Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, showing the table at each stage and state your final solution and its cost.

	Task X	**Task Y**	**Task Z**
Worker A	34	35	31
Worker B	26	31	27
Worker C	30	37	32

Solution:

$$\begin{pmatrix} 34 & 35 & 31 \\ 26 & 31 & 27 \\ 30 & 37 & 32 \end{pmatrix} \xrightarrow{\text{reducing rows}} \begin{pmatrix} 3 & 4 & 0 \\ 0 & 5 & 1 \\ 0 & 7 & 2 \end{pmatrix} \rightarrow$$

$$\text{reducing columns} \rightarrow \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 3 & 2 \end{pmatrix}$$

$$\text{Minimum uncovered is 1} \quad \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

Two solutions:

A – Y (35)		A – Z (31)		
B – Z (27)	or	B – Y (31)	cost 92	
C – X (30)		C – X (30)		

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, showing the table at each stage and state your final solution and its cost.

	Task A	Task B	Task C	Task D
Worker P	34	37	32	32
Worker Q	35	32	34	37
Worker R	42	35	37	36
Worker S	38	34	35	39

Solution:

$$\begin{pmatrix} 34 & 37 & 32 & 32 \\ 35 & 32 & 34 & 37 \\ 42 & 35 & 37 & 36 \\ 38 & 34 & 35 & 39 \end{pmatrix} \to \begin{matrix}\text{reducing} \\ \text{rows}\end{matrix} \to \begin{pmatrix} 2 & 5 & 0 & 0 \\ 3 & 0 & 2 & 5 \\ 7 & 0 & 2 & 1 \\ 4 & 0 & 1 & 5 \end{pmatrix}$$

$$\to \begin{matrix}\text{reducing} \\ \text{columns}\end{matrix} \begin{pmatrix} 0 & 5 & 0 & 0 \\ 1 & 0 & 2 & 5 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 5 \end{pmatrix}$$

$$\text{Minimum uncovered is 1} \begin{pmatrix} 0 & 6 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 4 & 0 & 1 & 0 \\ 1 & 0 & 0 & 4 \end{pmatrix}$$

Two solutions

$$\begin{matrix} P-A\ (34) \\ Q-B\ (32) \\ R-D\ (36) \\ S-C\ (35) \end{matrix} \quad \text{or} \quad \begin{matrix} P-D\ (32) \\ Q-A\ (35) \\ R-B\ (35) \\ S-C\ (35) \end{matrix} \quad \text{cost } 137$$

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Exercise A, Question 3

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, showing the table at each stage and state your final solution and its cost.

	Task R	Task S	Task T	Task U
Worker J	20	22	14	24
Worker K	20	19	12	20
Worker L	13	10	18	16
Worker M	22	23	9	28

Solution:

$$\begin{pmatrix} 20 & 22 & 14 & 24 \\ 20 & 19 & 12 & 20 \\ 13 & 10 & 18 & 16 \\ 22 & 23 & 9 & 28 \end{pmatrix} \rightarrow \begin{matrix}\text{reducing} \\ \text{rows}\end{matrix} \rightarrow \begin{pmatrix} 6 & 8 & 0 & 10 \\ 8 & 7 & 0 & 8 \\ 3 & 0 & 8 & 6 \\ 13 & 14 & 0 & 19 \end{pmatrix}$$

$$\begin{matrix}\text{reducing} \\ \text{columns}\end{matrix} \begin{pmatrix} 3 & 8 & 0 & 4 \\ 5 & 7 & 0 & 2 \\ 0 & 0 & 8 & 0 \\ 10 & 14 & 0 & 13 \end{pmatrix}$$

$$\text{Minimum uncovered element is 2} \begin{pmatrix} 1 & 6 & 0 & 2 \\ 3 & 5 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 8 & 12 & 0 & 11 \end{pmatrix}$$

$$\text{Minimum uncovered element is 1} \begin{pmatrix} 0 & 5 & 0 & 1 \\ 3 & 5 & 1 & 0 \\ 0 & 0 & 11 & 0 \\ 7 & 11 & 0 & 10 \end{pmatrix} \quad \text{or} \quad \begin{matrix}\text{Minimum} \\ \text{uncovered} \\ \text{element} \\ \text{is 1}\end{matrix} \quad \begin{pmatrix} 0 & 5 & 0 & 2 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 11 & 0 \\ 7 & 11 & 0 & 10 \end{pmatrix}$$

Solution

J – R (20)
K – U (20)
L – S (10)
M – T (9)

cost 59

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 4

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, showing the table at each stage and state your final solution and its cost.

	Task V	Task W	Task X	Task Y	Task Z
Worker D	85	95	97	87	80
Worker E	110	115	95	105	100
Worker F	90	95	86	93	105
Worker G	85	83	84	85	87
Worker H	100	100	105	120	95

Solution:

$$\begin{pmatrix} 85 & 95 & 97 & 87 & 80 \\ 110 & 115 & 95 & 105 & 100 \\ 90 & 95 & 86 & 93 & 105 \\ 85 & 83 & 84 & 85 & 87 \\ 100 & 100 & 105 & 120 & 95 \end{pmatrix} \xrightarrow[\text{rows}]{\text{reducing}} \begin{pmatrix} 5 & 15 & 17 & 7 & 0 \\ 15 & 20 & 0 & 10 & 5 \\ 4 & 9 & 0 & 7 & 19 \\ 2 & 0 & 1 & 2 & 4 \\ 5 & 5 & 10 & 25 & 0 \end{pmatrix}$$

$$\begin{matrix}\text{reducing} \\ \text{columns}\end{matrix} \begin{pmatrix} 3 & 15 & 17 & 5 & 0 \\ 13 & 20 & 0 & 8 & 5 \\ 2 & 9 & 0 & 5 & 19 \\ 0 & 0 & 1 & 0 & 4 \\ 3 & 5 & 10 & 23 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 2} \begin{pmatrix} 1 & 13 & 17 & 3 & 0 \\ 11 & 18 & 0 & 6 & 5 \\ 0 & 7 & 0 & 3 & 19 \\ 0 & 0 & 3 & 0 & 6 \\ 1 & 3 & 10 & 21 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 3} \begin{pmatrix} 1 & 10 & 17 & 0 & 0 \\ 11 & 15 & 0 & 3 & 5 \\ 0 & 4 & 0 & 0 & 19 \\ 3 & 0 & 6 & 0 & 9 \\ 1 & 0 & 10 & 18 & 0 \end{pmatrix}$$

There are two solutions

D – Z (80)	D – Y (87)	
E – X (95)	E – X (95)	
F – V (90)	F – V (90)	
G – Y (85)	G – W (83)	cost 450
H – W (100)	H – Z (95)	

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Exercise A, Question 5

Question:

	100 m	Hurdles	200 m	400 m
Ahmed	14	21	37	64
Ben	13	22	40	68
Chang	12	20	38	70
Davina	13	21	39	74

A junior school has to enter four pupils in an athletics competition comprising four events; 100 m sprint, hurdles, 200 m, 400 m. The rules are that each pupil may only enter one event and the winning team is the one whose total time for the four events is the least. The school holds trials and the table shows the time, in seconds, that each of the team members takes. Reducing rows first, use the Hungarian algorithm to determine who should participate in which event in order to minimise the total time.

Solution:

$$\begin{pmatrix} 14 & 21 & 37 & 64 \\ 13 & 22 & 40 & 68 \\ 12 & 20 & 38 & 70 \\ 13 & 21 & 39 & 74 \end{pmatrix} \rightarrow \begin{matrix}\text{reducing} \\ \text{rows}\end{matrix} \rightarrow \begin{pmatrix} 0 & 7 & 23 & 50 \\ 0 & 9 & 27 & 55 \\ 0 & 8 & 26 & 58 \\ 0 & 8 & 26 & 61 \end{pmatrix}$$

$$\begin{matrix}\text{reducing} \\ \text{columns}\end{matrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 5 \\ 0 & 1 & 3 & 8 \\ 0 & 1 & 3 & 11 \end{pmatrix}$$

$$\text{Minimum uncovered element is 1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 2 & 10 \end{pmatrix}$$

$$\text{Minimum uncovered element is 2} \begin{pmatrix} 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 8 \end{pmatrix}$$

Two solutions:

A – 400 m (64)	A – 400 m (64)	
B – 100 m (13)	B – 100 m (13)	time: 136
C – hurdles (20)	C – 200 m (38)	
D – 200 m (39)	D – hurdles (21)	

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Exercise A, Question 6

Question:

	Beech	Elm	Eucalyptus	Oak	Olive
A	153	87	62	144	76
B	162	105	87	152	88
C	159	84	75	165	79
D	145	98	63	170	85
E	149	94	70	138	82

The table shows the cost, in pounds, of purchasing trees from five local nurseries. A landscape gardener wishes to support each of these local nurseries for the year and so decides to use each nursery to supply one type of tree. He will use equal numbers of each type of tree throughout the year.

Reducing rows first, use the Hungarian algorithm to determine which type of tree should be supplied by which nursery in order to minimise the total cost.

Solution:

$$\begin{pmatrix} 153 & 87 & 62 & 144 & 76 \\ 162 & 105 & 87 & 152 & 88 \\ 159 & 84 & 75 & 165 & 79 \\ 145 & 98 & 63 & 170 & 85 \\ 149 & 94 & 70 & 138 & 82 \end{pmatrix}$$

$$\begin{array}{cc} \text{reducing rows} & \begin{pmatrix} 91 & 25 & 0 & 82 & 14 \\ 75 & 18 & 0 & 65 & 1 \\ 84 & 9 & 0 & 90 & 4 \\ 82 & 35 & 0 & 107 & 22 \\ 79 & 24 & 0 & 68 & 12 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} \text{reducing columns} & \begin{pmatrix} 16 & 16 & 0 & 17 & 13 \\ 0 & 9 & 0 & 0 & 0 \\ 9 & 0 & 0 & 25 & 3 \\ 7 & 26 & 0 & 42 & 21 \\ 4 & 15 & 0 & 3 & 11 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} \text{Minimum uncovered element is 3} & \begin{pmatrix} 13 & 13 & 0 & 14 & 10 \\ 0 & 9 & 3 & 0 & 0 \\ 9 & 0 & 3 & 25 & 3 \\ 4 & 23 & 0 & 39 & 18 \\ 1 & 12 & 0 & 0 & 8 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} \text{Minimum uncovered element is 4} & \begin{pmatrix} 9 & 9 & 0^* & 10 & 6 \\ 0 & 9 & 7 & 0 & 0^* \\ 9 & 0^* & 7 & 25 & 3 \\ 0^* & 19 & 0 & 35 & 14 \\ 1 & 12 & 4 & 0^* & 8 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} \text{reducing rows} & \begin{pmatrix} 91 & 25 & 0 & 82 & 14 \\ 75 & 18 & 0 & 65 & 1 \\ 84 & 9 & 0 & 90 & 4 \\ 82 & 35 & 0 & 107 & 22 \\ 79 & 24 & 0 & 68 & 12 \end{pmatrix} \end{array}$$

$$\text{reducing columns} \begin{pmatrix} 16 & 16 & 0 & 17 & 13 \\ 0 & 9 & 0 & 0 & 0 \\ 9 & 0 & 0 & 25 & 3 \\ 7 & 26 & 0 & 42 & 21 \\ 4 & 15 & 0 & 3 & 11 \end{pmatrix}$$

$$\begin{array}{c} \text{Minimum} \\ \text{uncovered} \\ \text{element is 3} \end{array} \begin{pmatrix} 13 & 13 & 0 & 14 & 10 \\ 0 & 9 & 3 & 0 & 0 \\ 9 & 0 & 3 & 25 & 3 \\ 4 & 23 & 0 & 39 & 18 \\ 1 & 12 & 0 & 0 & 8 \end{pmatrix}$$

$$\begin{array}{c} \text{Minimum} \\ \text{uncovered} \\ \text{element is 4} \end{array} \begin{pmatrix} 9 & 9 & 0^* & 10 & 6 \\ 0 & 9 & 7 & 0 & 0^* \\ 9 & 0^* & 7 & 25 & 3 \\ 0^* & 19 & 0 & 35 & 14 \\ 1 & 12 & 4 & 0^* & 8 \end{pmatrix}$$

A – Eucalyptus (62)
B – Olive (88)
C – Elm (84) Cost 517
D – Beech (145)
E – Oak (138)

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Exercise B, Question 1

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, show the table at each stage, and state your final solution and its cost.

	Task M	Task N
Worker J	23	26
Worker K	26	30
Worker L	29	28

Solution:

$$\begin{pmatrix} 23 & 26 & 0 \\ 26 & 30 & 0 \\ 29 & 28 & 0 \end{pmatrix} \begin{matrix} \text{reducing} \\ \text{columns} \end{matrix} \begin{pmatrix} 0 & 0 & 0 \\ 3 & 4 & 0 \\ 6 & 2 & 0 \end{pmatrix}$$

Minimum uncovered element is 2 $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 2 & 0 \\ 4 & 0 & 0 \end{pmatrix}$

Solution: J – M (23)
K – dummy
L – N (28)

cost: 51

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Exercise B, Question 2

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, show the table at each stage, and state your final solution and its cost.

	Task W	Task X	Task Y	Task Z
Worker A	31	43	19	35
Worker B	28	46	10	34
Worker C	24	42	13	33

Solution:

$$\begin{pmatrix} 31 & 43 & 19 & 35 \\ 28 & 46 & 10 & 34 \\ 24 & 42 & 13 & 33 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow[\text{rows}]{\text{reducing}} \begin{pmatrix} 12 & 24 & 0 & 16 \\ 18 & 36 & 0 & 24 \\ 11 & 29 & 0 & 20 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Minimum uncovered element is 11 $\begin{pmatrix} 1 & 13 & 0 & 5 \\ 7 & 25 & 0 & 13 \\ 0 & 18 & 0 & 9 \\ 0 & 0 & 11 & 0 \end{pmatrix}$

Minimum uncovered element is 5 $\begin{pmatrix} 1 & 8 & 0 & 0 \\ 7 & 20 & 0 & 8 \\ 0 & 13 & 0 & 4 \\ 5 & 0 & 16 & 0 \end{pmatrix}$

Solution:

A – Z (35)
B – Y (10)
C – W (24)
dummy – X (0)

cost: 69

Alternative solution

Minimum uncovered element is 1 $\begin{pmatrix} 0 & 12 & 0 & 4 \\ 6 & 24 & 0 & 12 \\ 0 & 18 & 1 & 9 \\ 0 & 0 & 12 & 0 \end{pmatrix}$

then

Minimum uncovered element is 4 $\begin{pmatrix} 0 & 8 & 0 & 0 \\ 6 & 20 & 0 & 8 \\ 0 & 14 & 1 & 5 \\ 4 & 0 & 16 & 0 \end{pmatrix}$

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Exercise B, Question 3

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, show the table at each stage, and state your final solution and its cost.

	Task R	Task S	Task T
Worker W	81	45	55
Worker X	67	32	48
Worker Y	87	38	58
Worker Z	73	37	60

Solution:

$$\begin{pmatrix} 81 & 45 & 55 & 0 \\ 67 & 32 & 48 & 0 \\ 87 & 38 & 58 & 0 \\ 73 & 37 & 60 & 0 \end{pmatrix} \begin{matrix} \text{reducing} \\ \text{columns} \end{matrix} \begin{pmatrix} 14 & 13 & 7 & 0 \\ 0 & 0 & 0 & 0 \\ 20 & 6 & 10 & 0 \\ 6 & 5 & 12 & 0 \end{pmatrix}$$

Minimum uncovered element is 5 $\begin{pmatrix} 9 & 8 & 2 & 0 \\ 0 & 0 & 0 & 5 \\ 15 & 1 & 5 & 0 \\ 1 & 0 & 7 & 0 \end{pmatrix}$

Minimum uncovered element is 1 $\begin{pmatrix} 8 & 8 & 1 & 0 \\ 0 & 1 & 0 & 6 \\ 14 & 1 & 4 & 0 \\ 0 & 0 & 6 & 0 \end{pmatrix}$

Minimum uncovered element is 1 $\begin{pmatrix} 7 & 7 & 0 & 0 \\ 0 & 2 & 0 & 7 \\ 13 & 0 & 3 & 0 \\ 0 & 0 & 6 & 1 \end{pmatrix}$

There are two solutions

W – dummy	W – T (55)	
X – T (48)	X – R (67)	cost 159
Y – S (38)	Y – dummy	
Z – R (73)	Z – S (37)	

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 4

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, show the table at each stage, and state your final solution and its cost.

	Task E	Task F	Task G	Task H
Worker P	24	42	32	31
Worker Q	22	39	30	35
Worker R	13	34	22	25
Worker S	19	41	27	29
Worker T	18	40	31	33

Solution:

$$\begin{pmatrix} 24 & 42 & 32 & 31 & 0 \\ 22 & 39 & 30 & 35 & 0 \\ 13 & 34 & 22 & 25 & 0 \\ 19 & 41 & 27 & 29 & 0 \\ 18 & 40 & 31 & 33 & 0 \end{pmatrix} \begin{array}{c} \text{reducing} \\ \text{columns} \end{array} \begin{pmatrix} 11 & 8 & 10 & 6 & 0 \\ 9 & 5 & 8 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 6 & 7 & 5 & 4 & 0 \\ 5 & 6 & 9 & 8 & 0 \end{pmatrix}$$

Minimum uncovered element is 4 $\begin{pmatrix} 7 & 4 & 6 & 2 & 0 \\ 5 & 1 & 4 & 6 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 2 & 3 & 1 & 0 & 0 \\ 1 & 2 & 5 & 4 & 0 \end{pmatrix}$

Either

Minimum uncovered element is 1 $\begin{pmatrix} 6 & 3 & 5 & 1 & 0 \\ 4 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \\ 2 & 3 & 1 & 0 & 1 \\ 0 & 1 & 4 & 3 & 0 \end{pmatrix}$ or $\begin{pmatrix} 6 & 3 & 5 & 2 & 0 \\ 4 & 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 1 & 5 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 4 & 4 & 0 \end{pmatrix}$

Solution:

P – dummy
Q – F (39)
R – G (22) cost 108
S – H (29)
T – E (18)

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
The crosses ' × ' indicate that that worker cannot be assigned to that task.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, show the table at each stage and state your final solution and its cost.

	Task L	Task M	Task N
Worker P	48	34	×
Worker Q	×	37	67
Worker R	53	43	56

Solution:

$$\begin{pmatrix} 48 & 34 & 140 \\ 140 & 37 & 67 \\ 53 & 43 & 56 \end{pmatrix} \begin{matrix} \text{reducing} \\ \text{rows} \end{matrix} \begin{pmatrix} 14 & 0 & 106 \\ 103 & 0 & 30 \\ 10 & 0 & 13 \end{pmatrix}$$

$$\text{reducing columns} \begin{pmatrix} 4 & 0 & 93 \\ 93 & 0 & 17 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 4} \begin{pmatrix} 0 & 0 & 89 \\ 89 & 0 & 13 \\ 0 & 4 & 0 \end{pmatrix}$$

Solution: P – L (48)

Q – M (37) cost 141

R – N (56)

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Exercise C, Question 2

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
The crosses '×' indicate that that worker cannot be assigned to that task.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, show the table at each stage and state your final solution and its cost.

	Task D	Task E	Task F	Task G
Worker R	38	47	55	53
Worker S	32	×	47	64
Worker T	×	53	43	×
Worker U	41	48	52	47

Solution:

$$\begin{pmatrix} 38 & 47 & 55 & 53 \\ 32 & 130 & 47 & 64 \\ 130 & 53 & 43 & 130 \\ 41 & 48 & 52 & 47 \end{pmatrix} \begin{matrix} \text{reducing} \\ \text{rows} \end{matrix} \begin{pmatrix} 0 & 9 & 17 & 15 \\ 0 & 98 & 15 & 32 \\ 87 & 10 & 0 & 87 \\ 0 & 7 & 11 & 6 \end{pmatrix}$$

$$\text{reducing columns} \begin{pmatrix} 0 & 2 & 17 & 9 \\ 0 & 91 & 15 & 26 \\ 87 & 3 & 0 & 81 \\ 0 & 0 & 11 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 2} \begin{pmatrix} 0 & 0 & 17 & 7 \\ 0 & 89 & 15 & 24 \\ 87 & 1 & 0 & 79 \\ 2 & 0 & 13 & 0 \end{pmatrix}$$

Solution:

R – E (47)
S – D (32)
T – F (43) cost 169
U – G (47)

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Exercise C, Question 3

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
The crosses '×' indicate that that worker cannot be assigned to that task.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, show the table at each stage and state your final solution and its cost.

	Task P	Task Q	Task R	Task S
Worker A	46	53	67	75
Worker B	48	×	61	78
Worker C	42	46	53	62
Worker D	39	50	×	73

Solution:

$$\begin{pmatrix} 46 & 53 & 67 & 75 \\ 48 & 150 & 61 & 78 \\ 42 & 46 & 53 & 62 \\ 39 & 50 & 150 & 73 \end{pmatrix} \text{reducing rows} \begin{pmatrix} 0 & 7 & 21 & 29 \\ 0 & 102 & 13 & 30 \\ 0 & 4 & 11 & 20 \\ 0 & 11 & 111 & 34 \end{pmatrix}$$

$$\text{reducing columns} \begin{pmatrix} 0 & 3 & 10 & 9 \\ 0 & 98 & 2 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 7 & 100 & 14 \end{pmatrix}$$

$$\text{Minimum uncovered element is 2} \begin{pmatrix} 0 & 1 & 8 & 7 \\ 0 & 96 & 0 & 8 \\ 2 & 0 & 0 & 0 \\ 0 & 5 & 98 & 12 \end{pmatrix}$$

$$\text{Minimum uncovered element is 1} \begin{pmatrix} 0 & 0 & 8 & 6 \\ 0 & 95 & 0 & 7 \\ 3 & 0 & 1 & 0 \\ 0 & 4 & 98 & 11 \end{pmatrix}$$

Solution: A – Q (53)
B – R (61)
C – S (62)
D – P (39)

cost 215

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 4

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
The crosses '×' indicate that that worker cannot be assigned to that task.
Reducing rows first, use the Hungarian algorithm to find an allocation that minimises the cost. You should make your method clear, show the table at each stage and state your final solution and its cost.

	Task R	Task S	Task T	Task U	Task V
Worker J	143	112	149	137	×
Worker K	149	106	153	115	267
Worker L	137	109	143	121	×
Worker M	157	×	×	134	290
Worker N	126	101	132	111	253

Solution:

$$\begin{pmatrix} 143 & 112 & 149 & 137 & 600 \\ 149 & 106 & 153 & 115 & 267 \\ 137 & 109 & 143 & 121 & 600 \\ 157 & 600 & 600 & 134 & 290 \\ 126 & 101 & 132 & 111 & 253 \end{pmatrix} \begin{matrix}\text{reducing} \\ \text{rows}\end{matrix} \begin{pmatrix} 31 & 0 & 37 & 25 & 488 \\ 43 & 0 & 47 & 9 & 161 \\ 28 & 0 & 34 & 12 & 491 \\ 23 & 466 & 466 & 0 & 156 \\ 25 & 0 & 31 & 10 & 152 \end{pmatrix}$$

$$\text{reducing columns} \begin{pmatrix} 8 & 0 & 6 & 25 & 336 \\ 20 & 0 & 16 & 9 & 9 \\ 5 & 0 & 3 & 12 & 339 \\ 0 & 466 & 435 & 0 & 4 \\ 2 & 0 & 0 & 10 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 3} \begin{pmatrix} 5 & 0 & 3 & 22 & 333 \\ 17 & 0 & 13 & 6 & 6 \\ 2 & 0 & 0 & 9 & 336 \\ 0 & 469 & 435 & 0 & 4 \\ 2 & 3 & 0 & 10 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 3} \begin{pmatrix} 2 & 0 & 0 & 19 & 330 \\ 14 & 0 & 10 & 3 & 3 \\ 2 & 3 & 0 & 9 & 336 \\ 0 & 472 & 435 & 0 & 4 \\ 2 & 6 & 0 & 10 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 2} \begin{pmatrix} 0 & 0 & 0 & 17 & 328 \\ 12 & 0 & 10 & 1 & 1 \\ 0 & 3 & 0 & 7 & 334 \\ 0 & 474 & 437 & 0 & 4 \\ 2 & 8 & 2 & 10 & 0 \end{pmatrix}$$

Solution:

J – R (143)
K – S (106)
L – T (143)
M – U (134)
N – V (253)

cost 779

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Edexcel AS and A Level Modular Mathematics

Exercise D, Question 1

Question:

The table shows the **profit**, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that maximises the profit. You should make your method clear, show the table at each stage and state your final solution and its profit.

	Task C	Task D	Task E
Worker L	37	15	12
Worker M	25	13	16
Worker N	32	41	35

Solution:

$$\begin{pmatrix} 37 & 15 & 12 \\ 25 & 13 & 16 \\ 32 & 41 & 35 \end{pmatrix} \text{ Subtracting all terms from 41 } \begin{pmatrix} 4 & 26 & 29 \\ 16 & 28 & 25 \\ 9 & 0 & 6 \end{pmatrix}$$

$$\text{reducing rows } \begin{pmatrix} 0 & 22 & 25 \\ 0 & 12 & 9 \\ 9 & 0 & 6 \end{pmatrix} \text{ reducing columns } \begin{pmatrix} 0 & 22 & 19 \\ 0 & 12 & 3 \\ 9 & 0 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 3 } \begin{pmatrix} 0 & 19 & 16 \\ 0 & 9 & 0 \\ 3 & 0 & 0 \end{pmatrix}$$

Solution $L-C(37)$
$M-E(16)$
$N-D(41)$

Profit 94

Solutionbank D2

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Exercise D, Question 2

Question:

The table shows the **profit**, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that maximises the profit. You should make your method clear, show the table at each stage and state your final solution and its profit.

	Task S	Task T	Task U	Task V
Worker C	36	34	32	35
Worker D	37	32	34	33
Worker E	42	35	37	36
Worker F	39	34	35	35

Solution:

$$\begin{pmatrix} 36 & 34 & 32 & 35 \\ 37 & 32 & 34 & 33 \\ 42 & 35 & 37 & 36 \\ 39 & 34 & 35 & 35 \end{pmatrix} \text{ Subtracting all terms from 42 } \begin{pmatrix} 6 & 8 & 10 & 7 \\ 5 & 10 & 8 & 9 \\ 0 & 7 & 5 & 6 \\ 3 & 8 & 7 & 7 \end{pmatrix}$$

$$\text{reducing rows } \begin{pmatrix} 0 & 2 & 4 & 1 \\ 0 & 5 & 3 & 4 \\ 0 & 7 & 5 & 6 \\ 0 & 5 & 4 & 4 \end{pmatrix} \text{ reducing columns } \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 3 \\ 0 & 5 & 2 & 5 \\ 0 & 3 & 1 & 3 \end{pmatrix}$$

$$\text{Minimum uncovered element is 3 } \begin{pmatrix} 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

There are two solutions

C – T (34)		C – V (35)	
D – U (34)	or	D – U (34)	Profit 145
E – S (42)		E – S (42)	
F – V (35)		F – T (34)	

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Exercise D, Question 3

Question:

The table shows the **profit**, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that maximises the profit. You should make your method clear, show the table at each stage and state your final solution and its profit.

	Task E	Task F	Task G	Task H
Worker R	20	22	14	24
Worker S	20	19	12	20
Worker T	13	10	18	16
Worker U	22	23	9	28

Solution:

$$\begin{pmatrix} 20 & 22 & 14 & 24 \\ 20 & 19 & 12 & 20 \\ 13 & 10 & 18 & 16 \\ 22 & 23 & 9 & 28 \end{pmatrix} \text{ Subtracting all terms from 28 } \begin{pmatrix} 8 & 6 & 14 & 4 \\ 8 & 9 & 16 & 8 \\ 15 & 18 & 10 & 12 \\ 6 & 5 & 19 & 0 \end{pmatrix}$$

$$\text{reducing rows } \begin{pmatrix} 4 & 2 & 10 & 0 \\ 0 & 1 & 8 & 0 \\ 5 & 8 & 0 & 2 \\ 6 & 5 & 19 & 0 \end{pmatrix} \text{ reducing columns } \begin{pmatrix} 4 & 1 & 10 & 0 \\ 0 & 0 & 8 & 0 \\ 5 & 7 & 0 & 2 \\ 6 & 4 & 19 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 1 } \begin{pmatrix} 3 & 0 & 9 & 0 \\ 0 & 0 & 8 & 1 \\ 5 & 7 & 0 & 3 \\ 5 & 3 & 18 & 0 \end{pmatrix}$$

Solution R – F (22)
S – E (20)
T – G (18)
U – H (28)

Profit 88

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Edexcel AS and A Level Modular Mathematics

Exercise D, Question 4

Question:

The table shows the **profit**, in pounds, of allocating workers to tasks.
Reducing rows first, use the Hungarian algorithm to find an allocation that maximises the profit. You should make your method clear, show the table at each stage and state your final solution and its profit.

	Task J	Task K	Task L	Task M	Task N
Worker A	85	95	86	87	97
Worker B	110	111	95	115	100
Worker C	90	95	86	93	105
Worker D	85	87	84	85	87
Worker E	100	100	105	120	95

Solution:

$$\begin{pmatrix} 85 & 95 & 86 & 87 & 97 \\ 110 & 111 & 95 & 115 & 100 \\ 90 & 95 & 86 & 93 & 105 \\ 85 & 87 & 84 & 85 & 87 \\ 100 & 100 & 105 & 120 & 95 \end{pmatrix} \text{ Subtracting all terms from 120 } \begin{pmatrix} 35 & 25 & 34 & 33 & 23 \\ 10 & 9 & 25 & 5 & 20 \\ 30 & 25 & 34 & 27 & 15 \\ 35 & 33 & 36 & 35 & 33 \\ 20 & 20 & 15 & 0 & 25 \end{pmatrix}$$

$$\text{reducing rows } \begin{pmatrix} 12 & 2 & 11 & 10 & 0 \\ 5 & 4 & 20 & 0 & 15 \\ 15 & 10 & 19 & 12 & 0 \\ 2 & 0 & 3 & 2 & 0 \\ 20 & 20 & 15 & 0 & 25 \end{pmatrix} \text{ reducing columns } \begin{pmatrix} 10 & 2 & 8 & 10 & 0 \\ 3 & 4 & 17 & 0 & 15 \\ 13 & 10 & 16 & 12 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 18 & 20 & 12 & 0 & 25 \end{pmatrix}$$

$$\text{Minimum uncovered element is 2 } \begin{pmatrix} 8 & 0 & 6 & 10 & 0 \\ 1 & 2 & 15 & 0 & 15 \\ 11 & 8 & 14 & 12 & 0 \\ 0 & 0 & 0 & 4 & 2 \\ 16 & 18 & 10 & 0 & 25 \end{pmatrix}$$

$$\text{Minimum uncovered element is 1 } \begin{pmatrix} 7 & 0 & 5 & 10 & 0 \\ 0 & 2 & 14 & 0 & 15 \\ 10 & 8 & 13 & 12 & 0 \\ 0 & 1 & 0 & 5 & 3 \\ 15 & 18 & 9 & 0 & 25 \end{pmatrix}$$

Solution A – K (95)
B – J (110)
C – N (105)
D – L (84)
E – M (120)

Profit 514

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Exercise E, Question 1

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
You wish to minimise the total cost.

Formulate this as a linear programming problem, defining your variables and making the objective and constraints clear.

	Task C	Task D	Task E
Worker L	37	15	12
Worker M	25	13	16
Worker N	32	41	35

Solution:

Let x_{ij} be 0 or 1

$$x_{ij} \begin{cases} 1 \text{ if worker, } i \text{ does task } j \\ 0 \text{ otherwise} \end{cases}$$

where $i \in \{\text{L}, \text{M}, \text{N}\}$ and $j \in \{\text{C}, \text{D}, \text{E}\}$

$$\text{Minimise } C = 37x_{\text{LC}} + 15x_{\text{LD}} + 12x_{\text{LE}} + 25x_{\text{MC}} + 13x_{\text{MD}} + 16x_{\text{ME}} + 32x_{\text{NC}} + 41x_{\text{ND}} + 35x_{\text{NE}}$$

Subject to:

$$\sum x_{\text{Lj}} = 1$$
$$\sum x_{\text{Mj}} = 1$$
$$\sum x_{\text{Nj}} = 1$$
$$\sum x_{\text{iC}} = 1$$
$$\sum x_{\text{iD}} = 1$$
$$\sum x_{\text{iE}} = 1$$

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Exercise E, Question 2

Question:

The table shows the cost, in pounds, of allocating workers to tasks.
You wish to minimise the total cost.

Formulate this as a linear programming problem, defining your variables and making the objective and constraints clear.

	Task S	Task T	Task U	Task V
Worker C	36	34	32	35
Worker D	37	32	34	33
Worker E	42	35	37	36
Worker F	39	34	35	35

Solution:

Let x_{ij} be 0 or 1

$$x_{ij}\begin{cases}1 \text{ if worker } i \text{ does task } j\\ 0 \text{ otherwise}\end{cases}$$

where $i \in \{C, D, E, F\}$ and $j \in \{S, T, U, V\}$.

$$\begin{aligned}\text{Minimise } C = {} & 36x_{CS} + 34x_{CT} + 32x_{CU} + 35x_{CV}\\ & +37x_{DS} + 32x_{DT} + 34x_{DU} + 33x_{DV}\\ & +42x_{ES} + 35x_{ET} + 37x_{EU} + 36x_{EV}\\ & +39x_{FS} + 34x_{FT} + 35x_{FU} + 35x_{FV}\end{aligned}$$

$$\begin{aligned}\text{Subject to: } & \sum x_{Cj} = 1\\ & \sum x_{Dj} = 1\\ & \sum x_{Ej} = 1\\ & \sum x_{Fj} = 1\\ & \sum x_{iS} = 1\\ & \sum x_{iT} = 1\\ & \sum x_{iU} = 1\\ & \sum x_{iV} = 1\end{aligned}$$

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Exercise E, Question 3

Question:

Repeat question 1, but take the entries to be the **profit** earned in allocating workers to tasks, and seek to maximise the total profit.

Solution:

Subtract all terms from 41 $\begin{pmatrix} 4 & 26 & 29 \\ 16 & 28 & 25 \\ 9 & 0 & 6 \end{pmatrix}$

Let x_{ij} be 0 or 1

$x_{ij} \begin{cases} 1 & \text{if worker; } i \text{ does task } j \\ 0 & \text{otherwise} \end{cases}$

where $i \in \{L, M, N\}$ and $j \in \{C, D, E\}$

Minimise $P = 4x_{LC} + 26x_{LD} + 29x_{LE}$
$+16x_{MC} + 28x_{MD} + 25x_{ME}$
$+9x_{NC} + 6x_{NE}$

Subject to:

$\sum x_{Lj} = 1$ $\sum x_{iC} = 1$
$\sum x_{Mj} = 1$ $\sum x_{iD} = 1$
$\sum x_{Nj} = 1$ $\sum x_{iE} = 1$

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Exercise E, Question 4

Question:

Repeat question 2, but take the entries to be the **profit** earned in allocating workers to tasks, and seek to maximise the total profit.

Solution:

Subtract all terms from 42

$$\begin{pmatrix} 6 & 8 & 10 & 7 \\ 5 & 10 & 8 & 9 \\ 0 & 7 & 5 & 6 \\ 3 & 8 & 7 & 7 \end{pmatrix}$$

Let x_{ij} be 0 or 1

$$x_{ij} \begin{cases} 1 & \text{if worker } i \text{ does task } j \\ 0 & \text{otherwise} \end{cases}$$

where $i \in \{C, D, E, F\}$ and $j \in \{S, T, U, V\}$

Minimise $P = 6x_{CS} + 8x_{CT} + 10x_{CU} + 7x_{CV}$

$5x_{DS} + 10x_{DT} + 8x_{DU} + 9x_{DV}$

$+7x_{ET} + 5x_{EU} + 6x_{EV}$

$3x_{FS} + 8x_{FT} + 7x_{FU} + 7x_{FV}$

Subject to:

$\sum x_{Cj} = 1 \qquad \sum x_{iS} = 1$

$\sum x_{Dj} = 1 \qquad \sum x_{iT} = 1$

$\sum x_{Ej} = 1 \qquad \sum x_{iU} = 1$

$\sum x_{Fj} = 1 \qquad \sum x_{iV} = 1$

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Exercise F, Question 1

Question:

	Airport	Depot	Docks	Station
Bring-it	322	326	326	328
Collect-it	318	325	324	325
Fetch-it	315	319	317	320
Haul-it	323	322	319	321

A museum is staging a special exhibition. They have been loaned exhibits from other museums and from private collectors. Seven days before the exhibition starts these exhibits will be arriving at the airport, road depot, docks and railway station and in each case the single load has to be transported to the museum. There are four local companies that could deliver the exhibits: Bring-it, Collect-it, Fetch-it and Haul-it. Since all four companies are helping to sponsor the exhibition, the museum wishes to use all four companies, allocating each company to just one arrival point.

The table shows the cost, in pounds, of using each company for each task. The museum wishes to minimise its transportation costs.

Reducing rows first, use the Hungarian algorithm to determine the allocation that minimises the total cost. You must make your method clear and show the table after each stage. State your final allocation and its cost.

Solution:

$$\begin{pmatrix} 322 & 326 & 326 & 328 \\ 318 & 325 & 324 & 325 \\ 315 & 319 & 317 & 320 \\ 323 & 322 & 319 & 321 \end{pmatrix} \text{reducing rows} \begin{pmatrix} 0 & 4 & 4 & 6 \\ 0 & 7 & 6 & 7 \\ 0 & 4 & 2 & 5 \\ 4 & 3 & 0 & 2 \end{pmatrix}$$

$$\text{reducing columns} \begin{pmatrix} 0 & 1 & 4 & 4 \\ 0 & 4 & 6 & 5 \\ 0 & 1 & 2 & 3 \\ 4 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 1} \begin{pmatrix} 0 & 0 & 3 & 3 \\ 0 & 3 & 5 & 4 \\ 0 & 0 & 1 & 2 \\ 5 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{Minimum uncovered element is 1} \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 6 & 1 & 0 & 0 \end{pmatrix}$$

Bring-it – Depot (326)
Collect-it – Airport (318)
Fetch-it – Docks (317)
Haul-it – Station (321)

Cost 1282

Solutionbank D2

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Exercise F, Question 2

Question:

	Back	Breast	Butterfly	Crawl
Jack	18	20	19	14
Kyle	19	21	19	14
Liam	17	20	20	16
Mike	20	21	20	15

A medley relay swimming team consists of four swimmers. The first member of the team swims one length of backstroke, then the second person swims a length of breaststroke, then the next a length of butterfly and finally the fourth person a length of crawl. Each member of the team must swim just one length. All the team members could swim any of the lengths, but some members of the team are faster at one or two particular strokes.

The table shows the time, in seconds, each member of the team took to swim each length using each type of stroke during the last training session

a Use the Hungarian algorithm, reducing rows first, to find an allocation that minimises the total time it takes the team to complete all four lengths.

b State the best time in which this team could complete the race.

c Show that there is more than one way of allocating the team so that they can achieve this best time.

In fact there are four optimal solutions to this problem.

Solution:

a

$$\begin{pmatrix} 18 & 20 & 19 & 14 \\ 19 & 21 & 19 & 14 \\ 17 & 20 & 20 & 16 \\ 20 & 21 & 20 & 15 \end{pmatrix} \xrightarrow[\text{rows}]{\text{reducing}} \begin{pmatrix} 4 & 6 & 5 & 0 \\ 5 & 7 & 5 & 0 \\ 1 & 4 & 4 & 0 \\ 5 & 6 & 5 & 0 \end{pmatrix}$$

$$\begin{matrix}\text{reducing} \\ \text{columns}\end{matrix} \begin{pmatrix} 3 & 2 & 1 & 0 \\ 4 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 2 & 1 & 0 \end{pmatrix}$$

Minimum uncovered element is 1 $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 \end{pmatrix}$

Minimum uncovered element is 1 $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 2 & 0 & 0 & 0 \end{pmatrix}$

b and **c**

There are 4 solutions each of duration 71 seconds

J – Br (20)	J – C (14)	J – Br (20)	J – Bu (19)
K – Bu (19)	K – Bu (19)	K – Cr (14)	K – Cr (14)
L – Ba (17)	L – Ba (17)	L – Ba (17)	L – Ba (17)
M – C (15)	M – Br (21)	M – Bu (20)	M – Br (21)

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Exercise F, Question 3

Question:

	Grand Hall	Dining Room	Gallery	Bedroom	Kitchen
Alf	8	19	11	14	12
Betty	12	17	14	18	20
Charlie	10	22	18	14	19
Donna	9	15	16	15	21
Eve	14	23	20	20	19

Five tour guides work at Primkal Mansion. They talk to groups of tourists about five particularly significant rooms. Each tour guide will be stationed in a particular room for the day, but may change rooms the next day. The tourists will listen to each talk before moving on to the next room. Once they have listened to all five talks they will head off to the gift shop.

The table shows the average length of each tour guide's talk in each room.

A tourist party arrives at the Mansion.

a Use the Hungarian algorithm, reducing rows first, to find the **quickest** time that the tour could take. You should state the optimal allocation and its length and show the state of the table at each stage.

b Adapt the table and re-apply the Hungarian algorithm, reducing rows first, to find the **longest** time that the tour could take. You should state the optimal allocation and its duration and show the state of the table at each stage.

Solution:

a

$$\begin{pmatrix} 8 & 19 & 11 & 14 & 12 \\ 12 & 17 & 14 & 18 & 20 \\ 10 & 22 & 18 & 14 & 19 \\ 9 & 15 & 16 & 15 & 21 \\ 14 & 23 & 20 & 20 & 19 \end{pmatrix} \text{ reducing rows } \begin{pmatrix} 0 & 11 & 3 & 6 & 4 \\ 0 & 5 & 2 & 5 & 8 \\ 0 & 12 & 8 & 7 & 9 \\ 0 & 6 & 7 & 6 & 12 \\ 0 & 9 & 6 & 6 & 5 \end{pmatrix}$$

$$\text{reducing columns } \begin{pmatrix} 0 & 6 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 7 & 6 & 0 & 5 \\ 0 & 1 & 5 & 2 & 8 \\ 0 & 4 & 4 & 2 & 1 \end{pmatrix}$$

$$\text{Minimum uncovered element is 1 } \begin{pmatrix} 0 & 5 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 & 5 \\ 0 & 6 & 5 & 0 & 5 \\ 0 & 0 & 4 & 2 & 8 \\ 0 & 3 & 3 & 2 & 1 \end{pmatrix}$$

Solution
Aif - Kitchen (12)
Betty - Gallery (14)
Charlie - Bedroom (14)
Donna - Dining room (15)
Eve - Grand Hall (14) Minimum time 69 minutes

b

$$\text{Subtracting all terms from 23 } \begin{pmatrix} 15 & 4 & 12 & 9 & 11 \\ 11 & 6 & 9 & 5 & 3 \\ 13 & 1 & 5 & 9 & 4 \\ 14 & 8 & 7 & 8 & 2 \\ 9 & 0 & 3 & 3 & 4 \end{pmatrix}$$

$$\text{reducing rows } \begin{pmatrix} 11 & 0 & 8 & 5 & 7 \\ 8 & 3 & 6 & 2 & 0 \\ 12 & 0 & 4 & 8 & 3 \\ 12 & 6 & 5 & 6 & 0 \\ 9 & 0 & 3 & 3 & 4 \end{pmatrix} \text{ reducing columns } \begin{pmatrix} 3 & 0 & 5 & 3 & 7 \\ 0 & 3 & 3 & 0 & 0 \\ 4 & 0 & 1 & 6 & 3 \\ 4 & 6 & 2 & 4 & 0 \\ 1 & 0 & 0 & 1 & 4 \end{pmatrix}$$

$$\text{Minimum uncovered element is 1 } \begin{pmatrix} 2 & 0 & 5 & 2 & 6 \\ 0 & 4 & 4 & 0 & 0 \\ 3 & 0 & 1 & 5 & 2 \\ 4 & 7 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

Minimum uncovered element is 1 $\begin{pmatrix} 1 & 0 & 4 & 1 & 6 \\ 0 & 5 & 4 & 0 & 1 \\ 2 & 0 & 0 & 4 & 2 \\ 3 & 7 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 & 4 \end{pmatrix}$

Solutions

Alf – Dining room (19)		Alf – Dining room (19)
Betty – Hall (12)		Betty – Bedroom (15)
Charlie – Gallery (18)	or	Charlie – Gallery (18)
Donna – Kitchen (21)		Donna – Kitchen (21)
Eve – Bedroom (20)		Eve – Hall (14)

Maximum time 90 minutes

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Exercise F, Question 4

Question:

	Award ceremony	Film premiere	Celebrity party
Denzel	245	378	459
Eun-Ling	250	387	467
Frank	224	350	442
Gabby	231	364	453

A company hires out chauffer-driven, luxury stretch-limousines. They have to provide cars for three events next Saturday night: an award ceremony, a film premiere and a celebrity party. The company has four chauffeurs available and the cost, in pounds, of assigning each of them to each event is shown in the table above. The company wishes to minimise its total costs.

a Explain why it is necessary to add a dummy event.

b Reducing rows first, use the Hungarian algorithm to determine the allocation that minimises the total cost. You should state the optimal allocation and its cost and show the state of the table at each stage.

Solution:

a There are 4 chauffeurs but only 3 tasks.

b
$$\begin{pmatrix} 245 & 378 & 459 & 0 \\ 250 & 387 & 467 & 0 \\ 224 & 350 & 442 & 0 \\ 231 & 364 & 453 & 0 \end{pmatrix} \begin{matrix} \text{reducing} \\ \text{columns} \end{matrix} \begin{pmatrix} 21 & 28 & 17 & 0 \\ 26 & 37 & 25 & 0 \\ 0 & 0 & 0 & 0 \\ 7 & 14 & 11 & 0 \end{pmatrix}$$

Minimum uncovered element is 7
$$\begin{pmatrix} 14 & 21 & 10 & 0 \\ 19 & 30 & 18 & 0 \\ 0 & 0 & 0 & 7 \\ 0 & 7 & 4 & 0 \end{pmatrix}$$

Minimum uncovered element is 10
$$\begin{pmatrix} 4 & 11 & 0 & 0 \\ 9 & 20 & 8 & 0 \\ 0 & 0 & 0 & 17 \\ 0 & 7 & 4 & 10 \end{pmatrix}$$

Solution:

D – Party (459)

E – Dummy

F – Film (350)

G – Award (231)

Cost £1040

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Exercise F, Question 5

Question:

	Catering	Cleaning	Computer	Copying	Post
Blue	No	863	636	628	739
Green	562	796	583	478	674
Orange	No	825	672	583	756
Red	635	881	650	538	No
Yellow	688	934	No	554	No

A large office block is to be serviced and supplied by five companies Blue supplies, Green services, Orange office supplies, Red Co and Yellow Ltd. These companies have each applied to take care of catering, cleaning, computer supplies/servicing, copying and postal services.

The table shows the daily cost of using each firm, in pounds.

For political reasons the owners of the office block will use all five companies, one for each of the five tasks.

Some of the companies cannot offer some services and this is indicated by 'No'.

Use the Hungarian algorithm, reducing rows first, to allocate the companies to the services in such a way as to minimise the total cost. You should state the optimal allocation and its cost and show the state of the table at each stage.

Solution:

$$\begin{pmatrix} 2000 & 863 & 636 & 628 & 739 \\ 562 & 796 & 583 & 478 & 674 \\ 2000 & 825 & 672 & 583 & 756 \\ 635 & 881 & 650 & 538 & 2000 \\ 688 & 934 & 2000 & 554 & 2000 \end{pmatrix} \begin{matrix} \text{reducing} \\ \text{rows} \end{matrix} \begin{pmatrix} 1372 & 235 & 8 & 0 & 111 \\ 84 & 318 & 105 & 0 & 196 \\ 1417 & 242 & 89 & 0 & 173 \\ 97 & 343 & 112 & 0 & 1462 \\ 134 & 380 & 1446 & 0 & 1446 \end{pmatrix}$$

$$\begin{matrix} \text{reducing} \\ \text{columns} \end{matrix} \begin{pmatrix} 1288 & 0 & 0 & 0 & 0 \\ 0 & 83 & 97 & 0 & 85 \\ 1333 & 7 & 81 & 0 & 62 \\ 13 & 108 & 104 & 0 & 1351 \\ 50 & 145 & 1433 & 0 & 1335 \end{pmatrix}$$

$$\text{Minimum uncovered element is 7} \begin{pmatrix} 1288 & 0 & 0 & 7 & 0 \\ 0 & 83 & 97 & 7 & 85 \\ 1326 & 0 & 74 & 0 & 55 \\ 6 & 101 & 97 & 0 & 1344 \\ 43 & 138 & 1426 & 0 & 1328 \end{pmatrix}$$

$$\text{Minimum uncovered element is 83} \begin{pmatrix} 1371 & 0 & 0 & 90 & 0 \\ 0 & 0 & 14 & 13 & 2 \\ 1409 & 0 & 74 & 83 & 55 \\ 6 & 18 & 14 & 0 & 1261 \\ 43 & 55 & 1343 & 0 & 1245 \end{pmatrix}$$

$$\text{Minimum uncovered element is 6} \begin{pmatrix} 1371 & 6 & 0 & 96 & 0 \\ 0 & 6 & 14 & 19 & 2 \\ 1403 & 0 & 68 & 83 & 49 \\ 0 & 18 & 8 & 0 & 1255 \\ 37 & 55 & 1337 & 0 & 1239 \end{pmatrix}$$

$$\text{Minimum uncovered element is 2} \begin{pmatrix} 1373 & 6 & 0 & 98 & 0 \\ 0 & 4 & 12 & 19 & 0 \\ 1405 & 0 & 68 & 85 & 49 \\ 0 & 16 & 6 & 0 & 1253 \\ 37 & 53 & 1335 & 0 & 1237 \end{pmatrix}$$

Blue – Computer (636)
Green – Post (674)
Orange – Cleaning (825)
Red – Catering (635)
Yellow – Copying (554)
Cost £3324

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Exercise F, Question 6

Question:

	Cafe	Coffee shop	Restaurant	Snack shop
Ghost train	834	365	580	648
Log flume	874	375	No	593
Roller coaster	743	289	No	665
Teddie's adventure	899	500	794	No

The owners of a theme park wish to provide a café, coffee shop, restaurant and snack shop at four sites: next to the ghost train, log flume, roller coaster and teddie's adventure. They employ a market researcher who estimates the daily profit of each type of catering at each site.

The market researcher also suggests that some types of catering are not suitable at some of the sites, these are indicated by 'No'.

Using the Hungarian algorithm, determine the allocation that provides the maximum daily profit.

This question is a maximising question and one with incomplete data. You need to chose numbers to put at the sites marked 'No' so that they become 'unattractive' to the algorithm after it has been altered to look for the maximum solution.

Solution:

$$\begin{pmatrix} 834 & 365 & 580 & 648 \\ 874 & 375 & 2000 & 593 \\ 743 & 289 & 2000 & 665 \\ 899 & 500 & 794 & 2000 \end{pmatrix} \begin{matrix} \text{Subtracting} \\ \text{all terms} \\ \text{from 1000} \end{matrix} \begin{pmatrix} 166 & 635 & 420 & 352 \\ 126 & 625 & 1000 & 407 \\ 257 & 711 & 1000 & 335 \\ 101 & 500 & 206 & 1000 \end{pmatrix}$$

$$\begin{matrix} \text{reducing} \\ \text{rows} \end{matrix} \begin{pmatrix} 0 & 469 & 254 & 186 \\ 0 & 499 & 874 & 281 \\ 0 & 454 & 743 & 78 \\ 0 & 399 & 105 & 899 \end{pmatrix} \begin{matrix} \text{reducing} \\ \text{columns} \end{matrix} \begin{pmatrix} 0 & 70 & 149 & 108 \\ 0 & 100 & 769 & 203 \\ 0 & 55 & 638 & 0 \\ 0 & 0 & 0 & 821 \end{pmatrix}$$

$$\text{Minimum uncovered element is 55} \begin{pmatrix} 0 & 15 & 94 & 108 \\ 0 & 45 & 714 & 203 \\ 0 & 0 & 583 & 0 \\ 55 & 0 & 0 & 876 \end{pmatrix}$$

$$\text{Minimum uncovered element is 15} \begin{pmatrix} 0 & 0 & 79 & 93 \\ 0 & 30 & 699 & 188 \\ 15 & 0 & 583 & 0 \\ 70 & 0 & 0 & 876 \end{pmatrix}$$

Ghost train – Coffee shop (365)
Log flume – Cafe (874)
Roller coaster – Snack shop (665)
Teddie's adventure – Restaurant (794)
Profit 2698

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Exercise F, Question 7

Question:

	1	2	3	4
P	143	243	247	475
Q	132	238	218	437
R	126	207	197	408
S	138	222	238	445

Four workers P, Q, R and S are to be assigned to four tasks 1, 2, 3 and 4. Each worker is to be assigned to one task and each task must be assigned to one worker. The cost, in pounds, of using each worker for each task is given in the table above. The cost is to be minimised.

Formulate this as a linear programming problem, defining your variables and making the objective and constraints clear.

Solution:

Let x_{ij} be 0 or 1

$$x_{ij} \begin{cases} 1 \text{ if worker } i \text{ does task } j \\ 0 \text{ otherwise} \end{cases}$$

where $i \in \{P, Q, R, S\}$ and $j \in \{1, 2, 3, 4\}$

Minimise $C = 143x_{P1} + 243x_{P2} + 247x_{P3} + 475x_{P4}$
$+132x_{Q1} + 238x_{Q2} + 218x_{Q3} + 437x_{Q4}$
$+126x_{R1} + 207x_{R2} + 197x_{R3} + 408x_{R4}$
$+138x_{S1} + 222x_{S2} + 238x_{S3} + 445x_{S4}$

Subject to: $\sum x_{Pj} = 1$
$\sum x_{Pj} = 1$
$\sum x_{Qj} = 1$
$\sum x_{Rj} = 1$
$\sum x_{Sj} = 1$
$\sum x_{i1} = 1$
$\sum x_{i2} = 1$
$\sum x_{i3} = 1$
$\sum x_{i4} = 1$

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Exercise F, Question 8

Question:

	A	B	C	D
P	13	17	15	18
Q	15	19	12	19
R	16	20	13	22
S	14	15	17	24

Krunchy Cereals Ltd will send four salesman P, Q, R and S to visit four store managers at A, B, C and D to take orders for their new products. Each salesman will visit only one store manager and each store manager will be visited by just one salesman. The expected value, in thousands of pounds, of the orders won is shown in the table above. The company wishes to maximise the value of the orders.

Formulate this as a linear programming problem, defining your variables and making the objective and constraints clear.

Solution:

$$\begin{pmatrix} 13 & 17 & 15 & 18 \\ 15 & 19 & 12 & 19 \\ 16 & 20 & 13 & 22 \\ 14 & 15 & 17 & 24 \end{pmatrix} \text{ Subtracting all terms from 24 } \begin{pmatrix} 11 & 7 & 9 & 6 \\ 9 & 5 & 12 & 5 \\ 8 & 4 & 11 & 2 \\ 10 & 9 & 7 & 0 \end{pmatrix}$$

Let x_{ij} be 0 or 1

$$x_{ij} \begin{cases} 1 \text{ if worker } i \text{ does task } j \\ 0 \text{ otherwise} \end{cases}$$

where $i \in \{P, Q, R, S\}$ and $j \in \{A, B, C, D\}$

$$\begin{aligned} \text{Minimise } P = {} & 11x_{PA} + 7x_{PB} + 9x_{PC} + 6x_{PD} \\ & + 9x_{QA} + 5x_{QB} + 12x_{QC} + 5x_{QD} \\ & + 8x_{RA} + 4x_{RB} + 11x_{RC} + 2x_{RD} \\ & + 10x_{SA} + 9x_{SB} + 7x_{SC} \end{aligned}$$

$$\begin{aligned} \text{Subject to: } & \sum x_{Pj} = 1 \\ & \sum x_{Qj} = 1 \\ & \sum x_{Rj} = 1 \\ & \sum x_{Sj} = 1 \\ & \sum x_{iA} = 1 \\ & \sum x_{iB} = 1 \\ & \sum x_{iC} = 1 \\ & \sum x_{iD} = 1 \end{aligned}$$