Solutionbank
Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise A, Question 1

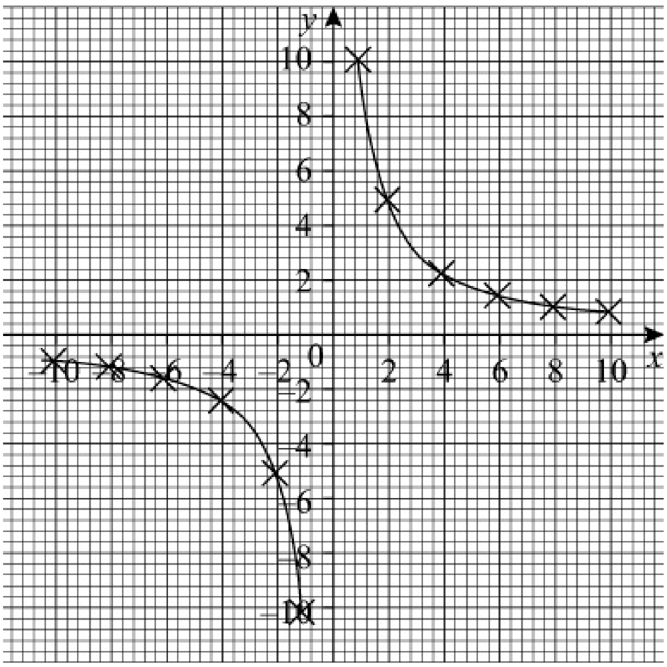
Question:

A curve is given by the parametric equations $x = 2t$, $y = \frac{5}{t}$ where $t \neq 0$. Complete the table and draw a graph of the curve for $-5 \leq t \leq 5$.

t	−5	−4	−3	−2	−1	−0.5	0.5	1	2	3	4	5
$x = 2t$	−10	−8				−1						
$y = \frac{5}{t}$	−1	−1.25					10					

Solution:

t	−5	−4	−3	−2	−1	−0.5	0.5	1	2	3	4	5
$x=2t$	−10	−8	−6	−4	−2	−1	1	2	4	6	8	10
$y=\frac{5}{t}$	−1	−1.25	−1.67	−2.5	−5	−10	10	5	2.5	1.67	1.25	1



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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise A, Question 2

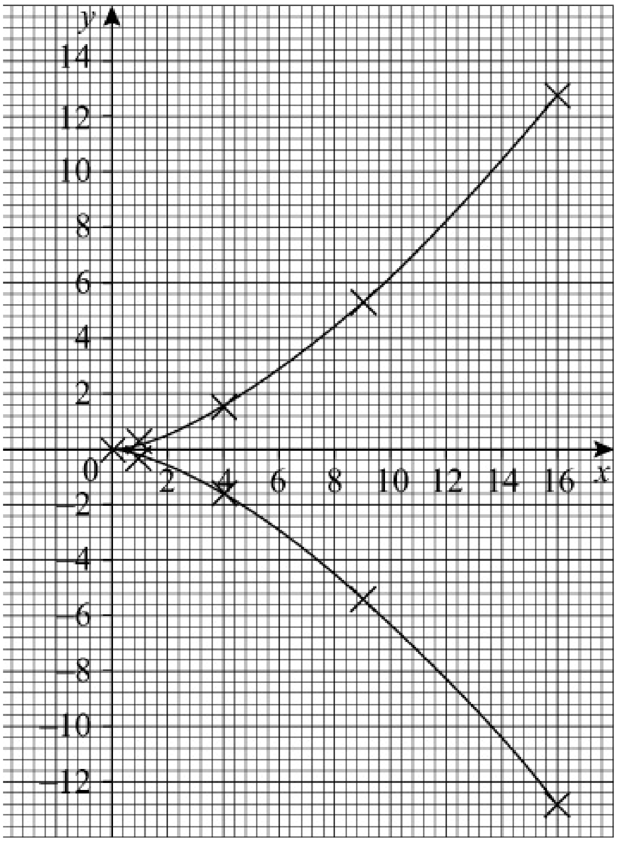
Question:

A curve is given by the parametric equations $x = t^2$, $y = \frac{t^3}{5}$. Complete the table and draw a graph of the curve for $-4 \leq t \leq 4$.

t	−4	−3	−2	−1	0	1	2	3	4
$x = t^2$	16								
$y = \frac{t^3}{5}$	−12.8								

Solution:

t	−4	−3	−2	−1	0	1	2	3	4
$x = t^2$	16	9	4	1	0	1	4	9	16
$y = \frac{t^3}{5}$	−12.8	−5.4	−1.6	−0.2	0	0.2	1.6	5.4	12.8



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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise A, Question 3

Question:

Sketch the curves given by these parametric equations:

(a) $x = t - 2$, $y = t^2 + 1$ for $-4 \leq t \leq 4$

(b) $x = t^2 - 2$, $y = 3 - t$ for $-3 \leq t \leq 3$

(c) $x = t^2$, $y = t(5 - t)$ for $0 \leq t \leq 5$

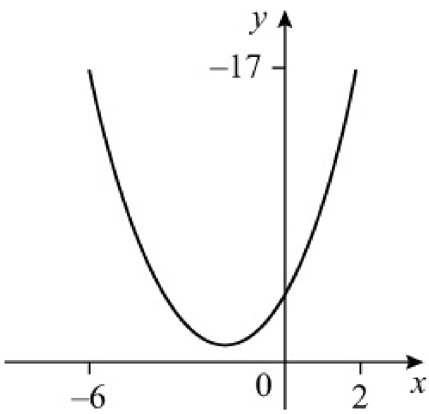
(d) $x = 3\sqrt{t}$, $y = t^3 - 2t$ for $0 \leq t \leq 2$

(e) $x = t^2$, $y = (2 - t)(t + 3)$ for $-5 \leq t \leq 5$

Solution:

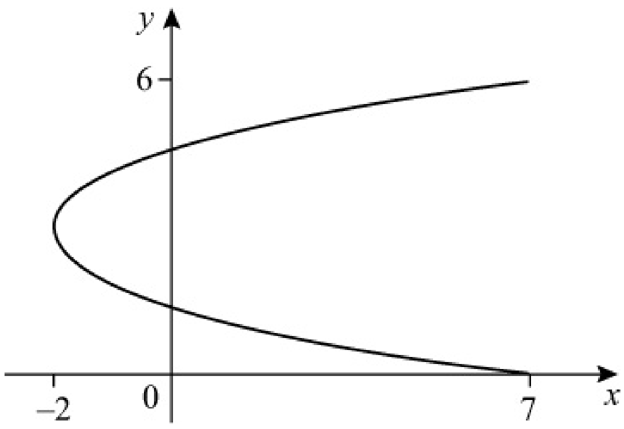
(a)

t	-4	-3	-2	-1	0	1	2	3	4
$x = t - 2$	-6	-5	-4	-3	-2	-1	0	1	2
$y = t^2 + 1$	17	10	5	2	1	2	5	10	17



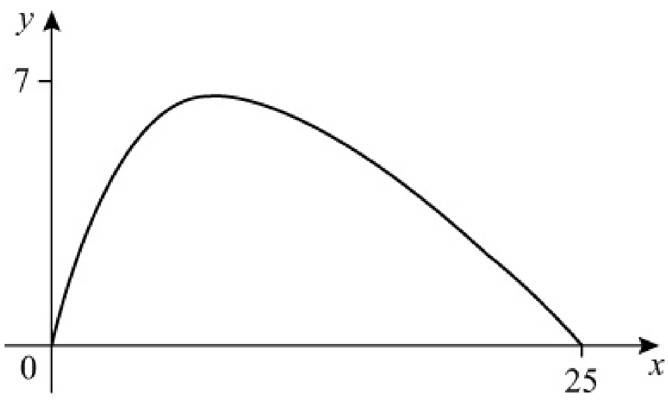
(b)

t	-3	-2	-1	0	1	2	3
$x = t^2 - 2$	7	2	-1	-2	-1	2	7
$y = 3 - t$	6	5	4	3	2	1	0



(c)

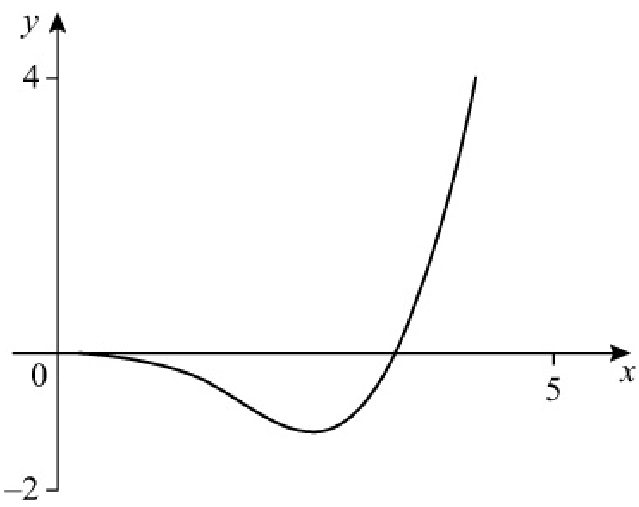
t	0	1	2	3	4	5
$x=t^2$	0	1	4	9	16	25
$y=t(5-t)$	0	4	6	6	4	0



(d)

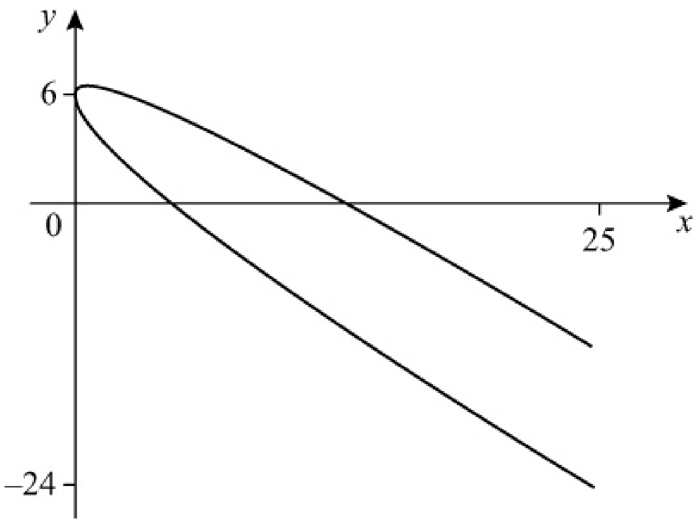
t	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$x=3\sqrt{t}$	0	1.5	2.12	2.60	3	3.35	3.67	3.97	4.24
$y=t^3-2t$	0	−0.48	−0.88	−1.08	−1	−0.55	0.38	1.86	4

Answers have been rounded to 2 d.p.



(e)

t	-5	-4	-3	-2	-1	0	1	2	3	4	5
$x = t^2$	25	16	9	4	1	0	1	4	9	16	25
$y = (2-t)(t+3)$	-14	-6	0	4	6	6	4	0	-6	-14	-24



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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise A, Question 4

Question:

Find the cartesian equation of the curves given by these parametric equations:

(a) $x = t - 2, y = t^2$

(b) $x = 5 - t, y = t^2 - 1$

(c) $x = \frac{1}{t}, y = 3 - t, t \neq 0$

(d) $x = 2t + 1, y = \frac{1}{t}, t \neq 0$

(e) $x = 2t^2 - 3, y = 9 - t^2$

(f) $x = \sqrt{t}, y = t (9 - t)$

(g) $x = 3t - 1, y = (t - 1) (t + 2)$

(h) $x = \frac{1}{t - 2}, y = t^2, t \neq 2$

(i) $x = \frac{1}{t + 1}, y = \frac{1}{t - 2}, t \neq - 1, t \neq 2$

(j) $x = \frac{t}{2t - 1}, y = \frac{t}{t + 1}, t \neq - 1, t \neq \frac{1}{2}$

Solution:

(a) $x = t - 2, y = t^2$

$x = t - 2$

$t = x + 2$

Substitute $t = x + 2$ into $y = t^2$

$y = (x + 2)^2$

So the cartesian equation of the curve is $y = (x + 2)^2$.

(b) $x = 5 - t, y = t^2 - 1$

$x = 5 - t$
$t = 5 - x$
Substitute $t = 5 - x$ into $y = t^2 - 1$
$y = (5 - x)^2 - 1$
$y = 25 - 10x + x^2 - 1$
$y = x^2 - 10x + 24$
So the cartesian equation of the curve is $y = x^2 - 10x + 24$.

(c) $x = \frac{1}{t}$, $y = 3 - t$

$x = \frac{1}{t}$

$t = \frac{1}{x}$

Substitute $t = \frac{1}{x}$ into $y = 3 - t$

$y = 3 - \frac{1}{x}$

So the cartesian equation of the curve is $y = 3 - \frac{1}{x}$.

(d) $x = 2t + 1$, $y = \frac{1}{t}$

$x = 2t + 1$
$2t = x - 1$
$t = \frac{x - 1}{2}$

Substitute $t = \frac{x - 1}{2}$ into $y = \frac{1}{t}$

$$y = \frac{1}{\left(\frac{x - 1}{2}\right)}$$

$$y = \frac{2}{x - 1} \qquad \left[\text{Note: This uses } \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a} \right]$$

So the cartesian equation of the curve is $y = \frac{2}{x - 1}$.

(e) $x = 2t^2 - 3, y = 9 - t^2$

$x = 2t^2 - 3$

$2t^2 = x + 3$

$t^2 = \frac{x + 3}{2}$

Substitute $t^2 = \frac{x+3}{2}$ into $y = 9 - t^2$

$y = 9 - \frac{x + 3}{2}$

$y = \frac{18 - (x + 3)}{2}$

$y = \frac{15 - x}{2}$

So the cartesian equation is $y = \frac{15 - x}{2}$.

(f) $x = \sqrt{t}, y = t (9 - t)$

$x = \sqrt{t}$

$t = x^2$

Substitute $t = x^2$ into $y = t (9 - t)$

$y = x^2 (9 - x^2)$

So the cartesian equation is $y = x^2 (9 - x^2)$.

(g) $x = 3t - 1, y = (t - 1) (t + 2)$

$x = 3t - 1$

$3t = x + 1$

$t = \frac{x + 1}{3}$

Substitute $t = \frac{x+1}{3}$ into $y = (t - 1) (t + 2)$

$y = \left(\frac{x + 1}{3} - 1 \right) \left(\frac{x + 1}{3} + 2 \right)$

$y = \left(\frac{x + 1}{3} - \frac{3}{3} \right) \left(\frac{x + 1}{3} + \frac{6}{3} \right)$

$y = \left(\frac{x + 1 - 3}{3} \right) \left(\frac{x + 1 + 6}{3} \right)$

$y = \left(\frac{x - 2}{3} \right) \left(\frac{x + 7}{3} \right)$

$y = \frac{1}{9}\left(x - 2\right)\left(x + 7\right)$

So the cartesian equation of the curve is $y = \frac{1}{9}\left(x - 2\right)\left(x + 7\right)$.

(h) $x = \frac{1}{t - 2}, y = t^2$

$x = \frac{1}{t - 2}$

$x\left(t - 2\right) = 1$

$t - 2 = \frac{1}{x}$

$t = \frac{1}{x} + 2$

$t = \frac{1}{x} + \frac{2x}{x}$

$t = \frac{1 + 2x}{x}$

Substitute $t = \frac{1 + 2x}{x}$ into $y = t^2$

$y = \left(\frac{1 + 2x}{x}\right)^2$

So the cartesian equation of the curve is $y = \left(\frac{1 + 2x}{x}\right)^2$.

(i) $x = \frac{1}{t + 1}, y = \frac{1}{t - 2}$

$x = \frac{1}{t + 1}$

$\left(t + 1\right)x = 1$

$t + 1 = \frac{1}{x}$

$t = \frac{1}{x} - 1$

Substitute $t = \frac{1}{x} - 1$ into $y = \frac{1}{t - 2}$

$y = \frac{1}{\left(\frac{1}{x} - 1\right) - 2}$

$$y = \frac{1}{\frac{1}{x} - 3}$$

$$y = \frac{1}{\frac{1}{x} - \frac{3x}{x}}$$

$$y = \frac{1}{\left(\frac{1 - 3x}{x}\right)}$$

$$y = \frac{x}{1 - 3x} \quad \left[\text{Note: This uses } \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a} \right]$$

So the cartesian equation of the curve is $y = \frac{x}{1 - 3x}$.

(j) $x = \frac{t}{2t - 1}, y = \frac{t}{t + 1}$

$x = \frac{t}{2t - 1}$

$x \times \left(2t - 1 \right) = \frac{t}{2t - 1} \times \left(2t - 1 \right)$ Multiply each side by $\left(2t - 1 \right)$

$x \left(2t - 1 \right) = t$ Simplify

$2tx - x = t$ Expand the brackets

$2tx = t + x$ Add x to each side

$2tx - t = x$ Subtract $2t$ from each side

$t \left(2x - 1 \right) = x$ Factorise t

$\frac{t \left(2x - 1 \right)}{\left(2x - 1 \right)} = \frac{x}{2x - 1}$ Divide each side by $\left(2x - 1 \right)$

$t = \frac{x}{2x - 1}$ Simplify

Substitute $t = \frac{x}{2x - 1}$ into $y = \frac{t}{t + 1}$

$$y = \frac{\left(\frac{x}{2x - 1} \right)}{\left(\frac{x}{2x - 1} + 1 \right)}$$

$$y = \frac{\left(\frac{x}{2x - 1} \right)}{\left(\frac{x}{2x - 1} + \frac{2x - 1}{2x - 1} \right)}$$

$$y = \frac{\left(\frac{x}{2x - 1} \right)}{\left(\frac{x + 2x - 1}{2x - 1} \right)}$$

$$y = \frac{\left(\frac{x}{2x - 1} \right)}{\left(\frac{3x - 1}{2x - 1} \right)}$$

$$y = \frac{x}{3x - 1} \qquad \left[\text{Note: This uses } \frac{\left(\frac{a}{b} \right)}{\left(\frac{c}{b} \right)} = \frac{a}{c} \right]$$

So the cartesian equation of the curve is $y = \frac{x}{3x - 1}$.

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Coordinate geometry in the (x, y) plane
Exercise A, Question 5

Question:

Show that the parametric equations:
(i) $x = 1 + 2t$, $y = 2 + 3t$

(ii) $x = \frac{1}{2t-3}$, $y = \frac{t}{2t-3}$, $t \neq \frac{3}{2}$

represent the same straight line.

Solution:

(i) $x = 1 + 2t$, $y = 2 + 3t$

$x = 1 + 2t$

$2t = x - 1$

$t = \frac{x-1}{2}$

Substitute $t = \frac{x-1}{2}$ into $y = 2 + 3t$

$y = 2 + 3\left(\frac{x-1}{2}\right)$

$y = 2 + 3\left(\frac{x}{2} - \frac{1}{2}\right)$

$y = 2 + \frac{3x}{2} - \frac{3}{2}$

$y = \frac{3x}{2} + \frac{1}{2}$

(ii) $x = \frac{1}{2t-3}$, $y = \frac{t}{2t-3}$

$\frac{y}{x} = \frac{\left(\frac{t}{2t-3}\right)}{\left(\frac{1}{2t-3}\right)}$ Note: $\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{b}\right)} = \frac{a}{c}$

$\frac{y}{x} = t$

Substitute $t = \frac{y}{x}$ into $x = \frac{1}{2t-3}$

$$x = \frac{1}{2\left(\frac{y}{x}\right) - 3}$$

$$x\left[2\left(\frac{y}{x}\right) - 3\right] = 1$$

$$2y - 3x = 1$$

$$2y = 3x + 1$$

$$y = \frac{3}{2}x + \frac{1}{2}$$

The cartesian equations of (i) and (ii) are the same, so they represent the same straight line.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 1

Question:

Find the coordinates of the point(s) where the following curves meet the x-axis:

(a) $x = 5 + t, y = 6 - t$

(b) $x = 2t + 1, y = 2t - 6$

(c) $x = t^2, y = (1 - t)(t + 3)$

(d) $x = \frac{1}{t}, y = \sqrt{(t - 1)(2t - 1)}, t \neq 0$

(e) $x = \frac{2t}{1 + t}, y = t - 9, t \neq -1$

Solution:

(a) $x = 5 + t, y = 6 - t$
When $y = 0$
$6 - t = 0$
so $t = 6$
Substitute $t = 6$ into $x = 5 + t$
$x = 5 + 6$
$x = 11$
So the curve meets the x-axis at (11, 0).

(b) $x = 2t + 1, y = 2t - 6$
When $y = 0$
$2t - 6 = 0$
$2t = 6$
so $t = 3$
Substitute $t = 3$ into $x = 2t + 1$
$x = 2(3) + 1$
$x = 6 + 1$
$x = 7$
So the curve meets the x-axis at (7, 0).

(c) $x = t^2, y = (1 - t)(t + 3)$
When $y = 0$

$(1-t)(t+3)=0$

so $t=1$ and $t=-3$

(1) Substitute $t=1$ into $x=t^2$

$x=1^2$

$x=1$

(2) Substitute $t=-3$ into $x=t^2$

$x=(-3)^2$

$x=9$

So the curve meets the x-axis at (1, 0) and (9, 0).

(d) $x=\frac{1}{t}$, $y=\sqrt{(t-1)(2t-1)}$

When $y=0$

$\sqrt{(t-1)(2t-1)}=0$

$(t-1)(2t-1)=0$

so $t=1$ and $t=\frac{1}{2}$

(1) Substitute $t=1$ into $x=\frac{1}{t}$

$x=\frac{1}{(1)}$

$x=1$

(2) Substitute $t=\frac{1}{2}$ into $x=\frac{1}{t}$

$x=\frac{1}{\left(\frac{1}{2}\right)}$

$x=2$

So the curve meets the x-axis at (1, 0) and (2, 0).

(e) $x=\frac{2t}{1+t}$, $y=t-9$

When $y=0$

$t-9=0$

so $t=9$

Substitute $t=9$ into $x=\frac{2t}{1+t}$

$x=\frac{2(9)}{1+(9)}$

$$x = \frac{18}{10}$$

$$x = \frac{9}{5}$$

So the curve meets the x-axis at $\left(\frac{9}{5},\ 0 \right)$.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 2

Question:

Find the coordinates of the point(s) where the following curves meet the y-axis:

(a) $x = 2t$, $y = t^2 - 5$

(b) $x = \sqrt{(3t - 4)}$, $y = \frac{1}{t^2}$, $t \neq 0$

(c) $x = t^2 + 2t - 3$, $y = t(t - 1)$

(d) $x = 27 - t^3$, $y = \frac{1}{t - 1}$, $t \neq 1$

(e) $x = \frac{t - 1}{t + 1}$, $y = \frac{2t}{t^2 + 1}$, $t \neq -1$

Solution:

(a) When $x = 0$
$2t = 0$
so $t = 0$
Substitute $t = 0$ into $y = t^2 - 5$
$y = (0)^2 - 5$
$y = -5$
So the curve meets the y-axis at $(0, -5)$.

(b) When $x = 0$
$\sqrt{3t - 4} = 0$
$3t - 4 = 0$
$3t = 4$
so $t = \frac{4}{3}$

Substitute $t = \frac{4}{3}$ into $y = \frac{1}{t^2}$

$$y = \frac{1}{\left(\frac{4}{3}\right)^2}$$

$$y = \frac{1}{\left(\frac{16}{9}\right)}$$

$$y = \frac{9}{16} \qquad \left[\text{Note: This uses } \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a} \right]$$

So the curve meets the y-axis at $\left(0 , \frac{9}{16} \right)$.

(c) When $x = 0$

$t^2 + 2t - 3 = 0$

$(t + 3) (t - 1) = 0$

so $t = -3$ and $t = 1$

(1) Substitute $t = -3$ into $y = t (t - 1)$

$y = (-3) [(-3) - 1]$

$y = (-3) \times (-4)$

$y = 12$

(2) Substitute $t = 1$ into $y = t (t - 1)$

$y = 1 (1 - 1)$

$y = 1 \times 0$

$y = 0$

So the curve meets the y-axis at (0, 0) and (0, 12).

(d) When $x = 0$

$27 - t^3 = 0$

$t^3 = 27$

$t = \sqrt[3]{27}$

so $t = 3$

Substitute $t = 3$ into $y = \frac{1}{t - 1}$

$$y = \frac{1}{(3) - 1}$$

$$y = \frac{1}{2}$$

So the curve meets the y-axis at $\left(0, \frac{1}{2}\right)$.

(e) When $x = 0$

$\frac{t-1}{t+1} = 0$

$t - 1 = 0$ $\left[\text{Note: } \frac{a}{b} = 0 \Rightarrow a = 0\right]$

So $t = 1$

Substitute $t = 1$ into $y = \frac{2t}{t^2 + 1}$

$y = \frac{2(1)}{(1)^2 + 1}$

$y = \frac{2}{2}$

$y = 1$

So the curve meets the y-axis at (0, 1).

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 3

Question:

A curve has parametric equations $x = 4at^2$, $y = a(2t - 1)$, where a is a constant. The curve passes through the point (4, 0). Find the value of a.

Solution:

When $y = 0$
$a(2t - 1) = 0$
$2t - 1 = 0$
$2t = 1$
$t = \frac{1}{2}$

When $t = \frac{1}{2}$, $x = 4$

So substitute $t = \frac{1}{2}$ and $x = 4$ into $x = 4at^2$

$4a\left(\frac{1}{2}\right)^2 = 4$

$4a \times \frac{1}{4} = 4$

$a = 4$
So the value of a is 4.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 4

Question:

A curve has parametric equations $x = b\left(2t - 3\right)$, $y = b\left(1 - t^2\right)$, where b is a constant. The curve passes through the point $\left(0, -5\right)$. Find the value of b.

Solution:

When $x = 0$
$b\left(2t - 3\right) = 0$
$2t - 3 = 0$
$2t = 3$
$t = \frac{3}{2}$

When $t = \frac{3}{2}$, $y = -5$

So substitute $t = \frac{3}{2}$ and $y = -5$ into $y = b\left(1 - t^2\right)$

$$b\left[1 - \left(\frac{3}{2}\right)^2\right] = -5$$

$$b\left(1 - \frac{9}{4}\right) = -5$$

$$b\left(\frac{-5}{4}\right) = -5$$

$$b = \frac{-5}{\left(\frac{-5}{4}\right)}$$

$b = 4$
So the value of b is 4.

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Coordinate geometry in the (x, y) plane
Exercise B, Question 5

Question:

A curve has parametric equations $x = p(2t - 1)$, $y = p(t^3 + 8)$, where p is a constant. The curve meets the x-axis at (2, 0) and the y-axis at A.

(a) Find the value of p.

(b) Find the coordinates of A.

Solution:

(a) When $y = 0$

$p(t^3 + 8) = 0$
$t^3 + 8 = 0$
$t^3 = -8$
$t = \sqrt[3]{-8}$
$t = -2$

When $t = -2$, $x = 2$

So substitute $t = -2$ and $x = 2$ into $x = p(2t - 1)$

$p[2(-2) - 1] = 2$
$p(-4 - 1) = 2$
$p(-5) = 2$
$p = -\frac{2}{5}$

(b) When $x = 0$

$p(2t - 1) = 0$
$2t - 1 = 0$
$2t = 1$
$t = \frac{1}{2}$

When the curve meets the y-axis $t = \frac{1}{2}$

So substitute $t = \frac{1}{2}$ into $y = p(t^3 + 8)$

$y = p\left[\left(\frac{1}{2}\right)^3 + 8\right]$

but $p = -\frac{2}{5}$

So $y = -\frac{2}{5}\left[\left(\frac{1}{2}\right)^3 + 8\right] = -\frac{2}{5}\left(\frac{1}{8} + 8\right) = -\frac{2}{5} \times \frac{65}{8} = -\frac{13}{4}$

So the coordinates of A are $\left(0, -\frac{13}{4}\right)$.

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 6

Question:

A curve is given parametrically by the equations $x = 3qt^2$, $y = 4 (t^3 + 1)$, where q is a constant. The curve meets the x-axis at X and the y-axis at Y. Given that $OX = 2OY$, where O is the origin, find the value of q.

Solution:

(1) When $y = 0$
$4 (t^3 + 1) = 0$
$t^3 + 1 = 0$
$t^3 = -1$
$t = \sqrt[3]{-1}$
$t = -1$
Substitute $t = -1$ into $x = 3qt^2$
$x = 3q (-1)^2$
$x = 3q$
So the coordinates of X are $(3q, 0)$.
(2) When $x = 0$
$3qt^2 = 0$
$t^2 = 0$
$t = 0$
Substitute $t = 0$ into $y = 4 (t^3 + 1)$
$y = 4 [(0)^3 + 1]$
$y = 4$
So the coordinates of Y are $(0, 4)$.
(3) Now $OX = 3q$ and $OY = 4$
As $OX = 2OY$
$(3q) = 2 (4)$
$3q = 8$
$q = \frac{8}{3}$

So the value of q is $\frac{8}{3}$.

Solutionbank
Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 7

Question:

Find the coordinates of the point of intersection of the line with parametric equations $x = 3t + 2$, $y = 1 - t$ and the line $y + x = 2$.

Solution:

(1) Substitute $x = 3t + 2$ and $y = 1 - t$ into $y + x = 2$

$(1 - t) + (3t + 2) = 2$

$1 - t + 3t + 2 = 2$

$2t + 3 = 2$

$2t = - 1$

$t = - \frac{1}{2}$

(2) Substitute $t = - \frac{1}{2}$ into $x = 3t + 2$

$x = 3 \left(- \frac{1}{2} \right) + 2 = - \frac{3}{2} + 2 = \frac{1}{2}$

(3) Substitute $t = - \frac{1}{2}$ into $y = 1 - t$

$y = 1 - \left(- \frac{1}{2} \right) = 1 + \frac{1}{2} = \frac{3}{2}$

So the coordinates of the point of intersection are $\left(\frac{1}{2} , \frac{3}{2} \right)$.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 8

Question:

Find the coordinates of the points of intersection of the curve with parametric equations $x = 2t^2 - 1$, $y = 3(t + 1)$ and the line $3x - 4y = 3$.

Solution:

(1) Substitute $x = 2t^2 - 1$ and $y = 3(t + 1)$ into $3x - 4y = 3$

$3(2t^2 - 1) - 4[3(t + 1)] = 3$

$3(2t^2 - 1) - 12(t + 1) = 3$

$6t^2 - 3 - 12t - 12 = 3$

$6t^2 - 12t - 15 = 3$

$6t^2 - 12t - 18 = 0 \qquad (\div 6)$

$t^2 - 2t - 3 = 0$

$(t - 3)(t + 1) = 0$

so $t = 3$ and $t = -1$

(2) Substitute $t = 3$ into $x = 2t^2 - 1$ and $y = 3(t + 1)$

$x = 2(3)^2 - 1 = 17$

$y = 3(3 + 1) = 12$

(3) Substitute $t = -1$ into $x = 2t^2 - 1$ and $y = 3(t + 1)$

$x = 2(-1)^2 - 1 = 1$

$y = 3(-1 + 1) = 0$

So the coordinates of the points of intersection are (17, 12) and (1, 0).

Solutionbank
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Coordinate geometry in the (x, y) plane
Exercise B, Question 9

Question:

Find the values of t at the points of intersection of the line $4x - 2y - 15 = 0$ with the parabola $x = t^2$, $y = 2t$ and give the coordinates of these points.

Solution:

(1) Substitute $x = t^2$ and $y = 2t$ into $4x - 2y - 15 = 0$

$4(t^2) - 2(2t) - 15 = 0$

$4t^2 - 4t - 15 = 0$

$(2t + 3)(2t - 5) = 0$

So $2t + 3 = 0 \quad \Rightarrow \quad 2t = -3 \quad \Rightarrow \quad t = \frac{-3}{2}$ and

$2t - 5 = 0 \quad \Rightarrow \quad 2t = 5 \quad \Rightarrow \quad t = \frac{5}{2}$

(2) Substitute $t = -\frac{3}{2}$ into $x = t^2$ and $y = 2t$

$x = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

$y = 2\left(-\frac{3}{2}\right) = -3$

(3) Substitute $t = \frac{5}{2}$ into $x = t^2$ and $y = 2t$

$x = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$y = 2\left(\frac{5}{2}\right) = 5$

So the coordinates of the points of intersection are $\left(\frac{9}{4}, -3\right)$ and $\left(\frac{25}{4}, 5\right)$.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise B, Question 10

Question:

Find the points of intersection of the parabola $x = t^2$, $y = 2t$ with the circle $x^2 + y^2 - 9x + 4 = 0$.

Solution:

(1) Substitute $x = t^2$ and $y = 2t$ into $x^2 + y^2 - 9x + 4 = 0$

$\left(t^2 \right)^2 + \left(2t \right)^2 - 9 \left(t^2 \right) + 4 = 0$

$t^4 + 4t^2 - 9t^2 + 4 = 0$

$t^4 - 5t^2 + 4 = 0$

$\left(t^2 - 4 \right) \left(t^2 - 1 \right) = 0$

So $t^2 - 4 = 0 \Rightarrow t^2 = 4 \Rightarrow t = \sqrt{4} \Rightarrow t = \pm 2$ and

$t^2 - 1 = 0 \Rightarrow t^2 = 1 \Rightarrow t = \sqrt{1} \Rightarrow t = \pm 1$

(2) Substitute $t = 2$ into $x = t^2$ and $y = 2t$

$x = \left(2 \right)^2 = 4$

$y = 2 \left(2 \right) = 4$

(3) Substitute $t = -2$ into $x = t^2$ and $y = 2t$

$x = \left(-2 \right)^2 = 4$

$y = 2 \left(-2 \right) = -4$

(4) Substitute $t = 1$ into $x = t^2$ and $y = 2t$

$x = \left(1 \right)^2 = 1$

$y = 2 \left(1 \right) = 2$

(5) Substitute $t = -1$ into $x = t^2$ and $y = 2t$

$x = \left(-1 \right)^2 = 1$

$y = 2 \left(-1 \right) = -2$

So the coordinates of the points of intersection are (4, 4), $\left(4 , -4 \right)$, (1, 2) and $\left(1 , -2 \right)$.

Solutionbank
Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise C, Question 1

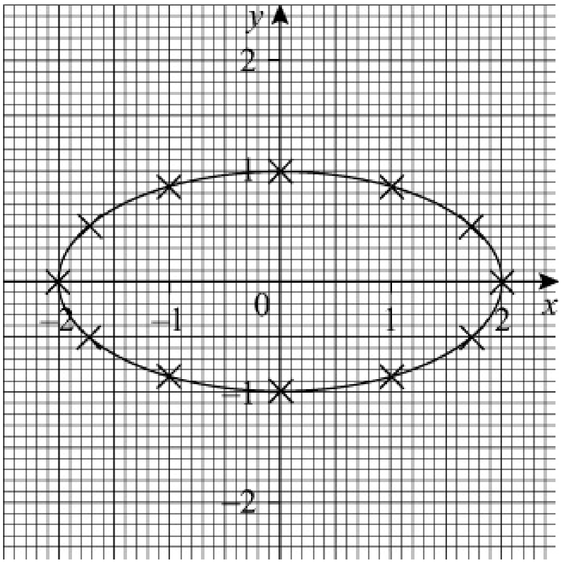
Question:

A curve is given by the parametric equations $x = 2 \sin t$, $y = \cos t$.
Complete the table and draw a graph of the curve for $0 \leq t \leq 2\pi$.

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$x = 2\sin t$			1.73		1.73			−1		−2			0
$y = \cos t$		0.87					−1		−0.5		0.5		

Solution:

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$x = 2\sin t$	0	1	1.73	2	1.73	1	0	−1	−1.73	−2	−1.73	−1	0
$y = \cos t$	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0	0.5	0.87	1



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Coordinate geometry in the (x, y) plane
Exercise C, Question 2

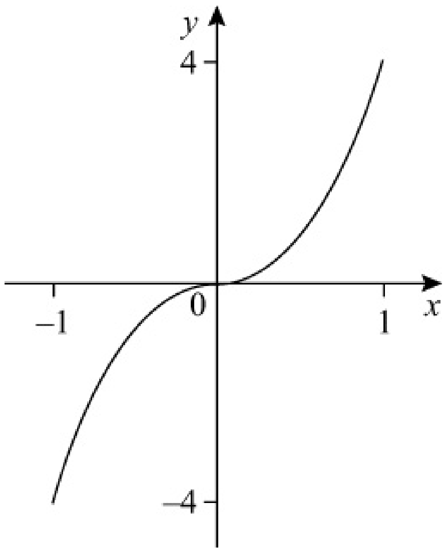
Question:

A curve is given by the parametric equations $x = \sin t$, $y = \tan t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$. Draw a graph of the curve.

Solution:

t	$\frac{-4\pi}{10}$	$\frac{-3\pi}{10}$	$\frac{-2\pi}{10}$	$\frac{-\pi}{10}$	0	$\frac{\pi}{10}$	$\frac{2\pi}{10}$	$\frac{3\pi}{10}$	$\frac{4\pi}{10}$
$x = \sin t$	−0.95	−0.81	−0.59	−0.31	0	0.31	0.59	0.81	0.95
$y = \tan t$	−3.08	−1.38	−0.73	−0.32	0	0.32	0.73	1.38	3.08

Answers are given to 2 d.p.



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Coordinate geometry in the (x, y) plane
Exercise C, Question 3

Question:

Find the cartesian equation of the curves given by the following parametric equations:

(a) $x = \sin t$, $y = \cos t$

(b) $x = \sin t - 3$, $y = \cos t$

(c) $x = \cos t - 2$, $y = \sin t + 3$

(d) $x = 2 \cos t$, $y = 3 \sin t$

(e) $x = 2 \sin t - 1$, $y = 5 \cos t + 4$

(f) $x = \cos t$, $y = \sin 2t$

(g) $x = \cos t$, $y = 2 \cos 2t$

(h) $x = \sin t$, $y = \tan t$

(i) $x = \cos t + 2$, $y = 4 \sec t$

(j) $x = 3 \cot t$, $y = \operatorname{cosec} t$

Solution:

(a) $x = \sin t$, $y = \cos t$

$x^2 = \sin^2 t$, $y^2 = \cos^2 t$

As $\sin^2 t + \cos^2 t = 1$

$x^2 + y^2 = 1$

(b) $x = \sin t - 3$, $y = \cos t$

$\sin t = x + 3$

$\sin^2 t = (x + 3) ^2$

$\cos t = y$

$\cos^2 t = y^2$

As $\sin^2 t + \cos^2 t = 1$

$(x + 3) ^2 + y^2 = 1$

(c) $x = \cos t - 2, y = \sin t + 3$

$\cos t = x + 2$

$\sin t = y - 3$

As $\sin^2 t + \cos^2 t = 1$

$(y - 3)^2 + (x + 2)^2 = 1$ or $(x + 2)^2 + (y - 3)^2 = 1$

(d) $x = 2 \cos t, y = 3 \sin t$

$\sin t = \frac{y}{3}$

$\cos t = \frac{x}{2}$

As $\sin^2 t + \cos^2 t = 1$

$\left(\frac{y}{3} \right)^2 + \left(\frac{x}{2} \right)^2 = 1$ or $\left(\frac{x}{2} \right)^2 + \left(\frac{y}{3} \right)^2 = 1$

(e) $x = 2 \sin t - 1, y = 5 \cos t + 4$

$2 \sin t - 1 = x$

$2 \sin t = x + 1$

$\sin t = \frac{x+1}{2}$

and

$5 \cos t + 4 = y$

$5 \cos t = y - 4$

$\cos t = \frac{y-4}{5}$

As $\sin^2 t + \cos^2 t = 1$

$\left(\frac{x+1}{2} \right)^2 + \left(\frac{y-4}{5} \right)^2 = 1$

(f) $x = \cos t, y = \sin 2t$

As $\sin 2t = 2 \sin t \cos t$

$y = 2 \sin t \cos t = (2 \sin t) \quad x$

Now $\sin^2 t + \cos^2 t = 1$

So $\sin^2 t + x^2 = 1$

$\Rightarrow \quad \sin^2 t = 1 - x^2$

$\Rightarrow \quad \sin t = \sqrt{1 - x^2}$

So $y = (2\sqrt{1 - x^2}) \quad x$ or $y = 2x\sqrt{1 - x^2}$

(g) $x = \cos t, y = 2 \cos 2t$

As $\cos 2t = 2 \cos^2 t - 1$

$y = 2\left(2\cos^2 t - 1\right)$

But $x = \cos t$

So $y = 2\left(2x^2 - 1\right)$

$y = 4x^2 - 2$

(h) $x = \sin t$, $y = \tan t$

As $\tan t = \frac{\sin t}{\cos t}$

$y = \frac{\sin t}{\cos t}$

But $x = \sin t$

So $y = \frac{x}{\cos t}$

Now $\cos t = \sqrt{1 - \sin^2 t} = \sqrt{1 - x^2}$ (from $\sin^2 t + \cos^2 t = 1$)

So $y = \frac{x}{\sqrt{1 - x^2}}$

(i) $x = \cos t + 2$, $y = 4\sec t$

As $\sec t = \frac{1}{\cos t}$

$y = 4 \times \frac{1}{\cos t} = \frac{4}{\cos t}$

Now $x = \cos t + 2 \Rightarrow \cos t = x - 2$

So $y = \frac{4}{x - 2}$

(j) $x = 3\cot t$, $y = \operatorname{cosec} t$

As $\sin^2 t + \cos^2 t = 1$

$\frac{\sin^2 t}{\sin^2 t} + \frac{\cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}$ $\left(\div \sin^2 t\right)$

$1 + \left(\frac{\cos t}{\sin t}\right)^2 = \left(\frac{1}{\sin t}\right)^2$

$1 + \cot^2 t = \operatorname{cosec}^2 t$

Now $x = 3\cot t \Rightarrow \cot t = \frac{x}{3}$

and $y = \operatorname{cosec} t$

So $1 + \left(\frac{x}{3}\right)^2 = \left(y\right)^2$ (using $1 + \cos^2 t = \operatorname{cosec}^2 t$)

or $y^2 = 1 + \left(\frac{x}{3}\right)^2$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise C, Question 4

Question:

A circle has parametric equations $x = \sin t - 5$, $y = \cos t + 2$.

(a) Find the cartesian equation of the circle.

(b) Write down the radius and the coordinates of the centre of the circle.

Solution:

(a) $x = \sin t - 5$, $y = \cos t + 2$

$\sin t = x + 5$ and $\cos t = y - 2$

As $\sin^2 t + \cos^2 t = 1$

$(x + 5)^2 + (y - 2)^2 = 1$

(b) This is a circle with centre $(-5 , 2)$ and radius 1.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise C, Question 5

Question:

A circle has parametric equations $x = 4 \sin t + 3$, $y = 4 \cos t - 1$. Find the radius and the coordinates of the centre of the circle.

Solution:

$x = 4 \sin t + 3$
$4 \sin t = x - 3$
$\sin t = \frac{x - 3}{4}$

and
$y = 4 \cos t - 1$
$4 \cos t = y + 1$
$\cos t = \frac{y + 1}{4}$

As $\sin^2 t + \cos^2 t = 1$

$\left(\frac{x - 3}{4} \right)^2 + \left(\frac{y + 1}{4} \right)^2 = 1$

$\frac{(x - 3)^2}{4^2} + \frac{(y + 1)^2}{4^2} = 1$

$\frac{(x - 3)^2}{16} + \frac{(y + 1)^2}{16} = 1$

$(x - 3)^2 + (y + 1)^2 = 16$ Multiply throughout by 16
So the centre of the circle is $(3, -1)$ and the radius is 4.

Solutionbank
Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 1

Question:

The following curves are given parametrically. In each case, find an expression for $y\frac{dx}{dt}$ in terms of t.

(a) $x = t + 3,\ y = 4t - 3$

(b) $x = t^3 + 3t,\ y = t^2$

(c) $x = (2t - 3)^2,\ y = 1 - t^2$

(d) $x = 6 - \frac{1}{t},\ y = 4t^3,\ t > 0$

(e) $x = \sqrt{t},\ y = 6t^3,\ t \geq 0$

(f) $x = \frac{4}{t^2},\ y = 5t^2,\ t < 0$

(g) $x = 5t^{\frac{1}{2}},\ y = 4t^{-\frac{3}{2}},\ t > 0$

(h) $x = t^{\frac{1}{3}} - 1,\ y = \sqrt{t},\ t \geq 0$

(i) $x = 16 - t^4,\ y = 3 - \frac{2}{t},\ t < 0$

(j) $x = 6t^{\frac{2}{3}},\ y = t^2$

Solution:

(a) $x = t + 3,\ y = 4t - 3$

$$\frac{dx}{dt} = 1$$

$$\text{So } y\frac{dx}{dt} = \left(4t - 3\right) \times 1 = 4t - 3$$

(b) $x = t^3 + 3t, y = t^2$

$\frac{dx}{dt} = 3t^2 + 3$

So $y \frac{dx}{dt} = t^2 \left(3t^2 + 3 \right) = 3t^2 \left(t^2 + 1 \right)$ Factorise 3

(c) $x = \left(2t - 3 \right)^2, y = 1 - t^2$

$x = 4t^2 - 12t + 9$

$\frac{dx}{dt} = 8t - 12$

So $y \frac{dx}{dt} = \left(1 - t^2 \right) \left(8t - 12 \right) = 4 \left(1 - t^2 \right) \left(2t - 3 \right)$

Factorise 4

(d) $x = 6 - \frac{1}{t}, y = 4t^3$

$x = 6 - t^{-1}$

$\frac{dx}{dt} = t^{-2}$

So $y \frac{dx}{dt} = 4t^3 \times t^{-2} = 4t$

(e) $x = \sqrt{t}, y = 6t^3$

$x = t^{\frac{1}{2}}$

$\frac{dx}{dt} = \frac{1}{2} t^{-\frac{1}{2}}$

So $y \frac{dx}{dt} = 6t^3 \times \frac{1}{2} t^{-\frac{1}{2}} = 3t^{3 - \frac{1}{2}} = 3t^{\frac{5}{2}}$

(f) $x = \frac{4}{t^2}, y = 5t^2$

$x = 4t^{-2}$

$\frac{dx}{dt} = -8t^{-3}$

So $y \frac{dx}{dt} = 5t^2 \times -8t^{-3} = -40t^{2-3} = -40t^{-1} = -\frac{40}{t}$

(g) $x = 5t^{\frac{1}{2}}, y = 4t^{-\frac{3}{2}}$

$$\frac{dx}{dt} = 5 \times \frac{1}{2}t^{-\frac{1}{2}} = \frac{5}{2}t^{-\frac{1}{2}}$$

$$\text{So } y\frac{dx}{dt} = 4t^{-\frac{3}{2}} \times \frac{5}{2}t^{-\frac{1}{2}} = 10t^{-\frac{3}{2}-\frac{1}{2}} = 10t^{-2}$$

(h) $x = t^{\frac{1}{3}} - 1,\ y = \sqrt{t}$

$$\frac{dx}{dt} = \frac{1}{3}t^{\frac{1}{3}-1} = \frac{1}{3}t^{-\frac{2}{3}}$$

$$\text{So } y\frac{dx}{dt} = \sqrt{t} \times \frac{1}{3}t^{-\frac{2}{3}} = t^{\frac{1}{2}} \times \frac{1}{3}t^{-\frac{2}{3}} = \frac{1}{3}t^{\frac{1}{2}-\frac{2}{3}} = \frac{1}{3}t^{-\frac{1}{6}}$$

(i) $x = 16 - t^4,\ y = 3 - \frac{2}{t}$

$$\frac{dx}{dt} = -4t^3$$

$$\text{So } y\frac{dx}{dt} = \left(3 - \frac{2}{t}\right)\left(-4t^3\right)$$

$$= 3 \times \left(-4t^3\right) + \frac{2}{t} \times 4t^3$$

$$= -12t^3 + 8t^2 \quad [\text{or } 8t^2 - 12t^3 \text{ or } 4t^2(2 - 3t)]$$

(j) $x = 6t^{\frac{2}{3}},\ y = t^2$

$$\frac{dx}{dt} = 6 \times \frac{2}{3}t^{\frac{2}{3}-1} = 4t^{-\frac{1}{3}}$$

$$\text{So } y\frac{dx}{dt} = t^2 \times 4t^{-\frac{1}{3}} = 4t^{2-\frac{1}{3}} = 4t^{\frac{5}{3}}$$

Solutionbank
Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 2

Question:

A curve has parametric equations $x = 2t - 5$, $y = 3t + 8$. Work out $\int_0^4 y \frac{dx}{dt} dt$.

Solution:

$x = 2t - 5$, $y = 3t + 8$

$\frac{dx}{dt} = 2$

So $y \frac{dx}{dt} = \left(3t + 8 \right) \times 2 = 6t + 16$

$\int_0^4 y \frac{dx}{dt} dt = \int_0^4 6t + 16 \quad dt$

$= \left[3t^2 + 16t \right]_0^4$

$= \left[3 (4)^2 + 16 (4) \right] - \left[3 (0)^2 + 16 (0) \right]$

$= (3 \times 16 + 16 \times 4) - 0$

$= 48 + 64$

$= 112$

Solutionbank
Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 3

Question:

A curve has parametric equations $x = t^2 - 3t + 1$, $y = 4t^2$. Work out $\int_{-1}^{5} y \frac{\mathrm{d}x}{\mathrm{d}t} \mathrm{d}t$.

Solution:

$x = t^2 - 3t + 1$, $y = 4t^2$

$\frac{\mathrm{d}x}{\mathrm{d}t} = 2t - 3$

So $y \frac{\mathrm{d}x}{\mathrm{d}t} = 4t^2 \left(2t - 3 \right) = 8t^3 - 12t^2$

$$\int_{-1}^{5} y \frac{\mathrm{d}x}{\mathrm{d}t} \mathrm{d}t = \int_{-1}^{5} 8t^3 - 12t^2 \quad \mathrm{d}t$$
$$= \left[2t^4 - 4t^3 \right]_{-1}^{5}$$
$$= \left[2 (5)^4 - 4 (5)^3 \right] - \left[2 (-1)^4 - 4 (-1)^3 \right]$$
$$= 750 - 6$$
$$= 744$$

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Edexcel AS and A Level Modular Mathematics

Coordinate geometry in the (x, y) plane
Exercise D, Question 4

Question:

A curve has parametric equations $x = 3t^2$, $y = \frac{1}{t} + t^3$, $t > 0$. Work out $\int_{0.5}^{3} y \frac{dx}{dt} dt$.

Solution:

$$x = 3t^2,\ y = \frac{1}{t} + t^3$$

$$\frac{dx}{dt} = 6t$$

$$\text{So } y\frac{dx}{dt} = \left(\frac{1}{t} + t^3\right) \times 6t = \frac{1}{t} \times 6t + t^3 \times 6t = 6 + 6t^4$$

$$\int_{0.5}^{3} y \frac{dx}{dt} dt = \int_{0.5}^{3} 6 + 6t^4 \ dt$$

$$= \left[6t + \frac{6}{5}t^5 \right]_{0.5}^{3}$$

$$= \left[6\left(3\right) + \frac{6}{5}(3)^5 \right] - \left[6\left(0.5\right) + \frac{6}{5}(0.5)^5 \right]$$

$$= 309.6 - 3.0375$$

$$= 306.5625 \quad (\text{or } 306\tfrac{9}{16})$$

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Coordinate geometry in the (x, y) plane
Exercise D, Question 5

Question:

A curve has parametric equations $x = t^3 - 4t$, $y = t^2 - 1$. Work out $\int_{-2}^{2} y \frac{dx}{dt} dt$.

Solution:

$x = t^3 - 4t,\ y = t^2 - 1$

$\frac{dx}{dt} = 3t^2 - 4$

So $y \frac{dx}{dt} = \left(t^2 - 1 \right) \times \left(3t^2 - 4 \right) = 3t^4 - 4t^2 - 3t^2 + 4 = 3t^4 - 7t^2 + 4$

$\int_{-2}^{2} 3t^4 - 7t^2 + 4 \ dt = \left[\frac{3}{5}t^5 - \frac{7}{3}t^3 + 4t \right]_{-2}^{2}$

$= \left[\frac{3}{5}(2)^5 - \frac{7}{3}(2)^3 + 4(2) \right] - \left[\frac{3}{5}(-2)^5 - \frac{7}{3}(-2)^3 + (-2) \right]$

$= 8\frac{8}{15} - \left(-8\frac{8}{15} \right)$

$= 17\frac{1}{15}$

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Coordinate geometry in the (x, y) plane
Exercise D, Question 6

Question:

A curve has parametric equations $x = 9t^{\frac{4}{3}}$, $y = t^{-\frac{1}{3}}$, $t > 0$.

(a) Show that $y\frac{dx}{dt} = a$, where a is a constant to be found.

(b) Work out $\int_3^5 y\frac{dx}{dt}dt$.

Solution:

(a) $x = 9t^{\frac{4}{3}}$, $y = t^{-\frac{1}{3}}$

$\frac{dx}{dt} = 9 \times \frac{4}{3}t^{\frac{4}{3}-1} = 9 \times \frac{4}{3}t^{\frac{1}{3}} = 12t^{\frac{1}{3}}$

So $y\frac{dx}{dt} = t^{-\frac{1}{3}} \times 12t^{\frac{1}{3}} = 12t^{-\frac{1}{3}+\frac{1}{3}} = 12t^0 = 12$

So $a = 12$

(b) $\int_3^5 y\frac{dx}{dt}dt = \int_3^5 12\ dt = \left[12t\right]_3^5 = 12\left(5\right) - 12\left(3\right) = 24$

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Coordinate geometry in the (x, y) plane
Exercise D, Question 7

Question:

A curve has parametric equations $x = \sqrt{t}$, $y = 4\sqrt{t^3}$, $t > 0$.

(a) Show that $y\frac{dx}{dt} = pt$, where p is a constant to be found.

(b) Work out $\int_1^6 y\frac{dx}{dt}dt$.

Solution:

(a) $x = \sqrt{t}$, $y = 4\sqrt{t^3}$

$x = t^{\frac{1}{2}}$

$\frac{dx}{dt} = \frac{1}{2}t^{\frac{1}{2}-1} = \frac{1}{2}t^{-\frac{1}{2}}$

$y\frac{dx}{dt} = 4\sqrt{t^3} \times \frac{1}{2}t^{-\frac{1}{2}}$

$= 4t^{\frac{3}{2}} \times \frac{1}{2}t^{-\frac{1}{2}}$

$= 2t^{\frac{3}{2}-\frac{1}{2}}$

$= 2t^1$

$= 2t$

So $p = 2$

(b) $\int_1^6 y\frac{dx}{dt}dt = \int_1^6 2t\,dt = \left[t^2 \right]_1^6 = (6)^2 - (1)^2 = 35$

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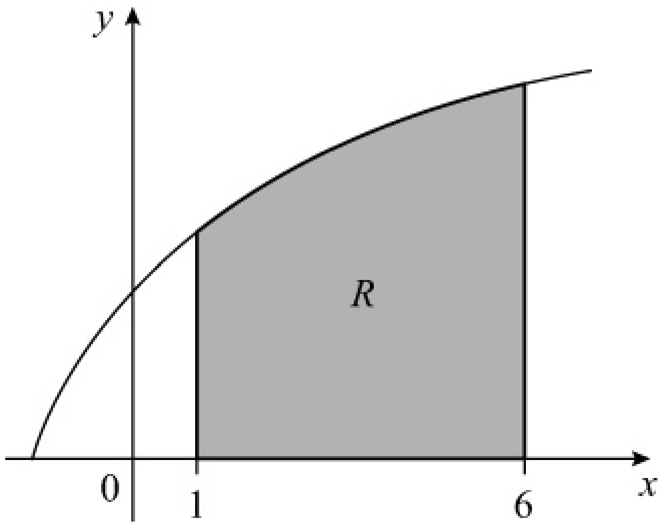
Coordinate geometry in the (x, y) plane
Exercise D, Question 8

Question:

The diagram shows a sketch of the curve with parametric equations $x = t^2 - 3$, $y = 3t$, $t > 0$. The shaded region R is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 6$.

(a) Find the value of t when
(i) $x = 1$
(ii) $x = 6$

(b) Find the area of R.



Solution:

(a) Substitute $x = 1$ into $x = t^2 - 3$

$t^2 - 3 = 1$

$t^2 = 4$

$t = 2$ (as $t > 0$)

Substitute $x = 6$ into $x = t^2 - 3$

$t^2 - 3 = 6$

$t^2 = 9$

$t = 3$ (as $t > 0$)

(b) $\int_1^6 y\,dx = \int_2^3 y \frac{dx}{dt} dt$

$$\frac{dx}{dt} = 2t$$

$$\text{So } y\frac{dx}{dt} = 3t \times 2t = 6t^2$$

$$\int_2^3 y\frac{dx}{dt}dt = \int_2^3 6t^2 \quad dt$$

$$= \left[2t^3 \right]_2^3$$

$$= 2(3)^3 - 2(2)^3$$

$$= 54 - 16$$

$$= 38$$

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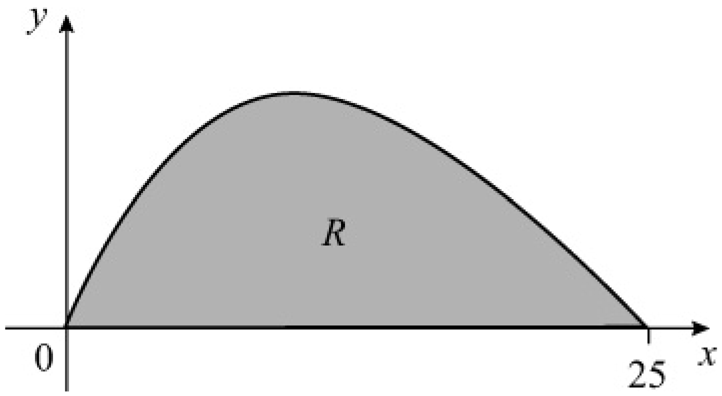
Coordinate geometry in the (x, y) plane
Exercise D, Question 9

Question:

The diagram shows a sketch of the curve with parametric equations $x = 4t^2$, $y = t(5 - 2t)$, $t \geq 0$. The shaded region R is bounded by the curve and the x-axis. Find the area of R.



Solution:

When $x = 0$
$4t^2 = 0$
$t^2 = 0$
$t = 0$
When $x = 25$
$4t^2 = 25$
$t^2 = \frac{25}{4}$
$t = \sqrt{\frac{25}{4}}$
$t = \frac{5}{2}$ (as $t \geq 0$)

So $\int_0^{25} y \, dx = \int_0^{\frac{5}{2}} y \frac{dx}{dt} dt$

$\frac{dx}{dt} = 8t$

So $y \frac{dx}{dt} = t(5 - 2t) \times 8t = 8t^2(5 - 2t) = 40t^2 - 16t^3$

$$\int_0^{\frac{5}{2}} y \frac{dx}{dt} dt = \int_0^{\frac{5}{2}} 40t^2 - 16t^3 \quad dt$$

$$= \left[\frac{40}{3}t^3 - 4t^4 \right]_0^{\frac{5}{2}}$$

$$= \left[\frac{40}{3}\left(\frac{5}{2}\right)^3 - 4\left(\frac{5}{2}\right)^4 \right] - \left[\frac{40}{3}(0)^3 - 4(0)^4 \right]$$

$$= 52\frac{1}{12} - 0$$

$$= 52\frac{1}{12}$$

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Coordinate geometry in the (x, y) plane
Exercise D, Question 10

Question:

The region R is bounded by the curve with parametric equations $x = t^3$, $y = \frac{1}{3t^2}$, the x-axis and the lines $x = -1$ and $x = -8$.

(a) Find the value of t when
(i) $x = -1$
(ii) $x = -8$

(b) Find the area of R.

Solution:

(a) (i) Substitute $x = -1$ into $x = t^3$

$t^3 = -1$

$t = \sqrt[3]{-1}$

$t = -1$

(ii) Substitute $x = -8$ into $x = t^3$

$t^3 = -8$

$t = \sqrt[3]{-8}$

$t = -2$

(b) $R = \int_{-8}^{-1} y \, dx = \int_{-2}^{-1} y \frac{dx}{dt} dt$

$\frac{dx}{dt} = 3t^2$

So $y \frac{dx}{dt} = \frac{1}{3t^2} \times 3t^2 = 1$

$\int_{-2}^{-1} y \frac{dx}{dt} dt = \int_{-2}^{-1} 1 \, dt = \left[t \right]_{-2}^{-1} = \left(-1 \right) - \left(-2 \right)$

$= -1 + 2 = 1$

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Coordinate geometry in the (x, y) plane
Exercise E, Question 1

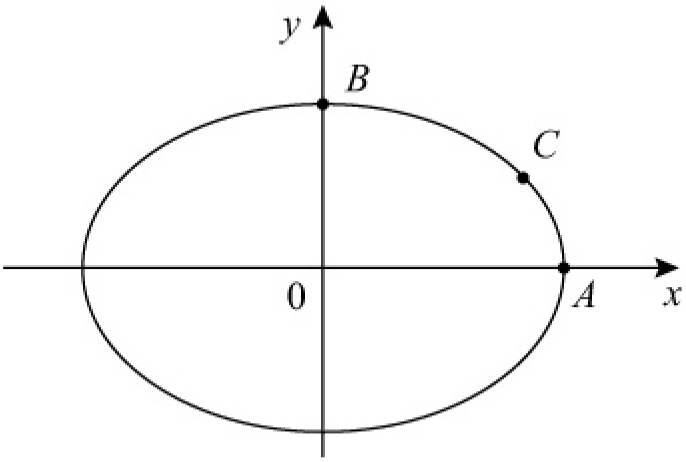
Question:

The diagram shows a sketch of the curve with parametric equations $x = 4 \cos t$, $y = 3 \sin t$, $0 \leq t < 2\pi$.

(a) Find the coordinates of the points A and B.

(b) The point C has parameter $t = \frac{\pi}{6}$. Find the exact coordinates of C.

(c) Find the cartesian equation of the curve.



Solution:

(a) (1) At A, $y = 0 \Rightarrow 3 \sin t = 0 \Rightarrow \sin t = 0$

So $t = 0$ and $t = \pi$

Substitute $t = 0$ and $t = \pi$ into $x = 4 \cos t$

$t = 0 \Rightarrow x = 4 \cos (0) = 4 \times 1 = 4$

$t = \pi \Rightarrow x = 4 \cos \pi = 4 \times (- 1) = - 4$

So the coordinates of A are (4, 0).

(2) At B, $x = 0 \Rightarrow 4 \cos t = 0 \Rightarrow \cos t = 0$

So $t = \frac{\pi}{2}$ and $t = \frac{3\pi}{2}$

Substitute $t = \frac{\pi}{2}$ and $t = \frac{3\pi}{2}$ into $y = 3 \sin t$

$t = \frac{\pi}{2} \quad \Rightarrow \quad y = 3 \sin\left(\frac{\pi}{2}\right) = 3 \times 1 = 3$

$t = \frac{3\pi}{2} \quad \Rightarrow \quad y = 3 \sin\left(\frac{3\pi}{2}\right) = 3 \times -1 = -3$

So the coordinates of B are (0, 3)

(b) Substitute $t = \frac{\pi}{6}$ into $x = 4 \cos t$ and $y = 3 \sin t$

$x = 4 \cos\left(\frac{\pi}{6}\right) = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

$y = 3 \sin\left(\frac{\pi}{6}\right) = 3 \times \frac{1}{2} = \frac{3}{2}$

So the coordinates of C are $\left(2\sqrt{3}, \frac{3}{2}\right)$

(c) $x = 4 \cos t$, $y = 3 \sin t$

$\cos t = \frac{x}{4}$ and $\sin t = \frac{y}{3}$

As $\sin^2 t + \cos^2 t = 1$

$\left(\frac{y}{3}\right)^2 + \left(\frac{x}{4}\right)^2 = 1 \quad$ or $\quad \left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

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Coordinate geometry in the (x, y) plane
Exercise E, Question 2

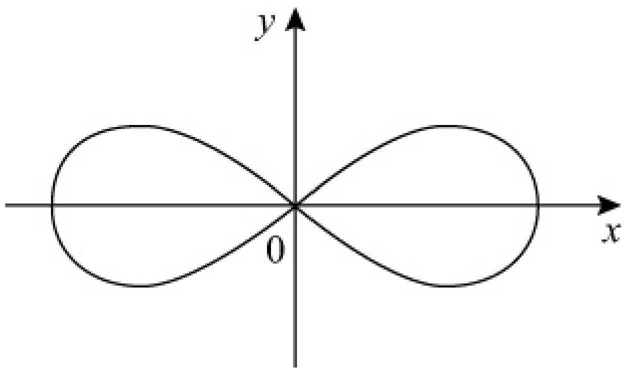
Question:

The diagram shows a sketch of the curve with parametric equations $x = \cos t$, $y = \frac{1}{2} \sin 2t$.

$0 \le t < 2\pi$. The curve is symmetrical about both axes.

(a) Copy the diagram and label the points having parameters $t = 0$, $t = \frac{\pi}{2}$, $t = \pi$ and $t = \frac{3\pi}{2}$.

(b) Show that the cartesian equation of the curve is $y^2 = x^2 (1 - x^2)$.



Solution:

(a) (1) Substitute $t = 0$ into $x = \cos t$ and $y = \frac{1}{2} \sin 2t$

$x = \cos 0 = 1$

$y = \frac{1}{2} \sin \left(2 \times 0 \right) = \frac{1}{2} \sin 0 = \frac{1}{2} \times 0 = 0$

So when $t = 0$, $(x , y) = (1 , 0)$

(2) Substitute $t = \frac{\pi}{2}$ into $x = \cos t$ and $y = \frac{1}{2} \sin 2t$

$x = \cos \frac{\pi}{2} = 0$

$y = \frac{1}{2} \sin \left(2 \times \frac{\pi}{2} \right) = \frac{1}{2} \sin \pi = \frac{1}{2} \times 0 = 0$

So when $t = \frac{\pi}{2}$, $(x, y) = (0, 0)$

(3) Substitute $t = \pi$ into $x = \cos t$ and $y = \frac{1}{2}\sin 2t$

$x = \cos\pi = -1$

$y = \frac{1}{2}\sin\left(2\pi\right) = \frac{1}{2} \times 0 = 0$

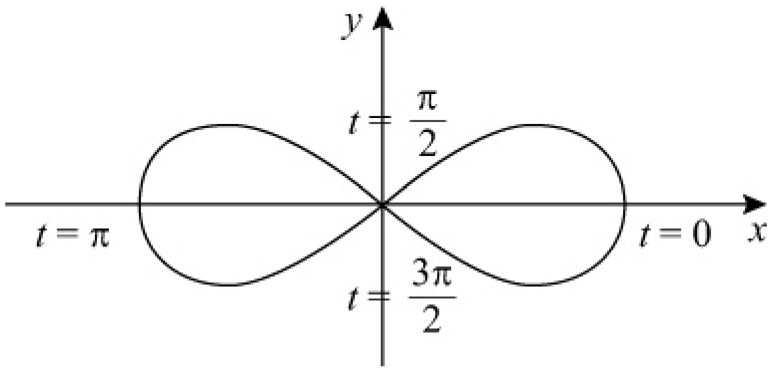
So when $t = \pi$, $(x, y) = (-1, 0)$

(4) Substitute $t = \frac{3\pi}{2}$ into $x = \cos t$ and $y = \frac{1}{2}\sin 2t$

$x = \cos\frac{3\pi}{2} = 0$

$y = \frac{1}{2}\sin\left(2 \times \frac{3\pi}{2}\right) = \frac{1}{2}\sin\left(3\pi\right) = \frac{1}{2} \times 0 = 0$

So when $t = \frac{3\pi}{2}$, $(x, y) = (0, 0)$



(b) $y = \frac{1}{2}\sin 2t = \frac{1}{2} \times 2\sin t\cos t = \sin t\cos t$

As $x = \cos t$

$y = \sin t \times x$

$y = x\sin t$

Now $\sin^2 t + \cos^2 t = 1$

So $\sin^2 t + x^2 = 1$

$\Rightarrow \quad \sin^2 t = 1 - x^2$

$\Rightarrow \quad \sin t = \sqrt{1 - x^2}$

So $y = x\sqrt{1 - x^2}$ or $y^2 = x^2(1 - x^2)$

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Coordinate geometry in the (x, y) plane
Exercise E, Question 3

Question:

A curve has parametric equations $x = \sin t$, $y = \cos 2t$, $0 \leq t < 2\pi$.

(a) Find the cartesian equation of the curve.
The curve cuts the x-axis at $(a, 0)$ and $(b, 0)$.

(b) Find the value of a and b.

Solution:

(a) $x = \sin t$, $y = \cos 2t$
As $\cos 2t = 1 - 2\sin^2 t$
$y = 1 - 2x^2$

(b) Substitute $y = 0$ into $y = 1 - 2x^2$
$0 = 1 - 2x^2$
$2x^2 = 1$
$x^2 = \frac{1}{2}$
$x = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{1}}{\sqrt{2}} = \pm \frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
So the curve meets the x-axis at $\left(\frac{\sqrt{2}}{2}, 0 \right)$ and $\left(-\frac{\sqrt{2}}{2}, 0 \right)$

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Coordinate geometry in the (x, y) plane
Exercise E, Question 4

Question:

A curve has parametric equations $x = \frac{1}{1+t}$, $y = \frac{1}{(1+t)(1-t)}$, $t \neq \pm 1$.
Express t in terms of x. Hence show that the cartesian equation of the curve is $y = \frac{x^2}{2x-1}$.

Solution:

(1) $x = \frac{1}{1+t}$

$x \times \left(1+t\right) = \frac{1}{(1+t)} \times \left(1+t\right)$ Multiply each side by $(1+t)$

$x(1+t) = 1$ Simplify

$\frac{x(1+t)}{x} = \frac{1}{x}$ Divide each side by x

$1 + t = \frac{1}{x}$ Simplify

So $t = \frac{1}{x} - 1$

Substitute $t = \frac{1}{x} - 1$ into $y = \frac{1}{(1+t)(1-t)}$

$$y = \frac{1}{(1+\frac{1}{x}-1)[1-(\frac{1}{x}-1)]}$$

$$= \frac{1}{\frac{1}{x}(1-\frac{1}{x}+1)}$$

$$= \frac{1}{\frac{1}{x}(2-\frac{1}{x})}$$

$$= \frac{1}{\frac{1}{x}\left(\frac{2x}{x} - \frac{1}{x}\right)}$$

$$= \frac{1}{\frac{1}{x}\left(\frac{2x-1}{x}\right)}$$

$$= \frac{1}{\left(\frac{2x-1}{x^2}\right)}$$

$$= \frac{x^2}{2x-1} \qquad \left(\text{Remember } \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a}\right)$$

So the cartesian equation of the curve is $y = \frac{x^2}{2x-1}$.

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Coordinate geometry in the (x, y) plane
Exercise E, Question 5

Question:

A circle has parametric equations $x = 4 \sin t - 3$, $y = 4 \cos t + 5$.

(a) Find the cartesian equation of the circle.

(b) Draw a sketch of the circle.

(c) Find the exact coordinates of the points of intersection of the circle with the y-axis.

Solution:

(a) $x = 4 \sin t - 3$, $y = 4 \cos t + 5$

$4 \sin t = x + 3$

$\sin t = \frac{x+3}{4}$

and

$4 \cos t = y - 5$

$\cos t = \frac{y-5}{4}$

As $\sin^2 t + \cos^2 t = 1$

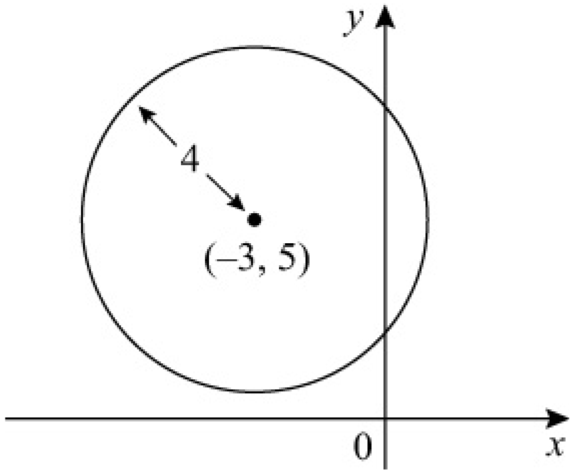
$\left(\frac{x+3}{4} \right)^2 + \left(\frac{y-5}{4} \right)^2 = 1$

$\frac{(x+3)^2}{4^2} + \frac{(y-5)^2}{4^2} = 1$

$\frac{(x+3)^2}{4^2} \times 4^2 + \frac{(y-5)^2}{4^2} \times 4^2 = 1 \times 4^2$

$(x+3)^2 + (y-5)^2 = 4^2$ or $(x+3)^2 + (y-5)^2 = 16$

(b) The circle $(x+3)^2 + (y-5)^2 = 4^2$ has centre $(-3, 5)$ and radius 4.



(c) Substitute $x = 0$ into $(x+3)^2 + (y-5)^2 = 4^2$

$(0+3)^2 + (y-5)^2 = 4^2$

$3^2 + (y-5)^2 = 4^2$

$9 + (y-5)^2 = 16$

$(y-5)^2 = 7$

$y - 5 = \pm\sqrt{7}$

$y = 5 \pm \sqrt{7}$

So the circle meets the y-axis at $(0, 5+\sqrt{7})$ and $(0, 5-\sqrt{7})$.

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Coordinate geometry in the (x, y) plane
Exercise E, Question 6

Question:

Find the cartesian equation of the line with parametric equations $x = \frac{2-3t}{1+t}$, $y = \frac{3+2t}{1+t}$, $t \neq -1$.

Solution:

$x = \frac{2-3t}{1+t}$

$x\left(1+t\right) = \frac{2-3t}{(1+t)} \times \left(1+t\right)$

$x(1+t) = 2-3t$

$x + xt = 2-3t$

$x + xt + 3t = 2$

$xt + 3t = 2-x$

$t(x+3) = 2-x$

$t\frac{(x+3)}{(x+3)} = \frac{2-x}{x+3}$

$t = \frac{2-x}{x+3}$

Substitute $t = \frac{2-x}{x+3}$ into $y = \frac{3+2t}{1+t}$

$$y = \frac{3+2\left(\frac{2-x}{x+3}\right)}{1+\left(\frac{2-x}{x+3}\right)}$$

$$= \frac{3+2\left(\frac{2-x}{x+3}\right)}{1+\left(\frac{2-x}{x+3}\right)} \times \frac{(x+3)}{(x+3)}$$

$$= \frac{3 \times (x+3) + 2\left(\frac{2-x}{x+3}\right) \times (x+3)}{1 \times (x+3) + \left(\frac{2-x}{x+3}\right) \times (x+3)}$$

$$= \frac{3(x+3) + 2(2-x)}{(x+3) + (2-x)}$$

$$= \frac{3x + 9 + 4 - 2x}{x + 3 + 2 - x}$$

$$= \frac{x + 13}{5}$$

So $y = \frac{x}{5} + \frac{13}{5}$

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Coordinate geometry in the (x, y) plane
Exercise E, Question 7

Question:

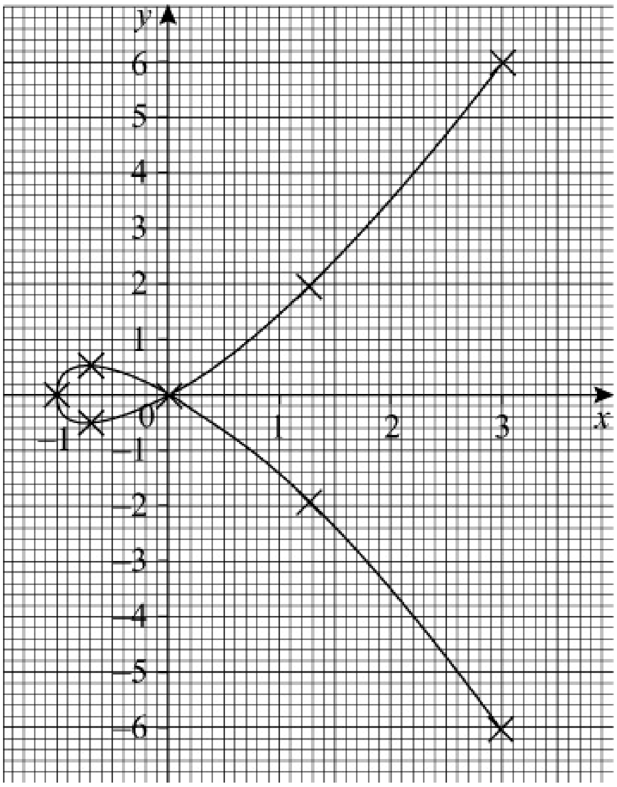
A curve has parametric equations $x = t^2 - 1$, $y = t - t^3$, where t is a parameter.

(a) Draw a graph of the curve for $-2 \leq t \leq 2$.

(b) Find the area of the finite region enclosed by the loop of the curve.

Solution:

t	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
$x = t^2 - 1$	3	1.25	0	−0.75	−1	−0.75	0	1.25	3
$y = t - t^3$	6	1.875	0	−0.375	0	0.375	0	−1.875	−6



(b) $A = 2\int_{-1}^{0} y\,dx = 2\int_{0}^{1} y\frac{dx}{dt}dt$, When $x = -1$, $t^2 - 1 = -1$, So $t = 0$

When $x = 0$, $t^2 - 1 = 0$, So $t = 1$

$$\frac{dx}{dt} = 2t$$

$$\text{So } y\frac{dx}{dt} = \left(t - t^3\right) \times 2t = 2t^2 - 2t^4$$

$$\text{Therefore } A = 2\int_{0}^{1} 2t^2 - 2t^4 dt$$

$$= 2\left[\frac{2}{3}t^3 - \frac{2}{5}t^5\right]_0^1$$

$$= 2\left(\left[\frac{2}{3}(1)^3 - \frac{2}{5}(1)^5\right] - \left[\frac{2}{3}(0)^3 - \frac{2}{5}(0)^5\right]\right)$$

$$= 2\left[\left(\frac{2}{3} - \frac{2}{5}\right) - 0\right]$$

$$= 2 \times \frac{4}{15}$$

$$= \frac{8}{15}$$

So the area of the loop is $\frac{8}{15}$.

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Coordinate geometry in the (x, y) plane
Exercise E, Question 8

Question:

A curve has parametric equations $x = t^2 - 2$, $y = 2t$, where $-2 \leq t \leq 2$.

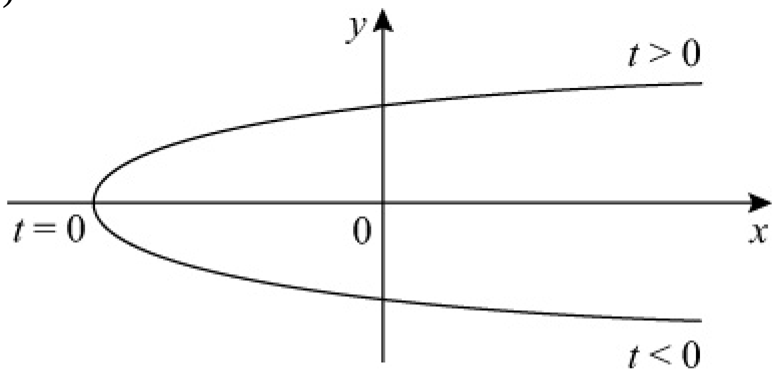
(a) Draw a graph of the curve.

(b) Indicate on your graph where
(i) $t = 0$
(ii) $t > 0$
(iii) $t < 0$

(c) Calculate the area of the finite region enclosed by the curve and the y-axis.

Solution:

(a)



(b) (i) When $t = 0$, $y = 2(0) = 0$.
This is where the curve meets the x-axis.
(ii) When $t > 0$, $y > 0$.
This is where the curve is above the x-axis.
(iii) When $t < 0$, $y < 0$.
This is where the curve is below the x-axis.

(c) $A = 2\int_{-2}^{0} y\,dx = 2\int_{0}^{\sqrt{2}} y \frac{dx}{dt}dt$, When $x = -2$, $t^2 - 2 = -2$, so $t = 0$
When $x = 0$, $t^2 - 2 = 0$, so $t = \sqrt{2}$

$$\frac{dx}{dt} = 2t$$

So $y\frac{\mathrm{d}x}{\mathrm{d}t} = 2t \times 2t = 4t^2$

Therefore $A = 2\int_0^{\sqrt{2}} 4t^2 \mathrm{d}t$

$$= 2\left[\frac{4}{3}t^3\right]_0^{\sqrt{2}}$$

$$= 2\left[\frac{4}{3}(\sqrt{2})^3 - \frac{4}{3}(0)^3\right]$$

$$= 2 \times \frac{4}{3}(\sqrt{2})^3$$

$$= \frac{8}{3}(\sqrt{2})^3$$

$$= \frac{16}{3}\sqrt{2}, \qquad \text{As } (\sqrt{2})^3 = (\sqrt{2} \times \sqrt{2}) \times \sqrt{2} = 2\sqrt{2}$$

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Coordinate geometry in the (x, y) plane
Exercise E, Question 9

Question:

Find the area of the finite region bounded by the curve with parametric equations $x = t^3$, $y = \frac{4}{t}$, $t \neq 0$, the x-axis and the lines $x = 1$ and $x = 8$.

Solution:

(1) When $x = 1$, $t^3 = 1$, so $t = \sqrt[3]{1} = 1$
When $x = 8$, $t^3 = 8$, so $t = \sqrt[3]{8} = 2$

(2) $A = \int_1^8 y\,\mathrm{d}x = \int_1^2 y \frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t$

(3) $\frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2$

So $y \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{4}{t} \times 3t^2 = 12t$

Therefore $A = \int_1^2 12t\,\mathrm{d}t$

$= \left[6t^2 \right]_1^2$
$= 6(2)^2 - 6(1)^2$
$= 24 - 6$
$= 18$

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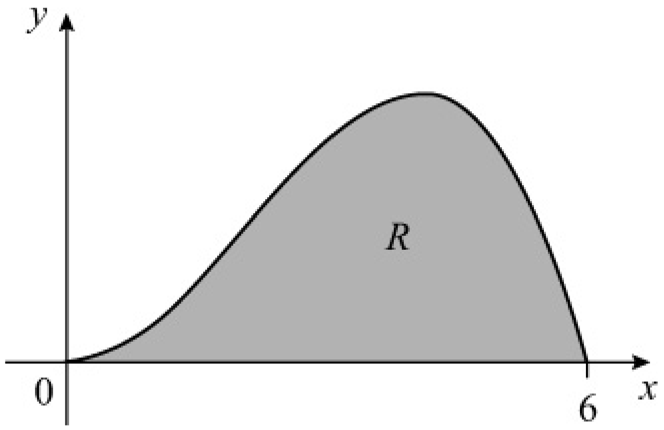
Coordinate geometry in the (x, y) plane
Exercise E, Question 10

Question:

The diagram shows a sketch of the curve with parametric equations $x = 3\sqrt{t}$, $y = t(4 - t)$, where $0 \le t \le 4$. The region R is bounded by the curve and the x-axis.

(a) Show that $y\frac{dx}{dt} = 6t^{\frac{1}{2}} - \frac{3}{2}t^{\frac{3}{2}}$.

(b) Find the area of R.



Solution:

(a) $x = 3\sqrt{t} = 3t^{\frac{1}{2}}$

$$\frac{dx}{dt} = \frac{1}{2} \times 3t^{\frac{1}{2} - 1} = \frac{3}{2}t^{-\frac{1}{2}}$$

$$y\frac{dx}{dt} = t\left(4 - t\right) \times \frac{3}{2}t^{-\frac{1}{2}}$$

$$= \left(4t - t^2\right) \times \frac{3}{2}t^{-\frac{1}{2}}$$

$$= 4t \times \frac{3}{2}t^{-\frac{1}{2}} - t^2 \times \frac{3}{2}t^{-\frac{1}{2}}$$

$$= 6t^{1 - \frac{1}{2}} - \frac{3}{2}t^{2 - \frac{1}{2}}$$

$$= 6t^{\frac{1}{2}} - \frac{3}{2}t^{\frac{3}{2}}$$

(b) $A = \int_0^4 y \frac{dx}{dt} dt$

$= \int_0^4 6t^{\frac{1}{2}} - \frac{3}{2}t^{\frac{3}{2}} dt$

$= \left[\frac{6t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{\frac{3}{2}t^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} \right]_0^4$

$= \left[4t^{\frac{3}{2}} - \frac{3}{5}t^{\frac{5}{2}} \right]_0^4$

$= \left[4(4)^{\frac{3}{2}} - \frac{3}{5}(4)^{\frac{5}{2}} \right] - \left[4(0)^{\frac{3}{2}} - \frac{3}{5}(0)^{\frac{5}{2}} \right]$

$= \left(4 \times 8 - \frac{3}{5} \times 32 \right) - 0$

$= 32 - 19\frac{1}{5}$

$= 12\frac{4}{5}$