales. You must make your method clear and show the table after each stage.

b State the value of the maximum sales.

c Show that there is a second allocation that maximises the sales. ***E***

Solution:

a To maximise, subtract all entries from $n \geq 30$

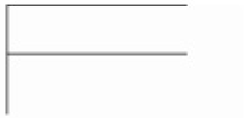
e.g. $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 5 & 3 & 6 \\ 0 & 3 & 5 & 9 \end{bmatrix}$

minimum uncovered
element is 1

so $\begin{bmatrix} 0 & 6 & 3 & 0 \\ 0 & 4 & 2 & 5 \\ 0 & 2 & 4 & 8 \end{bmatrix}$

or

minimum element is 2



minimum element is 2

$\begin{bmatrix} 7 & 0 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 8 \end{bmatrix}$ $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$

A − 2 B − 4 C − 3 D − 1
or A − 3 B − 4 C − 1 D − 2

b £1 160 000

c Gives other solution from part **a**.

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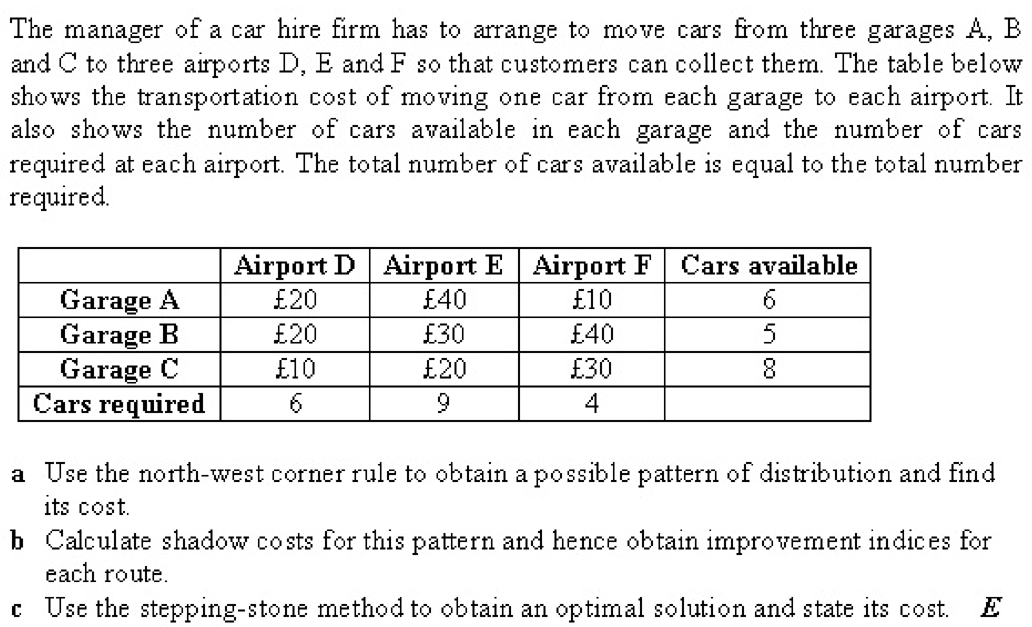
Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 24

Question:



The manager of a car hire firm has to arrange to move cars from three garages A, B and C to three airports D, E and F so that customers can collect them. The table below shows the transportation cost of moving one car from each garage to each airport. It also shows the number of cars available in each garage and the number of cars required at each airport. The total number of cars available is equal to the total number required.

	Airport D	**Airport E**	**Airport F**	**Cars available**
Garage A	£20	£40	£10	6
Garage B	£20	£30	£40	5
Garage C	£10	£20	£30	8
Cars required	6	9	4	

a Use the north-west corner rule to obtain a possible pattern of distribution and find its cost.

b Calculate shadow costs for this pattern and hence obtain improvement indices for each route.

c Use the stepping-stone method to obtain an optimal solution and state its cost. ***E***

Solution:

a *e.g.*

	D	E	F
A	6		
B	0	5	
C		4	4

or

	D	E	F
A	6	0	
B		5	
C		4	4

Cost £470

b $S_A = 0 \quad S_B = 0 \quad S_C = -10$

$D_D = 20 \quad D_E = 30 \quad D_F = 40$

$I_{AE} = 40 - 30 = 10$

$I_{AF} = 10 - 40 = -30$

$I_{BF} = 40 - 40 = 0$

$I_{CD} = 10 - 10 = 0$

$S_A = 0 \quad S_B = -10 \quad S_C = -20$

$D_D = 20 \quad D_E = 40 \quad D_F = 50$

$I_{AF} = 10 - 50 = -40$

$I_{BD} = 20 - 10 = 10$

$I_{BF} = 40 - 40 = 0$

$I_{CD} = 10 - 0 = 10$

c Choose A F as entering route

$AF(+) \to CF(-) \to CE(+) \to BE(-) \to BD(+) \to AD(-)$

Exiting route CF $\theta = 4$

	D	E	F
A	2		4
B	4	1	
C		8	

$S_A = 0 \quad S_B = 0 \quad S_C = -10$

$D_D = 20 \quad D_E = 30 \quad D_F = 10$

$I_{AE} = 10, I_{BF} = 30,$

$I_{CD} = 0 \quad I_{CF} = 30$

$\therefore$ optimal

cost = £350

$AF(+) \to CF(-) \to CE(+) \to AE(-)$

Exiting route AE $\theta = 0$

	D	E	F
A	6		0
B		5	
C		4	4

So $S_A = 0 \quad S_B = 30 \quad S_C = 20$

$D_D = 20 \quad D_E = 0 \quad D_F = 10$

$I_{AE} = 40, I_{BD} = -30, I_{BF} = 0, I_{CD} = -30$

e.g. $CD(+) \to AD(-) \to AF(+) \to CF(-)\ \theta = 4$

	D	E	F
A	2		4
B		5	
C	4	4	

$S_A = 0 \quad S_B = 0 \quad S_C = -10$

$D_D = 20 \quad D_E = 30 \quad D_F = 10$

$I_{AE} = 10, I_{BD} = 0 \quad I_{BF} = 30 \quad I_{CF} = 30$

$\therefore$ optimal cost £350

or $DB(+) \to BE(-) \to CE(+) \to CD(-)\ \theta = 4$
giving left hand solution table

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Review Exercise 1
Exercise A, Question 25

Question:

A chemical company makes 3 products X, Y and Z. It wishes to maximise its profit £P. The manager considers the limitations on the raw materials available and models the situation with the following linear programming problem.

Maximise $P = 3x + 6y + 4z$,
subject to $x + z \le 4$,
$x + 4y + 2z \le 6$,
$x + y + 2z \le 12$,
$x \ge 0, y \ge 0, z \ge 0$,

where x, y and z are the weights, in kg, of products X, Y and Z respectively.

A possible tableau is

Basic variable	x	y	z	r	s	t	Value
r	1	0	1	1	0	0	4
s	1	4	2	0	1	0	6
t	1	1	2	0	0	1	12
P	−3	−6	−4	0	0	0	0

a Explain
  i the purpose of the variables r, s and t,
  ii the final row of the tableau.
b Solve this linear programming problem by using the simplex algorithm.
Increase y for your first iteration and then increase x for your second iteration.
c Interpret your solution. *E*

Solution:

a **i** Slack variables used to enable us to write inequalities as equalities. All slack variable are ≥ 0

ii $P - 3x - 6y - 4z = 0$

b

b.v.	x	y	z	r	s	t	value	row ops
r	1	0	1	1	0	0	4	
s	1	④	2	0	1	0	6	
t	1	1	2	0	0	1	12	
P	-3	-6	-4	0	0	0	0	

b.v.	x	y	z	r	s	t	value	row ops
r	①	0	1	1	0	0	4	No change
Y	$\frac{1}{4}$	1	$\frac{1}{2}$	0	$\frac{1}{4}$	0	$1\frac{1}{2}$	R2 $\div$ 4
t	$\frac{3}{4}$	0	$1\frac{1}{2}$	0	$-\frac{1}{4}$	1	$10\frac{1}{2}$	R3 $-$R2
P	$-1\frac{1}{2}$	0	-1	0	$1\frac{1}{2}$	0	9	R4 + 6R2

b.v.	x	y	z	r	s	t	value	row ops
x	1	0	1	1	0	0	4	R1 $\div$ 1
y	0	1	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	R2 $-\frac{1}{4}$R1
t	0	0	$\frac{3}{4}$	$-\frac{3}{4}$	$-\frac{1}{4}$	1	$7\frac{1}{2}$	R3 $-\frac{3}{4}$R1
P	0	0	$\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	0	15	R4 $+1\frac{1}{2}$R1

c Maximum profit is £15

when $x = 4$ kg, $y = \frac{1}{2}$ kg, $z = 0$ kg

The first and second constraints have no slack

There is a slack of $7\frac{1}{2}$ in the third constraint.

Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 26

Question:

The table below shows the distances, in km, between six towns A, B, C, D, E and F.

	A	**B**	**C**	**D**	**E**	**F**
A	–	85	110	175	108	100
B	85	–	38	175	160	93
C	110	38	–	148	156	73
D	175	175	148	–	110	84
E	108	160	156	110	–	92
F	100	93	73	84	92	–

a Starting from A, use Prim's algorithm to find a minimum connector and draw the minimum spanning tree. You must make your method clear by stating the order in which the arcs are selected.

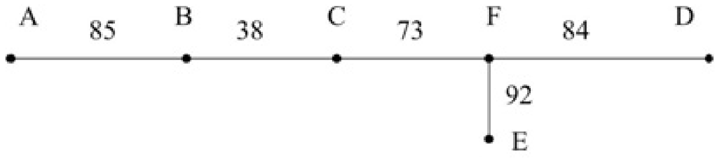
b **i** Using your answer to part **a** obtain an initial upper bound for the solution of the travelling salesman problem.

ii Use a short cut to reduce the upper bound to a value less than 680.

c Starting by deleting F, find a lower bound for the solution of the travelling salesman problem. ***E***

Solution:

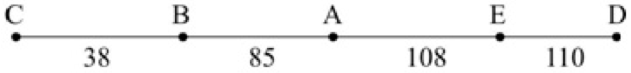
a Order of arcs: AB, BC, CF, FD, FE



b **i** $2 \times 372 = 744$

ii e.g. AD saves 105 giving 639
or AE saves 180 giving 564
AF saves 96 giving 648
DE saves 66 giving 678

c Residual M.S.T.
AB, BC, AE, ED



$$\text{Lower bound} = 341 + 73 + 84$$
$$= 498$$

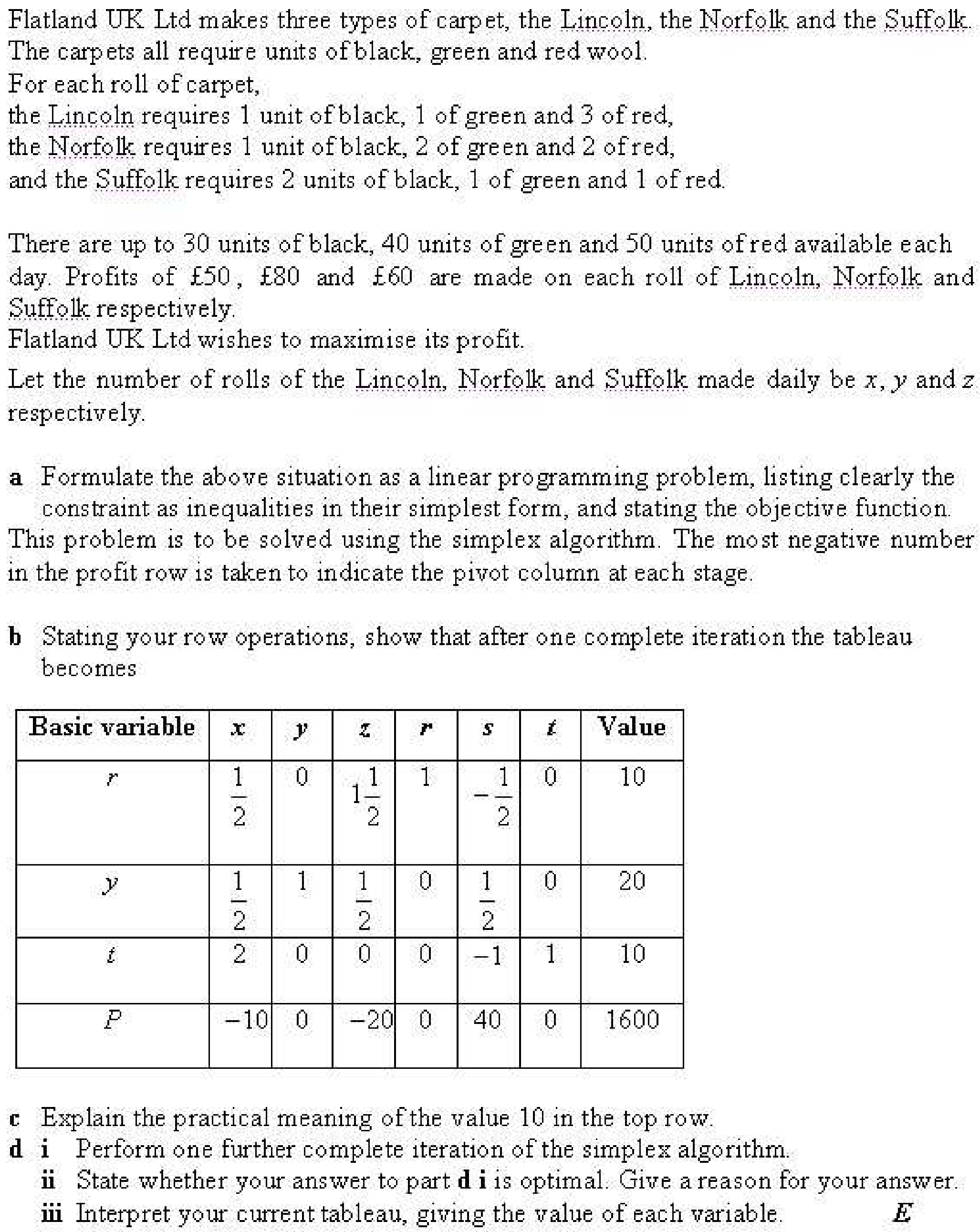
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 27

Question:



Flatland UK Ltd makes three types of carpet, the Lincoln, the Norfolk and the Suffolk.
The carpets all require units of black, green and red wool.
For each roll of carpet,
the Lincoln requires 1 unit of black, 1 of green and 3 of red,
the Norfolk requires 1 unit of black, 2 of green and 2 of red,
and the Suffolk requires 2 units of black, 1 of green and 1 of red.

There are up to 30 units of black, 40 units of green and 50 units of red available each day. Profits of £50, £80 and £60 are made on each roll of Lincoln, Norfolk and Suffolk respectively.
Flatland UK Ltd wishes to maximise its profit.

Let the number of rolls of the Lincoln, Norfolk and Suffolk made daily be x, y and z respectively.

a Formulate the above situation as a linear programming problem, listing clearly the constraint as inequalities in their simplest form, and stating the objective function.
This problem is to be solved using the simplex algorithm. The most negative number in the profit row is taken to indicate the pivot column at each stage.

b Stating your row operations, show that after one complete iteration the tableau becomes

Basic variable	x	y	z	r	s	t	Value
r	$\frac{1}{2}$	0	$1\frac{1}{2}$	1	$-\frac{1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	20
t	2	0	0	0	-1	1	10
P	-10	0	-20	0	40	0	1600

c Explain the practical meaning of the value 10 in the top row.

d **i** Perform one further complete iteration of the simplex algorithm.

ii State whether your answer to part **d i** is optimal. Give a reason for your answer.

iii Interpret your current tableau, giving the value of each variable. *E*

Solution:

a Maximise $P = 50x + 80y + 60z$

Subject to $x + y + 2z \le 30$

$x + 2y + z \le 40$

$3x + 2y + z \le 50$

where $x, y, z \ge 0$

b Initialising tableau

b.v.	x	y	z	r	s	t	value
r	1	1	2	1	0	0	30
s	1	②	1	0	1	0	40
t	3	2	1	0	0	1	50
P	-50	-80	-60	0	0	0	0

Chooses correct pivot, divide R2 by 2

State correct row operation R1 − R2, R3 − 2R2, R4 + 80R2, R2 ÷ 2

c The solution found after one iteration has a slack of 10 units of black per day

d **i**

b.v.	x	y	z	r	s	t	value
r	$\frac{1}{2}$	0	$\textcircled{\tfrac{3}{2}}$	1	$\frac{-1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	20
t	2	0	0	0	-1	1	10
P	-10	0	-20	0	40	0	1600

(given)

b.v.	x	y	z	r	s	t	value	
z	$\frac{1}{3}$	0	1	$\frac{2}{3}$	$\frac{-1}{3}$	0	$6\frac{2}{3}$	R1 ÷ $\frac{3}{2}$
y	$\frac{1}{3}$	1	0	$\frac{-1}{3}$	$\frac{2}{3}$	0	$16\frac{2}{3}$	R2 − $\frac{1}{2}$R1
t	2	0	0	0	-1	1	10	R3 − no change
P	$-3\frac{1}{3}$	0	0	$13\frac{1}{3}$	$33\frac{1}{3}$	0	$1733\frac{1}{3}$.	R4+20R1

ii Not optimal, a negative value in profit row

iii $x = 0 \quad y = 16\frac{2}{3} \quad z = 6\frac{2}{3}$

$P = £1733.33 \quad r = 0, s = 0, t = 10$

Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 28

Question:

	A	B	C	D	E	F
A	–	113	53	54	87	68
B	113	–	87	123	38	100
C	53	87	–	106	58	103
D	54	123	106	–	140	48
E	87	38	58	140	–	105
F	68	100	103	48	105	–

The table shows the distances, in km, between six towns A, B, C, D, E and F.

a Starting from A, use Prim's algorithm to find a minimum connector and draw the minimum spanning tree. You must make your method clear by stating the order in which the arcs are selected.

b **i** Hence form an initial upper bound for the solution to the travelling salesman problem.

ii Use a short cut to reduce the upper bound to a value below 360.

c By deleting A, find a lower bound for the solution to the travelling salesman problem.

d Use your answers to parts **b** and **c** to make a comment on the value of the optimal solution.

e Draw a diagram to show your best route. ***E***

Solution:

a AC (–53), AD(54), DF(–48), CE(–58), EB(–38)

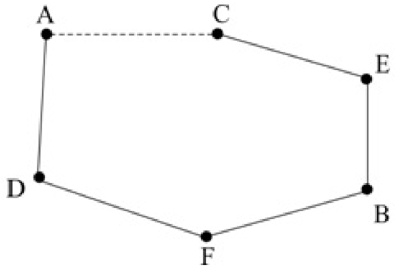
b **i** M.S.T. XZ = 251 × 2 = 502

ii Finding a shortcut to below 360, e.g. FB leaves 351

c M.S.T. is DF, CE, EB, FB length 244
The 2 shortest arcs are AC (–53) and AD (–54) giving a total of 351

d The optimal solution is 351 and is A–C–E–B–F–D–A

e



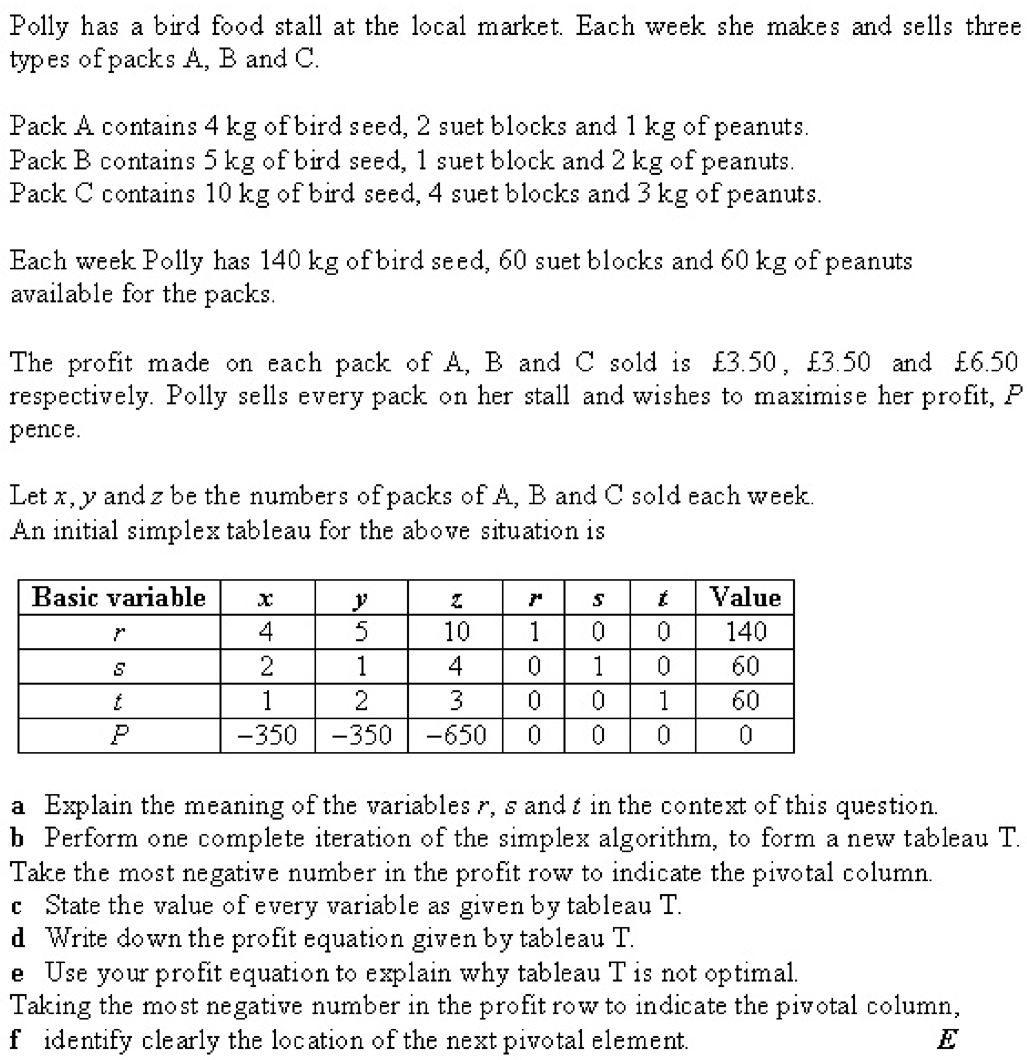
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 29

Question:



Polly has a bird food stall at the local market. Each week she makes and sells three types of packs A, B and C.

Pack A contains 4 kg of bird seed, 2 suet blocks and 1 kg of peanuts.
Pack B contains 5 kg of bird seed, 1 suet block and 2 kg of peanuts.
Pack C contains 10 kg of bird seed, 4 suet blocks and 3 kg of peanuts.

Each week Polly has 140 kg of bird seed, 60 suet blocks and 60 kg of peanuts available for the packs.

The profit made on each pack of A, B and C sold is £3.50, £3.50 and £6.50 respectively. Polly sells every pack on her stall and wishes to maximise her profit, P pence.

Let x, y and z be the numbers of packs of A, B and C sold each week.
An initial simplex tableau for the above situation is

Basic variable	x	y	z	r	s	t	Value
r	4	5	10	1	0	0	140
s	2	1	4	0	1	0	60
t	1	2	3	0	0	1	60
P	−350	−350	−650	0	0	0	0

a Explain the meaning of the variables r, s and t in the context of this question.
b Perform one complete iteration of the simplex algorithm, to form a new tableau T.
Take the most negative number in the profit row to indicate the pivotal column.
c State the value of every variable as given by tableau T.
d Write down the profit equation given by tableau T.
e Use your profit equation to explain why tableau T is not optimal.
Taking the most negative number in the profit row to indicate the pivotal column,
f identify clearly the location of the next pivotal element. ***E***

Solution:

a r, s and t are unused amounts of bird seed (in kg), suet blocks and peanuts (in kg) that Polly has at the end of each week after she has made up and sold her packs.

b

b.v.	x	y	z	r	s	t	value	
z	$\frac{2}{5}$	$\frac{1}{2}$	1	$\frac{1}{10}$	0	0	14	R1 ÷ 10
s	$\boxed{\frac{2}{5}}$	−1	0	$\frac{-2}{5}$	1	0	4	R2 − 4R1
t	$-\frac{1}{5}$	$\frac{1}{2}$	0	$-\frac{3}{10}$	0	1	18	R3 − 3R1
P	−90	−25	0	65	0	0	9100	R4 + 650R1

c $x = 0 \quad y = 0 \quad z = 14 \quad r = 0 \quad s = 4 \quad t = 18 \quad P = £91$

d $P - 90x - 25y + 65r = 9100$

e $P = 9100 + 90x + 25y - 65r$

So increasing x or y would increase the profit

f The $\frac{2}{5}$ in the x column and 2nd (s) row.

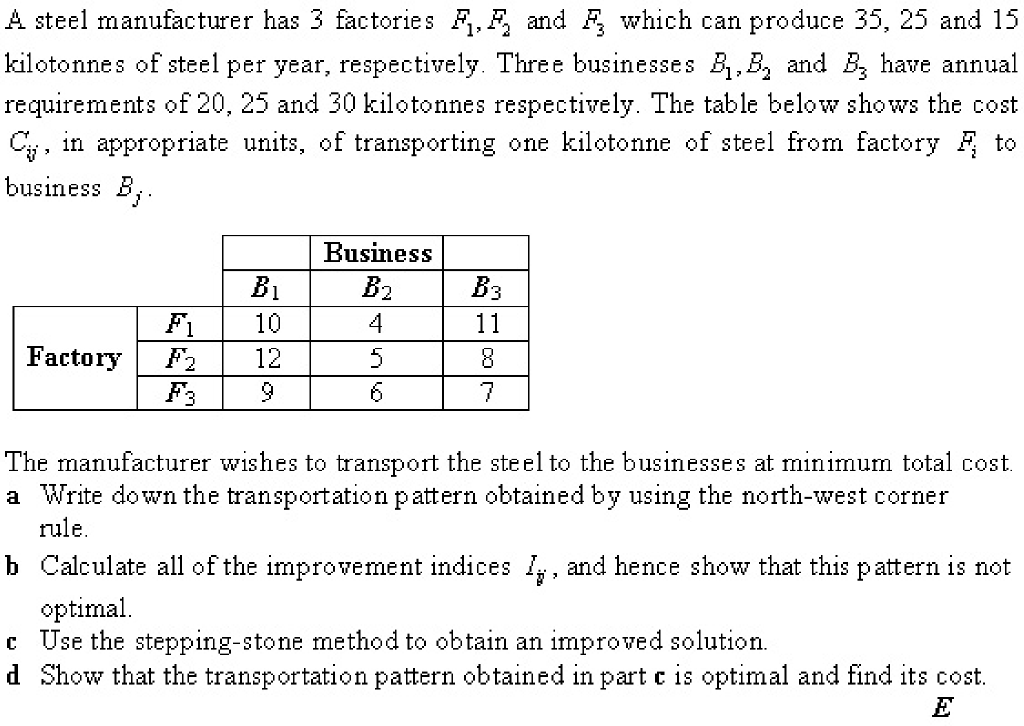
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 30

Question:



A steel manufacturer has 3 factories F_1, F_2 and F_3 which can produce 35, 25 and 15 kilotonnes of steel per year, respectively. Three businesses B_1, B_2 and B_3 have annual requirements of 20, 25 and 30 kilotonnes respectively. The table below shows the cost C_{ij}, in appropriate units, of transporting one kilotonne of steel from factory F_i to business B_j.

			Business	
		B_1	B_2	B_3
Factory	F_1	10	4	11
	F_2	12	5	8
	F_3	9	6	7

The manufacturer wishes to transport the steel to the businesses at minimum total cost.

a Write down the transportation pattern obtained by using the north-west corner rule.

b Calculate all of the improvement indices I_{ij}, and hence show that this pattern is not optimal.

c Use the stepping-stone method to obtain an improved solution.

d Show that the transportation pattern obtained in part **c** is optimal and find its cost.

E

Solution:

a

	B_1	B_2	B_3
F_1	20	15	
F_2		10	15
F_3			15

b

$S(F_1) = 0 \quad S(F_2) = 1 \quad S(F_3) = 0$

$D(B_1) = 10 \quad D(B_2) = 4 \quad D(B_3) = 7$

$I_{13} = 11 - 0 - 7 = 4$

$I_{21} = 12 - 1 - 10 = 1$

$I_{31} = 9 - 0 - 10 = -1$

$I_{32} = 6 - 0 - 4 = 2$

Since I_{31} is negative pattern is not optimal

c

	B_1	B_2	B_3
F_1	$20-\theta$	$15+\theta$	
F_2		$10-\theta$	$15+\theta$
F_3	θ		$15-\theta$

Entering square $F_3\,B_1$

Exiting square F_2B_2

$\theta = 10$

	B_1	B_2	B_3
F_1	10	25	
F_2			25
F_3	10		5

d

$S(F_1) = 0 \quad S(F_2) = 0 \quad S(F_3) = -1$

$D(B_1) = 10 \quad D(B_2) = 4 \quad D(B_3) = 8$

$I_{13} = 11 - 0 - 8 = 3$

$I_{21} = 12 - 0 - 10 = 2$

$I_{22} = 5 - 0 - 4 = 1$

$I_{32} = 6 - (-1) - 4 = 3$

all positions are positive $\therefore$ optimal

$\text{Cost} = (10\times10) + (25\times4) + (25\times8) + (10\times9) + (5\times7) = 525$ units

Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 31

Question:

A company makes three sizes of lamps, small, medium and large. The company is trying to determine how many of each size to make in a day, in order to maximise its profit. As part of the process the lamps need to be sanded, painted, dried and polished. A single machine carries out these tasks and is available 24 hours per day. A small lamp requires one hour on this machine, a medium lamp 2 hours and a large lamp 4 hours.

Let $x =$ number of small lamps made per day,
$y =$ number of medium lamps made per day,
$z =$ number of large lamps made per day,
where $x \geq 0, y \geq 0$ and $z \geq 0$.

a Write the information about this machine as a constraint.
b **i** Re-write your constraint from part **a** using a slack variable s.
ii Explain what s means in practical terms.

Another constraint and the objective function give the following simplex tableau. The profit P is stated in euros.

Basic variable	x	y	z	r	s	Value
r	3	5	6	1	0	50
s	1	2	4	0	1	24
p	−1	−3	−4	0	0	0

c Write down the profit on each small lamp.
d Use the simplex algorithm to solve this linear programming problem.
e Explain why the solution to part **d** is not practical.
f Find a practical solution which gives a profit of 30 euros. Verify that it is feasible. *E*

Solution:

a $x+2y+4z \le 24$

b **i** $x+2y+4z+s=24$

ii $s(\ge 0)$ is the slack time on the machine in hours

c 1 euro

d

b.v.	x	y	z	r	s	value	
r	$\frac{3}{2}$	2	0	1	$-\frac{3}{2}$	14	R1−6R2
z	$\frac{1}{4}$	$\frac{1}{2}$	1	0	$\frac{1}{4}$	6	R2÷4
P	0	−1	0	0	1	24	R3+4R2

b.v.	x	y	z	t	s	value	
y	$\frac{3}{4}$	1	0	$\frac{1}{2}$	$\frac{-3}{4}$	7	R1÷2
z	$\frac{-1}{8}$	0	1	$\frac{-1}{4}$	$\frac{5}{8}$	$\frac{5}{2}$	$\text{R2}-\frac{1}{2}\text{R1}$
P	$\frac{3}{4}$	0	0	$\frac{1}{2}$	$\frac{1}{4}$	31	R3+R1

Profit = 31 euros $y=7$ $z=2.5$ $x=r=s=0$

e Cannot make $\frac{1}{2}$ a lamp

f e.g. (0, 10, 0) or (0, 6, 3) or (1, 7, 2) checks in **both** inequalities

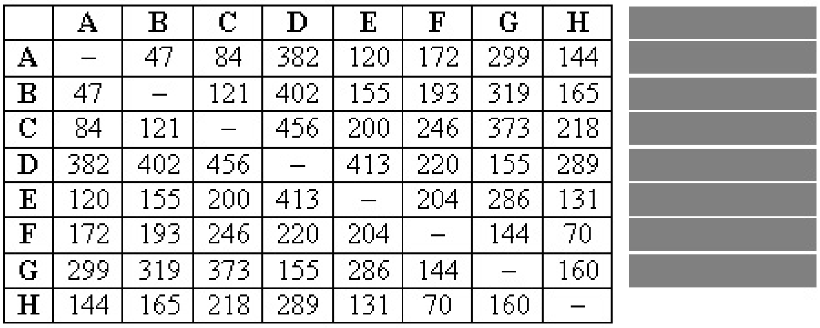
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

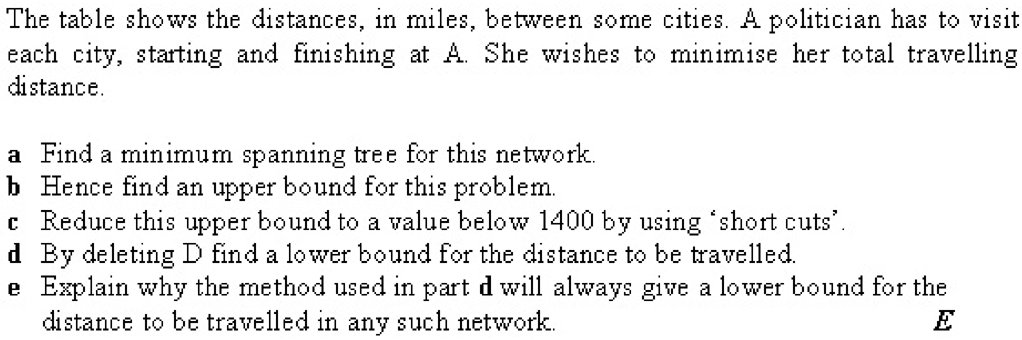
Review Exercise 1

Exercise A, Question 32

Question:



	A	B	C	D	E	F	G	H
A	–	47	84	382	120	172	299	144
B	47	–	121	402	155	193	319	165
C	84	121	–	456	200	246	373	218
D	382	402	456	–	413	220	155	289
E	120	155	200	413	–	204	286	131
F	172	193	246	220	204	–	144	70
G	299	319	373	155	286	144	–	160
H	144	165	218	289	131	70	160	–



The table shows the distances, in miles, between some cities. A politician has to visit each city, starting and finishing at A. She wishes to minimise her total travelling distance.

a Find a minimum spanning tree for this network.

b Hence find an upper bound for this problem.

c Reduce this upper bound to a value below 1400 by using ‘short cuts’.

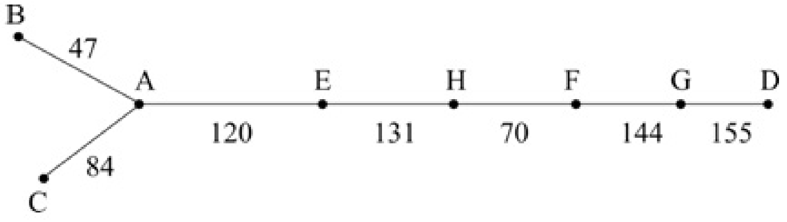
d By deleting D find a lower bound for the distance to be travelled.

e Explain why the method used in part **d** will always give a lower bound for the distance to be travelled in any such network. ***E***

Solution:

a

	A	B	C	D	E	F	G	H
A	–	47	84	382	120	172	299	144
B	47	–	121	402	155	193	319	165
C	84	121	–	456	200	246	373	218
D	382	402	456	–	413	220	155	289
E	120	155	200	413	–	204	286	131
F	172	193	246	220	204	–	144	70
G	299	319	373	155	286	144	–	160
H	144	165	218	289	131	70	160	–



b Upper bound $751 \times 2 = 1502$

c B to D saves 265, H to G saves 54, B to H saves 133 etc.

d Delete D minimum spanning tree 596
2 least paths $155 + 220 = 375$
$\therefore$ lower bound is $596 + 375 = 971$

e The non-deleted vertices form a minimum spanning tree so they do not form a cycle.
The optimum solution is a cycle.
Unless the 2 least paths complete the cycle it will not give the optimum solution.
In general this will not be the case so a lower bound will be formed, shorter than the optimum solution:

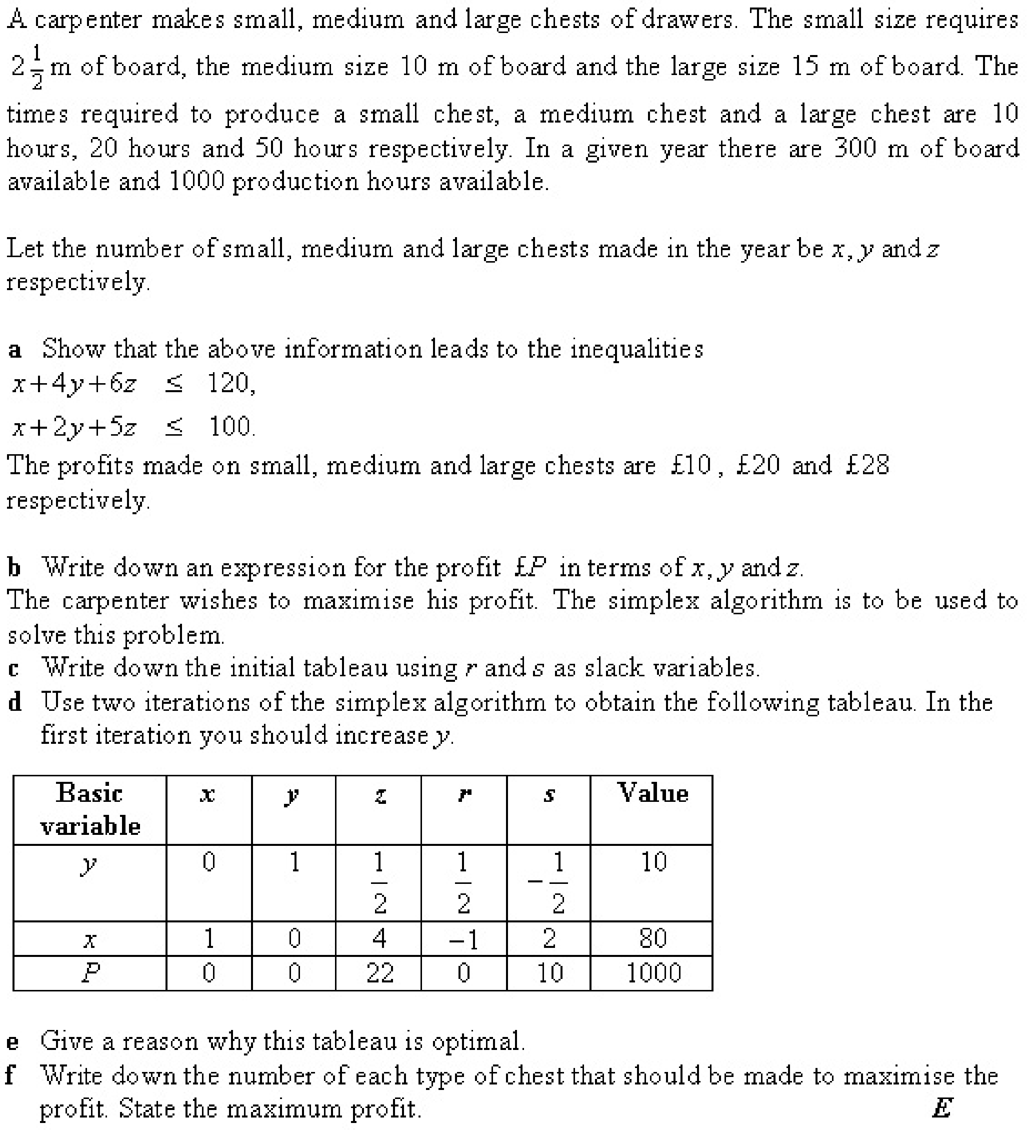
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 33

Question:



A carpenter makes small, medium and large chests of drawers. The small size requires $2\frac{1}{2}$ m of board, the medium size 10 m of board and the large size 15 m of board. The times required to produce a small chest, a medium chest and a large chest are 10 hours, 20 hours and 50 hours respectively. In a given year there are 300 m of board available and 1000 production hours available.

Let the number of small, medium and large chests made in the year be x, y and z respectively.

a Show that the above information leads to the inequalities

$$x+4y+6z \le 120,$$

$$x+2y+5z \le 100.$$

The profits made on small, medium and large chests are £10 , £20 and £28 respectively.

b Write down an expression for the profit £P in terms of x, y and z.

The carpenter wishes to maximise his profit. The simplex algorithm is to be used to solve this problem.

c Write down the initial tableau using r and s as slack variables.

d Use two iterations of the simplex algorithm to obtain the following tableau. In the first iteration you should increase y.

Basic variable	x	y	z	r	s	Value
y	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	10
x	1	0	4	-1	2	80
P	0	0	22	0	10	1000

e Give a reason why this tableau is optimal.

f Write down the number of each type of chest that should be made to maximise the profit. State the maximum profit. ***E***

Solution:

a

	Board (m)	Time (R)
Small (x)	$2\frac{1}{2}$	10
Medium (y)	10	20
Large (z)	15	50
Available	300	1,000

Board $2\frac{1}{2}x+10y+15z \le 300$

$$x+4y+6z \le 120$$

Time $10x+20y+50z \le 1000$

$$x+2y+5z \le 100$$

b $P=10x+20y+28z$

c

b.v.	x	y	z	r	s	values
r	1	4	6	1	0	120
s	1	2	5	0	1	100
P	-10	-20	-28	0	0	0

d $\theta_1 = 30, \theta_2 = 50$; pivot 4

b.v.	x	y	z	r	s	value
y	$\frac{1}{4}$	1	$1\frac{1}{2}$	$\frac{1}{4}$	0	30
s	$\frac{1}{2}$	0	2	$\frac{-1}{2}$	1	40
P	-5	0	2	5	0	600

$\theta_1 = 120, \theta_2 = 80$; pivot $\frac{1}{2}$

b.v.	x	y	z	r	s	Value
y	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	10
x	1	0	4	-1	2	80
P	0	0	22	0	10	1000

e This tableau is optimal as there are no negative numbers in the profit line.

f Small 80, medium 10; large 0
Profit £1000

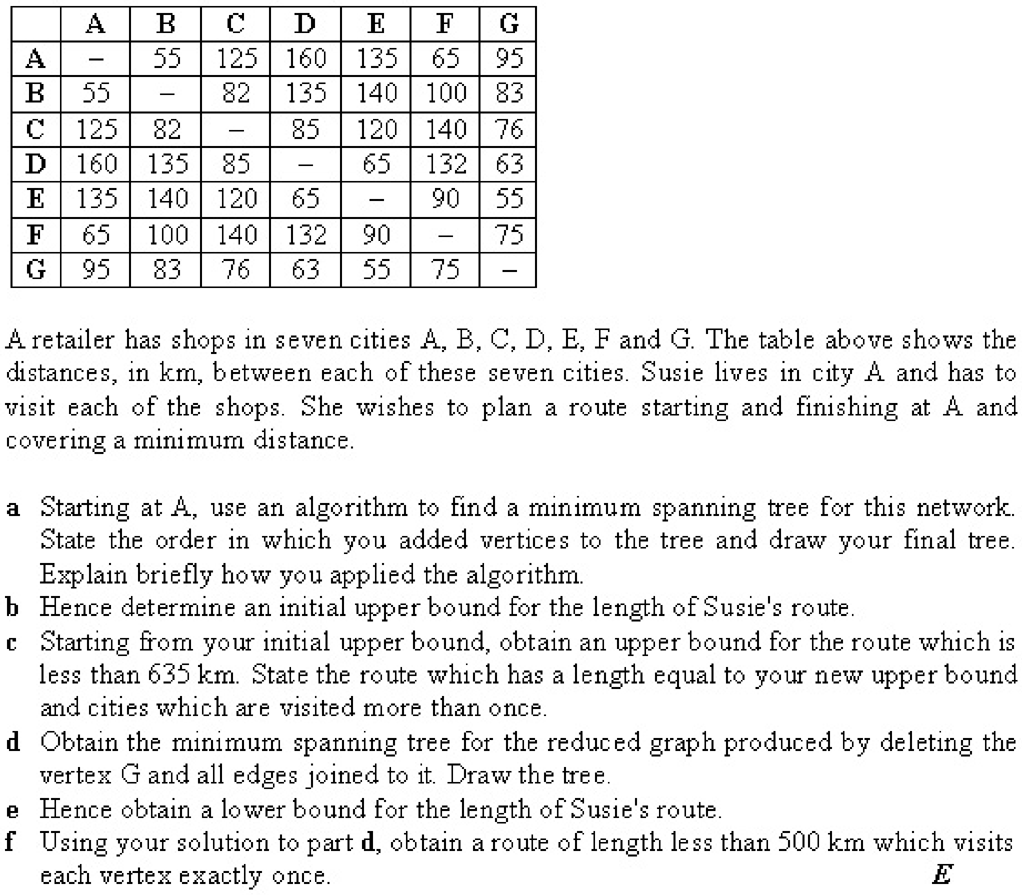
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 34

Question:



	A	B	C	D	E	F	G
A	–	55	125	160	135	65	95
B	55	–	82	135	140	100	83
C	125	82	–	85	120	140	76
D	160	135	85	–	65	132	63
E	135	140	120	65	–	90	55
F	65	100	140	132	90	–	75
G	95	83	76	63	55	75	–

A retailer has shops in seven cities A, B, C, D, E, F and G. The table above shows the distances, in km, between each of these seven cities. Susie lives in city A and has to visit each of the shops. She wishes to plan a route starting and finishing at A and covering a minimum distance.

a Starting at A, use an algorithm to find a minimum spanning tree for this network. State the order in which you added vertices to the tree and draw your final tree. Explain briefly how you applied the algorithm.

b Hence determine an initial upper bound for the length of Susie's route.

c Starting from your initial upper bound, obtain an upper bound for the route which is less than 635 km. State the route which has a length equal to your new upper bound and cities which are visited more than once.

d Obtain the minimum spanning tree for the reduced graph produced by deleting the vertex G and all edges joined to it. Draw the tree.

e Hence obtain a lower bound for the length of Susie's route.

f Using your solution to part **d**, obtain a route of length less than 500 km which visits each vertex exactly once. ***E***

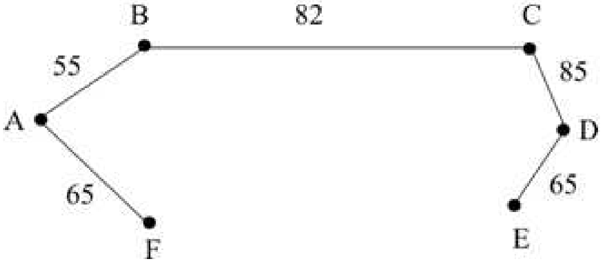
Solution:

a Label column A, delete row A.
Scan all labelled columns and choose least number.
Add that new vertex to the tree
Label the new vertex's column and delete its row
Repeat the 3 steps until all vertices added.
Applying algorithm
order of vertex selection A, B, F, G, E, D, C

b Initial upper band $= 2 \times 389 = 778$ km

c Reducing upper bound by short cuts
e.g. Using BC instead of $BA + AF + FG + GC$ leaves an upper bound of 589
Lists new route e.g. A B C G D G E G F A
States revisited vertices e.g. G

d



e Lower bound $= 352 + GD + GE$

$$= 352 + 63 + 55$$

$$= 470$$

f e.g. Use GE and GF (rather than GD)
length $= 352 + 55 + 75 = 482$ km
Route A B C D E G F A

Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Review Exercise 1

Exercise A, Question 35

Question:

T42 Co. Ltd produces three different blends of tea, Morning, Afternoon and Evening. The teas must be processed, blended and then packed for distribution.
The table below shows the time taken, in hours, for each stage of the production of a tonne of tea. It also shows the profit, in hundreds of pounds, on each tonne.

	Processing	**Blending**	**Packing**	**Profit** (£100)
Morning blend	3	1	2	4
Afternoon blend	2	3	4	5
Evening blend	4	2	3	3

The total times available each week for processing, blending and packing are 35, 20 and 24 hours respectively. T42 Co. Ltd wishes to maximise the weekly profit.
Let x, y and z be the number of tonnes of Morning, Afternoon and Evening blend produced each week.

a Formulate the above situation as a linear programming problem, listing clearly the objective function, and the constraints as inequalities.

An initial simplex tableau for the above situation is

Basic variable	x	y	z	r	s	t	**Value**
r	3	2	4	1	0	0	35
s	1	3	2	0	1	0	20
t	2	4	3	0	0	1	24
P	−4	−5	−3	0	0	0	0

b Solve this linear programming problem using the simplex algorithm. Take the most negative number in the profit row to indicate the pivot column at each stage.

T42 Co. Ltd wishes to increase its profit further and is prepared to increase the time available for processing or blending or packing or any two of these three.

c Use your answer to part **b** to advise the company as to which stage(s) should be allocated increased time. ***E***

Solution:

a Objective: Maximise $P = 4x + 5y + 3z$

Subject to $3x + 2y + 4z \le 35$

$x + 3y + 2z \le 20$

$2x + 4y + 3z \le 24$

b

b.v.	x	y	z	r	s	t	Value	
r	2	0	$\frac{5}{4}$	1	0	$-\frac{1}{2}$	23	R1 − 2R3
s	$-\frac{1}{2}$	0	$-\frac{1}{4}$	0	1	$-\frac{3}{4}$	2	R2 − 3R3
y	$\frac{1}{2}$	1	$\frac{3}{4}$	0	0	$\frac{1}{4}$	6	R3 ÷ 4
P	$-\frac{3}{2}$	0	$\frac{3}{4}$	0	0	$\frac{5}{4}$	30	R4 + 5R3

b.v.	x	y	z	r	s	t	Value	
x	1	0	$\frac{5}{4}$	$\frac{1}{2}$	0	$-\frac{1}{4}$	$\frac{23}{2}$	R1 ÷ 2
s	0	0	$\frac{3}{8}$	$\frac{1}{4}$	1	$-\frac{7}{8}$	$\frac{31}{4}$	R2 + $\frac{1}{2}$R1
y	0	1	$\frac{1}{8}$	$-\frac{1}{4}$	0	$\frac{3}{8}$	$\frac{1}{4}$	R3 − $\frac{1}{2}$R1
P	0	0	$\frac{21}{8}$	$\frac{3}{4}$	0	$\frac{7}{8}$	$\frac{189}{4}$	R4 + $\frac{3}{2}$R1

$P = 47\frac{1}{4} \quad x = 11\frac{1}{2}, y = \frac{1}{4}, z = 0$

c There is some slack $(7\frac{3}{4})$ on S, so *do not* increase blending: therefore increase Processing and Packing which are both at their limit at present.