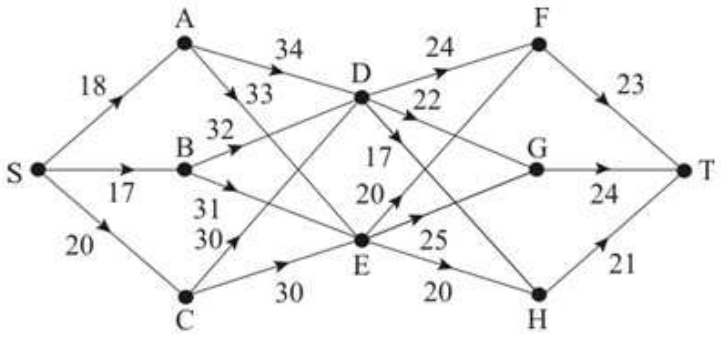
# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 1**

**Question:**

Use dynamic programming to find
**a** a shortest
**b** a longest
route from S to T in the network below. State the route and its length.




**Solution:**

**a Shortest**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | F | FT | T | 23* |
| | G | GT | T | 24* |
| | H | HT | T | 21* |
| 2 | D | DF | F | 24 + 23 = 47 |
| | | DG | G | 22 + 24 = 46 |
| | | DH | H | 17 + 21 = 38* |
| | E | EF | F | 20 + 23 = 43 |
| | | EG | G | 25 + 24 = 49 |
| | | EH | H | 20 + 21 = 41* |
| 3 | A | AD | D | 34 + 38 = 72* |
| | | AE | E | 33 + 41 = 74 |
| | B | BD | D | 32 + 38 = 70* |
| | | BE | E | 31 + 41 = 72 |
| | C | CD | D | 30 + 38 = 68* |
| | | CE | E | 30 + 41 = 71 |
| 4 | S | SA | A | 18 + 72 = 90 |
| | | SB | B | 17 + 70 = 87* |
| | | SC | C | 20 + 68 = 88 |

Shortest route SBDHT length 87

**b Longest**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | F | FT | T | 23* |
| | G | GT | T | 24* |
| | H | HT | T | 21* |
| 2 | D | DF | F | 24 + 23 = 47* |
| | | DG | G | 22 + 24 = 46 |
| | | DH | H | 17 + 21 = 38 |
| | E | EF | F | 20 + 23 = 43 |
| | | EG | G | 25 + 24 = 49* |
| | | EH | H | 20 + 21 = 41 |
| 3 | A | AD | D | 34 + 47 = 81 |
| | | AE | E | 33 + 49 = 82* |
| | B | BD | D | 32 + 47 = 79 |
| | | BE | E | 31 + 49 = 80* |
| | C | CD | D | 30 + 47 = 77 |
| | | CE | E | 30 + 49 = 79* |
| 4 | S | SA | A | 18 + 82 = 100* |
| | | SB | B | 17 + 80 = 97 |
| | | SC | C | 20 + 79 = 99 |

Longest route SAEGT length 100

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 2**

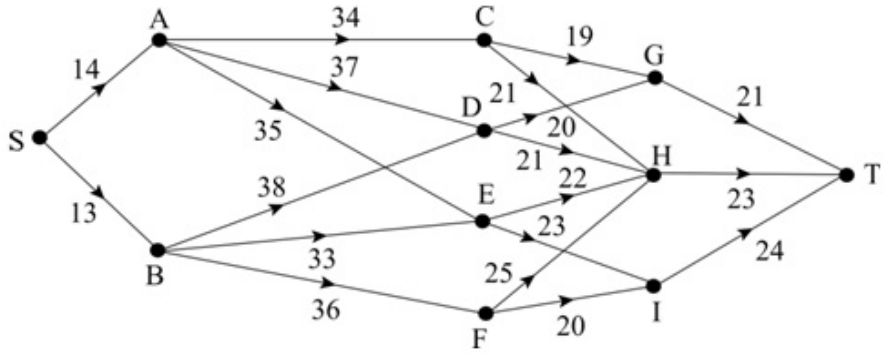
**Question:**

Use dynamic programming to find

**a** a shortest

**b** a longest

route from S to T in the network below. State the route and its length.




**Solution:**

**a Shortest**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | G | GT | T | 21* |
| | H | HT | T | 23* |
| | I | IT | T | 24* |
| 2 | C | CG | G | 19+21=40* |
| | | CH | H | 21+23=44 |
| | D | DG | G | 20+21=41* |
| | | DH | H | 21+23=44 |
| | E | EH | H | 22+23=45* |
| | | EI | I | 23+24=47 |
| | F | FH | H | 25+23=48 |
| | | FI | I | 20+24=44* |
| 3 | A | AC | C | 34+40=74* |
| | | AD | D | 37+41=78 |
| | | AE | E | 35+45=80 |
| | B | BD | D | 38+41=79 |
| | | BE | E | 33+45=78* |
| | | BF | F | 36+44=80 |
| 4 | S | SA | A | 14+74=88* |
| | | SB | B | 13+78=91 |

Shortest route length is 88 with route SACGT

**b Longest**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | G | GT | T | 21* |
| | H | HT | T | 23* |
| | I | IT | T | 24* |
| 2 | C | CG | G | 19+21=40 |
| | | CH | H | 21+23=44* |
| | D | DG | G | 20+21=41 |
| | | DH | H | 21+23=44* |
| | E | EH | H | 22+23=45 |
| | | EI | I | 23+24=47* |
| | F | FH | H | 25+23=48* |
| | | FI | I | 20+24=44 |
| 3 | A | AC | C | 34+44=78 |
| | | AD | D | 37+44=81 |
| | | AE | E | 35+47=82* |
| | B | BD | D | 38+44=82 |
| | | BE | E | 33+47=80 |
| | | BF | F | 36+48=84* |
| 4 | S | SA | A | 14+82=96 |
| | | SB | B | 13+84=97* |

Longest route length is 97 with route SBFHT

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 3**

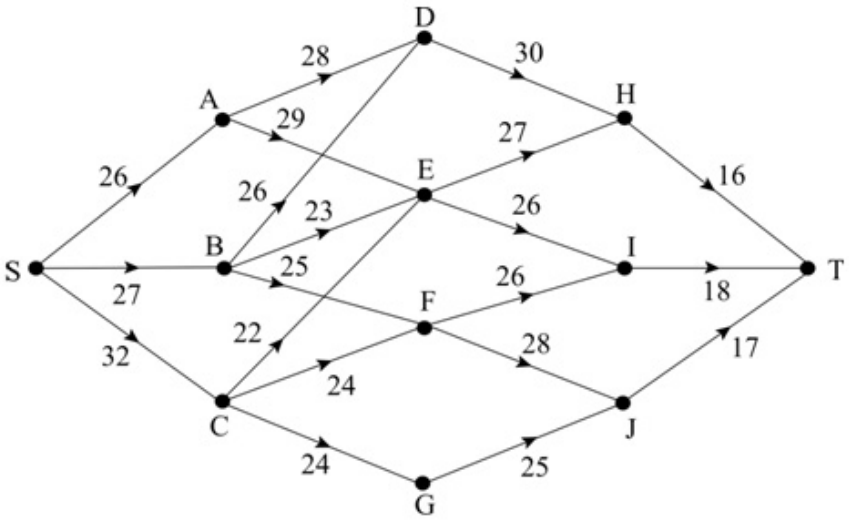
**Question:**

Use dynamic programming to find

**a** a shortest

**b** a longest

route from S to T in the network below. State the route and its length.




**Solution:**

**a Shortest**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | H | HT | T | 16* |
| | I | IT | T | 18* |
| | J | JT | T | 17* |
| 2 | D | DH | H | 30+16=46* |
| | E | EH | H | 27+16=43* |
| | | EI | I | 26+18=44 |
| | F | FI | I | 26+18=44* |
| | | FJ | J | 28+17=45 |
| | G | GJ | J | 25+17=42* |
| 3 | A | AD | D | 28+46=74 |
| | | AE | E | 29+43=72* |
| | B | BD | D | 26+46=72 |
| | | BE | E | 23+43=66* |
| | | BF | F | 25+44=69 |
| | C | CE | E | 22+43=65* |
| | | CF | F | 24+44=68 |
| | | CG | G | 24+42=66 |
| 4 | S | SA | A | 26+72=98 |
| | | SB | B | 27+66=93* |
| | | SC | C | 32+65=97 |

Shortest route length is 93 with route SBEHT

**b Longest**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | H | HT | T | 16* |
| | I | IT | T | 18* |
| | J | JT | T | 17* |
| 2 | D | DH | H | 30+16=46* |
| | E | EH | H | 27+16=43 |
| | | EI | I | 26+18=44* |
| | F | FI | I | 26+18=44 |
| | | FJ | J | 28+17=45* |
| | G | GJ | J | 25+17=42* |
| 3 | A | AD | D | 28+46=74* |
| | | AE | E | 29+44=73 |
| | B | BD | D | 26+46=72* |
| | | BE | E | 23+44=67 |
| | | BF | F | 25+45=70 |
| | C | CE | E | 22+44=66 |
| | | CF | F | 24+45=69* |
| | | CG | G | 24+42=66 |
| 4 | S | SA | A | 26+74=100 |
| | | SB | B | 27+72=99 |
| | | SC | C | 32+69=101* |

Longest route length is 101 with route SCFJT

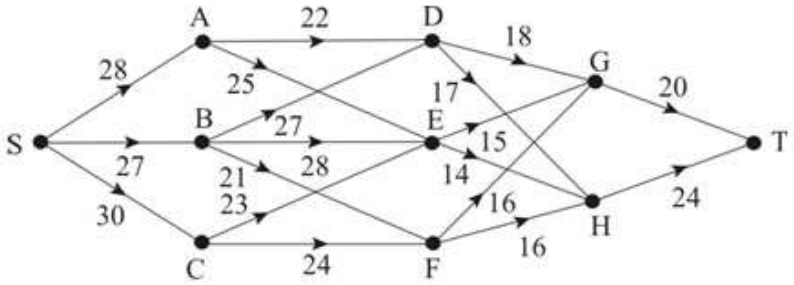
# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 4**

**Question:**

Use dynamic programming to find
**a** a shortest
**b** a longest
route from S to T in the network below. State the route and its length.




**Solution:**

**a Shortest**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | G | GT | T | 20* |
| | H | HT | T | 24* |
| 2 | D | DG | G | 18+20=38* |
| | | DH | H | 17+24=41 |
| | E | EG | G | 15+20=35* |
| | | EH | H | 14+24=38 |
| | F | FG | G | 16+20=36* |
| | | FH | H | 16+24=40 |
| 3 | A | AD | D | 22+38=60* |
| | | AE | E | 25+35=60* |
| | B | BD | D | 27+38=65 |
| | | BE | E | 28+35=63 |
| | | BF | F | 21+36=57* |
| | C | CE | E | 23+35=58* |
| | | CF | F | 24+36=60 |
| 4 | S | SA | A | 28+60=88 |
| | | SB | B | 27+57=84* |
| | | SC | C | 30+58=88 |

Shortest route length is 84 with route SBFGT

**b Longest**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | G | GT | T | 20* |
| | H | HT | T | 24* |
| 2 | D | DG | G | 18+20=38 |
| | | DH | H | 17+24=41* |
| | E | EG | G | 15+20=35 |
| | | EH | H | 14+24=38* |
| | F | FG | G | 16+20=36 |
| | | FH | H | 16+24=40* |
| 3 | A | AD | D | 22+41=63* |
| | | AE | E | 25+38=63* |
| | B | BD | D | 27+41=68* |
| | | BE | E | 28+38=66 |
| | | BF | F | 21+40=61 |
| | C | CE | E | 23+38=61 |
| | | CF | F | 24+40=64* |
| 4 | S | SA | A | 28+63=91 |
| | | SB | B | 27+68=95* |
| | | SC | C | 30+64=94 |

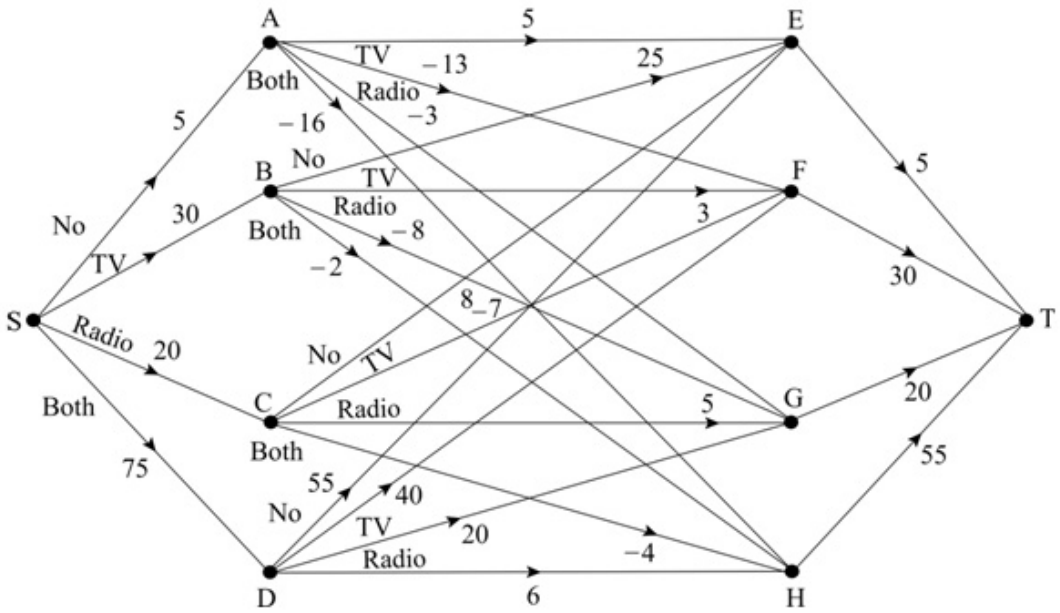
Longest route length is 95 with route SBDHT

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 5**

**Question:**




The diagram shows the effect on a company's profits, in £1000's, of taking various advertising decisions. The company wishes to create a two-year plan that will maximise its total profit.

Each year they must decide if they will not advertise (No), advertise through television only (TV), advertise through radio only (Radio), or advertise in both media (Both).

To determine the effectiveness of the strategy the company will estimate the value of its assets at the end of the two-year period.

Use dynamic programming to determine the advertising decisions that the directors should take.

**Solution:**

**Maximise**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | E | ET | T | 5* |
| Assets | F | FT | T | 30* |
| | G | GT | T | 20* |
| | H | HT | T | 55* |
| 2 | A | AE (No) | E | 5+5=10 |
| Year two | | AF (TV) | F | −13+30=17 |
| | | AG (Radio) | G | −3+20=17 |
| | | AH (Both) | H | −16+55=39* |
| | B | BE (No) | E | 25+5=30 |
| | | BF (TV) | F | 3+30=33 |
| | | BG (Radio) | G | 8+20=28 |
| | | BH (Both) | H | −2+55=53* |
| | C | CE (No) | E | 8+5=13 |
| | | CF (TV) | F | −7+30=23 |
| | | CG (Radio) | G | 5+20=25 |
| | | CH (Both) | H | −4+55=51* |
| | D | DE (No) | E | 55+5=60 |
| | | DF (TV) | F | 40+30=70* |
| | | DG (Radio) | G | 20+20=40 |
| | | DH (Both) | H | 6+55=61 |
| 3 | S | SA (No) | A | 5+39=44 |
| Year one | | SB (TV) | B | 30+53=83 |
| | | SC (Radio) | C | 20+51=71 |
| | | SD (Both) | D | 75+70=145* |

The maximum profit is £145 000
The maximum route is SDFT
In practical terms the company's strategy is:
Year 1 – advertise in both TV and Radio
Year 2 – advertise on TV only.

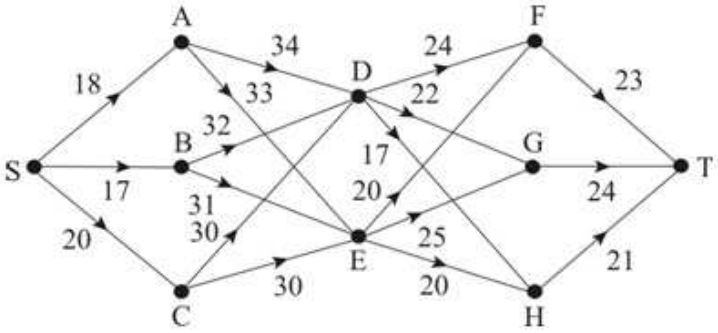
# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 1**

**Question:**

Use dynamic programming to find
**a** a minimax,
**b** a maximin,
route from S to T in the network below. State the route and its length.




**Solution:**

**a Minimax**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | F | FT | T | 23* |
| | G | GT | T | 24* |
| | H | HT | T | 21* |
| 2 | D | DF | F | Max(24, 23) = 24 |
| | | DG | G | Max(22, 24) = 24 |
| | | DH | H | Max(17, 21) = 21* |
| | E | EF | F | Max(20, 23) = 23 |
| | | EG | G | Max(25, 24) = 25 |
| | | EH | H | Max(20, 21) = 21* |
| 3 | A | AD | D | Max(34, 21) = 34 |
| | | AE | E | Max(33, 21) = 33* |
| | B | BD | D | Max(32, 21) = 32 |
| | | BE | E | Max(31, 21) = 31* |
| | C | CD | D | Max(30, 21) = 30* |
| | | CE | E | Max(30, 21) = 30* |
| 4 | S | SA | A | Max(18, 33) = 33 |
| | | SB | B | Max(17, 31) = 31 |
| | | SC | C | Max(20, 30) = 30* |

Minimax route SCDHT or SCEHT – both of value 30

**b Maximin**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | F | FT | T | 23* |
| | G | GT | T | 24* |
| | H | HT | T | 21* |
| 2 | D | DF | F | Min(24, 23) = 23* |
| | | DG | G | Min(22, 24) = 22 |
| | | DH | H | Min(17, 21) = 17 |
| | E | EF | F | Min(20, 23) = 20 |
| | | EG | G | Min(25, 24) = 24* |
| | | EH | H | Min(20, 21) = 20 |
| 3 | A | AD | D | Min(34, 23) = 23 |
| | | AE | E | Min(33, 24) = 24* |
| | B | BD | D | Min(32, 23) = 23 |
| | | BE | E | Min(31, 24) = 24* |
| | C | CD | D | Min(30, 23) = 23 |
| | | CE | E | Min(30, 24) = 24* |
| 4 | S | SA | A | Min(18, 24) = 18 |
| | | SB | B | Min(17, 24) = 17 |
| | | SC | C | Min(20, 24) = 20* |

Maximin route SCEGT of value 20

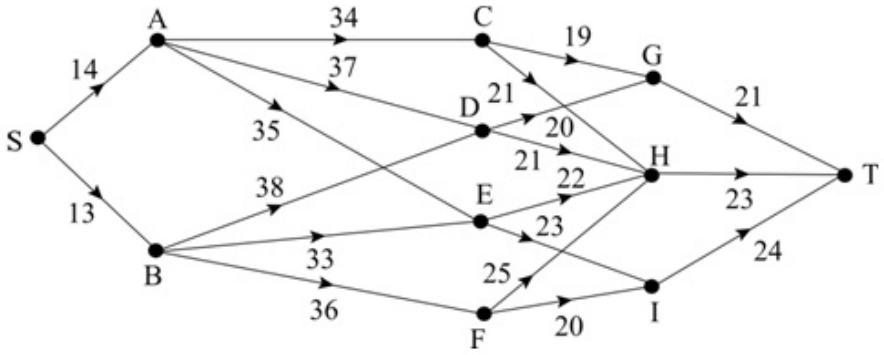
# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 2**

**Question:**

Use dynamic programming to find
**a** a minimax,
**b** a maximin,
route from S to T in the network below. State the route and its length.




**Solution:**

**a Minimax**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | G | GT | T | 21* |
| | H | HT | T | 23* |
| | I | IT | T | 24* |
| 2 | C | CG | G | Max(19, 21) = 21* |
| | | CH | H | Max(21, 23) = 23 |
| | D | DG | G | Max(20, 21) = 21* |
| | | DH | H | Max(21, 23) = 23 |
| | E | EH | H | Max(22, 23) = 23* |
| | | EI | I | Max(23, 24) = 24 |
| | F | FH | H | Max(25, 23) = 25 |
| | | FI | I | Max(20, 24) = 24* |
| 3 | A | AC | C | Max(34, 21) = 34* |
| | | AD | D | Max(37, 21) = 37 |
| | | AE | E | Max(35, 23) = 35 |
| | B | BD | D | Max(38, 21) = 38 |
| | | BE | E | Max(33, 23) = 33* |
| | | BF | F | Max(36, 24) = 36 |
| 4 | S | SA | A | Max(14, 34) = 34 |
| | | SB | B | Max(13, 33) = 33* |

Minimax route SBEHT of value 33

**b Maximin**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | G | GT | T | $21^*$ |
| | H | HT | T | $23^*$ |
| | I | IT | T | $24^*$ |
| 2 | C | CG | G | $\text{Min}(19, 21) = 19$ |
| | | CH | H | $\text{Min}(21, 23) = 21^*$ |
| | D | DG | G | $\text{Min}(20, 21) = 20$ |
| | | DH | H | $\text{Min}(21, 23) = 21^*$ |
| | E | EH | H | $\text{Min}(22, 23) = 22$ |
| | | EI | I | $\text{Min}(23, 24) = 23^*$ |
| | F | FH | H | $\text{Min}(25, 23) = 23^*$ |
| | | FI | I | $\text{Min}(20, 24) = 20$ |
| 3 | A | AC | C | $\text{Min}(34, 21) = 34$ |
| | | AD | D | $\text{Min}(37, 21) = 37^*$ |
| | | AE | E | $\text{Min}(35, 23) = 35$ |
| | B | BD | D | $\text{Min}(38, 21) = 38^*$ |
| | | BE | E | $\text{Min}(33, 23) = 33$ |
| | | BF | F | $\text{Min}(36, 24) = 36$ |
| 4 | S | SA | A | $\text{Min}(14, 37) = 37$ |
| | | SB | B | $\text{Min}(13, 38) = 38^*$ |

Maximin route SBDHT of value 38

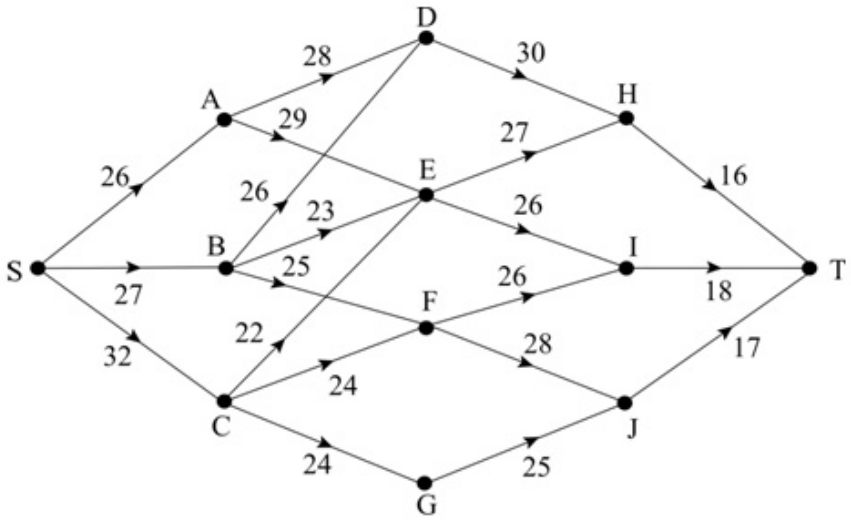
# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 3**

**Question:**

Use dynamic programming to find
**a** a minimax,
**b** a maximin,
route from S to T in the network below. State the route and its length.




**Solution:**

**a Minimax**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | H | HT | T | 16* |
| | I | IT | T | 18* |
| | J | JT | T | 17* |
| 2 | D | DH | H | Max(30,16) = 30* |
| | E | EH | H | Max(27,16) = 27 |
| | | EI | I | Max(26,18) = 26* |
| | F | FI | I | Max(26,18) = 26* |
| | | FJ | J | Max(28,17) = 28 |
| | G | GJ | J | Max(25,17) = 25* |
| 3 | A | AD | D | Max(28,30) = 30 |
| | | AE | E | Max(29,26) = 29* |
| | B | BD | D | Max(26,30) = 30 |
| | | BE | E | Max(23,26) = 26* |
| | | BF | F | Max(25,26) = 26* |
| | C | CE | E | Max(22,26) = 26 |
| | | CF | F | Max(24,26) = 26 |
| | | CG | G | Max(24,25) = 25* |
| 4 | S | SA | A | Max(26,29) = 29 |
| | | SB | B | Max(27,26) = 27* |
| | | SC | C | Max(32,25) = 32 |

Minimax route SBEIT or SBFIT – both of value 27

**b Maximin**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | H | HT | T | $16^*$ |
| | I | IT | T | $18^*$ |
| | J | JT | T | $17^*$ |
| 2 | D | DH | H | $\text{Min}(30,16)=16^*$ |
| | E | EH | H | $\text{Min}(27,16)=16$ |
| | | EI | I | $\text{Min}(26,18)=18^*$ |
| | F | FI | I | $\text{Min}(26,18)=18^*$ |
| | | FJ | J | $\text{Min}(28,17)=17$ |
| | G | GJ | J | $\text{Min}(25,17)=17^*$ |
| 3 | A | AD | D | $\text{Min}(28,16)=16$ |
| | | AE | E | $\text{Min}(29,18)=18^*$ |
| | B | BD | D | $\text{Min}(26,16)=16$ |
| | | BE | E | $\text{Min}(23,18)=18^*$ |
| | | BF | F | $\text{Min}(25,18)=18^*$ |
| | C | CE | E | $\text{Min}(22,18)=18^*$ |
| | | CF | F | $\text{Min}(24,18)=18^*$ |
| | | CG | G | $\text{Min}(24,17)=17$ |
| 4 | S | SA | A | $\text{Min}(26,18)=18^*$ |
| | | SB | B | $\text{Min}(27,18)=18^*$ |
| | | SC | C | $\text{Min}(32,18)=18^*$ |

Maximin routes SAEIT, SBEIT, SBFIT, SCEIT, SCFIT all of value 18

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 4**

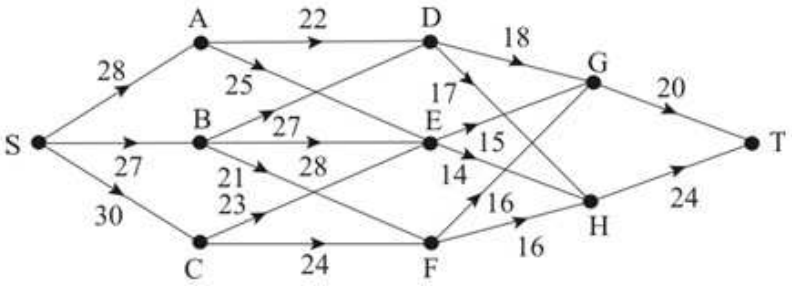
**Question:**

Use dynamic programming to find

**a** a minimax,

**b** a maximin,

route from S to T in the network below. State the route and its length.




**Solution:**

**a Minimax**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | G | GT | T | 20* |
| | H | HT | T | 24* |
| 2 | D | DG | G | Max(18, 20) = 20* |
| | | DH | H | Max(17, 24) = 24 |
| | E | EG | G | Max(15, 20) = 20* |
| | | EH | H | Max(14, 24) = 24 |
| | F | FG | G | Max(16, 20) = 20* |
| | | FH | H | Max(16, 24) = 24 |
| 3 | A | AD | D | Max(22, 20) = 22* |
| | | AE | E | Max(25, 20) = 25 |
| | B | BD | D | Max(27, 20) = 27 |
| | | BE | E | Max(28, 20) = 28 |
| | | BF | F | Max(21, 20) = 21* |
| | C | CE | E | Max(23, 20) = 23* |
| | | CF | F | Max(24, 20) = 24 |
| 4 | S | SA | A | Max(28, 22) = 28 |
| | | SB | B | Max(27, 21) = 27* |
| | | SC | C | Max(30, 23) = 30 |

Minimax route SBFGT of value 27

**b Maximin**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | G | GT | T | 20* |
| | H | HT | T | 24* |
| 2 | D | DG | G | $\text{Min}(18,20)=18^*$ |
| | | DH | H | $\text{Min}(17,24)=17$ |
| | E | EG | G | $\text{Min}(15,20)=15^*$ |
| | | EH | H | $\text{Min}(14,24)=14$ |
| | F | FG | G | $\text{Min}(16,20)=16^*$ |
| | | FH | H | $\text{Min}(16,24)=16^*$ |
| 3 | A | AD | D | $\text{Min}(22,18)=18^*$ |
| | | AE | E | $\text{Min}(25,15)=15$ |
| | B | BD | D | $\text{Min}(27,18)=18^*$ |
| | | BE | E | $\text{Min}(28,15)=15$ |
| | | BF | F | $\text{Min}(21,16)=16$ |
| | C | CE | E | $\text{Min}(23,15)=15$ |
| | | CF | F | $\text{Min}(24,16)=16^*$ |
| 4 | S | SA | A | $\text{Min}(28,18)=18^*$ |
| | | SB | B | $\text{Min}(27,18)=18^*$ |
| | | SC | C | $\text{Min}(30,16)=16$ |

Maximin routes SADGT and SBDGT both of value 18

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 1**

**Question:**

A company is created to sell holidays on an island. There are three new resorts A, B and C being created on the island and the company decides to introduce one new resort to its catalogue each year over the next three years. The costs of introducing each resort will be influenced by the number of resorts listed in the catalogue. The more resorts the company has listed, the smaller the cost of adding another resort. The estimates of annual costs are shown in the table below, in hundreds of pounds.

| Resorts listed | A | B | C |
|---|---|---|---|
| None | 60 | 60 | 55 |
| A | – | 50 | 60 |
| B | 40 | – | 55 |
| C | 35 | 50 | – |
| A and B | – | – | 50 |
| A and C | – | 45 | – |
| B and C | 30 | – | – |

For funding reasons the company needs to choose the order in which the resorts are introduced so that the greatest annual cost is as small as possible.
Dynamic programming will be used to determine the order in which the resorts are introduced.

**a** Explain the meaning of Stage, State and Action in this context.

**b** Find the order in which the resorts should be added and the greatest annual cost.

**Solution:**

**a** Stage – time, in years, remaining
State – resorts already created
Action – resort to be opened

**b** We require the route that gives the minimax value

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | AB | C | ABC | 50* |
| | AC | B | ABC | 45* |
| | BC | A | ABC | 30* |
| 2 | A | B | AB | Max(50,50) = 50* |
| | | C | AC | Max(60,45) = 60 |
| | B | A | AB | Max(40,50) = 50* |
| | | C | BC | Max(55,30) = 55 |
| | C | A | AC | Max(35,45) = 45* |
| | | B | BC | Max(50,30) = 50 |
| 3 | none | A | A | Max(60,50) = 60 |
| | | B | B | Max(60,50) = 60 |
| | | C | C | Max(55,45) = 55* |

The minimax route is CAB with a value of £5500
The order in which the results should be built is C then A then B

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 2**

**Question:**

A house renovation project is to be completed in 6 weeks (30 working days). The work is in four phases: clearance, repairing, modernisation and decorating, which must be undertaken in that order. The cost, in £1000, of each stage depends on the time taken to do it. These are shown in the table.

| Time for stage (days) | Clearance | Repairing | Modernisation | Decorating |
|---|---|---|---|---|
| 5 | 15 | 24 | 22 | 14 |
| 10 | 13 | 20 | 19 | 12 |
| 15 | 8 | 15 | 15 | 9 |
| 20 | 5 | 10 | 11 | 4 |
| 25 | 2 | 6 | 7 | 2 |

Dynamic programming will be used to solve this problem.

**a** Define the terms Stage, State, Action, Destination and Value in this context.

**b** Determine the number of days that should be allocated to each stage in order to minimise costs.

**Solution:**

**a** Stage – phase being considered
State – number of days remaining
Action – number of days allocated
Destination – number of days remaining
Value – total costs

**b**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| Decorating | 5 | 5 | 0 | $14^*$ |
| | 10 | 10 | 0 | $12^*$ |
| Modernisation | 10 | 5 | 5 | $22+14=36^*$ |
| | 15 | 10 | 5 | $19+14=33^*$ |
| | | 5 | 10 | $22+12=34$ |
| Repairing | 15 | 5 | 10 | $24+36=60^*$ |
| | 20 | 10 | 10 | $20+36=56^*$ |
| | | 5 | 15 | $24+33=57$ |
| Clearance | 25 | 10 | 15 | $13+60=73$ |
| | | 5 | 20 | $15+56=71^*$ |

The minimum cost is £71 000. The time should be allocated as follows

| Activity | Clearance | Repairing | Modernisation | Decorating |
|---|---|---|---|---|
| Number of days | 5 | 10 | 5 | 5 |

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 3**

**Question:**

A company makes aircraft. The order book over the next four months is shown in the table below.

| **Month** | March | April | May | June |
|---|---|---|---|---|
| **Number of aircraft ordered** | 1 | 2 | 3 | 2 |

The aircraft are delivered to customers at the end of each month.

Up to three aircraft can be made in any month, but if more than two are made in any one month additional equipment will need to be hired at £20 000 per month.

If any work is done in a month the overhead costs are £50 000.

Up to two aircraft can be held in secure hangers at a cost of £10 000 per aircraft per month.

There are no aircraft in store at the beginning of March and there should be no aircraft in store after the June delivery.

Use dynamic programming to determine the production schedule that minimises the costs.

**Solution:**

Stage – Month
State – number in storage
Action – number to be made

| Stage | State | Action | Destination | Value (in £10000) |
|---|---|---|---|---|
| June | 2 | 0 | 0 | $2=2^*$ |
| (2) | 1 | 1 | 0 | $5+1=6^*$ |
| | 0 | 2 | 0 | $5=5^*$ |
| May | 2 | 1 | 0 | $5+2+5=12$ |
| (3) | | 2 | 1 | $5+2+6=13$ |
| | | 3 | 2 | $2+5+2+2=11^*$ |
| | 1 | 2 | 0 | $5+1+5=11^*$ |
| | | 3 | 1 | $2+5+1+6=14$ |
| | 0 | 3 | 0 | $2+5+1+2=10^*$ |
| April | 2 | 0 | 0 | $2+10=12^*$ |
| (2) | | 1 | 1 | $5+2+11=18$ |
| | | 2 | 2 | $5+2+11=18$ |
| | 1 | 1 | 0 | $5+1+10=16^*$ |
| | | 2 | 1 | $5+1+11=17$ |
| | | 3 | 2 | $2+5+1+11=19$ |
| | 0 | 2 | 0 | $5+10=15^*$ |
| | | 3 | 1 | $2+5+11=18$ |
| March | 0 | 1 | 0 | $5+15=20$ |
| (1) | | 2 | 1 | $5+16=21$ |
| | | 3 | 2 | $2+5+12=19^*$ |

The minimum cost is £190 000, the aircraft should be built as follows.

| Month | March | April | May | June |
|---|---|---|---|---|
| Number of aircraft built in each month | 3 | 0 | 3 | 2 |

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 4**

**Question:**

A salesman will visit four shops in the next four days to demonstrate a new product. He will start at home and travel to the first shop and spend the first day there, then travel directly to the second shop for day 2, onto the third shop for day 3, then to the fourth shop for day 4 and then travel home.

Table 1 shows the shops he could visit on each day.
Table 2 shows the anticipated profit, in £100, from sales at each shop.
Table 3 shows the travelling expenses, in £100, that will be incurred.

The company employing the salesman wishes to maximise the income, after subtracting the travel costs, generated by the salesman's visits. Find his optimum route.

Table 1

| Monday | Tuesday | Wednesday | Thursday |
|---|---|---|---|
| A, B, C | D, E | F, G | H, I, J |

Table 2

| Shop | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| Profit | 8 | 9 | 8 | 12 | 14 | 10 | 11 | 14 | 13 | 11 |

Table 3

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| Home | 2 | 2 | 3 | | | | | 6 | 4 | 3 |
| A | | | | 3 | 4 | | | | | |
| B | | | | 4 | 6 | | | | | |
| C | | | | 4 | 4 | | | | | |
| D | | | | | | 5 | 5 | | | |
| E | | | | | | 4 | 7 | | | |
| F | | | | | | | | 5 | 4 | 4 |
| G | | | | | | | | 5 | 5 | 4 |

**Solution:**

Stage – day
State – shop being visited
Action – next journey to be undertaken

| Stage | State | Action | Destination | Value, in £ 100 |
|---|---|---|---|---|
| Thursday | H | H-home | home | $14-6=8^*$ |
| | I | I–home | home | $13-4=9^*$ |
| | J | J – home | home | $11-3=8^*$ |
| Wednesday | F | FH | H | $10-5+8=13$ |
| | | FI | I | $10-4+9=15^*$ |
| | | FJ | J | $10-4+8=14$ |
| | G | GH | H | $11-5+8=14$ |
| | | GI | I | $11-5+9=15^*$ |
| | | GJ | J | $11-4+8=15^*$ |
| Tuesday | D | DF | F | $12-5+15=22^*$ |
| | | DG | G | $12-5+15=22^*$ |
| | E | EF | F | $14-4+15=25^*$ |
| | | EG | G | $14-7+15=22$ |
| Monday | A | AD | D | $8-3+22=27$ |
| | | AE | E | $8-4+25=29^*$ |
| | B | BD | D | $9-4+22=27$ |
| | | BE | E | $9-6+25=28^*$ |
| | C | CD | D | $8-4+22=26$ |
| | | CE | E | $8-4+25=29^*$ |
| Sunday | Home | Home – A | | $-2+29=27$ |
| | | Home – B | | $-2+28=26^*$ |
| | | Home – C | | $-3+29=26^*$ |

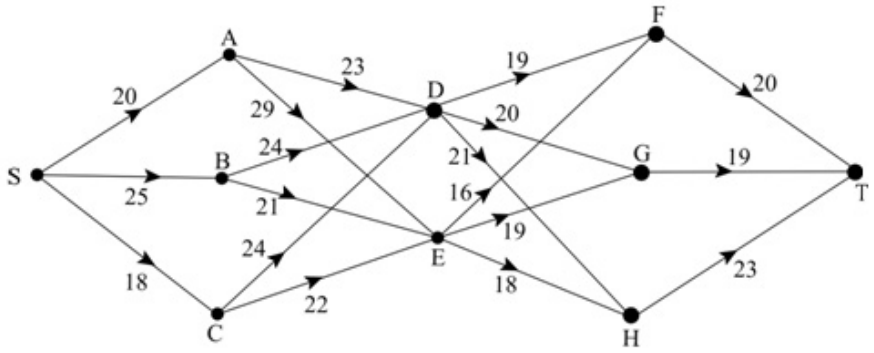
There are two possible minimum routes.
Home – B – E – F – I – Home
Home – C – E – F – I – Home
Each has a value of £2600

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 1**

**Question:**




Use dynamic programming to find the minimax route from S to T in the network above.

**Solution:**

Minimax

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | F | FT | T | 20* |
| | G | GT | T | 19* |
| | H | HT | T | 23* |
| 2 | D | DF | F | Max(19, 20) = 20* |
| | | DG | G | Max(20,19) = 20* |
| | | DH | H | Max(21, 23) = 23 |
| | E | EF | F | Max(16, 20) = 20 |
| | | EG | G | Max(19,19) = 19* |
| | | EH | H | Max(18, 23) = 23 |
| | A | AD | D | Max(23, 20) = 23* |
| | | AE | E | Max(29,19) = 29 |
| | B | BD | D | Max(24, 20) = 24 |
| | | BE | E | Max(21,19) = 21* |
| | C | CD | D | Max(24, 20) = 24 |
| | | CE | E | Max(22,19) = 22* |
| 3 | S | SA | A | Max(20, 23) = 23 |
| | | SB | B | Max(25, 21) = 25 |
| | | SC | C | Max(18, 22) = 22* |

Minimax route is SCEGT value 22

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 2**

**Question:**

Using the same diagram as in question 1, use dynamic programming to find the maximin route.

**Solution:**

Maximin

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 1 | F | FT | T | 20* |
| | G | GT | T | 19* |
| | H | HT | T | 23* |
| 2 | D | DF | F | Min(19, 20) = 19 |
| | | DG | G | Min(20, 19) = 19 |
| | | DH | H | Min(21, 23) = 21* |
| | E | EF | F | Min(16, 20) = 16 |
| | | EG | G | Min(19, 19) = 19* |
| | | EH | H | Min(18, 23) = 18 |
| | A | AD | D | Min(23, 21) = 21* |
| | | AE | E | Min(29, 19) = 19 |
| | B | BD | D | Min(24, 21) = 21* |
| | | BE | E | Min(21, 19) = 19 |
| | C | CD | D | Min(24, 21) = 21* |
| | | CE | E | Min(22, 19) = 19 |
| 3 | S | SA | A | Min(20, 21) = 20 |
| | | SB | B | Min(25, 21) = 21* |
| | | SC | C | Min(18, 21) = 18 |

The maximin route is SBDHT of value 21

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 3**

**Question:**

A dairy manufacturer can make butter, cheese and yoghurt. Up to five units of milk can be processed and the profits from the various allocations are shown in the table.

| **Number of units** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Butter** | 14 | 25 | 34 | 41 | 47 |
| **Cheese** | 12 | 30 | 40 | 45 | 49 |
| **Yoghurt** | 10 | 20 | 30 | 40 | 50 |

The manufacturer wishes to maximise his profit.

**a** Use dynamic programming to find an optimal solution and state the profit.

**b** Show that there is a second optimal solution.

**Solution:**

Maximum

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| Yoghurt | 5 | 5 | 0 | 50* |
| | 4 | 4 | 0 | 40* |
| | 3 | 3 | 0 | 30* |
| | 2 | 2 | 0 | 20* |
| | 1 | 1 | 0 | 10* |
| | 0 | 0 | 0 | 0* |
| Cheese | 5 | 0 | 5 | 0+50=50 |
| | | 1 | 4 | 12+40=52 |
| | | 2 | 3 | 30+30=60* |
| | | 3 | 2 | 40+20=60* |
| | | 4 | 1 | 45+10=55 |
| | | 5 | 0 | 49+0=49 |
| | 4 | 0 | 4 | 0+40=40 |
| | | 1 | 3 | 12+30=42 |
| | | 2 | 2 | 30+20=50* |
| | | 3 | 1 | 40+10=50* |
| | | 4 | 0 | 45+0=45 |
| | 3 | 0 | 3 | 0+30=30 |
| | | 1 | 2 | 12+20=32 |
| | | 2 | 1 | 30+10=40* |
| | | 3 | 0 | 40+0=40* |
| | 2 | 0 | 2 | 0+20=20 |
| | | 1 | 1 | 12+10=22 |
| | | 2 | 0 | 30+0=30* |
| | 1 | 0 | 1 | 0+10=10 |
| | | 1 | 0 | 12+0=12* |
| | 0 | 0 | 0 | 0+0=0* |
| Butter | 5 | 0 | 5 | 0+60=60 |
| | | 1 | 4 | 14+50=64 |
| | | 2 | 3 | 25+40=65* |
| | | 3 | 2 | 34+30=64 |
| | | 4 | 1 | 41+12=53 |
| | | 5 | 0 | 47+0=47 |

There are two possible courses of action each of value £65

| Product | Butter | Cheese | Yoghurt |
|---|---|---|---|
| Units to be used | 2 | 2 | 1 |
| Product | Butter | Cheese | Yoghurt |
| Units to be used | 2 | 3 | 0 |

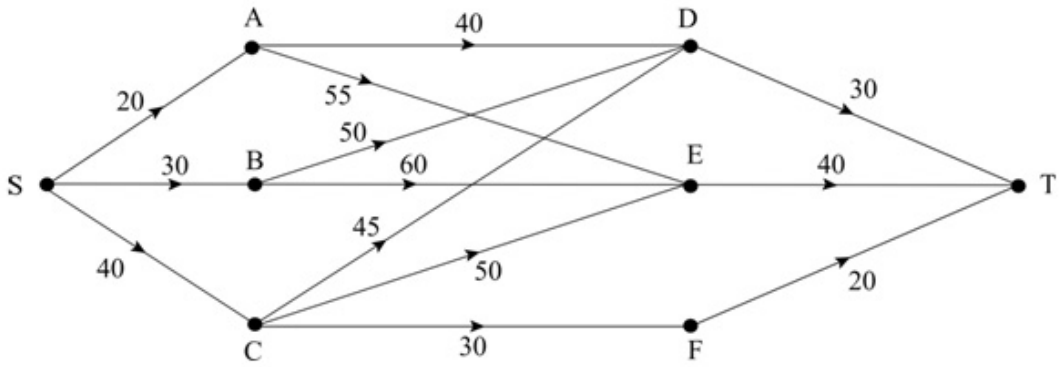
# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 4**

**Question:**

Jenny wishes to travel from S to T. There are several routes available. She wishes to choose the route on which the maximum altitude, above sea level, is as small as possible. This is called the minimax route.



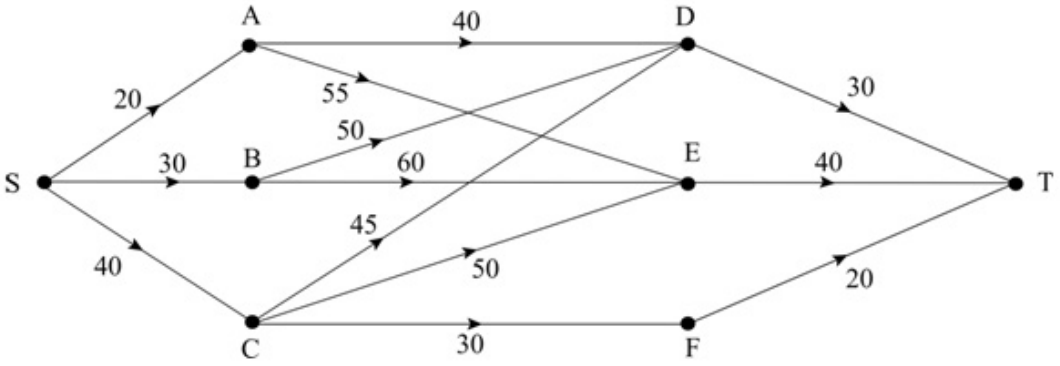

The diagram gives the possible routes and the weights on the edges give the maximum altitude on the road (in units of 100 feet).

Use dynamic programming, carefully defining the stages and states, to determine the route or routes Jenny should take. You should show your calculations in tabular form, using a table with columns labelled as shown below.

| Stage | Initial state | Action | Final state | Value |
|---|---|---|---|---|

**[*E*]**

**Solution:**




The states are the vertices.

| Stage | Initial state | Action | Final state | Value |
|---|---|---|---|---|
| 1 | D | DT | T | 30 |
| | E | ET | T | 40 |
| | F | FT | T | 20 |
| 2 | A | AD | D | max (40, 30) = 40* |
| | | AE | E | max (55, 40) = 55 |
| | B | BD | D | max (50, 30) = 50* |
| | | BE | E | max (60, 40) = 60 |
| | C | CD | D | max (45, 30) = 45 |
| | | CE | E | max (50, 40) = 50 |
| | | CF | F | max (30, 20) = 30* |
| 3 | S | SA | A | max (40, 20) = 40* |
| | | SB | B | max (50, 30) = 50 |
| | | SC | C | max (40, 30) = 40* |

Tracing back there are two routes
SC, CF, FT, $\Rightarrow$ SCFT
SA, AD, DT, $\Rightarrow$ SADT
Maximum altitude on these routes is 40 ($\times$ 100 ft) = 4000 ft.

# Solutionbank D2

## Edexcel AS and A Level Modular Mathematics

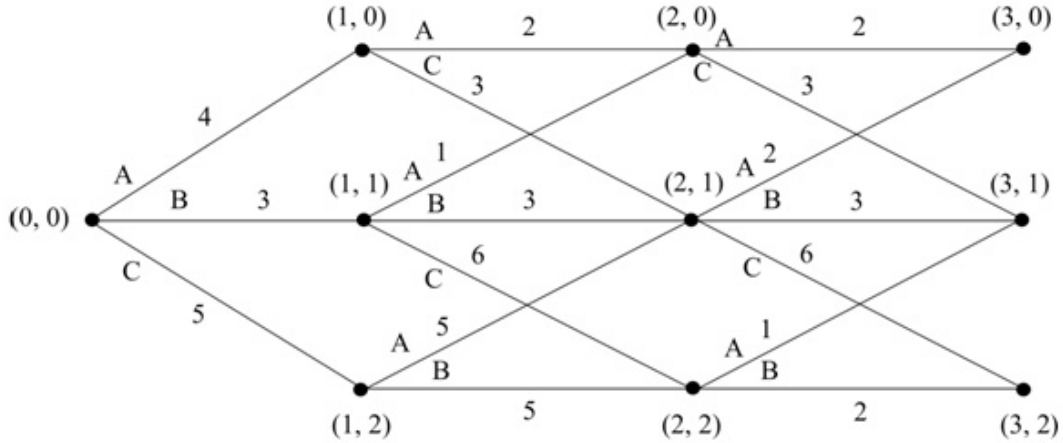
**Exercise D, Question 5**

**Question:**

At the beginning of each month an advertising manager must choose one of 3 adverts:
the previous advert;
the current advert;
a new advert.

She therefore has 3 options:
A: use the previous advert;
B: use the current advert;
C: run a new advert.

The possible choices are shown in the network below together with (stage, state) variables at the vertices and the expected profits, in thousands of pounds, on the arcs.




| September | October | November |
|---|---|---|
| | | |

The manager wants to maximise her profits for the three-month period.

**a** Complete the table on the worksheet.

**b** Hence obtain the sequence of decisions she should make to obtain the maximum profit. State the maximum profit.

**[*E*]**

**Solution:**

**a**

| Stage | State | Action | Cost | Total Cost |
|---|---|---|---|---|
| 2 | 0 | A | 2 | 2 |
| | | C | 3 | 3* |
| | 1 | A | 2 | 2 |
| | | B | 3 | 3 |
| | | C | 6 | 6* |
| | 2 | A | 1 | 1 |
| | | B | 2 | 2* |
| 1 | 0 | A | 2 | $2+3=5$ |
| | | C | 3 | $3+6=9$* |
| | 1 | A | 1 | $1+3=4$ |
| | | B | 3 | $3+6=9$* |
| | | C | 6 | $6+2=8$ |
| | 2 | A | 5 | $5+6=11$* |
| | | B | 5 | $5+2=7$ |
| 0 | 0 | A | 4 | $4+9=13$ |
| | | B | 3 | $3+9=12$ |
| | | C | 5 | $5+11=16$ |

**b** Hence maximum profit is 16
Tracing back through calculations the optimal strategy is CAC