Solutionbank D1

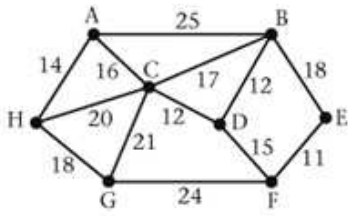
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

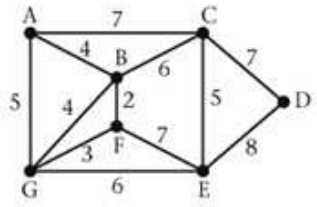
Question:

Use Kruskal's algorithm to find minimum spanning trees for each of these networks. State the weight of each tree. You must list the arcs in the order in which you consider them.

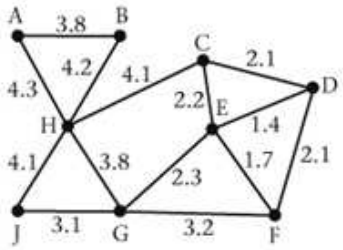
a



b

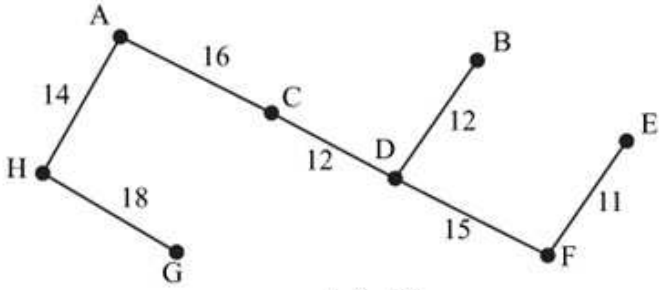


c



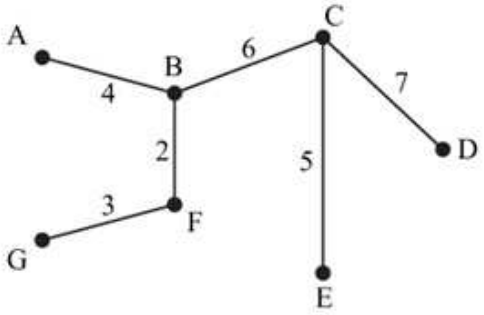
Solution:

a EF (11) add to tree
BD (12) add to tree
CD (12) add to tree
AH (14) add to tree
DF (15) add to tree
AC (16) add to tree
BC (17) reject
GH (18) add to tree
BE (18)
CH (20)
CG (21)
FG (24)
AB (25)
reject all remaining arcs.



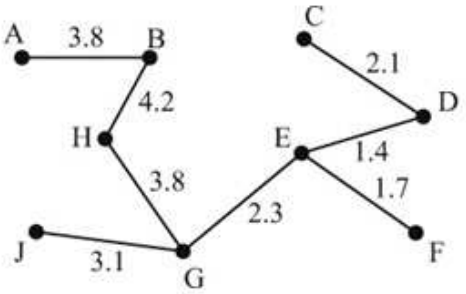
weight: 98

b BF (2) add to tree
FG (3) add to tree
AB (4) add to tree
BG (4) reject
AG (5) reject
CE (5) add to tree
BC (6) add to tree
EG (6) reject
AC (7) reject
CD (7) add to tree
EF (7)
DE (8)
reject all remaining arcs



weight: 27

c DE (1.4) add to tree
EF (1.7) add to tree
CD (2.1) add to tree
DF (2.1) reject
CE (2.2) reject
EG (2.3) add to tree
GJ (3.1) add to tree
FG (3.2) reject
AB (3.8) add to tree
GH (3.8) add to tree
GH (4.1) reject
HJ (4.1) reject
BH (4.2) add to tree
AH (4.3) reject



weight: 22.4

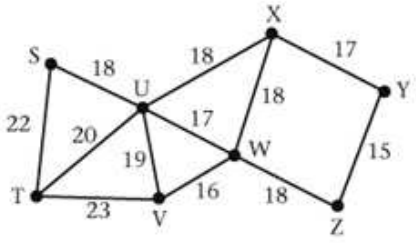
Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

Use Kruskal's algorithm to find the three possible minimum connectors (MSTs) for this network. You must list the arcs in the order in which you consider them.

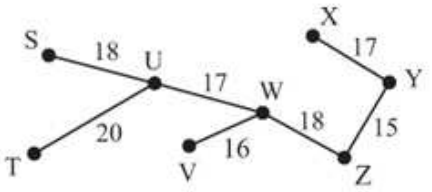


Solution:

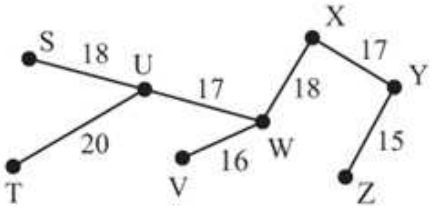
YZ (15) add to tree
VW (16) add to tree
XY (17) add to tree
UW (17) add to tree

WZ (18)
WX (18) reject 2 of these and add one to tree
UX (18)

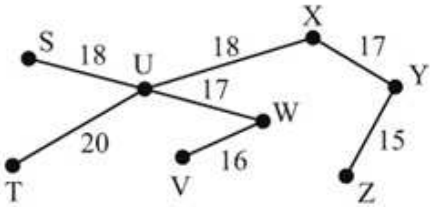
SU (18) add to tree
UV (19) reject
TU (20) add to tree
ST (22) reject
TV (23) reject



solution 1



solution 2



solution 3

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 3

Question:

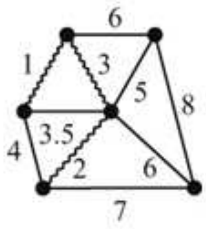
Draw a network in which

a the three shortest arcs from part of the minimum connector (MST),

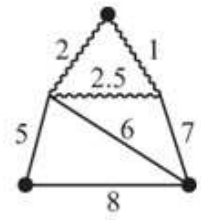
b not all of the three shortest arcs from part of the minimum connector.

Solution:

a For example



b For example



Solutionbank D1

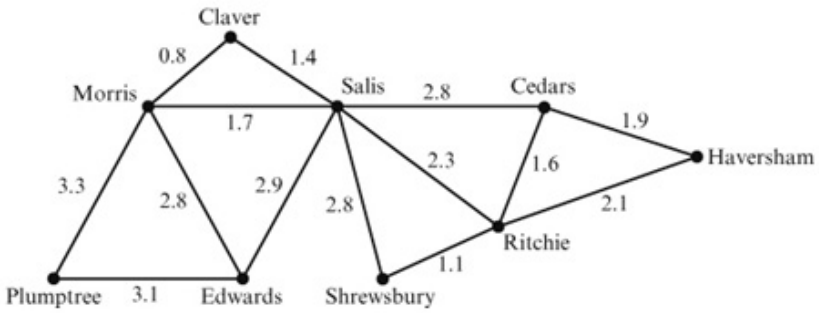
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 4

Question:

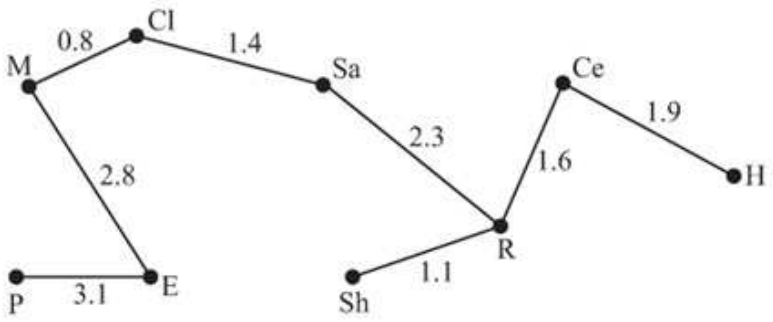
The diagram shows nine estates and the distances, in km, between them. A cable TV company wishes to link up the estates. Find a minimum spanning tree for the network using Kruskal's algorithm. You must list the arcs in the order in which you consider them. State the weight of your tree.

a



Solution:

Cl – M (0.8) add to tree
R – Sh (1.1) add to tree
Cl – Sa (1.4) add to tree
Ce – R (1.6) add to tree
Sa – M (1.7) reject
Ce – H (1.9) add to tree
H – R (2.1) reject
R – Sa (2.3) add to tree
Ce – Sa (2.8) reject
Sa – Sh (2.8) reject
E – M (2.8) add to tree
E – Sa (2.9) reject
E – P (3.1) add to tree
M – P (3.3) reject



weight: 15

Solutionbank D1

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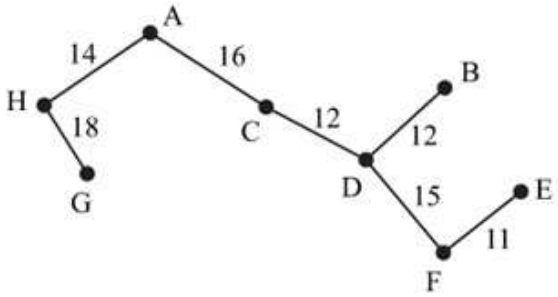
Exercise B, Question 1

Question:

Repeat Question 1 in Exercise 3A using Prim's algorithm. Start at vertex A each time.

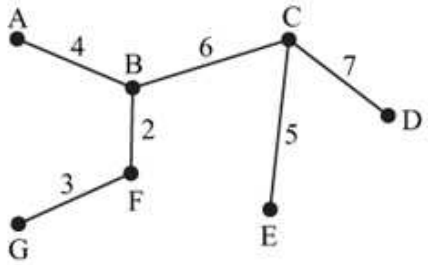
Solution:

a Arcs must be chosen in this order:
AH, AC, CD, BD, DF, FE, GH



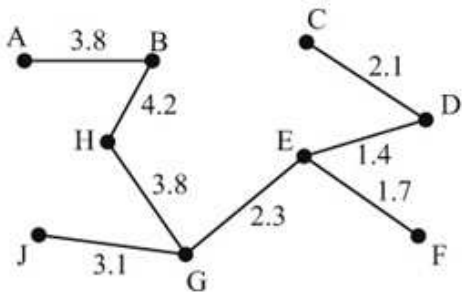
weight: 98

b Arcs must be chosen in this order:
BF, FG, AB, BC, CE, CD



weight: 27

c Arcs must be chosen in this order:
AB, BH, GH, GE, DE, EF, CD, GJ



weight: 22.4

Solutionbank D1

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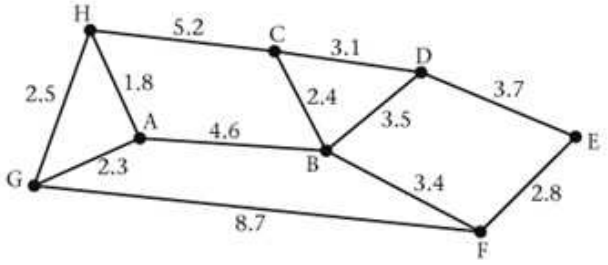
Exercise B, Question 2

Question:



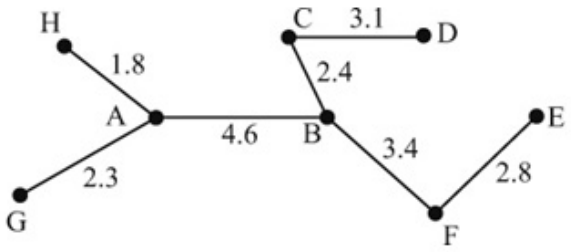
The network shows the distance, in kilometres, between eight weather monitoring stations. The eight stations need to be linked together with underground cables.

a Use Prim's algorithm, starting at A, to find a minimum spanning tree. You must make your order of arc selection clear.

b Given that cable costs £850 per kilometre to lay, find the cost of linking these weather stations.

Solution:

a Arcs must be chosen in this order
AH, AG, AB, BC, CD, BF, FE



b cost $= 20.4 \times 850$
$= £17\,340$

Solutionbank D1

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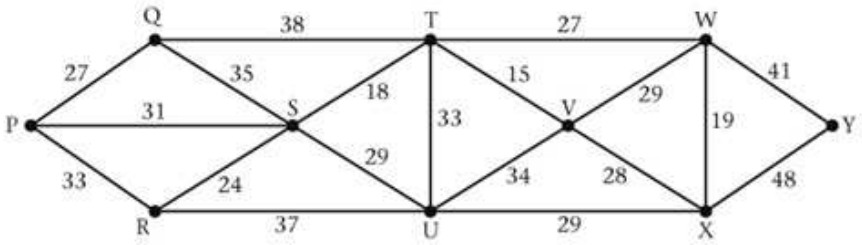
Exercise B, Question 3

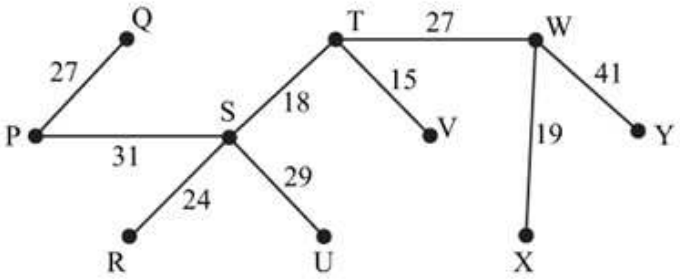
Question:



The network shows ten villages and the costs, in thousands of pounds, of connecting them with a new energy supply.
Use Prim's algorithm starting at P, to find the minimum cost energy supply network that would connect all ten villages.
Draw your minimum connector and state its cost.

Solution:

Arcs must be chosen in this order
either
PS, ST, TV, RS, TW, WX, PQ, SU, WY
or
PS, ST, TV, RS, PQ, TW, WX, SU, WY.
Cost: £231 000



Solutionbank D1

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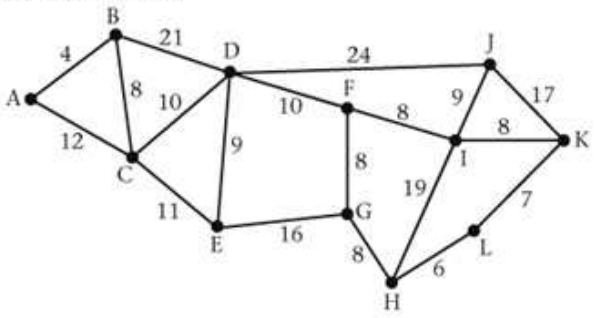
Exercise B, Question 4

Question:

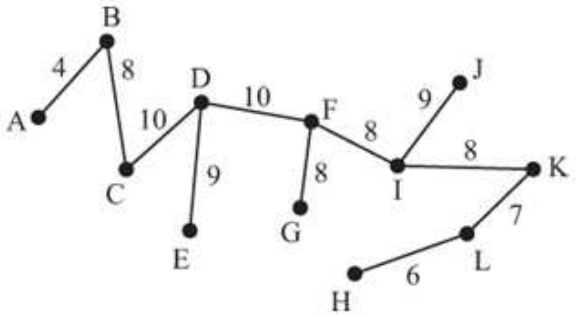
Use Prim's algorithm, starting at A, to find four distance minimum connectors for the network below. In each case draw your spanning tree, and make your order of arc selection clear.



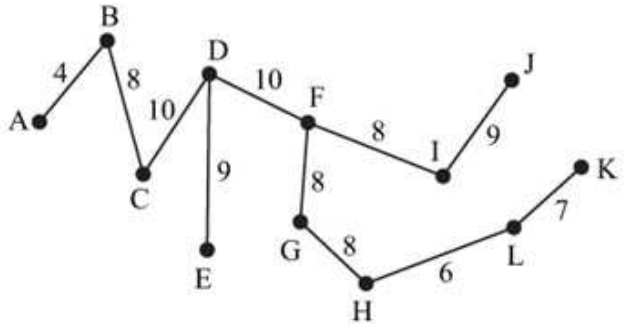
Solution:

Solution 1

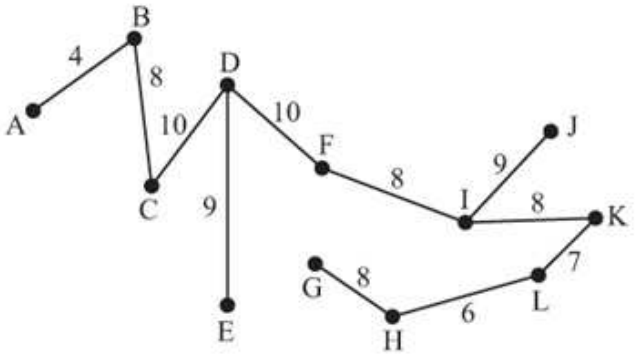
AB, BC, CD, DE, DF
then *either*
FG FI IK KL HL IJ
or FI FG IK KL HL IJ
or FI IK KL HL FG IJ



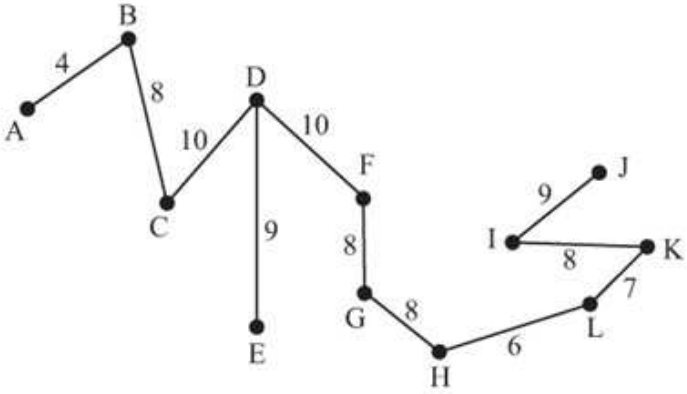
Solution 2
AB BC CD DE DF
then *either* FI FG GH HL LK IJ
or FG GH HL LK FI IJ
or FG FI GH HL LK IJ



Solution 3
AB BC CD DE DF FI
IK KL HL GH IJ



Solution 4
AB BC CD DE DF FG
GH HL KL IK IJ



Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

Apply Prim's algorithm to the distance matrices below. List the arcs in order of selection and state the weight of your tree.

a

	A	B	C	D	E	F
A	–	15	20	34	25	9
B	15	–	36	38	28	14
C	20	36	–	43	38	22
D	34	38	43	–	26	40
E	25	28	38	26	–	31
F	9	14	22	40	31	–

b

	R	S	T	U	V
R	–	28	30	31	41
S	28	–	16	19	43
T	30	16	–	22	41
U	31	19	22	–	37
V	41	43	41	37	–

Solution:

a

	↓1 A	↓3 B	↓4 C	↓6 D	↓5 E	↓2 F
A	–	15	20	34	25	9
B	15	–	36	38	28	(14)
C	(20)	36	–	43	38	22
D	34	38	43	–	(26)	40
E	(25)	28	38	26	–	31
F	(9)	14	22	40	31	–

Arcs in order
AF (9)
FB (14)
AC (20)
AE (25)
DE (26)
weight $= 9+14+20+25+26$
$= 94$

b

	↓1 R	↓2 S	↓3 T	↓4 U	↓5 V
R	–	28	30	31	41
S	(28)	–	16	19	43
T	30	(16)	–	22	41
U	31	(19)	22	–	37
V	41	43	41	(37)	–

Arcs in order
RS (28)
ST (16)
SU (19)
UV (37)
weight $= 28+16+19+37$
$= 100$

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 2

Question:

	Birmingham	Nottingham	Lincoln	Stoke	Manchester
Birmingham	–	164	100	49	88
Nottingham	164	–	37	56	74
Lincoln	100	37	–	90	86
Stoke	49	56	90	–	44
Manchester	88	74	86	44	–

The table shows the distance, in miles, between 5 cities. It is intended to link these 5 cities to a transit system.
Use Prim's algorithm, starting at Birmingham, to find a minimum spanning tree for this network. You must list the arcs in order of selection and state the weight of your tree.

Solution:

	↓1	↓4	↓5	↓2	↓3
	B	N	L	S	M
B	–	164	100	49	88
N	164	–	37	(56)	74
L	100	(37)	–	90	86
S	(49)	56	90	–	44
M	88	74	86	(44)	–

Arcs in order
BS (49)
SM (44)
SN (56)
NL (37)
weight = 186

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

	A	B	C	D	E	F	G	H
A	–	84	53	35	–	47	–	42
B	84	–	71	113	142	61	75	–
C	53	71	–	–	–	–	59	–
D	35	113	–	–	58	67	151	–
E	–	142	–	58	–	168	159	48
F	47	61	–	67	168	–	–	73
G	–	75	59	151	159	–	–	52
H	42	–	–	–	48	73	52	–

The table shows the costs, in euros per 1000 words, of translating DVD player instruction manuals between eight languages.

a Use Prim's algorithm, starting from D, to find the cost of translating an instruction manual of 3000 words from D into the seven other languages.

b Draw your minimum spanning tree.

From the table we see that it costs 159 euros per 1000 words to translate from language E to G. A manual is written in language E and needs to be translated into language G.

c Give a reason why
i it might be decided *not* to translate directly from E to G,
ii it might be decided to translate directly.

Solution:

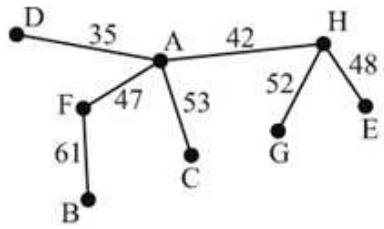
a

	↓2	↓8	↓7	↓1	↓5	↓4	↓6	↓3
	A	B	C	D	E	F	G	H
A	–	84	53	(35)	–	47	–	42
B	84	–	71	113	142	(61)	75	–
C	(53)	71	–	–	–	–	59	–
D	35	113	–	–	58	67	151	–
E	–	142	–	58	–	168	159	(48)
F	(47)	61	–	67	168	–	–	73
G	–	75	59	151	159	–	–	(52)
H	(42)	–	–	–	48	73	52	–

Arcs in order
DA (35)
AH (42)
AF (47)
HE (48)
HG (52)
AC (53)
FB (61)

weight = 338
∴ cost = 3×338
= € 1014

b



c **i** It is cheaper to translate from E to H then from H to G at a cost of 48 + 52 = 100 euro rather than 159 euro per 1000 words.

ii A direct translation is likely to be more accurate than a translation via another language.

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 4

Question:

	X	A	B	C	D	E	F	G	H	I
X	–	65	80	89	74	26	71	41	41	74
A	65	–	27	41	22	37	20	29	25	43
B	80	27	–	30	24	55	16	46	40	42
C	89	41	30	–	50	84	24	70	49	26
D	74	22	24	50	–	51	35	34	47	63
E	26	37	55	84	51	–	52	18	23	68
F	71	20	16	24	35	52	–	45	31	27
G	41	29	46	70	34	18	45	–	25	64
H	41	25	40	49	47	23	31	25	–	44
I	74	43	42	26	63	68	27	64	44	–

The table shows the distances, in miles, between nine oil rigs and the depot X. Pipes are to be laid to connect the rigs and the depot.

a Use Prim's algorithm, starting at X, to find a minimum connector for the network. You must make the order of arc selection clear.

Oil rig A exhausts its supply and is closed down.

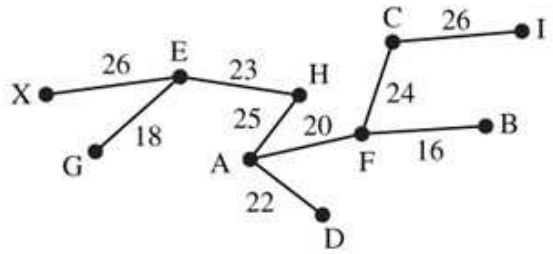
b Use Prim's algorithm to find a minimum connector excluding A. You must make the order of arc selection clear.

Solution:

a

	1	5	7	9	8	2	6	3	4	10
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
	X	A	B	C	D	E	F	G	H	I
X	–	65	80	89	74	26	71	41	41	74
A	65	–	27	41	22	37	20	29	(25)	43
B	80	27	–	30	24	55	(16)	46	40	42
C	89	41	30	–	50	84	(24)	70	49	26
D	74	(22)	24	50	–	51	35	34	47	63
E	(26)	37	55	84	51	–	52	18	23	68
F	71	(20)	16	24	35	52	–	45	31	27
G	41	29	46	70	34	(18)	45	–	25	64
H	41	25	40	49	47	(23)	31	25	–	44
I	74	43	42	(26)	63	68	27	64	44	–

order of arcs
XE (26)
EG (18)
EH (23)
HA (25)
AF (20)
FB (16)
AD (22)
FC (24)
CI (26)

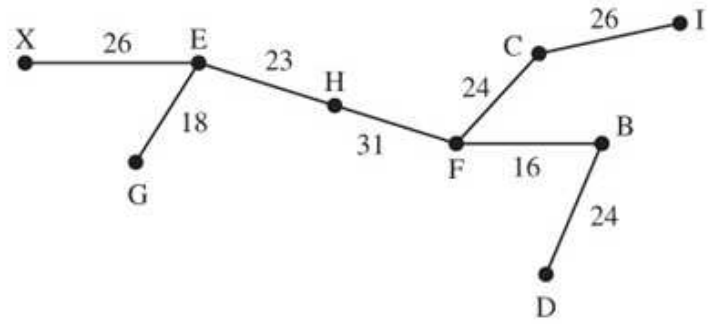


b

	1	6	7 or 8	8 or 9	2	5	3	4	9
	↓	↓	↓	↓	↓	↓	↓	↓	↓
	X	B	C	D	E	F	G	H	I
X	–	80	89	74	26	71	41	41	74
B	80	–	30	24	55	(16)	46	40	42
C	89	30	–	50	84	(24)	70	49	26
D	74	(24)	50	–	51	35	34	47	63
E	(26)	55	84	51	–	52	18	23	68
F	71	16	24	35	52	–	45	(31)	27
G	41	46	70	34	(18)	45	–	25	64
H	41	40	49	47	(23)	31	25	–	44
I	74	42	(26)	63	68	27	64	44	–

order of arcs

XE (26)
EG (18)
EH (23)
HF (31)
FB (16)
FC (24)
BD (24)
CI (26)



Solutionbank D1

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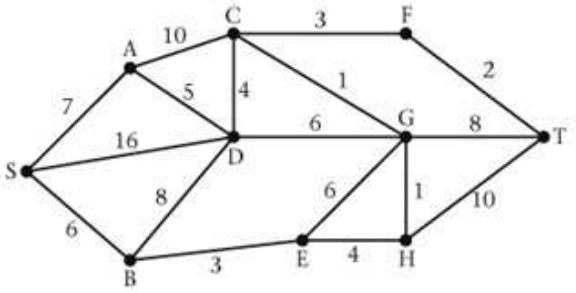
Exercise D, Question 1

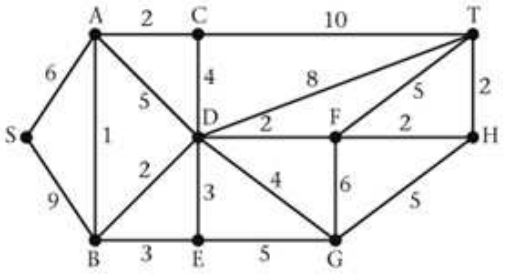
Question:

Use Dijkstra's algorithm to find a shortest route from S to T in each of the following networks. Show your working. State your shortest routes and their lengths. You should show how you obtained your shortest route from your labelled diagrams.

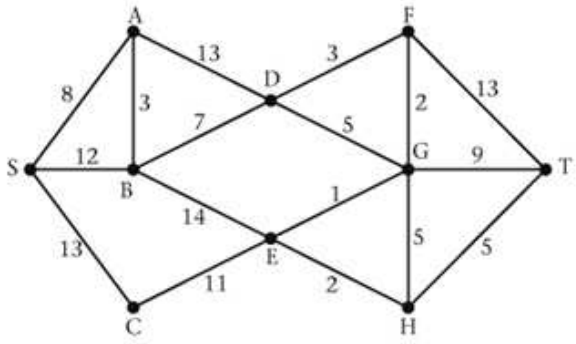
a



b

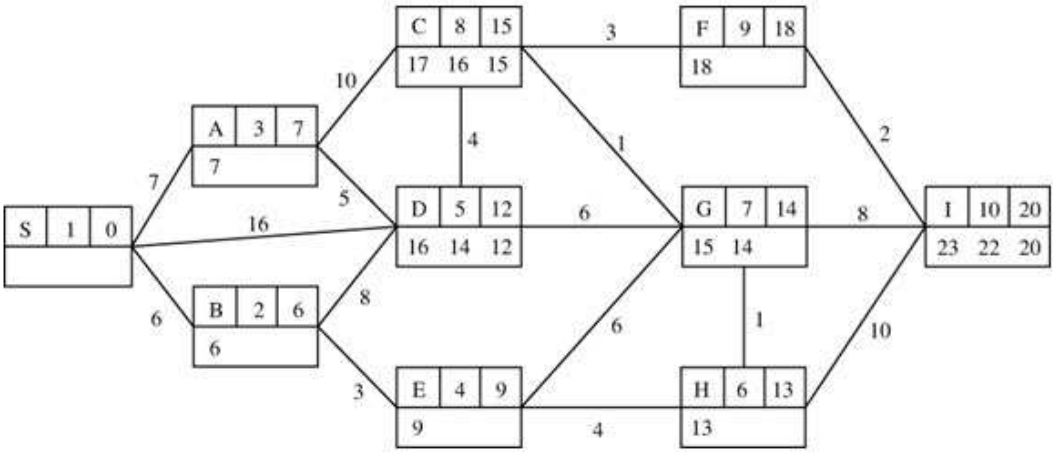


c



Solution:

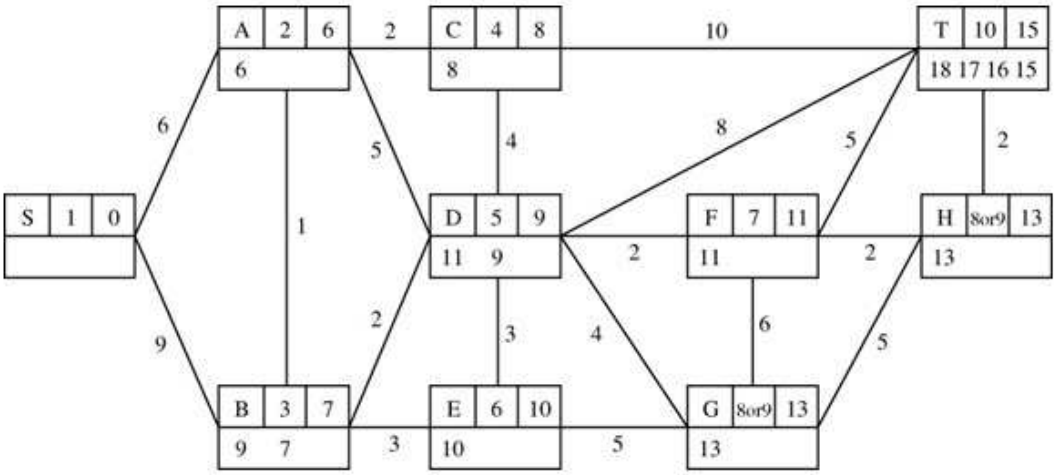
a



20 − 2 = 18 FI
18 − 3 = 15 CF
15 − 1 = 14 GC
14 − 1 = 13 HG
13 − 4 = 9 EH
9 − 3 = 6 BE
6 − 6 = 0 SB

Shortest route: S − B − E − H − G − C − F − T Length of shortest route: 20

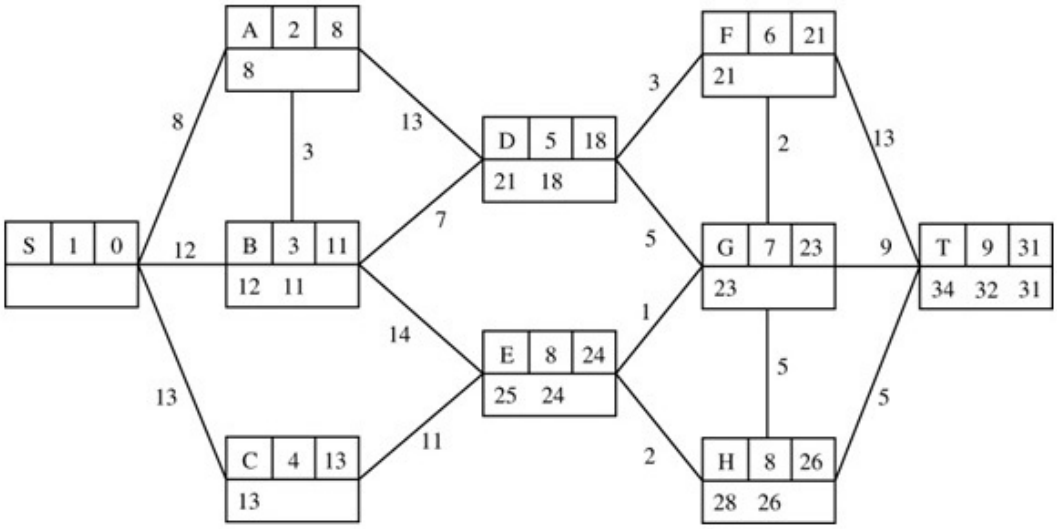
b



15 − 2 = 13 HT
13 − 2 = 11 FH
11 − 2 = 9 DF
9 − 2 = 7 BD
7 − 1 = 6 AB
6 − 6 = 0 SA

Shortest route: S − A − B − D − F − H − T Length of shortest route: 15

c



$31-5=26$ HT
$26-2=24$ EH
$24-11=13$ CE
$13-13=0$ SC

Shortest route: S – C – E – H – T

Length of shortest route: 31

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 2

Question:

The network shows part of a road network in a city. The number on each arc gives the travel time, in minutes, it takes to travel along that arc.

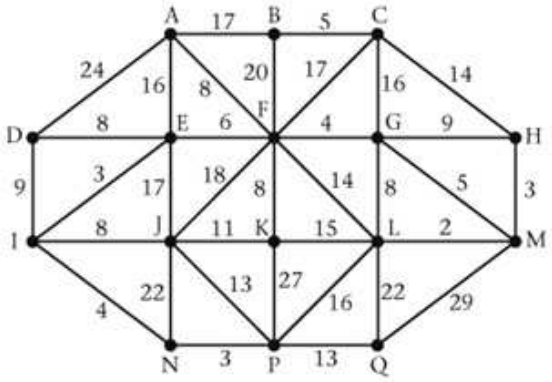
Find

a the quickest route from A to Q and its length,

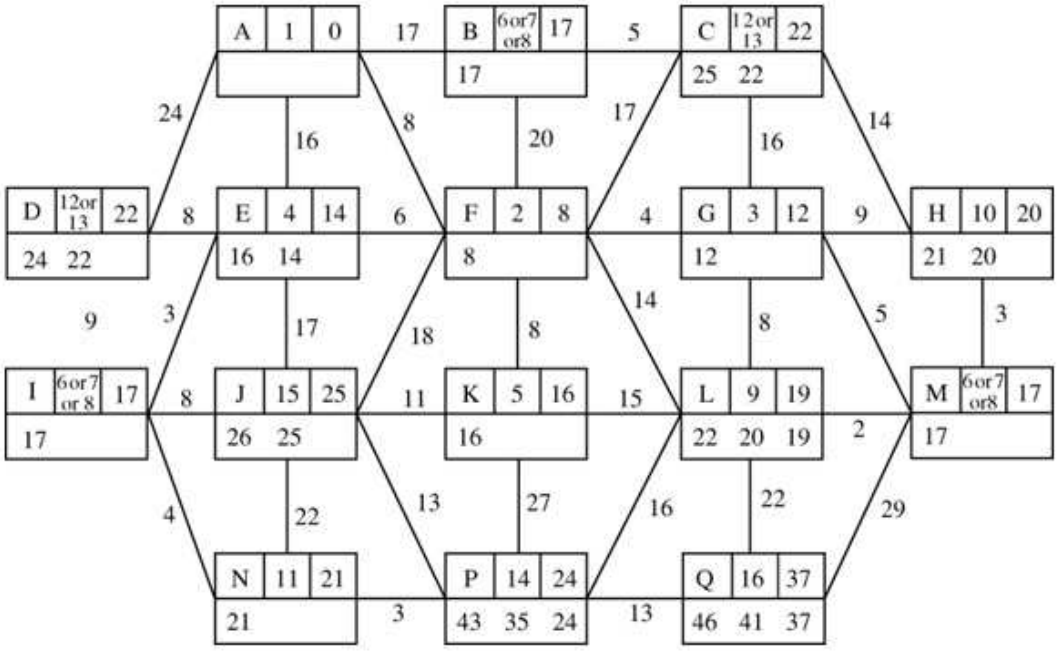
b the quickest route from A to L and its length,

c the quickest route from M to A and its length,

d the quickest route from P to A and its length.



Solution:



a	A to Q	A – F – E – I – N – P – Q	Length	37
b	A to L	A – F – G – M – L	Length	19
c	M to A	M – G – F – A	Length	17
d	P to A	P – N – I – E – F – A	Length	24

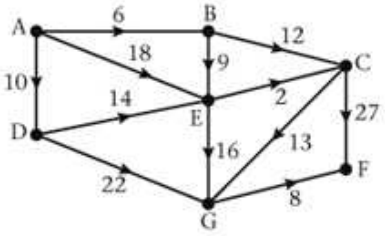
Solutionbank D1

Edexcel AS and A Level Modular Mathematics

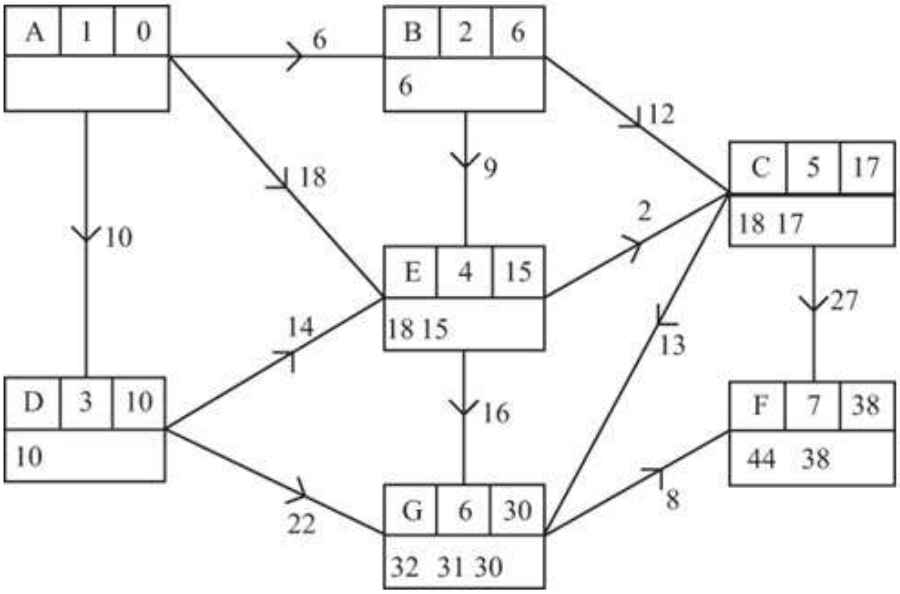
Exercise D, Question 3

Question:

Use Dijkstra's algorithm to find the shortest route, and its length, from A to F in the directed network opposite.



Solution:



Shortest route: A – B – E – C – G – F Length 38

Solutionbank D1

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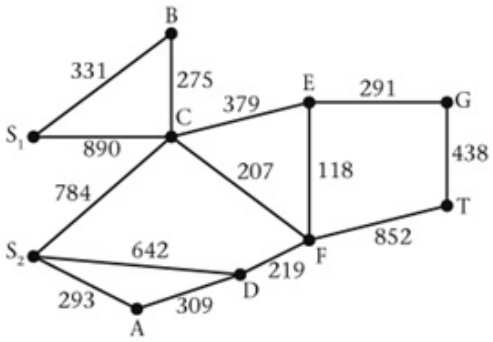
Exercise D, Question 4

Question:

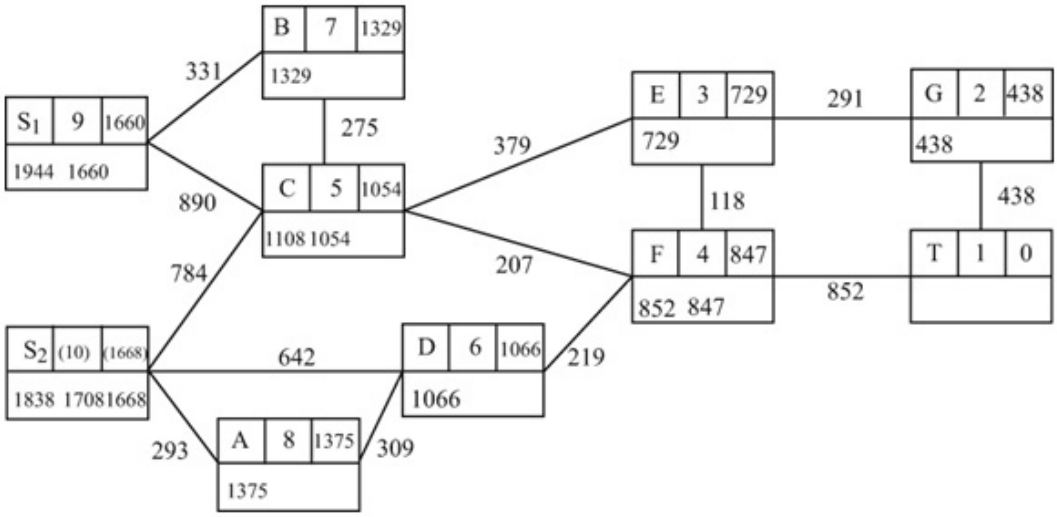
The network represents the distances, in metres, of all roads in a building site. A crane is needed for one day at T. There are two cranes available on site, one at S_1, and the other at S_2. One of these two cranes will be moved to T. In order to minimise the cost it is decided to move the crane that is closest to T. Use Dijkstra's algorithm to determine which crane should be moved.



It is possible to solve this problem with only one application of Dijkstra's algorithm. Think carefully about the starting point.

Solution:

Start at T and work back to S_1 and S_2.



Shortest route $S_1 - B - C - F - E - G - T$

Length of shortest route 1660

Solutionbank D1

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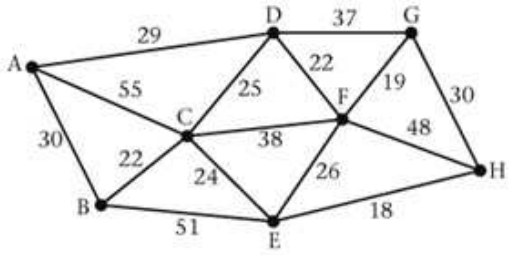
Exercise D, Question 5

Question:

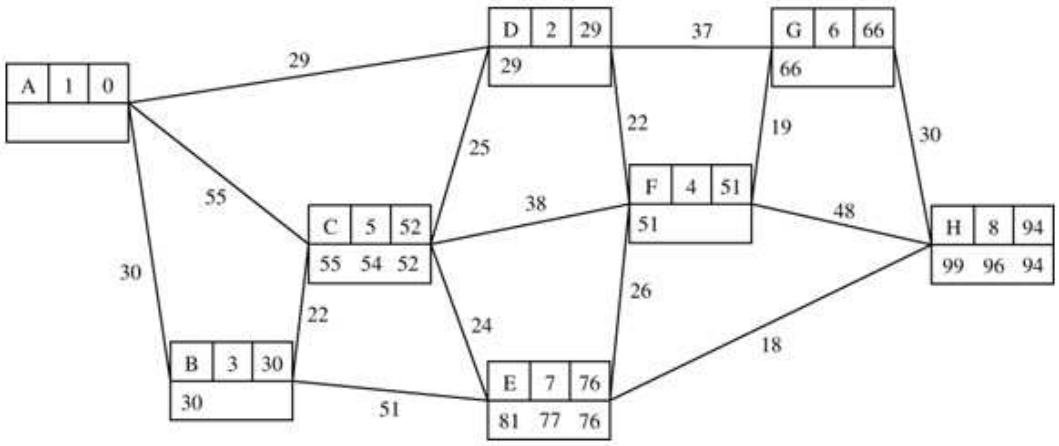


a Use Dijkstra's algorithm to find the shortest route from A to H. Indicate how you obtained your shortest route from your labelled diagram.

b Find the shortest route from A to H via G.

c Find the shortest route from A to H, not using CE.

Solution:



a $94 - 18 = 76$ EH
$76 - 24 = 52$ CE
$52 - 22 = 30$ BC
$30 - 30 = 0$ AB
Shortest route A to H: A – B – C – E – H Length 94

b Shortest route A to H via G: A – D – G – H Length 96

c Shortest route A to H not using CE: A – D – F – E – H Length 95

Solutionbank D1

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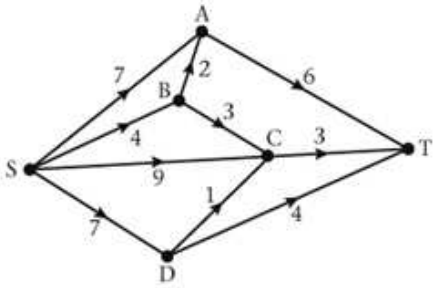
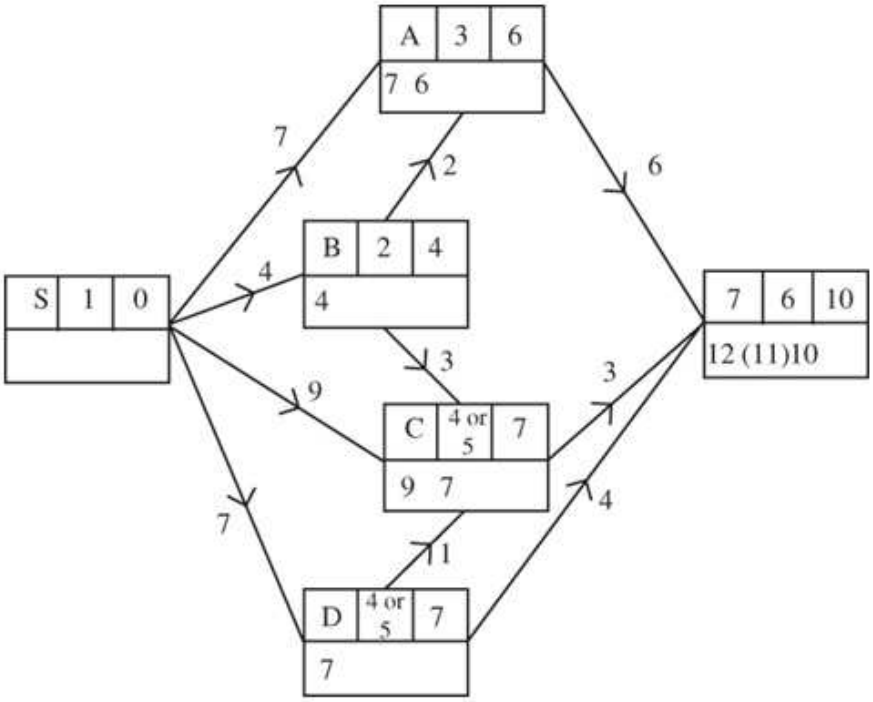
Exercise D, Question 6

Question:



Use Dijkstra's algorithm to find the shortest route from S to T. State the length of your route.

Solution:



Shortest route: S – B – C – T
Length of shortest route: 10

Solutionbank D1

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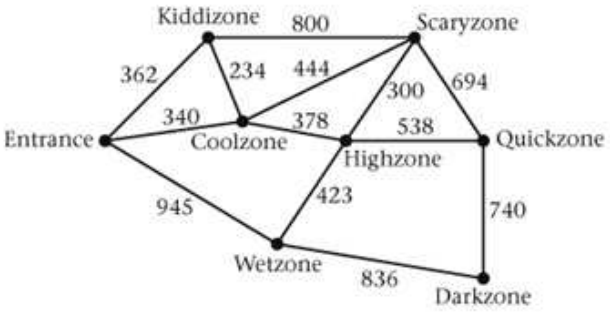
Exercise E, Question 1

Question:

The network represents a theme park with seven zones. The number on each arc shows a distance in metres.



Tramways are to be built to link the seven zones and the car park at the Entrance.

a Find a minimum connector using
- **i** Kruskal's algorithm,
- **ii** Prim's algorithm, starting at the Entrance.

You must make your order of arc selection clear.

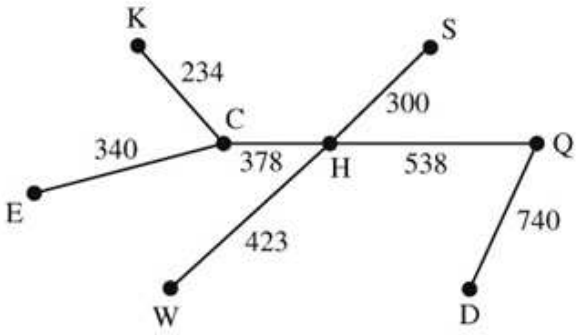
b Draw your tree and state its weight.

Solution:

a **i** Arcs are labeled with initial letters of the nodes.
CK add to tree
SH add to tree
CE add to tree
EK reject
CH add to tree
HW add to tree
CS reject
HQ add to tree
QS reject
QD add to tree
KS reject
DW reject
EW reject

ii EC
CK
CH
HS
HW
HQ
QD

b



weight: 2953

Solutionbank D1

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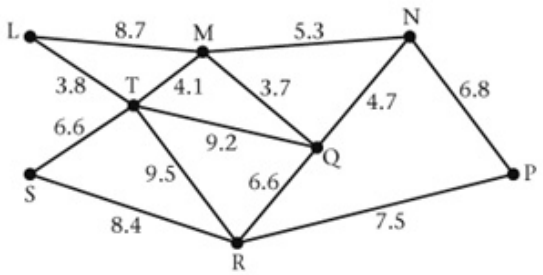
Exercise E, Question 2

Question:

The network represents eight observation points in a wildlife reserve and the possible paths connecting them. The number on each arc is the distance, in kilometres, along that path. It is decided to link the observation points by paths, but in order to minimise the impact on the wildlife reserve, we wish to use the least total length of path.



a Find a minimum spanning tree for the network using
- **i** Prim's algorithm, starting at L,
- **ii** Kruskal's algorithm,

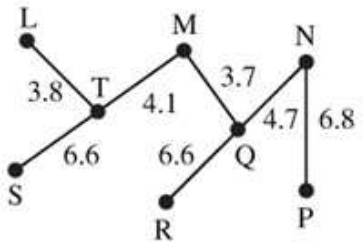
In each case list the arcs in the order in which you consider them.

Given that paths TQ and RP already exist and so will form part of the tree,

b State which algorithm, Prim's or Kruskal's, you would select to complete the spanning tree. Give a reason for your answer.

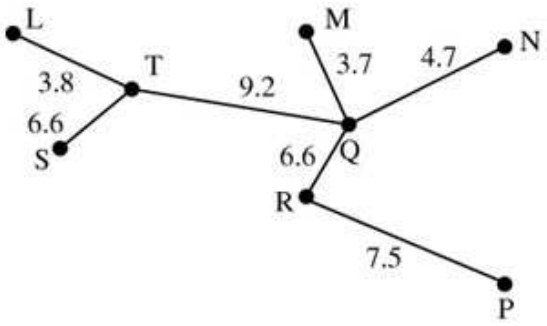
Solution:

a **i** LT
MT
MQ
NQ
ST
QR
NP



ii MQ (3.7) add to tree
LT (3.8) add to tree
MT (4.1) add to tree
NQ (4.7) add to tree
MN (5.3) reject
ST (6.6) add to tree
QR (6.6) add to tree
NP (6.8) add to tree
reject remaining arcs

b Start off the tree with QT and PR then apply Kruskal's algorithm.
Prim's algorithm requires the 'growing' tree to be connected at all times. When using Kruskal's algorithm the tree can be built from non-connected sub-trees.



Solutionbank D1

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Exercise E, Question 3

Question:

	A	B	C	D	E	F
A	–	124	52	87	58	97
B	124	–	114	111	115	84
C	52	114	–	67	103	98
D	87	111	67	–	41	117
E	58	115	103	41	–	121
F	97	84	98	117	121	–

The table shows the distances, in mm, between six nodes A to F in a network.

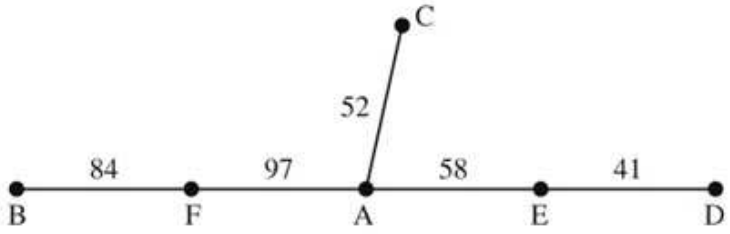
a Use Prim's algorithm, starting at A, to solve the minimum connector problem for this table of distances. You must explain your method carefully and indicate clearly the order in which you selected the arcs.

b Draw a sketch showing the minimum spanning tree and find its length.

Solution:

	↓1	↓6	↓2	↓4	↓3	↓5
	A	B	C	D	E	F
A	–	124	52	87	58	97
B	124	–	114	111	115	(84)
C	(52)	114	–	67	103	98
D	87	111	67	–	(41)	117
E	(58)	115	103	41	–	121
F	(97)	84	98	117	121	–

Arcs in order
AC
AE
DE
AF
BF



Length 332 mm

Solutionbank D1

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Exercise E, Question 4

Question:

It is intended to network five computers at a large theme park. There is one computer at the office and one at each of the four different entrances. Cables need to be laid to link the computers. Cable laying is expensive, so a minimum total length of cable is required. The table shows the shortest distances, in metres, between the various sites.

	Office	Entrance 1	Entrance 2	Entrance 3	Entrance 4
Office	–	1514	488	980	945
Entrance 1	1514	–	1724	2446	2125
Entrance 2	488	1724	–	884	587
Entrance 3	980	2446	884	–	523
Entrance 4	945	2125	587	523	–

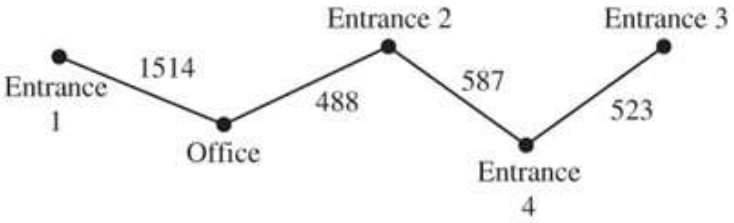
a Starting at Entrance 2, demonstrate the use of Prim's algorithm and hence find a minimum spanning tree. You must make your method clear, indicating the order in which you selected the arcs in your final tree.

b Calculate the minimum total length of cable required.

Solution:

	↓2	↓5	↓1	↓4	↓3
	Office	Entrance 1	Entrance 2	Entrance 3	Entrance 4
Office	–	1514	(488)	980	945
Entrance 1	(1514)	–	1724	2446	2125
Entrance 2	488	1724	–	884	587
Entrance 3	980	2446	884	–	(523)
Entrance 4	945	2125	(587)	523	–

Arcs in order: Entrance 2–office; Entrance 2–Entrance 4; Entrance 4–Entrance 3; Office–Entrance 1



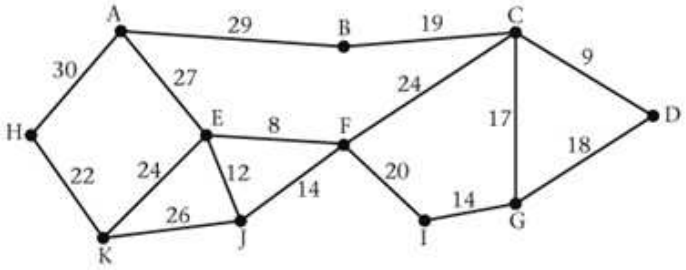
Length: 3112 m

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 5

Question:



You are to use Kruskal's algorithm to find a minimum spanning tree for the network shown.

a **i** Write down the order in which you selected the arcs.

ii Sketch your minimum spanning tree.

iii State the weight of your minimum spanning tree.

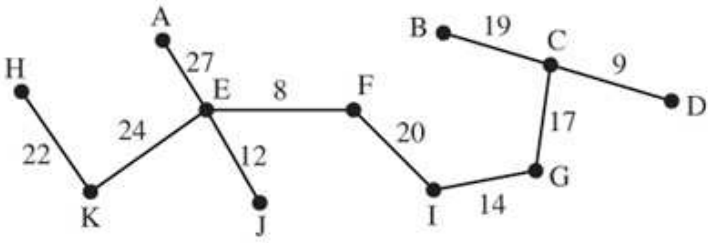
For any connected network,
E = the number of edges in the minimum spanning tree, and
V = the number of vertices in the network.

b Write down the relationship between E and V.

Solution:

a **i** Order of arcs
EF (8) add to tree
CD (9) add to tree
EJ (12) add to tree
FJ (14) reject
GI (14) add to tree
CG (17) add to tree
DG (18) reject
BC (19) add to tree
FI (20) add to tree
HK (22) add to tree
EK (24) add to tree
CF (24) reject
JK (26) reject
AE (27) add to tree
AB (29), AH (30) } reject remaining arcs

ii



iii weight: 172

b $V = E + 1$

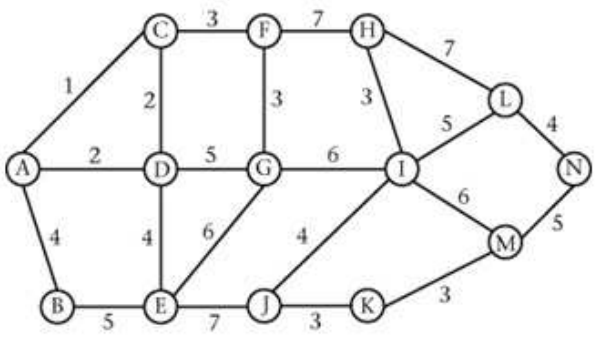
Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 6

Question:

A company is to install power lines to buildings on a large industrial estate. The lines are to be laid by the side of the roads on the estate. The estate is shown as a network opposite. The buildings are designated A, B, C, ..., N and the distances between them are given in hundreds of metres. The manager wants to minimise the total length of power line to be used.



a Use Kruskal's algorithm to obtain a minimum spanning tree for the network and hence determine the minimum length of power line needed.

Owing to a change of circumstances, the company modifies its plans for the estate. The result is that the road from F to G now has a length of 700 metres.

b Determine the new minimum total length of power line.

Solution:

a Order of arcs

AC (1) add to tree

AD (2) add to tree

CD (2) reject

CF (3) add to tree

FG (3) add to tree

HI (3) add to tree

KM (3) add to tree

JK (3) add to tree

AB (4) add to tree

DE (4) add to tree

IJ (4) add to tree

LN (4) add to tree

DG (5) reject

BE (5) reject

IL (5) add to tree

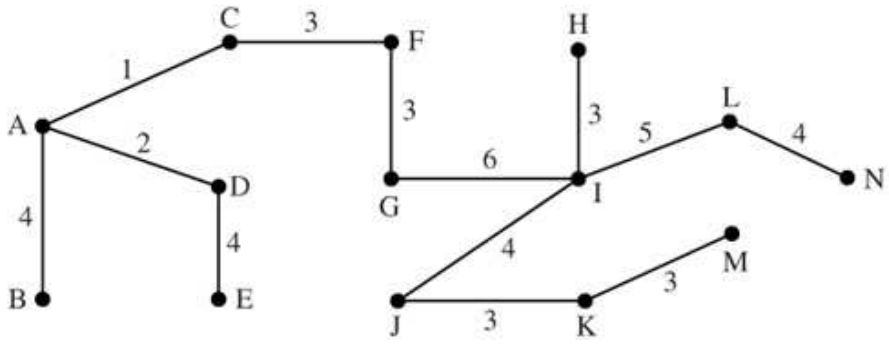
MN (5) reject

EG (6) reject

GI (6) add to tree

IM (6), FH (7), HL (7), EJ (7) reject remaining arcs

weight = 45 so 4500 m needed



b Remove FG (7) and replace with DG (5) weight = 47 so 4700 m

Solutionbank D1

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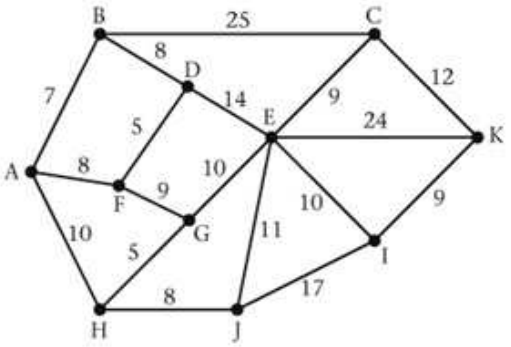
Exercise E, Question 7

Question:



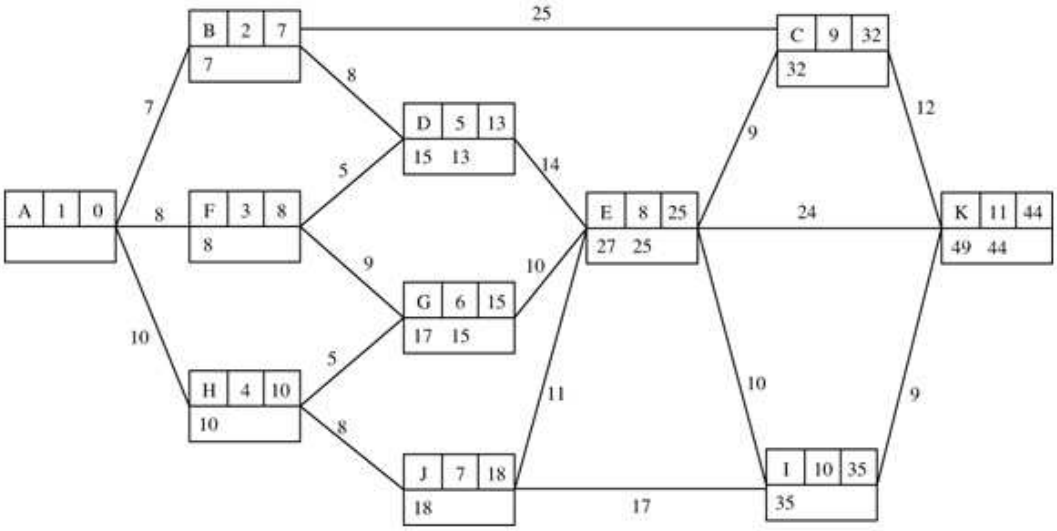
A weighted network is shown above. The number on each arc indicates the weight of that arc.

a Use Dijkstra's algorithm to find a path of least weight from A to K.
State clearly
i the order in which the vertices were labelled.
ii how you determined the path of least weight from your labelling.

b List all alternative paths of least weight.

c Describe a practical problem that could be modelled by the above network and solved using Dijkstra's algorithm.

Solution:



a **i** Possible paths are A–H–G–E–I–K
and A–H–J–I–K
and A–B–C–K
} Any one accepted

ii

44 − 9 = 35	IK		44 − 9 = 35	IK		44 − 12 = 32	CK
35 − 10 = 25	EI		35 − 17 = 18	JI		32 − 25 = 7	BC
25 − 10 = 15	GE	or	18 − 8 = 10	MJ	or	7 − 7 = 0	AB
15 − 5 = 10	HG		10 − 10 = 0	AH			
10 − 10 = 0	AH						

b A–H–G–E–I–K and A–H–J–I–K and A–B–C–K

c The arcs could be roads.
The nodes could be junctions
The number on each arc could be the distance in km.

The network, together with Dijkstra's algorithm,
could be used to find the shortest route from A to K.

Solutionbank D1

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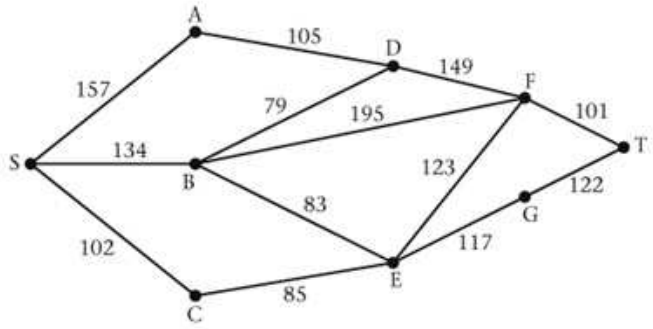
Exercise E, Question 8

Question:



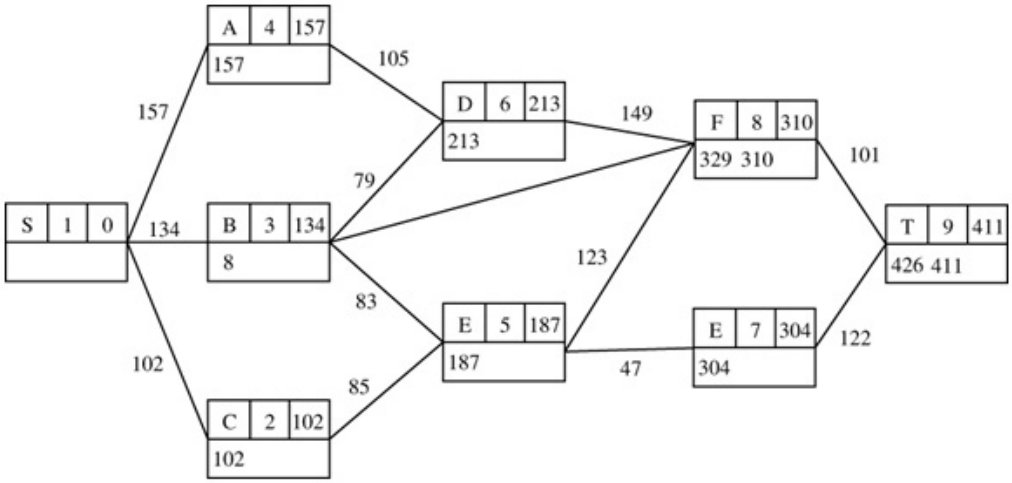
The network above shows the distances, in miles, between nine cities. Use Dijkstra's algorithm to determine the shortest route, and its length, between cities S and T. You must indicate clearly

i the order in which the vertices are labelled,

ii how you used your labelled diagram to decide which cities to include in the shortest route.

Solution:

a



Order of vertex labelling: S C B A E D G F T

Route : S–C–E–F–T

411 − 101 = 310 FT
310 − 123 = 187 EF
187 − 85 = 102 CE
102 − 102 = 0 SC