Solutionbank D1
Edexcel AS and A Level Modular Mathematics

1 Review Exercise
Exercise A, Question 1

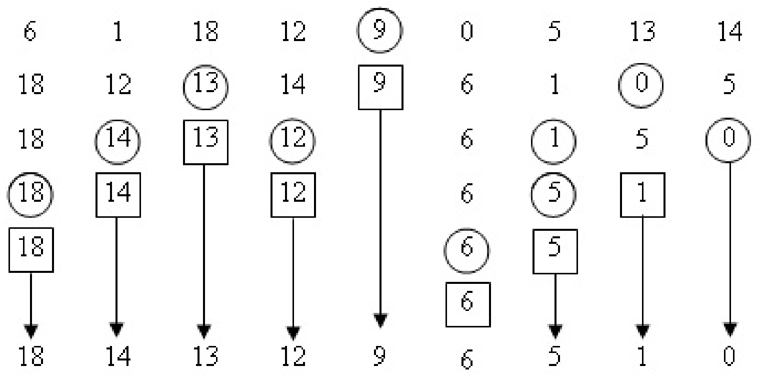
Question:

The table opposite shows the points obtained by each of the teams in a football league after they had each played 6 games. The teams are listed in alphabetical order. Carry out a quick sort to produce a list of teams in descending order of points obtained. ***E***

Ashford	6
Colnbrook	1
Datchet	18
Feltham	12
Halliford	9
Laleham	0
Poyle	5
Staines	13
Wraysbury	14

Solution:

For example,



Datchet (18), Wraysbury (14), Staines (13) Feltham (12), Halliford (9) Ashford (6), Poyle (5), Colnbrooke (1), Laleham (0)

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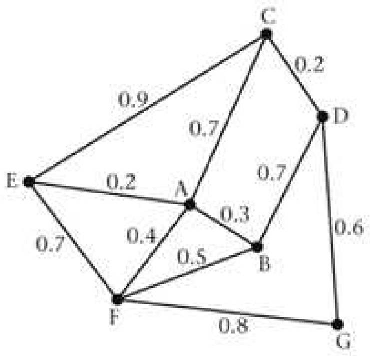
1 Review Exercise
Exercise A, Question 2

Question:



A local council is responsible for maintaining pavements in a district. The roads for which it is responsible are represented by arcs in the diagram. The road junctions are labelled A, B, C, ..., G. The number on each arc represents the length of that road in km.
The council has received a number of complaints about the condition of the pavements. In order to inspect the pavements, a council employee needs to walk along each road twice (once on each side of the road) starting and ending at the council offices at C. The length of the route is to be minimal. Ignore the widths of the roads.

a Explain how this situation differs from the standard route inspection problem.

b Find a route of minimum length and state its length. ***E***

Solution:

a All arcs are to be traversed twice, this is, in effect, repeating each arc. So all valencies are even.

b E.g. A–B–D–G–F–G–D–C–E–A–E–C–A–F–E–F–B–F–A–B–D–C–A
(all correct routes will have 23 letters in their name)
length $= 2 \times 6 = 12$ km

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1 Review Exercise
Exercise A, Question 3

Question:

a Use the binary search algorithm to try to locate the name *SABINE* in the following alphabetical list. Explain each step of the algorithm.

1	*ABLE*
2	*BROWN*
3	*COOKE*
4	*DANIEL*
5	*DOUBLE*
6	*FEW*
7	*OSBORNE*
8	*PAUL*
9	*SWIFT*
10	*TURNER*

b Find the maximum number of iterations of the binary search algorithm needed to locate a name in a list of 1000 names. ***E***

Solution:

a 1st pivot is $\left[\frac{1+10}{2}\right] \rightarrow 6$ FEW, SABINE after FEW

Rejecting: 1–6 list is now

7 OSBORNE
8 PAUL
9 SWIFT
10 TURNER

2nd pivot is $\left[\frac{7+10}{2}\right] \rightarrow 9$ SWIFT, SABINE before SWIFT.

Rejecting 9 and 10 list is now

7 OSBORNE
8 PAUL

3rd Pivot is $\left[\frac{7+8}{2}\right] \rightarrow 8$ PAUL, SABINE is after PAUL. Rejecting 7 and 8 list is now empty

So SABINE is not in the list

b Either:

maximum length of list remaining is
1000, 500, 250, 125, 63, 31, 15, 7, 3, 1
so 10 iterations.

or

we seek the smallest integer value of n such that

$$2^n > 1000$$
$$\log 2^n > \log 1000$$
$$n \log 2 > \log 1000$$
$$n > \frac{\log 1000}{\log 2}$$
$$n > 9.966$$
$$\therefore n \text{ is } 10$$

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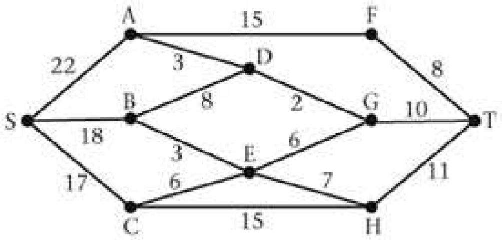
1 Review Exercise
Exercise A, Question 4

Question:



The diagram shows a network of roads.
The number on each edge gives the time, in minutes, to travel along that road.
Avinash wishes to travel from S to T as quickly as possible.

a Use Dijkstra's algorithm to find the shortest time to travel from S to T.

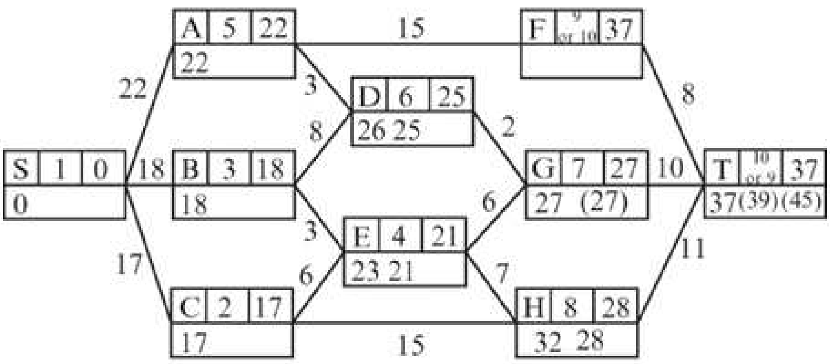
b Find a route for Avinash to travel from S to T in the shortest time. State, with a reason, whether this route is a unique solution.

On a particular day Avinash must include C in his route.

c Find a route of minimal time from S to T that includes C, and state its time. ***E***

Solution:

a



Shortest time: 37 minutes

b either S − A − D − G − T or S − B − E − G − T
The route is not unique; there are two of them (S − A − D − G − T and S − B − E − G − T)

c S − C − E − G − T length 39 minutes.

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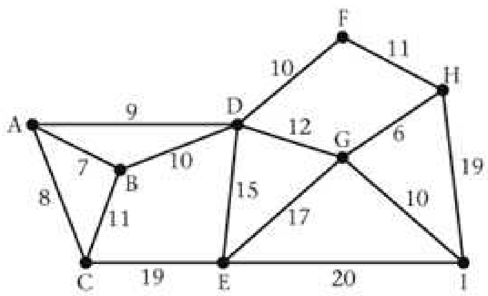
1 Review Exercise
Exercise A, Question 5

Question:

a State briefly
 i Prim's algorithm,
 ii Kruskal's algorithm.

b Find a minimum spanning tree for the network below using
 i Prim's algorithm, starting with vertex G,
 ii Kruskal's algorithm.

In each case write down the order in which you made your selection of arcs.



c State the weight of a minimum spanning tree.

d State, giving a reason for your answer, which algorithm is preferable for a large network. ***E***

Solution:

a **i** Prim
- Select any vertex to start the tree.
- Select the shortest arc that joins a vertex already in the tree to a vertex not yet in the tree. Add it to the tree.
- Continue selecting shortest arcs until all vertices are in the tree.

ii Kruskal
- Sort all arcs into ascending order of weight.
- Select the arc at least weight to start the tree.
- Take arcs in order of weight.
  - Reject the arc if it would form a cycle.
  - Add it to the tree if it does not form a cycle.
- Continue taking each arc in turn until all vertices are connected.

b **i** GH, GI, HF, FD, DA, AB, AC, DE

ii GH, AB, AC, AD, reject BD, DF, GI, reject BC, FH, reject DG, DE, reject EG, reject HI, reject CE.

c weight is 76

d Prim's algorithm
- It is easily converted into matrix form.
- It is difficult to use Kruskal's algorithm because it is difficult to check for cycles in a large network.

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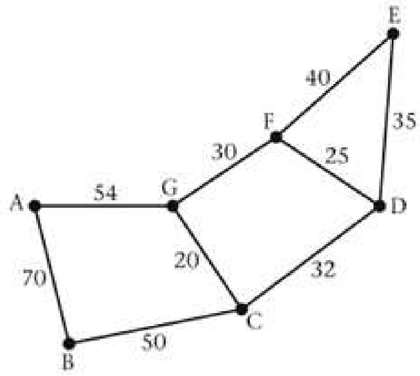
1 Review Exercise
Exercise A, Question 6

Question:



The diagram shows 7 locations A, B, C, D, E, F and G which are to be connected by pipelines. The arcs show the possible routes. The number on each arc gives the cost, in thousands of pounds, of laying that particular section.

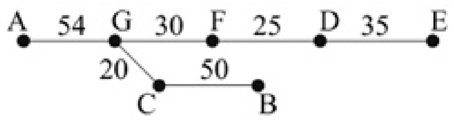
a Use Kruskal's algorithm to obtain a minimum spanning tree for the network, giving the order in which you selected the arcs.

b Draw your minimum spanning tree and find the least cost of pipelines. ***E***

Solution:

a GC, FD, GF, reject CD, ED, reject EF, BC, AG, reject AB.

b



$$\text{cost} = (20+25+30+35+50+54)\times 1000 = £214\,000$$

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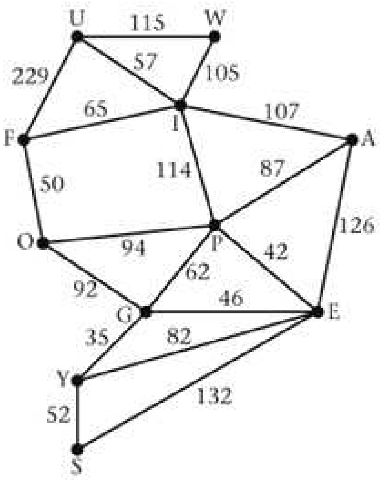
1 Review Exercise
Exercise A, Question 7

Question:



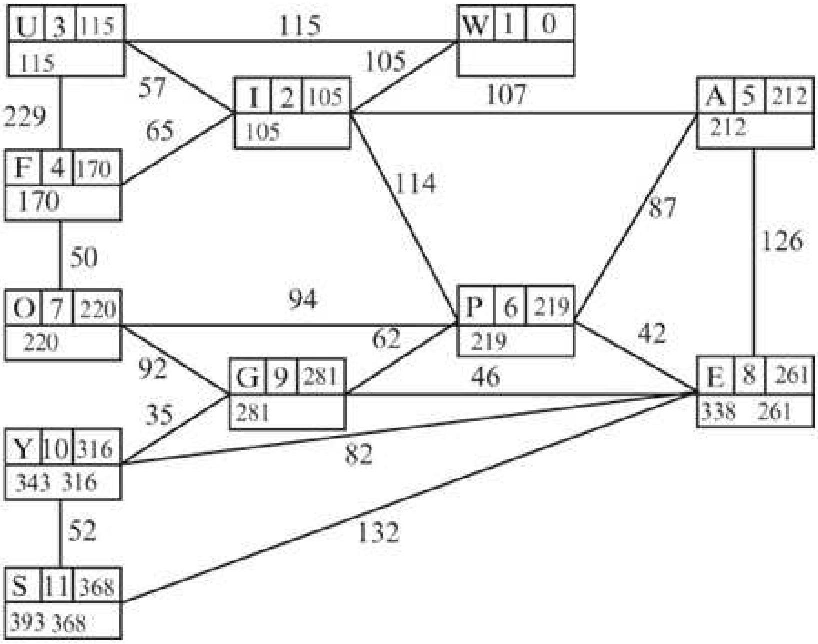
The network above represents the distances, in miles between eleven places A, E, F, G, I, O, P, S, U, W and Y.

a Use Dijkstra's algorithm to find the shortest route from W to S. State clearly
  i the order in which you labelled the vertices,
  ii how you determined the shortest route from your labelling,
  iii the places on the shortest route,
  iv the shortest distance.

b Explain how part **a** could have been completed so that the distance from A to S could also have been obtained without further calculation. (You are not required to find this distance.) ***E***

Solution:

a **i**



ii

$368-52 = 316$ SY

$316-35 = 281$ GS

$281-62 = 219$ PG

$219-114 = 105$ IP

$105-105 = 0$ WI

iii Shortest route is W–I–P–G–Y–S

iv length 368 miles

b If we started at S, the algorithm would find the least distance from S to each vertex, so in finding S to W we could have found S to A.

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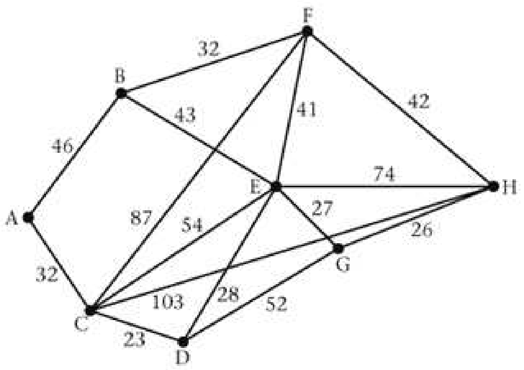
1 Review Exercise
Exercise A, Question 8

Question:

The network shows the possible routes between cities A, B, C, D, E, F, G and H.



The number on each arc gives the cost, in pounds, of taking that part of the route.
Use Dijkstra's algorithm to determine the cheapest route from A to H and its cost.

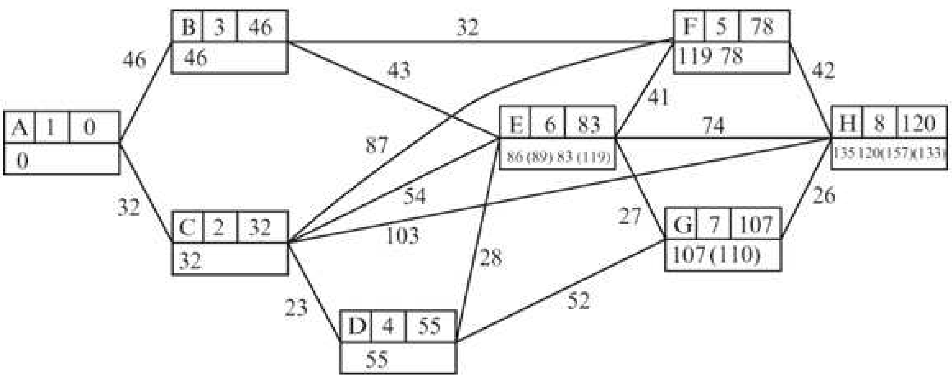
Your solution must indicate clearly how you have applied the algorithm.
State clearly

a the order in which the vertices are labelled,

b how you used your labelled diagram to decide on the cheapest route. *E*

Solution:

a



b $120-42=78$ FH
$78-32=46$ BF
$46-46=0$ AB
route is A – B – F – H cost £120

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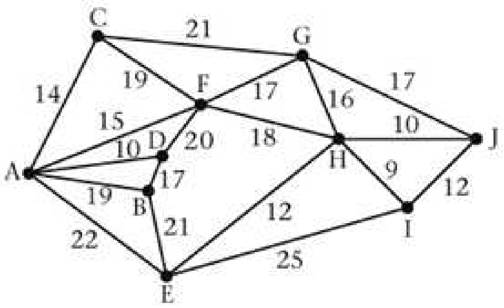
1 Review Exercise
Exercise A, Question 9

Question:



The network above models the roads linking ten towns A, B, C, D, E, F, G, H, I and J. The number on each arc is the journey time in minutes, along the road.

Alice lives in town A and works in town J.

a Use Dijkstra's algorithm to find the quickest route for Alice to travel to work each morning.

State clearly

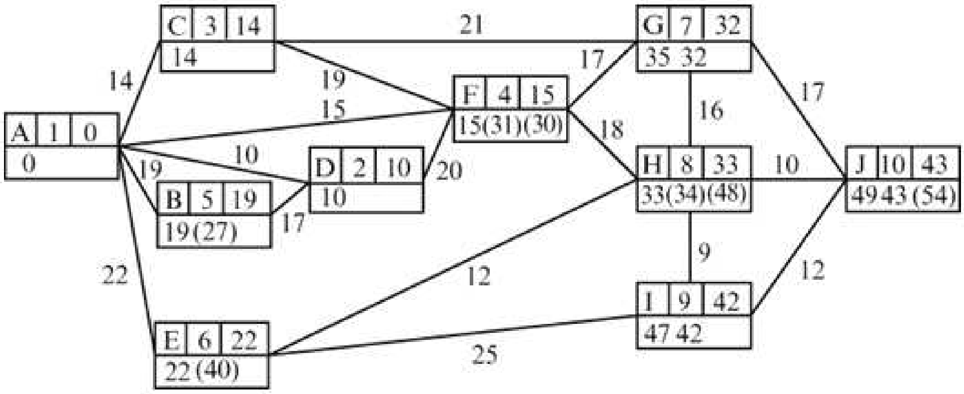
i the order in which all the vertices were labelled,

ii how you determined the quickest route from your labelling.

b On her return journey from work one day Alice wishes to call in at the supermarket located in town C.
Explain briefly how you would find the quickest route in this case. ***E***

Solution:

a **i**



ii $43-10=33$ HJ

$33-18=15$ FH

$15-15=0$ AF

route is A – F – H – J length 43 minutes

b Use the algorithm starting at J to find the shortest route from J to C, then add arc CA.

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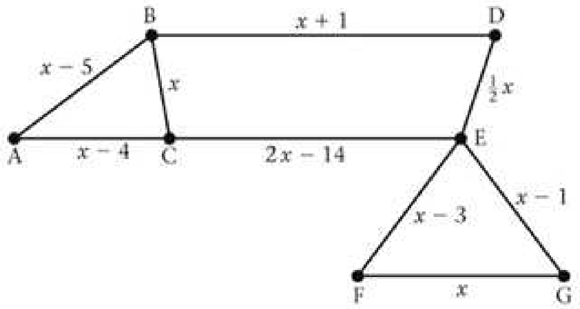
1 Review Exercise
Exercise A, Question 10

Question:

a Explain why it is impossible to draw a network with exactly three odd vertices.



The route inspection problem is solved for the network above and the length of the route is found to be 100.

b Determine the value of x, showing your working clearly. ***E***

Solution:

a Each edge contributes 1 to the order of the vertices at each end. So each edge contributes 2 to the total sum of the orders.
The sum of the orders is therefore an even number.
If there were an odd number of vertices with odd order the sum would be odd. So there must always be an even (or zero) number of vertices of odd order.

b The only vertices of odd order are B and C, we have to repeat the shortest path between B and C.

If $x \geq 9$ the shortest path is BC (direct).

Weight of network $+\text{BC} = 100$

$$(9\frac{1}{2}x - 26) + x = 100 \Rightarrow x = 12$$

If $x < 9$ the shortest path is BAC of length $2x - 9$
$(9\frac{1}{2}x - 26) + 2x - 9 = 100 \Rightarrow x = 11\frac{17}{23} \not< 9$ so inconsistent.

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1 Review Exercise
Exercise A, Question 11

Question:

a

	A	B	C	D	E	F
A	–	10	12	13	20	9
B	10	–	7	15	11	7
C	12	7	–	11	18	3
D	13	15	11	–	27	8
E	20	11	18	27	–	18
F	9	7	3	8	18	–

The table shows the distances, in metres, between six nodes A, B, C, D, E and F of a network.

i Use Prim's algorithm, starting at A, to solve the minimum connector problem for this table of distances. Explain your method and indicate the order in which you selected the edges.
ii Draw your minimum spanning tree and find its total length.
iii State whether your minimum spanning tree is unique. Justify your answer.

b A connected network N has seven vertices.
i State the number of edges in a minimum spanning tree for N.

A minimum spanning tree for a connected network has n edges.
ii State the number of vertices in the network. ***E***

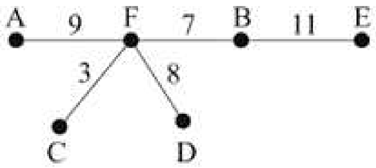
Solution:

a **i** Method:

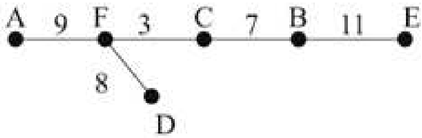
- Start at A and use this to start the tree.
- Choose the shortest edge that connects a vertex already in the tree to a vertex not yet in the tree. Add it to the tree.
- Continue adding edges until all vertices are in the tree.

$$\text{AF, FC}\begin{Bmatrix}\text{FB}\\ \text{or}\\ \text{BC}\end{Bmatrix}\text{, FD, EB}$$

ii



or



iii The tree is not unique, there are 2 of them (see above).

b **i** number of edges $= 7 - 1 = 6$

ii number of vertices $= n + 1$

In a tree the number of edges is always one less than the number of nodes.

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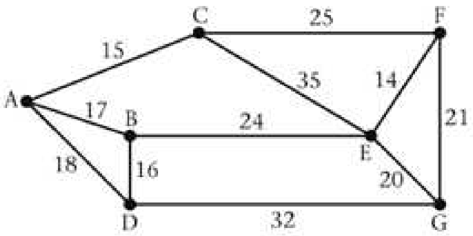
1 Review Exercise
Exercise A, Question 12

Question:

a Describe the differences between Prim's algorithm and Kruskal's algorithm for finding a minimum connector of a network.



b Listing the arcs in the order that you select them, find a minimum connector for the network above, using

i Prim's algorithm,
ii Kruskal's algorithm. ***E***

Solution:

a For example

- In Prim the tree ‘grows’ in a connected fashion.
- There is no need to check for cycles when using Prim.
- Prim can be adapted to matrix form.
- Prim starts with a vertex, Kruskal with an edge.
- Kruskal must start with the least edge, Prim can start with any verte

b **i** For example AC, AB, BD, EB, EF, EG

ii EF, AC, BD, AB, reject AD, EG, reject FG, BE

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1 Review Exercise
Exercise A, Question 13

Question:

45 56 37 79 46 18 90 81 51

a Using the quick sort algorithm, perform one complete iteration towards sorting these numbers into ascending order.

b Using the bubble sort algorithm, perform one complete pass towards sorting the original list into descending order.

Another list of numbers, in ascending order, is

7 23 31 37 41 44 50 62 71 73 94

c Use the binary search algorithm to locate the number 73 in this list. ***E***

Solution:

a For example, 45 37 18 $\boxed{46}$ 56 79 90 81 51

b For example, 56 45 79 46 37 90 81 51 18

c 1st pivot $\left[\frac{1+11}{2}\right] \to$ 6th number (44) 73 > 44 so reject 1st to 6th numbers

2nd pivot $\left[\frac{7+11}{2}\right] \to$ 9th number (71) 73 > 71 so reject 7th to 9th numbers

3rd pivot $\left[\frac{10+11}{2}\right] \to$ 11th number (94) 73 < 94 so reject 11th number

4th pivot 10th number (73) item found

73 was found as the 10th number in the list.

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1 Review Exercise
Exercise A, Question 14

Question:

The following list gives the names of some students who have represented Britain in the International Mathematics Olympiad.

Roper (R), Palmer (P), Boase (B), Young (Y), Thomas (T), Kenney (K), Morris (M), Halliwell (H), Wicker (W), Garesalingam (G).

a Use the quick sort algorithm to sort the names above into alphabetical order.

b Use the binary search algorithm to locate the name Kenney. ***E***

Solution:

a For example,

R	P	B	Y	T	K	M	H	W	G
B	H	G	$\boxed{K}$	R	P	Y	T	M	W
B	G	$\boxed{H}$		R	P	M	$\boxed{T}$	Y	W
B	$\boxed{G}$			M	$\boxed{P}$	R		$\boxed{W}$	Y
B	G	H	K	M	P	R	T	W	Y

list is in order

b $\left[\frac{10+1}{2}\right] \rightarrow 6$ Palmer Kenney $<$ Palmer reject 6 to 10

list is now

1 Boase
2 Garesalingham
3 Halliwell
4 Kenney
5 Morris

2nd pivot $\left[\frac{1+5}{2}\right] \rightarrow 3.$ Halliwell Kenney $>$ Halliwell reject 1 to 3

list is now
4 Kenney
5 Morris

3rd pivot $\left[\frac{4+5}{2}\right] \rightarrow 5.$ Morris Kenney $<$ Morris reject 5

list is now
4 Kenney
item found

Kenney was found as the 4th name in the list.

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1 Review Exercise
Exercise A, Question 15

Question:

1	Glasgow
2	Newcastle
3	Manchester
4	York
5	Leicester
6	Birmingham
7	Cardiff
8	Exeter
9	Southampton
10	Plymouth

A binary search is to be performed on the names in the list above to locate the name Newcastle.

a Explain why a binary search cannot be performed with the list in its present form.

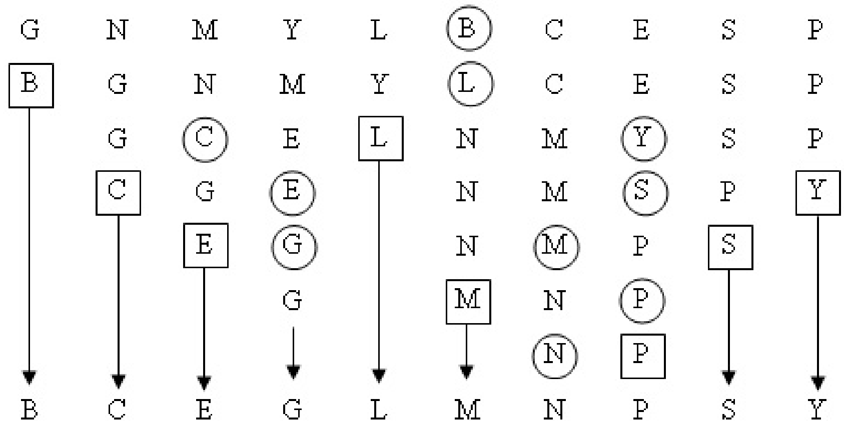
b Using an appropriate algorithm, alter the list so that a binary search can be performed. State the name of the algorithm you use.

c Use the binary search algorithm on your new list to locate the name Newcastle. ***E***

Solution:

a The list is not in alphabetical order.

b For example, quick sort.



c 1 Birmingham
2 Cardiff
3 Exeter
4 Glasgow
5 Leicester
6 Manchester
7 Newcastle
8 Plymouth
9 Southampton
10 York

$\left[\frac{1+10}{2}\right] \rightarrow 6$ Manchester Newcastle > Manchester so reject 1 to 6

list is now:
7 Newcastle
8 Plymouth
9 Southampton
10 York

$\left[\frac{7+10}{2}\right] \rightarrow 9$ Southampton Newcastle < Southampton so reject 9 to 10

list is now
7 Newcastle
8 Plymouth

$\left[\frac{7+8}{2}\right] \rightarrow 8$ Plymouth Newcastle < Plymouth so reject 8

list is now
7 Newcastle
Final term is 7, Newcastle $\therefore$ name found at 7.

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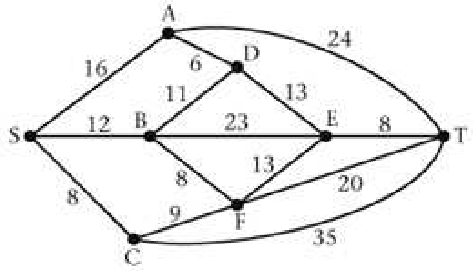
1 Review Exercise
Exercise A, Question 16

Question:



The weighted network shown above models the area in which Bill lives. Each vertex represents a town. The edges represent the roads between the towns. The weights are the lengths, in km, of the roads.

a Use Dijkstra's algorithm to find the shortest route from Bill's home at S to T. Complete all the boxes on the answer sheet and explain clearly how you determined the path of least weight from your labelling.

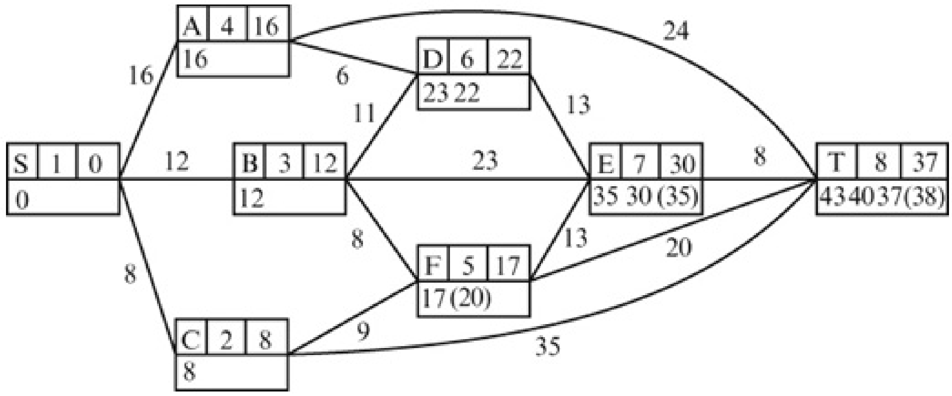
Bill decides that on the way to T he must visit a shop in town E.

b Obtain his shortest route now, giving its length and explaining your method clearly.

E

Solution:

a



For example

$37 - 20 = 17$ FT

$17 - 9 = 8$ CF

$8 - 8 = 0$ SC

shortest path is $S - C - F - T$ length 37

b Need shortest path S to $E + ET$

So $S - C - F - E - T$ length 38

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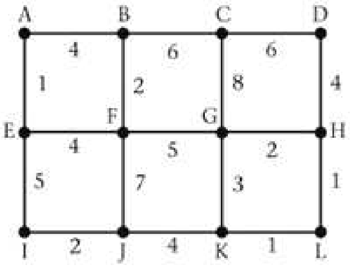
1 Review Exercise
Exercise A, Question 17

Question:



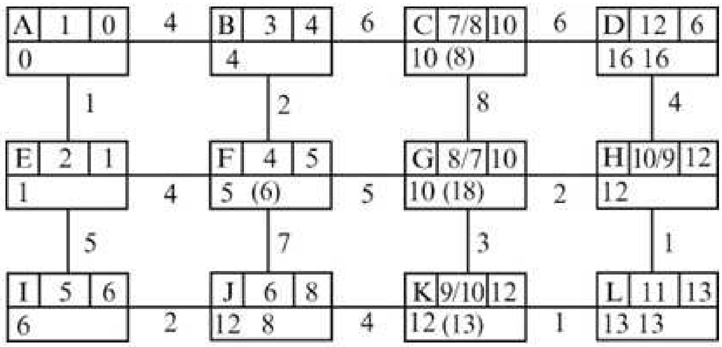
The diagram shows a network of roads. Erica wishes to travel from A to L as quickly as possible. The number on each edge gives the time, in minutes, to travel along the road.

a Use Dijkstra's algorithm to find the quickest route from A to L. Complete all the boxes on the answer sheet and explain clearly how you determined the quickest route from your labelling.

b Show that there is another route which also takes the minimum time. ***E***

Solution:

a



For example

$13-1=12$	HL	or	$13-1=12$	KL	
$12-2=10$	GH		$12-4=8$	JK	
$10-5=5$	FG		$8-2=6$	IJ	
$5-4=1$	EF		$6-5=1$	EI	
$1-1=0$	AE		$1-1=0$	AE	

Shortest path is $\left\{\begin{array}{l}A-E-F-G-H-L\\A-E-I-J-K-L\end{array}\right\}$ length 13

b State the other path given above in part **a**.

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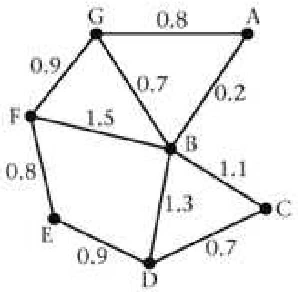
1 Review Exercise
Exercise A, Question 18

Question:



An engineer needs to check the state of a number of roads to see whether they need resurfacing. The roads that need to be checked are represented by the arcs in the diagram. The number on each arc represents the length of that road in km. To check all the roads, he needs to travel along each road at least once. He wishes to minimise the total distance travelled.

The engineer's office is at G, so he starts and ends his journey at G.

a Use an appropriate algorithm to find a route for the engineer to follow. State your route and its length.

The engineer lives at D. He believes he can reduce the distance travelled by starting from home and inspecting all the roads on the way to his office at G.

b State whether the engineer is correct in his belief. If so, calculate how much shorter his new route is. If not, explain why not. ***E***

Solution:

a $BD + FG = 1.3 + 0.9 = 2.2$ ← The odd valencies are at B, D, F, G.

$BF + DG = 1.5 + 2.0 = 3.5$

$BG + DF = 0.7 + 1.7 = 2.4$

Repeat BD and FG

Route for example $G - F - E - D - B - C - D - B - F - G - A - B - G$
(13 letters in route)
length $8.9 + 2.2 = 11.1$ km

b It would now only be necessary to repeat BF of length $1.5 < 2.2$
length $= 8.9 + 1.5 = 10.4$ km, saving 0.7 km.

Solutionbank D1
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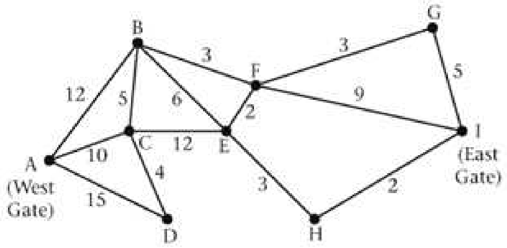
1 Review Exercise
Exercise A, Question 19

Question:



The diagram shows the network of paths in a country park. The number on each path gives its length in km. The vertices A and I represent the two gates in the park and the vertices B, C, D, E, F, G and H represent places of interest.

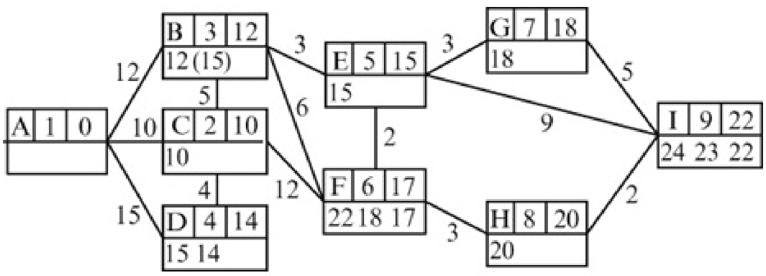
a Use Dijkstra's algorithm to find the shortest route from A to I. Show all necessary working in the boxes on the answer sheet and state your shortest route and its length.

The park warden wishes to inspect each of the paths to check for frost damage. She has to cycle along each path at least once, starting and finishing at A.

b i Use an appropriate algorithm to find which paths will be covered twice and state these paths.
ii Find a route of minimum length.
iii Find the total length of this shortest route. ***E***

Solution:

a



Shortest route is A − B − E − F − H − I, length 22 km

b i Odd vertices are A and I (only), so we need to repeat the shortest route from A to I. This was found in **a**.
so repeat AB, BE, EF, FH, HI.

ii For example A − B − C − A − D − C − E − H − I − H − E − F − I − G − F − E − B − F − B − A (20 letters in route)

iii 91 + 22 = 113 km

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1 Review Exercise
Exercise A, Question 20

Question:

90 50 55 40 20 35 30 25 45

a Use the bubble sort algorithm to sort the list of numbers above into descending order showing the rearranged order after each pass.

Jessica wants to record a number of television programmes onto video tapes. Each tape is 2 hours long. The lengths, in minutes, of the programmes she wishes to record are:

55 45 20 30 30 40 20 90 25 50 35 and 35

b Find the total length of programmes to be recorded and hence determine a lower bound for the number of tapes required.

c Use the first fit decreasing algorithm to fit the programmes onto her 2-hour tapes.

Jessica's friend Amy says she can fit all the programmes onto 4 tapes.

d Show how this is possible. ***E***

Solution:

a For example bubbling left to right

90	50	55	40	20	35	30	25	45
90	55	50	40	35	30	25	45	20
90	55	50	40	35	30	45	25	20
90	55	50	40	35	45	30	25	20
90	55	50	40	45	35	30	25	20
90	55	50	45	40	35	30	25	20

No more changes, list sorted.

b $\frac{475}{120} = 3.96$ so lower bound is 4 tapes

c Bin 1: $90 + 30$
Bin 2: $55 + 50$
Bin 3: $45 + 40 + 35$
Bin 4: $35 + 30 + 25 + 20$
Bin 5: 20

d For example
Bin 1: $90 + 30$
Bin 2: $55 + 35 + 30$
Bin 3: $45 + 40 + 35$
Bin 4: $50 + 25 + 20 + 20$

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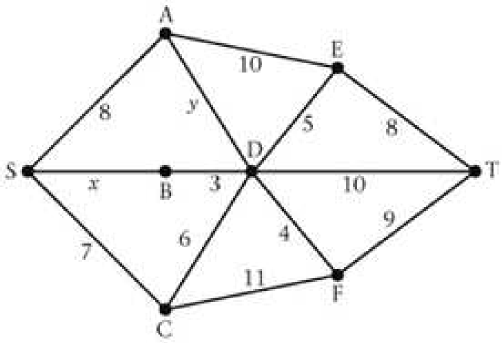
1 Review Exercise
Exercise A, Question 21

Question:



A weighted network is shown above.

Given that the shortest path from S to T is 17 and that $x \geq 0, y \geq 0$:

a **i** explain why A and C cannot lie on the shortest path,
ii find the value of x.

b Given that $x = 12$ and $y \geq 0$, find the possible range of values for the length of the shortest path.

c Give an example of a practical problem that could be solved by drawing a network and finding the shortest path through it. ***E***

Solution:

a **i** Shortest path through A is $18 + y$ or 26 both of which are greater than 17. Shortest path through C is 23, which is greater than 17. So shortest path cannot go through A or C.

ii Shortest path must go through B.

$$S - B - D - T = 13 + x$$
$$13 + x = 17$$
$$x = 4$$

b If $y = 0$ shortest path is $S - A - D - T = 18$
If $y = 5$ shortest path is $S - C - D - T = 23$
so range is 18 to 23.

c For example, a person seeking the quickest route from home to work through a city. The arcs are the roads that may be chosen, the number the time, in minutes, to journey along that road. The nodes represent junctions.

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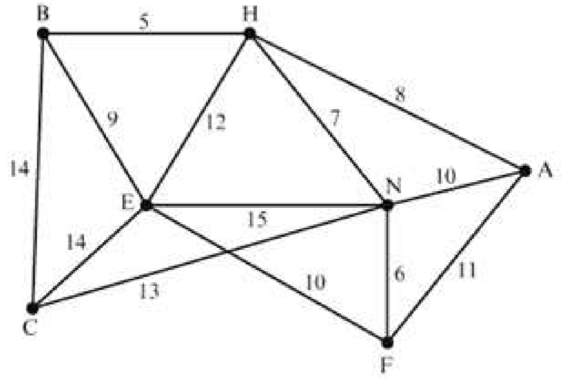
1 Review Exercise
Exercise A, Question 22

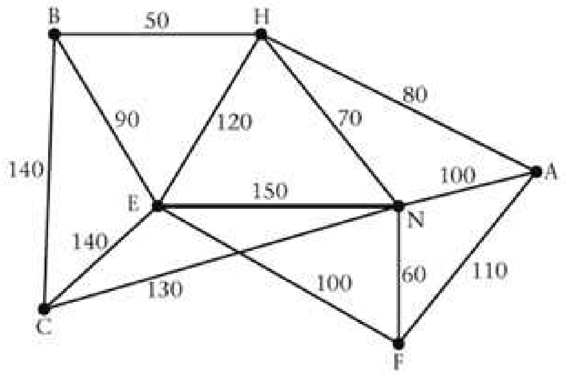
Question:

a Define the terms
  i tree,
  ii spanning tree,
  iii minimum spanning tree.

b State one difference between Kruskal's algorithm and Prim's algorithm, to find a minimum spanning tree.



c Use Kruskal's algorithm to find the minimum spanning tree for the network shown above. State the order in which you included the arcs. Draw the minimum spanning tree and state its length.



This network models a car park. Currently there are two pay-stations, one at E and one at N. These two are linked by a cable as shown. New pay-stations are to be installed at B, H, A, F and C. The number on each arc represents the distance between the pay-stations in metres. All of the pay-stations need to be connected by cables, either directly or indirectly. The current cable between E and N must be included in the final network. The minimum amount of new cable is to be used.

d Using your answer to part **c**, or otherwise, determine the minimum amount of new cable needed. Draw a diagram to show where these cables should be installed. State the minimum amount of new cable needed. ***E***

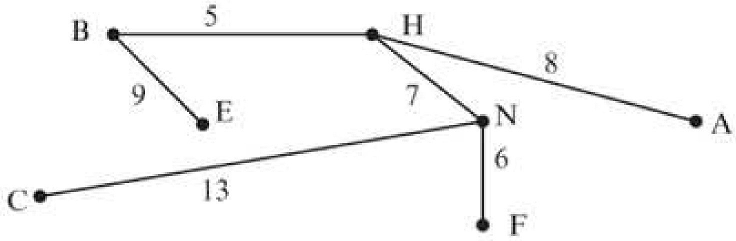
Solution:

a **i** A connected graph with no cycles, loops or multiple edges.
ii A tree that includes all vertices.
iii A spanning tree of minimum total weight.

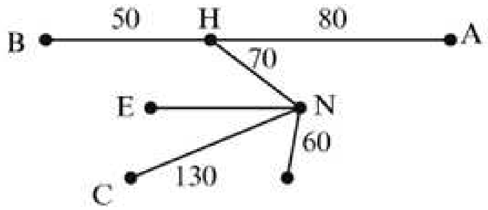
b For example,

- There is no need to check for cycles when using Prim's algorithm.
- Prim's algorithm can start at any vertex, Kruskal's algorithm starts with the shortest arc.
- In Prim's algorithm the tree 'grows' in a connected fashion, with Kruskal's algorithm the tree may not be connected until the end.
- When using Kruskal's algorithm the shortest *arc* is added to the tree (unless it completes a cycle); with Prim's algorithm the nearest unattached *vertex* is added.

c BH, NF, HN, AH, BE, AN, EF, AF, HE, CN, CE, BC, EN
Use BH, NF, HN, AH, BE, reject NA, reject EF, reject AF, reject HE, CN



d



New cable: 390 m

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1 Review Exercise
Exercise A, Question 23

Question:

Nine pieces of wood are required to build a small cabinet. The lengths, in cm, of the pieces of wood are listed below.

20 20 20 35 40 50 60 70 75

Planks, one metre in length, can be purchased at a cost of £3 each.

a The first fit decreasing algorithm is used to determine how many of these planks are to be purchased to make this cabinet. Find the total cost and the amount of wood wasted.

Planks of wood can also be bought in 1.5 m lengths, at a cost of £4 each. The cabinet can be built using a mixture of 1 m and 1.5 m planks.

b Find the minimum cost of making this cabinet. Justify your answer. ***E***

Solution:

a 75 70 60 50 40 35 20 20 20
Bin 1: $75+20$
Bin 2: $70+20$
Bin 3: $60+40$
Bin 4: $50+35$
Bin 5: 20
5 planks needed at a cost of $5\times £3 = £15$
wastage is $5+10+0+15+80=110$ cm

b For example Bin 1 (1.5 m): $75+70$
Bin 2 (1.5 m): $60+50+40$
Bin 3 (1 m): $35+20+20+20$
Cost: $2\times £4+£3=£11$.

or

Bin 1 (1 m): $75+20$
Bin 2 (1.5 m): $70+60+20$
Bin 3 (1.5 m): $50+40+35+20$

1.5 m lengths are better value than 1 m lengths, therefore the solution using as many 1.5 m lengths as possible is preferred.

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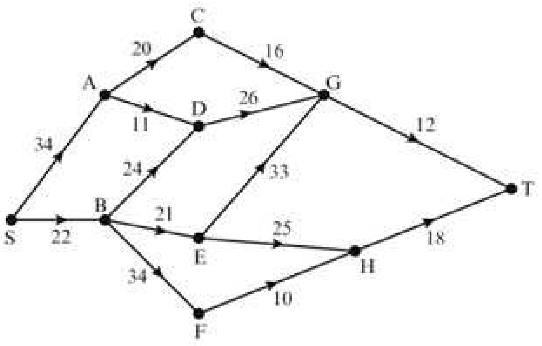
1 Review Exercise
Exercise A, Question 24

Question:



a Use Dijkstra's algorithm to find the shortest route from S to T in this network. Show all necessary working by drawing a diagram. State your shortest route and its length.

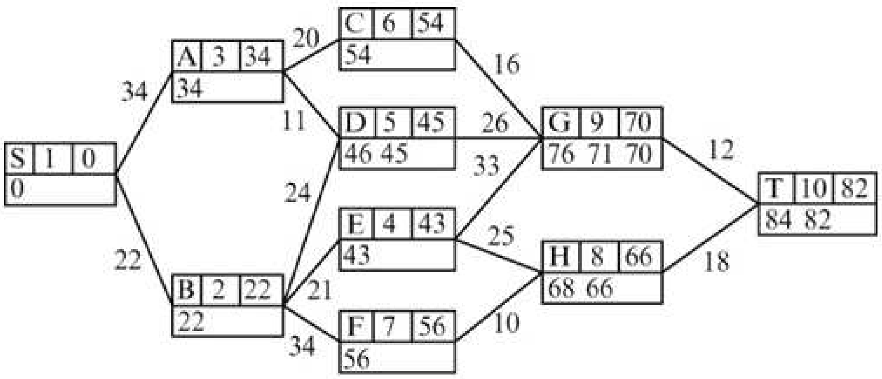
b Explain how you determined the shortest route from your labelling.

c It is now necessary to go from S to T via H. Obtain the shortest route and its length.

E

Solution:

a



Route: S – A – C – G – T length: 82

b For example

$82 - 12 = 70$ GT

$70 - 16 = 54$ CG

$54 - 20 = 34$ AC

$34 - 34 = 0$ SA

c Shortest route from S to H + HT

S – B – F – H – T length: 84

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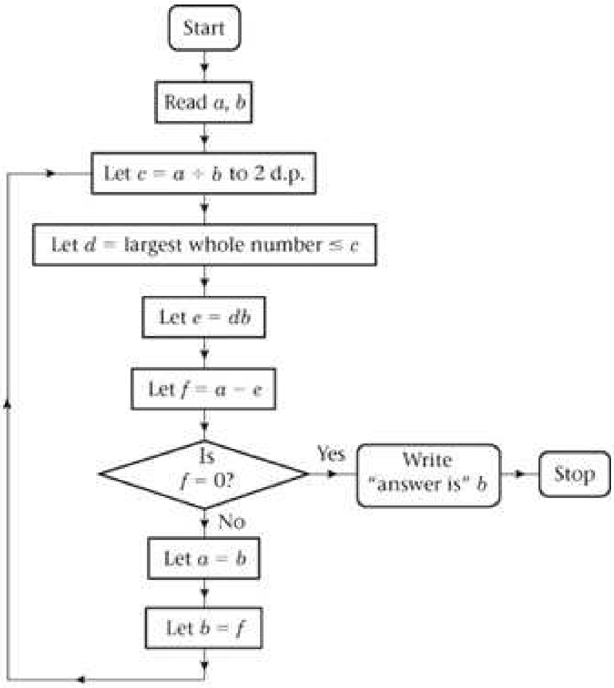
1 Review Exercise
Exercise A, Question 25

Question:

An algorithm is described by the flow chart below.



a Given that $a = 645$ and $b = 255$, draw a table to show the results obtained at each step when the algorithm is applied.

b Explain how your solution to part a would be different if you had been given that $a = 255$ and $b = 645$.

c State what the algorithm achieves. ***E***

Solution:

a

a	b	c	d	e	f	$f = 0$?
645	255	2.53	2	510	135	No
255	135	1.89	1	135	120	No
135	120	1.13	1	120	15	No
120	15	8	8	120	0	Yes

answer is 15

b The first row would be

255 645 0.40 0 0 255 No

but the second row would then be the same as the first row in the table above. So in effect this new first line would just be an additional row at the start of the solution.

c Finds the Highest Common Factor of a and b.

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1 Review Exercise
Exercise A, Question 26

Question:

55 80 25 84 25 34 17 75 3 5

a The list of numbers above is to be sorted into descending order. Perform a bubble sort to obtain the sorted list, giving the state of the list after each complete pass.

The numbers in the list represent weights, in grams, of objects which are to be packed into bins that hold up to 100 g.

b Determine the least number of bins needed.

c Use the first-fit decreasing algorithm to fit the objections into bins which hold up to 100 g. ***E***

Solution:

a For example, left to right

55	80	25	84	25	34	17	75	3	5
80	55	84	25	34	25	75	17	5	3
80	84	55	34	25	75	25	17	5	3
84	80	55	34	75	25	25	17	5	3
84	80	55	75	34	25	25	17	5	3
84	80	75	55	34	25	25	17	5	3

No more exchanges, sort complete.

b $403 \div 100 = 4.03$ so 5 bins are needed.

c Bin 1: $84 + 5 + 3$
Bin 2: $80 + 17$
Bin 3: $75 + 25$
Bin 4: $55 + 34$
Bin 5: 25

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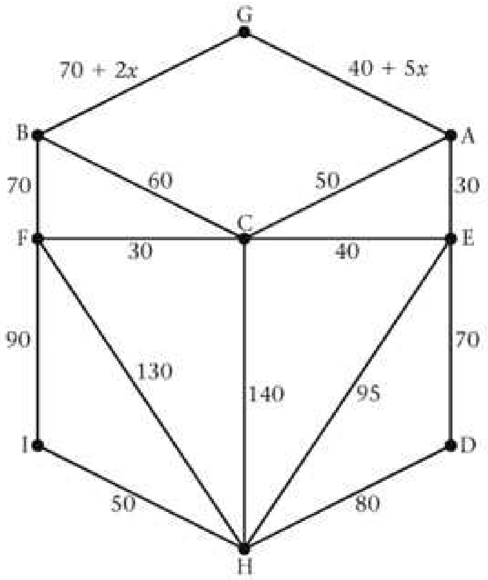
1 Review Exercise
Exercise A, Question 27

Question:



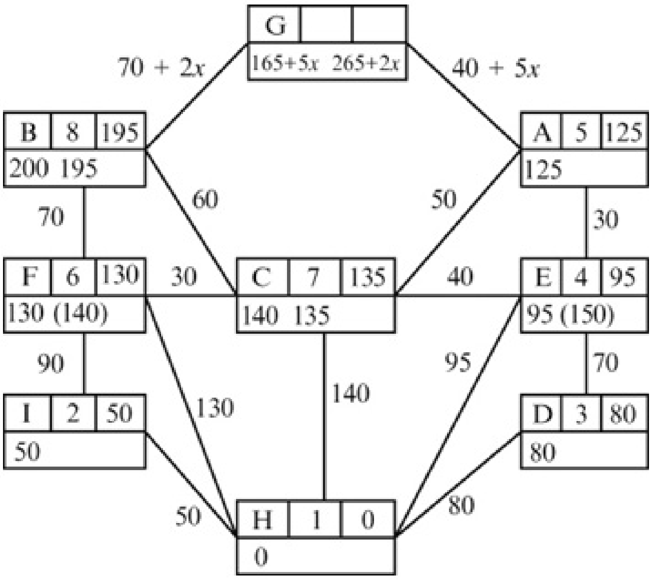
Peter wishes to minimise the time spent driving from his home at H, to a campsite at G. The network above shows a number of towns and the time, in minutes, taken to drive along these roads is expressed in terms of x, where $x \geq 0$.

a Use Dijkstra's algorithm to find two routes from H to G (one via A and one via B) that minimise the travelling time from H to G. State the length of each route in terms of x.

b Find the range of values of x for which Peter should follow the route via A. ***E***

Solution:

a



Via A H – E – A – G length $165+5x$
Via B H – E – C – B – G length $265+2x$

b $165+5x = 265+2x \Rightarrow x = 33\frac{1}{3}$

So range is $0 \le x \le 33\frac{1}{3}$

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1 Review Exercise
Exercise A, Question 28

Question:

	A	B	C	D	E	F
A	–	7	3	–	8	11
B	7	–	4	2	–	7
C	3	4	–	5	9	–
D	–	2	5	–	6	3
E	8	–	9	6	–	–
F	11	7	–	3	–	–

The matrix represents a network of roads between six villages A, B, C, D, E and F. The value in each cell represents the distance, in km, along these roads.

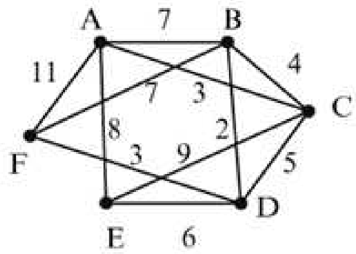
a Show this information on a diagram.

b Use Kruskal's algorithm to determine the minimum spanning tree. State the order in which you include the arcs and the length of the minimum spanning tree.

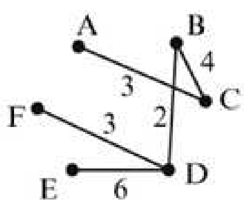
c Starting at D, use Prim's algorithm on the matrix given to find the minimum spanning tree. State the order in which you include the arcs. ***E***

Solution:

a



b BD, $\begin{Bmatrix} AC \\ DF \end{Bmatrix}$, BC, reject CD, DE



c DB, DF, BC, CA, DE
length of tree 18 km

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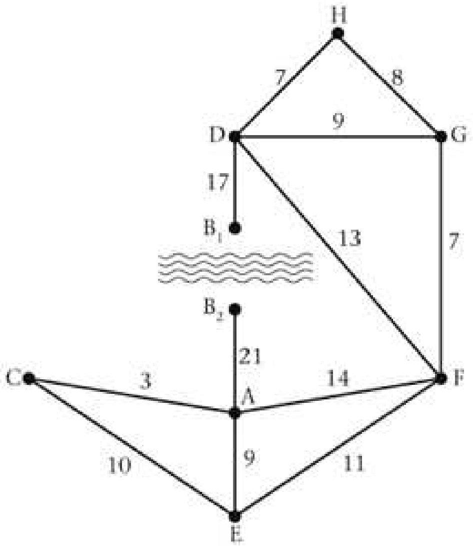
1 Review Exercise
Exercise A, Question 29

Question:



The diagram shows a network of roads connecting villages. The length of each road, in km, is shown. Village B has only a small footbridge over the river which runs through the village. It can be accessed by two roads, from A and D.

The driver of a snowplough, based at F, is planning a route to enable her to clear all the roads of snow. The route should be of minimum length. Each road can be cleared by driving along it once. The snowplough cannot cross the footbridge.

Showing all your working and using an appropriate algorithm,

a find the route the driver should follow, starting and ending at F, to clear all the roads of snow. Give the length of this route.

The local authority decides to build a road bridge over the river at B. The snowplough will be able to cross the road bridge.

b Reapply the algorithm to find the minimum distance the snowplough will have to travel (ignore the length of the new bridge). ***E***

Solution:

a Odd vertices are B_1, B_2, E, G

$B_1B_2 + EG = 65 + 18 = 83$

$B_1E + B_2G = 41 + 42 = 83$

$B_1G + B_2E = 26 + 30 = 56$

Repeat $B_1D, DG,\quad B_2A, AE$

Route: For example,

$F - A - E - A - B_2 - A - C - E - F - G - D - H - G - D - B_1 - D - F$

(All correct routes have 17 letters in their 'word')

length $= 129 + 56 = 185$ km

b Now only the route between E and G needs repeating

so repeat $EF + FG = 18$

length of new route $= 129 + 18$

$= 147$ km

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1 Review Exercise
Exercise A, Question 30

Question:

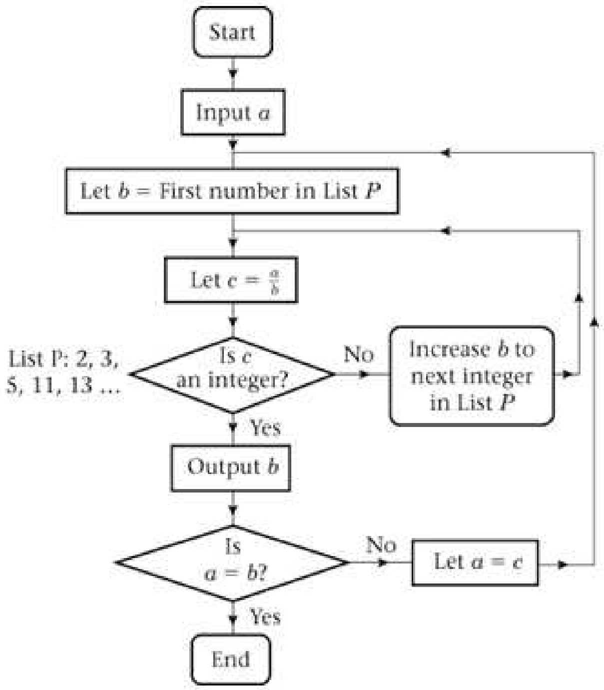
This diagram describes an algorithm in the form of a flow chart, where a is a positive integer.

List P, which is referred to in the flow chart, comprise the prime numbers 2, 3, 5, 7, 11, 13, 17, ...

a Starting with $a = 90$, implement this algorithm. Show your working in the table.

b Explain the significance of the output list.

c Write down the final value of c for any initial value of a.



E

Solution:

a

a	b	c	Integer?	Output list	$a = b$?
90	2	45	Y	2	N
45	2	22.5	N		
45	3	15	Y	3	N
15	2	7.5	N		
15	3	5	Y	3	N
5	2	2.5	N		
5	3	$1\frac{2}{3}$	N		
5	5	1	Y	5	Y

Output list: 2, 3, 3, 5

b Expresses a as a product of prime factors

c $c = 1$ (since we stop when $a = b$ and $c = \frac{a}{b}$)

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1 Review Exercise
Exercise A, Question 31

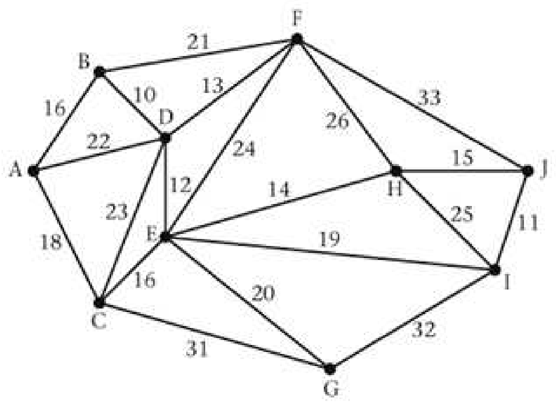
Question:

The network opposite represents the journey time, in minutes, between ten Midland towns.

a Use Dijkstra's algorithm to find the quickest route between A and J. Your solution must indicate clearly how you applied the algorithm, including

i the order in which the vertices were labelled,

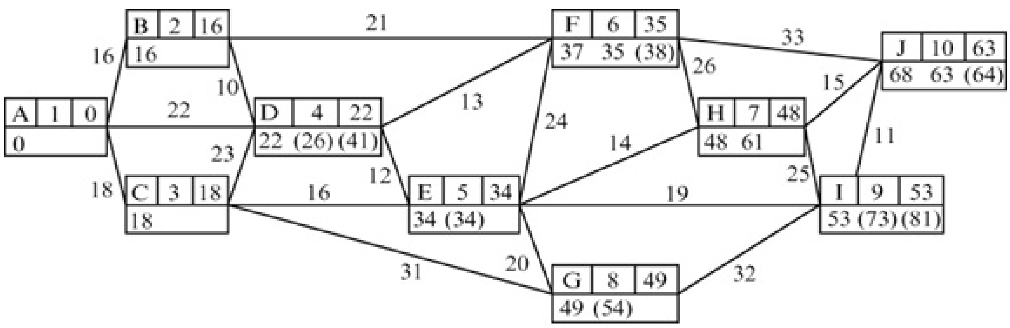
ii how you determined your quickest route from your labelling.



b Is the route you have found the only quickest route? Give a reason for your answer. ***E***

Solution:

a i



ii For example

$63-15=48$	HJ		$63-15=48$	HJ
$48-14=34$	EH	*or*	$48-14=34$	EH
$34-16=18$	CE		$34-12=22$	DE
$18-18=0$	AC		$22-22=0$	AD

Route A – C – E – H – J or A – D – E – H – J length 63

b No, it is not the only route, there are two shortest routes (see above).