Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Formulate the following problem as a linear programming problem. You must define your variables, state your objective and write your constraints as **equations**.

A company makes three types of metal box, round, square and rectangular. Each box has to pass through two machines to be cut and formed. The round, square and rectangular boxes need 4, 2 and 3 minutes respectively on the cutter and 2, 3 and 3 on the former. Both machines are available for 6 hours per day.
The profit, in pence, made on each round, square and rectangular box is 12, 10 and 11 respectively. The company wishes to maximise its profit.

Solution:

Let x_1, x_2 and x_3 be the number of round, square and rectangular boxes respectively.

Maximise $P = 12x_1 + 10x_2 + 11x_3$

Subject to:

$4x_1 + 2x_2 + 3x_3 + r = 360$

$2x_1 + 3x_2 + 3x_3 + s = 360$

$x_1, x_2, x_3, r, s \geq 0$

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Exercise A, Question 2

Question:

Formulate the following problem as a linear programming problem. You must define your variables, state your objective and write your constraints as **equations**.

A company makes four different types of backpack, A, B, C and D. Each type A uses 2.5 units of material, needs 10 minutes of cutting time and 5 minutes of stitching time. These figures, together with those for types B, C and D are shown in the table

	A	B	C	D
Material in units	2.5	3	2	4
Cutting time in minutes	10	12	8	15
Stitching time in minutes	5	7	4	9

There are 1400 units of material available each week, 150 hours per week available on the cutting machine and 80 hours available on the stitching machine.
Market research says that they will sell at most 500 backpacks each week.
The profit, in pounds, is 8, 7, 6 and 9 for types A, B, C and D respectively. The company wishes to maximise its profit.

Solution:

Let x_A, x_B, x_C and x_D be the number of type A, B, C and D backpacks made

Maximise $P = 8x_A + 7x_B + 6x_C + 9x_D$

Subject to:

$$2.5x_A + 3x_B + 2x_C + 4x_D + r = 1400$$
$$10x_A + 12x_B + 8x_C + 15x_D + s = 9000$$
$$5x_A + 7x_B + 4x_C + 9x_D + t = 4800$$
$$x_A, x_B, x_C, x_D, r, s, t \geq 0$$

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Exercise A, Question 3

Question:

Formulate the following problem as a linear programming problem. You must define your variables, state your objective and write your constraints as **equations**.

The annual subscription to a bowls club is £40 for adults £10 for children and £20 for seniors.
The total number of members is restricted to 100.
At most half the club must be children and at least a third must be adults.
The club wishes to maximise its income.

Solution:

Let x_A, x_C and x_s be the number of adults, children and senior members

Maximise $P = 40x_A + 10x_C + 20x_s$

Subject to:

$$\begin{aligned}
x_A + x_C + x_s + r &= 100 \\
-x_A + x_C - x_s + s &= 0 \\
-2x_A + x_C + x_s + t &= 0 \\
x_A, x_C, x_s, r, s, t &\geq 0
\end{aligned}$$

$$\begin{aligned}
x_C &\leq \frac{1}{2}(x_A + x_C + x_s) \\
\frac{1}{2}x_C &\leq \frac{1}{2}x_A + \frac{1}{2}x_s \\
x_C &\leq x_A + x_s \\
x_C - x_A - x_s &\leq 0
\end{aligned}$$

$$\begin{aligned}
x_A &\geq \frac{1}{3}(x_A + x_C + x_s) \\
3x_A &\geq x_A + x_C + x_s \\
2x_A &\geq x_C + x_s \\
x_C + x_s - 2x_A &\leq 0
\end{aligned}$$

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Exercise A, Question 4

Question:

Formulate the following problem as a linear programming problem. You must define your variables, state your objective and write your constraints as **equations**.

Mrs Brown was rather alarmed to discover from her children at bedtime that (a week ago) they had promised she would make at least 100 small cakes for a cake sale at school the next day. Not wishing to let her children down, she puts the oven on and checks her cupboards and finds she has 3 kg of flour, 2 kg of butter and 1.5 kg of sugar, as well as other ingredients. Mrs Brown finds three cake recipes for rock cakes, fairy cakes and muffins. The recipe for rock cakes uses 220 g of flour, 100 g butter and 50 g sugar and makes 8 cakes. The recipe for fairy cakes uses 100 g each of flour, butter and sugar and makes 18 cakes. The recipe for muffins uses 250 g of flour, 50 g butter and 75 g sugar and makes 12 muffins. Each batch of rock cakes, fairy cakes and muffins take 10 minutes, 20 minutes and 15 minutes respectively to prepare.
Mrs Brown wishes to minimise her preparation time.

Solution:

Let x_r, x_f and x_m be the number of batches of rock cakes, fairy cakes and muffin made.

Minimise $T = 10x_r + 20x_f + 15x_m$

Subject to:

$$\begin{aligned}
220x_r + 100x_f + 250x_m + r &= 3000 \\
100x_r + 100x_f + 50x_m + s &= 2000 \\
50x_r + 100x_f + 75x_m + t &= 1500 \\
8x_r + 18x_f + 12x_m - u &= 100 \\
x_r, x_f, x_m, r, s, t, u &\geq 0
\end{aligned}$$

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Exercise A, Question 5

Question:

Formulate the following problem as a linear programming problem. You must define your variables, state your objective and write your constraints as **equations**.

Roma is moving house. She needs to pack all her extensive collection of china into special cardboard boxes which will be sold to her by the removal company. There are three sizes of box, small, medium and large. The small boxes have a capacity of 0.1 m^3 and will hold a maximum of 3 kg. The medium boxes have a capacity of 0.3 m^3 and will hold a maximum weight of 8 kg. The large boxes have a capacity of 0.7 m^3 and will hold a maximum weight of 18 kg. An expert from the removal company informs her that she allow for at least 28 m^3 packing capacity and for at last 600 kg.

Roma decides that at least half of the boxes she uses should be small and that she should use at least twice as many medium as large.

She will be able to fill the boxes she buys and the cost of each small, medium and large box is 30p, 50p and 80p.

Roma wishes to minimise the cost of the boxes she buys.

Solution:

Let x_s, x_m and x_l be the number of small, medium and large boxes.

Minimise $C = 0.3x_s + 0.5x_m + 0.8x_l$

Subject to:

$$x_s + 3x_m + 7x_l - r = 280$$

$$3x_s + 8x_m + 18x_l - s = 600$$

$$-x_s + x_m + x_l + t = 0$$

$$-x_m + 2x_l + u = 0$$

$$x_s, x_m, x_l, r, s, t, u \geq 0$$

$$x_s \geq \frac{1}{2}(x_s + x_m + x_l)$$

$$x_s \geq x_m + x_l$$

$$x_m + x_l - x_s \leq 0$$

$$x_m \geq 2x_l$$

$$2x_l - x_m \leq 0$$

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Exercise B, Question 1

Question:

Solve this linear programming problem using the simplex tableau algorithm.

Maximise $P = 5x + 6y + 4z$
Subject to

$$x + 2y + r = 6$$
$$5x + 3y + 3z + s = 24$$
$$x, y, z, r, s \geq 0$$

Solution:

b.v.	x	y	z	r	s	value	θ values
r	1	2	0	1	0	6	3 *
s	5	3	3	0	1	24	8
P	-5	-6	-4	0	0	0	

b.v.	x	y	z	r	s	value	Row operations
y	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	3	R1 ÷ 2
s	$\frac{7}{2}$	0	3	$\frac{-3}{2}$	1	15	R2 − 3R1
P	-2	0	-4	3	0	18	R3 + 6R1

b.v.	x	y	z	r	s	value	Row operations
y	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	3	R1 (no change)
z	$\frac{7}{6}$	0	1	$\frac{-1}{2}$	$\frac{1}{3}$	5	R2 ÷ 3
P	$\frac{8}{3}$	0	0	1	$\frac{4}{3}$	38	R3 + 4R2

$P = 38 \quad x = 0 \quad y = 3 \quad z = 5 \quad r = 0 \quad s = 0$

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Exercise B, Question 2

Question:

Solve this linear programming problem using the simplex tableau algorithm.

Maximise $P = 3x + 4y + 10z$

Subject to

$$x + 2y + 2z + r = 100$$
$$x + 4z + s = 40$$
$$x, y, z, r, s \geq 0$$

Solution:

b.v.	x	y	z	r	s	value	θ values
r	1	2	2	1	0	100	50
s	1	0	4	0	1	40	10*
P	−3	−4	−10	0	0	0	

b.v.	x	y	z	r	s	value	Row operations
r	$\frac{1}{2}$	2	0	1	$\frac{-1}{2}$	80	R1− 2R2
z	$\frac{1}{4}$	0	1	0	$\frac{1}{4}$	10	R2 ÷ 4
P	$\frac{-1}{2}$	−4	0	0	$\frac{5}{2}$	100	R3+10R2

b.v.	x	y	z	r	s	value	Row operations
y	$\frac{1}{4}$	1	0	$\frac{1}{2}$	$\frac{-1}{4}$	40	R1÷ 2
z	$\frac{1}{4}$	0	1	0	$\frac{1}{4}$	10	R2 (no change)
P	$\frac{1}{2}$	0	0	2	$\frac{3}{2}$	260	R3+4R1

$P = 260 \quad x = 0 \quad y = 40 \quad z = 10 \quad r = 0 \quad s = 0$

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Exercise B, Question 3

Question:

Solve this linear programming problem using the simplex tableau algorithm.

Maximise $P = 3x + 5y + 2z$

Subject to

$$3x + 4y + 5z + r = 10$$
$$x + 3y + 10z + s = 5$$
$$x - 2y + t = 1$$
$$x, y, z, r, s, t \geq 0$$

Solution:

b.v.	x	y	z	r	s	t	value	θ values
r	3	4	5	1	0	0	10	2.5
s	1	3	10	0	1	0	5	$1\frac{2}{3}$ *
t	1	-2	0	0	0	1	1	negative pivot
P	-3	-5	-2	0	0	0	0	

b.v.	x	y	z	r	s	t	value	Row operations
R	$\frac{5}{3}$	0	$\frac{-25}{3}$	1	$\frac{-4}{3}$	0	$\frac{10}{3}$	R1 − 4R2
Y	$\frac{1}{3}$	1	$\frac{10}{3}$	0	$\frac{1}{3}$	0	$\frac{5}{3}$	R2 ÷ 3
T	$\frac{5}{3}$	0	$\frac{20}{3}$	0	$\frac{2}{3}$	1	$\frac{13}{3}$	R3 + 2R2
P	$\frac{-4}{3}$	0	$\frac{44}{3}$	0	$\frac{5}{3}$	0	$\frac{25}{3}$	R4 + 5R2

b.v.	x	y	z	r	s	t	value	Row operations
x	1	0	-5	$\frac{3}{5}$	$\frac{-4}{5}$	0	2	$R1 \div \frac{5}{3}$
y	0	1	5	$\frac{-1}{5}$	$\frac{3}{5}$	0	1	$R2 - \frac{1}{3}R1$
t	0	0	15	-1	2	1	1	$R3 - \frac{5}{3}R1$
P	0	0	8	$\frac{4}{5}$	$\frac{3}{5}$	0	11	$R4 + \frac{4}{3}R1$

$P = 11$ $x = 2$ $y = 1$ $z = 0$ $r = 0$ $s = 0$ $t = 1$

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Exercise B, Question 4

Question:

Solve this linear programming problem using the simplex tableau algorithm.

Maximise $P = 3x + 6y + 32z$

Subject to

$$x + 6y + 24z + r = 672$$
$$3x + y + 24z + s = 336$$
$$x + 3y + 16z + t = 168$$
$$2x + 3y + 32z + u = 352$$
$$x, y, z, r, s, t, u \geq 0$$

Solution:

b.v.	x	y	z	r	s	t	u	value	θ values
r	1	6	24	1	0	0	0	672	28
s	3	1	24	0	1	0	0	336	14
t	1	3	16	0	0	1	0	168	$10\frac{1}{2}$ *
u	2	3	32	0	0	0	1	352	11
P	−3	−6	−32	0	0	0	0	0	

b.v.	x	y	z	r	s	t	u	value	row operation
r	$\frac{-1}{2}$	$\frac{3}{2}$	0	1	0	$\frac{-3}{2}$	0	420	R1 − 24R3
s	$\frac{3}{2}$	$\frac{-7}{2}$	0	0	1	$\frac{-3}{2}$	0	84	R2 − 24R3
z	$\frac{1}{16}$	$\frac{3}{16}$	1	0	0	$\frac{1}{16}$	0	$\frac{21}{2}$	R3 ÷ 16
u	0	−3	0	0	0	−2	1	16	R4 − 32R3
P	−1	0	0	0	0	2	0	336	R5 + 32R3

b.v.	x	y	z	r	s	t	u	value	Row operations
r	0	$\frac{1}{3}$	0	1	$\frac{1}{3}$	−2	0	448	$R1+\frac{1}{2}R2$
x	1	$\frac{-7}{3}$	0	0	$\frac{2}{3}$	−1	0	56	$R2\div\frac{3}{2}$
z	0	$\frac{1}{3}$	1	0	$\frac{-1}{24}$	$\frac{1}{8}$	0	7	$R3-\frac{1}{16}R2$
u	0	−3	0	0	0	−2	1	16	R4 (No change)
P	0	$\frac{-7}{3}$	0	0	$\frac{2}{3}$	1	0	392	R5 + R2

b.v.	x	y	z	r	s	t	u	value	row operations
r	0	0	−1	1	$\frac{3}{8}$	$\frac{-17}{8}$	0	441	$R1-\frac{1}{3}R3$
x	1	0	7	0	$\frac{3}{8}$	$\frac{-1}{8}$	0	105	$R2+\frac{7}{3}R3$
y	0	1	3	0	$\frac{-1}{8}$	$\frac{3}{8}$	0	21	$R3\div\frac{1}{3}$
u	0	0	9	0	$\frac{-3}{8}$	$\frac{-7}{8}$	1	79	R4 + 3R3
P	0	0	7	0	$\frac{3}{8}$	$\frac{15}{8}$	0	441	$R5+\frac{7}{3}R3$

$P = 441 \quad x = 105 \quad y = 21 \quad z = 0 \quad r = 441 \quad s = 0 \quad t = 0 \quad u = 79$

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Exercise B, Question 5

Question:

Solve this linear programming problem using the simplex tableau algorithm.

Maximise $P = 4x_1 + 3x_2 + 2x_3 + 3x_4$

Subject to

$$x_1 + 4x_2 + 3x_3 + x_4 + r = 95$$
$$2x_1 + x_2 + 2x_3 + 3x_4 + s = 67$$
$$x_1 + 3x_2 + 2x_3 + 2x_4 + t = 75$$
$$3x_1 + 2x_2 + x_3 + 2x_4 + u = 72$$
$$x_1, x_2, x_3, x_4, r, s, t, u \geq 0$$

Solution:

b.v.	x_1	x_2	x_3	x_4	r	s	t	u	value	θ values
r	1	4	3	1	1	0	0	0	95	95
s	2	1	2	3	0	1	0	0	67	63.5
t	1	3	2	2	0	0	1	0	75	75
u	3	2	1	2	0	0	0	1	72	24
P	−4	−3	−2	−3	0	0	0	0	0	

b.v.	x_1	x_2	x_3	x_4	r	s	t	u	value	Row operation
r	0	$\frac{10}{3}$	$\frac{8}{3}$	$\frac{1}{3}$	1	0	0	$\frac{-1}{3}$	71	R1 − R4
s	0	$\frac{-1}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	0	1	0	$\frac{-2}{3}$	19	R2 − 2R4
t	0	$\frac{7}{3}$	$\frac{5}{3}$	$\frac{4}{3}$	0	0	1	$\frac{-1}{3}$	51	R3 − R4
x_1	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$	24	R4 ÷ 3
P	0	$\frac{-1}{3}$	$\frac{-2}{3}$	$\frac{-1}{3}$	0	0	0	$\frac{4}{3}$	96	R5 + 4R4

b.v.	x_1	x_2	x_3	x_4	r	s	t	u	value	Row operation
r	0	4	0	−3	1	−2	0	1	33	$R1-\frac{8}{3}R2$
x_3	0	$\frac{-1}{4}$	1	$\frac{5}{4}$	0	$\frac{3}{4}$	0	$\frac{-1}{2}$	$\frac{57}{4}$	$R2\div\frac{4}{3}$
t	0	$\frac{11}{4}$	0	$\frac{-3}{4}$	0	$\frac{-5}{4}$	1	$\frac{1}{2}$	$\frac{109}{4}$	$R3-\frac{5}{3}R2$
x_1	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	$\frac{-1}{4}$	0	$\frac{1}{2}$	$\frac{77}{4}$	$R4-\frac{1}{3}R2$
P	0	$\frac{-1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	1	$\frac{211}{2}$	$R5+\frac{2}{3}R2$

b.v.	x_1	x_2	x_3	x_4	r	s	t	u	value	Row operation
x_2	0	1	0	$\frac{-3}{4}$	$\frac{1}{4}$	$\frac{-1}{2}$	0	$\frac{1}{4}$	$\frac{33}{4}$	R1 ÷ 4
x_3	0	0	1	$\frac{17}{16}$	$\frac{1}{16}$	$\frac{5}{8}$	0	$\frac{-7}{16}$	$\frac{261}{16}$	$R2+\frac{1}{4}R1$
t	0	0	0	$\frac{21}{16}$	$\frac{-11}{16}$	$\frac{1}{8}$	1	$\frac{-3}{16}$	$\frac{73}{16}$	$R3-\frac{11}{4}R1$
x_1	1	0	0	$\frac{13}{16}$	$\frac{-3}{16}$	$\frac{1}{8}$	0	$\frac{5}{16}$	$\frac{209}{16}$	$R4-\frac{3}{4}R1$
P	0	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	0	$\frac{9}{8}$	$\frac{877}{8}$	$R5+\frac{1}{2}R1$

$P=\frac{877}{8}\quad x_1=\frac{209}{16}\quad x_2=\frac{33}{4}\quad x_3=\frac{261}{16}\quad x_4=0\quad r=0\quad s=0\quad t=\frac{73}{16}\quad u=0$

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Exercise B, Question 6

Question:

For each of the above questions 1 to 5:

a verify, using the original equations, that your solution is feasible,

b write down the final set of equations given by your optimal tableau,

c use the profit equation, written in part **b**, to explain why your solution is optimal.

Solution:

For Q1

a

$$P = 5x + 6y + 42 \qquad \text{P} = 5(0) + 6(3) + 4(5) = 38$$

$$x + 2y + r = 6 \qquad 0 + 2(3) + 0 = 6$$

$$5x + 3y + 32 + 5 = 24 \qquad 5(0) + 3(3) + 3(5) + 0 = 24$$

b

$$P + 12x + r + \frac{4}{3}s = 38$$

$$\frac{1}{2}x + y + \frac{1}{2}r = 3$$

$$\frac{7}{6}x + z - \frac{1}{2}r + \frac{1}{3}s = 5$$

c $P = 38 - 12x - r - \dfrac{4}{3}s$ so increasing x, r or s would decrease P.

For Q2

a

$$P = 3x + 4y + 10z \Rightarrow 3(0) + 4(10) + 10(10) = 260$$

$$x + 2y + 2z + r = 100 \Rightarrow 0 + 2(40) + 2(10) + 0 = 100$$

$$x + 4z + s = 40 \Rightarrow 0 + 4(10) + 0 = 40$$

b

$$P + \frac{1}{2}x + 2r + \frac{3}{2}s = 260$$

$$\frac{1}{4}x + y + \frac{1}{2}r - \frac{1}{4}s = 40$$

$$\frac{1}{4}x + z + \frac{1}{4}s = 10$$

c $P = 260 - \dfrac{1}{2}x - 2r - \dfrac{3}{2}s$, so increasing x, r, or s would decrease P.

For Q3

a

$$P = 3x + 5y + 2z \Rightarrow 3(2) + 5(1) + 2(0) = 11$$

$$3x + 4y + 5z + r = 10 \Rightarrow 3(2) + 4(1) + 5(0) + 0 = 10$$

$$x + 3y + 10z + s = 5 \Rightarrow 2 + 3(1) + 10(0) + 0 = 5$$

$$x - 2y + t = 1 \Rightarrow 2 - 2(1) + 1 = 1$$

b

$$\text{P} + 8z + \frac{4}{5}r + \frac{3}{5}s = 11$$

$$x - 5z + \frac{3}{5}r - \frac{4}{5}s = 2$$

$$y + 5z - \frac{1}{5}r + \frac{3}{5}s = 1$$

$$15z - r + 2s + t = 1$$

c $P = 11 - 8z - \dfrac{4}{5}r - \dfrac{3}{5}s$, so increasing z, r or s would decrease P.

For Q4

a
$$P = 3x + 6y + 32z \Rightarrow 3(105) + 6(21) + 32(0) = 441$$
$$x + 6y + 24z + r = 672 \Rightarrow 105 + 6(21) + 24(0) + 441 = 672$$
$$3x + y + 24z + s = 336 \Rightarrow 3(105) + 21 + 24(0) + 0 = 336$$
$$x + 3y + 16z + t = 168 \Rightarrow 105 + 3(21) + 16(0) + 0 = 168$$
$$2x + 3y + 32z + u = 352 \Rightarrow 2(105) + 3(21) + 32(0) + 79 = 352$$

b
$$P + 7z + \frac{3}{8}s + \frac{15}{8}t = 441$$
$$-z + r + \frac{3}{8}s - \frac{17}{8}t = 441$$
$$x + 7z + \frac{3}{8}s - \frac{1}{8}t = 105$$
$$y + 3z - \frac{1}{8}s + \frac{3}{8}t = 21$$
$$9z - \frac{3}{8}s - \frac{7}{8}t + u = 79$$

c $P = 441 - 7z - \frac{3}{8}s - \frac{15}{8}t$, so increasing z, s or t would decrease P.

For Q5

a
$$P = 4x_1 + 3x_2 + 2x_3 + 3x_4 \Rightarrow 4\left(\frac{209}{16}\right) + 3\left(\frac{33}{4}\right) + 2\left(\frac{261}{16}\right) + 3(0) = \frac{877}{8}$$
$$x_1 + 4x_2 + 3x_3 + x_4 + r = 95 \Rightarrow \frac{209}{16} + 4\left(\frac{33}{4}\right) + 3\left(\frac{261}{16}\right) + (0) + 0 = 95$$
$$2x_1 + x_2 + 2x_3 + 3x_4 + s = 67 \Rightarrow 2\left(\frac{209}{16}\right) + \left(\frac{33}{4}\right) + 2\left(\frac{261}{16}\right) + 3(0) + 0 = 67$$
$$x_1 + 3x_2 + 2x_3 + 2x_4 + t = 75 \Rightarrow \frac{209}{16} + 3\left(\frac{33}{4}\right) + 2\left(\frac{261}{16}\right) + 2(0) + \frac{73}{16} = 75$$
$$3x_1 + 2x_2 + x_3 + 2x_4 + u = 72 \Rightarrow 3\left(\frac{209}{16}\right) + 2\left(\frac{33}{4}\right) + \left(\frac{261}{16}\right) + 2(0) + 0 = 72$$

b
$$P + \frac{1}{8}x_4 + \frac{1}{8}r + \frac{1}{4}s + \frac{9}{8}u = \frac{877}{8}$$
$$x_2 - \frac{3}{4}x_4 + \frac{1}{4}r - \frac{1}{2}s + \frac{1}{4}u = \frac{33}{4}$$
$$x_3 + \frac{17}{16}x_4 + \frac{1}{16}r + \frac{5}{8}s - \frac{7}{16}u = \frac{261}{16}$$
$$\frac{21}{16}x_4 - \frac{11}{16}r + \frac{1}{8}s + t - \frac{41}{48}u = \frac{73}{16}$$
$$x_1 + \frac{13}{16}x_4 - \frac{3}{16}r + \frac{1}{8}s + \frac{5}{16}u = \frac{209}{16}$$

c $P = \frac{877}{8} - \frac{1}{8}x_4 - \frac{1}{8}r - \frac{1}{4}s - \frac{9}{8}u$, so increasing, x_4, r, s or u would decrease P.

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Exercise C, Question 1

Question:

In a particular factory 3 types of product, A, B and C, are made. The number of each of the products made is x, y and z respectively and P is the profit in pounds. There are two machines involved in making the products which have only a limited time available. These time limitations produce two constraints.
In the process of using the simplex algorithm the following tableau is obtained, where r and s are slack variables.

Basic variable	x	y	z	r	s	Value
z	$\frac{1}{3}$	0	1	−8	1	75
y	$\frac{2}{11}$	1	0	$\frac{17}{11}$	0	56
P	$\frac{3}{2}$	0	0	$\frac{3}{4}$	0	840

a Give one reason why this tableau can be seen to be optimal (final).

b By writing out the profit equation, or otherwise, explain why a further increase in profit is not possible under these constraints.

c From this tableau deduce

i the maximum profit,

ii the optimum number of type A, B and C that should be produced to maximise the profit.

Solution:

a There are no negative numbers in the profit row.

b $P+\frac{3}{2}x+\frac{3}{4}r=840$

So $P=840-\frac{3}{2}x-\frac{3}{4}r$

Increasing x or r would decrease P.

c **i** Maximum profit = £840

ii Optimum number of A = 0, B = 56 and C = 75

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Exercise C, Question 2

Question:

A sweet manufacturer produces packets of orange and lemon flavoured sweets.
The manufacturer can produce up to 25 000 orange sweets and up to 36 000 lemon sweets per day.

Small packets contain 5 orange and 5 lemon sweets.
Medium packets contain 8 orange and 6 lemon sweets.
Large packets contain 10 orange and 15 lemon sweets.

The manufacturer makes a profit of 14p, 20p and 30p on each of the small, medium and large packets respectively. He wishes to maximise his total daily profit.

Use x, y and z to represent the number of small, medium and large packets respectively, produced each day.

a Formulate this information as a linear programming problem, making your objective function and constraints clear. Change any inequalities to equations using r and s as slack variables.

The tableau below is obtained after one complete iteration of the simplex algorithm.

Basic variable	x	y	z	r	s	Value
r	$1\frac{2}{3}$	4	0	1	$-\frac{2}{3}$	1000
z	$\frac{1}{3}$	$\frac{2}{5}$	1	0	$\frac{1}{15}$	2400
P	−4	−8	0	0	2	72 000

b Start from this tableau and continue the simplex algorithm by increasing y, until you have either completed two complete iterations or found an optimal solution.

From your final tableau:

c **i** write down the numbers of small, medium and large packets indicated,

ii write down the profit,

iii state whether this is an optimal solution, giving your reason. **[*E*]**

Solution:

a Maximise $P = 14x + 20y + 30z$

Subject to:

$5x + 8y + 10z + r = 25\,000$

$5x + 6y + 15z + s = 36\,000$

where r and s are slack variables $x, y, z, r, s \geq 0$

b

b.v.	x	y	z	r	s	value
r	$1\frac{2}{3}$	4	0	1	$-\frac{2}{3}$	1000
z	$\frac{1}{3}$	$\frac{2}{5}$	1	0	$\frac{1}{15}$	2400
P	-4	-8	0	0	2	72 000

b.v.	x	y	z	r	s	value	Row operation
y	$\frac{5}{12}$	1	0	$\frac{1}{4}$	$-\frac{1}{6}$	250	$R1 \div 4$
z	$\frac{1}{6}$	0	1	$-\frac{1}{10}$	$\frac{2}{15}$	23 00	$R2 - \frac{2}{5}R1$
P	$-\frac{2}{3}$	0	0	2	$\frac{2}{3}$	74 000	$R3 + 8R1$

b.v.	x	y	z	r	s	Value	Row operations
x	1	$2\frac{2}{5}$	0	$\frac{3}{5}$	$-\frac{2}{5}$	600	$R1 \div \frac{5}{12}$
z	0	$-\frac{2}{5}$	1	$-\frac{1}{5}$	$\frac{1}{5}$	2200	$R2 - \frac{1}{6}R1$
p	0	$1\frac{3}{5}$	0	$2\frac{2}{5}$	$\frac{2}{5}$	74 400	$R3 + \frac{2}{3}R1$

c **i** $x = 600$ $y = 0$ $z = 2200$

ii Profit is = £ 744

iii The solution is optimal since there are no negative numbers in the profit row.

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Exercise C, Question 3

Question:

Tables are to be bought for a new restaurant. The owners may buy small, medium and large tables that seat 2, 4 and 6 people respectively.
The owners require at most 20% of the total number of tables to be medium sized.
The tables cost £60, £100 and £160 respectively for small, medium and large. The owners have a budget of £2000 for buying tables.
Let the number of small, medium and large tables be x, y and z respectively.

a Write down 5 inequalities implied by the constraints. Simplify these where appropriate.

The owners wish to maximise the total seating capacity, S, of the restaurant.

b Write down the objective function for S in terms of x, y and z.

c Explain why it is not appropriate to use a graphical method to solve this problem.

It is decided to use the simplex algorithm to solve this problem.

d Show that a possible initial tableau is

Basic variable	x	y	z	r	t	Value
r	−1	4	−1	1	0	0
t	3	5	8	0	1	100
S	−2	−4	−6	0	0	0

It is decided to increase z first.

e Show that, after one complete iteration, the tableau becomes

Basic variable	x	y	z	r	t	Value
r	$-\frac{5}{8}$	$\frac{37}{8}$	0	1	$\frac{1}{8}$	$\frac{25}{2}$
t	$\frac{3}{8}$	$\frac{5}{8}$	1	0	$\frac{1}{8}$	$\frac{25}{2}$
S	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$\frac{3}{4}$	75

f Perform one further complete iteration.

g Explain how you can decide if your tableau is now final.

h Find the number of each type of table the restaurant should buy and their total cost. **[E]**

Solution:

a $\frac{1}{5}(x+y+z) \geq y \Rightarrow -x+4y-z \leq 0$

$60x+100y+160z \leq 2000 \Rightarrow 3x+5y+8z \leq 100$

$x \geq 0 \; y \geq 0 \quad z \geq 0$

b $S = 2x+4y+6z$

c There are three variables.

d

b.v.	x	y	z	r	t	value
r	-1	4	-1	1	0	0
t	3	5	8	0	1	100
S	-2	-4	-6	0	0	0

e

b.v.	x	y	z	r	t	value	Row operations
r	$\frac{-5}{8}$	$4\frac{5}{8}$	0	1	$\frac{1}{8}$	$12\frac{1}{2}$	R1+R2
z	$\frac{3}{8}$	$\frac{5}{8}$	1	0	$\frac{1}{8}$	$12\frac{1}{2}$	R2 ÷ 8
S	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$\frac{3}{4}$	75	R3+6R2

f

b.v.	x	y	z	r	t	value	Row operations
y	$-\frac{5}{37}$	1	0	$\frac{8}{37}$	$\frac{1}{37}$	$2\frac{26}{37}$	$\text{R1} \div 4\frac{5}{8}$
z	$\frac{17}{37}$	0	1	$\frac{-5}{37}$	$\frac{4}{37}$	$10\frac{30}{37}$	$\text{R2} - \frac{5}{8}\text{R1}$
S	$\frac{8}{37}$	0	0	$\frac{2}{37}$	$\frac{28}{37}$	$75\frac{25}{37}$	$\text{R3} + \frac{1}{4}\text{R1}$

g There are no negative numbers in the objective row.

h 0 small, 2 medium and 11 large tables (seating 74) at a cost of £1960

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Exercise C, Question 4

Question:

Kuddly Pals Co. Ltd. make two types of soft toy: bears and cats. The quantity of material needed and the time taken to make each type of toy is given in the table.

Toy	Material (m^2)	Time (minutes)
Bear	0.05	12
Cat	0.08	8

Each day the company can process up to $20\,m^2$ of material and there are 48 worker hours available to assemble the toys.
Let x be the number of bears made and y the number of cats made each day.

a Show that this situation can be modelled by the inequalities

$5x+8y \le 2000$,

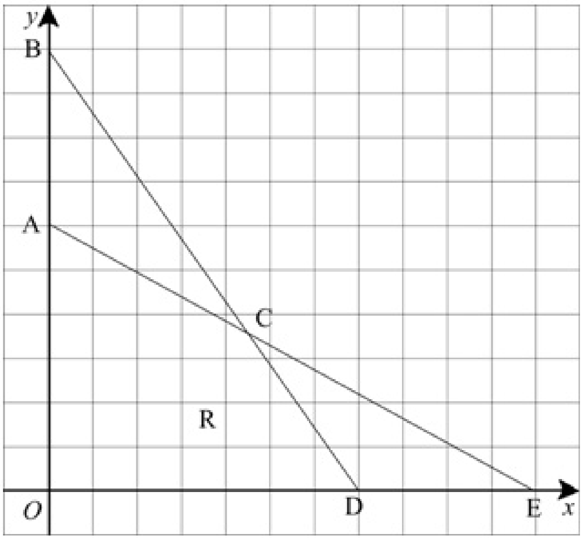
$3x+2y \le 720$,

in addition to $x \ge 0, y \ge 0$.

The profit made on each bear is £1.50 and on each cat £1.75. Kuddly Pals Co. Ltd. wishes to maximise its daily profit.

b Set up an initial simplex tableau for this problem.
c Solve the problem using the simplex algorithm.

The diagram shows a graphical representation of the feasible region.



d Relate each stage of the simplex tableau to the corresponding point in the diagram. **[E]**

Solution:

a *Material*

(×100) $0.05x + 0.08y \leq 20$

$5x + 8y \leq 2000$

Time (÷4) $12x + 8y \leq 2880$

$3x + 2y \leq 720$

b

b.v.	x	y	r	s	value
r	5	8	1	0	2000
s	3	2	0	1	720
P	−1.5	−1.75	0	0	0

c

b.v.	x	y	r	s	value	Row operations
Y	$\frac{5}{8}$	1	$\frac{1}{8}$	0	250	R1 ÷ 8
s	$1\frac{3}{4}$	0	$-\frac{1}{4}$	1	220	R2
P	$\frac{-13}{32}$	0	$\frac{7}{32}$	0	$437\frac{1}{2}$	$R3 - 1\frac{3}{4}R1$

b.v.	x	y	r	s	value	Row operations
y	0	1	$\frac{3}{14}$	$-\frac{5}{14}$	$171\frac{3}{7}$	$R1 - \frac{5}{8}R2$
x	1	0	$-\frac{1}{7}$	$\frac{4}{7}$	$125\frac{5}{7}$	$R2 \div 1\frac{3}{4}$
P	0	0	$\frac{9}{56}$	$\frac{13}{56}$	$488\frac{4}{7}$	$R3 \div 1\frac{13}{32}R2$

c Optimal solution $x = 125\frac{5}{7}$ $y = 171\frac{3}{7}$

Integer solutions needed, so point testing gives $x = 126$ $y = 171$

d The first point is A if y is increased first

(D if x is increased first),

The second point is C.

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Exercise C, Question 5

Question:

A clocksmith makes three types of luxury wristwatch. The mechanism for each watch is assembled by hand by a skilled watchmaker and then the complete watch is formed, weatherproofed and packaged for sale by a fitter.
The table shows the times, in minutes, for each stage of the process.

Watch type	Watchmaker	Fitter
A	54	60
B	72	36
C	36	48

The watchmaker works for a maximum of 30 hours per week and the fitter for a maximum of 25 hours per week.
Let the number of type A, B and C watches made per week be x, y and z.

a Show that the above information leads to the two inequalities

$$3x + 4y + 2z \leq 100,$$
$$5x + 3y + 4z \leq 125.$$

The profit made on type A, B and C watches is £12, £24 and £20 respectively.

b Write down an expression for the profit, P, in pounds, in terms of x, y and z.
The clocksmith wishes to maximise his weekly profit. It is decided to use the simplex algorithm to solve this problem.
c Write down the initial tableau using r and s as the slack variables.
d Increasing y first, show that after two complete iterations of the simplex algorithm the tableau becomes

Basic variable	x	y	z	r	s	Value
y	$\frac{1}{5}$	1	0	$\frac{2}{5}$	$-\frac{1}{5}$	15
z	$\frac{11}{10}$	0	1	$-\frac{3}{10}$	$\frac{2}{5}$	20
P	$\frac{74}{5}$	0	0	$\frac{18}{5}$	$\frac{16}{5}$	760

e Give a reason why this tableau is optimal (final).
f Write down the numbers of each type of watch that should be made to maximise the profit. State the maximum profit. **[*E*]**

Solution:

a Watchmaker

$$54x+72y+36z \le 1800$$

$(\div 418)$

$$3x+4y+2z \le 100$$

Fitter

$$60x+36y+48z \le 1500$$

$(\div 12)$

$$5x+3y+4z \le 125$$

b $P = 12x+24y+20z$

c

b.v.	x	y	z	r	s	value
r	3	4	2	1	0	100
s	5	3	4	0	1	125
P	−12	−24	−20	0	0	0

d

b.v.	x	y	z	r	s	value	Row operations
y	$\frac{3}{4}$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	25	R1 ÷ 4
s	$2\frac{3}{4}$	0	$2\frac{1}{2}$	$\frac{-3}{4}$	1	50	R2 − 3R1
P	6	0	−8	6	0	600	R3 + 24R1

b.v.	x	y	z	r	s	value	Raw operations
y	$\frac{1}{5}$	1	0	$\frac{2}{5}$	$-\frac{1}{5}$	15	$\text{R1} - \frac{1}{2}\text{R2}$
z	$\frac{11}{10}$	0	1	$-\frac{3}{10}$	$\frac{2}{5}$	20	$\text{R2} \div 2\frac{1}{2}$
P	$14\frac{4}{5}$	0	0	$3\frac{3}{5}$	$3\frac{1}{5}$	760	R3 + 8R2

e There are no negative numbers in the profit row

f Type A = 0 Type B = 15 Type C = 20

Profit = £760

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Exercise C, Question 6

Question:

A craftworker makes three types of wooden animals for sale in wildlife parks. Each animal has to be carved and then sanded.
Each Lion takes 2 hours to carve and 25 minutes to sand.
Each Giraffe takes $2\frac{1}{2}$ hours to carve and 20 minutes to sand.
Each Elephant takes $1\frac{1}{2}$ hours to carve and 30 minutes to sand.
Each day the craftworker wishes to spend at most 3 hours carving and at most 2 hours sanding.
Let x be the number of Lions, y the number of Giraffes and z the number of Elephants he produces each day.
The craftworker makes a profit of £14 on each Lion, £12 on each Giraffe and £13 on each Elephant. He wishes to maximise his profit, P.

a Model this as a linear programming problem, simplifying your expressions so that they have integer coefficients.

It is decided to use the simplex algorithm to solve this problem.

b Explaining the purpose of r and s, show that the initial tableau can be written as:

Basic variable	x	y	z	r	s	Value
r	4	5	3	1	0	16
t	5	4	6	0	1	24
P	−14	−12	−13	0	0	0

c Choosing to increase x first, work out the next complete tableau, where the x column includes two zeros.

d Explain what this first iteration means in practical terms. **[*E*]**

Solution:

a Maximise $P = 14x + 12y + 13z$

Subject to:
Carving $2x + 2.5y + 1.5z \le 8 \Rightarrow 4x + 5y + 3z \le 16$
Sanding $25x + 20y + 30z \le 120 \Rightarrow 5x + 4y + 6z \le 24$
$x, y, z \ge 0$

b r and s are numbers which indicate the slack time
Profit: $P - 14x - 12y - 13z = 0$
Constraints:
$4x + 5y + 3z + r = 16$
$5x + 4y + 6z + s = 24$

b.v.	x	y	z	r	s	value
r	4	5	3	1	0	16
s	5	4	6	0	1	24
P	−14	−12	−13	0	0	0

c

b.v.	x	y	z	r	s	value	Row operations
x	1	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	4	R1 ÷ 4
s	0	$\frac{-9}{4}$	$\frac{9}{4}$	$\frac{-5}{4}$	1	4	R2 − 5R1
P	0	$\frac{11}{2}$	$\frac{-5}{2}$	$\frac{7}{2}$	0	56	R3 + 14R1

d From a zero stock situation, if we increase the number of lions to 4, we are increasing the profit from 0 to £56.