Solutionbank D1
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

a Implement this algorithm with the fractions

i $2\frac{1}{4}$

ii $1\frac{1}{3}$.

1 Write the fractions in the form $\frac{a}{b}$ and $\frac{c}{d}$.

2 Let $e = ad$.

3 Let $f = bc$.

4 Print 'answer is $\frac{e}{f}$'.

b What does this algorithm do?

Solution:

a **1** $\frac{a}{b} = \frac{9}{4}$ $\frac{c}{d} = \frac{4}{3}$ $a = 9, b = 4, c = 4, d = 3$

2 $e = ad = 9 \times 3 = 27$

3 $f = bc = 4 \times 4 = 16$

4 answer is $\frac{27}{16}$

b It divides the first fraction by the second fraction

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Exercise A, Question 2

Question:

a Implement this algorithm.

1 Let $A = 1, n = 1$.

2 Print A.

3 $A = A + 2n + 1$.

4 Let $n = n + 1$.

5 If $n \leq 10$ go to 2.

6 Stop.

b What does this algorithm produce?

Solution:

a **1** $A = 1 \quad n = 1$
2 Print 1
3 $A = 1 + 2 \times 1 + 1 = 4$
4 $n = 2$
5 $2 \leq 10$ go to 2
2 Print 4.
3 $A = 4 + 2 \times 2 + 1 = 9$
4 $n = 3$
5 $3 \leq 10$ go to 2
2 Print 9
3 $A = 9 + 2 \times 3 + 1 = 16$
4 $n = 4$
5 $4 \leq 10$ go to 2
2 Print 16
3 $A = 16 + 2 \times 4 + 1 = 25$
4 $n = 5$
5 $5 \leq 10$ go to 2
2 Print 25
3 $A = 25 + 2 \times 5 + 1 = 36$
4 $n = 6$
5 $6 \leq 10$ go to 2
2 Print 36
3 $A = 36 + 2 \times 6 + 1 = 49$
4 $n = 7$
5 $7 \leq 10$ go to 2
2 Print 49
3 $A = 49 + 2 \times 7 + 1 = 64$
4 $n = 8$
5 $8 \leq 10$ go to 2
2 Print 64
3 $A = 64 + 2 \times 8 + 1 = 81$
4 $n = 9$
5 $9 \leq 10$ go to 2
2 Print 81
3 $A = 81 + 2 \times 9 + 1 = 100$
4 $n = 10$
5 $10 \leq 10$ go to 2
2 Print 100
3 $A = 100 + 2 \times 10 + 1 = 121$
4 $n = 11$
5 $11 \not\leq 10$
6 stop

Output 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

b It finds the squares of the first 10 natural numbers.

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Exercise A, Question 3

Question:

a Use a trace table to implement the following algorithm when

i $A = 253$ and $r = 12$,

ii $A = 79$ and $r = 10$,

iii $A = 4275$ and $r = 50$.

1 Input A, r.

2 Let $C = \frac{A}{r}$ to 3 decimal places.

3 If $|r - C| \leq 10^{-2}$ go to 7.

4 Let $s = \frac{1}{2}(r + C)$ to 3 decimal places.

5 Let $r = s$.

6 Go to 2.

7 Print r.

8 Stop.

b What does the algorithm produce?

> This requires you to use the modulus function. If $x \neq y$, $|x - y|$ is the positive difference between x and y. For example, $|3.2 - 7| = 3.8$.

Solution:

a i

Step	A	r	c	$\|r-c\|$	s	Print r
1	253	12				
2			21.083			
3				9.083		
4					16.542	
5		16.542				
$6 \rightarrow 2$			15.294			
3				1.248		
4					15.918	
5		15.918				
$6 \rightarrow 2$			15.894			
3				0.024		
4					15.906	
5		15.906				
$6 \rightarrow 2$			15.906			
$3 \rightarrow 7$				0		
7						$r = 15.906$
8 stop						

ii

Step	A	r	c	$\|r-c\|$	s	Print r
1	79	10				
2				7.9		
3				2.1		
4					8.95	
5		8.95				
$6 \rightarrow 2$			8.827			
3				0.123		
4					8.889	
5		8.889				
$6 \rightarrow 2$			8.887			
$3 \rightarrow 7$				0.002		
7						Print 8.889

iii

Step	A	r	c	$\|r-c\|$	s	Print r
1	4275	50				
2			85.5			
3				35.5		
4					67.75	
5		67.75				
$6 \to 2$			63.100			
3				4.65		
4					65.425	
5		65.425				
$6 \to 2$			65.342			
3				0.083		
4					65.384	
5		65.384				
$6 \to 2$			65.383			
$3 \to 7$				0.001		
7						Print 65.384

b Finds the square root of A.

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Exercise A, Question 4

Question:

Use the algorithm in Example 3 to evaluate

a 244×125

b 125×244

c 256×123.

Solution:

a

A	*B*
~~244~~	~~125~~
~~122~~	~~250~~
61	500
~~30~~	~~1000~~
15	2000
7	4000
3	8000
1	16 000
Total	30 500

b

A	*B*
125	244
~~62~~	~~488~~
31	976
15	1952
7	3904
3	7808
1	15 616
Total	30 500

c

A	*B*
~~256~~	~~123~~
~~128~~	~~246~~
~~64~~	~~492~~
~~32~~	~~984~~
~~16~~	~~1968~~
~~8~~	~~3936~~
~~4~~	~~7872~~
~~2~~	~~15744~~
1	31 488
Total	31 488

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Exercise B, Question 1

Question:

Apply the flow chart in Example 5 to the following equations.

a $4x^2 - 12x + 9 = 0$

b $-6x^2 + 13x + 5 = 0$

c $3x^2 - 8x + 11 = 0$

Solution:

a

a	b	c	d	$d < 0$?	$d = 0$?	x
4	−12	9	0	No	Yes	1.5

Equal roots $x = 1.5$

b

a	b	c	d	$d < 0$?	$d = 0$?	x_1	x_2
−6	13	5	289	No	No	$-\frac{1}{3}$	$\frac{5}{2}$

Roots are $-\frac{1}{3}$ and $\frac{5}{2}$

c

a	b	c	d	$d < 0$?
3	−8	11	−68	Yes

No real roots

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Exercise B, Question 2

Question:

a Apply the flow chart in Example 6 to the following data.

i $u_1 = 28, u_2 = 26, u_3 = 23, u_4 = 25, u_5 = 21$

ii $u_1 = 11, u_2 = 8, u_3 = 9, u_4 = 8, u_5 = 5$

b If box 4 is altered to ⟨Is T > A?⟩, how will this affect the output?

c Which box would need to be altered if the algorithm had to be applied to a list of 8 numbers?

Solution:

a i

	n	A	T	$T < A$?	$n < 5$?
box 1	1	28			
box 2	2				
box 3			26		
box 4				Yes	
box 5		26			
box 6					Yes
box 2	3				
box 3			23		
box 4				Yes	
box 5		23			
box 6					Yes
box 2	4				
box 3			25		
box 4				No	
box 6					Yes
box 2	5				
box 3			21		
box 4				Yes	
box 5		21			
box 6					No
box 7	output is 21				

ii

	n	A	T	$T < A$?	$n < 5$?
box 1	1	11			
box 2	2				
box 3			8		
box 4				Yes	
box 5		8			
box 6					Yes
box 2	3				
box 3			9		
box 4				No	
box 6					Yes
box 2	4				
box 3			8		
box 4				No	
box 6					Yes
box 2	5				
box 3			5		
box 4				Yes	
box 5		5			
box 6					No
box 7	output is 5				

b It will find the largest number in the list.

c box 6 – changed to 'Is $n < 8$?'

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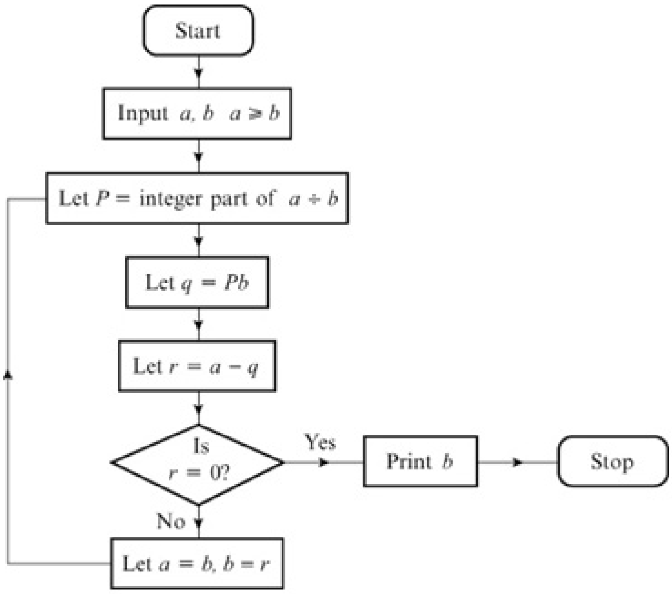
Exercise B, Question 3

Question:

Euclid's algorithm is applied to two non-zero integers a and b.

a Apply Euclid's algorithm to
  i 507, 52
  ii 884, 85
  iii 4845, 3795

b What does the algorithm do?



Solution:

a i

a	b	p	q	r	$r = 0$?
507	52				
		9			
			468		
				39	
					No
52	39				
		1			
			39		
				13	
					No
39	13				
		3			
			39		
				0	
					Yes

Print 13

ii

a	b	p	q	r	$r = 0$?
884	85				
		10			
			850		
				34	
					No
85	34				
		2			
			68		
				17	
					No
34	17				
		2			
			34		
				0	
					Yes

Print 17

iii

a	*b*	*p*	*q*	*r*	*r* = 0 ?
4845	3795				
		1			
			3795		
				1050	
					No
3795	1050				
		3			
			3150		
				645	
					No
1050	645				
		1			
			645		
				405	
					No
645	405				
		1			
			405		
				240	
					No
405	240				
		1			
			240		
				165	
					No
240	165				
		1			
			165		
				75	
					No
165	75				
		2			
			150		
				15	
					No
75	15				
		5			
			75		
				0	
					Yes

Print 15

b Finds the HCF.

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Exercise B, Question 4

Question:

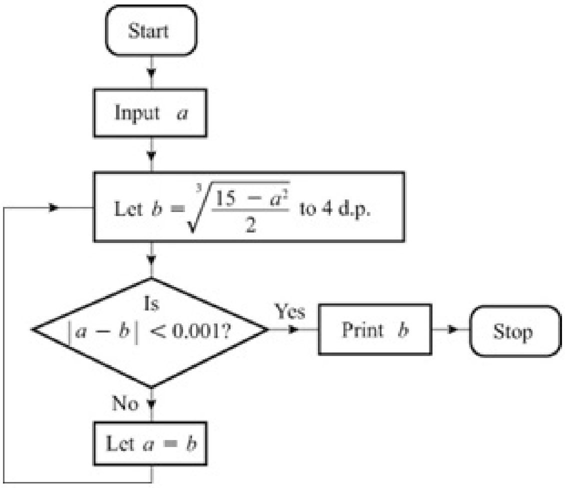
The equation $2x^3 + x^2 - 15 = 0$ may be solved by the iteration

$$x_{n+1} = \sqrt[3]{\frac{15 - x^2}{2}}$$

using the chart opposite.

a Use $a = 2$ to find a root of the equation.

b Use $a = 20$ to find a root of the equation. What do you notice?



Solution:

a

a	b	$\lvert a-b\rvert$	$\lvert a-b\rvert < 0.001$?
2	1.7652	0.2348	No
1.7652	1.8112	0.046	No
1.8112	1.8029	0.0083	No
1.8029	1.8044	0.0015	No
1.8044	1.8041	0.0003	Yes

output: 1.8041

b

a	b	$\lvert a-b\rvert$	$\lvert a-b\rvert < 0.001$?
20	−5.7740	25.774	No
−5.7740	−2.0931	3.6809	No
−2.0931	1.7446	3.8377	No
1.7446	1.8149	0.0703	No
1.8149	1.8022	0.0127	No
1.8022	1.8045	0.0023	No
1.8045	1.8041	0.0004	Yes

output 1.8041 – same root

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Exercise C, Question 1

Question:

Use the bubble sort to arrange the list

8 3 4 6 5 7 2

into

a ascending order,

b descending order.

Solution:

a

8 3 4 6 5 7 2

Bubbling left to right

1st pass	3	4	6	5	7	2	8
2nd pass	3	4	5	6	2	7	8
3rd pass	3	4	5	2	6	7	8
4th pass	3	4	2	5	6	7	8
5th pass	3	2	4	5	6	7	8
6th pass	2	3	4	5	6	7	8

sort complete

b Bubbling left to right

1st pass	8	4	6	5	7	3	2
2nd pass	8	6	5	7	4	3	2
3rd pass	8	6	7	5	4	3	2
4th pass	8	7	6	5	4	3	2

sort complete

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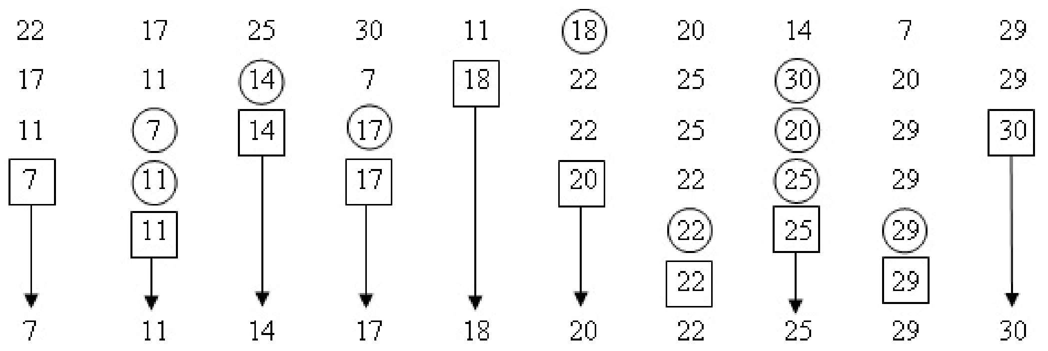
Exercise C, Question 2

Question:

Use a quick sort to arrange the list
22 17 25 30 11 18 20 14 7 29
into
a ascending order,
b descending order.

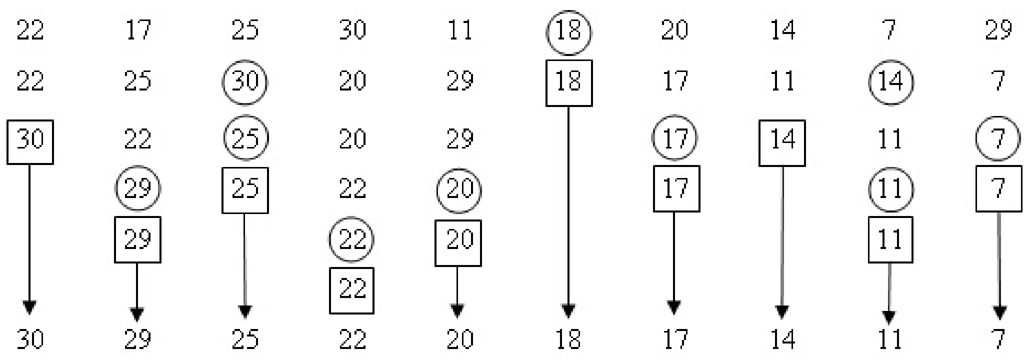
Solution:

a



sort complete

b



sort complete

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Exercise C, Question 3

Question:

Sort the letters below into alphabetical order using

a a bubble sort,

b a quick sort.

N H R K S C J E M P L

Solution:

a

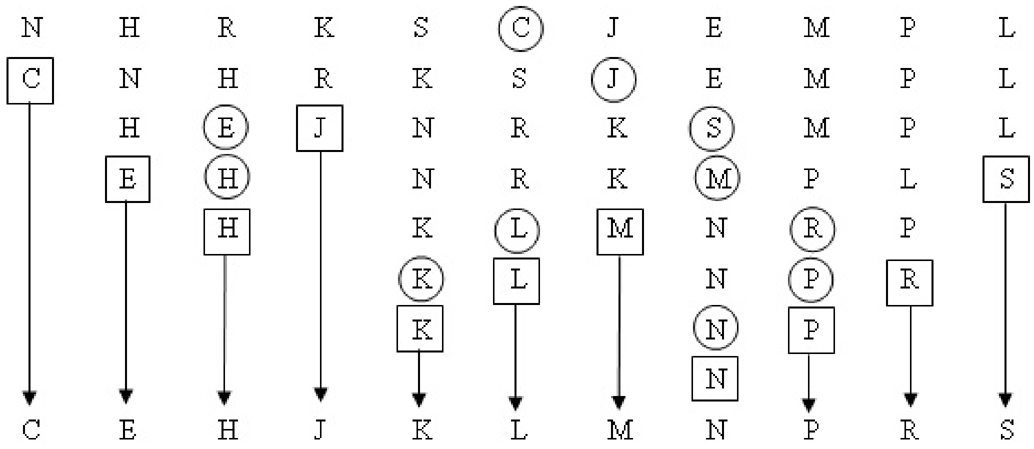
N H R K S C J E M P L

Bubbling left to right

H	N	K	R	C	J	E	M	P	L	S
H	K	N	C	J	E	M	P	L	R	S
H	K	C	J	E	M	N	L	P	R	S
H	C	J	E	K	M	L	N	P	R	S
C	H	E	J	K	L	M	N	P	R	S
C	E	H	J	K	L	M	N	P	R	S

sort complete

b



sort complete

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Exercise C, Question 4

Question:

The list shows the test results of a group of students.

Alex	33	Hugo	9
Alison	56	Janelle	89
Amy	93	Josh	37
Annie	51	Lucy	57
Dom	77	Myles	19
Greg	91	Sam	29
Harry	49	Sophie	77

Produce a list of students, in descending order of their marks, using

a a bubble sort.

b a quick sort.

Solution:

a

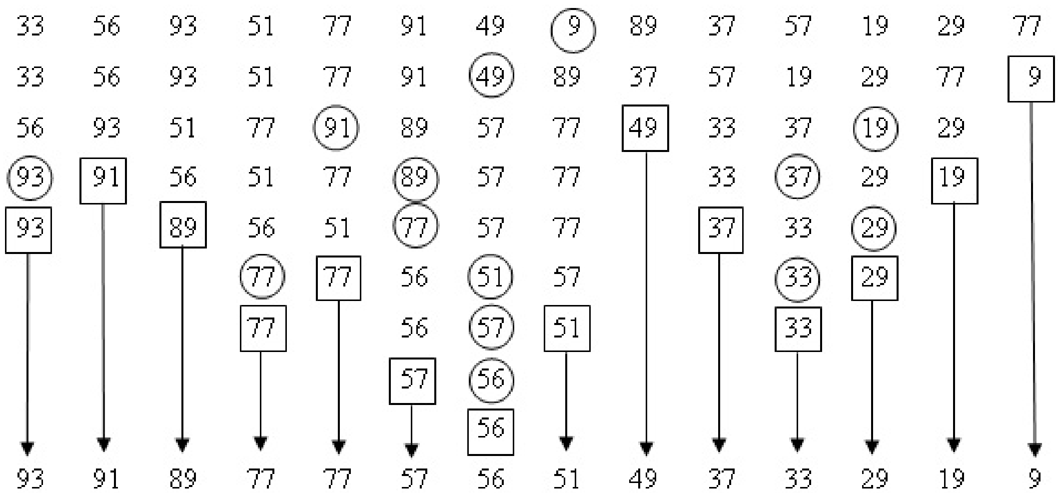
33	56	93	51	77	91	49	9	89	37	57	19	29	77
56	93	51	77	91	49	33	89	37	57	19	29	77	9
93	56	77	91	51	49	89	37	57	33	29	77	19	9
93	77	91	56	51	89	49	57	37	33	77	29	19	9
93	91	77	56	89	51	57	49	37	77	33	29	19	9
93	91	77	89	56	57	51	49	77	37	33	29	19	9
93	91	89	77	57	56	51	77	49	37	33	29	19	9
93	91	89	77	57	56	77	51	49	37	33	29	19	9
93	91	89	77	57	77	56	51	49	37	33	29	19	9
93	91	89	77	77	57	56	51	49	37	33	29	19	9

sort complete

So list is:

Amy	93	Annie	51
Greg	91	Harry	49
Janelle	89	Josh	37
Sophie }	77	Alex	33
Dom }		Sam	29
Lucy	57	Myles	19
Alison	56	Hugo	9

b



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Exercise C, Question 5

Question:

Sort the numbers listed below into ascending order using

a a bubble sort,

b a quick sort.

> This question gives you a good comparison between the efficiencies of the two algorithms.

453	330	405	792	516	162	465	870	431
927	129	348	34	107	64	253	382	411
147	389	597	414	620	425	73	275	212
482	302	52	868	144	65	471	930	766
243	578	274	630	281	732	114	517	322
748	517	492	331					

Solution:

The sort is left as an exercise for the reader.

The quick sort should be notably faster than the bubble sort.

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Exercise D, Question 1

Question:

Use the binary search algorithm to try to locate

a Connock,

b Walkey,

c Peabody.

in the list below.

1 Berry
2 Connock
3 Ladley
4 Sully
5 Tapner
6 Walkey
7 Ward
8 Wilson

Solution:

a 1st pivot $\left[\frac{1+8}{2}\right]=5$ Tapner. Connock before Tapner reject $5\to 8$

2nd pivot $\left[\frac{1+4}{2}\right]=3$ Ladley. Connock before Ladley reject $3\to 4$

3rd pivot $\left[\frac{1+2}{2}\right]=2$ Connock. Target found at position 2.

b 1st pivot $\left[\frac{1+8}{2}\right]=5$ Tapner. Walkey after Tapner reject $1\to 5$

2nd pivot $\left[\frac{6+8}{2}\right]=7$ Ward. Walkey before Ward reject $7\to 8$

3rd pivot 6 Walkey. Target found at position 6.

c 1st pivot $\left[\frac{1+8}{2}\right]=5$ Tapner. Peabody before Tapner reject $5\to 8$

2nd pivot $\left[\frac{1+4}{2}\right]=3$ Ladley. Peabody after Ladley reject $1\to 3$

3rd pivot 4 Sully. Sully is not Peabody reject 4.

List empty. Peabody not in list.

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Exercise D, Question 2

Question:

Use the binary search algorithm to try to locate
a 21,
b 5
in the list below.
1 3
2 4
3 7
4 9
5 10
6 13
7 15
8 17
9 18
10 20
11 21
12 24

Solution:

a 1st pivot $\left[\frac{1+12}{2}\right]=7$ (number 15) $21>15$ reject $1\to7$

2nd pivot $\left[\frac{8+12}{2}\right]=10$ (number 20) $21>20$ reject $8\to10$

3rd pivot $\left[\frac{11+12}{2}\right]=12$ (number 24) $21<24$ reject 12

4th pivot 11 (number 21). Target found at position 11.

b 1st pivot $\left[\frac{1+12}{2}\right]=7$ (number 15) $5<15$ reject $7\to12$

2nd pivot $\left[\frac{1+6}{2}\right]=4$ (number 9) $5<9$ reject $4\to6$

3rd pivot $\left[\frac{1+3}{2}\right]=2$ (number 4) $2<5$ reject $1\to2$

4th pivot 3 (number 7) $5\neq7$ reject 3

List empty. 5 not in list.

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Exercise D, Question 3

Question:

Use the binary search algorithm to try to locate
a Fredco,
b Matt,
c Elliot
in the list below.
1 Adam
2 Alex
3 Des
4 Doug
5 Ed
6 Emily
7 Fredco
8 George
9 Harry
10 Jess
11 Katie
12 Leo
13 Lottie
14 Louis
15 Matt
16 Miranda
17 Oli
18 Ramin
19 Saul
20 Simon

Solution:

a 1st pivot $\left[\frac{1+20}{2}\right]=11$ Katie. Fredco before Katie, reject $11 \rightarrow 20$

2nd pivot $\left[\frac{1+10}{2}\right]=6$ Emily. Fredco after Emily, reject $1 \rightarrow 6$

3rd pivot $\left[\frac{7+10}{2}\right]=9$ Harry. Fredco before Harry, reject $9 \rightarrow 10$

4th pivot $\left[\frac{7+8}{2}\right]=8$ George. Fredco before George, reject 8

5th pivot 7 Fredco Target found at position 7.

b 1st pivot $\left[\frac{1+20}{2}\right]=11$ Katie. Matt after Katie, reject $1 \rightarrow 11$

2nd pivot $\left[\frac{12+20}{2}\right]=16$ Miranda. Matt before Miranda, reject $16 \rightarrow 20$

3rd pivot $\left[\frac{12+15}{2}\right]=14$ Louis. Matt after Louis, reject $12 \rightarrow 14$

4th pivot 15 Matt. Target found at position 15.

c 1st pivot $\left[\frac{1+20}{2}\right]=11$ Katie. Elliot before Katie, reject $11 \rightarrow 20$

2nd pivot $\left[\frac{1+10}{2}\right]=6$ Emily. Elliot before Emily, reject $6 \rightarrow 10$

3rd pivot $\left[\frac{1+5}{2}\right]=3$ Des. Elliot after Des, reject $1 \rightarrow 3$

4th pivot $\left[\frac{4+5}{2}\right]=5$ Ed. Elliot after Ed, reject $4 \rightarrow 5$

List empty. Elliot not in list.

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Exercise D, Question 4

Question:

The 26 letters of the English alphabet are listed, in order.

a Apply the binary search algorithm to locate the letter P.

b What is the maximum number of iterations needed to locate any letter?

Solution:

a 1st pivot $\left[\frac{1+26}{2}\right] = 14$ N. P after N, reject $1 \rightarrow 14$

2nd pivot $\left[\frac{15+26}{2}\right] = 21$ U. P before U, reject $21 \rightarrow 26$

3rd pivot $\left[\frac{16+20}{2}\right] = 18$ R. P after R, reject $18 \rightarrow 20$

4th pivot $\left[\frac{16+17}{2}\right] = 17$ Q. P before Q, reject 17

5th pivot 16 P. Target found at position 16.

b After each pass the list halves.
After 1 pass at most 13 letters remain in the list.
After 2 passes at most 6 letters remain in the list.
After 3 passes at most 3 letters remain in the list.
After 4 passes at most 1 letter remains in the list.
After 5 passes the final letter is examined.
so at most 5 passes.

Alternative method

Each pass halves the list. So we use need to find the smallest value of n for which

$$\begin{aligned} 2^n &> 26 \\ n\log 2 &> \log 26 \\ n &> \frac{\log 26}{\log 2} \\ n &> 4.7 \\ \text{so } n &= 5. \end{aligned}$$

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Exercise D, Question 5

Question:

The binary search algorithm is applied to an ordered list of n items. Determine the maximum number of iterations needed when n is equal to

a 100

b 1000

c 10 000.

You may find it helpful to record the maximum length of the list after each iteration.

Solution:

a

Pass	1	2	3	4	5	6	7
Maximum number of items remaining in list	50	25	12	6	3	1	0

So 7 passes
alternative

$$2^m > 100$$
$$\log 2^m > \log 100$$
$$m \log 2 > \log 100$$
$$m > \frac{\log 100}{\log 2}$$
$$m > 6.64 \text{ so 7 passes}$$

b

Pass	1	2	3	4	5	6	7	8	9	10
Maximum number of items remainings in list	500	250	125	62	31	15	7	3	1	0

So 10 passes
alternative

$$2^m > 1000$$
$$\log 2^m > \log 1000$$
$$m \log 2 > \log 1000$$
$$m > \frac{\log 1000}{\log 2}$$
$$m > 9.97 \text{ so 10 passes}$$

c

$$2^m > 10\,000$$
$$\log 2^m > \log 10\,000$$
$$m \log 2 > \log 10\,000$$
$$m > \frac{\log 10\,000}{\log 2}$$
$$m > 13.3 \text{ so 14 passes}$$

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Exercise E, Question 1

Question:

18 4 23 8 27 19 3 26 30 35 32

The above items are to be packed in bins of size 50.

a Calculate the lower bound for the number of bins.

b Pack the items into the bins using

i the first-fit algorithm,

ii the first-fit decreasing algorithm,

iii the full-bin algorithm.

Solution:

a Lower bound $= \frac{18+4+23+8+27+19+3+26+30+35+32}{50} = \frac{225}{50}$

$= 4.5$

Therefore 5 bins (4 bins will be insufficient)

b **i** Bin 1: $18+4+23+3$

Bin 2: $8+27$

Bin 3: $19+26$

Bin 4: 30

Bin 5: 35

Bin 6: 32

ii Putting list into descending order

35 32 30 27 26 23 19 18 8 4 3

Bin 1: $35+8+4+3$

Bin 2: $32+18$

Bin 3: $30+19$

Bin 4: $27+23$

Bin 5: 26

iii For example

Bin 1: $32+18$ } Full bins

Bin 2: $27+23$ } Full bins

Bin 3: $35+8+4+3$ } Full bins

Bin 4: $19+26$

Bin 5: 30

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Exercise E, Question 2

Question:

Laura wishes to record the following television programmes onto DVDs, each of which can hold up to 3 hours of programmes.

Programme	A	B	C	D	E	F	G	H	I	J	K	L	M
Length (minutes)	30	30	30	45	45	60	60	60	60	75	90	120	120

a Apply the first-fit algorithm, in the order A to M, to determine the number of DVDs that need to be used. State which programmes should be recorded on each disc.

b Repeat part **a** using the first-fit decreasing algorithm.

c Is your answer to part **b** optimal? Give a reason for your answer.

Laura finds that her DVDs will only hold up to 2 hours of programmes.

d Use the full-bin algorithm to determine the number of DVDs she needs to use. State which programmes should be recorded on each disc.

Solution:

a Bin 1: A(30) + B(30) + C(30) + D(45) + E(45)
Bin 2: F(60) + G(60) + H(60)
Bin 3: I(60) + J(75)
Bin 4: K(90)
Bin 5: L(120)
Bin 6: M(120)

b Bin 1: M(120) + I(60)
Bin 2: L(120) + H(60)
Bin 3: K(90) + J(75)
Bin 4: G(60) + F(60) + E(45)
Bin 5: D(45) + C(30) + B(30) + A(30)

c $$\text{Lower bound} = \frac{30+30+30+45+45+60+60+60+60+75+90+120+120}{180}$$
$$= \frac{825}{180}$$
$= 4.5$ so 5 tapes needed at least

Since a minimum of 5 tapes are needed and **b** uses 5 tapes it is optimal

d For example
Bin 1: M(120)
Bin 2: L(120)
Bin 3: A(30) + K(90)
Bin 4: F(60) + G(60)
Bin 5: H(60) + I(60)
Bin 6: J(75) + E(45)
(Bins 1–6: Full bins)
Bin 7: B(30) + C(30) + D(45)

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Exercise E, Question 3

Question:

A small ferry has three car lanes, each 30 m long. There are 10 vehicles waiting to use the ferry.

	Vehicle	Length (m)
A	car	4 m
B	car + trailer	7 m
C	lorry	13 m
D	van	6 m
E	lorry	13 m

	Vehicle	Length (m)
F	car	4 m
G	lorry	12 m
H	lorry	14 m
I	van	6 m
J	lorry	11 m

a Apply the first-fit algorithm, in the order A to J. Is it possible to load all the vehicles using this method?

b Apply the first-fit decreasing algorithm. Is it possible to load all the vehicles using this method?

c Use full-bin packing to load all of the vehicles.

Solution:

a Bin 1: A(4) + B(7) + C(13) + D(6)
Bin 2: E(13) + F(4) + G(12)
Bin 3: H(14) + I(6)
Bin 4: J(11)
So not possible is 3 bins.

b Re-ordering list
H(14) C(13) E(13) G(12) J(11) B(7) D(6) I(6) A(4) F(4)
Bin 1: H(14) + C(13)
Bin 2: E(13) + G(12) + A(4)
Bin 3: J(11) + B(7) + D(6) + I(6)
Bin 4: F(4)
Still not possible in 3 bins

c Bin 1: H(14) + G(12) + A(4)
Bin 2: C(13) + E(13) + F(4)
Bin 3: J(11) + B(7) + D(6) + I(6)

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Exercise E, Question 4

Question:

The ground floor of an office block is to be fully recarpeted, with specially made carpet which incorporate the firm's logo. The carpet comes in rolls of 15 m.

The following lengths are required.

A	3 m	D	4 m	G	5 m	J	7 m
B	3 m	E	4 m	H	5 m	K	8 m
C	4 m	F	4 m	I	5 m	L	8 m

Determine how the lengths should be cut from the rolls using

a the first-fit algorithm A to L,

b the first-fit decreasing algorithm,

c full-bin packing.

In each case, state the number of rolls used and the amount of wasted carpet.

Solution:

a Bin 1: A(3) B(3) C(4) D(4)
Bin 2: E(4) F(4) G(5)
Bin 3: H(5) I(5)
Bin 4: J(7) K(8)
Bin 5: L(8)
5 rolls used and 15 m wasted.

b For example
Bin 1: L(8) J(7)
Bin 2: K(8) I(5)
Bin 3: H(5) G(5) F(4)
Bin 4: E(4) D(4) C(4) B(3)
Bin 5: A(3)
5 rolls used and 15 m wasted.

c For example
Bin 1: L(8) J(7)
Bin 2: G(5) H(5) I(5)
Bin 3: C(4) D(4) E(4) B(3)
Bin 4: K(8) F(4) A(3)
4 rolls used and no wastage.

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Exercise E, Question 5

Question:

Eight computer programs need to be recorded onto 40 MB discs.
The size of each program is given below.

Programme	A	B	C	D	E	F	G	H
Size (MB)	8	16	17	21	22	24	25	25

a Use the first-fit decreasing algorithm to determine which programs should be recorded onto each disc.

b Calculate a lower bound for the number of discs needed.

c Explain why it is not possible to record these programs on the number of discs found in part **b**.

Consider the programs over 20 MB in size.

Solution:

a Bin 1: H(25) A(8)
Bin 2: G(25)
Bin 3: F(24) B(16)
Bin 4: E(22) C(17)
Bin 5: D(21)

b $\frac{8+16+17+21+22+24+25+25}{40}=\frac{158}{40}=3.95$

$\therefore$ Lower bound is 4

c There are 5 programs over 20 MB. It is not possible for any two of these to share a bin. So at least 5 bins will be needed, so 4 will be insufficient.

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Exercise F, Question 1

Question:

Use the bubble-sort algorithm to sort, in ascending order, the list:

27 15 2 38 16 1

giving the state of the list at each stage. [*E*]

Solution:

Bubbling left to right

Initial list	27	15	2	38	16	1
1st pass	15	2	27	16	1	38
2nd pass	2	15	16	1	27	38
3rd pass	2	15	1	16	27	38
4th pass	2	1	15	16	27	38
5th pass	1	2	15	16	27	38

No further changes $\therefore$ sorted

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Exercise F, Question 2

Question:

a Use the bubble-sort algorithm to sort, in descending order, the list:
25 42 31 22 26 41
giving the state of the list on each occasion when two values are interchanged.

b Find the *maximum* number of interchanges needed to sort a list of six pieces of data using the bubble-sort algorithm. [*E*]

Solution:

a Bubbling left to right

Initial list	25	42	31	22	26	41
1st pass	42	31	25	26	41	22
2nd pass	42	31	26	41	25	22
3rd pass	42	31	41	26	25	22
4th pass	42	41	31	26	25	22

No further changes ∴ sorted

b 15

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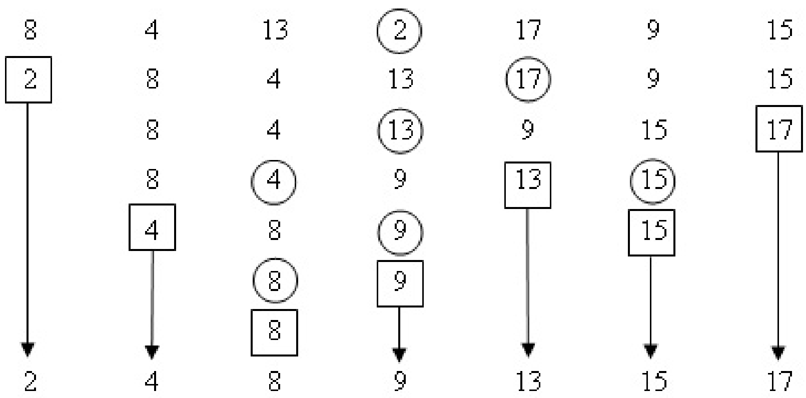
Exercise F, Question 3

Question:

8 4 13 2 17 9 15
This list of numbers is to be sorted into desending order.
Perform a quick sort to obtain the sorted list, giving the state of the list after each rearrangement. [*E*]

Solution:



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Exercise F, Question 4

Question:

111 103 77 81 98 68 82 115 93

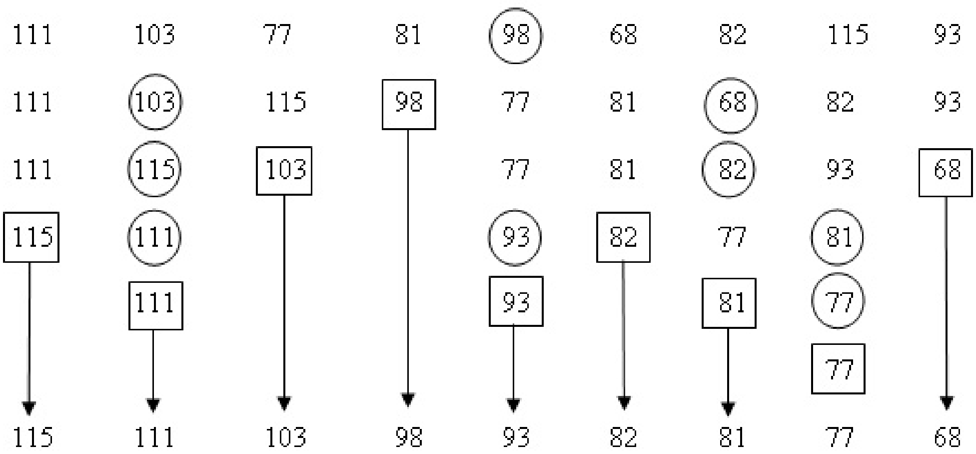
a The list of numbers above is to be sorted into descending order. Perform a quick-sort to obtain the sorted list, giving the state of the list after each rearrangement and indicating the pivot elements used.

b **i** Use the first-fit decreasing bin-packing algorithm to fit the data into bins of size 200.

ii Explain how you decided in which bin to place the number 77. [*E*]

Solution:

a



b **i** Bin1: 115 + 82
Bin2: 111 + 81
Bin3: 103 + 93
Bin4: 98 + 77
Bin5: 68

ii No room in bin 1(3 left) or bin 2 (8 left) or bin 3 (4 left) but room in bin 4.

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Exercise F, Question 5

Question:

Trishna wishes to video eight television programmes. The lengths of the programmes, in minutes, are:

75 100 52 92 30 84 42 60

Trishna decides to use 2-hour (120-minute) video tapes only to record all of these programmes.

a Explain how to use a first-fit decreasing bin-packing algorithm to find the solution that uses the fewest tapes and determine the total amount of unused tape.

b Determine whether it is possible for Trishna to record an additional two 25-minute programmes on these 2-hour tapes, without using another video tape. [*E*]

Solution:

a Rank the times in descending order and use them in this order. Number the bins, starting at 1.
Place each recording time into the first available bin, starting with bin 1 each time.

100 92 84 75 60 52 42 30

Bin 1: 100
Bin 2: 92
Bin 3: 84 + 30
Bin 4: 75 + 42
Bin 5: 60 + 52

$$\begin{aligned}\text{unused tape} &= 5\times120-(100+92+84+75+60+52+42+30)\\ &= 600-535\\ &= 65\text{ minutes}\end{aligned}$$

b There is room on tape 2 for one of the 25-minute programmes but no room on any tape for the second programme.

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Exercise F, Question 6

Question:

A DIY enthusiast requires the following 14 pieces of wood as shown in the table.

Length in metres	0.4	0.6	1	1.2	1.4	1.6
Number of pieces	3	4	3	2	1	1

The DIY store sells wood in 2 m and 2.4 m lengths. He considers buying six 2 m lengths of wood.

a Explain why he will not be able to cut all of the lengths he requires from these six 2 m lengths.

b He eventually decides to buy 2.4 m lengths. Use a first-fit decreasing bin-packing algorithm to show how he could use six 2.4 m lengths to obtain the pieces he requires.

c Obtain a solution that only requires five 2.4 m lengths. [*E*]

Solution:

a For example, the length total 12 m so no wastage is permitted. We are therefore seeking a full bin solution.

The two 1.2 m length can not be 'made up' to 2 m bins since these are only 3×0.4 m length. Two of these can be used to make a full bin, $1.2+0.4+0.4$, but the second 1.2 m can not be made up to 2 m since there is only 1 remaining 0.4 m length.

b Bin 1: $1.6 + 0.6$
Bin 2: $1.4 + 1$
Bin 3: $1.2 + 1.2$
Bin 4: $1 + 1 + 0.4$
Bin 5: $0.6 + 0.6 + 0.6 + 0.4$
Bin 6: 0.4

c For example
Bin 1: $1.6+0.4+0.4$
Bin 2: $1.4+1$
Bin 3: $1.2+1.2$
Bin 4: $1+1+0.4$
Bin 5: $0.6+0.6+0.6+0.6$

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Exercise F, Question 7

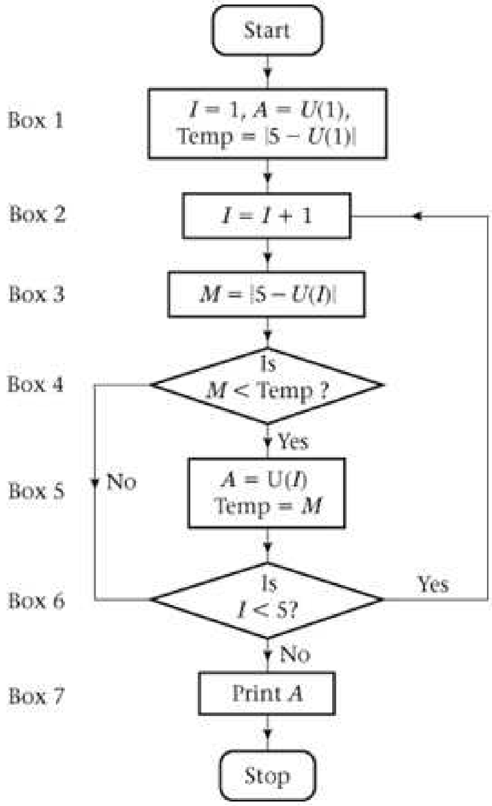
Question:

Note: This question uses the modulus function. If $x \neq y, |x-y|$ is the positive difference between x and y, e.g. $|5-6.1|=1.1$. The algorithm described by the flow chart below is to be applied to the five pieces of data below.

$U(1)=6.1, U(2)=6.9, U(3)=5.7, U(4)=4.8, U(5)=5.3$

a Obtain the final output of the algorithm using the five values given for $U(1)$ to $U(5)$.

b In general, for any set of values $U(1)$ to $U(5)$, explain what the algorithm achieves.



c If Box 4 in the flow chart is altered to
'Is M > Temp ?'
state what the algorithm now achieves. [*E*]

Solution:

a

	I	*M*	Box 4	*A*	Temp	Box 6
Initial conditions	1	-		6.1	1.1	
1st pass	2	1.9	No	6.1	1.1	Yes
2nd pass	3	0.7	Yes	5.7	0.7	Yes
3rd pass	4	0.2	Yes	4.8	0.2	Yes
4th pass	5	0.3	No			No

output 4.8

b It selects the number nearest to 5.

c It would select the number furthest from 5.