

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 1

#### Question:

A student makes the mistake of thinking that  $\sin(A + B) \equiv \sin A + \sin B$ . Choose non-zero values of  $A$  and  $B$  to show that this statement is not true for all values of  $A$  and  $B$ .

#### Solution:

Example: Take  $A = 30^\circ$ ,  $B = 60^\circ$

$$\sin A = \frac{1}{2}$$

$$\sin B = \frac{\sqrt{3}}{2}$$

$$\sin A + \sin B \neq 1$$

$$\text{but } \sin(A + B) = \sin 90^\circ = 1.$$

This proves that  $\sin(A + B) = \sin A + \sin B$  is *not* true for all values. There will be many values of  $A$  and  $B$  for which it is true, e.g.  $A = -30^\circ$  and  $B = +30^\circ$ , and that is the danger of trying to prove a statement by taking particular examples. To prove a statement requires a sound argument; to disprove only requires one example.

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## Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

### Question:

Using the expansion of  $\cos(A - B)$  with  $A = B = \theta$ , show that  $\sin^2 \theta + \cos^2 \theta \equiv 1$ .

### Solution:

$$\cos(A - B) \equiv \cos A \cos B + \sin A \sin B$$

Set  $A = \theta, B = \theta$

$$\Rightarrow \cos(\theta - \theta) \equiv \cos \theta \cos \theta + \sin \theta \sin \theta$$

$$\Rightarrow \cos 0 \equiv \cos^2 \theta + \sin^2 \theta$$

$$\text{So } \cos^2 \theta + \sin^2 \theta \equiv 1 \quad (\text{since } \cos 0 = 1)$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 3

#### Question:

(a) Use the expansion of  $\sin(A - B)$  to show that  $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ .

(b) Use the expansion of  $\cos(A - B)$  to show that  $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ .

#### Solution:

$$(a) \sin(A - B) \equiv \sin A \cos B - \cos A \sin B$$

$$\text{Set } A = \frac{\pi}{2}, B = \theta$$

$$\begin{aligned} &\Rightarrow \sin\left(\frac{\pi}{2} - \theta\right) \equiv \sin \frac{\pi}{2} \cos \theta - \cos \frac{\pi}{2} \sin \theta \\ &\Rightarrow \sin\left(\frac{\pi}{2} - \theta\right) \equiv \cos \theta \quad (\text{since } \sin \frac{\pi}{2} = 1, \cos \frac{\pi}{2} = 0) \end{aligned}$$

$$(b) \cos(A - B) \equiv \cos A \cos B + \sin A \sin B$$

$$\text{Set } A = \frac{\pi}{2}, B = \theta$$

$$\begin{aligned} &\Rightarrow \cos\left(\frac{\pi}{2} - \theta\right) \equiv \cos \frac{\pi}{2} \cos \theta + \sin \frac{\pi}{2} \sin \theta \\ &\Rightarrow \cos\left(\frac{\pi}{2} - \theta\right) \equiv \sin \theta \quad (\text{since } \cos \frac{\pi}{2} = 0, \sin \frac{\pi}{2} = 1) \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 4

#### Question:

Express the following as a single sine, cosine or tangent:

(a)  $\sin 15^\circ \cos 20^\circ + \cos 15^\circ \sin 20^\circ$

(b)  $\sin 58^\circ \cos 23^\circ - \cos 58^\circ \sin 23^\circ$

(c)  $\cos 130^\circ \cos 80^\circ - \sin 130^\circ \sin 80^\circ$

(d) 
$$\frac{\tan 76^\circ - \tan 45^\circ}{1 + \tan 76^\circ \tan 45^\circ}$$

(e)  $\cos 2\theta \cos \theta + \sin 2\theta \sin \theta$

(f)  $\cos 4\theta \cos 3\theta - \sin 4\theta \sin 3\theta$

(g)  $\sin \frac{1}{2}\theta \cos 2\frac{1}{2}\theta + \cos \frac{1}{2}\theta \sin 2\frac{1}{2}\theta$

(h) 
$$\frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta \tan 3\theta}$$

(i)  $\sin(A + B) \cos B - \cos(A + B) \sin B$

(j)  $\cos \left( \frac{3x+2y}{2} \right) \cos \left( \frac{3x-2y}{2} \right) - \sin \left( \frac{3x+2y}{2} \right) \sin \left( \frac{3x-2y}{2} \right)$

#### Solution:

(a) Using  $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$

$$\sin 15^\circ \cos 20^\circ + \cos 15^\circ \sin 20^\circ \equiv \sin(15^\circ + 20^\circ) \equiv \sin 35^\circ$$

(b) Using  $\sin(A - B) \equiv \sin A \cos B - \cos A \sin B$

$$\sin 58^\circ \cos 23^\circ - \cos 58^\circ \sin 23^\circ \equiv \sin(58^\circ - 23^\circ) \equiv \sin 35^\circ$$

(c) Using  $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$

$$\cos 130^\circ \cos 80^\circ - \sin 130^\circ \sin 80^\circ \equiv \cos(130^\circ + 80^\circ) \equiv \cos 210^\circ$$

(d) Using  $\tan(A - B) \equiv \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\frac{\tan 76^\circ - \tan 45^\circ}{1 + \tan 76^\circ \tan 45^\circ} \equiv \tan(76^\circ - 45^\circ) \equiv \tan 31^\circ$$

(e) Using  $\cos(A - B) \equiv \cos A \cos B + \sin A \sin B$   
 $\cos 2\theta \cos \theta + \sin 2\theta \sin \theta \equiv \cos(2\theta - \theta) \equiv \cos \theta$

(f) Using  $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$   
 $\cos 4\theta \cos 3\theta - \sin 4\theta \sin 3\theta \equiv \cos(4\theta + 3\theta) \equiv \cos 7\theta$

(g) Using  $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$   
 $\sin \frac{1}{2}\theta \cos 2\frac{1}{2}\theta + \cos \frac{1}{2}\theta \sin 2\frac{1}{2}\theta \equiv \sin \left( \frac{1}{2}\theta + 2\frac{1}{2}\theta \right) \equiv \sin 3\theta$

(h) Using  $\tan(A + B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta \tan 3\theta} \equiv \tan(2\theta + 3\theta) \equiv \tan 5\theta$$

(i) Using  $\sin(P - Q) \equiv \sin P \cos Q - \cos P \sin Q$   
 $\sin(A + B) \cos B - \cos(A + B) \sin B \equiv \sin[(A + B) - B] \equiv \sin A$

(j) Using  $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$   
 $\cos \left( \frac{3x+2y}{2} \right) \cos \left( \frac{3x-2y}{2} \right) - \sin \left( \frac{3x+2y}{2} \right) \sin \left( \frac{3x-2y}{2} \right)$   
 $\equiv \cos \left[ \left( \frac{3x+2y}{2} \right) + \left( \frac{3x-2y}{2} \right) \right] \equiv \cos \left( \frac{6x}{2} \right) \equiv \cos 3x$

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## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 5**
**Question:**

Calculate, without using your calculator, the exact value of:

(a)  $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$

(b)  $\cos 110^\circ \cos 20^\circ + \sin 110^\circ \sin 20^\circ$

(c)  $\sin 33^\circ \cos 27^\circ + \cos 33^\circ \sin 27^\circ$

(d)  $\cos \frac{\pi}{8} \cos \frac{\pi}{8} - \sin \frac{\pi}{8} \sin \frac{\pi}{8}$

(e)  $\sin 60^\circ \cos 15^\circ - \cos 60^\circ \sin 15^\circ$

(f)  $\cos 70^\circ (\cos 50^\circ - \tan 70^\circ \sin 50^\circ)$

(g) 
$$\frac{\tan 45^\circ + \tan 15^\circ}{1 - \tan 45^\circ \tan 15^\circ}$$

(h) 
$$\frac{1 - \tan 15^\circ}{1 + \tan 15^\circ}$$

(i) 
$$\frac{\tan(\frac{7\pi}{12}) - \tan(\frac{\pi}{3})}{1 + \tan(\frac{7\pi}{12}) \tan(\frac{\pi}{3})}$$

(j)  $\sqrt{3} \cos 15^\circ - \sin 15^\circ$

**Solution:**

(a) Using  $\sin(A + B)$  expansion

$$\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = \sin(30 + 60)^\circ = \sin 90^\circ = 1$$

(b)  $\cos 110^\circ \cos 20^\circ + \sin 110^\circ \sin 20^\circ = \cos(110 - 20)^\circ = \cos 90^\circ = 0$

(c)  $\sin 33^\circ \cos 27^\circ + \cos 33^\circ \sin 27^\circ = \sin(33 + 27)^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$

(d)  $\cos \frac{\pi}{8} \cos \frac{\pi}{8} - \sin \frac{\pi}{8} \sin \frac{\pi}{8} = \cos\left(\frac{\pi}{8} + \frac{\pi}{8}\right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

(e)  $\sin 60^\circ \cos 15^\circ - \cos 60^\circ \sin 15^\circ = \sin(60 - 15)^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$$(f) \cos 70^\circ \cos 50^\circ - \cos 70^\circ \tan 70^\circ \sin 50^\circ \\ = \cos 70^\circ \cos 50^\circ - \sin 70^\circ \sin 50^\circ$$

$$\begin{aligned} & \left( \cos \theta \times \tan \theta = \cancel{\cos \theta} \times \frac{\sin \theta}{\cancel{\cos \theta}} = \sin \theta \right) \\ & = \cos (120^\circ + 30^\circ) \\ & = \cos 120^\circ \\ & = -\cos 60^\circ = -\frac{1}{2} \end{aligned}$$

$$(g) \frac{\tan 45^\circ + \tan 15^\circ}{1 - \tan 45^\circ \tan 15^\circ} = \tan (45^\circ + 15^\circ) = \tan 60^\circ = \sqrt{3}$$

$$\begin{aligned} (h) \frac{1 - \tan 15^\circ}{1 + \tan 15^\circ} &= \frac{\tan 45^\circ - \tan 15^\circ}{1 + \tan 45^\circ \tan 15^\circ} \quad (\text{using } \tan 45^\circ = 1) \\ &= \tan (45^\circ - 15^\circ) = \tan 30^\circ = \frac{\sqrt{3}}{3} \end{aligned}$$

$$(i) \frac{\tan \left( \frac{7\pi}{12} \right) - \tan \left( \frac{1}{3}\pi \right)}{1 + \tan \left( \frac{7\pi}{12} \right) \tan \left( \frac{1}{3}\pi \right)} = \tan \left( \frac{7\pi}{12} - \frac{\pi}{3} \right) = \tan \frac{3\pi}{12} = \tan \frac{\pi}{4} = 1$$

$$(j) \text{ This is very similar to part (e) but you need to rewrite it as } 2 \left( \frac{\sqrt{3}}{2} \cos 15^\circ - \frac{1}{2} \sin 15^\circ \right) \text{ to appreciate it!}$$

$$\begin{aligned} \sqrt{3} \cos 15^\circ - \sin 15^\circ &\equiv 2 \left( \frac{\sqrt{3}}{2} \cos 15^\circ - \frac{1}{2} \sin 15^\circ \right) \\ &\equiv 2 (\sin 60^\circ \cos 15^\circ - \cos 60^\circ \sin 15^\circ) \\ &\equiv 2 \sin (60^\circ - 15^\circ) \\ &\equiv 2 \sin 45^\circ \\ &= \sqrt{2} \end{aligned}$$

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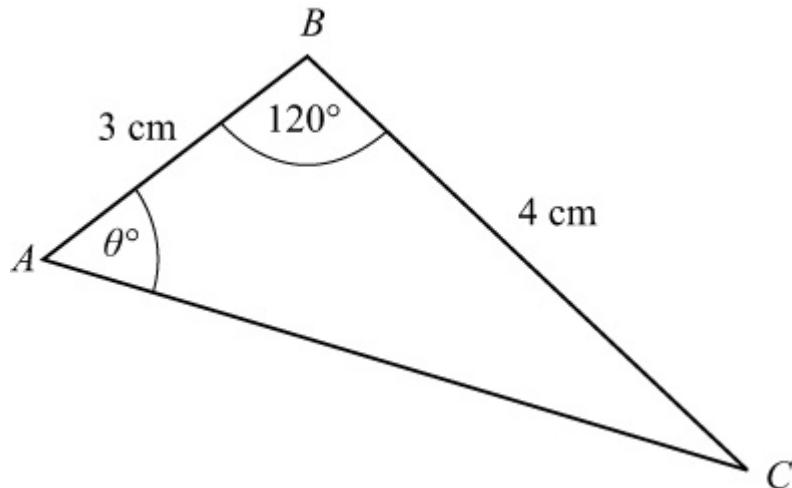
## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 6**
**Question:**

Triangle  $ABC$  is such that  $AB = 3\text{ cm}$ ,  $BC = 4\text{ cm}$ ,  $\angle ABC = 120^\circ$  and  $\angle BAC = \theta^\circ$ .

(a) Write down, in terms of  $\theta$ , an expression for  $\angle ACB$ .

(b) Using the sine rule, or otherwise, show that  $\tan \theta^\circ = \frac{2\sqrt{3}}{5}$ .

**Solution:**


$$(a) \angle ACB = 180^\circ - 120^\circ - \theta^\circ = (60 - \theta)^\circ$$

$$(b) \text{Using sine rule: } \frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\Rightarrow \frac{\sin(60 - \theta)^\circ}{3} = \frac{\sin \theta^\circ}{4}$$

$$\Rightarrow 4 \sin(60 - \theta)^\circ = 3 \sin \theta^\circ$$

$$\Rightarrow 4 \sin 60^\circ \cos \theta^\circ - 4 \cos 60^\circ \sin \theta^\circ = 3 \sin \theta^\circ$$

$$\Rightarrow 2\sqrt{3} \cos \theta^\circ - 2 \sin \theta^\circ = 3 \sin \theta^\circ \quad \left( \sin 60^\circ = \frac{\sqrt{3}}{2} \right)$$

$$\Rightarrow 5 \sin \theta^\circ = 2\sqrt{3} \cos \theta^\circ$$

$$\Rightarrow \frac{\sin \theta^\circ}{\cos \theta^\circ} = \frac{2\sqrt{3}}{5}$$

$$\Rightarrow \tan \theta^\circ = \frac{2\sqrt{3}}{5}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise A, Question 7

### Question:

Prove the identities

$$(a) \sin(A + 60^\circ) + \sin(A - 60^\circ) \equiv \sin A$$

$$(b) \frac{\cos A}{\sin B} - \frac{\sin A}{\cos B} \equiv \frac{\cos(A + B)}{\sin B \cos B}$$

$$(c) \frac{\sin(x + y)}{\cos x \cos y} \equiv \tan x + \tan y$$

$$(d) \frac{\cos(x + y)}{\sin x \sin y} + 1 \equiv \cot x \cot y$$

$$(e) \cos\left(\theta + \frac{\pi}{3}\right) + \sqrt{3}\sin\theta \equiv \sin\left(\theta + \frac{\pi}{6}\right)$$

$$(f) \cot(A + B) \equiv \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$(g) \sin^2(45 + \theta)^\circ + \sin^2(45 - \theta)^\circ \equiv 1$$

$$(h) \cos(A + B)\cos(A - B) \equiv \cos^2 A - \sin^2 B$$

### Solution:

$$\begin{aligned} (a) \text{L.H.S.} &\equiv \sin(A + 60^\circ) + \sin(A - 60^\circ) \\ &\equiv \sin A \cos 60^\circ + \cos A \sin 60^\circ + \sin A \cos 60^\circ - \cos A \sin 60^\circ \\ &\equiv 2 \sin A \cos 60^\circ \\ &\equiv \sin A \quad (\text{since } \cos 60^\circ = \frac{1}{2}) \\ &\equiv \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} (b) \text{L.H.S.} &\equiv \frac{\cos A}{\sin B} - \frac{\sin A}{\cos B} \\ &\equiv \frac{\cos A \cos B - \sin A \sin B}{\sin B \cos B} \\ &\equiv \frac{\cos(A + B)}{\sin B \cos B} \\ &\equiv \text{R.H.S.} \end{aligned}$$

$$(c) \text{L.H.S.} \equiv \frac{\sin(x + y)}{\cos x \cos y}$$

$$\begin{aligned}
 &\equiv \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y} \\
 &\equiv \frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y} \\
 &\equiv \frac{\sin x}{\cos x} + \frac{\sin y}{\cos y} \\
 &\equiv \tan x + \tan y \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{d}) \text{ L.H.S.} &\equiv \frac{\cos(x+y)}{\sin x \sin y} + 1 \\
 &\equiv \frac{\cos(x+y) + \sin x \sin y}{\sin x \sin y} \\
 &\equiv \frac{\cos x \cos y - \sin x \sin y + \sin x \sin y}{\sin x \sin y} \\
 &\equiv \frac{\cos x \cos y}{\sin x \sin y} \\
 &\equiv \cot x \cot y \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{e}) \text{ L.H.S.} &\equiv \cos \left( \theta + \frac{\pi}{3} \right) + \sqrt{3} \sin \theta \\
 &\equiv \cos \theta \cos \frac{\pi}{3} - \sin \theta \sin \frac{\pi}{3} + \sqrt{3} \sin \theta \\
 &\equiv \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta + \sqrt{3} \sin \theta \\
 &\equiv \frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta \\
 &\equiv \sin \theta \cos \frac{\pi}{6} + \cos \theta \sin \frac{\pi}{6} \quad \left( \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}, \sin \frac{\pi}{6} = \frac{1}{2} \right) \\
 &\equiv \sin \left( \theta + \frac{\pi}{6} \right) \quad [ \sin(A+B) ] \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

(f)

$$\begin{aligned}
 \text{R.H.S.} &\equiv \frac{\cot A \cot B - 1}{\cot A + \cot B} \\
 &\equiv \frac{\frac{1}{\tan A} \times \frac{1}{\tan B} - 1}{\frac{1}{\tan A} + \frac{1}{\tan B}} \\
 &\equiv \frac{\frac{1 - \tan A \tan B}{\tan A \tan B}}{\frac{\tan B - \tan A}{\tan A \tan B}} \\
 &\equiv \frac{1 - \tan A \tan B}{\tan A + \tan B} \times \frac{\tan A \tan B}{\tan A + \tan B} \\
 &\equiv \frac{1 - \tan A \tan B}{\tan A + \tan B} \\
 &\equiv \frac{1}{\frac{\tan A + \tan B}{1 - \tan A \tan B}} \\
 &\equiv \frac{1}{\tan(A + B)} \\
 &\equiv \cot(A + B) \\
 &\equiv \text{L.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(g) L.H.S.} &\equiv \sin^2(45^\circ + \theta) + \sin^2(45^\circ - \theta) \\
 &\equiv (\sin 45^\circ \cos \theta + \cos 45^\circ \sin \theta)^2 + (\sin 45^\circ \cos \theta - \cos 45^\circ \sin \theta)^2
 \end{aligned}$$

As  $\sin 45^\circ = \cos 45^\circ$  it is easier to take out as a common factor.

$$\begin{aligned}
 &\equiv (\sin 45^\circ)^2 [(\cos \theta + \sin \theta)^2 + (\cos \theta - \sin \theta)^2] \\
 &\equiv \frac{1}{2} \left[ \cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \right. \\
 &\quad \left. \cos \theta + \sin^2 \theta \right] \\
 &\equiv \frac{1}{2} \left[ 2 \left( \sin^2 \theta + \cos^2 \theta \right) \right] \\
 &\equiv \frac{1}{2} \times 2 \quad (\sin^2 \theta + \cos^2 \theta \equiv 1) \\
 &\equiv 1 \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

Alternatively:

$$\text{as } \sin(90^\circ - x^\circ) \equiv \cos x^\circ,$$

$$\text{if } x = 45^\circ + \theta^\circ \text{ then } \sin(45^\circ - \theta^\circ) \equiv \cos(45^\circ + \theta^\circ)$$

and original L.H.S. becomes  $\sin^2(45^\circ + \theta) + \cos^2(45^\circ + \theta)^\circ$

which is identically = 1

$$\begin{aligned}
 \text{(h) L.H.S.} &\equiv \cos(A + B) \cos(A - B) \\
 &\equiv (\cos A \cos B - \sin A \sin B)(\cos A \cos B + \sin A \sin B) \\
 &\equiv \cos^2 A \cos^2 B - \sin^2 A \sin^2 B
 \end{aligned}$$

$$\begin{aligned}&\equiv \cos^2 A (1 - \sin^2 B) - (1 - \cos^2 A) \sin^2 B \\&\equiv \cos^2 A - \cos^2 A \sin^2 B - \sin^2 B + \cos^2 A \sin^2 B \\&\equiv \cos^2 A - \sin^2 B \\&\equiv \text{R.H.S.}\end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise A, Question 8

### Question:

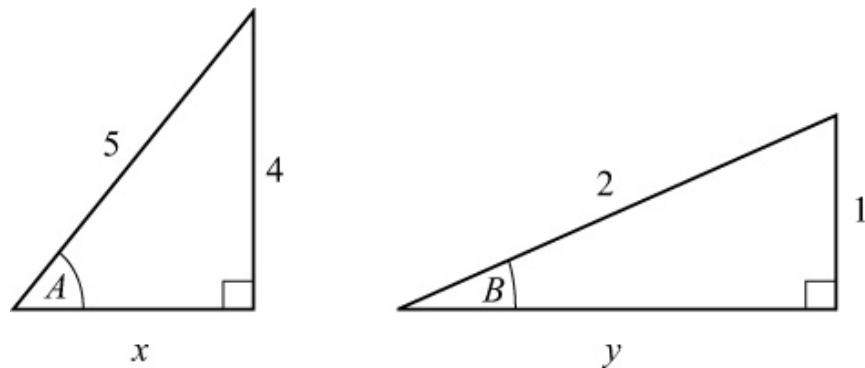
Given that  $\sin A = \frac{4}{5}$  and  $\sin B = \frac{1}{2}$ , where  $A$  and  $B$  are both acute angles, calculate the exact values of

(a)  $\sin(A + B)$

(b)  $\cos(A - B)$

(c)  $\sec(A - B)$

### Solution:



Using Pythagoras' theorem  $x = 3$  and  $y = \sqrt{3}$

$$(a) \sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{4}{5} \times \frac{\sqrt{3}}{2} + \frac{3}{5} \times \frac{1}{2} = \frac{4\sqrt{3} + 3}{10}$$

$$(b) \cos(A - B) = \cos A \cos B + \sin A \sin B = \frac{3}{5} \times \frac{\sqrt{3}}{2} + \frac{4}{5} \times \frac{1}{2} = \frac{3\sqrt{3} + 4}{10}$$

$$\begin{aligned} (c) \sec(A - B) &= \frac{1}{\cos(A - B)} = \frac{10}{3\sqrt{3} + 4} \\ &= \frac{10(3\sqrt{3} - 4)}{(3\sqrt{3} + 4)(3\sqrt{3} - 4)} \\ &= \frac{10(3\sqrt{3} - 4)}{27 - 16} \\ &= \frac{10(3\sqrt{3} - 4)}{11} \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 9

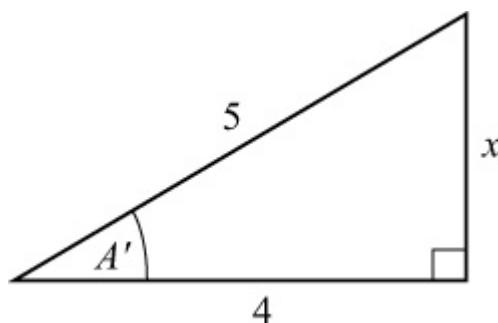
#### Question:

Given that  $\cos A = -\frac{4}{5}$ , and  $A$  is an obtuse angle measured in radians, find the exact value of

- (a)  $\sin A$
- (b)  $\cos(\pi + A)$
- (c)  $\sin\left(\frac{\pi}{3} + A\right)$
- (d)  $\tan\left(\frac{\pi}{4} + A\right)$

#### Solution:

Draw a right-angled triangle where  $\cos A' = \frac{4}{5}$



Using Pythagoras' theorem  $x = 3$

$$\text{So } \sin A' = \frac{3}{5}, \tan A' = \frac{3}{4}$$

(a) As  $A$  is in the 2nd quadrant,  $\sin A = \sin A'$

$$\sin A = \frac{3}{5}$$

(b)  $\cos(\pi + A) = \cos \pi \cos A - \sin \pi \sin A = -\cos A \quad (\cos \pi = -1, \sin \pi = 0)$

$$\cos(\pi + A) = + \frac{4}{5}$$

$$\begin{aligned}
 (\text{c}) \sin\left(\frac{\pi}{3} + A\right) &= \sin \frac{\pi}{3} \cos A + \cos \frac{\pi}{3} \sin A \\
 &= \left(\frac{\sqrt{3}}{2}\right) \left(-\frac{4}{5}\right) + \left(\frac{1}{2}\right) \left(\frac{3}{5}\right) \\
 &= \frac{3 - 4\sqrt{3}}{10}
 \end{aligned}$$

$$(\text{d}) \text{ As } A \text{ is in 2nd quadrant, } \tan A = - \tan A' = - \frac{3}{4}$$

$$\tan\left(\frac{\pi}{4} + A\right) = \frac{\tan \frac{\pi}{4} + \tan A}{1 - \tan \frac{\pi}{4} \tan A} = \frac{1 + \tan A}{1 - \tan A} = \frac{\frac{1}{4}}{\frac{7}{4}} = \frac{1}{7}$$

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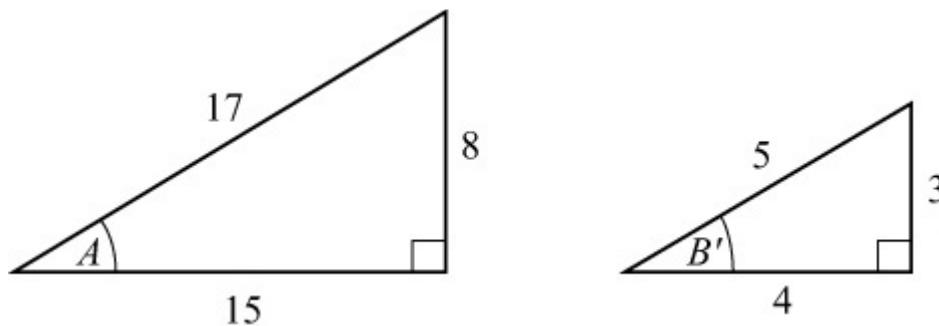
**Exercise A, Question 10**
**Question:**

Given that  $\sin A = \frac{8}{17}$ , where  $A$  is acute, and  $\cos B = -\frac{4}{5}$ , where  $B$  is obtuse, calculate the exact value of

(a)  $\sin(A - B)$

(b)  $\cos(A - B)$

(c)  $\cot(A - B)$

**Solution:**


$$\sin B = \sin B', \tan B = -\tan B'$$

By Pythagoras' theorem, the remaining sides are 15 and 3.

$$\text{So } \sin A = \frac{8}{17}, \cos A = \frac{15}{17}, \tan A = \frac{8}{15}$$

$$\text{and } \sin B = \frac{3}{5}, \cos B = -\frac{4}{5}, \tan B = -\frac{3}{4}$$

$$(a) \sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\begin{aligned} &= \left( \frac{8}{17} \right) \left( -\frac{4}{5} \right) - \left( \frac{15}{17} \right) \left( \frac{3}{5} \right) \\ &= \frac{-32 - 45}{85} = -\frac{77}{85} \end{aligned}$$

$$(b) \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= \left( \frac{15}{17} \right) \left( -\frac{4}{5} \right) + \left( \frac{8}{17} \right) \left( \frac{3}{5} \right)$$

$$= \frac{-60 + 24}{85} = -\frac{36}{85}$$

$$(c) \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{24}{60}} = \frac{\frac{77}{60}}{\frac{36}{60}} = \frac{77}{36}$$

$$\text{So } \cot(A - B) = \frac{1}{\tan(A - B)} = \frac{36}{77}$$

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## Edexcel AS and A Level Modular Mathematics

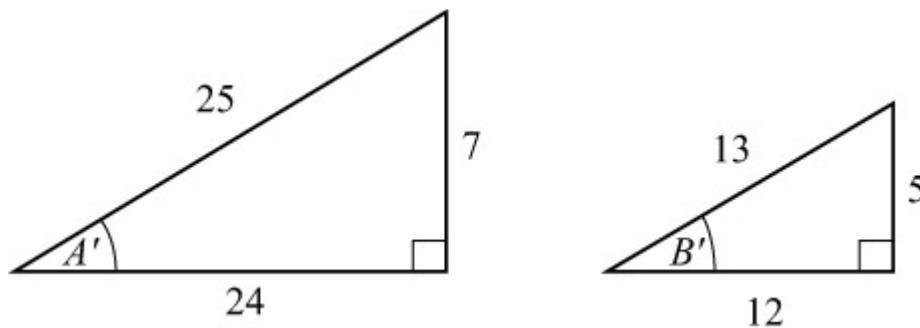
### Exercise A, Question 11

#### Question:

Given that  $\tan A = \frac{7}{24}$ , where  $A$  is reflex, and  $\sin B = \frac{5}{13}$ , where  $B$  is obtuse, calculate the exact value of

- (a)  $\sin(A + B)$
- (b)  $\tan(A - B)$
- (c)  $\operatorname{cosec}(A + B)$

#### Solution:



Using Pythagoras' theorem, the remaining sides are 25 and 12.  
As  $A$  is in the 3rd quadrant ( $\tan A$  is +ve, and  $A$  is reflex),  
 $\sin A = -\sin A'$ ,  $\cos A = -\cos A'$

$$\text{So } \sin A = -\frac{7}{25}, \cos A = -\frac{24}{25}, \tan A = \frac{7}{24}$$

As  $B$  is in the 2nd quadrant,  
 $\cos B = -\cos B'$ ,  $\tan B = -\tan B'$

$$\text{So } \sin B = \frac{5}{13}, \cos B = -\frac{12}{13}, \tan B = -\frac{5}{12}$$

(a)

$$\begin{aligned} \sin(A + B) &= \sin A \cos B + \cos A \sin B \\ &= \left(-\frac{7}{25}\right) \left(-\frac{12}{13}\right) + \left(-\frac{24}{25}\right) \left(\frac{5}{13}\right) \\ &= \frac{84 - 120}{325} = -\frac{36}{325} \end{aligned}$$

$$(b) \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\frac{7}{24} + \frac{5}{12}}{1 - (\frac{7}{24})(\frac{5}{12})} = \frac{\frac{17}{24}}{\frac{253}{288}} = \frac{204}{253}$$

$$(c) \operatorname{cosec}(A + B) = \frac{1}{\sin(A + B)} = -\frac{325}{36}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 12

#### Question:

Write the following as a single trigonometric function, assuming that  $\theta$  is measured in radians:

(a)  $\cos^2 \theta - \sin^2 \theta$

(b)  $2 \sin 4\theta \cos 4\theta$

(c)  $\frac{1 + \tan \theta}{1 - \tan \theta}$

(d)  $\frac{1}{\sqrt{2}} (\sin \theta + \cos \theta)$

#### Solution:

$$(a) \cos^2 \theta - \sin^2 \theta = \cos \theta \cos \theta - \sin \theta \sin \theta = \cos (\theta + \theta) = \cos 2\theta$$

$$\begin{aligned} (b) 2 \sin 4\theta \cos 4\theta &= \sin 4\theta \cos 4\theta + \sin 4\theta \cos 4\theta \\ &= \sin 4\theta \cos 4\theta + \cos 4\theta \sin 4\theta \\ &= \sin (4\theta + 4\theta) \\ &= \sin 8\theta \end{aligned}$$

$$\begin{aligned} (c) \frac{1 + \tan \theta}{1 - \tan \theta} &= \frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} \quad (\text{as } \tan \frac{\pi}{4} = 1) \\ &= \tan \left( \frac{\pi}{4} + \theta \right) \end{aligned}$$

$$\begin{aligned} (d) \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta &= \sin \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4} \quad (\text{as } \frac{1}{\sqrt{2}} = \cos \frac{\pi}{4} = \sin \frac{\pi}{4}) \\ &= \sin \left( \theta + \frac{\pi}{4} \right) \end{aligned}$$

[ **Note:** (d) could be  $\cos \left( \theta - \frac{\pi}{4} \right)$  ]

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### Exercise A, Question 13

#### Question:

Solve, in the interval  $0^\circ \leq \theta < 360^\circ$ , the following equations. Give answers to the nearest  $0.1^\circ$ .

$$(a) 3\cos\theta = 2\sin(\theta + 60^\circ)$$

$$(b) \sin(\theta + 30^\circ) + 2\sin\theta = 0$$

$$(c) \cos(\theta + 25^\circ) + \sin(\theta + 65^\circ) = 1$$

$$(d) \cos\theta = \cos(\theta + 60^\circ)$$

$$(e) \tan(\theta - 45^\circ) = 6\tan\theta$$

$$(f) \sin\theta + \cos\theta = 1$$

#### Solution:

$$(a) 3\cos\theta = 2\sin(\theta + 60^\circ)$$

$$\Rightarrow 3\cos\theta = 2(\sin\theta\cos 60^\circ + \cos\theta\sin 60^\circ)$$

$$\Rightarrow 3\cos\theta = 2\left(\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right) = \sin\theta + \sqrt{3}\cos\theta$$

$$\Rightarrow (3 - \sqrt{3})\cos\theta = \sin\theta$$

$$\Rightarrow \tan\theta = 3 - \sqrt{3} \quad \left( \tan\theta = \frac{\sin\theta}{\cos\theta} \right)$$

As  $\tan\theta$  is +ve,  $\theta$  is in 1st and 3rd quadrants.

$$\theta = \tan^{-1}(3 - \sqrt{3}), 180^\circ + \tan^{-1}(3 - \sqrt{3}) = 51.7^\circ, 231.7^\circ$$

$$(b) \sin(\theta + 30^\circ) + 2\sin\theta = 0$$

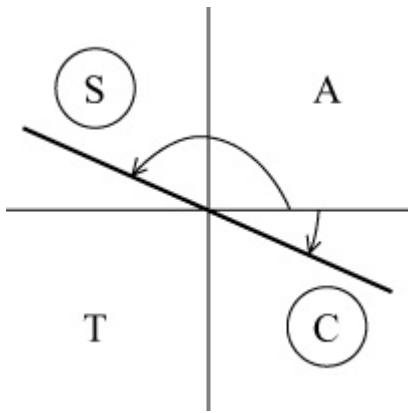
$$\Rightarrow \sin\theta\cos 30^\circ + \cos\theta\sin 30^\circ + 2\sin\theta = 0$$

$$\Rightarrow \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta + 2\sin\theta = 0$$

$$\Rightarrow (4 + \sqrt{3})\sin\theta = -\cos\theta$$

$$\Rightarrow \tan\theta = -\frac{1}{4 + \sqrt{3}}$$

As  $\tan\theta$  is -ve,  $\theta$  is in the 2nd and 4th quadrants.



$$\theta = \tan^{-1} \left( -\frac{1}{4 + \sqrt{3}} \right) + 180^\circ, \tan^{-1} \left( -\frac{1}{4 + \sqrt{3}} \right) + 360^\circ \\ = 170.1^\circ, 350.1^\circ.$$

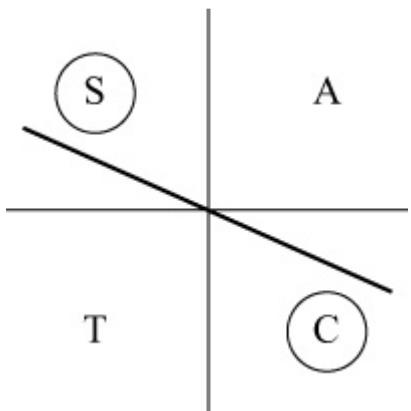
(c)  $\cos(\theta + 25^\circ) + \sin(\theta + 65^\circ) = 1$   
 $\Rightarrow \cos\theta\cos 25^\circ - \sin\theta\sin 25^\circ + \sin\theta\cos 65^\circ + \cos\theta\sin 65^\circ = 1$

As  $\sin(90 - x)^\circ = \cos x^\circ$  and  $\cos(90 - x)^\circ = \sin x^\circ$   
 $\cos 25^\circ = \sin 65^\circ$  and  $\sin 25^\circ = \cos 65^\circ$   
 $\Rightarrow 2\cos\theta\sin 65^\circ = 1$   
 $\Rightarrow \cos\theta = \frac{1}{2\sin 65^\circ} = 0.55169$

$$\theta = \cos^{-1}(0.55169), 360^\circ - \cos^{-1}(0.55169) = 56.5^\circ, 303.5^\circ$$

(d)  $\cos\theta = \cos(\theta + 60^\circ)$   
 $\Rightarrow \cos\theta = \cos\theta\cos 60^\circ - \sin\theta\sin 60^\circ = \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$   
 $\Rightarrow \cos\theta = -\sqrt{3}\sin\theta$   
 $\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}} \quad \left( \tan\theta = \frac{\sin\theta}{\cos\theta} \right)$

$\theta$  is in the 2nd and 4th quadrants.



$\tan^{-1} \left( -\frac{1}{\sqrt{3}} \right)$  is not in given interval.

$$\theta = \tan^{-1} \left( -\frac{1}{\sqrt{3}} \right) + 180^\circ, \tan^{-1} \left( -\frac{1}{\sqrt{3}} \right) + 360^\circ = 150^\circ, 330^\circ$$

$$(e) \tan(\theta - 45^\circ) = 6 \tan \theta$$

$$\Rightarrow \frac{\tan \theta - \tan 45^\circ}{1 + \tan \theta \tan 45^\circ} = 6 \tan \theta$$

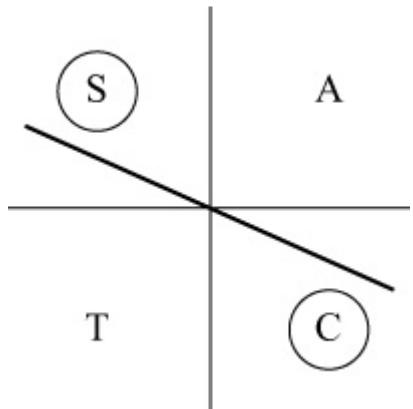
$$\Rightarrow \frac{\tan \theta - 1}{1 + \tan \theta} = 6 \tan \theta$$

$$\Rightarrow \tan \theta - 1 = 6 \tan \theta + 6 \tan^2 \theta$$

$$\Rightarrow 6 \tan^2 \theta + 5 \tan \theta + 1 = 0$$

$$\Rightarrow (3 \tan \theta + 1)(2 \tan \theta + 1) = 0$$

$$\Rightarrow \tan \theta = -\frac{1}{3} \text{ or } \tan \theta = -\frac{1}{2}$$



$$\tan \theta = -\frac{1}{3} \Rightarrow \theta = \tan^{-1} \left( -\frac{1}{3} \right) + 180^\circ, \tan^{-1} \left( -\frac{1}{3} \right) + 360^\circ = 161.6^\circ, 341.6^\circ$$

$$\tan \theta = -\frac{1}{2} \Rightarrow \theta = \tan^{-1} \left( -\frac{1}{2} \right) + 180^\circ, \tan^{-1} \left( -\frac{1}{2} \right) + 360^\circ = 153.4^\circ, 333.4^\circ$$

Set of solutions:  $153.4^\circ, 161.6^\circ, 333.4^\circ, 341.6^\circ$

$$(f) \sin \theta + \cos \theta = 1$$

$$\Rightarrow \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos 45^\circ \sin \theta + \sin 45^\circ \cos \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin(\theta + 45^\circ) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta + 45^\circ = 45^\circ, 135^\circ$$

$$\Rightarrow \theta = 0^\circ, 90^\circ$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 14

#### Question:

(a) Solve the equation  $\cos \theta \cos 30^\circ - \sin \theta \sin 30^\circ = 0.5$ , for  $0^\circ \leq \theta \leq 360^\circ$ .

(b) Hence write down, in the same interval, the solutions of  $\sqrt{3} \cos \theta - \sin \theta = 1$ .

#### Solution:

$$(a) \cos \theta \cos 30^\circ - \sin \theta \sin 30^\circ = 0.5$$

$$\Rightarrow \cos(\theta + 30^\circ) = 0.5$$

$$\Rightarrow \theta + 30^\circ = 60^\circ, 300^\circ$$

$$\Rightarrow \theta = 30^\circ, 270^\circ$$

$$(b) \cos \theta \cos 30^\circ - \sin \theta \sin 30^\circ \equiv \cos \theta \times \frac{\sqrt{3}}{2} - \sin \theta \times \frac{1}{2}$$

$$\text{So } \cos \theta \cos 30^\circ - \sin \theta \sin 30^\circ = \frac{1}{2}$$

is identical to  $\sqrt{3} \cos \theta - \sin \theta = 1$

Solutions are same as (a), i.e.  $30^\circ, 270^\circ$

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## Edexcel AS and A Level Modular Mathematics

Exercise A, Question 15

### Question:

(a) Express  $\tan(45 + 30)^\circ$  in terms of  $\tan 45^\circ$  and  $\tan 30^\circ$ .

(b) Hence show that  $\tan 75^\circ = 2 + \sqrt{3}$ .

### Solution:

$$(a) \tan(45 + 30)^\circ = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$$

(b)

$$\tan 75^\circ = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$\begin{aligned} &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \\ &= \frac{(\sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} \\ &= \frac{4 + 2\sqrt{3}}{2} \\ &= 2 + \sqrt{3} \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 16

#### Question:

Show that  $\sec 105^\circ = -\sqrt{2}(1 + \sqrt{3})$

#### Solution:

$$\begin{aligned}\cos(60 + 45)^\circ &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\&= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \\&= \frac{1 - \sqrt{3}}{2\sqrt{2}}\end{aligned}$$

$$\begin{aligned}\text{So } \sec 105^\circ &= \frac{1}{\cos 105^\circ} = \frac{2\sqrt{2}}{1 - \sqrt{3}} \\&= \frac{2\sqrt{2}(1 + \sqrt{3})}{(1 - \sqrt{3})(1 + \sqrt{3})} \\&= \frac{2\sqrt{2}(1 + \sqrt{3})}{-2} \\&= -\sqrt{2}(1 + \sqrt{3})\end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 17**
**Question:**

Calculate the exact values of

- (a)  $\cos 15^\circ$
- (b)  $\sin 75^\circ$
- (c)  $\sin (120 + 45)^\circ$
- (d)  $\tan 165^\circ$

**Solution:**

$$\begin{aligned}
 (a) \cos 15^\circ &= \cos (45 - 30)^\circ \\
 &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\
 &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} \\
 &= \frac{\sqrt{2}}{4} (\sqrt{3} + 1)
 \end{aligned}$$

$$\begin{aligned}
 (b) \sin 75^\circ &= \sin (45 + 30)^\circ \\
 &= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ \\
 &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} \\
 &= \frac{\sqrt{2}}{4} (\sqrt{3} + 1)
 \end{aligned}$$

$$[\sin 75^\circ = \cos (90 - 75^\circ) = \cos 15^\circ]$$

$$\begin{aligned}
 (c) \sin (120 + 45)^\circ &= \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ \\
 &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right) \frac{\sqrt{2}}{2} \\
 &= \frac{\sqrt{2}}{4} (\sqrt{3} - 1)
 \end{aligned}$$

$$\begin{aligned}
 (d) \tan 165^\circ &= \tan (120 + 45)^\circ = \frac{\tan 120^\circ + \tan 45^\circ}{1 - \tan 120^\circ \tan 45^\circ} \quad (\tan 120^\circ \\
 &= -\sqrt{3})
 \end{aligned}$$

$$\begin{aligned} &= \frac{-\sqrt{3+1}}{1+\sqrt{3}} \\ &= \frac{(-\sqrt{3+1})(\sqrt{3}-1)}{(\sqrt{3+1})(\sqrt{3}-1)} \\ &= \frac{-4+2\sqrt{3}}{2} \\ &= -2 + \sqrt{3} \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 18

#### Question:

- (a) Given that  $3 \sin(x - y) - \sin(x + y) = 0$ , show that  $\tan x = 2 \tan y$ .
- (b) Solve  $3 \sin(x - 45^\circ) - \sin(x + 45^\circ) = 0$ , for  $0^\circ \leq x \leq 360^\circ$ .

#### Solution:

$$\begin{aligned} (a) \quad & 3 \sin(x - y) - \sin(x + y) = 0 \\ \Rightarrow & 3 \sin x \cos y - 3 \cos x \sin y - \sin x \cos y - \cos x \sin y = 0 \\ \Rightarrow & 2 \sin x \cos y = 4 \cos x \sin y \\ \Rightarrow & \frac{2 \sin x \cos y}{\cos x \cos y} = \frac{4 \cos x \sin y}{\cos x \cos y} \\ \Rightarrow & \frac{2 \sin x}{\cos x} = \frac{4 \sin y}{\cos y} \\ \Rightarrow & 2 \tan x = 4 \tan y \end{aligned}$$

So  $\tan x = 2 \tan y$

$$\begin{aligned} (b) \quad & \text{Put } y = 45^\circ \Rightarrow \tan x = 2 \\ \text{So } x = \tan^{-1} 2, 180^\circ + \tan^{-1} 2 &= 63.4^\circ, 243.4^\circ \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 19

#### Question:

Given that  $\sin x (\cos y + 2 \sin y) = \cos x (2 \cos y - \sin y)$ , find the value of  $\tan(x + y)$ .

#### Solution:

$$\begin{aligned}\sin x (\cos y + 2 \sin y) &= \cos x (2 \cos y - \sin y) \\ \Rightarrow \sin x \cos y + 2 \sin x \sin y &= 2 \cos x \cos y - \cos x \sin y \\ \Rightarrow \sin x \cos y + \cos x \sin y &= 2 (\cos x \cos y - \sin x \sin y) \\ \Rightarrow \sin(x + y) &= 2 \cos(x + y) \\ \Rightarrow \frac{\sin(x + y)}{\cos(x + y)} &= 2 \\ \Rightarrow \tan(x + y) &= 2\end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise A, Question 20

### Question:

Given that  $\tan(x - y) = 3$ , express  $\tan y$  in terms of  $\tan x$ .

### Solution:

As  $\tan(x - y) = 3$

$$\text{so } \frac{\tan x - \tan y}{1 + \tan x \tan y} = 3$$

$$\Rightarrow \tan x - \tan y = 3 + 3 \tan x \tan y$$

$$\Rightarrow 3 \tan x \tan y + \tan y = \tan x - 3$$

$$\Rightarrow \tan y (3 \tan x + 1) = \tan x - 3$$

$$\Rightarrow \tan y = \frac{\tan x - 3}{3 \tan x + 1}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 21

#### Question:

In each of the following, calculate the exact value of  $\tan x^\circ$ .

$$(a) \tan(x - 45)^\circ = \frac{1}{4}$$

$$(b) \sin(x - 60)^\circ = 3 \cos(x + 30)^\circ$$

$$(c) \tan(x - 60)^\circ = 2$$

#### Solution:

$$(a) \tan(x - 45)^\circ = \frac{1}{4}$$

$$\Rightarrow \frac{\tan x^\circ - \tan 45^\circ}{1 + \tan x^\circ \tan 45^\circ} = \frac{1}{4}$$

$$\Rightarrow 4 \tan x^\circ - 4 = 1 + \tan x^\circ \quad (\tan 45^\circ = 1)$$

$$\Rightarrow 3 \tan x^\circ = 5$$

$$\Rightarrow \tan x^\circ = \frac{5}{3}$$

$$(b) \sin(x - 60)^\circ = 3 \cos(x + 30)^\circ$$

$$\Rightarrow \sin x^\circ \cos 60^\circ - \cos x^\circ \sin 60^\circ = 3 \cos x^\circ \cos 30^\circ - 3 \sin x^\circ \sin 30^\circ$$

$$\Rightarrow \sin x^\circ \times \frac{1}{2} - \cos x^\circ \times \frac{\sqrt{3}}{2} = 3 \cos x^\circ \times \frac{\sqrt{3}}{2} - 3 \sin x^\circ \times \frac{1}{2}$$

$$\Rightarrow 4 \sin x^\circ = 4 \sqrt{3} \cos x^\circ$$

$$\Rightarrow \frac{\sin x^\circ}{\cos x^\circ} = \frac{4\sqrt{3}}{4}$$

$$\Rightarrow \tan x^\circ = \sqrt{3}$$

$$(c) \tan(x - 60)^\circ = 2$$

$$\Rightarrow \frac{\tan x^\circ - \tan 60^\circ}{1 + \tan x^\circ \tan 60^\circ} = 2$$

$$\begin{aligned}\Rightarrow \quad & \frac{\tan x^\circ - \sqrt{3}}{1 + \sqrt{3} \tan x^\circ} = 2 \\ \Rightarrow \quad & \tan x^\circ - \sqrt{3} = 2 + 2\sqrt{3} \tan x^\circ \\ \Rightarrow \quad & (2\sqrt{3} - 1) \tan x^\circ = - (2 + \sqrt{3}) \\ \Rightarrow \quad \tan x^\circ = & - \frac{(2 + \sqrt{3})}{2\sqrt{3} - 1} = - \frac{(2 + \sqrt{3})(2\sqrt{3} + 1)}{(2\sqrt{3} - 1)(2\sqrt{3} + 1)} = - \frac{8 + 5\sqrt{3}}{11}\end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise A, Question 22

**Question:**

Given that  $\tan A^\circ = \frac{1}{5}$  and  $\tan B^\circ = \frac{2}{3}$ , calculate, without using your calculator, the value of  $A + B$ ,

- (a) where  $A$  and  $B$  are both acute,
- (b) where  $A$  is reflex and  $B$  is acute.

**Solution:**

$$(a) \tan(A^\circ + B^\circ) = \frac{\tan A^\circ + \tan B^\circ}{1 - \tan A^\circ \tan B^\circ}$$

$$= \frac{\frac{1}{5} + \frac{2}{3}}{1 - \frac{1}{5} \times \frac{2}{3}} = \frac{\frac{13}{15}}{\frac{15-2}{15}} = \frac{\frac{13}{15}}{\frac{13}{15}} = 1$$

As  $\tan(A + B)^\circ$  is +ve,  $A + B$  is in the 1st or 3rd quadrants, but as they are both acute  $A + B$  cannot be in the 3rd quadrant.

$$\text{So } (A + B)^\circ = \tan^{-1} 1 = 45^\circ$$

$$\text{i.e. } A + B = 45$$

(b)  $A$  is reflex but  $\tan A^\circ$  is +ve, so  $A$  is in 3rd quadrant,

$$\text{i.e. } 180^\circ < A^\circ < 270^\circ$$

$$\text{and } 0^\circ < B^\circ < 90^\circ$$

$(A + B)^\circ$  must be in the 3rd quadrant as  $\tan(A + B)^\circ$  is +ve.

$$\text{So } A + B = 225$$

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## Edexcel AS and A Level Modular Mathematics

Exercise A, Question 23

### Question:

Given that  $\cos y = \sin(x + y)$ , show that  $\tan y = \sec x - \tan x$ .

### Solution:

$$\begin{aligned}\cos y &= \sin(x + y) \\ \Rightarrow \cos y &= \sin x \cos y + \cos x \sin y\end{aligned}$$

Divide throughout by  $\cos x \cos y$

$$\frac{\cancel{\cos y}}{\cos x \cancel{\cos y}} = \frac{\sin x \cancel{\cos y}}{\cos x \cancel{\cos y}} + \frac{\cancel{\cos x} \sin y}{\cancel{\cos x} \cos y}$$

$$\begin{aligned}\Rightarrow \sec x &= \tan x + \tan y \\ \Rightarrow \tan y &= \sec x - \tan x\end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise A, Question 24

#### Question:

Given that  $\cot A = \frac{1}{4}$  and  $\cot(A + B) = 2$ , find the value of  $\cot B$ .

#### Solution:

$$\cot(A + B) = 2$$

$$\Rightarrow \tan(A + B) = \frac{1}{2}$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{1}{2}$$

But as  $\cot A = \frac{1}{4}$ , then  $\tan A = 4$ .

$$\text{So } \frac{4 + \tan B}{1 - 4 \tan B} = \frac{1}{2}$$

$$\Rightarrow 8 + 2 \tan B = 1 - 4 \tan B$$

$$\Rightarrow 6 \tan B = -7$$

$$\Rightarrow \tan B = -\frac{7}{6}$$

$$\text{So } \cot B = \frac{1}{\tan B} = -\frac{6}{7}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise A, Question 25

**Question:**

Given that  $\tan \left( x + \frac{\pi}{3} \right) = \frac{1}{2}$ , show that  $\tan x = 8 - 5\sqrt{3}$ .

**Solution:**

$$\tan \left( x + \frac{\pi}{3} \right) = \frac{1}{2}$$

$$\Rightarrow \frac{\tan x + \tan \frac{\pi}{3}}{1 - \tan x \tan \frac{\pi}{3}} = \frac{1}{2}$$

$$\Rightarrow \frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} = \frac{1}{2} \quad \left( \tan \frac{\pi}{3} = \sqrt{3} \right)$$

$$\Rightarrow 2\tan x + 2\sqrt{3} = 1 - \sqrt{3}\tan x$$

$$\Rightarrow (2 + \sqrt{3})\tan x = 1 - 2\sqrt{3}$$

$$\Rightarrow \tan x = \frac{1 - 2\sqrt{3}}{2 + \sqrt{3}} = \frac{(1 - 2\sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} = \frac{2 - 4\sqrt{3} - \sqrt{3} + 6}{1} = 8 - 5\sqrt{3}$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 1**
**Question:**

Write the following expressions as a single trigonometric ratio:

(a)  $2 \sin 10^\circ \cos 10^\circ$

(b)  $1 - 2 \sin^2 25^\circ$

(c)  $\cos^2 40^\circ - \sin^2 40^\circ$

(d) 
$$\frac{2 \tan 5^\circ}{1 - \tan^2 5^\circ}$$

(e) 
$$\frac{1}{2 \sin(24 \frac{1}{2})^\circ \cos(24 \frac{1}{2})^\circ}$$

(f)  $6 \cos^2 30^\circ - 3$

(g) 
$$\frac{\sin 8^\circ}{\sec 8^\circ}$$

(h)  $\cos^2 \frac{\pi}{16} - \sin^2 \frac{\pi}{16}$

**Solution:**

(a)  $2 \sin 10^\circ \cos 10^\circ = \sin 20^\circ$  (using  $2 \sin A \cos A \equiv \sin 2A$ )

(b)  $1 - 2 \sin^2 25^\circ = \cos 50^\circ$  (using  $\cos 2A \equiv 1 - 2 \sin^2 A$ )

(c)  $\cos^2 40^\circ - \sin^2 40^\circ = \cos 80^\circ$  (using  $\cos 2A \equiv \cos^2 A - \sin^2 A$ )

(d) 
$$\frac{2 \tan 5^\circ}{1 - \tan^2 5^\circ} = \tan 10^\circ$$
 (using  $\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$ )

$$(e) \frac{1}{2 \sin(24 \frac{1}{2})^\circ \cos(24 \frac{1}{2})^\circ} = \frac{1}{\sin 49^\circ} = \operatorname{cosec} 49^\circ$$

$$(f) 6 \cos^2 30^\circ - 3 = 3(2 \cos^2 30^\circ - 1) = 3 \cos 60^\circ$$

$$(g) \frac{\sin 8^\circ}{\sec 8^\circ} = \sin 8^\circ \cos 8^\circ = \frac{1}{2}(2 \sin 8^\circ \cos 8^\circ) = \frac{1}{2} \sin 16^\circ$$

$$(h) \cos^2 \frac{\pi}{16} - \sin^2 \frac{\pi}{16} = \cos \frac{2\pi}{16} = \cos \frac{\pi}{8}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 2

#### Question:

Without using your calculator find the exact values of:

$$(a) 2 \sin \left( 22 \frac{1}{2} \right)^\circ \cos \left( 22 \frac{1}{2} \right)^\circ$$

$$(b) 2 \cos^2 15^\circ - 1$$

$$(c) (\sin 75^\circ - \cos 75^\circ)^2$$

$$(d) \frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$$

#### Solution:

$$(a) 2 \sin \left( 22 \frac{1}{2} \right)^\circ \cos \left( 22 \frac{1}{2} \right)^\circ = \sin 2 \times 22 \frac{1}{2}^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$(b) 2 \cos^2 15^\circ - 1 = \cos (2 \times 15^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} (c) (\sin 75^\circ - \cos 75^\circ)^2 &= \sin^2 75^\circ + \cos^2 75^\circ - 2 \sin 75^\circ \cos 75^\circ \\ &= 1 - \sin (2 \times 75^\circ) \quad (\sin^2 75^\circ + \cos^2 75^\circ = 1) \\ &= 1 - \sin 150^\circ \\ &= 1 - \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

$$(d) \frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}} = \tan \left( 2 \times \frac{\pi}{8} \right) = \tan \frac{\pi}{4} = 1$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 3**
**Question:**

Write the following in their simplest form, involving only one trigonometric function:

(a)  $\cos^2 3\theta - \sin^2 3\theta$

(b)  $6 \sin 2\theta \cos 2\theta$

(c) 
$$\frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

(d)  $2 - 4 \sin^2 \frac{\theta}{2}$

(e)  $\sqrt{1 + \cos 2\theta}$

(f)  $\sin^2 \theta \cos^2 \theta$

(g)  $4 \sin \theta \cos \theta \cos 2\theta$

(h) 
$$\frac{\tan \theta}{\sec^2 \theta - 2}$$

(i)  $\sin^4 \theta - 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta$

**Solution:**

(a)  $\cos^2 3\theta - \sin^2 3\theta = \cos(2 \times 3\theta) = \cos 6\theta$

(b)  $6 \sin 2\theta \cos 2\theta = 3(2 \sin 2\theta \cos 2\theta) = 3 \sin(2 \times 2\theta) = 3 \sin 4\theta$

(c) 
$$\frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} = \tan \left( 2 \times \frac{\theta}{2} \right) = \tan \theta$$

$$(d) 2 - 4 \sin^2 \left( \frac{1}{2}\theta \right) = 2 \left[ 1 - 2 \sin^2 \left( \frac{1}{2}\theta \right) \right] = 2 \cos \left( 2 \times \frac{1}{2}\theta \right) \\ = 2 \cos \theta$$

$$(e) \sqrt{1 + \cos 2\theta} = \sqrt{1 + (2 \cos^2 \theta - 1)} = \sqrt{2 \cos^2 \theta} = \sqrt{2} \cos \theta$$

$$(f) \sin^2 \theta \cos^2 \theta = \frac{1}{4} (4 \sin^2 \theta \cos^2 \theta) = \frac{1}{4} (2 \sin \theta \cos \theta)^2 = \frac{1}{4} \sin^2 2\theta$$

$$(g) 4 \sin \theta \cos \theta \cos 2\theta \\ = 2 (2 \sin \theta \cos \theta) \cos 2\theta \\ = 2 \sin 2\theta \cos 2\theta \\ = \sin 4\theta \quad (\sin 2A = 2 \sin A \cos A \text{ with } A = 2\theta)$$

$$(h) \frac{\tan \theta}{\sec^2 \theta - 2} = \frac{\tan \theta}{(1 + \tan^2 \theta) - 2} \\ = \frac{\tan \theta}{\tan^2 \theta - 1} \\ = - \frac{\tan \theta}{1 - \tan^2 \theta} \\ = - \frac{1}{2} \left( \frac{2 \tan \theta}{1 - \tan^2 \theta} \right) \\ = - \frac{1}{2} \tan 2\theta$$

$$(i) \cos^4 \theta - 2 \sin^2 \theta \cos^2 \theta + \sin^4 \theta = (\cos^2 \theta - \sin^2 \theta)^2 = \cos^2 2\theta$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise B, Question 4

### Question:

Given that  $\cos x = \frac{1}{4}$ , find the exact value of  $\cos 2x$ .

### Solution:

$$\cos 2x = 2 \cos^2 x - 1 = 2 \left( \frac{1}{4} \right)^2 - 1 = \frac{1}{8} - 1 = -\frac{7}{8}$$

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# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise B, Question 5

**Question:**

Find the possible values of  $\sin \theta$  when  $\cos 2\theta = \frac{23}{25}$ .

**Solution:**

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\text{So } \frac{23}{25} = 1 - 2 \sin^2 \theta$$

$$\Rightarrow 2 \sin^2 \theta = 1 - \frac{23}{25} = \frac{2}{25}$$

$$\Rightarrow \sin^2 \theta = \frac{1}{25}$$

$$\Rightarrow \sin \theta = \pm \frac{1}{5}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 6

#### Question:

Given that  $\cos x + \sin x = m$  and  $\cos x - \sin x = n$ , where  $m$  and  $n$  are constants, write down, in terms of  $m$  and  $n$ , the value of  $\cos 2x$ .

#### Solution:

$$\cos x + \sin x = m$$

$$\cos x - \sin x = n$$

Multiply the equations:

$$(\cos x + \sin x)(\cos x - \sin x) = mn$$

$$\Rightarrow \cos^2 x - \sin^2 x = mn$$

$$\Rightarrow \cos 2x = mn$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

## Exercise B, Question 7

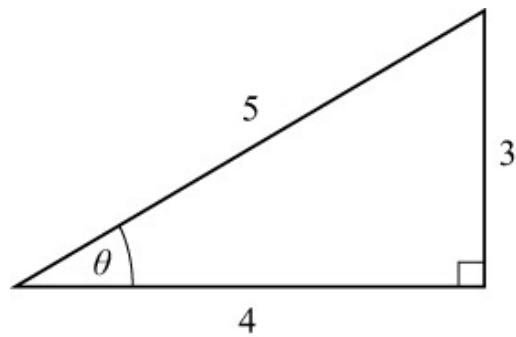
**Question:**

Given that  $\tan \theta = \frac{3}{4}$ , and that  $\theta$  is acute:

(a) Find the exact value of

- (i)  $\tan 2\theta$
- (ii)  $\sin 2\theta$
- (iii)  $\cos 2\theta$

(b) Deduce the value of  $\sin 4\theta$ .

**Solution:**

The hypotenuse is 5,

$$\text{so } \sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}, \tan \theta = \frac{3}{4}$$

$$(a) (i) \tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta} = \frac{\frac{3}{2}}{1 - \frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} \times \frac{16}{7} = \frac{24}{7}$$

$$(ii) \sin 2\theta = 2 \sin \theta \cos \theta = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

$$(iii) \cos 2\theta = \cos^2 \theta - \sin^2 \theta = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$(b) \sin 4\theta = 2 \sin 2\theta \cos 2\theta = 2 \times \frac{24}{25} \times \frac{7}{25} = \frac{336}{625}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 8**
**Question:**

Given that  $\cos A = -\frac{1}{3}$ , and that  $A$  is obtuse:

(a) Find the exact value of

- (i)  $\cos 2A$
- (ii)  $\sin A$
- (iii)  $\operatorname{cosec} 2A$

(b) Show that  $\tan 2A = \frac{4\sqrt{2}}{7}$ .

**Solution:**

$$(a) (i) \cos 2A = 2\cos^2 A - 1 = 2 \left( -\frac{1}{3} \right)^2 - 1 = \frac{2}{9} - 1 = -\frac{7}{9}$$

$$(ii) \cos 2A = 1 - 2\sin^2 A$$

$$\Rightarrow -\frac{7}{9} = 1 - 2\sin^2 A$$

$$\Rightarrow 2\sin^2 A = 1 + \frac{7}{9} = \frac{16}{9}$$

$$\Rightarrow \sin^2 A = \frac{8}{9}$$

$$\Rightarrow \sin A = \pm \frac{2\sqrt{2}}{3} \quad (\sqrt{8} = 2\sqrt{2})$$

but  $A$  is in 2nd quadrant  $\Rightarrow \sin A$  is +ve.

$$\text{So } \sin A = \frac{2\sqrt{2}}{3}$$

$$(iii) \operatorname{cosec} 2A = \frac{1}{\sin 2A} = \frac{1}{2\sin A \cos A} = \frac{1}{2 \times \frac{2\sqrt{2}}{3} \times \left( -\frac{1}{3} \right)} = -\frac{9}{4\sqrt{2}} = -\frac{9\sqrt{2}}{8}$$

$$(b) \tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{-\frac{4\sqrt{2}}{9}}{-\frac{7}{9}} = \frac{-4\sqrt{2}}{9} \times \frac{-9}{7} = \frac{4\sqrt{2}}{7}$$

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# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise B, Question 9

**Question:**

Given that  $\pi < \theta < \frac{3\pi}{2}$ , find the value of  $\tan \frac{\theta}{2}$  when  $\tan \theta = \frac{3}{4}$ .

**Solution:**

$$\text{Using } \tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$\Rightarrow \frac{3}{4} = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$\Rightarrow 3 - 3 \tan^2 \frac{\theta}{2} = 8 \tan \frac{\theta}{2}$$

$$\Rightarrow 3 \tan^2 \frac{\theta}{2} + 8 \tan \frac{\theta}{2} - 3 = 0$$

$$\Rightarrow \left( 3 \tan \frac{\theta}{2} - 1 \right) \left( \tan \frac{\theta}{2} + 3 \right) = 0$$

$$\text{so } \tan \frac{\theta}{2} = \frac{1}{3} \text{ or } \tan \frac{\theta}{2} = -3$$

$$\text{but } \pi < \theta < \frac{3\pi}{2}$$

$$\text{so } \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$$

i.e.  $\frac{\theta}{2}$  is in the 2nd quadrant

So  $\tan \frac{\theta}{2}$  is -ve.

$$\Rightarrow \tan \frac{\theta}{2} = -3$$

# Solutionbank

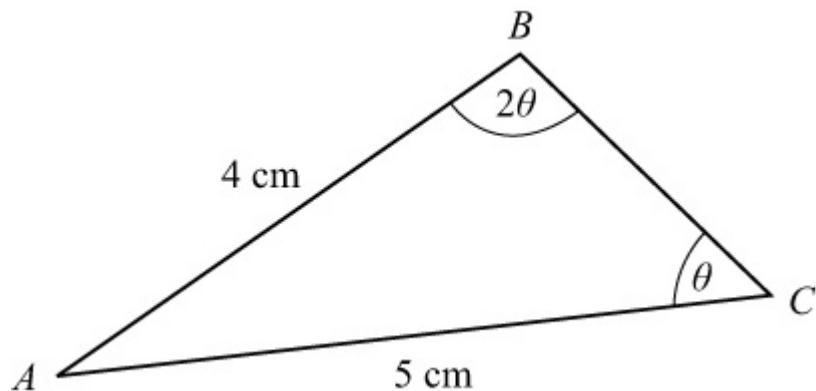
## Edexcel AS and A Level Modular Mathematics

Exercise B, Question 10

**Question:**

In  $\triangle ABC$ ,  $AB = 4 \text{ cm}$ ,  $AC = 5 \text{ cm}$ ,  $\angle ABC = 2\theta$  and  $\angle ACB = \theta$ . Find the value of  $\theta$ , giving your answer, in degrees, to 1 decimal place.

**Solution:**



Using sine rule with  $\frac{\sin B}{b} = \frac{\sin C}{c}$

$$\Rightarrow \frac{\sin 2\theta}{5} = \frac{\sin \theta}{4}$$

$$\Rightarrow \frac{2 \sin \theta \cos \theta}{5} = \frac{\sin \theta}{4}$$

Cancel  $\sin \theta$  as  $\theta \neq 0^\circ$  or  $180^\circ$

$$\text{So } 2 \cos \theta = \frac{5}{4}$$

$$\Rightarrow \cos \theta = \frac{5}{8}$$

$$\text{So } \theta = \cos^{-1} \frac{5}{8} = 51.3^\circ$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise B, Question 11

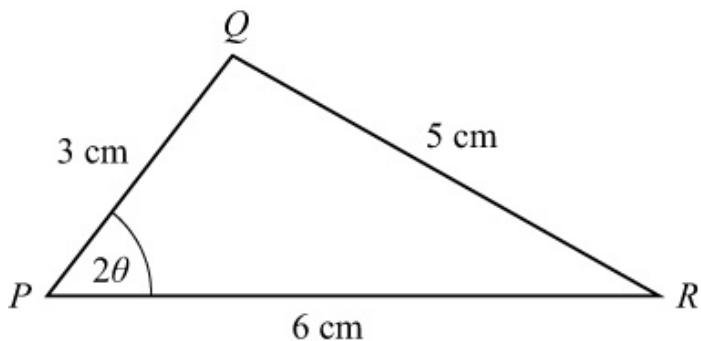
**Question:**

In  $\triangle PQR$ ,  $PQ = 3\text{ cm}$ ,  $PR = 6\text{ cm}$ ,  $QR = 5\text{ cm}$  and  $\angle QPR = 2\theta$ .

(a) Use the cosine rule to show that  $\cos 2\theta = \frac{5}{9}$ .

(b) Hence find the exact value of  $\sin \theta$ .

**Solution:**



(a) Using cosine rule with  $\cos P = \frac{q^2 + r^2 - p^2}{2qr}$

$$\cos 2\theta = \frac{36 + 9 - 25}{2 \times 6 \times 3} = \frac{20}{36} = \frac{5}{9}$$

(b) Using  $\cos 2\theta = 1 - 2\sin^2 \theta$

$$\frac{5}{9} = 1 - 2\sin^2 \theta$$

$$\Rightarrow 2\sin^2 \theta = 1 - \frac{5}{9} = \frac{4}{9}$$

$$\Rightarrow \sin^2 \theta = \frac{2}{9}$$

$$\Rightarrow \sin \theta = \pm \frac{\sqrt{2}}{3}$$

but  $\sin \theta$  cannot be negative for  $\theta$  in a triangle

$$\text{so } \sin \theta = \frac{\sqrt{2}}{3}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise B, Question 12

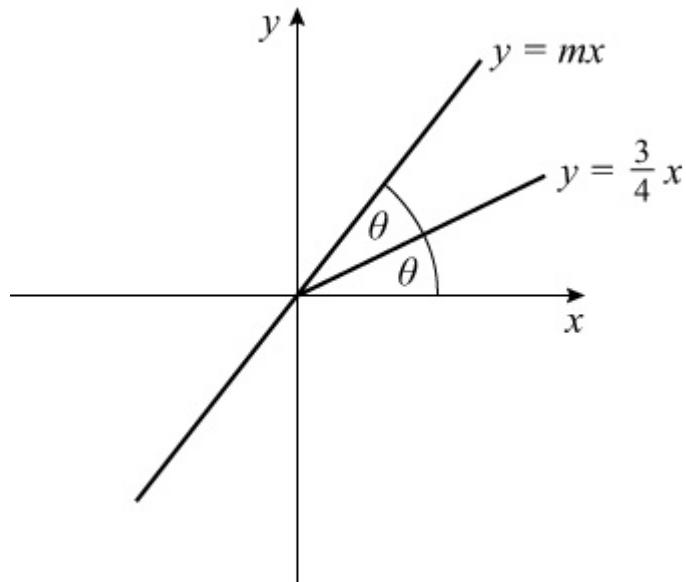
#### Question:

The line  $l$ , with equation  $y = \frac{3}{4}x$ , bisects the angle between the  $x$ -axis and the line  $y = mx$ ,  $m > 0$ . Given that the scales on each axis are the same, and that  $l$  makes an angle  $\theta$  with the  $x$ -axis,

(a) write down the value of  $\tan \theta$ .

(b) Show that  $m = \frac{24}{7}$ .

#### Solution:



(a) The gradient of line  $l$  is  $\frac{3}{4}$ , which is  $\tan \theta$ .

$$\text{So } \tan \theta = \frac{3}{4}$$

(b) The gradient of  $y = mx$  is  $m$ , and as  $y = \frac{3}{4}x$  bisects the angle between  $y = mx$  and  $x$ -axis

$$m = \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \times \frac{3}{4}}{1 - (\frac{3}{4})^2} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} \times \frac{16}{7} = \frac{24}{7}$$

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# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

### Question:

Prove the following identities:

$$(a) \frac{\cos 2A}{\cos A + \sin A} \equiv \cos A - \sin A$$

$$(b) \frac{\sin B}{\sin A} - \frac{\cos B}{\cos A} \equiv 2 \operatorname{cosec} 2A \sin(B - A)$$

$$(c) \frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$$

$$(d) \frac{\sec^2 \theta}{1 - \tan^2 \theta} \equiv \sec 2\theta$$

$$(e) 2(\sin^3 \theta \cos \theta + \cos^3 \theta \sin \theta) \equiv \sin 2\theta$$

$$(f) \frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \equiv 2$$

$$(g) \operatorname{cosec} \theta - 2 \cot 2\theta \cos \theta \equiv 2 \sin \theta$$

$$(h) \frac{\sec \theta - 1}{\sec \theta + 1} \equiv \tan^2 \frac{\theta}{2}$$

$$(i) \tan \left( \frac{\pi}{4} - x \right) \equiv \frac{1 - \sin 2x}{\cos 2x}$$

### Solution:

$$\begin{aligned} (a) \text{L.H.S.} &\equiv \frac{\cos 2A}{\cos A + \sin A} \\ &\equiv \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A} \\ &\equiv \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A + \sin A} \\ &\equiv \cos A - \sin A \equiv \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} (b) \text{L.H.S.} &\equiv \frac{\sin B}{\sin A} - \frac{\cos B}{\cos A} \\ &\equiv \frac{\sin B \cos A - \cos B \sin A}{\sin A \cos A} \end{aligned}$$

$$\begin{aligned}
 &\equiv \frac{\sin(B - A)}{\frac{1}{2}(2\sin A \cos A)} \\
 &\equiv \frac{2\sin(B - A)}{\sin 2A} \\
 &\equiv 2 \operatorname{cosec} 2A \sin(B - A) \equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{c}) \text{ L.H.S.} &\equiv \frac{1 - \cos 2\theta}{\sin 2\theta} \\
 &\equiv \frac{1 - (1 - 2\sin^2 \theta)}{2\sin \theta \cos \theta} \\
 &\equiv \frac{2\sin^2 \theta}{2\sin \theta \cos \theta} \\
 &\equiv \frac{\sin \theta}{\cos \theta} \\
 &\equiv \tan \theta \equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{d}) \text{ L.H.S.} &\equiv \frac{\sec^2 \theta}{1 - \tan^2 \theta} \\
 &\equiv \frac{1}{\cos^2 \theta (1 - \tan^2 \theta)} \\
 &\equiv \frac{1}{\cos^2 \theta - \sin^2 \theta} \quad (\text{as } \tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}) \\
 &\equiv \frac{1}{\cos 2\theta} \\
 &\equiv \sec 2\theta \equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{e}) \text{ L.H.S.} &\equiv 2(\sin^3 \theta \cos \theta + \cos^3 \theta \sin \theta) \\
 &\equiv 2\sin \theta \cos \theta (\sin^2 \theta + \cos^2 \theta) \\
 &\equiv \sin 2\theta \quad (\text{since } \sin^2 \theta + \cos^2 \theta \equiv 1) \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{f}) \text{ L.H.S.} &\equiv \frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \\
 &\equiv \frac{\sin 3\theta \cos \theta - \cos 3\theta \sin \theta}{\sin \theta \cos \theta} \\
 &\equiv \frac{\sin(3\theta - \theta)}{\frac{1}{2}\sin 2\theta}
 \end{aligned}$$

$$\equiv \frac{\sin 2\theta}{\frac{1}{2} \sin 2\theta}$$

$$\equiv 2 \equiv \text{R.H.S.}$$

(g) L.H.S.  $\equiv \operatorname{cosec} \theta - 2 \cot 2\theta \cos \theta$

$$\equiv \operatorname{cosec} \theta - 2 \frac{\cos 2\theta}{\sin 2\theta} \cos \theta$$

$$\equiv \operatorname{cosec} \theta - \frac{2 \cos 2\theta \cos \theta}{2 \sin \theta \cos \theta}$$

$$\equiv \frac{1}{\sin \theta} - \frac{\cos 2\theta}{\sin \theta}$$

$$\equiv \frac{1 - \cos 2\theta}{\sin \theta}$$

$$\equiv \frac{1 - (1 - 2\sin^2 \theta)}{\sin \theta}$$

$$\equiv \frac{2\sin^2 \theta}{\sin \theta}$$

$$\equiv 2 \sin \theta \equiv \text{R.H.S.}$$

(h)

$$\text{L.H.S.} \equiv \frac{\sec \theta - 1}{\sec \theta + 1}$$

$$\equiv \frac{\frac{1}{\cos \theta} - 1}{\frac{1}{\cos \theta} + 1}$$

$$\equiv \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\equiv \frac{1 - \left(1 - 2\sin^2 \frac{\theta}{2}\right)}{1 + \left(2\cos^2 \frac{\theta}{2} - 1\right)}$$

$$\equiv \frac{2\sin^2 \frac{\theta}{2}}{2\cos^2 \frac{\theta}{2}}$$

$$\equiv \tan^2 \frac{\theta}{2} \equiv \text{R.H.S.}$$

(i)

$$\begin{aligned}\text{L.H.S.} &\equiv \tan\left(\frac{\pi}{4} - x\right) \\ &\equiv \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} \\ &\equiv \frac{1 - \tan x}{1 + \tan x} \\ &\equiv \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} \\ &\equiv \frac{\cos x - \sin x}{\cos x + \sin x}\end{aligned}$$

Multiply 'top and bottom' by  $\cos x - \sin x$

$$\begin{aligned}&\equiv \frac{\cos^2 x + \sin^2 x - 2\sin x \cos x}{\cos^2 x - \sin^2 x} \\ &\equiv \frac{1 - \sin 2x}{\cos 2x} \equiv \text{R.H.S.}\end{aligned}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 2

#### Question:

- (a) Show that  $\tan \theta + \cot \theta \equiv 2 \operatorname{cosec} 2\theta$ .
- (b) Hence find the value of  $\tan 75^\circ + \cot 75^\circ$ .

#### Solution:

$$\begin{aligned}
 \text{(a) L.H.S. } &\equiv \tan \theta + \cot \theta \\
 &\equiv \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\
 &\equiv \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\
 &\equiv \frac{2}{2 \sin \theta \cos \theta} \quad (\sin^2 \theta + \cos^2 \theta \equiv 1) \\
 &\equiv \frac{2}{\sin 2\theta} \\
 &\equiv 2 \operatorname{cosec} 2\theta \equiv \text{R.H.S.}
 \end{aligned}$$

$$\text{(b) Use } \theta = 75^\circ$$

$$\Rightarrow \tan 75^\circ + \cot 75^\circ = 2 \operatorname{cosec} 150^\circ = 2 \times \frac{1}{\sin 150^\circ} = 2 \times \frac{1}{\frac{1}{2}} = 4$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 3**
**Question:**

Solve the following equations, in the interval shown in brackets. Give answers to 1 decimal place where appropriate.

(a)  $\sin 2\theta = \sin \theta \quad \{ 0 \leq \theta \leq 2\pi \}$

(b)  $\cos 2\theta = 1 - \cos \theta \quad \{ -180^\circ < \theta \leq 180^\circ \}$

(c)  $3 \cos 2\theta = 2 \cos^2 \theta \quad \{ 0 \leq \theta < 360^\circ \}$

(d)  $\sin 4\theta = \cos 2\theta \quad \{ 0 \leq \theta \leq \pi \}$

(e)  $2 \tan 2y \tan y = 3 \quad \{ 0 \leq y < 360^\circ \}$

(f)  $3 \cos \theta - \sin \frac{\theta}{2} - 1 = 0 \quad \left\{ \begin{array}{l} 0 \leq \theta < 720^\circ \end{array} \right\}$

(g)  $\cos^2 \theta - \sin 2\theta = \sin^2 \theta \quad \{ 0 \leq \theta \leq \pi \}$

(h)  $2 \sin \theta = \sec \theta \quad \{ 0 \leq \theta \leq 2\pi \}$

(i)  $2 \sin 2\theta = 3 \tan \theta \quad \{ 0 \leq \theta < 360^\circ \}$

(j)  $2 \tan \theta = \sqrt{3} (1 - \tan \theta) (1 + \tan \theta) \quad \{ 0 \leq \theta \leq 2\pi \}$

(k)  $5 \sin 2\theta + 4 \sin \theta = 0 \quad \{ -180^\circ < \theta \leq 180^\circ \}$

(l)  $\sin^2 \theta = 2 \sin 2\theta \quad \{ -180^\circ < \theta \leq 180^\circ \}$

(m)  $4 \tan \theta = \tan 2\theta \quad \{ 0 \leq \theta < 360^\circ \}$

**Solution:**

(a)  $\sin 2\theta = \sin \theta, 0 \leq \theta \leq 2\pi$

$$\Rightarrow 2 \sin \theta \cos \theta = \sin \theta$$

$$\Rightarrow 2 \sin \theta \cos \theta - \sin \theta = 0$$

$$\Rightarrow \sin \theta (2 \cos \theta - 1) = 0$$

$$\Rightarrow \sin \theta = 0 \text{ or } \cos \theta = \frac{1}{2}$$

Solution set:  $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

$$(b) \cos 2\theta = 1 - \cos \theta, -180^\circ < \theta \leq 180^\circ$$

$$\Rightarrow 2\cos^2 \theta - 1 = 1 - \cos \theta$$

$$\Rightarrow 2\cos^2 \theta + \cos \theta - 2 = 0$$

$$\Rightarrow \cos \theta = \frac{-1 \pm \sqrt{17}}{4}$$

$$\text{As } \frac{-1 - \sqrt{17}}{4} < -1, \cos \theta = \frac{-1 + \sqrt{17}}{4}$$

As  $\cos \theta$  is +ve,  $\theta$  is in 1st and 4th quadrants.

$$\text{Calculator solution is } \cos^{-1} \left( \frac{-1 + \sqrt{17}}{4} \right) = 38.7^\circ.$$

Solutions are  $\pm 38.7^\circ$

$$(c) 3\cos 2\theta = 2\cos^2 \theta, 0 \leq \theta < 360^\circ$$

$$\Rightarrow 3(2\cos^2 \theta - 1) = 2\cos^2 \theta$$

$$\Rightarrow 6\cos^2 \theta - 3 = 2\cos^2 \theta$$

$$\Rightarrow 4\cos^2 \theta = 3$$

$$\Rightarrow \cos^2 \theta = \frac{3}{4}$$

$$\Rightarrow \cos \theta = \pm \frac{\sqrt{3}}{2}$$

$\theta$  will be in all four quadrants.

Solution set:  $30^\circ, 150^\circ, 210^\circ, 330^\circ$

$$(d) \sin 4\theta = \cos 2\theta, 0 \leq \theta \leq \pi$$

$$\Rightarrow 2\sin 2\theta \cos 2\theta = \cos 2\theta$$

$$\Rightarrow \cos 2\theta (2\sin 2\theta - 1) = 0$$

$$\Rightarrow \cos 2\theta = 0 \text{ or } \sin 2\theta = \frac{1}{2}$$

$$\cos 2\theta = 0 \text{ in } 0 \leq 2\theta \leq 2\pi$$

$$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\sin 2\theta = \frac{1}{2} \text{ in } 0 \leq 2\theta \leq 2\pi$$

$$\Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}$$

Solution set:  $\frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$

(e)  $2\tan 2y \tan y = 3, 0 \leq y < 360^\circ$

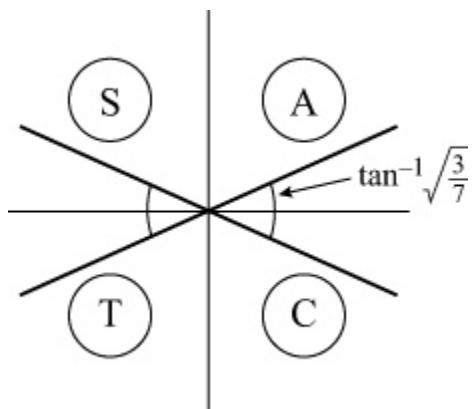
$$\Rightarrow \frac{4\tan y}{1 - \tan^2 y} \tan y = 3$$

$$\Rightarrow 4\tan^2 y = 3 - 3\tan^2 y$$

$$\Rightarrow 7\tan^2 y = 3$$

$$\Rightarrow \tan^2 y = \frac{3}{7}$$

$$\Rightarrow \tan y = \pm \sqrt{\frac{3}{7}}$$



$y$  is in all four quadrants.

$$y = \tan^{-1} \sqrt{\frac{3}{7}}, 180^\circ + \tan^{-1} \left( -\sqrt{\frac{3}{7}} \right), 180^\circ + \tan^{-1} \sqrt{\frac{3}{7}}, 360^\circ + \tan^{-1} \left( -\sqrt{\frac{3}{7}} \right)$$

$$y = 33.2^\circ, 146.8^\circ, 213.2^\circ, 326.8^\circ$$

(f)  $3\cos\theta - \sin\frac{\theta}{2} - 1 = 0, 0 \leq \theta \leq 720^\circ$

$$\Rightarrow 3 \left( 1 - 2\sin^2 \frac{\theta}{2} \right) - \sin \frac{\theta}{2} - 1 = 0$$

$$\Rightarrow 6\sin^2 \frac{\theta}{2} + \sin \frac{\theta}{2} - 2 = 0$$

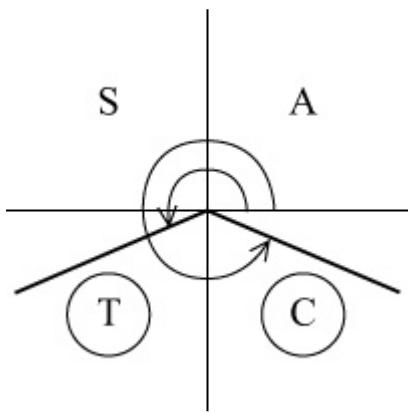
$$\Rightarrow \left( 3\sin \frac{\theta}{2} + 2 \right) \left( 2\sin \frac{\theta}{2} - 1 \right) = 0$$

$$\Rightarrow \sin \frac{\theta}{2} = -\frac{2}{3} \text{ or } \sin \frac{\theta}{2} = \frac{1}{2}$$

$$\sin \frac{\theta}{2} = \frac{1}{2} \text{ in } 0^\circ \leq \frac{\theta}{2} \leq 360^\circ$$

$$\Rightarrow \frac{\theta}{2} = 30^\circ, 150^\circ \Rightarrow \theta = 60^\circ, 300^\circ$$

$$\sin \frac{\theta}{2} = -\frac{2}{3} \text{ in } 0^\circ \leq \frac{\theta}{2} \leq 360^\circ$$



$$\Rightarrow \frac{\theta}{2} = 180^\circ - \sin^{-1} \left( -\frac{2}{3} \right), 360^\circ + \sin^{-1} \left( -\frac{2}{3} \right) = 221.8^\circ,$$

$$318.2^\circ$$

$$\Rightarrow \theta = 443.6^\circ, 636.4^\circ$$

Solution set:  $60^\circ, 300^\circ, 443.6^\circ, 636.4^\circ$

$$(g) \cos^2 \theta - \sin 2\theta = \sin^2 \theta, 0^\circ \leq \theta \leq \pi$$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta = \sin 2\theta$$

$$\Rightarrow \cos 2\theta = \sin 2\theta$$

$$\Rightarrow \tan 2\theta = 1 \quad (\text{divide both sides by } \cos 2\theta)$$

$$\tan 2\theta = 1 \text{ in } 0^\circ \leq 2\theta \leq 2\pi$$

$$\Rightarrow 2\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\Rightarrow \theta = \frac{\pi}{8}, \frac{5\pi}{8}$$

$$(h) 2 \sin \theta = \sec \theta, 0^\circ \leq \theta \leq 2\pi$$

$$\Rightarrow 2 \sin \theta = \frac{1}{\cos \theta}$$

$$\Rightarrow 2 \sin \theta \cos \theta = 1$$

$$\Rightarrow \sin 2\theta = 1$$

$$\begin{aligned}\sin 2\theta &= 1 \text{ in } 0 \leq 2\theta \leq 4\pi \\ \Rightarrow 2\theta &= \frac{\pi}{2}, \frac{5\pi}{2} \quad (\text{see graph}) \\ \Rightarrow \theta &= \frac{\pi}{4}, \frac{5\pi}{4}\end{aligned}$$

(i)  $2 \sin 2\theta = 3 \tan \theta, 0 \leq \theta < 360^\circ$

$$\begin{aligned}\Rightarrow 4 \sin \theta \cos \theta &= \frac{3 \sin \theta}{\cos \theta} \\ \Rightarrow 4 \sin \theta \cos^2 \theta &= 3 \sin \theta \\ \Rightarrow \sin \theta (4 \cos^2 \theta - 3) &= 0 \\ \Rightarrow \sin \theta = 0 \text{ or } \cos^2 \theta &= \frac{3}{4}\end{aligned}$$

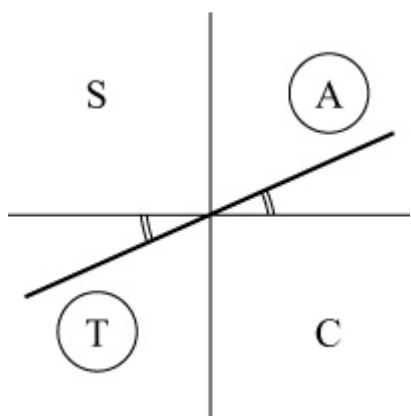
$$\sin \theta = 0 \Rightarrow \theta = 0^\circ, 180^\circ$$

$$\cos^2 \theta = \frac{3}{4} \Rightarrow \cos \theta = \pm \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

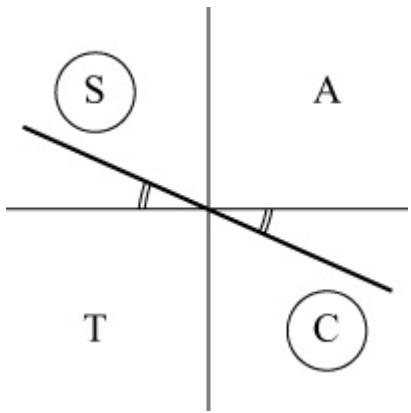
Solution set:  $0^\circ, 30^\circ, 150^\circ, 180^\circ, 210^\circ, 330^\circ$

$$\begin{aligned}(j) 2 \tan \theta &= \sqrt{3} (1 - \tan \theta) (1 + \tan \theta), 0 \leq \theta \leq 2\pi \\ \Rightarrow 2 \tan \theta &= \sqrt{3} (1 - \tan^2 \theta) \\ \Rightarrow \sqrt{3} \tan^2 \theta + 2 \tan \theta - \sqrt{3} &= 0 \\ \Rightarrow (\sqrt{3} \tan \theta - 1) (\tan \theta + \sqrt{3}) &= 0 \\ \Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \text{ or } \tan \theta &= -\sqrt{3}\end{aligned}$$

$$\tan \theta = \frac{1}{\sqrt{3}}, 0 \leq \theta \leq 2\pi$$



$$\begin{aligned}\Rightarrow \theta &= \tan^{-1} \frac{1}{\sqrt{3}}, \pi + \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}, \frac{7\pi}{6} \\ \tan \theta &= -\sqrt{3}, 0 \leq \theta \leq 2\pi\end{aligned}$$



$$\Rightarrow \theta = \pi + \tan^{-1}(-\sqrt{3}), 2\pi + \tan^{-1}(-\sqrt{3}) = \frac{2\pi}{3}, \frac{5\pi}{3}$$

Solution set:  $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

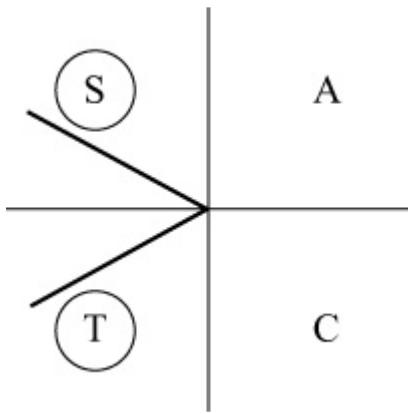
(k)  $5\sin 2\theta + 4\sin \theta = 0, -180^\circ < \theta \leq 180^\circ$

$$\Rightarrow 10\sin \theta \cos \theta + 4\sin \theta = 0$$

$$\Rightarrow 2\sin \theta (5\cos \theta + 2) = 0$$

$$\Rightarrow \sin \theta = 0 \text{ or } \cos \theta = -\frac{2}{5}$$

$$\sin \theta = 0 \Rightarrow \theta = 0^\circ, 180^\circ \quad (\text{from graph})$$



Calculator value for  $\cos^{-1}\left(-\frac{2}{5}\right)$  is  $113.6^\circ$

$$\Rightarrow \theta = \pm 113.6^\circ$$

Solution set:  $-113.6^\circ, 0^\circ, 113.6^\circ, 180^\circ$

(l)  $\sin^2 \theta = 2\sin 2\theta, -180^\circ < \theta \leq 180^\circ$

$$\Rightarrow \sin^2 \theta = 4\sin \theta \cos \theta$$

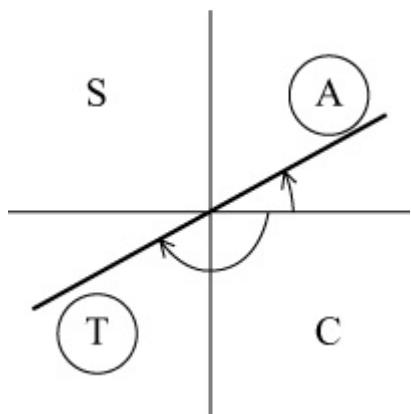
$$\Rightarrow \sin \theta (\sin \theta - 4\cos \theta) = 0$$

$$\Rightarrow \sin \theta = 0 \text{ or } \sin \theta = 4 \cos \theta$$

$$\Rightarrow \sin \theta = 0 \text{ or } \tan \theta = 4$$

$$\sin \theta = 0 \Rightarrow \theta = 0^\circ, 180^\circ$$

$$\tan \theta = 4 \Rightarrow \theta = \tan^{-1} 4, -180^\circ + \tan^{-1} 4 = 76.0^\circ, -104.0^\circ$$



Solution set:  $-104.0^\circ, 0^\circ, 76.0^\circ, 180^\circ$

$$(m) 4 \tan \theta = \tan 2\theta, 0^\circ \leq \theta < 360^\circ$$

$$\Rightarrow 4 \tan \theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\Rightarrow 2 \tan \theta (1 - \tan^2 \theta) = \tan \theta$$

$$\Rightarrow \tan \theta (2 - 2 \tan^2 \theta - 1) = 0$$

$$\Rightarrow \tan \theta (1 - 2 \tan^2 \theta) = 0$$

$$\Rightarrow \tan \theta = 0 \text{ or } \tan \theta = \pm \sqrt{\frac{1}{2}}$$

$$\tan \theta = 0 \Rightarrow \theta = 0^\circ, 180^\circ$$

$$\tan \theta = \pm \sqrt{\frac{1}{2}} \Rightarrow \theta = 35.3^\circ, 144.7^\circ, 215.3^\circ, 324.7^\circ$$

Solution set:  $0^\circ, 35.3^\circ, 144.7^\circ, 180^\circ, 215.3^\circ, 324.7^\circ$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 4

**Question:**

Given that  $p = 2 \cos \theta$  and  $q = \cos 2\theta$ , express  $q$  in terms of  $p$ .

**Solution:**

$$p = 2 \cos \theta \Rightarrow \cos \theta = \frac{p}{2}$$

$$\cos 2\theta = q$$

Using  $\cos 2\theta = 2 \cos^2 \theta - 1$

$$q = 2 \left( \frac{p}{2} \right)^2 - 1$$

$$\Rightarrow q = \frac{p^2}{2} - 1$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 5

#### Question:

Eliminate  $\theta$  from the following pairs of equations:

(a)  $x = \cos^2 \theta, y = 1 - \cos 2\theta$

(b)  $x = \tan \theta, y = \cot 2\theta$

(c)  $x = \sin \theta, y = \sin 2\theta$

(d)  $x = 3 \cos 2\theta + 1, y = 2 \sin \theta$

#### Solution:

(a)  $\cos^2 \theta = x, \cos 2\theta = 1 - y$

Using  $\cos 2\theta = 2 \cos^2 \theta - 1$

$$\Rightarrow 1 - y = 2x - 1$$

$$\Rightarrow y = 2 - 2x = 2(1 - x) \quad (\text{any form})$$

(b)  $y = \cot 2\theta \Rightarrow \tan 2\theta = \frac{1}{y}$

$x = \tan \theta$

Using  $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

$$\Rightarrow \frac{1}{y} = \frac{2x}{1 - x^2}$$

$$\Rightarrow 2xy = 1 - x^2 \quad (\text{any form})$$

(c)  $x = \sin \theta, y = 2 \sin \theta \cos \theta$

$$\Rightarrow y = 2x \cos \theta$$

$$\Rightarrow \cos \theta = \frac{y}{2x}$$

Using  $\sin^2 \theta + \cos^2 \theta \equiv 1$

$$\Rightarrow x^2 + \frac{y^2}{4x^2} = 1$$

$$\Rightarrow 4x^4 + y^2 = 4x^2 \text{ or } y^2 = 4x^2(1 - x^2) \quad (\text{any form})$$

$$(d) x = 3 \cos 2\theta + 1 \Rightarrow \cos 2\theta = \frac{x-1}{3}$$

$$y = 2 \sin \theta \Rightarrow \sin \theta = \frac{y}{2}$$

Using  $\cos 2\theta = 1 - 2 \sin^2 \theta$

$$\Rightarrow \frac{x-1}{3} = 1 - \frac{2y^2}{4} = 1 - \frac{y^2}{2}$$

$$\Rightarrow 2(x-1) = 6 - 3y^2 \quad (\times 6)$$

$$\Rightarrow 3y^2 = 6 - 2(x-1) = 8 - 2x$$

$$\Rightarrow y^2 = \frac{2(4-x)}{3} \quad (\text{any form})$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise C, Question 6

**Question:**

(a) Prove that  $(\cos 2\theta - \sin 2\theta)^2 \equiv 1 - \sin 4\theta$ .

(b) Use the result to solve, for  $0 \leq \theta < \pi$ , the equation  $\cos 2\theta - \sin 2\theta = -\frac{1}{\sqrt{2}}$ .

Give your answers in terms of  $\pi$ .

**Solution:**

$$\begin{aligned} \text{(a) L.H.S. } &\equiv (\cos 2\theta - \sin 2\theta)^2 \\ &\equiv \cos^2 2\theta - 2\sin 2\theta \cos 2\theta + \sin^2 2\theta \\ &\equiv (\cos^2 2\theta + \sin^2 2\theta) - (2\sin 2\theta \cos 2\theta) \\ &\equiv 1 - \sin 4\theta \quad (\sin^2 A + \cos^2 A \equiv 1, \sin 2A \equiv 2\sin A \cos A) \\ &\equiv \text{R.H.S.} \end{aligned}$$

(b) You can use  $(\cos 2\theta - \sin 2\theta)^2 = \frac{1}{2}$

but this also solves  $\cos 2\theta - \sin 2\theta = -\frac{1}{\sqrt{2}}$

so you need to check your final answers.

As  $(\cos 2\theta - \sin 2\theta)^2 \equiv 1 - \sin 4\theta$

$$\Rightarrow \frac{1}{2} = 1 - \sin 4\theta$$

$$\Rightarrow \sin 4\theta = \frac{1}{2}$$

$0 \leq \theta < \pi$ , so  $0 \leq 4\theta < 4\pi$

$$\Rightarrow 4\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$\Rightarrow \theta = \frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}$$

Checking these values in  $\cos 2\theta - \sin 2\theta = -\frac{1}{\sqrt{2}}$

eliminates  $\frac{5\pi}{24}, \frac{13\pi}{24}$  which apply to  $\cos 2\theta - \sin 2\theta = \frac{1}{\sqrt{2}}$

Solutions are  $\frac{\pi}{24}, \frac{17\pi}{24}$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 7

#### Question:

(a) Show that:

$$(i) \sin \theta \equiv \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$(ii) \cos \theta \equiv \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

(b) By writing the following equations as quadratics in  $\tan \frac{\theta}{2}$ , solve, in the interval  $0^\circ \leq \theta \leq 360^\circ$ :

$$(i) \sin \theta + 2 \cos \theta = 1 \quad (ii) 3 \cos \theta - 4 \sin \theta = 2$$

Give answers to 1 decimal place.

#### Solution:

$$(a) (i) \text{R.H.S.} \equiv \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\equiv \frac{2 \tan \frac{\theta}{2}}{\sec^2 \frac{\theta}{2}}$$

$$\equiv \frac{2 \sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} \times \cos^2 \frac{\theta}{2}$$

$$\equiv 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$\equiv \sin \theta \quad (\sin 2A = 2 \sin A \cos A) \\ \equiv \text{L.H.S.}$$

$$\begin{aligned}
 \text{(ii) R.H.S.} &\equiv \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \\
 &\equiv \frac{1 - \tan^2 \frac{\theta}{2}}{\sec^2 \frac{\theta}{2}} \\
 &\equiv \cos^2 \frac{\theta}{2} \left( 1 - \tan^2 \frac{\theta}{2} \right) \\
 &\equiv \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \quad \left| \begin{array}{l} \tan^2 \frac{\theta}{2} = \frac{\sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2}} \end{array} \right. \\
 &\equiv \cos \theta \quad (\cos 2A = \cos^2 A - \sin^2 A) \\
 &\equiv \text{L.H.S.}
 \end{aligned}$$

(b) Let  $\tan \frac{\theta}{2} = t$

$$\text{(i) } \sin \theta + 2 \cos \theta = 1$$

$$\begin{aligned}
 &\Rightarrow \frac{2t}{1+t^2} + \frac{2(1-t^2)}{1+t^2} = 1 \\
 &\Rightarrow 2t + 2 - 2t^2 = 1 + t^2 \\
 &\Rightarrow 3t^2 - 2t - 1 = 0 \\
 &\Rightarrow (3t+1)(t-1) = 0 \\
 &\Rightarrow \tan \frac{\theta}{2} = -\frac{1}{3} \text{ or } \tan \frac{\theta}{2} = 1 \quad 0^\circ \leq \frac{\theta}{2} \leq 180^\circ
 \end{aligned}$$

$$\tan \frac{\theta}{2} = 1 \Rightarrow \frac{\theta}{2} = 45^\circ \Rightarrow \theta = 90^\circ$$

$$\tan \frac{\theta}{2} = -\frac{1}{3} \Rightarrow \frac{\theta}{2} = 161.56^\circ \Rightarrow \theta = 323.1^\circ$$

Solution set:  $90^\circ, 323.1^\circ$

$$\text{(ii) } 3 \cos \theta - 4 \sin \theta = 2$$

$$\begin{aligned}
 &\Rightarrow \frac{3(1-t^2)}{1+t^2} - \frac{4 \times 2t}{1+t^2} = 2 \\
 &\Rightarrow 3(1-t^2) - 8t = 2(1+t^2) \\
 &\Rightarrow 5t^2 + 8t - 1 = 0
 \end{aligned}$$

$$\Rightarrow t = \frac{-8 \pm \sqrt{84}}{10}$$

$$\text{For } \tan \frac{\theta}{2} = \frac{-8 + \sqrt{84}}{10} \quad 0^\circ \leq \frac{\theta}{2} \leq 180^\circ$$

$$\frac{\theta}{2} = 6.65^\circ \Rightarrow \theta = 13.3^\circ$$

$$\text{For } \tan \frac{\theta}{2} = \frac{-8 - \sqrt{84}}{10} \quad 0^\circ \leq \frac{\theta}{2} \leq 180^\circ$$

$$\frac{\theta}{2} = 120.2^\circ \Rightarrow \theta = 240.4^\circ$$

Solution set:  $13.3^\circ, 240.4^\circ$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 8

#### Question:

(a) Using  $\cos 2A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$ , show that:

$$(i) \cos^2 \frac{x}{2} \equiv \frac{1 + \cos x}{2}$$

$$(ii) \sin^2 \frac{x}{2} \equiv \frac{1 - \cos x}{2}$$

(b) Given that  $\cos \theta = 0.6$ , and that  $\theta$  is acute, write down the values of:

$$(i) \cos \frac{\theta}{2}$$

$$(ii) \sin \frac{\theta}{2}$$

$$(iii) \tan \frac{\theta}{2}$$

$$(c) \text{Show that } \cos^4 \frac{A}{2} \equiv \frac{1}{8} (3 + 4\cos A + \cos 2A)$$

#### Solution:

$$(a) (i) \text{Using } \cos 2A \equiv 2\cos^2 A - 1 \text{ with } A = \frac{x}{2}$$

$$\Rightarrow \cos x \equiv 2\cos^2 \frac{x}{2} - 1$$

$$\Rightarrow 2\cos^2 \frac{x}{2} \equiv 1 + \cos x$$

$$\Rightarrow \cos^2 \frac{x}{2} \equiv \frac{1 + \cos x}{2}$$

$$(ii) \text{Using } \cos 2A \equiv 1 - 2\sin^2 A$$

$$\Rightarrow \cos x \equiv 1 - 2\sin^2 \frac{x}{2}$$

$$\Rightarrow 2\sin^2 \frac{x}{2} \equiv 1 - \cos x$$

$$\Rightarrow \sin^2 \frac{x}{2} \equiv \frac{1 - \cos x}{2}$$

(b) Given that  $\cos \theta = 0.6$  and  $\theta$  acute

$$(i) \text{ using (a) (i)} \cos^2 \frac{\theta}{2} = \frac{1.6}{2} = 0.8 = \frac{4}{5}$$

$$\Rightarrow \cos \frac{\theta}{2} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \quad (\text{as } \frac{\theta}{2} \text{ acute})$$

$$(ii) \text{ using (a) (ii)} \sin^2 \frac{\theta}{2} = \frac{0.4}{2} = 0.2 = \frac{1}{5}$$

$$\Rightarrow \sin \frac{\theta}{2} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$$

$$(iii) \tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \frac{\frac{\sqrt{5}}{5}}{\frac{2\sqrt{5}}{5}} = \frac{1}{2}$$

(c) Using (a) (i) and squaring

$$\cos^4 \frac{A}{2} = \left( \frac{1 + \cos A}{2} \right)^2 = \frac{1 + 2\cos A + \cos^2 A}{4}$$

but using (a) (i) again

$$\cos^2 A = \frac{1}{2} (1 + \cos 2A)$$

$$\text{So } \cos^4 \frac{A}{2} = \frac{1 + 2\cos A + \frac{1}{2} (1 + \cos 2A)}{4} = \frac{2 + 4\cos A + 1 + \cos 2A}{8} =$$

$$\frac{3 + 4\cos A + \cos 2A}{8}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 9**
**Question:**

(a) Show that  $3\cos^2 x - \sin^2 x \equiv 1 + 2\cos 2x$ .

(b) Hence sketch, for  $-\pi \leq x \leq \pi$ , the graph of  $y = 3\cos^2 x - \sin^2 x$ , showing the coordinates of points where the curve meets the axes.

**Solution:**

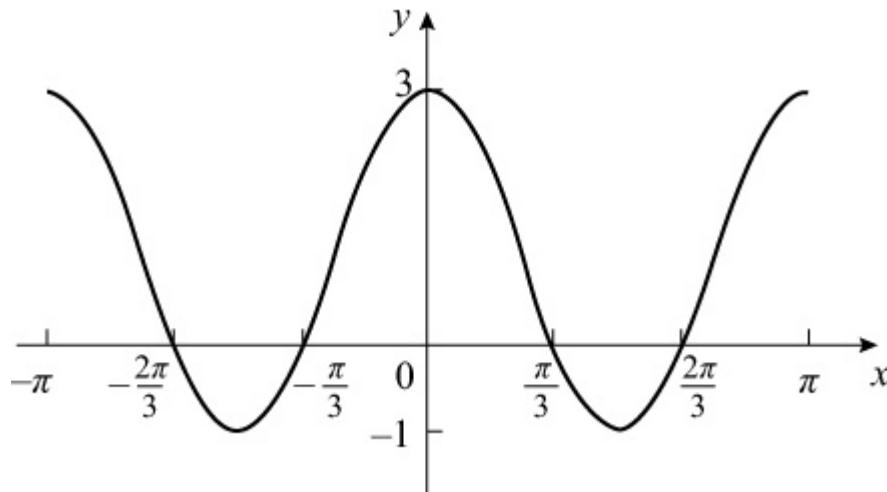
$$\begin{aligned}
 \text{(a) R.H.S.} &\equiv 1 + 2\cos 2x \\
 &\equiv 1 + 2(\cos^2 x - \sin^2 x) \\
 &\equiv 1 + 2\cos^2 x - 2\sin^2 x \\
 &\equiv \cos^2 x + \sin^2 x + 2\cos^2 x - 2\sin^2 x \quad (\text{using } \sin^2 x + \cos^2 x \equiv 1) \\
 &\equiv 3\cos^2 x - \sin^2 x \\
 &\equiv \text{L.H.S.}
 \end{aligned}$$

(b)  $y = 3\cos^2 x - \sin^2 x$

is the same as  $y = 1 + 2\cos 2x$

Using your work on transformations this curve is the result of

- (i) stretching  $y = \cos x$  by scale factor  $\frac{1}{2}$  in the  $x$  direction, then
- (ii) stretching the result by scale factor 2 in the  $y$  direction, then
- (iii) translating by 1 in the +ve  $y$  direction.

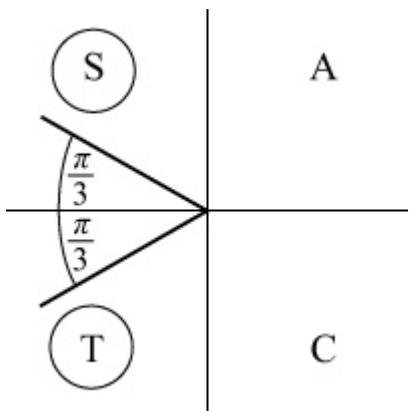


The curve crosses y-axis at (0, 3).

It crosses  $x$ -axis where  $y = 0$

i.e. where  $1 + 2\cos 2x = 0 \quad -\pi \leq x \leq \pi$

$$\Rightarrow \cos 2x = -\frac{1}{2} \quad -2\pi \leq 2x \leq 2\pi$$



$$\text{So } 2x = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\Rightarrow x = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 10

**Question:**

- (a) Express  $2\cos^2 \frac{\theta}{2} - 4\sin^2 \frac{\theta}{2}$  in the form  $a\cos\theta + b$ , where  $a$  and  $b$  are constants.
- (b) Hence solve  $2\cos^2 \frac{\theta}{2} - 4\sin^2 \frac{\theta}{2} = -3$ , in the interval  $0^\circ \leq \theta < 360^\circ$ , giving answers to 1 decimal place.

**Solution:**

$$(a) \cos^2 \frac{\theta}{2} = \frac{1 + \cos\theta}{2}, \sin^2 \frac{\theta}{2} = \frac{1 - \cos\theta}{2}$$

$$\text{So } 2\cos^2 \frac{\theta}{2} - 4\sin^2 \frac{\theta}{2} = (1 + \cos\theta) - 2(1 - \cos\theta) = 3\cos\theta - 1$$

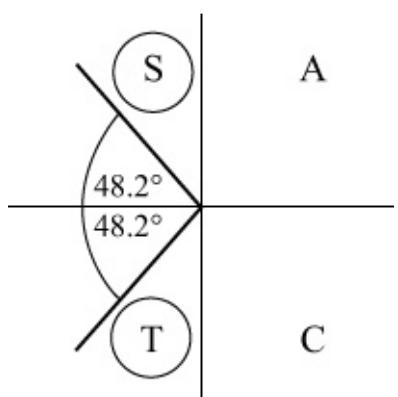
$$(b) \text{Hence solve } 3\cos\theta - 1 = -3, 0^\circ \leq \theta < 360^\circ$$

$$\Rightarrow 3\cos\theta = -2$$

$$\Rightarrow \cos\theta = -\frac{2}{3}$$

As  $\cos\theta$  is - ve,  $\theta$  is in 2nd and 3rd quadrants.

$$\text{Calculator value is } \cos^{-1}\left(-\frac{2}{3}\right) = 131.8^\circ$$



Solutions are  $131.8^\circ, 360^\circ - 131.8^\circ = 131.8^\circ, 228.2^\circ$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise C, Question 11

#### Question:

- (a) Use the identity  $\sin^2 A + \cos^2 A \equiv 1$  to show that  $\sin^4 A + \cos^4 A \equiv \frac{1}{2} (2 - \sin^2 2A)$ .
- (b) Deduce that  $\sin^4 A + \cos^4 A \equiv \frac{1}{4} (3 + \cos 4A)$ .
- (c) Hence solve  $8\sin^4 \theta + 8\cos^4 \theta = 7$ , for  $0 < \theta < \pi$ .

#### Solution:

$$(a) \text{ As } \sin^2 A + \cos^2 A \equiv 1$$

$$\text{so } (\sin^2 A + \cos^2 A)^2 \equiv 1$$

$$\Rightarrow \sin^4 A + \cos^4 A + 2\sin^2 A \cos^2 A \equiv 1$$

$$\Rightarrow \sin^4 A + \cos^4 A \equiv 1 - 2\sin^2 A \cos^2 A$$

$$\equiv 1 - \frac{1}{2} (4\sin^2 A \cos^2 A)$$

$$\equiv 1 - \frac{1}{2} [(2\sin A \cos A)^2]$$

$$\equiv 1 - \frac{1}{2} \sin^2 2A$$

$$\equiv \frac{1}{2} (2 - \sin^2 2A)$$

$$(b) \text{ As } \cos 2A \equiv 1 - 2\sin^2 A$$

$$\text{so } \cos 4A \equiv 1 - 2\sin^2 2A$$

$$\text{so } \sin^2 2A \equiv \frac{1 - \cos 4A}{2}$$

$$\Rightarrow \text{ from (a) } \sin^4 A + \cos^4 A \equiv \frac{1}{2} \left( 2 - \frac{1 - \cos 4A}{2} \right) \equiv \frac{1}{2} \left( \frac{4 - 1 + \cos 4A}{2} \right)$$

$$\left. \right\} \equiv \frac{1}{4} \left( 3 + \cos 4A \right)$$

(c) Using part (b)

$$8 \sin^4 \theta + 8 \cos^4 \theta = 7$$

$$\Rightarrow 8 \times \frac{1}{4} (3 + \cos 4\theta) = 7$$

$$\Rightarrow 3 + \cos 4\theta = \frac{7}{2}$$

$$\Rightarrow \cos 4\theta = \frac{1}{2}$$

Solve  $\cos 4\theta = \frac{1}{2}$  in  $0 < 4\theta < 4\pi$

$$\Rightarrow 4\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise C, Question 12

**Question:**

- (a) By expanding  $\cos(2A + A)$  show that  $\cos 3A \equiv 4\cos^3 A - 3\cos A$ .
- (b) Hence solve  $8\cos^3 \theta - 6\cos \theta - 1 = 0$ , for  $\{0 \leq \theta \leq 360^\circ\}$ .

**Solution:**

$$\begin{aligned}
 (a) \cos(2A + A) &\equiv \cos 2A \cos A - \sin 2A \sin A \\
 &\equiv (2\cos^2 A - 1) \cos A - (2\sin A \cos A) \sin A \\
 &\equiv 2\cos^3 A - \cos A - 2\sin^2 A \cos A \\
 &\equiv 2\cos^3 A - \cos A - 2(1 - \cos^2 A) \cos A \\
 &\equiv 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A \\
 &\equiv 4\cos^3 A - 3\cos A
 \end{aligned}$$

$$\begin{aligned}
 (b) 8\cos^3 \theta - 6\cos \theta - 1 &= 0 \quad 0 \leq \theta \leq 360^\circ \\
 \Rightarrow 2(4\cos^3 \theta - 3\cos \theta) - 1 &= 0 \\
 \Rightarrow 2\cos 3\theta - 1 &= 0 \quad [\text{using part (a)}] \\
 \Rightarrow \cos 3\theta &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{Solve } \cos 3\theta = \frac{1}{2} \text{ in } 0 \leq 3\theta \leq 1080^\circ \\
 \Rightarrow 3\theta &= 60^\circ, 300^\circ, 420^\circ, 660^\circ, 780^\circ, 1020^\circ \\
 \Rightarrow \theta &= 20^\circ, 100^\circ, 140^\circ, 220^\circ, 260^\circ, 340^\circ
 \end{aligned}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise C, Question 13

**Question:**

(a) Show that  $\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ .

(b) Given that  $\theta$  is acute such that  $\cos\theta = \frac{1}{3}$ , show that  $\tan 3\theta = \frac{10\sqrt{2}}{23}$ .

**Solution:**

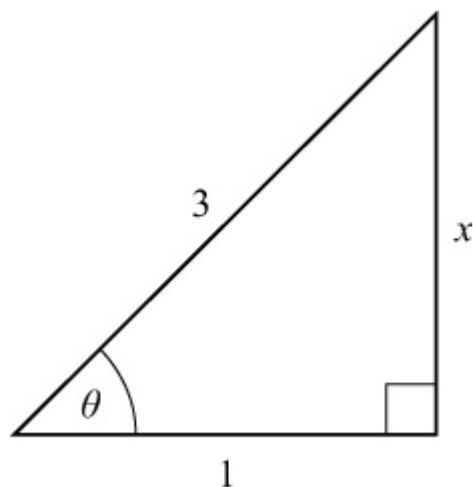
$$(a) \tan 3\theta \equiv \tan \left( 2\theta + \theta \right) \equiv \frac{\tan 2\theta + \tan \theta}{1 - \tan 2\theta \tan \theta}$$

$$\text{Numerator} = \frac{2\tan\theta}{1 - \tan^2\theta} + \tan\theta \equiv \frac{2\tan\theta + \tan\theta - \tan^3\theta}{1 - \tan^2\theta} \equiv \frac{3\tan\theta - \tan^3\theta}{1 - \tan^2\theta}$$

$$\text{Denominator} = 1 - \frac{2\tan\theta}{1 - \tan^2\theta} \tan\theta \equiv \frac{1 - \tan^2\theta - 2\tan^2\theta}{1 - \tan^2\theta} \equiv \frac{1 - 3\tan^2\theta}{1 - \tan^2\theta}$$

$$\text{So } \tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - \tan^2\theta} \times \frac{1 - \tan^2\theta}{1 - 3\tan^2\theta} \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

(b) Draw a right-angled triangle.



Using Pythagoras' theorem

$$x^2 = 9 - 1 = 8$$

$$\text{So } x = 2\sqrt{2}$$

$$\text{So } \tan\theta = 2\sqrt{2}$$

Using part (a)

$$\tan 3\theta = \frac{3(2\sqrt{2}) - (2\sqrt{2})^3}{1 - 3(2\sqrt{2})^2} = \frac{6\sqrt{2} - 16\sqrt{2}}{1 - 24} = \frac{-10\sqrt{2}}{-23} = \frac{10\sqrt{2}}{23}$$

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# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 1

#### Question:

Given that  $5 \sin \theta + 12 \cos \theta \equiv R \sin (\theta + \alpha)$ , find the value of  $R$ ,  $R > 0$ , and the value of  $\tan \alpha$ .

#### Solution:

$$5 \sin \theta + 12 \cos \theta \equiv R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$$

$$\text{Comparing } \sin \theta : R \cos \alpha = 5$$

$$\text{Comparing } \cos \theta : R \sin \alpha = 12$$

Divide the equations:

$$\frac{\sin \alpha}{\cos \alpha} = \frac{12}{5} \Rightarrow \tan \alpha = 2 \frac{2}{5}$$

Square and add the equations:

$$R^2 \cos^2 \alpha + R^2 \sin^2 \alpha = 5^2 + 12^2$$

$$R^2 (\cos^2 \alpha + \sin^2 \alpha) = 13^2$$

$$R = 13$$

$$\text{since } \cos^2 \alpha + \sin^2 \alpha \equiv 1$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 2

#### Question:

Given that  $\sqrt{3} \sin \theta + \sqrt{6} \cos \theta \equiv 3 \cos (\theta - \alpha)$ , where  $0 < \alpha < 90^\circ$ , find the value of  $\alpha$  to the nearest  $0.1^\circ$ .

#### Solution:

$$\sqrt{3} \sin \theta + \sqrt{6} \cos \theta \equiv 3 \cos \theta \cos \alpha + 3 \sin \theta \sin \alpha$$

Comparing  $\sin \theta$ :  $\sqrt{3} = 3 \sin \alpha \quad ①$

Comparing  $\cos \theta$ :  $\sqrt{6} = 3 \cos \alpha \quad ②$

Divide ① by ②:

$$\tan \alpha = \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}}$$

$$\text{So } \alpha = 35.3^\circ$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 3

#### Question:

Given that  $2 \sin \theta - \sqrt{5} \cos \theta \equiv -3 \cos(\theta + \alpha)$ , where  $0 < \alpha < 90^\circ$ , find the value of  $\alpha$  to the nearest  $0.1^\circ$ .

#### Solution:

$$2 \sin \theta - \sqrt{5} \cos \theta \equiv -3 \cos \theta \cos \alpha + 3 \sin \theta \sin \alpha$$

$$\text{Comparing } \sin \theta : \quad 2 = 3 \sin \alpha \quad \textcircled{1}$$

$$\text{Comparing } \cos \theta : \quad -\sqrt{5} = -3 \cos \alpha \quad \textcircled{2}$$

Divide \textcircled{1} by \textcircled{2}:

$$\tan \alpha = \frac{2}{\sqrt{5}}$$

$$\text{So } \alpha = 41.8^\circ$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 4

#### Question:

Show that:

$$(a) \cos \theta + \sin \theta \equiv \sqrt{2} \sin \left( \theta + \frac{\pi}{4} \right)$$

$$(b) \sqrt{3} \sin 2\theta - \cos 2\theta \equiv 2 \sin \left( 2\theta - \frac{\pi}{6} \right)$$

#### Solution:

$$\begin{aligned} (a) \text{R.H.S.} &\equiv \sqrt{2} \sin \left( \theta + \frac{\pi}{4} \right) \\ &\equiv \sqrt{2} \left( \sin \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4} \right) \\ &\equiv \sqrt{2} \left( \sin \theta \times \frac{1}{\sqrt{2}} + \cos \theta \times \frac{1}{\sqrt{2}} \right) \\ &\equiv \sin \theta + \cos \theta \\ &\equiv \text{L.H.S.} \end{aligned}$$

$$\begin{aligned} (b) \text{R.H.S.} &\equiv 2 \sin \left( 2\theta - \frac{\pi}{6} \right) \\ &\equiv 2 \left( \sin 2\theta \cos \frac{\pi}{6} - \cos 2\theta \sin \frac{\pi}{6} \right) \\ &\equiv 2 \left( \sin 2\theta \times \frac{\sqrt{3}}{2} - \cos 2\theta \times \frac{1}{2} \right) \\ &\equiv \sqrt{3} \sin 2\theta - \cos 2\theta \\ &\equiv \text{L.H.S.} \end{aligned}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 5

#### Question:

Prove that  $\cos 2\theta - \sqrt{3} \sin 2\theta \equiv 2 \cos \left( 2\theta + \frac{\pi}{3} \right) \equiv -2 \sin \left( 2\theta - \frac{\pi}{6} \right)$ .

#### Solution:

Let  $\cos 2\theta - \sqrt{3} \sin 2\theta \equiv R \cos (2\theta + \alpha) \equiv R \cos 2\theta \cos \alpha - R \sin 2\theta \sin \alpha$

$$\text{Compare } \cos 2\theta : R \cos \alpha = 1 \quad \textcircled{1}$$

$$\text{Compare } \sin 2\theta : R \sin \alpha = \sqrt{3} \quad \textcircled{2}$$

Divide \textcircled{2} by \textcircled{1}:

$$\tan \alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$

Square and add equations:

$$R^2 = 1 + 3 = 4$$

$$\Rightarrow R = 2$$

$$\text{So } \cos 2\theta - \sqrt{3} \sin 2\theta \equiv 2 \cos \left( 2\theta + \frac{\pi}{3} \right)$$

$$\begin{aligned} \cos (2\theta + \frac{\pi}{3}) &\equiv \cos 2\theta \cos \frac{\pi}{3} - \sin 2\theta \sin \frac{\pi}{3} \\ &\equiv \cos 2\theta \times \frac{1}{2} - \sin 2\theta \times \frac{\sqrt{3}}{2} \\ &\equiv \cos 2\theta \sin \frac{\pi}{6} - \sin 2\theta \cos \frac{\pi}{6} \\ &\equiv -(\sin 2\theta \cos \frac{\pi}{6} - \cos 2\theta \sin \frac{\pi}{6}) \\ &\equiv -\sin (2\theta - \frac{\pi}{6}) \end{aligned}$$

$$\text{So } \cos 2\theta - \sqrt{3} \sin 2\theta \equiv 2 \cos \left( 2\theta + \frac{\pi}{3} \right) \equiv -2 \sin \left( 2\theta - \frac{\pi}{6} \right)$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 6

#### Question:

Give all angles to the nearest  $0.1^\circ$  and non-exact values of  $R$  in surd form.  
 Find the value of  $R$ , where  $R > 0$ , and the value of  $\alpha$ , where  $0 < \alpha < 90^\circ$ , in each of the following cases:

$$(a) \sin \theta + 3 \cos \theta \equiv R \sin (\theta + \alpha)$$

$$(b) 3 \sin \theta - 4 \cos \theta \equiv R \sin (\theta - \alpha)$$

$$(c) 2 \cos \theta + 7 \sin \theta \equiv R \cos (\theta - \alpha)$$

$$(d) \cos 2\theta - 2 \sin 2\theta \equiv R \cos (2\theta + \alpha)$$

#### Solution:

$$(a) \sin \theta + 3 \cos \theta \equiv R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$$

$$\text{Compare } \sin \theta : R \cos \alpha = 1 \quad ①$$

$$\text{Compare } \cos \theta : R \sin \alpha = 3 \quad ②$$

Dividing ② by ①:

$$\tan \alpha = 3 \Rightarrow \alpha = 71.6^\circ$$

Square and add equations:

$$R^2 = 3^2 + 1^2 \Rightarrow R = \sqrt{10}$$

$$(b) 3 \sin \theta - 4 \cos \theta \equiv R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$$

$$\text{Compare } \sin \theta : R \cos \alpha = 3 \quad ①$$

$$\text{Compare } \cos \theta : R \sin \alpha = 4 \quad ②$$

Divide ② by ①:

$$\tan \alpha = \frac{4}{3} \Rightarrow \alpha = 53.1^\circ$$

Square and add equations:

$$R^2 = 3^2 + 4^2 \Rightarrow R = 5$$

$$(c) 2 \cos \theta + 7 \sin \theta \equiv R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$$

$$\text{Compare } \cos \theta : R \cos \alpha = 2 \quad ①$$

$$\text{Compare } \sin \theta : R \sin \alpha = 7 \quad ②$$

Divide ② by ①:

$$\tan \alpha = \frac{7}{2} \Rightarrow \alpha = 74.1^\circ$$

Square and add equations:

$$R^2 = 2^2 + 7^2 = 53 \Rightarrow R = \sqrt{53}$$

$$(d) \cos 2\theta - 2 \sin 2\theta \equiv R \cos 2\theta \cos \alpha - R \sin 2\theta \sin \alpha$$

$$\text{Compare } \cos 2\theta : R \cos \alpha = 1 \quad ①$$

$$\text{Compare } \sin 2\theta : R \sin \alpha = 2 \quad ②$$

Divide ② by ①:

$$\tan \alpha = 2 \Rightarrow \alpha = 63.4^\circ$$

Square and add equations:

$$R^2 = 1^2 + 2^2 = 5 \Rightarrow R = \sqrt{5}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 7

#### Question:

(a) Show that  $\cos \theta - \sqrt{3} \sin \theta$  can be written in the form  $R \cos (\theta + \alpha)$ , with  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ .

(b) Hence sketch the graph of  $y = \cos \theta - \sqrt{3} \sin \theta$ ,  $0 < \theta < 2\pi$ , giving the coordinates of points of intersection with the axes.

#### Solution:

(a) Let  $\cos \theta - \sqrt{3} \sin \theta \equiv R \cos (\theta + \alpha) \equiv R \cos \theta \cos \alpha - R \sin \theta \sin \alpha$

$$\text{Compare } \cos \theta : R \cos \alpha = 1 \quad \textcircled{1}$$

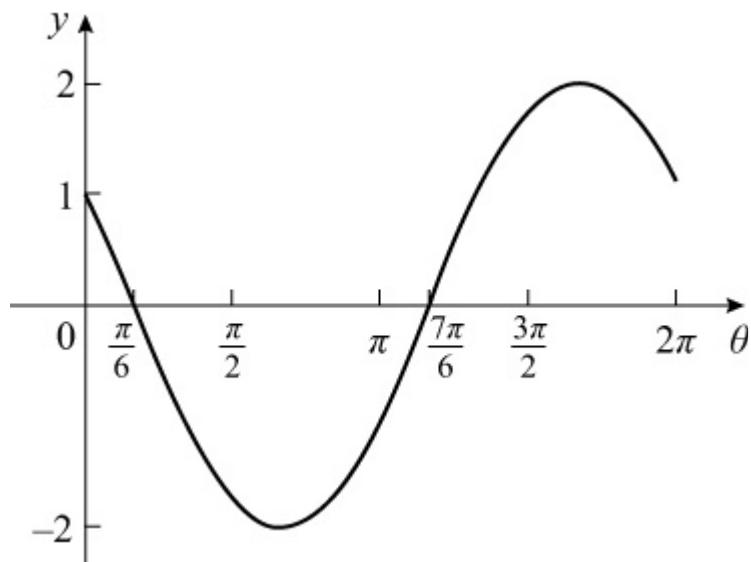
$$\text{Compare } \sin \theta : R \sin \alpha = \sqrt{3} \quad \textcircled{2}$$

$$\text{Divide } \textcircled{2} \text{ by } \textcircled{1} : \tan \alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$

$$\text{Square and add: } R^2 = 1 + 3 = 4 \Rightarrow R = 2$$

$$\text{So } \cos \theta - \sqrt{3} \sin \theta \equiv 2 \cos \left( \theta + \frac{\pi}{3} \right)$$

(b) This is the graph of  $y = \cos \theta$ , translated by  $\frac{\pi}{3}$  to the left and then stretched in the  $y$  direction by scale factor 2.



Meets  $y$ -axis at  $(0, 1)$

Meets  $x$ -axis at  $\left( \frac{\pi}{6}, 0 \right)$ ,  $\left( \frac{7\pi}{6}, 0 \right)$

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# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 8

#### Question:

(a) Show that  $3 \sin 3\theta - 4 \cos 3\theta$  can be written in the form  $R \sin (3\theta - \alpha)$ , with  $R > 0$  and  $0 < \alpha < 90^\circ$ .

(b) Deduce the minimum value of  $3 \sin 3\theta - 4 \cos 3\theta$  and work out the smallest positive value of  $\theta$  (to the nearest  $0.1^\circ$ ) at which it occurs.

#### Solution:

(a) Let  $3 \sin 3\theta - 4 \cos 3\theta \equiv R \sin (3\theta - \alpha) \equiv R \sin 3\theta \cos \alpha - R \cos 3\theta \sin \alpha$

$$\text{Compare } \sin 3\theta : R \cos \alpha = 3 \quad ①$$

$$\text{Compare } \cos 3\theta : R \sin \alpha = 4 \quad ②$$

$$\text{Divide } ② \text{ by } ①: \tan \alpha = \frac{4}{3} \Rightarrow \alpha = 53.1^\circ$$

$$\text{Square and add: } R^2 = 3^2 + 4^2 \Rightarrow R = 5$$

$$\text{So } 3 \sin 3\theta - 4 \cos 3\theta \equiv 5 \sin (3\theta - 53.1^\circ)$$

(b) Minimum value occurs when  $\sin (3\theta - 53.1^\circ) = -1$

So minimum value is  $-5$

To find smallest +ve value of  $\theta$  solve  $\sin (3\theta - 53.1^\circ) = -1$

$$\text{So } 3\theta - 53.1^\circ = 270^\circ$$

$$\Rightarrow 3\theta = 323.1^\circ$$

$$\Rightarrow \theta = 107.7^\circ$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 9

#### Question:

(a) Show that  $\cos 2\theta + \sin 2\theta$  can be written in the form  $R \sin(2\theta + \alpha)$ , with  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ .

(b) Hence solve, in the interval  $0 \leq \theta < 2\pi$ , the equation  $\cos 2\theta + \sin 2\theta = 1$ , giving your answers as rational multiples of  $\pi$ .

#### Solution:

(a) Let  $\cos 2\theta + \sin 2\theta \equiv R \sin(2\theta + \alpha) \equiv R \sin 2\theta \cos \alpha + R \cos 2\theta \sin \alpha$

$$\text{Compare } \cos 2\theta : R \sin \alpha = 1 \quad \textcircled{1}$$

$$\text{Compare } \sin 2\theta : R \cos \alpha = 1 \quad \textcircled{2}$$

$$\text{Divide } \textcircled{1} \text{ by } \textcircled{2}: \tan \alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$$

$$\text{Square and add: } R^2 = 1^2 + 1^2 = 2 \Rightarrow R = \sqrt{2}$$

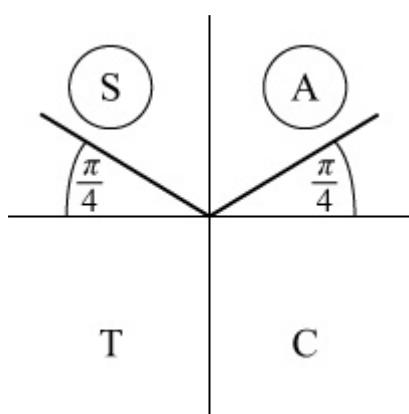
$$\text{So } \cos 2\theta + \sin 2\theta \equiv \sqrt{2} \sin \left( 2\theta + \frac{\pi}{4} \right)$$

$$(b) \text{ Solve } \sqrt{2} \sin \left( 2\theta + \frac{\pi}{4} \right) = 1, 0 \leq \theta < 2\pi$$

$$\text{so } \sin \left( 2\theta + \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}}, \frac{\pi}{4} \leq 2\theta + \frac{\pi}{4} < \frac{17\pi}{4}$$

As  $\sin \left( 2\theta + \frac{\pi}{4} \right)$  is +ve,  $\left( 2\theta + \frac{\pi}{4} \right)$  is in 1st and 2nd quadrants.

$$\text{Calculator value is } \sin^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$



$$\text{So } 2\theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$$

$$\Rightarrow 2\theta = 0, \frac{\pi}{2}, 2\pi, \frac{5\pi}{2}$$

$$\Rightarrow \theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$$

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# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 10

#### Question:

- (a) Express  $7\cos\theta - 24\sin\theta$  in the form  $R\cos(\theta + \alpha)$ , with  $R > 0$  and  $0 < \alpha < 90^\circ$ . Give  $\alpha$  to the nearest  $0.1^\circ$ .
- (b) The graph of  $y = 7\cos\theta - 24\sin\theta$  meets the  $y$ -axis at P. State the coordinates of P.
- (c) Write down the maximum and minimum values of  $7\cos\theta - 24\sin\theta$ .
- (d) Deduce the number of solutions, in the interval  $0 < \theta < 360^\circ$ , of the following equations:
- $7\cos\theta - 24\sin\theta = 15$
  - $7\cos\theta - 24\sin\theta = 26$
  - $7\cos\theta - 24\sin\theta = -25$

#### Solution:

(a) Let  $7\cos\theta - 24\sin\theta \equiv R\cos(\theta + \alpha) \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

Compare  $\cos\theta$ :  $R\cos\alpha = 7$       ①

Compare  $\sin\theta$ :  $R\sin\alpha = 24$       ②

Divide ② by ①:  $\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 73.7^\circ$

Square and add:  $R^2 = 24^2 + 7^2 \Rightarrow R = 25$

So  $7\cos\theta - 24\sin\theta \equiv 25\cos(\theta + 73.7^\circ)$

(b) Graph meets  $y$ -axis where  $\theta = 0$ ,

i.e.  $y = 7\cos 0^\circ - 24\sin 0^\circ = 7$

so coordinates are  $(0, 7)$

(c) Maximum value of  $25\cos(\theta + 73.7^\circ)$  is when  $\cos(\theta + 73.7^\circ) = 1$

So maximum is 25

Minimum value is  $25(-1) = -25$

(d) (i) The line  $y = 15$  will meet the graph twice in  $0 < \theta < 360^\circ$ , so there are 2 solutions.

(ii) As the maximum value is 25 it can never be 26, so there are 0 solutions.

(iii) As  $-25$  is a minimum, line  $y = -25$  only meets curve once, so only 1 solution.



# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 11

#### Question:

(a) Express  $5\sin^2 \theta - 3\cos^2 \theta + 6\sin\theta\cos\theta$  in the form  $a\sin 2\theta + b\cos 2\theta + c$ , where  $a$ ,  $b$  and  $c$  are constants.

(b) Hence find the maximum and minimum values of  $5\sin^2 \theta - 3\cos^2 \theta + 6\sin\theta\cos\theta$ .

#### Solution:

$$(a) \text{As } \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \text{ and } \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\begin{aligned} \text{so } 5\sin^2 \theta - 3\cos^2 \theta + 6\sin\theta\cos\theta \\ \equiv 5 \frac{1 - \cos 2\theta}{2} - 3 \frac{1 + \cos 2\theta}{2} + 3(2\sin\theta\cos\theta) \\ \equiv \frac{5}{2} - \frac{5}{2}\cos 2\theta - \frac{3}{2} - \frac{3}{2}\cos 2\theta + 3\sin 2\theta \\ \equiv 1 - 4\cos 2\theta + 3\sin 2\theta \end{aligned}$$

(b) Write  $3\sin 2\theta - 4\cos 2\theta$  in the form  $R\sin(2\theta - \alpha)$

The maximum value of  $R\sin(2\theta - \alpha)$  is  $R$

The minimum value of  $R\sin(2\theta - \alpha)$  is  $-R$

You know that  $R^2 = 3^2 + 4^2$  so  $R = 5$

So maximum value of  $1 - 4\cos 2\theta + 3\sin 2\theta$  is  $1 + 5 = 6$

and minimum value of  $1 - 4\cos 2\theta + 3\sin 2\theta$  is  $1 - 5 = -4$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 12

#### Question:

Solve the following equations, in the interval given in brackets. Give all angles to the nearest  $0.1^\circ$ .

$$(a) 6 \sin x + 8 \cos x = 5 \sqrt{3} [0, 360^\circ]$$

$$(b) 2 \cos 3\theta - 3 \sin 3\theta = -1 [0, 90^\circ]$$

$$(c) 8 \cos \theta + 15 \sin \theta = 10 [0, 360^\circ]$$

$$(d) 5 \sin \frac{x}{2} - 12 \cos \frac{x}{2} = -6.5 [-360^\circ, 360^\circ]$$

#### Solution:

(a) Write  $6 \sin x + 8 \cos x$  in the form  $R \sin(x + \alpha)$ , where  $R > 0, 0 < \alpha < 90^\circ$   
 so  $6 \sin x + 8 \cos x \equiv R \sin x \cos \alpha + R \cos x \sin \alpha$

$$\text{Compare } \sin x : R \cos \alpha = 6 \quad ①$$

$$\text{Compare } \cos x : R \sin \alpha = 8 \quad ②$$

$$\text{Divide } ② \text{ by } ①: \tan \alpha = \frac{4}{3} \Rightarrow \alpha = 53.13^\circ$$

$$R^2 = 6^2 + 8^2 \Rightarrow R = 10$$

$$\text{So } 6 \sin x + 8 \cos x \equiv 10 \sin(x + 53.13^\circ)$$

$$\text{Solve } 10 \sin(x + 53.13^\circ) = 5 \sqrt{3}, 0^\circ \leq x \leq 360^\circ$$

$$\text{so } \sin(x + 53.13^\circ) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x + 53.13^\circ = 60^\circ, 120^\circ$$

$$\Rightarrow x = 6.9^\circ, 66.9^\circ$$

(b) Let  $2 \cos 3\theta - 3 \sin 3\theta \equiv R \cos(3\theta + \alpha) \equiv R \cos 3\theta \cos \alpha - R \sin 3\theta \sin \alpha$

$$\text{Compare } \cos 3\theta : R \cos \alpha = 2 \quad ①$$

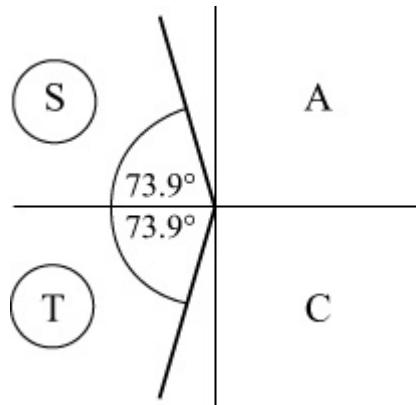
$$\text{Compare } \sin 3\theta : R \sin \alpha = 3 \quad ②$$

$$\text{Divide } ② \text{ by } ①: \tan \alpha = \frac{3}{2} \Rightarrow \alpha = 56.31^\circ$$

$$R^2 = 2^2 + 3^2 \Rightarrow R = \sqrt{13}$$

$$\text{Solve } \sqrt{13} \cos(3\theta + 56.31^\circ) = -1, 0^\circ \leq \theta \leq 90^\circ$$

so  $\cos(3\theta + 56.31^\circ) = -\frac{1}{\sqrt{13}}$  for  
 $56.31^\circ \leq 3\theta + 56.31^\circ \leq 326.31^\circ$



$$\Rightarrow 3\theta + 56.31^\circ = 106.1^\circ, 253.9^\circ$$

$$\Rightarrow 3\theta = 49.8^\circ, 197.6^\circ$$

$$\Rightarrow \theta = 16.6^\circ, 65.9^\circ$$

(c) Let  $8\cos\theta + 15\sin\theta \equiv R\cos(\theta - \alpha) \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$$\text{Compare } \cos\theta : R\cos\alpha = 8 \quad ①$$

$$\text{Compare } \sin\theta : R\sin\alpha = 15 \quad ②$$

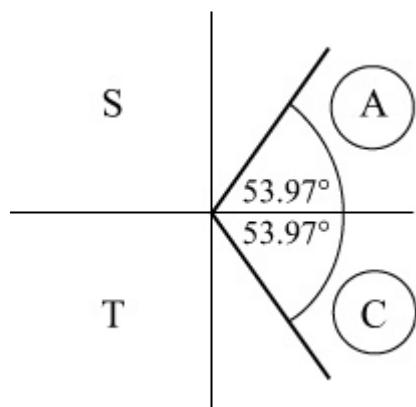
$$\text{Divide } ② \text{ by } ① : \tan\alpha = \frac{15}{8} \Rightarrow \alpha = 61.93^\circ$$

$$R^2 = 8^2 + 15^2 \Rightarrow R = 17$$

$$\text{Solve } 17\cos(\theta - 61.93^\circ) = 10, 0 \leq \theta \leq 360^\circ$$

$$\text{so } \cos(\theta - 61.93^\circ) = \frac{10}{17}, -61.93^\circ \leq \theta - 61.93^\circ \leq 298.1^\circ$$

$$\cos^{-1}\left(\frac{10}{17}\right) = 53.97^\circ$$



$$\text{So } \theta - 61.93^\circ = -53.97^\circ, +53.97^\circ$$

$$\Rightarrow \theta = 8.0^\circ, 115.9^\circ$$

(d) Let  $5 \sin \frac{x}{2} - 12 \cos \frac{x}{2} \equiv R \sin \left( \frac{x}{2} - \alpha \right) \equiv R \sin \frac{x}{2} \cos \alpha - R \cos \frac{x}{2} \sin \alpha$

$$\text{Compare } \sin \frac{x}{2} : \quad R \cos \alpha = 5 \quad \textcircled{1}$$

$$\text{Compare } \cos \frac{x}{2} : \quad R \sin \alpha = 12 \quad \textcircled{2}$$

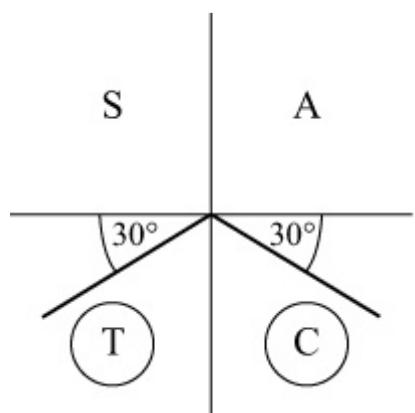
$$\text{Divide } \textcircled{2} \text{ by } \textcircled{1}: \quad \tan \alpha = \frac{12}{5} \Rightarrow \alpha = 67.38^\circ$$

$$R = 13$$

$$\text{Solve } 13 \sin \left( \frac{x}{2} - 67.38^\circ \right) = -6.5, -360^\circ \leq x \leq 360^\circ$$

$$\text{so } \sin \left( \frac{x}{2} - 67.38^\circ \right) = -\frac{1}{2}, -247.4^\circ \leq$$

$$\frac{x}{2} - 67.4^\circ \leq 112.6^\circ$$



From quadrant diagram:

$$\frac{x}{2} - 67.4^\circ = -150^\circ, -30^\circ$$

$$\Rightarrow \frac{x}{2} = -82.6^\circ, 37.4^\circ$$

$$\Rightarrow x = -165.2^\circ, 74.8^\circ$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 13**
**Question:**

Solve the following equations, in the interval given in brackets. Give all angles to the nearest  $0.1^\circ$ .

(a)  $\sin x \cos x = 1 - 2.5 \cos 2x$  [0,  $360^\circ$ ]

(b)  $\cot \theta + 2 = \operatorname{cosec} \theta$  [0 <  $\theta < 360$ ,  $\theta \neq 180$ ]

(c)  $\sin \theta = 2 \cos \theta - \sec \theta$  [0,  $180^\circ$ ]

(d)  $\sqrt{2} \cos \left( \theta - \frac{\pi}{4} \right) + \left( \sqrt{3} - 1 \right) \sin \theta = 2$  [0,  $2\pi$ ]

**Solution:**

(a)  $\sin x \cos x = 1 - 2.5 \cos 2x$ ,  $0 \leq x \leq 360^\circ$

$$\Rightarrow \frac{1}{2} \sin 2x = 1 - 2.5 \cos 2x$$

$$\Rightarrow \sin 2x + 5 \cos 2x = 2$$

Let  $\sin 2x + 5 \cos 2x \equiv R \sin (2x + \alpha) \equiv R \sin 2x \cos \alpha + R \cos 2x \sin \alpha$

Compare  $\sin 2x$ :  $R \cos \alpha = 1$  ①

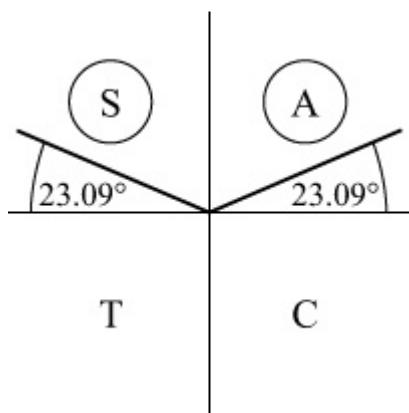
Compare  $\cos 2x$ :  $R \sin \alpha = 5$  ②

Divide ② by ①:  $\tan \alpha = 5 \Rightarrow \alpha = \tan^{-1} 5 = 78.7^\circ$

$$R^2 = 5^2 + 1^2 \Rightarrow R = \sqrt{26}$$

Solve  $\sqrt{26} \sin (2x + 78.7^\circ) = 2$ ,  $0 \leq x \leq 360^\circ$

$$\Rightarrow \sin \left( 2x + 78.7^\circ \right) = \frac{2}{\sqrt{26}}, 78.7^\circ \leq 2x + 78.7^\circ \leq 798.7^\circ$$



$$\Rightarrow 2x + 78.7^\circ = 156.9^\circ, 383.1^\circ, 516.9^\circ, 743.1^\circ$$

$$\Rightarrow 2x = 78.2^\circ, 304.4^\circ, 438.2^\circ, 664.4^\circ$$

$$\Rightarrow x = 39.1^\circ, 152.2^\circ, 219.1^\circ, 332.2^\circ$$

$$(b) \cot \theta + 2 = \operatorname{cosec} \theta, 0^\circ < \theta < 360^\circ, \theta \neq 180^\circ$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} + 2 = \frac{1}{\sin \theta} \quad (\text{as } \sin \theta \neq 0)$$

$$\Rightarrow \cos \theta + 2 \sin \theta = 1$$

$$\text{Let } \cos \theta + 2 \sin \theta \equiv R \cos(\theta - \alpha) \equiv R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$$

$$\text{Compare } \cos \theta : R \cos \alpha = 1 \quad ①$$

$$\text{Compare } \sin \theta : R \sin \alpha = 2 \quad ②$$

$$\text{Divide } ② \text{ by } ①: \tan \alpha = 2 \Rightarrow \alpha = 63.43^\circ$$

$$R^2 = 2^2 + 1^2 \Rightarrow R = \sqrt{5}$$

$$\text{Solve } \sqrt{5} \cos(\theta - 63.43^\circ) = 1, 0^\circ < \theta < 360^\circ$$

$$\Rightarrow \cos(\theta - 63.43^\circ) = \frac{1}{\sqrt{5}}, -63.43^\circ < \theta - 63.43^\circ < 296.6^\circ$$

$$\Rightarrow \theta - 63.43^\circ = 63.43^\circ$$

$$\Rightarrow \theta = 126.9^\circ$$

$$(c) \sin \theta = 2 \cos \theta - \sec \theta, 0^\circ \leq \theta \leq 180^\circ$$

$$\Rightarrow \sin \theta \cos \theta = 2 \cos^2 \theta - 1 \quad (\times \cos \theta)$$

$$\Rightarrow \frac{1}{2} \sin 2\theta = \cos 2\theta$$

$$\Rightarrow \tan 2\theta = 2, 0^\circ \leq 2\theta \leq 360^\circ$$

$$\Rightarrow 2\theta = \tan^{-1} 2, 180^\circ + \tan^{-1} 2 = 63.43^\circ, 243.43^\circ$$

$$\Rightarrow \theta = 31.7^\circ, 121.7^\circ$$

$$(d) \sqrt{2} \cos \left( \theta - \frac{\pi}{4} \right) + (\sqrt{3} - 1) \sin \theta$$

$$\equiv \sqrt{2} \left( \cos \theta \cos \frac{\pi}{4} + \sin \theta \sin \frac{\pi}{4} \right) + (\sqrt{3} - 1) \sin \theta$$

$$\equiv \cos \theta + \sin \theta + \sqrt{3} \sin \theta - \sin \theta$$

$$\equiv \cos \theta + \sqrt{3} \sin \theta$$

$$\text{Let } \cos \theta + \sqrt{3} \sin \theta \equiv R \cos(\theta - \alpha) \equiv R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$$

$$\text{Compare } \cos \theta : R \cos \alpha = 1 \quad ①$$

$$\text{Compare } \sin \theta : R \sin \alpha = \sqrt{3} \quad ②$$

$$\text{Divide } ② \text{ by } ①: \tan \alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$

$$R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$$

$$\text{Solve } 2\cos\left(\theta - \frac{\pi}{3}\right) = 2, 0 \leq \theta \leq 2\pi$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{3}\right) = 1, -\frac{\pi}{3} \leq \theta - \frac{\pi}{3} \leq \frac{5\pi}{3}$$

$$\Rightarrow \theta - \frac{\pi}{3} = 0$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 14

#### Question:

Solve, if possible, in the interval  $0 < \theta < 360^\circ$ ,  $\theta \neq 180^\circ$ , the equation

$$\frac{4 - 2\sqrt{2}\sin\theta}{1 + \cos\theta} = k \text{ in the case when } k \text{ is equal to:}$$

(a) 4

(b) 2

(c) 1

(d) 0

(e) -1

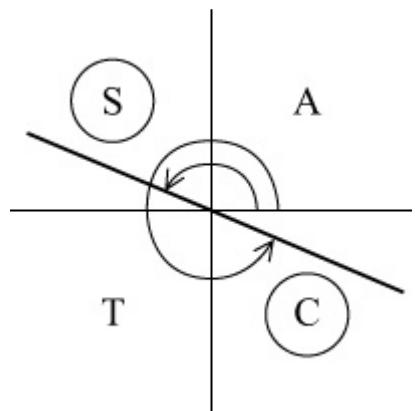
Give all angles to the nearest  $0.1^\circ$ .

#### Solution:

(a) When  $k = 4$ ,  $4 - 2\sqrt{2}\sin\theta = 4 + 4\cos\theta$

$$\Rightarrow -2\sqrt{2}\sin\theta = 4\cos\theta$$

$$\Rightarrow \tan\theta = -\frac{4}{2\sqrt{2}} = -\sqrt{2}$$



$$\theta = 180^\circ + \tan^{-1}(-\sqrt{2}), 360^\circ + \tan^{-1}(-\sqrt{2}) = 125.3^\circ, 305.3^\circ$$

(b) When  $k = 2$ ,  $4 - 2\sqrt{2}\sin\theta = 2 + 2\cos\theta$

$$\Rightarrow 2\cos\theta + 2\sqrt{2}\sin\theta = 2$$

$$\Rightarrow \cos\theta + \sqrt{2}\sin\theta = 1$$

Using the 'R formula' L.H.S.  $\equiv \sqrt{3}\cos(\theta - 54.74^\circ)$

$$\begin{aligned} \text{Solve } \sqrt{3} \cos(\theta - 54.74^\circ) &= 1 \\ \Rightarrow \cos(\theta - 54.74^\circ) &= \frac{1}{\sqrt{3}} \\ \Rightarrow \theta - 54.74^\circ &= 54.74^\circ \\ \Rightarrow \theta &= 109.5^\circ \end{aligned}$$

(c) When  $k = 1$ ,  $4 - 2\sqrt{2}\sin\theta = 1 + \cos\theta$   
 $\Rightarrow \cos\theta + 2\sqrt{2}\sin\theta = 3$

Using the R formula,  $\cos\theta + 2\sqrt{2}\sin\theta \equiv 3\cos(\theta - 70.53^\circ)$

$$\begin{aligned} \text{Solve } 3\cos(\theta - 70.53^\circ) &= 3 \\ \Rightarrow \cos(\theta - 70.53^\circ) &= 1 \\ \Rightarrow \theta - 70.53^\circ &= 0^\circ \\ \Rightarrow \theta &= 70.5^\circ \end{aligned}$$

(d) When  $k = 0$ ,  $4 - 2\sqrt{2}\sin\theta = 0$   
 $\Rightarrow \sin\theta = \sqrt{2}$

No solutions as  $-1 \leq \sin\theta \leq 1$

(e) When  $k = -1$ ,  $4 - 2\sqrt{2}\sin\theta = -1 - \cos\theta$   
 $\Rightarrow \cos\theta - 2\sqrt{2}\sin\theta = -5$

Using the R formula,  $\cos\theta - 2\sqrt{2}\sin\theta \equiv 3\cos(\theta + 70.53^\circ)$   
This lies between  $-3$  and  $+3$ , so there can be no solutions.

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise D, Question 15

#### Question:

Give all angles to the nearest  $0.1^\circ$  and non-exact values of  $R$  in surd form.  
A class were asked to solve  $3\cos\theta = 2 - \sin\theta$  for  $0^\circ \leq \theta \leq 360^\circ$ . One student expressed the equation in the form  $R\cos(\theta - \alpha) = 2$ , with  $R > 0$  and  $0 < \alpha < 90^\circ$ , and correctly solved the equation.

- (a) Find the values of  $R$  and  $\alpha$  and hence find her solutions.  
Another student decided to square both sides of the equation and then form a quadratic equation in  $\sin\theta$ .
- (b) Show that the correct quadratic equation is  $10\sin^2\theta - 4\sin\theta - 5 = 0$ .
- (c) Solve this equation, for  $0^\circ \leq \theta < 360^\circ$ .
- (d) Explain why not all of the answers satisfy  $3\cos\theta = 2 - \sin\theta$ .

#### Solution:

(a) Let  $3\cos\theta + \sin\theta \equiv R\cos(\theta - \alpha) \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

Compare  $\cos\theta$ :  $R\cos\alpha = 3$     ①

Compare  $\sin\theta$ :  $R\sin\alpha = 1$     ②

Divide ② by ①:  $\tan\alpha = \frac{1}{3} \Rightarrow \alpha = 18.43^\circ$

$$R^2 = 3^2 + 1^2 = 10 \Rightarrow R = \sqrt{10} = 3.16$$

$$\text{Solve } \sqrt{10}\cos(\theta - 18.43^\circ) = 2, 0^\circ \leq \theta \leq 360^\circ$$

$$\Rightarrow \cos(\theta - 18.43^\circ) = \frac{2}{\sqrt{10}}$$

$$\Rightarrow \theta - 18.43^\circ = 50.77^\circ, 309.23^\circ$$

$$\Rightarrow \theta = 69.2^\circ, 327.7^\circ$$

(b) Squaring  $3\cos\theta = 2 - \sin\theta$

gives  $9\cos^2\theta = 4 + \sin^2\theta - 4\sin\theta$

$$\Rightarrow 9(1 - \sin^2\theta) = 4 + \sin^2\theta - 4\sin\theta$$

$$\Rightarrow 10\sin^2\theta - 4\sin\theta - 5 = 0$$

(c)  $10\sin^2\theta - 4\sin\theta - 5 = 0$

$$\Rightarrow \sin \theta = \frac{4 \pm \sqrt{216}}{20}$$

For  $\sin \theta = \frac{4 + \sqrt{216}}{20}$ ,  $\sin \theta$  is +ve, so  $\theta$  is in 1st and 2nd quadrants.

$$\Rightarrow \theta = 69.2^\circ, 180^\circ - 69.2^\circ = 69.2^\circ, 110.8^\circ$$

For  $\sin \theta = \frac{4 - \sqrt{216}}{20}$ ,  $\sin \theta$  is -ve, so  $\theta$  is in 3rd and 4th quadrants.

$$\Rightarrow \theta = 180^\circ - (-32.3^\circ), 360^\circ + (-32.3^\circ) = 212.3^\circ, 327.7^\circ$$

So solutions of quadratic in (b) are  $69.2^\circ, 110.8^\circ, 212.3^\circ, 327.7^\circ$

(d) In squaring the equation, you are also including the solutions to  $3\cos\theta = -(2 - \sin\theta)$ , which when squared produces the same quadratic.

The extra two solutions satisfying this equation.

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 1

### Question:

(a) Show that  $\sin(A + B) + \sin(A - B) \equiv 2 \sin A \cos B$ .

(b) Deduce that  $\sin P + \sin Q \equiv 2 \sin\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right)$ .

(c) Use part (a) to express the following as the sum of two sines:

$$(i) 2 \sin 7\theta \cos 2\theta$$

$$(ii) 2 \sin 12\theta \cos 5\theta$$

(d) Use the result in (b) to solve, in the interval  $0^\circ \leq \theta \leq 180^\circ$ ,  $\sin 3\theta + \sin \theta = 0$ .

(e) Prove that  $\frac{\sin 7\theta + \sin \theta}{\sin 5\theta + \sin 3\theta} \equiv \frac{\cos 3\theta}{\cos \theta}$ .

### Solution:

$$\begin{aligned} (a) \sin(A + B) + \sin(A - B) &\equiv \sin A \cos B + \cos A \sin B + \sin A \cos B - \cos A \sin B \\ &\equiv 2 \sin A \cos B \end{aligned}$$

$$(b) \text{Let } P = A + B \text{ and } Q = A - B, \text{ so } A = \frac{P+Q}{2}, B = \frac{P-Q}{2}$$

$$\text{Substitute in (a): } \sin P + \sin Q \equiv 2 \sin\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right)$$

$$\begin{aligned} (c) (i) 2 \sin 7\theta \cos 2\theta &\equiv \sin(7\theta + 2\theta) + \sin(7\theta - 2\theta) \quad [\text{from (a)}] \\ &\equiv \sin 9\theta + \sin 5\theta \end{aligned}$$

$$(ii) 2 \sin 12\theta \sin 5\theta \equiv \sin(12\theta + 5\theta) + \sin(12\theta - 5\theta) \equiv \sin 17\theta + \sin 7\theta$$

$$(d) \sin 3\theta + \sin \theta = 0 \Rightarrow 2 \sin\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{3\theta - \theta}{2}\right) = 0$$

$$\text{so } 2 \sin 2\theta \cos \theta = 0$$

$$\Rightarrow \sin 2\theta = 0 \text{ or } \cos \theta = 0$$

$$\sin 2\theta = 0 \text{ in } 0^\circ \leq 2\theta \leq 360^\circ$$

$$\Rightarrow 2\theta = 0^\circ, 180^\circ, 360^\circ$$

$$\Rightarrow \theta = 0^\circ, 90^\circ, 180^\circ$$

$$\cos \theta = 0 \text{ in } 0^\circ \leq \theta \leq 180^\circ \Rightarrow \theta = 90^\circ$$

Solution set:  $0^\circ, 90^\circ, 180^\circ$

(e)

$$\frac{\sin 7\theta + \sin \theta}{\sin 5\theta + \sin 3\theta} \equiv \frac{2 \sin 4\theta \cos 3\theta}{2 \sin 4\theta \cos \theta} \text{ [using (b)]} \equiv \frac{\cos 3\theta}{\cos \theta}$$

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# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 2

### Question:

(a) Show that  $\sin(A + B) - \sin(A - B) \equiv 2\cos A \sin B$ .

(b) Express the following as the difference of two sines:

(i)  $2\cos 5x \sin 3x$

(ii)  $\cos 2x \sin x$

(iii)  $6\cos \frac{3}{2}x \sin \frac{1}{2}x$

(c) Using the result in (a) show that  $\sin P - \sin Q \equiv 2\cos \left( \frac{P+Q}{2} \right) \sin \left( \frac{P-Q}{2} \right)$ .

(d) Deduce that  $\sin 56^\circ - \sin 34^\circ = \sqrt{2} \sin 11^\circ$ .

### Solution:

$$\begin{aligned} \sin(A + B) - \sin(A - B) &\equiv \sin A \cos B + \cos A \sin B - \\ &(\sin A \cos B - \cos A \sin B) \\ &\equiv 2\cos A \sin B \end{aligned}$$

$$(b) (i) 2\cos 5x \sin 3x \equiv \sin(5x + 3x) - \sin(5x - 3x) \equiv \sin 8x - \sin 2x$$

$$(ii) \cos 2x \sin x \equiv \frac{1}{2} [\sin(2x + x) - \sin(2x - x)] \equiv \frac{1}{2} (\sin 3x - \sin x)$$

$$\begin{aligned} (iii) 6\cos \frac{3}{2}x \sin \frac{1}{2}x &\equiv 3 \left[ \sin \left( \frac{3}{2}x + \frac{1}{2}x \right) - \sin \left( \frac{3}{2}x - \frac{1}{2}x \right) \right] \equiv 3 \\ &(\sin 2x - \sin x) \end{aligned}$$

(c) In (a) let  $P = A + B$  and  $Q = A - B$ , so  $A = \frac{P+Q}{2}$ ,  $B = \frac{P-Q}{2}$

$$\text{So } \sin P - \sin Q \equiv 2\cos \left( \frac{P+Q}{2} \right) \sin \left( \frac{P-Q}{2} \right)$$

$$\begin{aligned} (d) \sin 56^\circ - \sin 34^\circ &= 2\cos \left( \frac{56^\circ + 34^\circ}{2} \right) \sin \left( \frac{56^\circ - 34^\circ}{2} \right) \\ &= 2\cos 45^\circ \sin 11^\circ = 2 \times \frac{1}{\sqrt{2}} \sin 11^\circ = \sqrt{2} \sin 11^\circ \end{aligned}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 3

### Question:

(a) Show that  $\cos(A + B) + \cos(A - B) \equiv 2\cos A \cos B$ .

(b) Express as a sum of cosines (i)  $2\cos \frac{5\theta}{2} \cos \frac{\theta}{2}$

(ii)  $5\cos 2x \cos 3x$

(c) Show that  $\cos P + \cos Q \equiv 2\cos \left( \frac{P+Q}{2} \right) \cos \left( \frac{P-Q}{2} \right)$ .

(d) Prove that  $\frac{\sin 3\theta - \sin \theta}{\cos 3\theta + \cos \theta} \equiv \tan \theta$ .

### Solution:

(a)  $\cos(A + B) + \cos(A - B) \equiv \cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B$   
 $\equiv 2\cos A \cos B$

(b) Hence, using (a),

(i)  $2\cos \frac{5\theta}{2} \cos \frac{\theta}{2} \equiv \cos \left( \frac{5\theta}{2} + \frac{\theta}{2} \right) + \cos \left( \frac{5\theta}{2} - \frac{\theta}{2} \right) \equiv \cos 3\theta + \cos 2\theta$

(ii)  $5\cos 2x \cos 3x \equiv \frac{5}{2} (2\cos 3x \cos 2x)$

$$\equiv \frac{5}{2} [\cos(3x + 2x) + \cos(3x - 2x)] \equiv \frac{5}{2}$$

$(\cos 5x + \cos x)$

(c) In (a) let  $P = A + B$ ,  $Q = A - B$ , so  $A = \frac{P+Q}{2}$ ,  $B = \frac{P-Q}{2}$

So  $\cos P + \cos Q \equiv 2\cos \left( \frac{P+Q}{2} \right) \cos \left( \frac{P-Q}{2} \right)$

$$\begin{aligned} \text{L.H.S.} \equiv \frac{\sin 3\theta - \sin \theta}{\cos 3\theta + \cos \theta} &\equiv \frac{\cancel{2} \cos \left( \frac{3\theta + \theta}{2} \right) \sin \left( \frac{3\theta - \theta}{2} \right)}{\cancel{2} \cos \left( \frac{3\theta + \theta}{2} \right) \cos \left( \frac{3\theta - \theta}{2} \right)} \\ &\equiv \tan \theta \end{aligned}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 4

**Question:**

(a) Show that  $\cos(A + B) - \cos(A - B) \equiv -2\sin A \sin B$ .

(b) Hence show that  $\cos P - \cos Q \equiv -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$ .

(c) Deduce that  $\cos 2\theta - 1 \equiv -2\sin^2 \theta$ .

(d) Solve, in the interval  $0^\circ \leq \theta \leq 180^\circ$ ,  $\cos 3\theta + \sin 2\theta - \cos \theta = 0$ .

**Solution:**

$$\begin{aligned} (a) \cos(A + B) - \cos(A - B) &\equiv \cos A \cos B - \sin A \sin B - (\cos A \cos B + \sin A \sin B) \\ &\equiv \cos A \cos B - \sin A \sin B - \cos A \cos B - \sin A \sin B \equiv -2\sin A \sin B \end{aligned}$$

$$(b) \text{Let } P = A + B, Q = A - B, \text{ so } A = \frac{P+Q}{2}, B = \frac{P-Q}{2}$$

$$\text{then } \cos P - \cos Q \equiv -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

$$(c) \text{Let } P = 2\theta, Q = 0$$

$$\text{then } \cos 2\theta - \cos 0 \equiv -2\sin \theta \sin \theta$$

$$\Rightarrow \cos 2\theta - 1 \equiv -2\sin^2 \theta$$

$$(d) \text{As } \cos 3\theta - \cos \theta \equiv -2\sin\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)$$

$$\cos 3\theta - \cos \theta \equiv -2\sin 2\theta \sin \theta$$

$$\text{So } \cos 3\theta + \sin 2\theta - \cos \theta = 0$$

$$\Rightarrow \sin 2\theta - 2\sin 2\theta \sin \theta = 0$$

$$\Rightarrow \sin 2\theta(1 - 2\sin \theta) = 0$$

$$\Rightarrow \sin 2\theta = 0 \text{ or } \sin \theta = \frac{1}{2}$$

$$\text{For } \sin \theta = \frac{1}{2}, \theta = 30^\circ, 150^\circ$$

$$\text{For } \sin 2\theta = 0, 2\theta = 0^\circ, 180^\circ, 360^\circ$$

$$\text{So } \theta = 0^\circ, 90^\circ, 180^\circ$$

$$\text{Solution set: } 0^\circ, 30^\circ, 90^\circ, 150^\circ, 180^\circ$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 5**
**Question:**

Express the following as a sum or difference of sines or cosines:

(a)  $2 \sin 8x \cos 2x$

(b)  $\cos 5x \cos x$

(c)  $3 \sin x \sin 7x$

(d)  $\cos 100^\circ \cos 40^\circ$

(e)  $10 \cos \frac{3x}{2} \sin \frac{x}{2}$

(f)  $2 \sin 30^\circ \cos 10^\circ$

**Solution:**

$$\begin{aligned} \text{(a)} \quad 2 \sin 8x \cos 2x &\equiv \sin(8x + 2x) + \sin(8x - 2x) \\ &\equiv \sin 10x + \sin 6x \quad [\text{question 1(a)}] \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \cos 5x \cos x &\equiv \frac{1}{2} (2 \cos 5x \cos x) \quad [\text{question 3(a)}] \\ &\equiv \frac{1}{2} [\cos(5x + x) + \cos(5x - x)] \equiv \frac{1}{2} \\ &(\cos 6x + \cos 4x) \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 3 \sin x \sin 7x &\equiv -\frac{3}{2} (-2 \sin 7x \sin x) \quad [\text{question 4(a)}] \\ &\equiv -\frac{3}{2} [\cos(7x + x) - \cos(7x - x)] \equiv -\frac{3}{2} \\ &(\cos 8x - \cos 6x) \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad \cos 100^\circ \cos 40^\circ &\equiv \frac{1}{2} (2 \cos 100^\circ \cos 40^\circ) \\ &\equiv \frac{1}{2} [\cos(100^\circ + 40^\circ) + \cos(100^\circ - 40^\circ)] \equiv \\ &\frac{1}{2} (\cos 140^\circ + \cos 60^\circ) \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad 10 \cos \frac{3x}{2} \sin \frac{x}{2} &\equiv 5 \left( 2 \cos \frac{3x}{2} \sin \frac{x}{2} \right) \quad [\text{question 2(a)}] \\ &\equiv 5 \left[ \sin \left( \frac{3x}{2} + \frac{x}{2} \right) - \sin \left( \frac{3x}{2} - \frac{x}{2} \right) \right] \equiv 5 \left( \right. \\ &\quad \left. \sin 2x - \sin x \right) \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad 2 \sin 30^\circ \cos 10^\circ &\equiv \sin (30^\circ + 10^\circ) + \sin (30^\circ - 10^\circ) \\ &\equiv \sin 40^\circ + \sin 20^\circ \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 6

**Question:**

Show, without using a calculator, that  $2 \sin 82 \frac{1}{2}^\circ \cos 37 \frac{1}{2}^\circ = \frac{1}{2} \left( \sqrt{3} + \sqrt{2} \right)$ .

**Solution:**

$$\begin{aligned} 2 \sin 82 \frac{1}{2}^\circ \cos 37 \frac{1}{2}^\circ &= \sin \left( 82 \frac{1}{2}^\circ + 37 \frac{1}{2}^\circ \right) + \sin \left( 82 \frac{1}{2}^\circ - 37 \frac{1}{2}^\circ \right) \\ &= \sin 120^\circ + \sin 45^\circ \\ &= \sin 60^\circ + \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \\ &= \frac{1}{2} \left( \sqrt{3} + \sqrt{2} \right) \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 7

### Question:

Express, in their simplest form, as a product of sines and/or cosines:

(a)  $\sin 12x + \sin 8x$

(b)  $\cos(x + 2y) - \cos(2y - x)$

(c)  $(\cos 4x + \cos 2x) \sin x$

(d)  $\sin 95^\circ - \sin 5^\circ$

(e)  $\cos \frac{\pi}{15} + \cos \frac{\pi}{12}$

(f)  $\sin 150^\circ + \sin 20^\circ$

### Solution:

$$\begin{aligned} \text{(a)} \sin 12x + \sin 8x &\equiv 2 \sin \left( \frac{12x + 8x}{2} \right) \cos \left( \frac{12x - 8x}{2} \right) \\ &\equiv 2 \sin 10x \cos 2x \end{aligned}$$

$$\begin{aligned} \text{(b)} \cos(x + 2y) - \cos(2y - x) &\equiv -2 \sin \left[ \frac{(x + 2y) + (2y - x)}{2} \right] \sin \left[ \frac{(x + 2y) - (2y - x)}{2} \right] \\ &\equiv -2 \sin 2y \sin x \end{aligned}$$

$$\begin{aligned} \text{(c)} \cos 4x + \cos 2x &\equiv 2 \cos \left( \frac{4x + 2x}{2} \right) \cos \left( \frac{4x - 2x}{2} \right) \\ &\equiv 2 \cos 3x \cos x \end{aligned}$$

$$\begin{aligned} \text{So } (\cos 4x + \cos 2x) \sin x &\equiv 2 \cos 3x \cos x \sin x \\ &\equiv \cos 3x (2 \sin x \cos x) \equiv \sin 2x \cos 3x \end{aligned}$$

$$\begin{aligned} \text{(d)} \sin 95^\circ - \sin 5^\circ &\equiv 2 \cos \left( \frac{95^\circ + 5^\circ}{2} \right) \sin \left( \frac{95^\circ - 5^\circ}{2} \right) \\ &\equiv 2 \cos 50^\circ \sin 45^\circ \equiv \sqrt{2} \cos 50^\circ \end{aligned}$$

$$(e) \cos \frac{\pi}{15} + \cos \frac{\pi}{12} \equiv 2 \cos \left( \frac{\frac{\pi}{15} + \frac{\pi}{12}}{2} \right) \cos \left( \frac{\frac{\pi}{15} - \frac{\pi}{12}}{2} \right)$$
$$\equiv 2 \cos \frac{9\pi}{120} \cos \left( - \frac{\pi}{120} \right) \equiv 2 \cos \frac{9\pi}{120} \cos \frac{\pi}{120}$$

$$(f) \sin 150^\circ + \sin 20^\circ \equiv 2 \sin \left( \frac{150^\circ + 20^\circ}{2} \right) \cos \left( \frac{150^\circ - 20^\circ}{2} \right)$$
$$\equiv 2 \sin 85^\circ \cos 65^\circ$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise E, Question 8

#### Question:

Using the identity  $\cos P + \cos Q \equiv 2 \cos \left( \frac{P+Q}{2} \right) \cos \left( \frac{P-Q}{2} \right)$ , show that

$$\cos \theta + \cos \left( \theta + \frac{2\pi}{3} \right) + \cos \left( \theta + \frac{4\pi}{3} \right) = 0.$$

#### Solution:

$$\begin{aligned}
 & \cos \theta + \cos \left( \theta + \frac{2\pi}{3} \right) + \cos \left( \theta + \frac{4\pi}{3} \right) \\
 & \equiv \left[ \cos \left( \theta + \frac{4\pi}{3} \right) + \cos \theta \right] + \cos \left( \theta + \frac{2\pi}{3} \right) \\
 & \equiv 2 \cos \left[ \frac{\left( \theta + \frac{4\pi}{3} \right) + \theta}{2} \right] \cos \left[ \frac{\left( \theta + \frac{4\pi}{3} \right) - \theta}{2} \right] + \cos \left( \theta + \frac{2\pi}{3} \right) \\
 & \equiv 2 \cos \left( \theta + \frac{2\pi}{3} \right) \cos \frac{2\pi}{3} + \cos \left( \theta + \frac{2\pi}{3} \right) \\
 & \equiv 2 \cos \left( \theta + \frac{2\pi}{3} \right) \left( -\frac{1}{2} \right) + \cos \left( \theta + \frac{2\pi}{3} \right) \\
 & \equiv -\cos \left( \theta + \frac{2\pi}{3} \right) + \cos \left( \theta + \frac{2\pi}{3} \right) \\
 & \equiv 0
 \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 9

**Question:**

Prove that  $\frac{\sin 75^\circ + \sin 15^\circ}{\cos 15^\circ - \cos 75^\circ} = \sqrt{3}$ .

**Solution:**

$$\begin{aligned}\sin 75^\circ + \sin 15^\circ &= 2 \sin \left( \frac{75+15}{2} \right)^\circ \cos \left( \frac{75-15}{2} \right)^\circ = 2 \sin 45^\circ \\ \cos 15^\circ - \cos 75^\circ &= -(\cos 75^\circ - \cos 15^\circ) \\ &= - \left[ -2 \sin \left( \frac{75+15}{2} \right)^\circ \sin \left( \frac{75-15}{2} \right)^\circ \right] \\ &= 2 \sin 45^\circ \sin 30^\circ \\ \text{So } \frac{\sin 75^\circ + \sin 15^\circ}{\cos 15^\circ - \cos 75^\circ} &= \frac{2 \sin 45^\circ \cos 30^\circ}{2 \sin 45^\circ \sin 30^\circ} = \cot 30^\circ = \frac{1}{\tan 30^\circ} = \sqrt{3}\end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise E, Question 10

#### Question:

Solve the following equations:

$$(a) \cos 4x = \cos 2x, \text{ for } 0^\circ \leq x \leq 180^\circ$$

$$(b) \sin 3\theta - \sin \theta = 0, \text{ for } 0^\circ \leq \theta \leq 2\pi$$

$$(c) \sin(x + 20^\circ) + \sin(x - 10^\circ) = \cos 15^\circ, \text{ for } 0^\circ \leq x \leq 360^\circ$$

$$(d) \sin 3\theta - \sin \theta = \cos 2\theta, \text{ for } 0^\circ \leq \theta \leq 2\pi$$

#### Solution:

$$(a) \cos 4x - \cos 2x = 0$$

$$\Rightarrow -2 \sin\left(\frac{4x+2x}{2}\right) \sin\left(\frac{4x-2x}{2}\right) = 0$$

$$\Rightarrow \sin 3x \sin x = 0$$

$$\Rightarrow \sin 3x = 0 \text{ or } \sin x = 0, 0^\circ \leq x \leq 180^\circ$$

$$\sin x = 0, 0^\circ \leq x \leq 180^\circ$$

$$\Rightarrow x = 0^\circ, 180^\circ$$

$$\sin 3x = 0, 0^\circ \leq 3x \leq 540^\circ$$

$$\Rightarrow 3x = 0^\circ, 180^\circ, 360^\circ, 540^\circ$$

$$\Rightarrow x = 0^\circ, 60^\circ, 120^\circ, 180^\circ$$

Solution set:  $0^\circ, 60^\circ, 120^\circ, 180^\circ$

$$(b) \sin 3\theta - \sin \theta = 0$$

$$\Rightarrow 2 \cos\left(\frac{3\theta+\theta}{2}\right) \sin\left(\frac{3\theta-\theta}{2}\right) = 0$$

$$\Rightarrow 2 \cos 2\theta \sin \theta = 0, 0^\circ \leq \theta \leq 2\pi$$

$$\sin \theta = 0, 0^\circ \leq \theta \leq 2\pi \Rightarrow \theta = 0, \pi, 2\pi$$

$$\cos 2\theta = 0, 0^\circ \leq 2\theta \leq 4\pi$$

$$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

Solution set:  $0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$

$$(c) \sin \left( x + 20^\circ \right) + \sin \left( x - 10^\circ \right) \equiv 2 \sin \left( \frac{x + 20^\circ + x - 10^\circ}{2} \right) \cos \left[ \frac{x + 20^\circ - (x - 10^\circ)}{2} \right] \\ \equiv 2 \sin (x + 5^\circ) \cos 15^\circ$$

$$\text{So } \sin (x + 20^\circ) + \sin (x - 10^\circ) = \cos 15^\circ, 0 \leq x \leq 360^\circ \\ \Rightarrow 2 \sin (x + 5^\circ) = 1$$

$$\text{So } \sin (x + 5^\circ) = \frac{1}{2}, 5^\circ \leq (x + 5^\circ) \leq 365^\circ \\ \Rightarrow x + 5^\circ = 30^\circ, 150^\circ \\ \Rightarrow x = 25^\circ, 145^\circ$$

$$(d) \sin 3\theta - \sin \theta \equiv 2 \cos \left( \frac{3\theta + \theta}{2} \right) \sin \left( \frac{3\theta - \theta}{2} \right) \\ \equiv 2 \cos 2\theta \sin \theta$$

$$\text{So } \sin 3\theta - \sin \theta = \cos 2\theta \\ \Rightarrow 2 \cos 2\theta \sin \theta = \cos 2\theta \\ \Rightarrow \cos 2\theta (2 \sin \theta - 1) = 0 \\ \Rightarrow \cos 2\theta = 0 \text{ or } \sin \theta = \frac{1}{2}$$

$$\sin \theta = \frac{1}{2}, 0 \leq \theta \leq 2\pi$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\cos 2\theta = 0, 0 \leq 2\theta \leq 4\pi$$

$$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\text{Solution set: } \frac{\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 11

### Question:

Prove the identities

$$(a) \frac{\sin 7\theta - \sin 3\theta}{\sin \theta \cos \theta} \equiv 4 \cos 5\theta$$

$$(b) \frac{\cos 2\theta + \cos 4\theta}{\sin 2\theta - \sin 4\theta} \equiv -\cot \theta$$

$$(c) \sin^2(x+y) - \sin^2(x-y) \equiv \sin 2x \sin 2y$$

$$(d) \cos x + 2 \cos 3x + \cos 5x \equiv 4 \cos^2 x \cos 3x$$

### Solution:

$$\begin{aligned} (a) \text{L.H.S.} &\equiv \frac{\sin 7\theta - \sin 3\theta}{\sin \theta \cos \theta} \\ &\equiv \frac{2 \cos \frac{1}{2}(7\theta + 3\theta) \sin \frac{1}{2}(7\theta - 3\theta)}{\frac{1}{2}(2 \sin \theta \cos \theta)} \end{aligned}$$

$$\begin{aligned} &\equiv \frac{2 \cos 5\theta \sin 2\theta}{\frac{1}{2} \sin 2\theta} \\ &\equiv 4 \cos 5\theta \\ &\equiv \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} (b) \text{L.H.S.} &\equiv \frac{\cos 2\theta + \cos 4\theta}{\sin 2\theta - \sin 4\theta} \\ &\equiv \frac{2 \cos \frac{1}{2}(4\theta + 2\theta) \cos \frac{1}{2}(4\theta - 2\theta)}{2 \cos \frac{1}{2}(2\theta + 4\theta) \sin \frac{1}{2}(2\theta - 4\theta)} \\ &\equiv \frac{2 \cos 3\theta \cos \theta}{2 \cos 3\theta \sin(-\theta)} \end{aligned}$$

$$\begin{aligned}
 &\equiv \frac{\cos \theta}{-\sin \theta} \quad [\text{as } \sin(-\theta) \equiv -\sin \theta] \\
 &\equiv -\cot \theta \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{c}) \text{ L.H.S.} &\equiv \sin^2(x+y) - \sin^2(x-y) \\
 &\equiv [\sin(x+y) + \sin(x-y)] [\sin(x+y) - \sin(x-y)]
 \end{aligned}$$

$$\begin{aligned}
 &\equiv \left[ 2 \sin \left( \frac{x+y+x-y}{2} \right) \cos \left( \frac{x+y-x+y}{2} \right) \right] \left[ 2 \cos \left( \frac{x+y+x-y}{2} \right) \sin \left( \frac{x+y-x+y}{2} \right) \right] \\
 &\equiv (2 \sin x \cos y) (2 \cos x \sin y) \\
 &\equiv (2 \sin x \cos x) (2 \sin y \cos y) \\
 &\equiv \sin 2x \sin 2y \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{d}) \text{ L.H.S.} &\equiv \cos x + 2 \cos 3x + \cos 5x \\
 &\equiv \cos 5x + \cos x + 2 \cos 3x \\
 &\equiv 2 \cos \left( \frac{5x+x}{2} \right) \cos \left( \frac{5x-x}{2} \right) + 2 \cos 3x \\
 &\equiv 2 \cos 3x \cos 2x + 2 \cos 3x \\
 &\equiv 2 \cos 3x (\cos 2x + 1) \\
 &\equiv 2 \cos 3x (2 \cos^2 x - 1 + 1) \quad (\cos 2x \equiv 2 \cos^2 x - 1) \\
 &\equiv 2 \cos 3x \times 2 \cos^2 x \\
 &\equiv 4 \cos^2 x \cos 3x \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

Exercise E, Question 12

**Question:**

- (a) Prove that  $\cos \theta + \sin 2\theta - \cos 3\theta \equiv \sin 2\theta (1 + 2\sin \theta)$ .
- (b) Hence solve, for  $0 \leq \theta \leq 2\pi$ ,  $\cos \theta + \sin 2\theta = \cos 3\theta$ .

**Solution:**

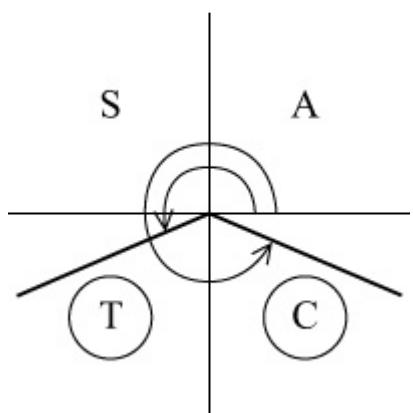
$$\begin{aligned}
 \text{(a) L.H.S.} &\equiv \cos \theta + \sin 2\theta - \cos 3\theta \\
 &\equiv -(\cos 3\theta - \cos \theta) + \sin 2\theta \\
 &\equiv -\left[-2 \sin\left(\frac{3\theta + \theta}{2}\right) \sin\left(\frac{3\theta - \theta}{2}\right)\right] + \sin 2\theta \\
 &\equiv 2 \sin 2\theta \sin \theta + \sin 2\theta \\
 &\equiv \sin 2\theta (2 \sin \theta + 1) \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

- (b) So to solve  $\cos \theta + \sin 2\theta = \cos 3\theta$   
or  $\cos \theta + \sin 2\theta - \cos 3\theta = 0$   
solve  $\sin 2\theta (1 + 2\sin \theta) = 0$  [using (a)]  
Either  $\sin 2\theta = 0$

$$\Rightarrow 2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$\Rightarrow \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

$$\text{or } \sin \theta = -\frac{1}{2}$$



$$\Rightarrow \theta = \pi - \sin^{-1}\left(-\frac{1}{2}\right), 2\pi + \sin^{-1}\left(-\frac{1}{2}\right) = \pi + \frac{\pi}{6}, 2\pi -$$

$$\frac{\pi}{6} = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Solution set:  $0, \frac{\pi}{2}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}, 2\pi$

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## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 1

#### Question:

The lines  $l_1$  and  $l_2$ , with equations  $y = 2x$  and  $3y = x - 1$  respectively, are drawn on the same set of axes. Given that the scales are the same on both axes and that the angles that  $l_1$  and  $l_2$  make with the positive  $x$ -axis are  $A$  and  $B$  respectively,

- write down the value of  $\tan A$  and the value of  $\tan B$ ;
- without using your calculator, work out the acute angle between  $l_1$  and  $l_2$ .

#### Solution:

(a)  $\tan A = 2$ ,  $\tan B = \frac{1}{3}$  since  $y = \frac{1}{3}x - \frac{1}{3}$

(b) The angle required is  $(A - B)$ .

$$\text{Using } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{2 - \frac{1}{3}}{1 + 2 \times \frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{3}} = 1$$

$$\Rightarrow A - B = 45^\circ$$

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## Edexcel AS and A Level Modular Mathematics

Exercise F, Question 2

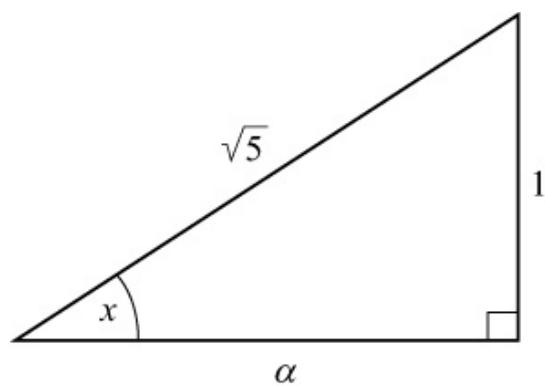
**Question:**

Given that  $\sin x = \frac{1}{\sqrt{5}}$  where  $x$  is acute, and that  $\cos(x - y) = \sin y$ , show that  $\tan y = \frac{\sqrt{5} + 1}{2}$ .

**Solution:**

$$\begin{aligned} \text{As } \cos(x - y) &= \sin y \\ \cos x \cos y + \sin x \sin y &= \sin y \quad \textcircled{1} \end{aligned}$$

Draw a right-angled triangle where  $\sin x = \frac{1}{\sqrt{5}}$



Using Pythagoras' theorem,

$$a^2 = (\sqrt{5})^2 - 1^2 = 4 \Rightarrow a = 2$$

$$\text{So } \cos x = \frac{2}{\sqrt{5}}$$

Substitute into ①:

$$\begin{aligned} \frac{2}{\sqrt{5}} \cos y + \frac{1}{\sqrt{5}} \sin y &= \sin y \\ \Rightarrow 2 \cos y + \sin y &= \sqrt{5} \sin y \\ \Rightarrow 2 \cos y &= \sin y (\sqrt{5} - 1) \\ \Rightarrow \frac{2}{\sqrt{5} - 1} &= \tan y \quad \left( \tan y = \frac{\sin y}{\cos y} \right) \\ \Rightarrow \tan y &= \frac{2(\sqrt{5} + 1)}{(\sqrt{5} - 1)(\sqrt{5} + 1)} = \frac{2(\sqrt{5} + 1)}{4} = \frac{\sqrt{5} + 1}{2} \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 3

#### Question:

Using  $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$  with an appropriate value of  $\theta$ ,

(a) show that  $\tan \frac{\pi}{8} = \sqrt{2} - 1$ .

(b) Use the result in (a) to find the exact value of  $\tan \frac{3\pi}{8}$ .

#### Solution:

(a) Using  $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$  with  $\theta = \frac{\pi}{8}$

$$\Rightarrow \tan \frac{\pi}{4} = \frac{2\tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$$

Let  $t = \tan \frac{\pi}{8}$

So  $1 = \frac{2t}{1 - t^2}$

$$\Rightarrow 1 - t^2 = 2t$$

$$\Rightarrow t^2 + 2t - 1 = 0$$

$$\Rightarrow t = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

As  $\frac{\pi}{8}$  is acute,  $\tan \frac{\pi}{8}$  is +ve, so  $\tan \frac{\pi}{8} = \sqrt{2} - 1$

$$\begin{aligned}
 (b) \tan \frac{3\pi}{8} &= \tan \left( \frac{\pi}{4} + \frac{\pi}{8} \right) = \frac{\tan \frac{\pi}{4} + \tan \frac{\pi}{8}}{1 - \tan \frac{\pi}{4} \tan \frac{\pi}{8}} \\
 &= \frac{1 + (\sqrt{2} - 1)}{1 - (\sqrt{2} - 1)} = \frac{\sqrt{2}}{2 - \sqrt{2}} = \frac{\sqrt{2}(2 + \sqrt{2})}{(2 - \sqrt{2})(2 + \sqrt{2})} = \frac{\sqrt{2}}{2}
 \end{aligned}$$

$$(2 + \sqrt{2}) = \sqrt{2} + 1$$

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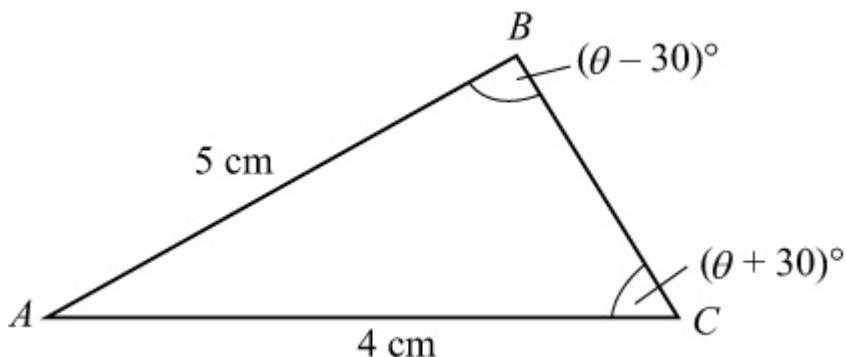
## Edexcel AS and A Level Modular Mathematics

Exercise F, Question 4

**Question:**

In  $\triangle ABC$ ,  $AB = 5 \text{ cm}$  and  $AC = 4 \text{ cm}$ ,  $\angle ABC = (\theta - 30)^\circ$  and  $\angle ACB = (\theta + 30)^\circ$ . Using the sine rule, show that  $\tan \theta = 3\sqrt{3}$ .

**Solution:**



$$\begin{aligned} \text{Using } \frac{\sin B}{b} = \frac{\sin C}{c} \\ \Rightarrow \frac{\sin(\theta - 30)^\circ}{4} = \frac{\sin(\theta + 30)^\circ}{5} \\ \Rightarrow 5 \sin(\theta - 30)^\circ = 4 \sin(\theta + 30)^\circ \\ \Rightarrow 5(\sin \theta \cos 30^\circ - \cos \theta \sin 30^\circ) = 4(\sin \theta \cos 30^\circ \\ + \cos \theta \sin 30^\circ) \\ \Rightarrow \sin \theta \cos 30^\circ = 9 \cos \theta \sin 30^\circ \\ \Rightarrow \frac{\sin \theta}{\cos \theta} = 9 \frac{\sin 30^\circ}{\cos 30^\circ} = 9 \tan 30^\circ \\ \Rightarrow \tan \theta = 9 \times \frac{\sqrt{3}}{3} = 3\sqrt{3} \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 5**
**Question:**

Two of the angles,  $A$  and  $B$ , in  $\triangle ABC$  are such that  $\tan A = \frac{3}{4}$ ,  $\tan B = \frac{5}{12}$ .

(a) Find the exact value of

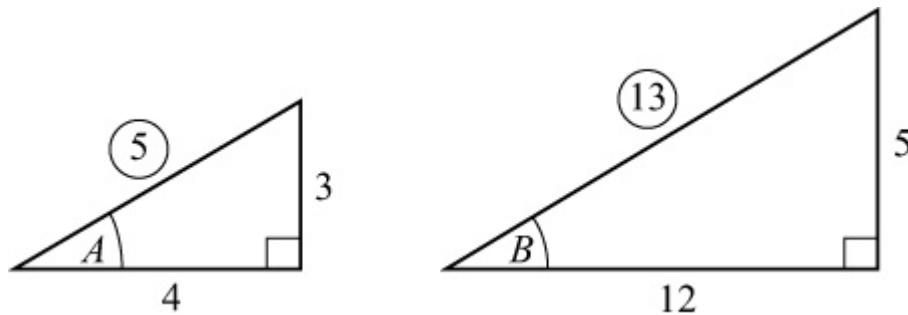
(i)  $\sin(A + B)$

(ii)  $\tan 2B$

(b) By writing  $C$  as  $180^\circ - (A + B)$ , show that  $\cos C = -\frac{33}{65}$ .

**Solution:**

(a) Draw right-angled triangles.



$$\sin A = \frac{3}{5}, \cos A = \frac{4}{5} \quad \sin B = \frac{5}{13}, \cos B = \frac{12}{13}$$

$$(i) \sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} = \frac{56}{65}$$

$$(ii) \tan 2B = \frac{2 \tan B}{1 - \tan^2 B} = \frac{2 \times \frac{5}{12}}{1 - (\frac{5}{12})^2} = \frac{\frac{5}{6}}{\frac{119}{144}} = \frac{5}{6} \times \frac{144}{119} = \frac{120}{119}$$

$$(b) \cos C = \cos [180^\circ - (A + B)] = -\cos(A + B)$$

$$= - (\cos A \cos B - \sin A \sin B) = - \left( \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} \right)$$

$$= -$$

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# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise F, Question 6

**Question:**

Show that

$$(a) \sec \theta \operatorname{cosec} \theta \equiv 2 \operatorname{cosec} 2\theta$$

$$(b) \frac{1 - \cos 2x}{1 + \cos 2x} \equiv \sec^2 x - 1$$

$$(c) \cot \theta - 2 \cot 2\theta \equiv \tan \theta$$

$$(d) \cos^4 2\theta - \sin^4 2\theta \equiv \cos 4\theta$$

$$(e) \tan \left( \frac{\pi}{4} + x \right) - \tan \left( \frac{\pi}{4} - x \right) \equiv 2 \tan 2x$$

$$(f) \sin(x+y) - \sin(x-y) \equiv \cos^2 y - \cos^2 x$$

$$(g) 1 + 2 \cos 2\theta + \cos 4\theta \equiv 4 \cos^2 \theta \cos 2\theta$$

**Solution:**

$$(a) \text{L.H.S.} \equiv \sec \theta \operatorname{cosec} \theta$$

$$\equiv \frac{1}{\cos \theta} \times \frac{1}{\sin \theta}$$

$$\equiv \frac{2}{2 \sin \theta \cos \theta}$$

$$\equiv \frac{2}{\sin 2\theta}$$

$$\equiv 2 \operatorname{cosec} 2\theta$$

$$\equiv \text{R.H.S.}$$

$$(b) \text{L.H.S.} \equiv \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$\equiv \frac{1 - (1 - 2 \sin^2 x)}{1 + (2 \cos^2 x - 1)}$$

$$\equiv \frac{2 \sin^2 x}{2 \cos^2 x}$$

$$\begin{aligned}
 &\equiv \tan^2 x \\
 &\equiv \sec^2 x - 1 \quad (1 + \tan^2 x \equiv \sec^2 x) \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{c}) \text{ L.H.S.} &\equiv \cot \theta - 2 \cot 2\theta \\
 &\equiv \frac{1}{\tan \theta} - \frac{2}{\tan 2\theta} \\
 &\equiv \frac{1}{\tan \theta} - \frac{2(1 - \tan^2 \theta)}{2 \tan \theta} \\
 &\equiv \frac{1 - 1 + \tan^2 \theta}{\tan \theta} \\
 &\equiv \frac{\tan^2 \theta}{\tan \theta} \\
 &\equiv \tan \theta \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{d}) \text{ L.H.S.} &\equiv \cos^4 2\theta - \sin^4 2\theta \\
 &\equiv (\cos^2 2\theta + \sin^2 2\theta)(\cos^2 2\theta - \sin^2 2\theta) \\
 &\equiv (1)(\cos 4\theta) \quad (\cos^2 A + \sin^2 A \equiv 1, \\
 &\cos^2 A - \sin^2 A \equiv \cos 2A) \\
 &\equiv \cos 4\theta \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 (\text{e}) \text{ L.H.S.} &\equiv \tan \left( \frac{\pi}{4} + x \right) - \tan \left( \frac{\pi}{4} - x \right) \\
 &\equiv \frac{1 + \tan x}{1 - \tan x} - \frac{1 - \tan x}{1 + \tan x} \\
 &\equiv \frac{(1 + \tan x)^2 - (1 - \tan x)^2}{(1 - \tan x)(1 + \tan x)} \\
 &\equiv \frac{1 + 2\tan x + \tan^2 x - (1 - 2\tan x + \tan^2 x)}{1 - \tan^2 x} \\
 &\equiv \frac{4\tan x}{1 - \tan^2 x} \\
 &\equiv 2 \left( \frac{2\tan x}{1 - \tan^2 x} \right) \\
 &\equiv 2 \tan 2x \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

$$(\text{f}) \text{ R.H.S.} \equiv \cos^2 y - \cos^2 x$$

$$\begin{aligned}
 &\equiv (\cos y + \cos x) (\cos y - \cos x) \\
 &\equiv \left[ 2\cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) \right] \left[ -2\sin\left(\frac{x+y}{2}\right) \right. \\
 &\quad \left. \sin\left(\frac{y-x}{2}\right) \right] \\
 &\equiv \left[ 2\cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) \right] \left[ 2\sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right) \right] \quad [\text{as } \sin(-\theta) = -\sin\theta] \\
 &\equiv \left[ 2\sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x+y}{2}\right) \right] \left[ 2\sin\left(\frac{x-y}{2}\right) \cos\left(\frac{x-y}{2}\right) \right] \\
 &\equiv \sin 2\left(\frac{x+y}{2}\right) \sin 2\left(\frac{x-y}{2}\right) \\
 &\equiv \sin(x+y) \sin(x-y) \\
 &\equiv \text{L.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(g) L.H.S.} &\equiv 1 + 2\cos 2\theta + \cos 4\theta \\
 &\equiv 1 + 2\cos 2\theta + (2\cos^2 2\theta - 1) \\
 &\equiv 2\cos 2\theta + 2\cos^2 2\theta \\
 &\equiv 2\cos 2\theta(1 + \cos 2\theta) \\
 &\equiv 2\cos 2\theta [1 + (2\cos^2 \theta - 1)] \\
 &\equiv 4\cos^2 \theta \cos 2\theta \\
 &\equiv \text{R.H.S.}
 \end{aligned}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 7

#### Question:

The angles  $x$  and  $y$  are acute angles such that  $\sin x = \frac{2}{\sqrt{5}}$  and  $\cos y = \frac{3}{\sqrt{10}}$

(a) Show that  $\cos 2x = -\frac{3}{5}$ .

(b) Find the value of  $\cos 2y$ .

(c) Show without using your calculator, that

(i)  $\tan(x + y) = 7$

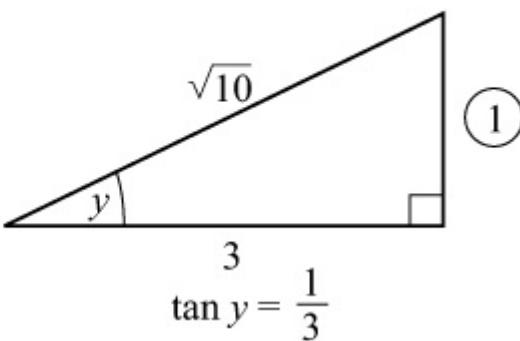
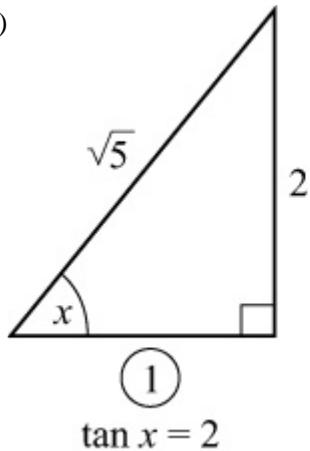
(ii)  $x - y = \frac{\pi}{4}$

#### Solution:

(a)  $\cos 2x \equiv 1 - 2 \sin^2 x = 1 - 2 \left( \frac{2}{\sqrt{5}} \right)^2 = 1 - \frac{8}{5} = -\frac{3}{5}$

(b)  $\cos 2y \equiv 2 \cos^2 y - 1 = 2 \left( \frac{3}{\sqrt{10}} \right)^2 - 1 = 2 \left( \frac{9}{10} \right) - 1 = \frac{4}{5}$

(c)



$$(i) \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{2}{\sqrt{5}} + \frac{3}{\sqrt{10}}}{1 - \frac{2}{\sqrt{5}} \cdot \frac{3}{\sqrt{10}}} = \frac{\frac{2}{\sqrt{5}} + \frac{3}{\sqrt{10}}}{1 - \frac{6}{\sqrt{50}}} = \frac{\frac{2}{\sqrt{5}} + \frac{3}{\sqrt{10}}}{1 - \frac{3}{\sqrt{10}}} = \frac{\frac{2\sqrt{10}}{10} + \frac{3\sqrt{5}}{10}}{1 - \frac{3\sqrt{5}}{10}} = \frac{\frac{2\sqrt{10} + 3\sqrt{5}}{10}}{\frac{10 - 3\sqrt{5}}{10}} = \frac{2\sqrt{10} + 3\sqrt{5}}{10 - 3\sqrt{5}} = \frac{(2\sqrt{10} + 3\sqrt{5})(10 + 3\sqrt{5})}{(10 - 3\sqrt{5})(10 + 3\sqrt{5})} = \frac{20\sqrt{10} + 6\sqrt{50} + 30\sqrt{5} + 9\sqrt{25}}{100 - 90} = \frac{20\sqrt{10} + 30\sqrt{2} + 30\sqrt{5} + 45}{10} = \frac{20\sqrt{10} + 30\sqrt{2} + 30\sqrt{5} + 45}{10} = 7$$

$$(ii) \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} = \frac{\frac{5}{3} - \frac{3}{5}}{1 + \frac{5}{3} \cdot \frac{3}{5}} = 1$$

As  $x$  and  $y$  are acute,  $x - y = \frac{\pi}{4}$  (it cannot be  $\frac{5\pi}{4}$ )

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 8

#### Question:

Given that  $\sin x \cos y = \frac{1}{2}$  and  $\cos x \sin y = \frac{1}{3}$ ,

(a) show that  $\sin(x + y) = 5 \sin(x - y)$ .

Given also that  $\tan y = k$ , express in terms of  $k$ :

(b)  $\tan x$

(c)  $\tan 2x$

#### Solution:

$$(a) \sin(x + y) \equiv \sin x \cos y + \cos x \sin y = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$5 \sin(x - y) \equiv 5(\sin x \cos y - \cos x \sin y) = 5 \left( \frac{1}{2} - \frac{1}{3} \right) = 5 \times \frac{1}{6} = \frac{5}{6}$$

$$(b) \frac{\sin x \cos y}{\cos x \sin y} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}$$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{3}{2}$$

$$\text{so } \tan x = \frac{3}{2} \tan y = \frac{3}{2}k$$

$$(c) \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{3k}{1 - \frac{9}{4}k^2} \quad \left( = \frac{12k}{4 - 9k^2} \right)$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 9**
**Question:**

Solve the following equations in the interval given in brackets:

$$(a) \sqrt{3} \sin 2\theta + 2 \sin^2 \theta = 1 \quad \{ 0 \leq \theta \leq \pi \}$$

$$(b) \sin 3\theta \cos 2\theta = \sin 2\theta \cos 3\theta \quad \{ 0 \leq \theta \leq 2\pi \}$$

$$(c) \sin(\theta + 40^\circ) + \sin(\theta + 50^\circ) = 0 \quad \{ 0 \leq \theta \leq 360^\circ \}$$

$$(d) \sin^2 \frac{\theta}{2} = 2 \sin \theta \quad \{ 0 \leq \theta \leq 360^\circ \}$$

$$(e) 2 \sin \theta = 1 + 3 \cos \theta \quad \{ 0 \leq \theta \leq 360^\circ \}$$

$$(f) \cos 5\theta = \cos 3\theta \quad \{ 0 \leq \theta \leq \pi \}$$

$$(g) \cos 2\theta = 5 \sin \theta \quad \{ -\pi \leq \theta \leq \pi \} .$$

**Solution:**

$$(a) \sqrt{3} \sin 2\theta = 1 - 2 \sin^2 \theta, 0 \leq \theta \leq \pi \\ \Rightarrow \sqrt{3} \sin 2\theta = \cos 2\theta$$

$$\Rightarrow \tan 2\theta = \frac{1}{\sqrt{3}}, 0 \leq 2\theta \leq 2\pi$$

$$\Rightarrow 2\theta = \frac{\pi}{6}, \pi + \frac{\pi}{6} = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$\Rightarrow \theta = \frac{\pi}{12}, \frac{7\pi}{12}$$

$$(b) \sin 3\theta \cos 2\theta - \cos 3\theta \sin 2\theta = 0$$

$$\Rightarrow \sin(3\theta - 2\theta) = 0$$

$$\Rightarrow \sin \theta = 0, 0 \leq \theta \leq 2\pi$$

$$\Rightarrow \theta = 0, \pi, 2\pi$$

$$(c) \sin(\theta + 40^\circ) + \sin(\theta + 50^\circ) = 0, 0 \leq \theta \leq 360^\circ$$

$$\Rightarrow 2 \sin \left[ \frac{(\theta + 40^\circ) + (\theta + 50^\circ)}{2} \right] \cos \left[ \frac{(\theta + 40^\circ) - (\theta + 50^\circ)}{2} \right]$$

$$= 0$$

$$\Rightarrow 2 \sin(\theta + 45^\circ) \cos(-5^\circ) = 0$$

$$\Rightarrow \sin(\theta + 45^\circ) = 0, 45^\circ \leq \theta + 45^\circ \leq 405^\circ$$

$$\Rightarrow \theta + 45^\circ = 180^\circ, 360^\circ$$

$$\Rightarrow \theta = 135^\circ, 315^\circ$$

$$(d) \sin^2 \frac{\theta}{2} = 2 \left( 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right) \quad \left( \sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right)$$

$$\Rightarrow \sin \frac{\theta}{2} \left( \sin \frac{\theta}{2} - 4 \cos \frac{\theta}{2} \right) = 0$$

$$\Rightarrow \sin \frac{\theta}{2} = 0 \text{ or } \sin \frac{\theta}{2} = 4 \cos \frac{\theta}{2}, \text{i.e. } \tan \frac{\theta}{2} = 4$$

$$\text{For } \sin \frac{\theta}{2} = 0 \Rightarrow \frac{\theta}{2} = 0^\circ, 180^\circ \Rightarrow \theta = 0^\circ, 360^\circ$$

$$\text{For } \tan \frac{\theta}{2} = 4 \Rightarrow \frac{\theta}{2} = \tan^{-1} 4 = 75.96^\circ \Rightarrow \theta = 151.9^\circ$$

Solution set:  $0^\circ, 151.9^\circ, 360^\circ$

$$(e) 2 \sin \theta - 3 \cos \theta = 1$$

$$\text{Let } 2 \sin \theta - 3 \cos \theta \equiv R \sin(\theta - \alpha) \equiv R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$$

$$\Rightarrow R \cos \alpha = 2 \text{ and } R \sin \alpha = 3$$

$$\Rightarrow \tan \alpha = \frac{3}{2} (\Rightarrow \alpha = 56.3^\circ), R = \sqrt{13}$$

$$\Rightarrow \sqrt{13} \sin(\theta - 56.3^\circ) = 1$$

$$\Rightarrow \sin(\theta - 56.3^\circ) = \frac{1}{\sqrt{13}}$$

$$\Rightarrow \theta - 56.3^\circ = \sin^{-1} \frac{1}{\sqrt{13}}, 180^\circ - \sin^{-1} \frac{1}{\sqrt{13}} = 16.1^\circ, 163.9^\circ$$

$$\Rightarrow \theta = 72.4^\circ, 220.2^\circ$$

$$(f) \cos 5\theta - \cos 3\theta = 0$$

$$\Rightarrow -2 \sin \left( \frac{5\theta + 3\theta}{2} \right) \sin \left( \frac{5\theta - 3\theta}{2} \right) = 0$$

$$\Rightarrow \sin 4\theta \sin \theta = 0, 0 \leq \theta \leq \pi$$

$$\Rightarrow \sin \theta = 0 \Rightarrow \theta = 0, \pi$$

$$\text{or } \sin 4\theta = 0 \Rightarrow 4\theta = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$\Rightarrow \theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$$

Solution set:  $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

(g)  $\cos 2\theta = 5 \sin \theta$

$$\Rightarrow 1 - 2 \sin^2 \theta = 5 \sin \theta$$

$$\Rightarrow 2 \sin^2 \theta + 5 \sin \theta - 1 = 0$$

$$\Rightarrow \sin \theta = \frac{-5 \pm \sqrt{33}}{4}$$

$$\text{As } -1 \leq \sin \theta \leq 1, \sin \theta = \frac{-5 + \sqrt{33}}{4}$$

In radian mode:  $\theta = 0.187, \pi - 0.187 = 0.187, 2.95$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 10

#### Question:

The first three terms of an arithmetic series are  $\sqrt{3}\cos\theta$ ,  $\sin(\theta - 30^\circ)$  and  $\sin\theta$ , where  $\theta$  is acute. Find the value of  $\theta$ .

#### Solution:

As the three values are consecutive terms of an arithmetic progression,  
 $\sin(\theta - 30^\circ) - \sqrt{3}\cos\theta = \sin\theta - \sin(\theta - 30^\circ)$

$$\Rightarrow 2\sin(\theta - 30^\circ) = \sin\theta + \sqrt{3}\cos\theta$$

$$\Rightarrow 2(\sin\theta\cos 30^\circ - \cos\theta\sin 30^\circ) = \sin\theta + \sqrt{3}\cos\theta$$

$$\Rightarrow \sqrt{3}\sin\theta - \cos\theta = \sin\theta + \sqrt{3}\cos\theta$$

$$\Rightarrow \sin\theta(\sqrt{3} - 1) = \cos\theta(\sqrt{3} + 1)$$

$$\Rightarrow \tan\theta = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Calculator value is  $\theta = \tan^{-1} \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = 75^\circ$

No other values as  $\theta$  is acute.

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise F, Question 11

**Question:**

Solve, for  $0^\circ \leq \theta \leq 360^\circ$ ,  $\cos(\theta + 40^\circ) \cos(\theta - 10^\circ) = 0.5$ .

**Solution:**

$$\begin{aligned}2\cos(\theta + 40^\circ)\cos(\theta - 10^\circ) &= 1 \\ \Rightarrow \cos\left[\left(\theta + 40^\circ\right) + \left(\theta - 10^\circ\right)\right] + \cos\left[\left(\theta + 40^\circ\right) \\ - \left(\theta - 10^\circ\right)\right] &= 1 \\ \Rightarrow \cos(2\theta + 30^\circ) + \cos 50^\circ &= 1 \\ \Rightarrow \cos(2\theta + 30^\circ) &= 1 - \cos 50^\circ = 0.3572 \\ \Rightarrow 2\theta + 30^\circ &= 69.07^\circ, 290.9^\circ, 429.07^\circ, 650.9^\circ \\ \Rightarrow 2\theta &= 39.07^\circ, 260.9^\circ, 399.07^\circ, 620.9^\circ \\ \Rightarrow \theta &= 19.5^\circ, 130.5^\circ, 199.5^\circ, 310.5^\circ\end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 12

#### Question:

Without using calculus, find the maximum and minimum value of the following expressions. In each case give the smallest positive value of  $\theta$  at which each occurs.

(a)  $\sin \theta \cos 10^\circ - \cos \theta \sin 10^\circ$

(b)  $\cos 30^\circ \cos \theta - \sin 30^\circ \sin \theta$

(c)  $\sin \theta + \cos \theta$

#### Solution:

(a)  $\sin \theta \cos 10^\circ - \cos \theta \sin 10^\circ = \sin(\theta - 10^\circ)$  [  $\sin(A - B)$  ]

Maximum value = +1 when  $\theta - 10^\circ = 90^\circ \Rightarrow \theta = 100^\circ$

Minimum value = -1 when  $\theta - 10^\circ = 270^\circ \Rightarrow \theta = 280^\circ$

(b)  $\cos 30^\circ \cos \theta - \sin 30^\circ \sin \theta = \cos(\theta + 30^\circ)$

Maximum value = +1 when  $\theta + 30^\circ = 360^\circ \Rightarrow \theta = 330^\circ$

Minimum value = -1 when  $\theta + 30^\circ = 180^\circ \Rightarrow \theta = 150^\circ$

(c)  $\sin \theta + \cos \theta$

$$= \sqrt{2} \left( \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta \right)$$

$$= \sqrt{2} (\sin \theta \cos 45^\circ + \cos \theta \sin 45^\circ)$$

$$= \sqrt{2} \sin(\theta + 45^\circ)$$

Maximum value =  $\sqrt{2}$  when  $\theta + 45^\circ = 90^\circ \Rightarrow \theta = 45^\circ$

Minimum value = - $\sqrt{2}$  when  $\theta + 45^\circ = 270^\circ \Rightarrow \theta = 225^\circ$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise F, Question 13

**Question:**

(a) Express  $\sin x - \sqrt{3} \cos x$  in the form  $R \sin(x - \alpha)$ , with  $R > 0$  and  $0 < \alpha < 90^\circ$ .

(b) Hence sketch the graph of  $y = \sin x - \sqrt{3} \cos x$  { $-360^\circ \leq x \leq 360^\circ$ }, giving the coordinates of all points of intersection with the axes.

**Solution:**

(a) Let  $\sin x - \sqrt{3} \cos x \equiv R \sin(x - \alpha) \equiv R \sin x \cos \alpha - R \cos x \sin \alpha$   
 $R > 0, 0 < \alpha < 90^\circ$

$$\text{Compare } \sin x : R \cos \alpha = 1 \quad \textcircled{1}$$

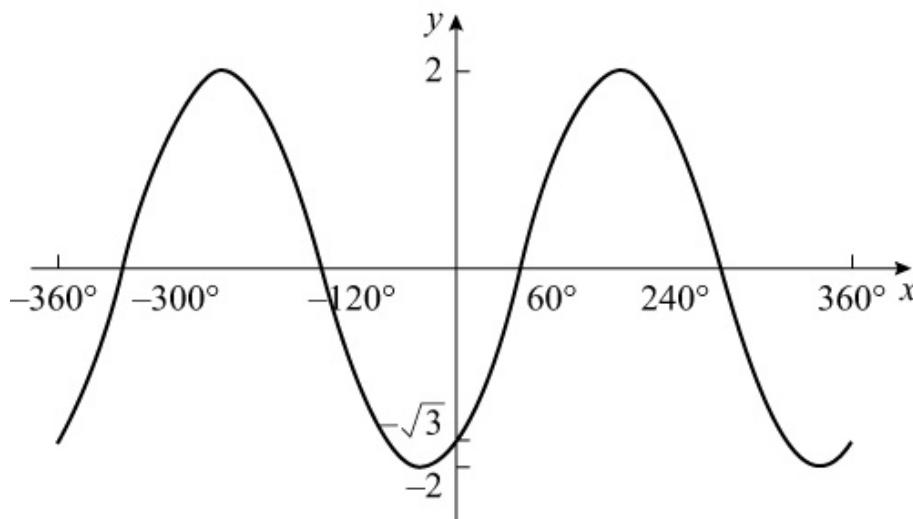
$$\text{Compare } \cos x : R \sin \alpha = \sqrt{3} \quad \textcircled{2}$$

$$\text{Divide } \textcircled{2} \text{ by } \textcircled{1} : \tan \alpha = \sqrt{3} \Rightarrow \alpha = 60^\circ$$

$$R^2 = (\sqrt{3})^2 + 1^2 = 4 \Rightarrow R = 2$$

$$\text{So } \sin x - \sqrt{3} \cos x \equiv 2 \sin(x - 60^\circ)$$

(b) Sketch  $y = 2 \sin(x - 60^\circ)$  by first translating  $y = \sin x$  by  $60^\circ$  to the right and then stretching the result in the  $y$  direction by scale factor 2.



Graph meets  $y$ -axis when  $x = 0$ , i.e.  $y = 2 \sin(-60^\circ) = -\sqrt{3}$

Graph meets  $x$ -axis when  $y = 0$ , i.e.  $(-300^\circ, 0), (-120^\circ, 0), (60^\circ, 0), (240^\circ, 0)$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 14

#### Question:

Given that  $7\cos 2\theta + 24\sin 2\theta \equiv R\cos(2\theta - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ , find:

(a) the value of  $R$  and the value of  $\alpha$ , to 2 decimal places

(b) the maximum value of  $14\cos^2\theta + 48\sin\theta\cos\theta$

#### Solution:

(a) Let  $7\cos 2\theta + 24\sin 2\theta \equiv R\cos(2\theta - \alpha) \equiv R\cos 2\theta \cos \alpha + R\sin 2\theta \sin \alpha$   
 $R > 0, 0 < \alpha < \frac{\pi}{2}$

$$\text{Compare } \cos 2\theta : \quad R\cos \alpha = 7 \quad \textcircled{1}$$

$$\text{Compare } \sin 2\theta : \quad R\sin \alpha = 24 \quad \textcircled{2}$$

$$\text{Divide } \textcircled{2} \text{ by } \textcircled{1}: \quad \tan \alpha = \frac{24}{7} \Rightarrow \alpha = 1.29 \quad (1.287)$$

$$R^2 = 24^2 + 7^2 \Rightarrow R = 25$$

$$\text{So } 7\cos 2\theta + 24\sin 2\theta \equiv 25\cos(2\theta - 1.29)$$

$$\begin{aligned} (b) & 14\cos^2\theta + 48\sin\theta\cos\theta \\ & \equiv 14 \left( \frac{1 + \cos 2\theta}{2} \right) + 24(2\sin\theta\cos\theta) \\ & \equiv 7(1 + \cos 2\theta) + 24\sin 2\theta \\ & \equiv 7 + 7\cos 2\theta + 24\sin 2\theta \end{aligned}$$

The maximum value of  $7\cos 2\theta + 24\sin 2\theta$  is 25 [using (a) with  $\cos(2\theta - 1.29) = 1$ ].

So maximum value of  $7 + 7\cos 2\theta + 24\sin 2\theta = 7 + 25 = 32$ .

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise F, Question 15

**Question:**

(a) Given that  $\alpha$  is acute and  $\tan \alpha = \frac{3}{4}$ , prove that

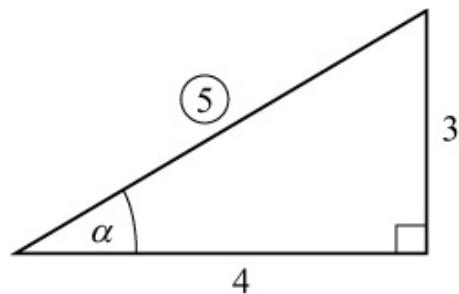
$$3 \sin (\theta + \alpha) + 4 \cos (\theta + \alpha) \equiv 5 \cos \theta$$

(b) Given that  $\sin x = 0.6$  and  $\cos x = -0.8$ , evaluate  $\cos(x + 270)^\circ$  and  $\cos(x + 540)^\circ$ .

[E]

**Solution:**

(a) Draw a right-angled triangle and find  $\sin \alpha$  and  $\cos \alpha$ .



$$\Rightarrow \sin \alpha = \frac{3}{5}, \cos \alpha = \frac{4}{5}$$

$$\begin{aligned} \text{So } & 3 \sin(\theta + \alpha) + 4 \cos(\theta + \alpha) \\ & \equiv 3(\sin \theta \cos \alpha + \cos \theta \sin \alpha) + 4(\cos \theta \cos \alpha - \sin \theta \sin \alpha) \\ & \equiv 3\left(\frac{4}{5} \sin \theta + \frac{3}{5} \cos \theta\right) + 4\left(\frac{4}{5} \cos \theta - \frac{3}{5} \sin \theta\right) \\ & \equiv \frac{12}{5} \sin \theta + \frac{9}{5} \cos \theta + \frac{16}{5} \cos \theta - \frac{12}{5} \sin \theta \\ & \equiv \frac{25}{5} \cos \theta \\ & \equiv 5 \cos \theta \end{aligned}$$

$$\begin{aligned} \text{(b) } & \cos(x + 270)^\circ \equiv \cos x^\circ \cos 270^\circ - \sin x^\circ \sin 270^\circ \\ & \quad = (-0.8)(0) - (0.6)(-1) = 0 + 0.6 = 0.6 \\ \cos(x + 540)^\circ & \equiv \cos x^\circ \cos 540^\circ - \sin x^\circ \sin 540^\circ \\ & \quad = (-0.8)(-1) - (0.6)(0) = 0.8 - 0 = 0.8 \end{aligned}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 16

#### Question:

(a) Without using a calculator, find the values of:

$$(i) \sin 40^\circ \cos 10^\circ - \cos 40^\circ \sin 10^\circ$$

$$(ii) \frac{1}{\sqrt{2}} \cos 15^\circ - \frac{1}{\sqrt{2}} \sin 15^\circ$$

$$(iii) \frac{1 - \tan 15^\circ}{1 + \tan 15^\circ}$$

(b) Find, to 1 decimal place, the values of  $x$ ,  $0^\circ \leq x \leq 360^\circ$ , which satisfy the equation  $2 \sin x = \cos(x - 60)$

[E]

#### Solution:

$$(a) (i) \sin 40^\circ \cos 10^\circ - \cos 40^\circ \sin 10^\circ = \sin(40^\circ - 10^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$(ii) \frac{1}{\sqrt{2}} \cos 15^\circ - \frac{1}{\sqrt{2}} \sin 15^\circ \\ = \cos 45^\circ \cos 15^\circ - \sin 45^\circ \sin 15^\circ = \cos(45^\circ + 15^\circ) = \cos 60^\circ \\ = \frac{1}{2}$$

$$(iii) \frac{1 - \tan 15^\circ}{1 + \tan 15^\circ} = \frac{\tan 45^\circ - \tan 15^\circ}{1 + \tan 45^\circ \tan 15^\circ} \\ = \tan(45^\circ - 15^\circ) = \tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$(b) 2 \sin x = \cos(x - 60^\circ)$$

$$\Rightarrow 2 \sin x = \cos x \cos 60^\circ + \sin x \sin 60^\circ$$

$$\Rightarrow 2 \sin x = \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$$

$$\Rightarrow (4 - \sqrt{3}) \sin x = \cos x$$

$$\Rightarrow \frac{\sin x}{\cos x} = \frac{1}{4 - \sqrt{3}}$$

$$\Rightarrow \tan x = \frac{1}{4 - \sqrt{3}}$$

$$\text{So } x = \tan^{-1} \left( \frac{1}{4 - \sqrt{3}} \right), 180^\circ + \tan^{-1} \left( \frac{1}{4 - \sqrt{3}} \right)$$
$$\Rightarrow x = 23.8^\circ, 203.8^\circ$$

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# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 17

#### Question:

(a) Prove, by counter example, that the statement  
 $\sec(A + B) \equiv \sec A + \sec B$ , for all  $A$  and  $B$   
is false.

(b) Prove that  $\tan \theta + \cot \theta \equiv 2 \operatorname{cosec} 2\theta$ ,  $\theta \neq \frac{n\pi}{2}$ ,  $n \in \mathbb{Z}$ .

[E]

#### Solution:

(a) One example is sufficient to disprove a statement.

E.g.  $A = 60^\circ$ ,  $B = 0^\circ$

$$\sec(A + B) = \sec(60^\circ + 0^\circ) = \sec 60^\circ = \frac{1}{\cos 60^\circ} = 2$$

$$\sec A = \sec 60^\circ = \frac{1}{\cos 60^\circ} = 2$$

$$\sec B = \sec 0^\circ = \frac{1}{\cos 0^\circ} = 1$$

So  $\sec(60^\circ + 0^\circ) \neq \sec 60^\circ + \sec 0^\circ$

$\Rightarrow \sin(A + B) \equiv \sec A + \sec B$  not true for all values of  $A, B$ .

(b) L.H.S.  $\equiv \tan \theta + \cot \theta$

$$\equiv \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\equiv \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\equiv \frac{1}{\frac{1}{2} \sin 2\theta} \quad (\sin^2 \theta + \cos^2 \theta \equiv 1, \sin 2\theta \equiv 2 \sin \theta \cos \theta)$$

$$\equiv \frac{2}{\sin 2\theta}$$

$$\equiv 2 \operatorname{cosec} 2\theta$$

$$\equiv \text{R.H.S.}$$



# Solutionbank

## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 18

#### Question:

Using the formula  $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$ :

- Show that  $\cos(A - B) - \cos(A + B) \equiv 2 \sin A \sin B$ .
- Hence show that  $\cos 2x - \cos 4x \equiv 2 \sin 3x \sin x$ .
- Find all solutions in the range  $0 \leq x \leq \pi$  of the equation  $\cos 2x - \cos 4x = \sin x$  giving all your solutions in multiples of  $\pi$  radians.

[E]

#### Solution:

$$(a) \cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

$$\Rightarrow \cos(A - B) \equiv \cos A \cos(-B) - \sin A \sin(-B)$$

$$\equiv \cos A \cos B + \sin A \sin B$$

$$\text{so } \cos(A + B) - \cos(A - B) \equiv (\cos A \cos B - \sin A \sin B) - (\cos A \cos B + \sin A \sin B) \\ \equiv -2 \sin A \sin B$$

$$(b) \text{Let } A + B = 2x, A - B = 4x$$

$$\text{Add: } 2A = 6x \Rightarrow A = 3x$$

$$\text{Subtract: } 2B = -2x \Rightarrow B = -x$$

$$\text{Using (a) } \cos 2x - \cos 4x \equiv -2 \sin 3x \sin(-x) \equiv 2 \sin 3x \sin x \\ \text{as } \sin(-x) = -\sin x$$

$$(c) \text{Solve } 2 \sin 3x \sin x = \sin x$$

$$\Rightarrow \sin x (2 \sin 3x - 1) = 0$$

$$\Rightarrow \sin x = 0 \text{ or } \sin 3x = \frac{1}{2}$$

$$\sin x = 0 \Rightarrow x = 0, \pi$$

$$\sin 3x = \frac{1}{2}, 0 \leq 3x \leq 3\pi \Rightarrow 3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$$

Solution set:  $0, \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \pi$

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## Edexcel AS and A Level Modular Mathematics

### Exercise F, Question 19

#### Question:

(a) Given that  $\cos(x + 30^\circ) = 3\cos(x - 30^\circ)$ , prove that  $\tan x = -\frac{\sqrt{3}}{2}$ .

(b) (i) Prove that  $\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \theta$ .

(ii) Verify that  $\theta = 180^\circ$  is a solution of the equation  $\sin 2\theta = 2 - 2\cos 2\theta$ .

(iii) Using the result in part (i), or otherwise, find the two other solutions,  $0 < \theta < 360^\circ$ , of the equation  $\sin 2\theta = 2 - 2\cos 2\theta$ .

[E]

#### Solution:

$$(a) \cos(x + 30^\circ) = 3\cos(x - 30^\circ)$$

$$\Rightarrow \cos x \cos 30^\circ - \sin x \sin 30^\circ = 3(\cos x \cos 30^\circ + \sin x \sin 30^\circ)$$

$$\Rightarrow -2\cos x \cos 30^\circ = 4\sin x \sin 30^\circ$$

$$\Rightarrow -2\cos x \times \frac{\sqrt{3}}{2} = 4\sin x \times \frac{1}{2}$$

$$\Rightarrow -\frac{\sqrt{3}}{2} = \frac{\sin x}{\cos x}$$

$$\Rightarrow \tan x = -\frac{\sqrt{3}}{2}$$

$$(b) (i) L.H.S. \equiv \frac{1 - \cos 2\theta}{\sin 2\theta}$$

$$\equiv \frac{1 - (1 - 2\sin^2 \theta)}{2\sin \theta \cos \theta}$$

$$\equiv \frac{2\sin^2 \theta}{2\sin \theta \cos \theta}$$

$$\equiv \frac{\sin \theta}{\cos \theta}$$

$$\equiv \tan \theta$$

$$(ii) L.H.S. = \sin 360^\circ = 0$$

$$R.H.S. = 2 - 2\cos 360^\circ = 2 - 2(1) = 0 \checkmark$$

(iii) Using (i) this is equivalent to solving  $\tan \theta = \frac{1}{2}$ .

From (i)  $1 - \cos 2\theta = \sin 2\theta \tan \theta$

So  $\sin 2\theta = 2 - 2 \cos 2\theta \Rightarrow \sin 2\theta = 2 \sin 2\theta \tan \theta$

$\sin 2\theta = 0$  gives  $\theta = 180^\circ$ , so  $\tan \theta = \frac{1}{2} \Rightarrow \theta = 26.6^\circ, 206.6^\circ$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

Exercise F, Question 20

**Question:**

(a) Express  $1.5 \sin 2x + 2 \cos 2x$  in the form  $R \sin(2x + \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ , giving your values of  $R$  and  $\alpha$  to 3 decimal places where appropriate.

(b) Express  $3 \sin x \cos x + 4 \cos^2 x$  in the form  $a \sin 2x + b \cos 2x + c$ , where  $a$ ,  $b$  and  $c$  are constants to be found.

(c) Hence, using your answer to part (a), deduce the maximum value of  $3 \sin x \cos x + 4 \cos^2 x$ .

[E]

**Solution:**

(a) Let  $1.5 \sin 2x + 2 \cos 2x \equiv R \sin(2x + \alpha) \equiv R \sin 2x \cos \alpha + R \cos 2x \sin \alpha$

$$R > 0, 0 < \alpha < \frac{\pi}{2}$$

$$\text{Compare } \sin 2x : R \cos \alpha = 1.5 \quad ①$$

$$\text{Compare } \cos 2x : R \sin \alpha = 2 \quad ②$$

$$\text{Divide } ② \text{ by } ① : \tan \alpha = \frac{4}{3} \Rightarrow \alpha = 0.927$$

$$R^2 = 2^2 + 1.5^2 \Rightarrow R = 2.5$$

$$\begin{aligned} (b) 3 \sin x \cos x + 4 \cos^2 x &\equiv \frac{3}{2} (2 \sin x \cos x) + 4 \left( \frac{1 + \cos 2x}{2} \right) \\ &\equiv \frac{3}{2} \sin 2x + 2 + 2 \cos 2x \equiv \frac{3}{2} \sin 2x + 2 \cos 2x + 2 \end{aligned}$$

$$(c) \text{From part (a)} \frac{3}{2} \sin 2x + 2 \cos 2x \equiv 2.5 \sin(2x + 0.927)$$

$$\text{So maximum value of } \frac{3}{2} \sin 2x + 2 \cos 2x = 2.5 \times 1 = 2.5$$

$$\text{So maximum value of } 3 \sin x \cos x + 4 \cos^2 x = 2.5 + 2 = 4.5$$