Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Sketch the graph of each of the following. In each case, write down the coordinates of any points at which the graph meets the coordinate axes.

(a) $y = |x - 1|$

(b) $y = |2x + 3|$

(c) $y = \left|\frac{1}{2}x - 5\right|$

(d) $y = |7 - x|$

(e) $y = |x^2 - 7x - 8|$

(f) $y = |x^2 - 9|$

(g) $y = |x^3 + 1|$

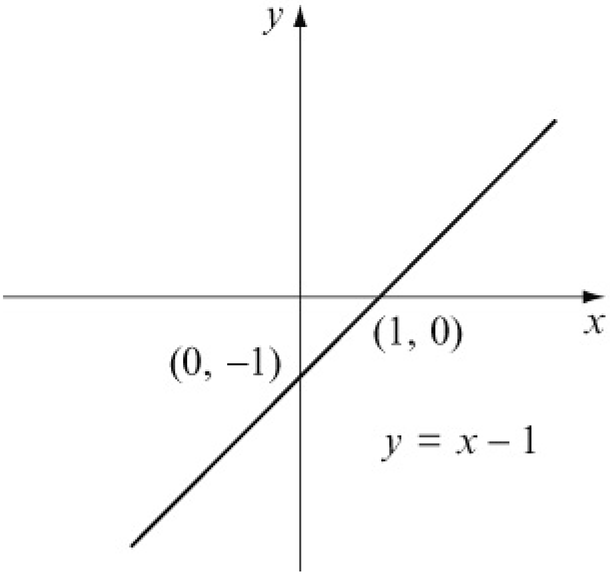
(h) $y = \left|\frac{12}{x}\right|$

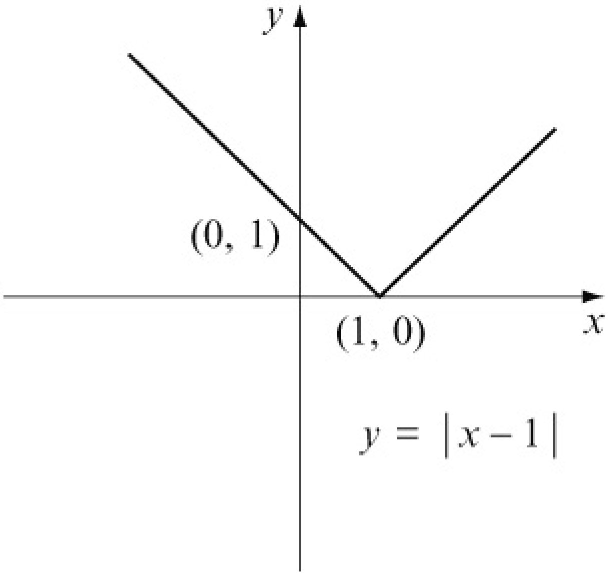
(i) $y = -|x|$

(j) $y = -|3x - 1|$

Solution:

(a)



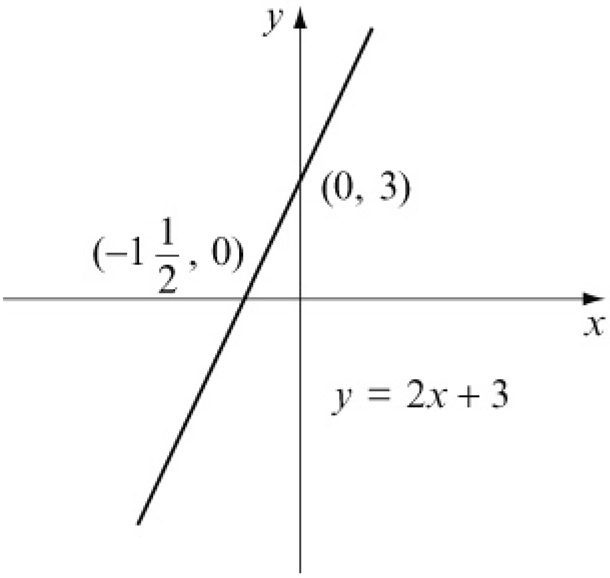


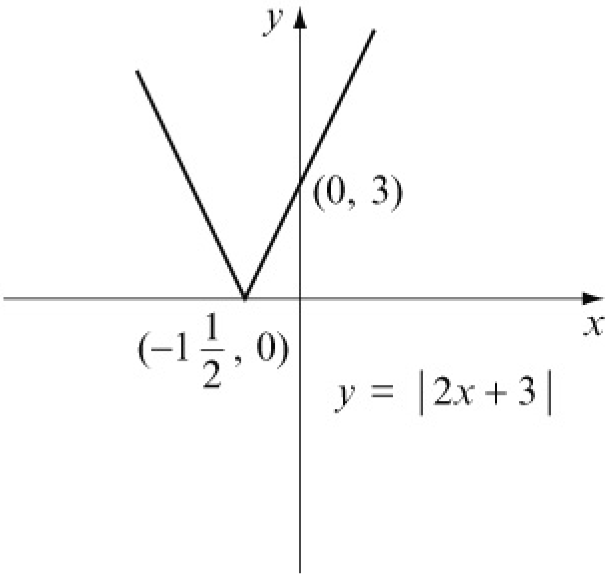
For $y = |x-1|$:

When $x = 0$, $y = |-1| = 1$ $(0, 1)$

When $y = 0$, $x - 1 = 0 \Rightarrow x = 1$ $(1, 0)$

(b)



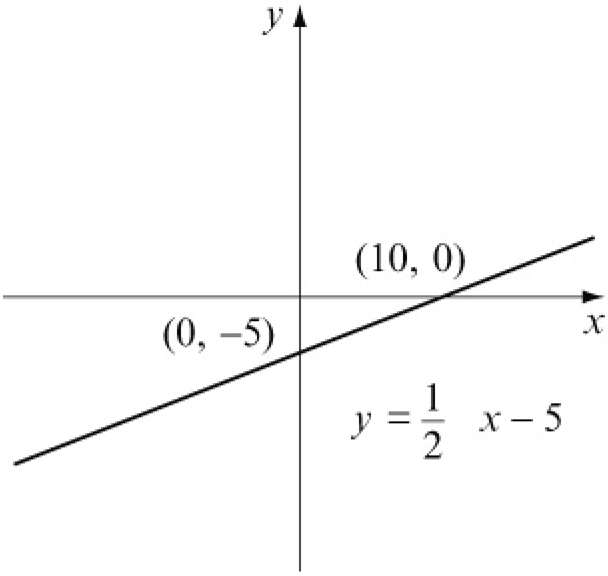


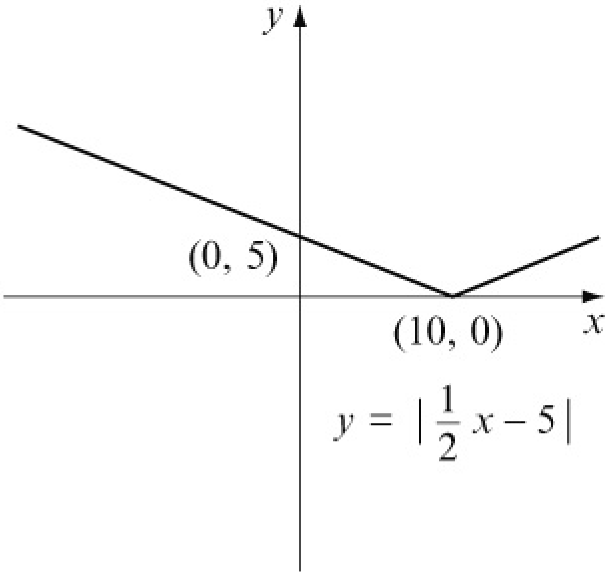
For $y = |2x+3|$:

When $x = 0$, $y = |3| = 3$ $(0, 3)$

When $y = 0$, $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$ $\left(-1\frac{1}{2}, 0\right)$

(c)



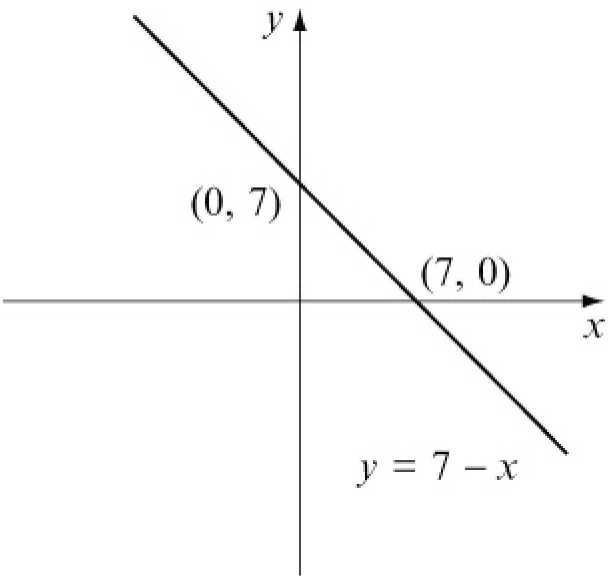


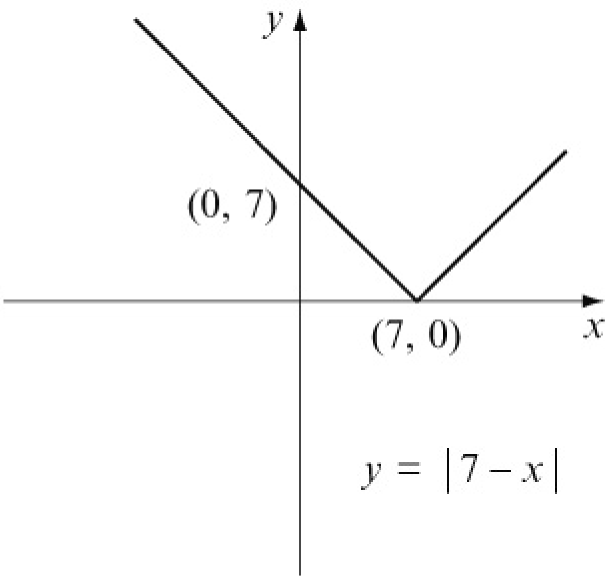
For $y = \left| \frac{1}{2}x - 5 \right|$:

When $x = 0$, $y = | -5 | = 5$ $\quad (0, 5)$

When $y = 0$, $\frac{1}{2}x - 5 = 0 \Rightarrow x = 10$ $\quad (10, 0)$

(d)





For $y = | 7 - x |$:

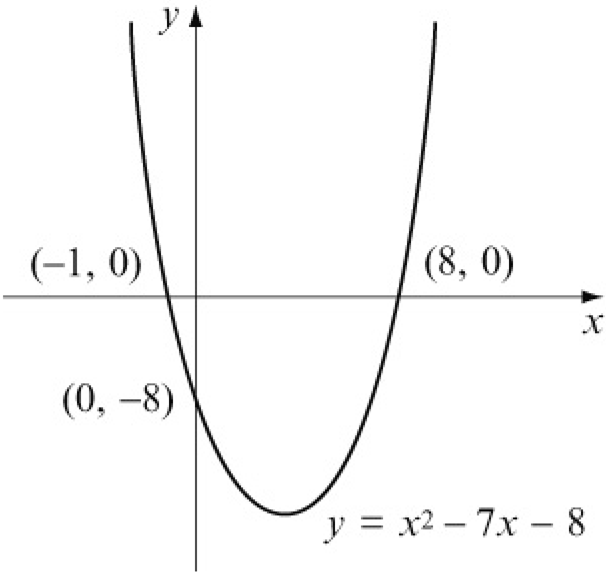
When $x = 0$, $y = | 7 | = 7$ $\quad (0, 7)$

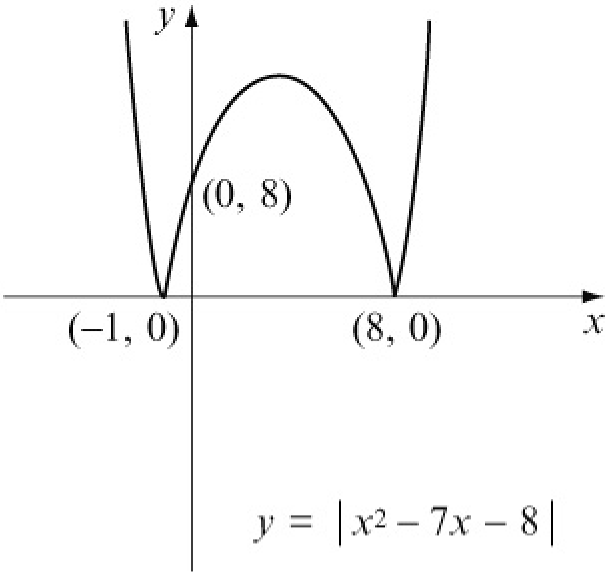
When $y = 0$, $7 - x = 0 \Rightarrow x = 7$ $\quad (7, 0)$

(e) $x^2 - 7x - 8 = (x + 1)(x - 8)$

When $y = 0$, $(x + 1)(x - 8) = 0 \Rightarrow x = -1$ and $x = 8$

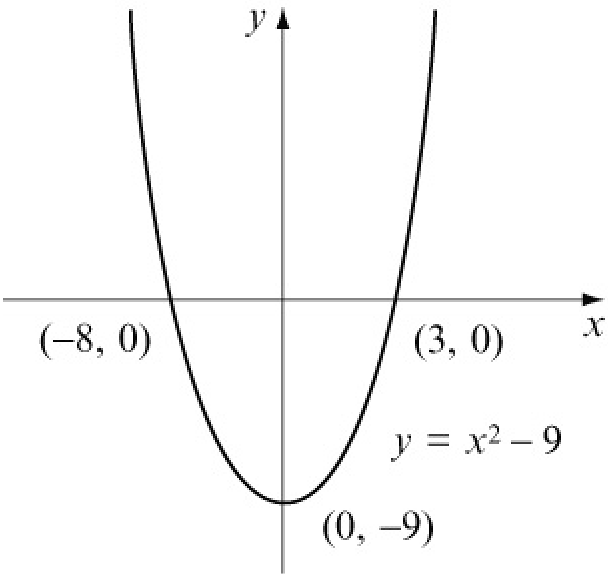
Curve crosses x-axis at $(-1, 0)$ and $(8, 0)$

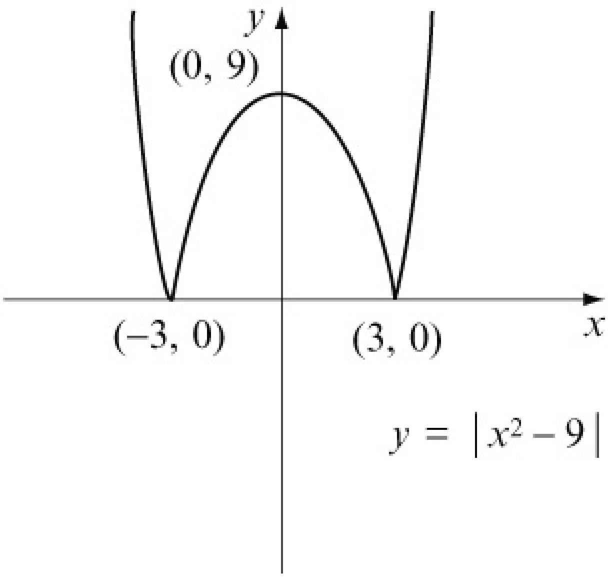




For $y = \left| x^2 - 7x - 8 \right|$:
When $x = 0$, $y = \left| -8 \right| = 8$ $\left(0 , 8 \right)$

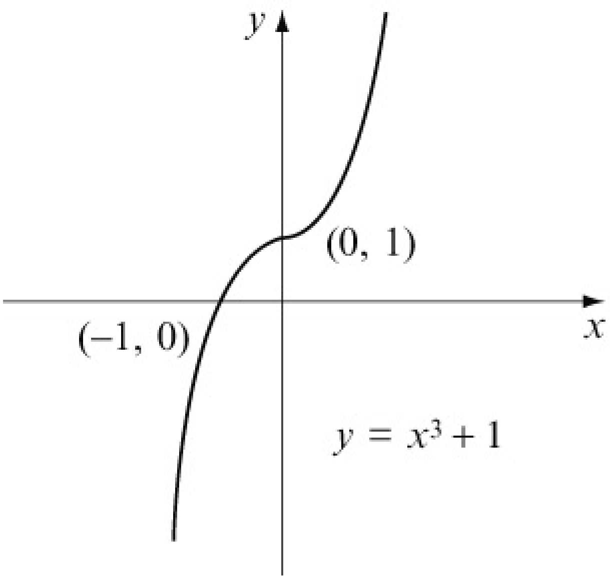
(f) $x^2 - 9 = \left(x + 3 \right) \left(x - 3 \right)$
When $y = 0$, $\left(x + 3 \right) \left(x - 3 \right) = 0 \Rightarrow x = -3$ and $x = 3$
Curve crosses x-axis at $\left(-3 , 0 \right)$ and (3, 0)

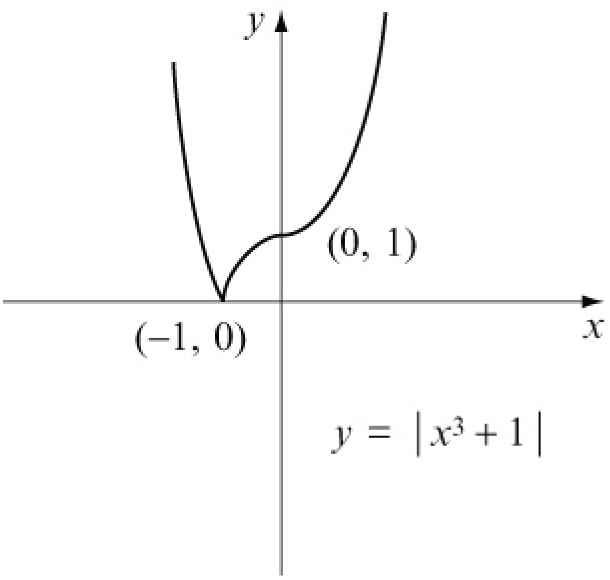




For $y = \left| x^2 - 9 \right|$:
When $x = 0$, $y = \left| -9 \right| = 9$ $\left(0 , 9 \right)$

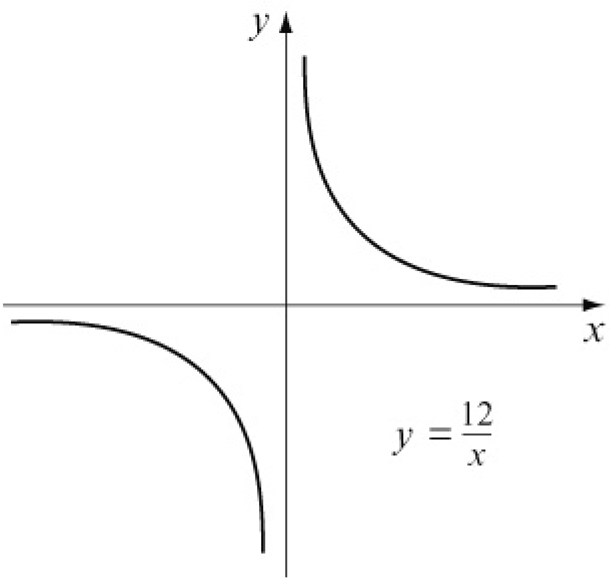
(g) The graph of $y = x^3 + 1$ is found by translating $y = x^3$ by +1 parallel to the y-axis.

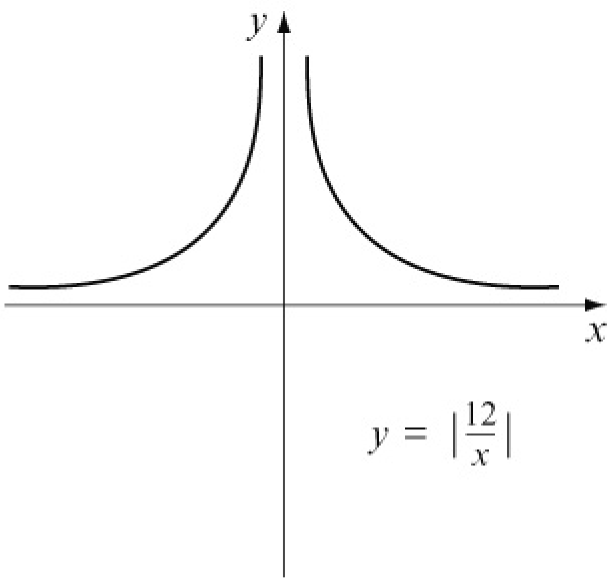




For $y = |x^3 + 1|$:
When $x = 0$, $y = |1| = 1$ $(0, 1)$
When $y = 0$, $x^3 + 1 = 0 \Rightarrow x^3 = -1 \Rightarrow x = -1$ $(-1, 0)$

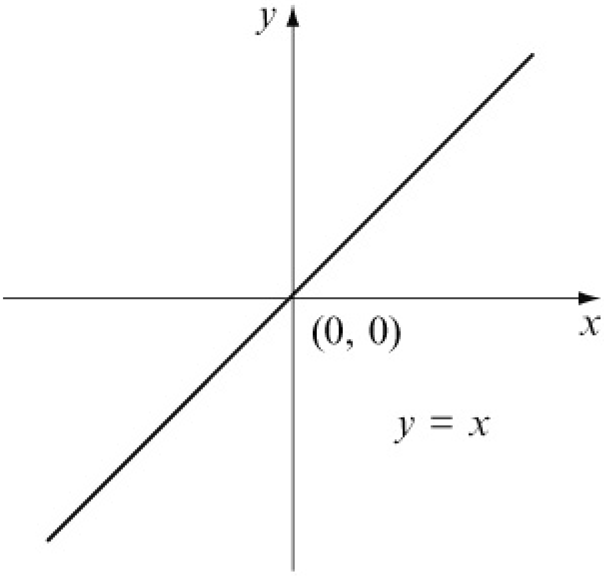
(h)

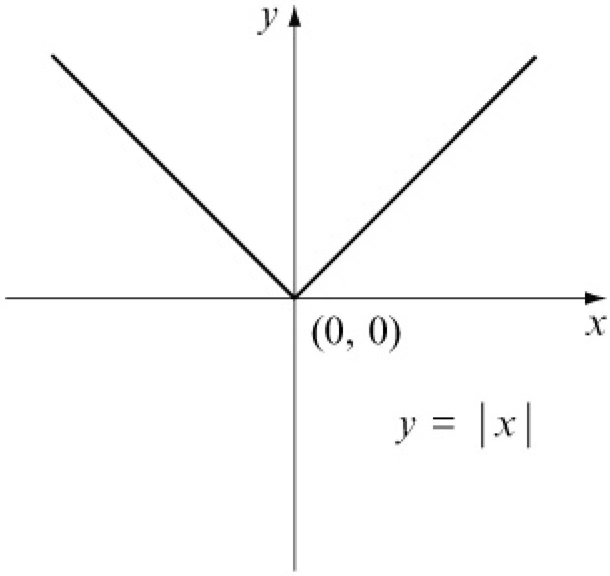


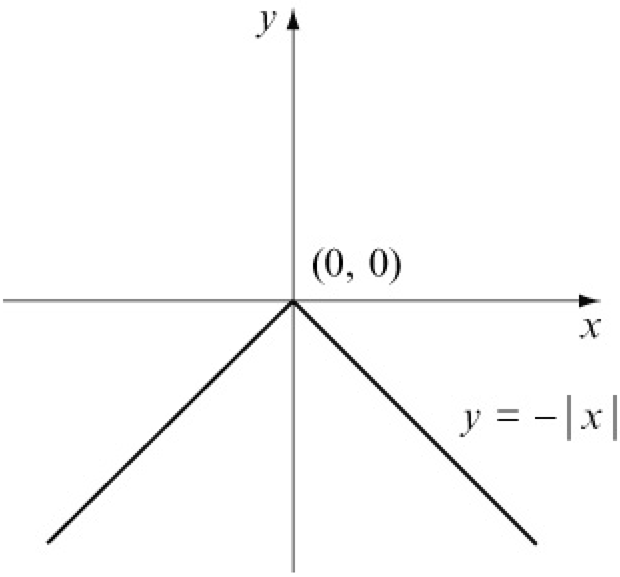


No intersections with the axes (the axes are asymptotes).

(i)

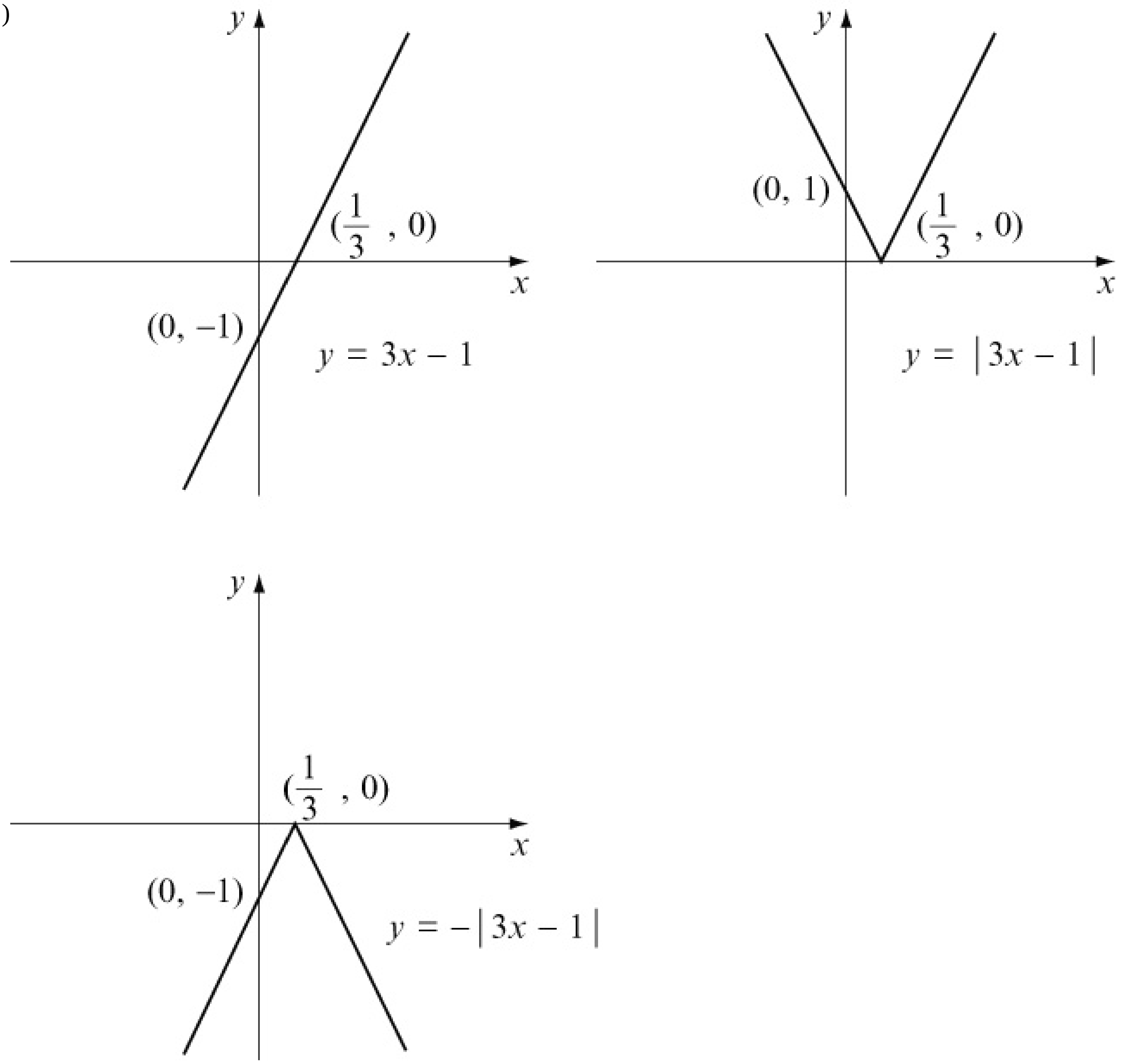






Passes through the origin (0, 0)

(j)



For $y = -|3x - 1|$:
When $x = 0$, $y = -|-1| = -1$ $(0, -1)$
When $y = 0$, $3x - 1 = 0 \Rightarrow x = \frac{1}{3}$ $\left(\frac{1}{3}, 0\right)$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

Sketch the graph of each of the following. In each case, write down the coordinates of any points at which the graph meets the coordinate axes.

(a) $y = |\cos x|, 0 \leq x \leq 2\pi$

(b) $y = |\ln x|, x > 0$

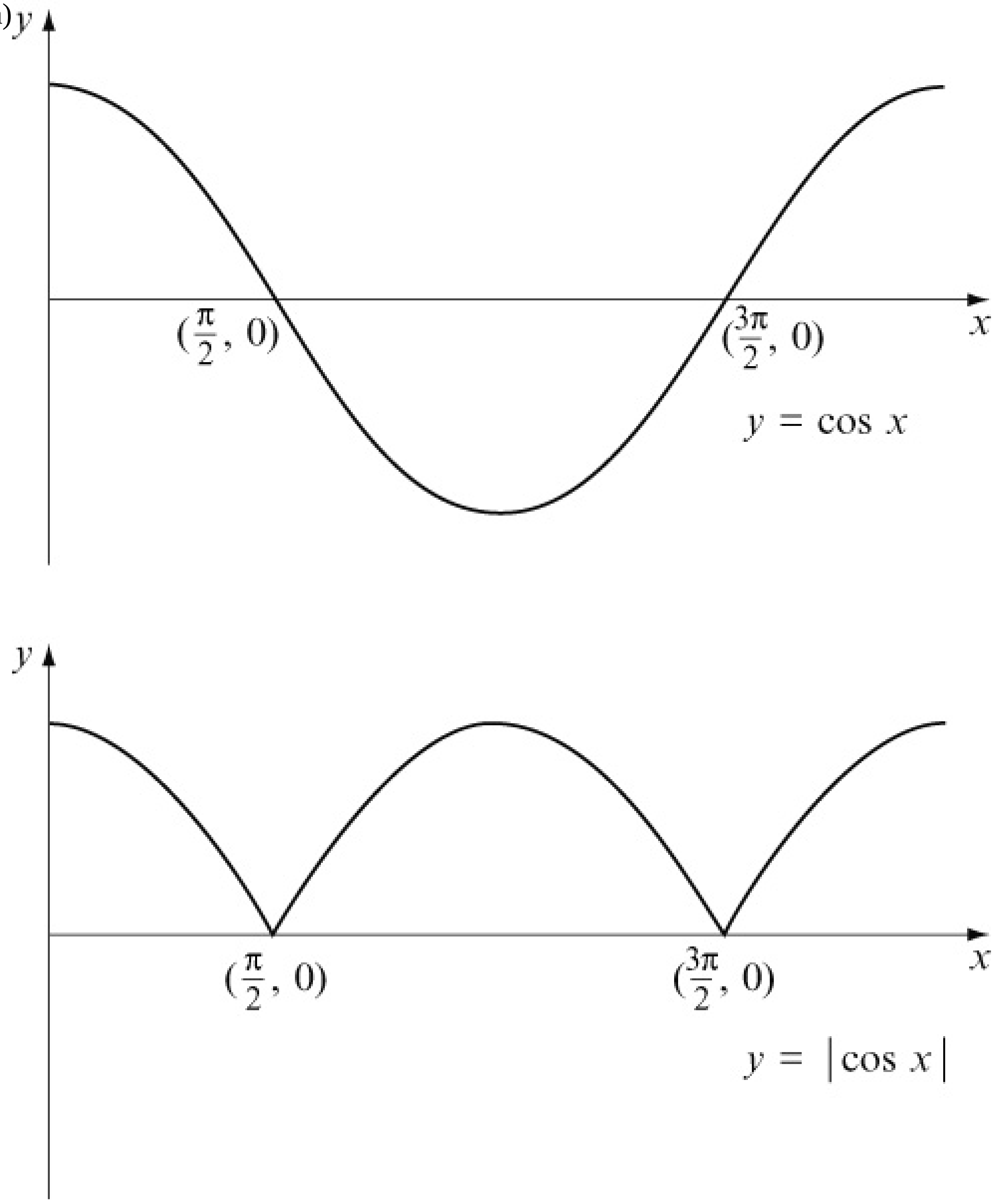
(c) $y = |2^x - 2|$

(d) $y = |100 - 10^x|$

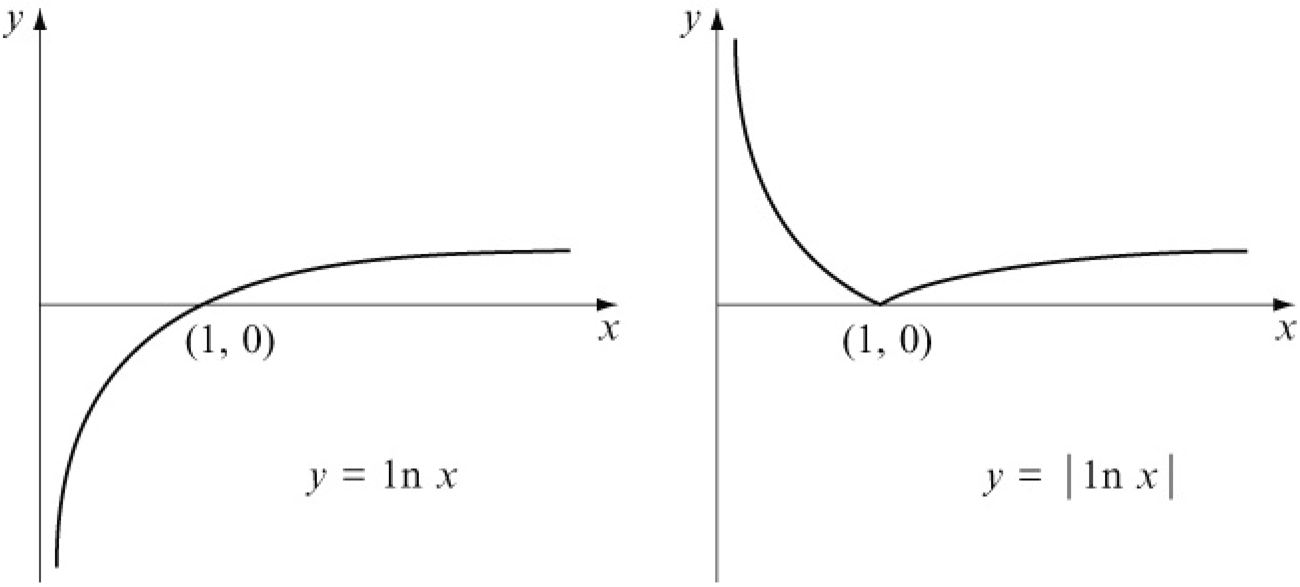
(e) $y = |\tan 2x|, 0 < x < 2\pi$

Solution:

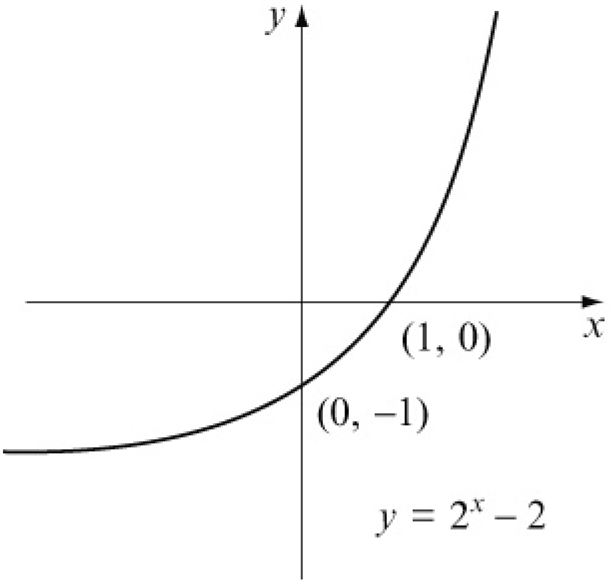
(a)

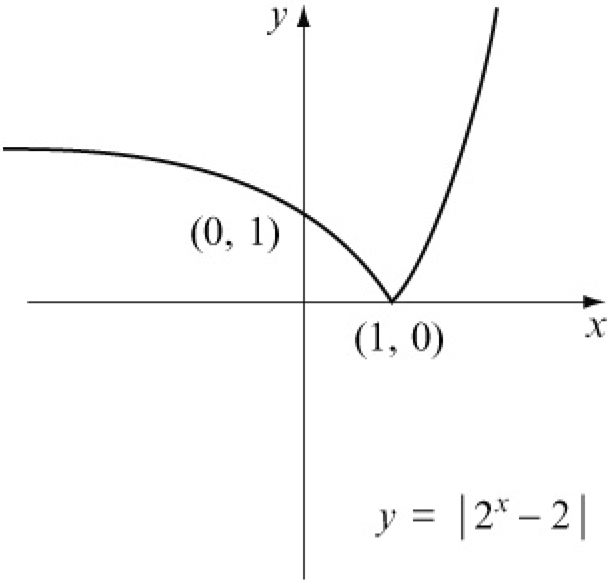


(b)



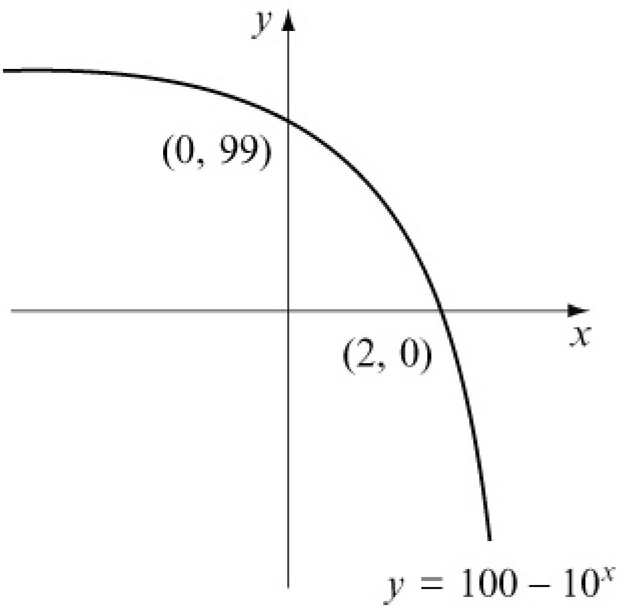
(c)

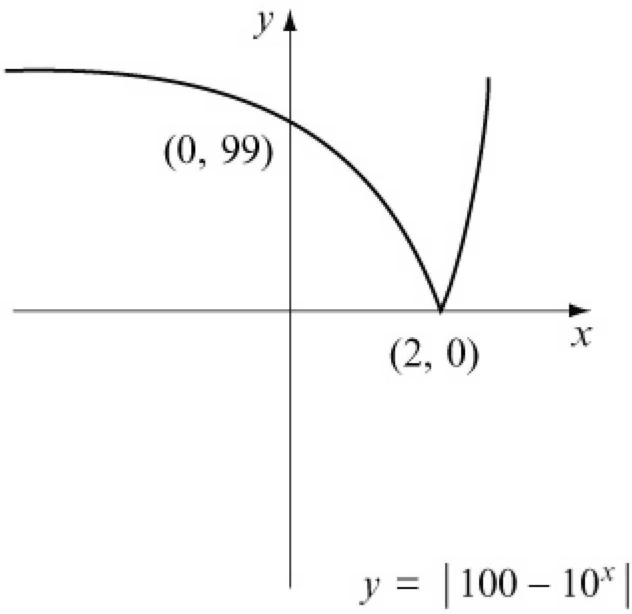




For $y = \mid 2^x - 2 \mid$:
When $x = 0$, $y = \mid 2^0 - 2 \mid = \mid -1 \mid = 1$ (0, 1)
When $y = 0$, $2^x - 2 = 0 \Rightarrow 2^x = 2 \Rightarrow x = 1$ (1, 0)

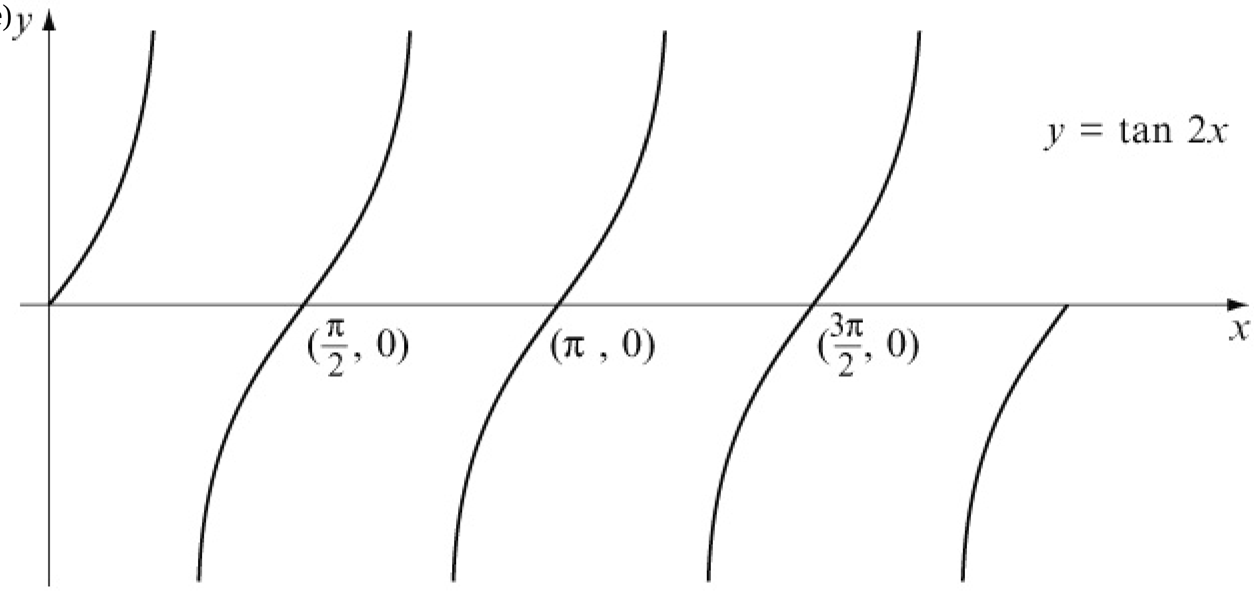
(d)

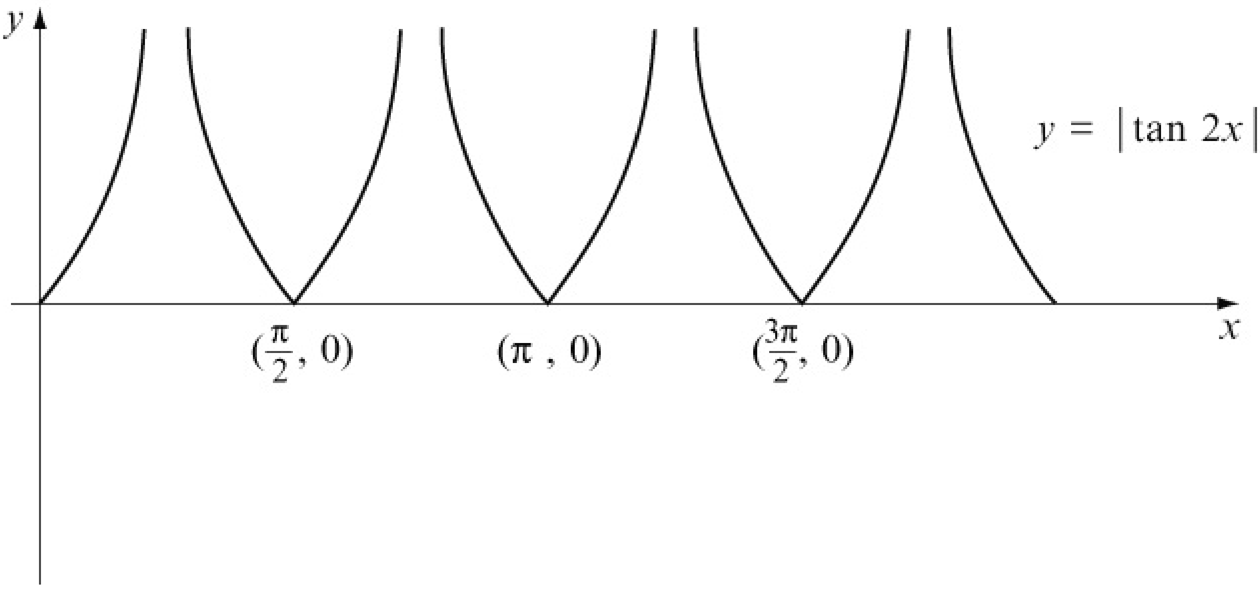




For $y = \mid 100 - 10^x \mid$:
When $x = 0$, $y = \mid 100 - 10^0 \mid = \mid 99 \mid = 99$ (0, 99)
When $y = 0$, $100 - 10^x = 0 \Rightarrow 10^x = 100 \Rightarrow x = 2$ (2, 0)

(e)





For $y = |\tan 2x|$:
When $x = 0$, $y = |\tan 0| = 0$
When $y = 0$, $\tan 2x = 0$

$\Rightarrow \quad x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \quad \ldots$ $\quad \left(\frac{\pi}{2}, 0 \right), (\pi, 0), \left(\frac{3\pi}{2}, 0 \right)$

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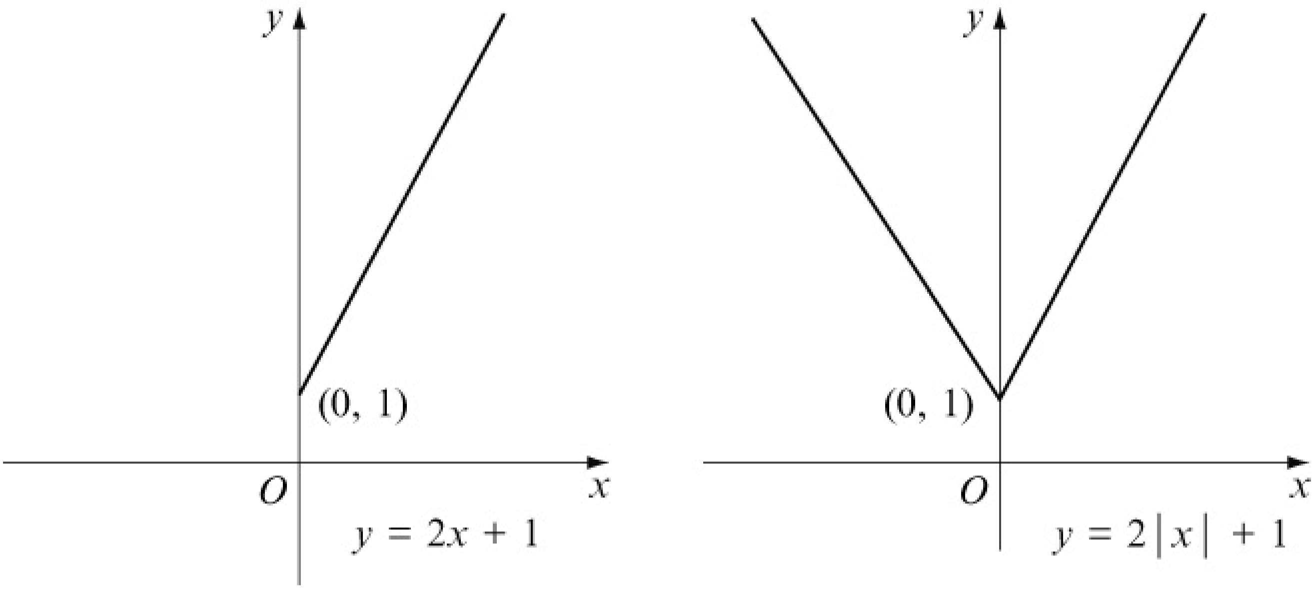
Exercise B, Question 1

Question:

Sketch the following graph and write down the coordinates of any points at which the graph meets the coordinate axes.

$y = 2 \mid x \mid + 1$

Solution:



For $y = 2 \mid x \mid + 1$:

When $x = 0$, $y = 1$ $(0 , 1)$

When $y = 0$, $2 \mid x \mid + 1 = 0$

$\Rightarrow \mid x \mid = - \frac{1}{2}$

No values ($\mid x \mid$ cannot be negative).

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Edexcel AS and A Level Modular Mathematics

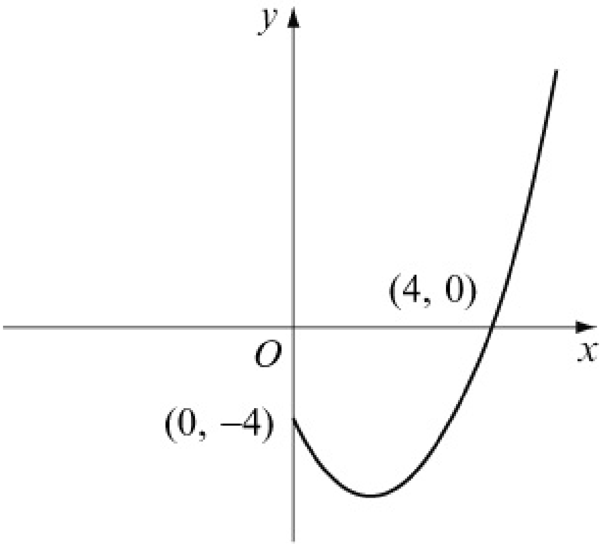
Exercise B, Question 2

Question:

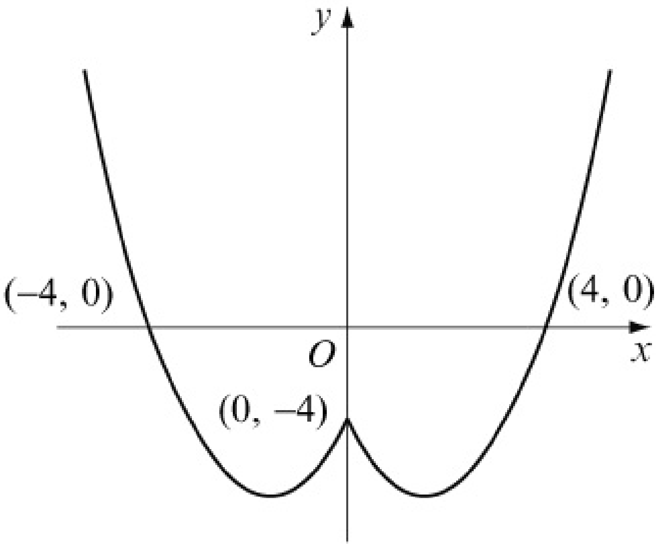
Sketch the following graph and write down the coordinates of any points at which the graph meets the coordinate axes.

$y = |x|^2 - 3|x| - 4$

Solution:



$y = x^2 - 3x - 4$



$y = |x|^2 - 3|x| - 4$

For $y = |x|^2 - 3|x| - 4$:

When $x = 0$, $y = -4$ $(0, -4)$

When $y = 0$, $|x|^2 - 3|x| - 4 = 0$

$\Rightarrow (|x| + 1)(|x| - 4) = 0$

$\Rightarrow |x| = 4$

$\Rightarrow x = 4$ or -4 $(-4, 0)$ and $(4, 0)$

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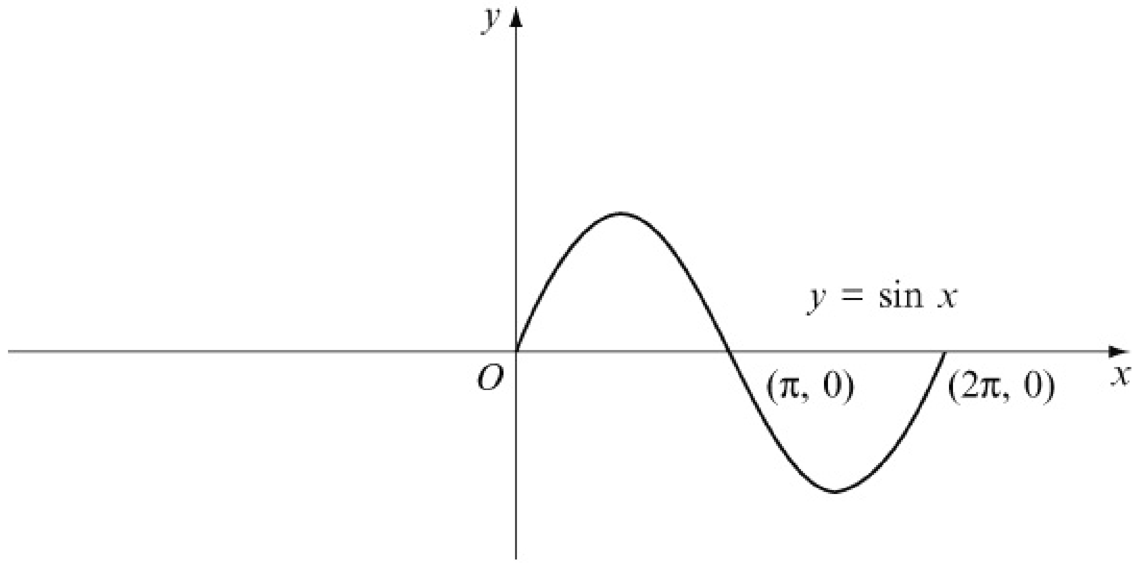
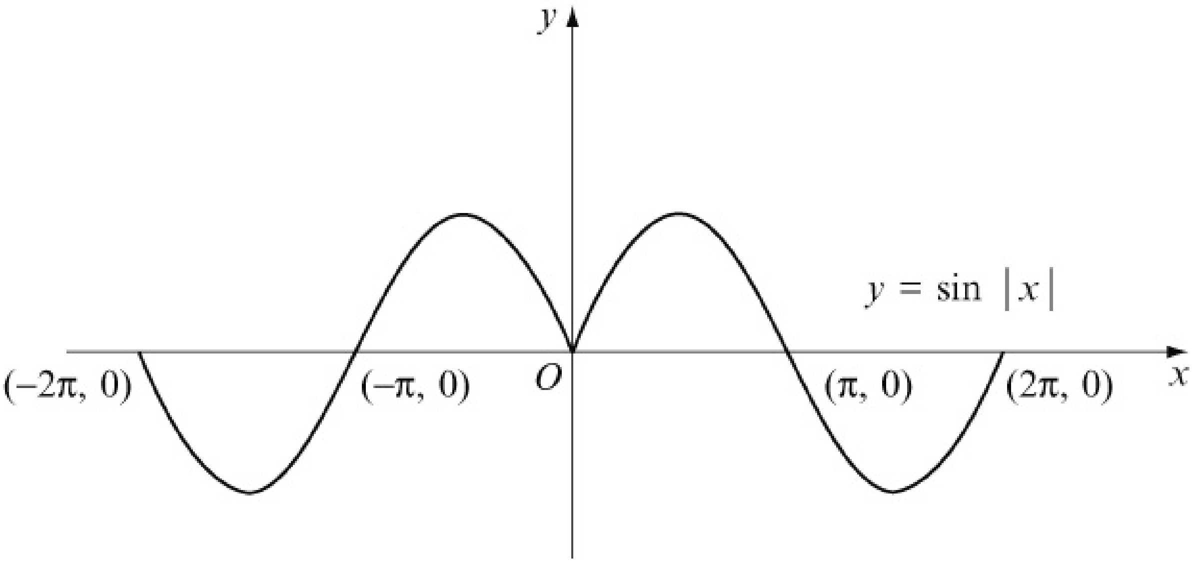
Exercise B, Question 3

Question:

Sketch the following graph and write down the coordinates of any points at which the graph meets the coordinate axes.
$y = \sin | x | , -2\pi \leq x \leq 2\pi$

Solution:





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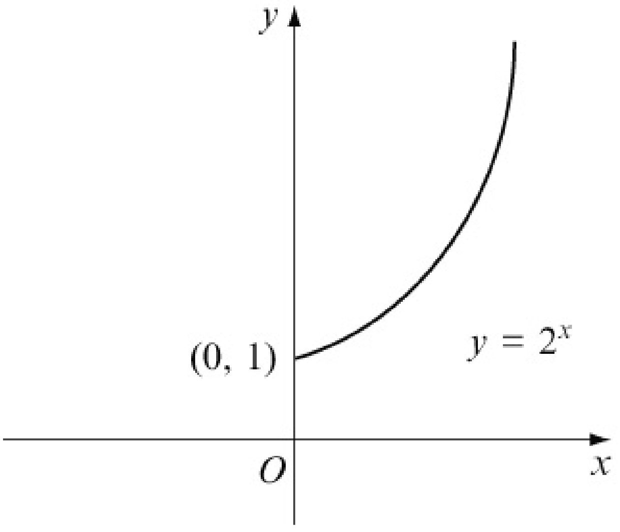
Exercise B, Question 4

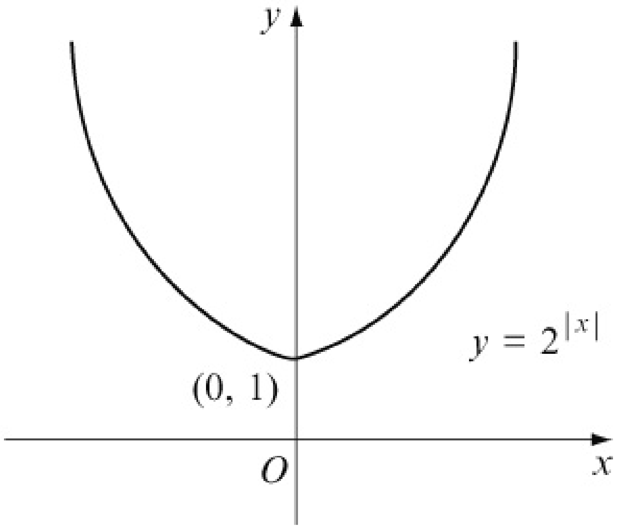
Question:

Sketch the following graph and write down the coordinates of any points at which the graph meets the coordinate axes.

$y = 2^{|x|}$

Solution:





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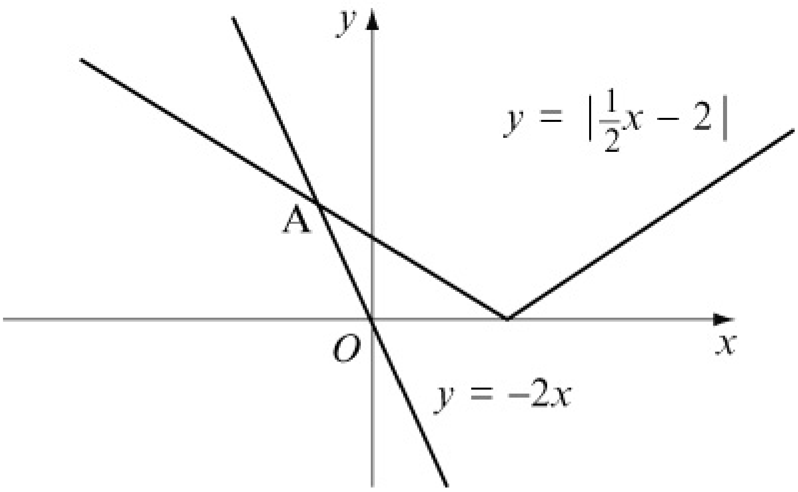
Exercise C, Question 1

Question:

On the same diagram, sketch the graphs of $y = -2x$ and $y = \left| \frac{1}{2}x - 2 \right|$.

Solve the equation $-2x = \left| \frac{1}{2}x - 2 \right|$.

Solution:



Intersection point A is on the reflected part of $y = \frac{1}{2}x - 2$.

$-\left(\frac{1}{2}x - 2\right) = -2x$

$-\frac{1}{2}x + 2 = -2x$

$2x - \frac{1}{2}x = -2$

$\frac{3}{2}x = -2$

$x = -\frac{4}{3}$

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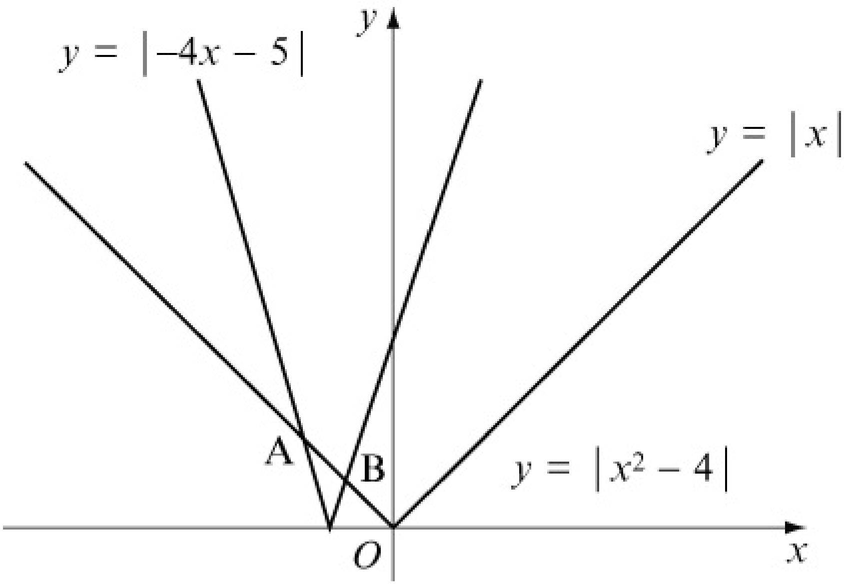
Exercise C, Question 2

Question:

On the same diagram, sketch the graphs of $y = |x|$ and $y = |-4x-5|$.
Solve the equation $|x| = |-4x-5|$.

Solution:



Intersection point A is on the reflected part of $y = x$.

$-x = -4x - 5$

$4x - x = -5$

$3x = -5$

$x = -\frac{5}{3}$

Intersection point B is on the reflected part of $y = x$ and also on the reflected part of $y = -4x - 5$.

$-x = -(-4x-5)$

$-x = 4x + 5$

$-x - 4x = 5$

$-5x = 5$

$x = -1$

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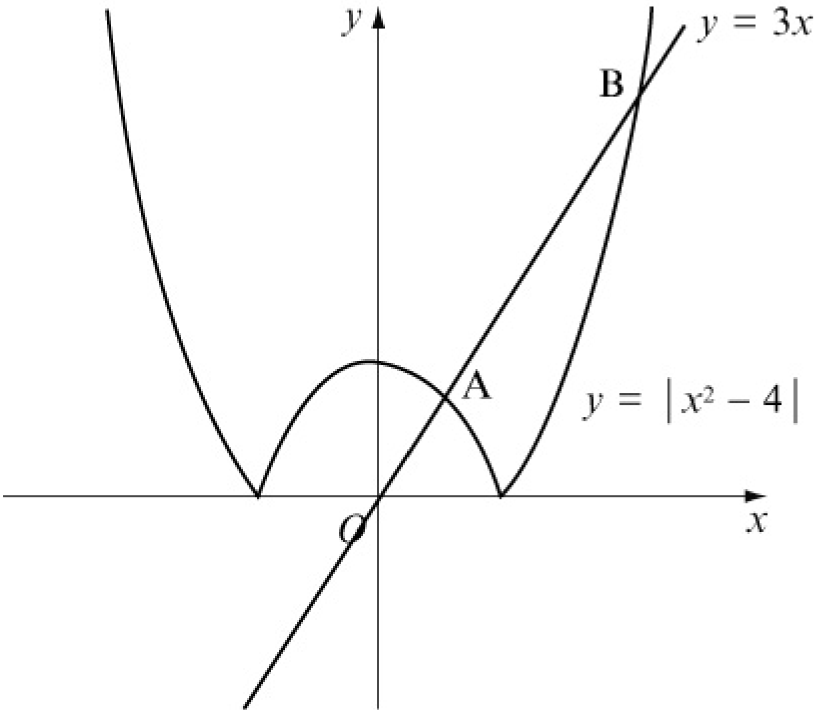
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

On the same diagram, sketch the graphs of $y = 3x$ and $y = | x^2 - 4 |$. Solve the equation $3x = | x^2 - 4 |$.

Solution:



Intersection point A is on the reflected part of $y = x^2 - 4$.

$3x = -(x^2 - 4)$

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$ ($x = -4$ is not valid)

$x = 1$

Intersection point B:

$3x = x^2 - 4$

$0 = x^2 - 3x - 4$

$0 = (x - 4)(x + 1)$ ($x = -1$ is not valid)

$x = 4$

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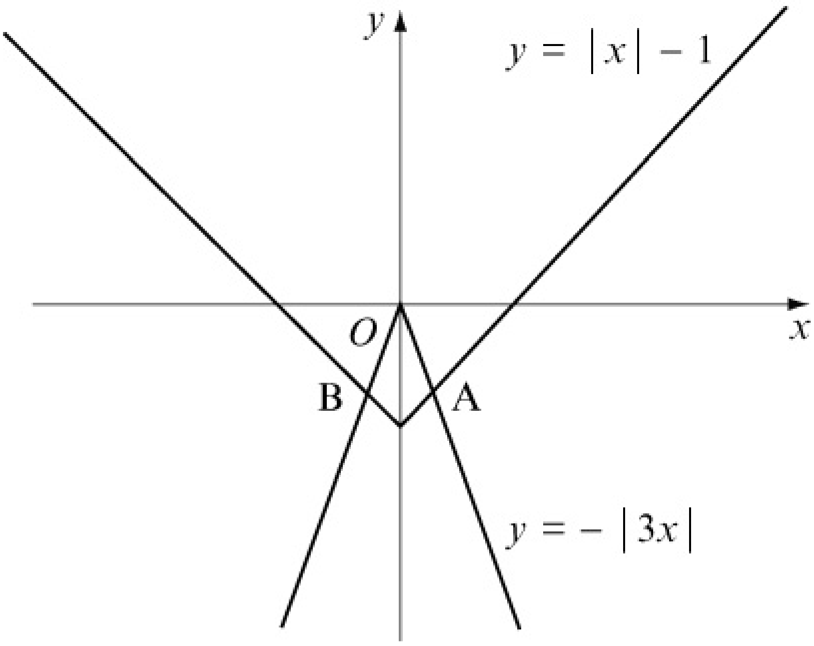
Exercise C, Question 4

Question:

On the same diagram, sketch the graphs of $y = |x| - 1$ and $y = -|3x|$.
Solve the equation $|x| - 1 = -|3x|$.

Solution:



Intersection point A:
$x - 1 = -3x$
$3x + x = 1$
$x = \frac{1}{4}$

Intersection point B is on the reflected part of both graphs.
$-(x) - 1 = -(-3x)$
$-x - 1 = 3x$
$-4x = 1$
$x = -\frac{1}{4}$

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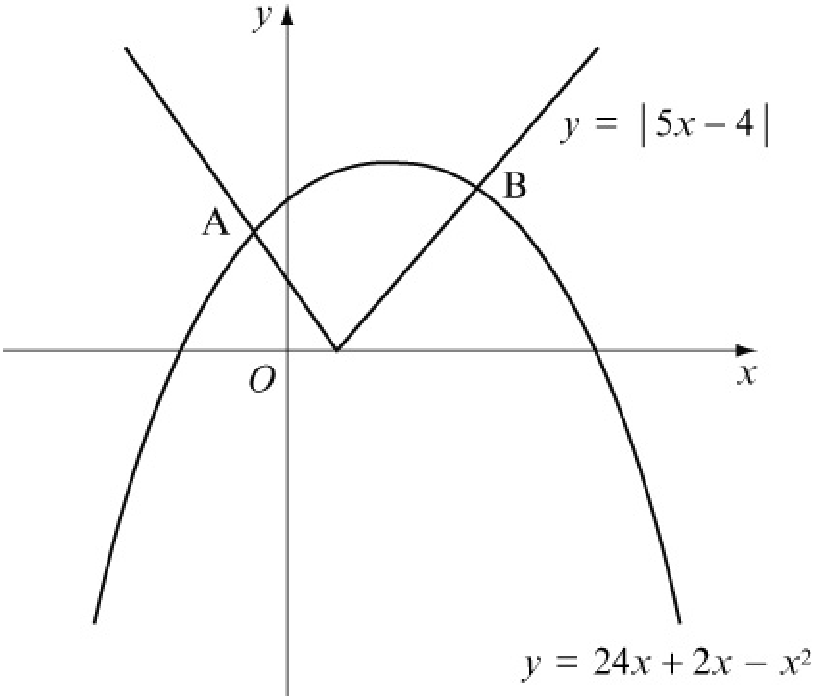
Exercise C, Question 5

Question:

On the same diagram, sketch the graphs of $y = 24 + 2x - x^2$ and $y = | 5x - 4 |$.
Solve the equation $24 + 2x - x^2 = | 5x - 4 |$. (Answers to 2 d.p. where appropriate).

Solution:



Intersection point A is on the reflected part of $y = 5x - 4$.

$-(5x - 4) = 24 + 2x - x^2$

$-5x + 4 = 24 + 2x - x^2$

$x^2 - 7x - 20 = 0$

$x = \frac{7 \pm \sqrt{49 + 80}}{2}$ (positive solution not valid)

$x = -2.18$ (2 d.p.)

Intersection point B:

$5x - 4 = 24 + 2x - x^2$

$x^2 + 3x - 28 = 0$

$(x + 7)(x - 4) = 0$ ($x = -7$ is not valid)

$x = 4$

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Exercise D, Question 1

Question:

Using combinations of transformations, sketch the graph of each of the following:

(a) $y = 2x^2 - 4$

(b) $y = 3(x + 1)^2$

(c) $y = \frac{3}{x} - 2$

(d) $y = \frac{3}{x - 2}$

(e) $y = 5\sin(x + 30°)$, $0 \le x \le 360°$

(f) $y = \frac{1}{2}e^x + 4$

(g) $y = |4x| + 1$

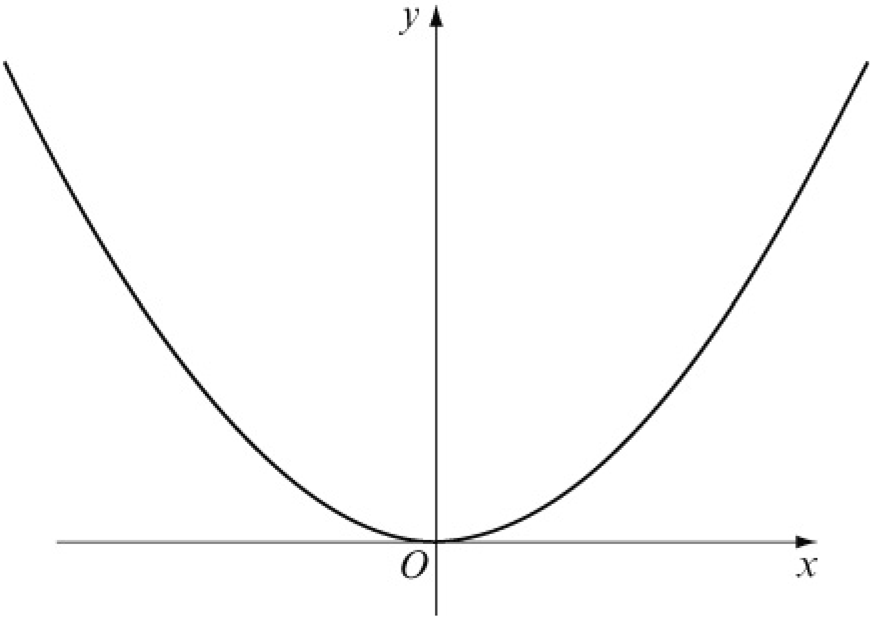
(h) $y = 2x^3 - 3$

(i) $y = 3\ln(x - 2)$, $x > 2$

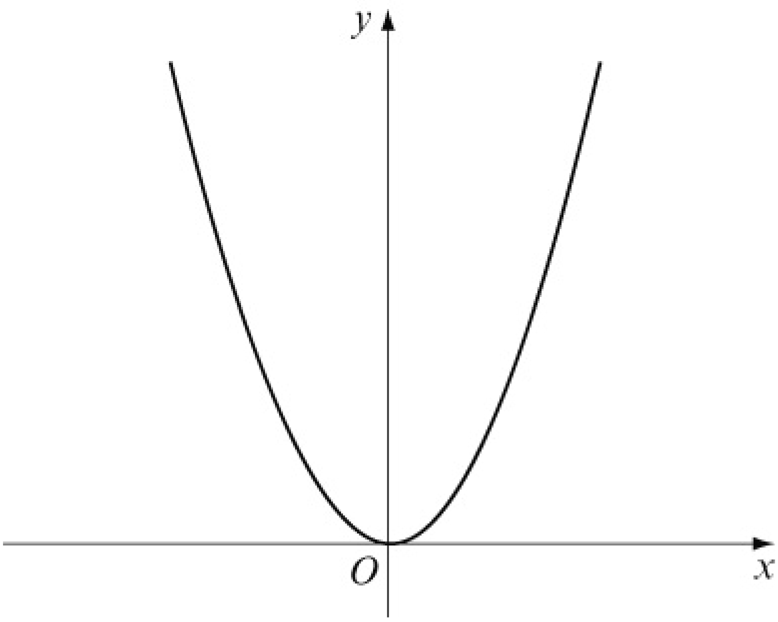
(j) $y = |2e^x - 3|$

Solution:

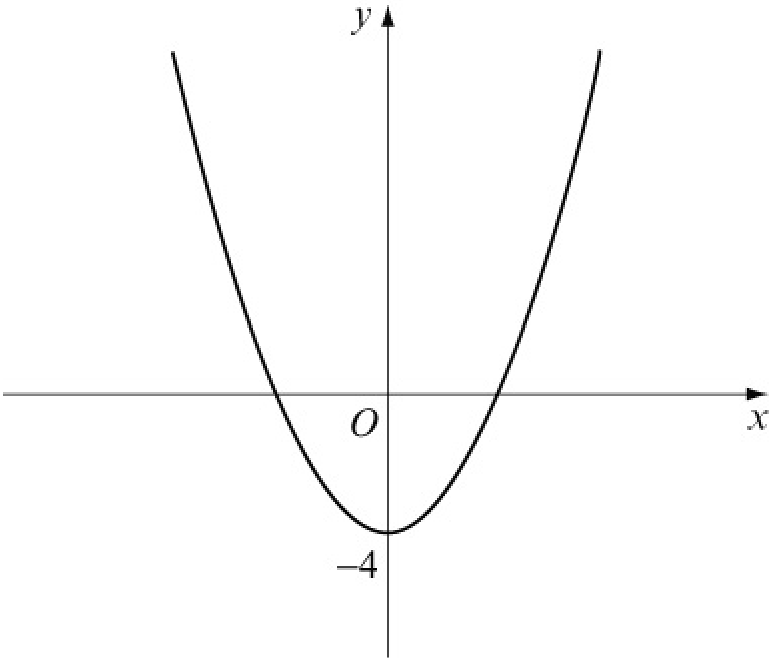
(a) $y = x^2$



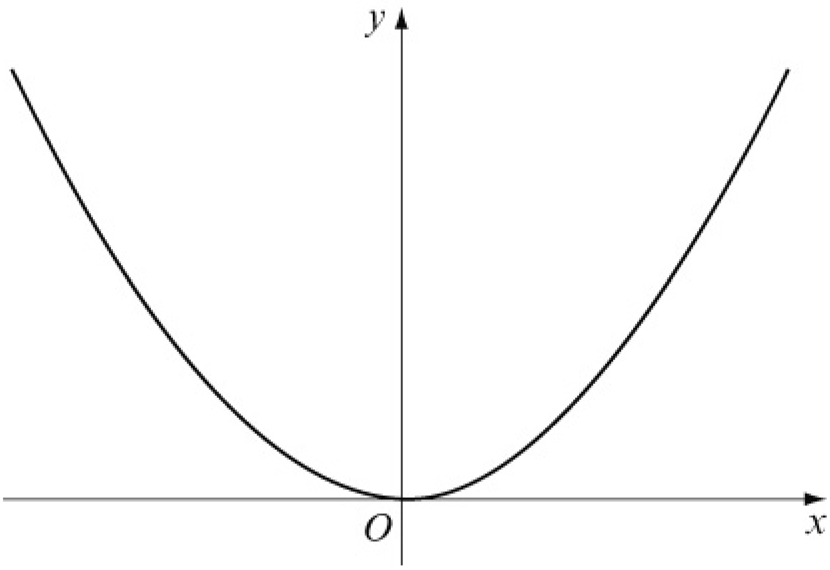
$y = 2x^2$. Vertical stretch, scale factor 2.



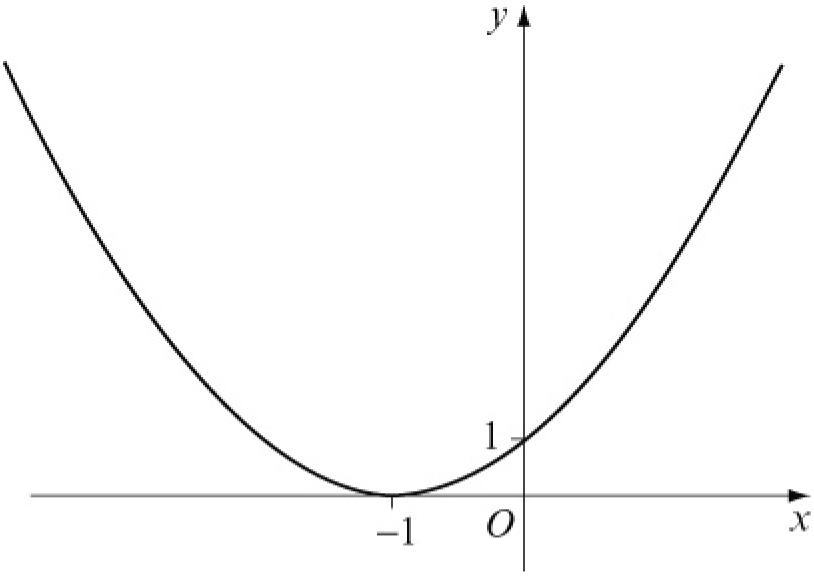
$y = 2x^2 - 4$. Vertical translation of -4.



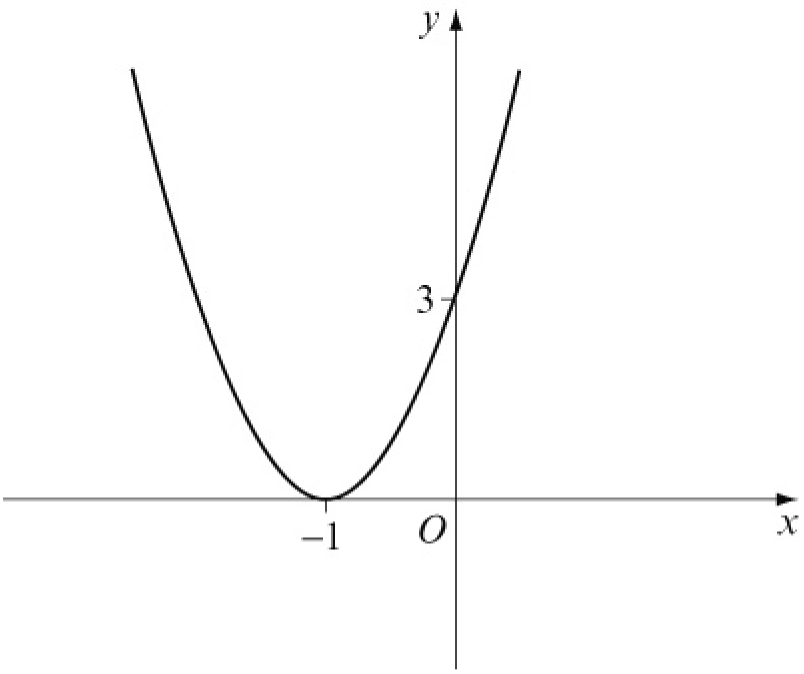
(b) $y = x^2$



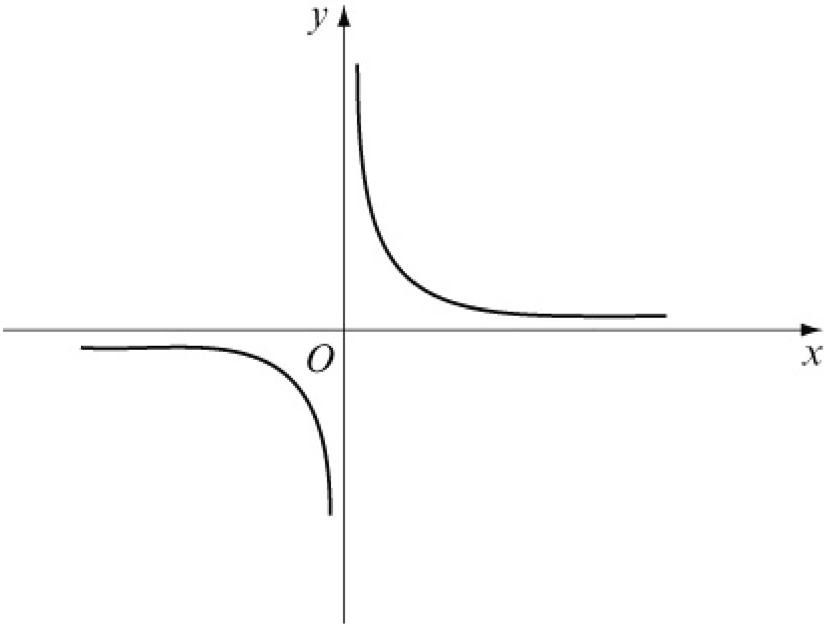
$y = (x + 1)^2$. Horizontal translation of -1.



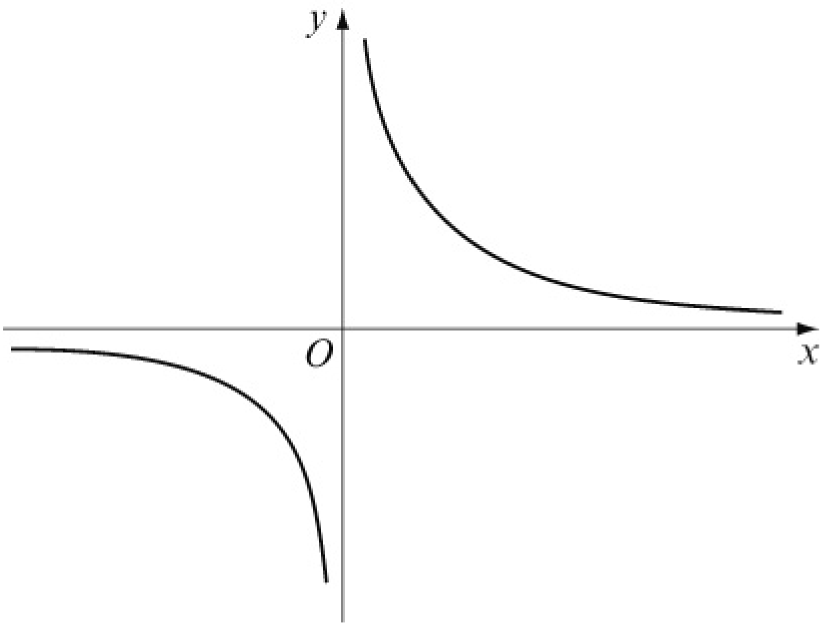
$y = 3 (x + 1)^2$. Vertical stretch, scale factor 3.



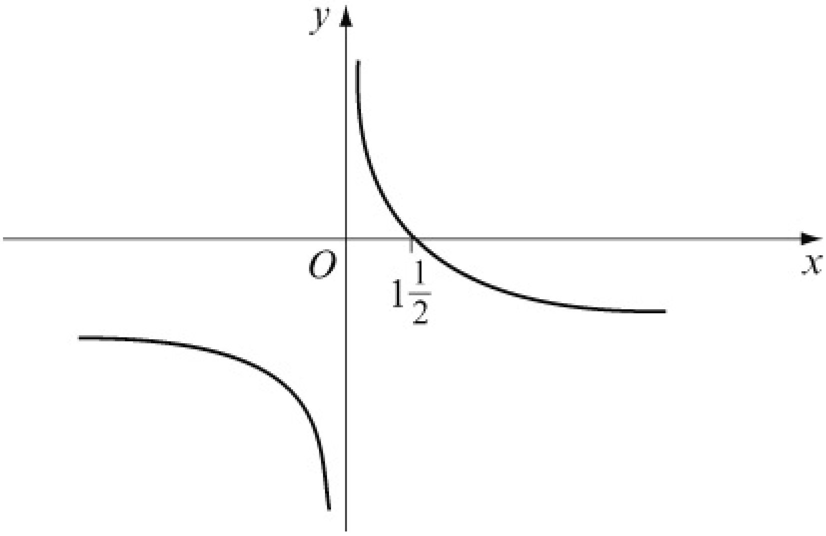
(c) $y = \frac{1}{x}$



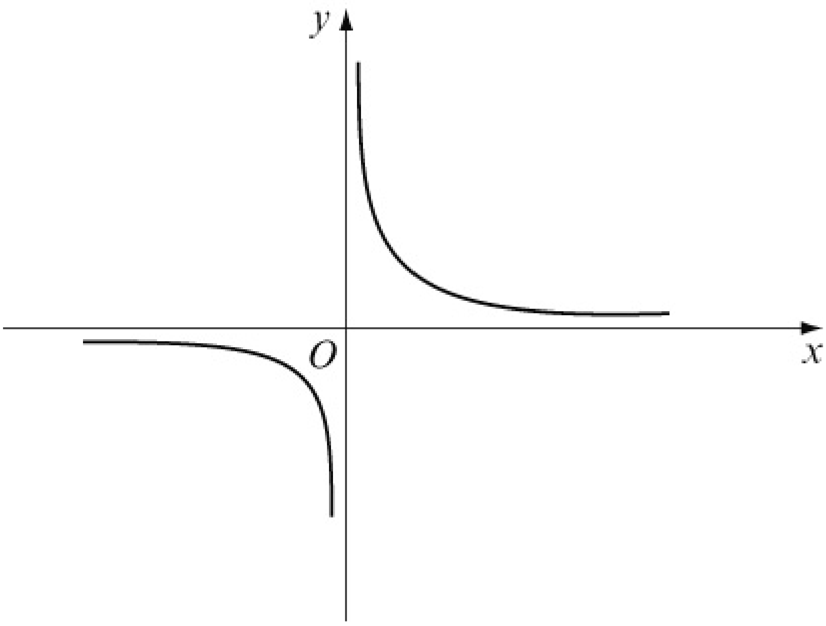
$y = \frac{3}{x}$. Vertical stretch, scale factor 3.



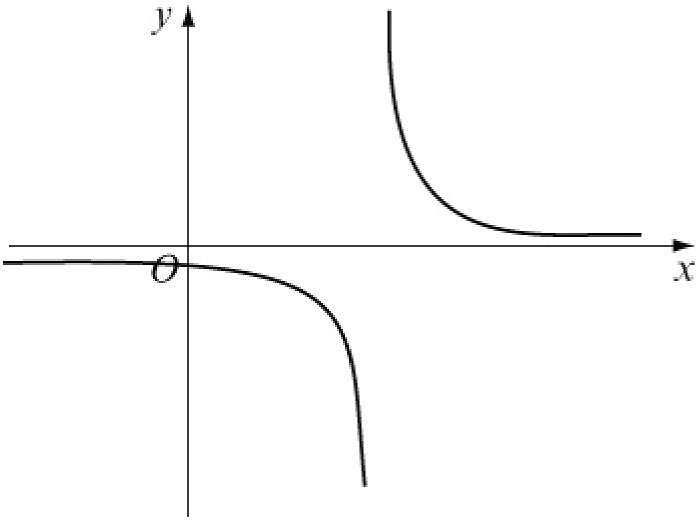
$y = \frac{3}{x} - 2$. Vertical translation of -2.



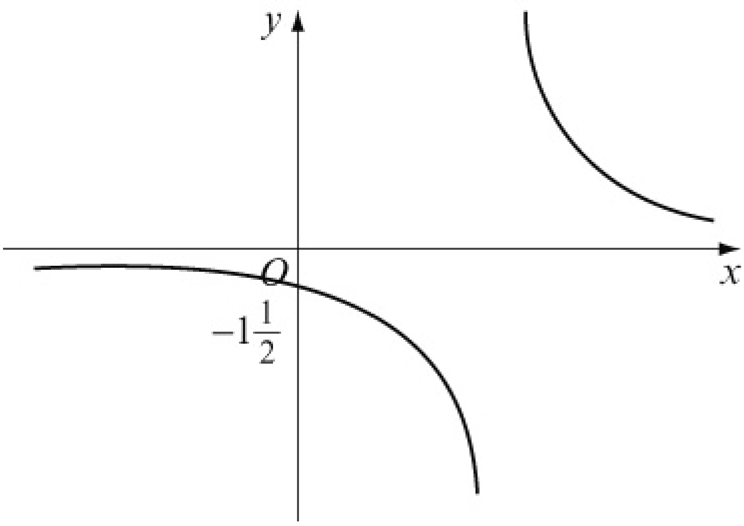
(d) $y = \frac{1}{x}$



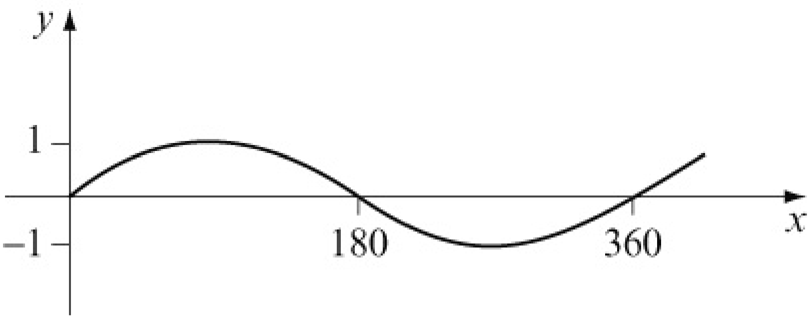
$y = \frac{1}{x-2}$. Horizontal translation of +2.



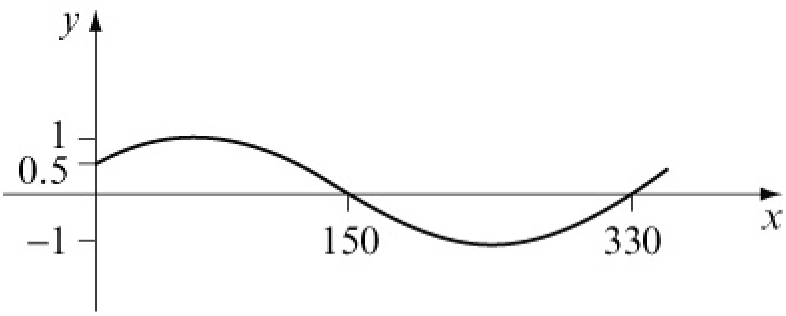
$y = \frac{3}{x-2}$. Vertical stretch, scale factor 3.



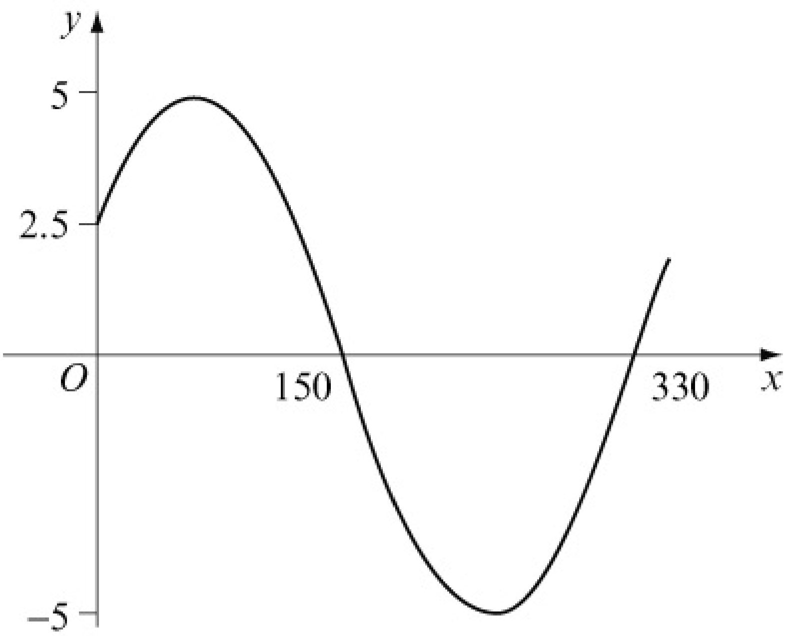
(e) $y = \sin x$



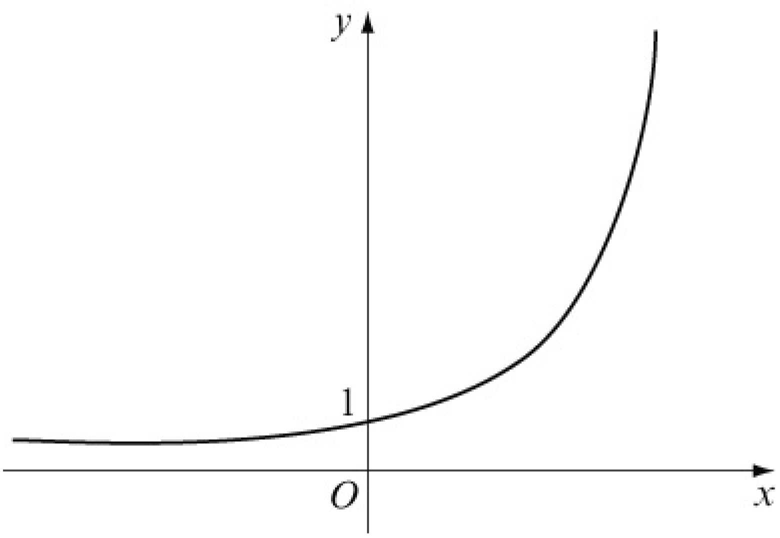
$y = \sin(x + 30°)$. Horizontal translation of $-30°$



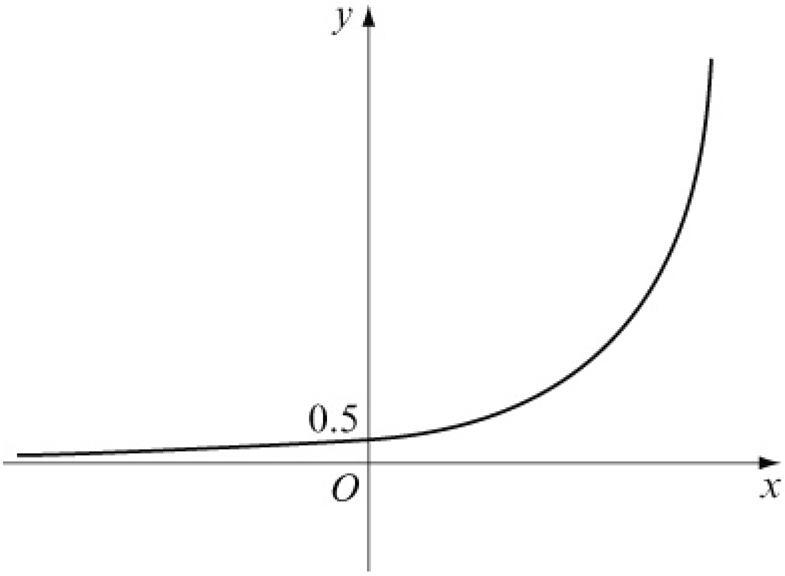
$y = 5 \sin (x + 30°)$. Vertical stretch, scale factor 5.



(f) $y = e^x$

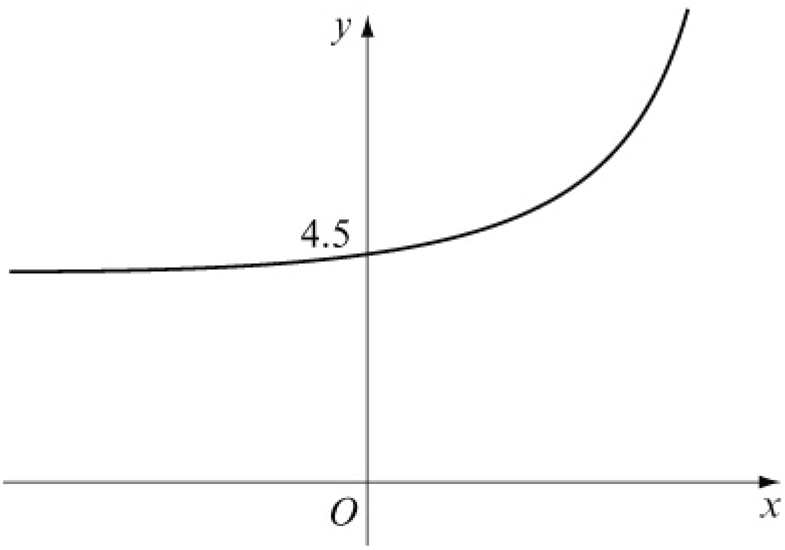


$y = \frac{1}{2}e^x$. Vertical stretch, scale factor $\frac{1}{2}$.

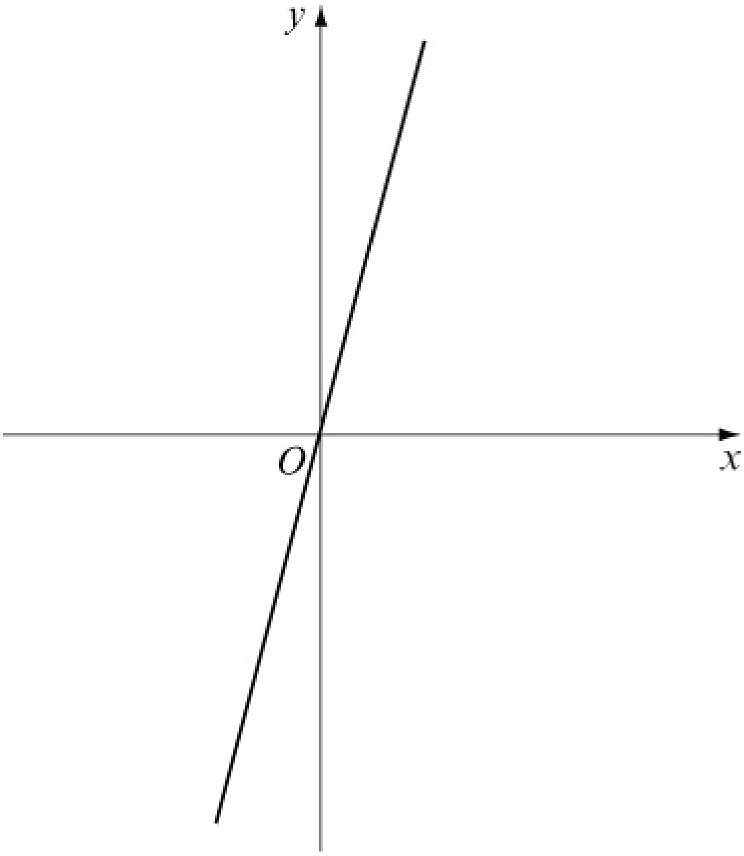


$y = \frac{1}{2}e^x + 4$. Vertical translation of $+ 4$.

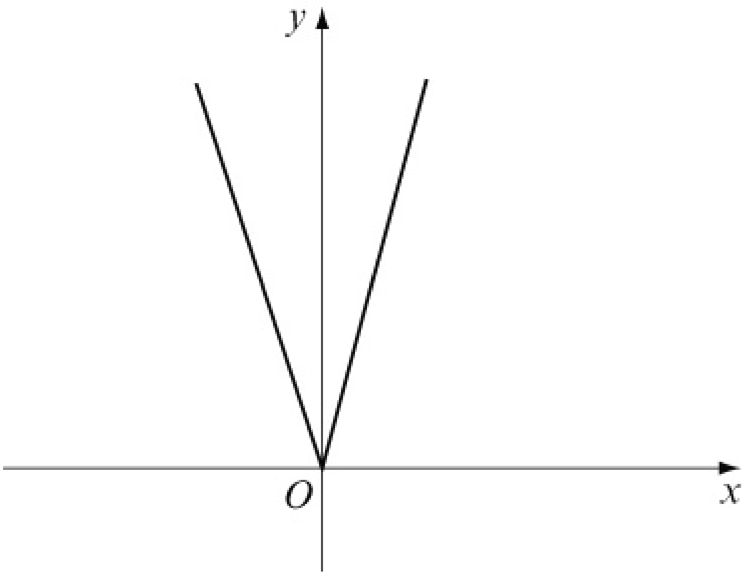
(When $x = 0$, $y = \frac{1}{2}e^0 + 4 = 4.5$).



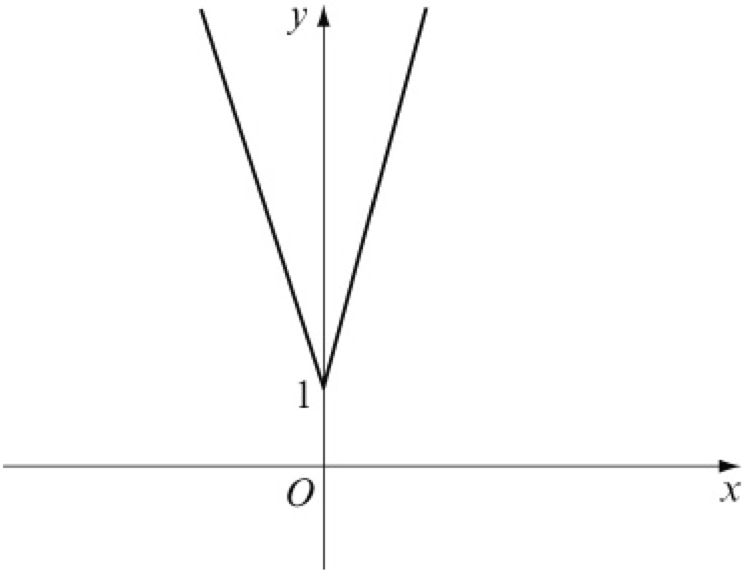
(g) $y = 4x$



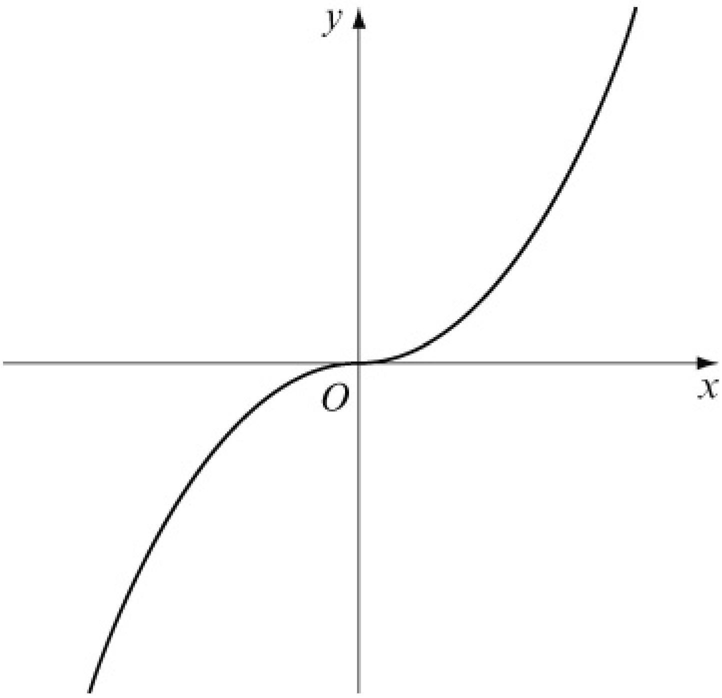
$y = |4x|$. For the part below the x-axis, reflect in the x-axis.



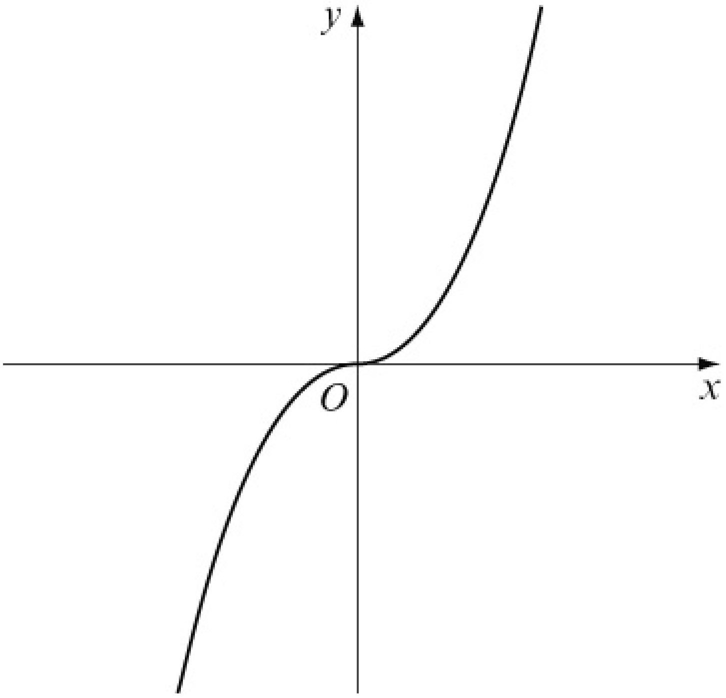
$y = |4x| + 1$. Vertical translation of $+1$.



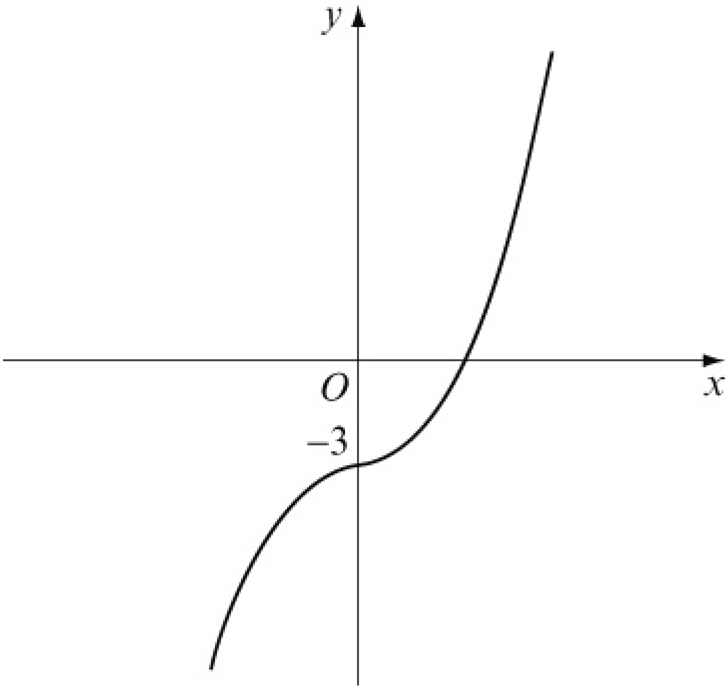
(h) $y = x^3$



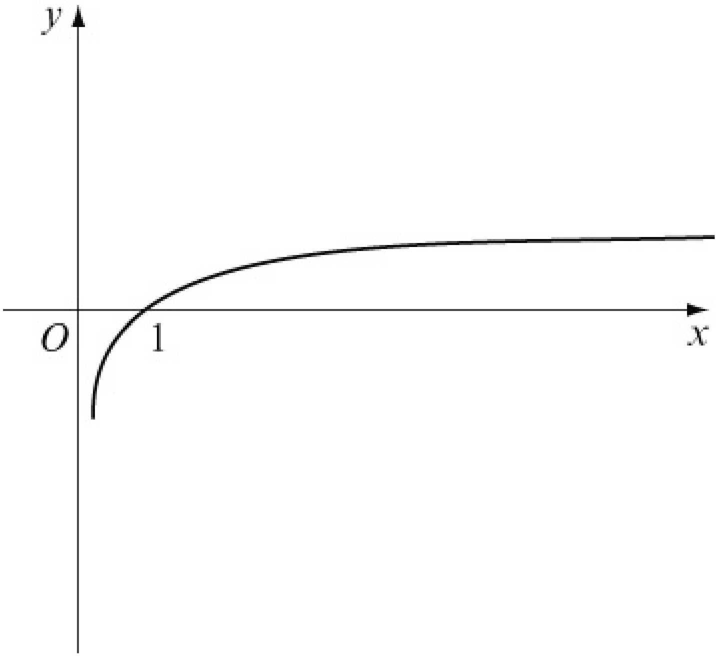
$y = 2x^3$. Vertical stretch, scale factor 2.



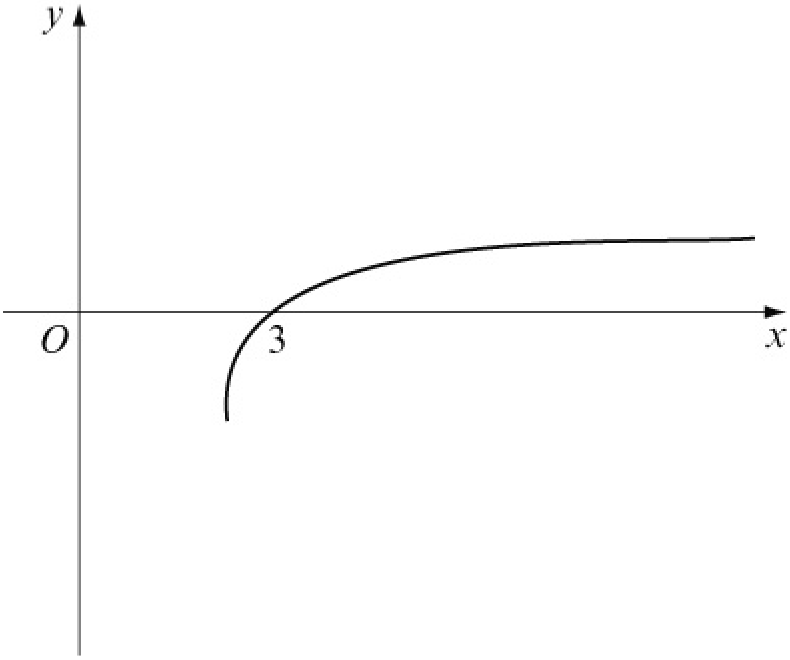
$y = 2x^3 - 3$. Vertical translation of -3.



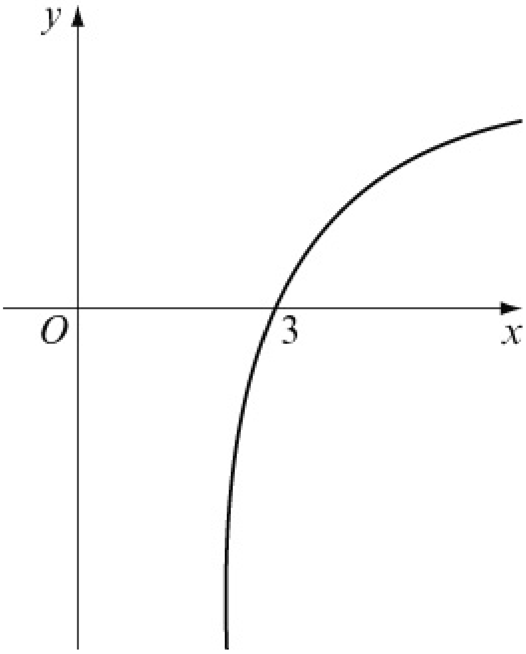
(i) $y = \ln x$



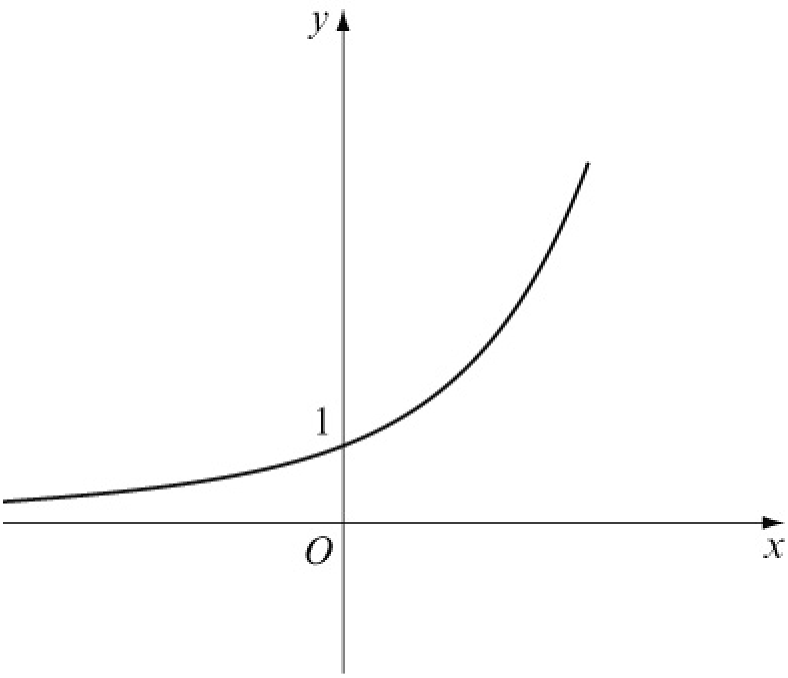
$y = \ln(x - 2)$. Horizontal translation of +2.



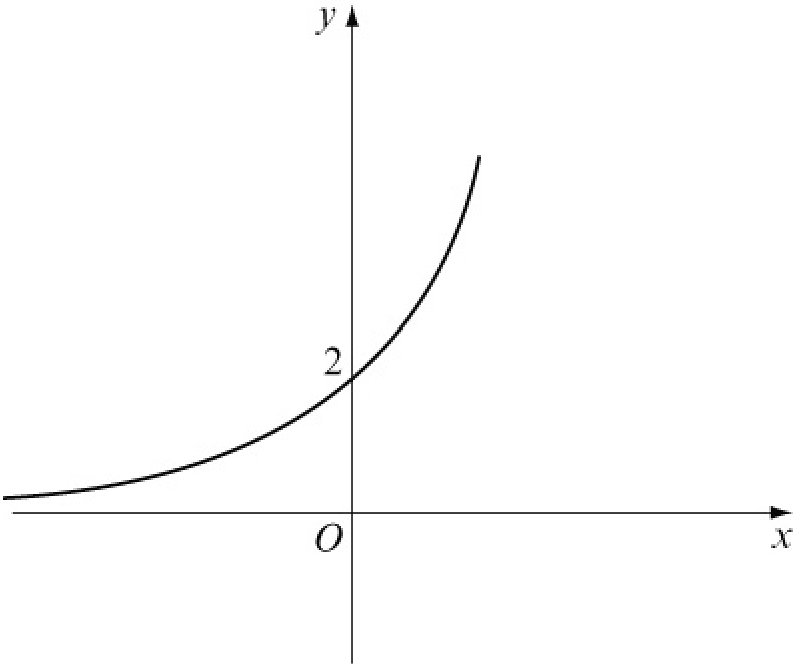
$y = 3\ln(x - 2)$. Vertical stretch, scale factor 3.



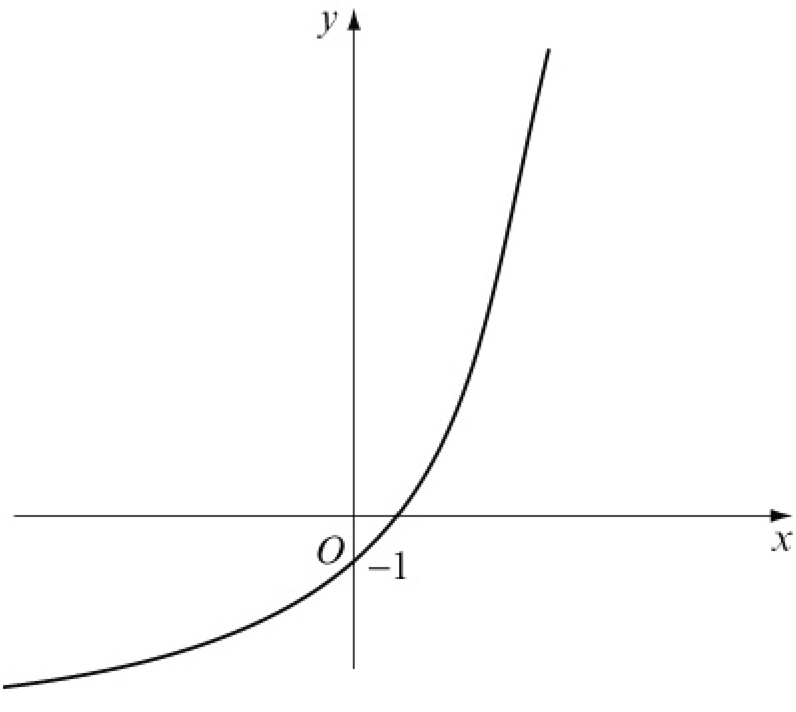
(j) $y = \mathrm{e}^{x}$



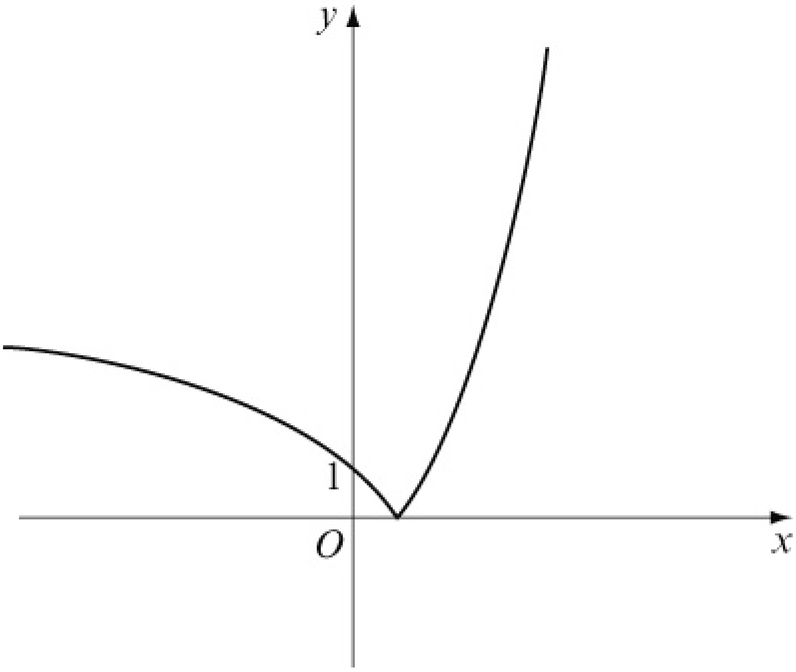
$y = 2\mathrm{e}^{x}$. Vertical stretch, scale factor 2.



$y = 2e^x - 3$. Vertical translation of -3.



$y = |2e^x - 3|$. For the part below the *x*-axis, reflect in the *x*-axis.


y
1
O
x

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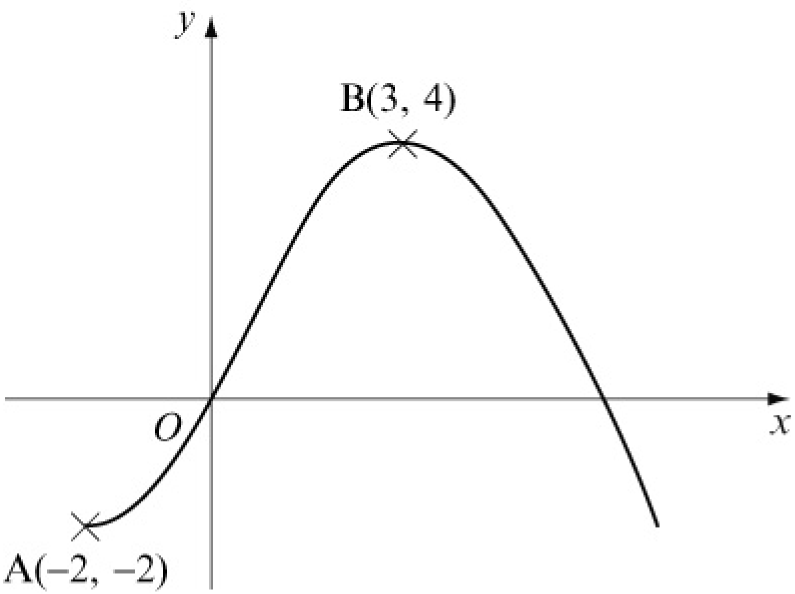
Exercise E, Question 1

Question:

The diagram shows a sketch of the graph of $y = f(x)$.
The curve passes through the origin O, the point $A(-2, -2)$ and the point $B(3, 4)$.



Sketch the graph of:

(a) $y = 3f(x) + 2$

(b) $y = f(x - 2) - 5$

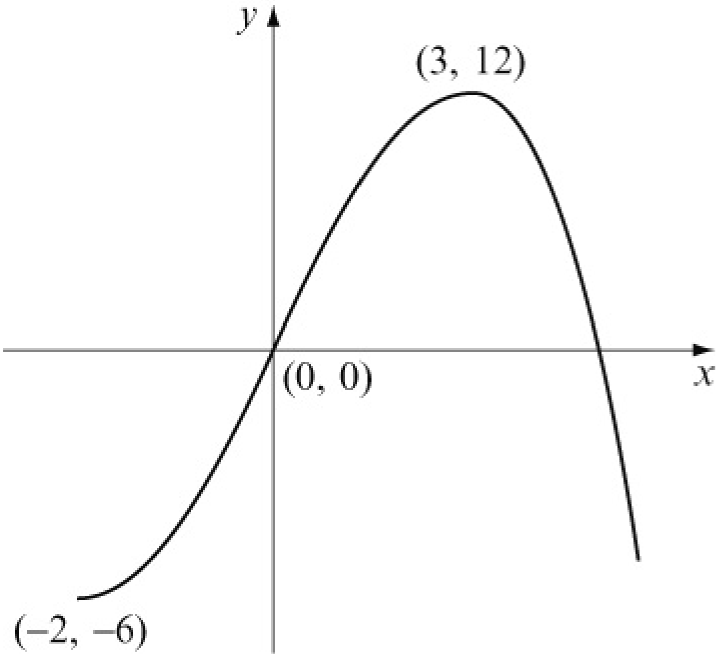
(c) $y = \frac{1}{2}f(x + 1)$

(d) $y = -f(2x)$

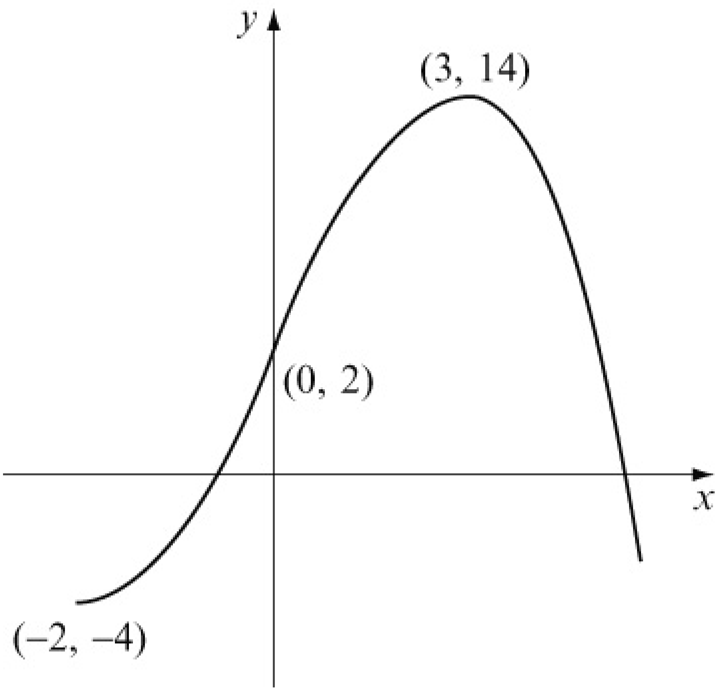
In each case, find the coordinates of the images of the points O, A and B.

Solution:

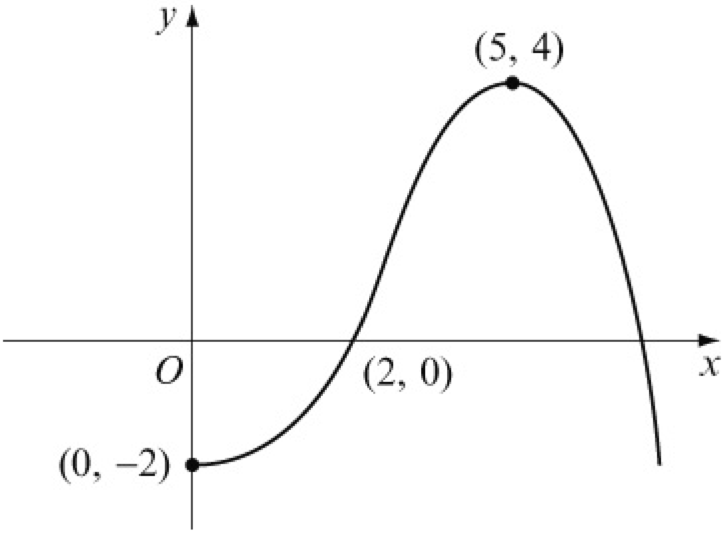
(a) $y = 3f(x)$. Vertical stretch, scale factor 3.



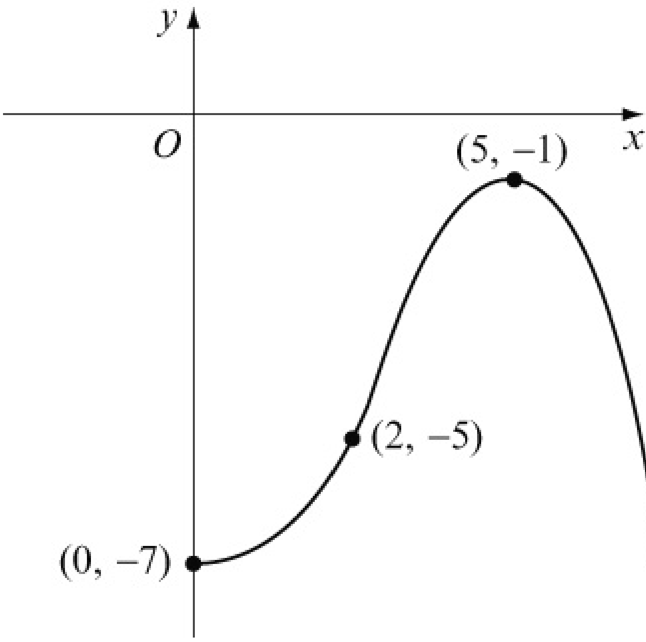
$y = 3\mathrm{f}(x) + 2$. Vertical translation of +2.



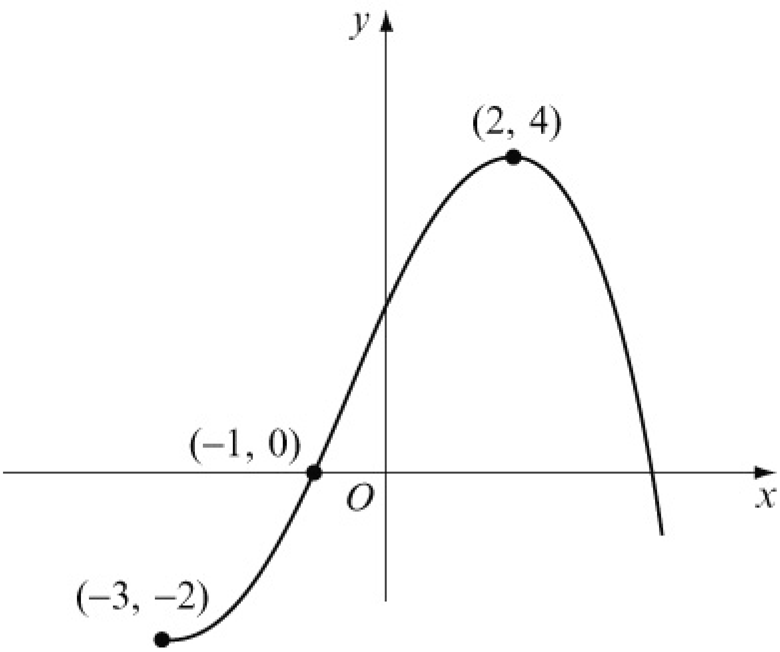
(b) $y = \mathrm{f}(x - 2)$. Horizontal translation of +2.



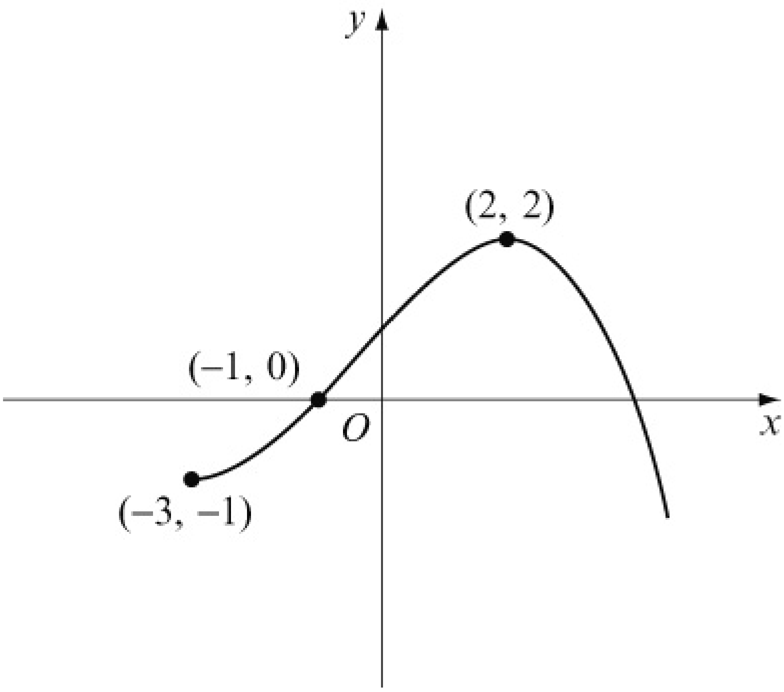
$y = \mathrm{f}(x - 2) - 5$. Vertical translation of -5.



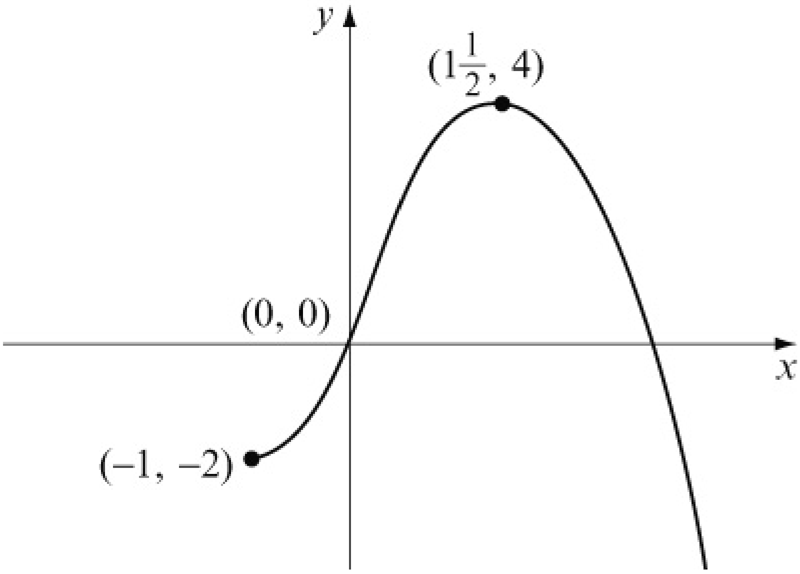
(c) $y = \mathrm{f}(x + 1)$. Horizontal translation of -1.



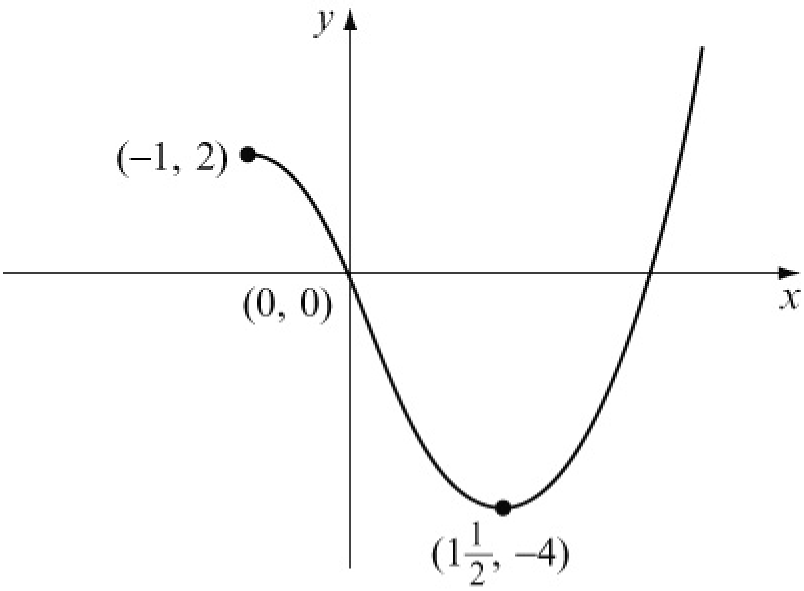
$y = \frac{1}{2}\text{f}(x + 1)$. Vertical stretch, scale factor $\frac{1}{2}$.



(d) $y = \text{f}(2x)$. Horizontal stretch, scale factor $\frac{1}{2}$.



$y = -f(2x)$. Reflection in the x-axis.
(Vertical stretch, scale factor -1).



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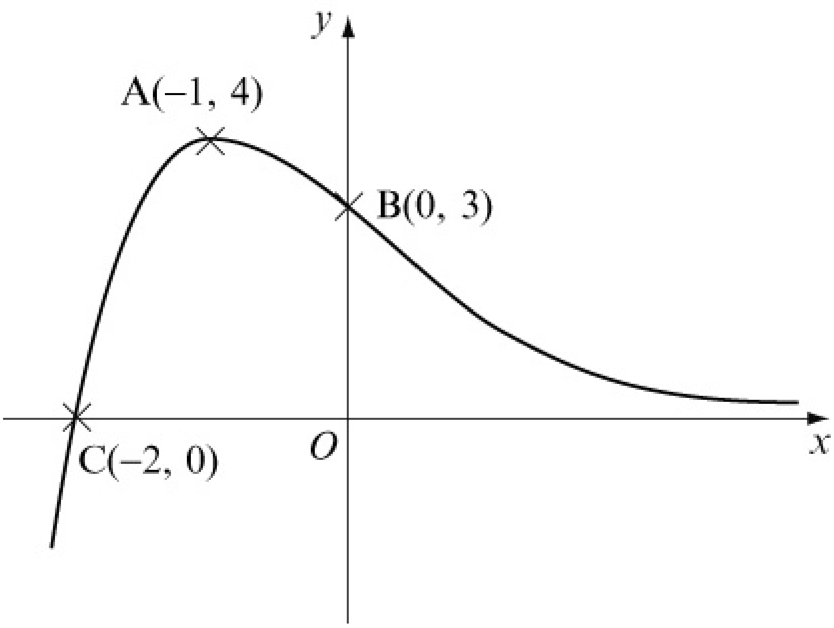
Exercise E, Question 2

Question:

The diagram shows a sketch of the graph of $y = f(x)$. The curve has a maximum at the point $A(-1, 4)$ and crosses the axes at the points B(0, 3) and $C(-2, 0)$.



Sketch the graph of:
(a) $y = 3f(x-2)$

(b) $y = \frac{1}{2}f\left(\frac{1}{2}x\right)$

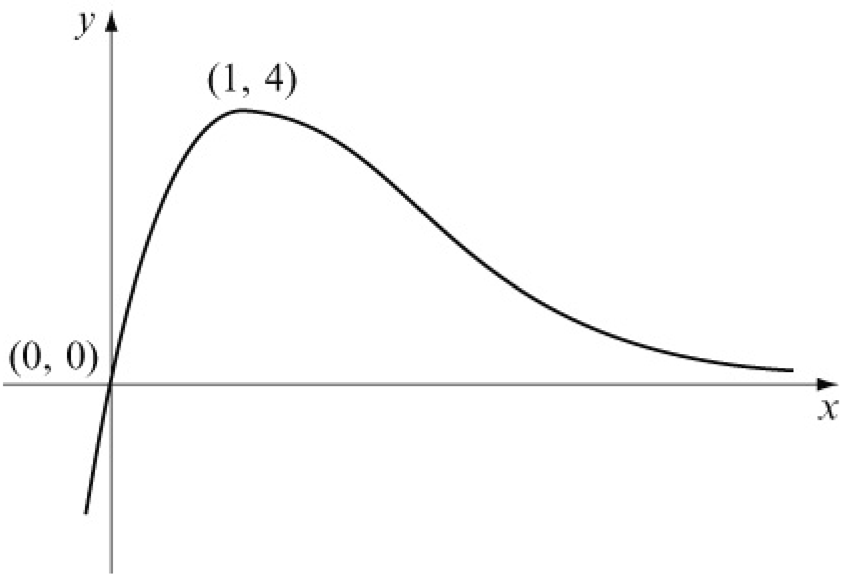
(c) $y = -f(x) + 4$

(d) $y = -2f(x+1)$

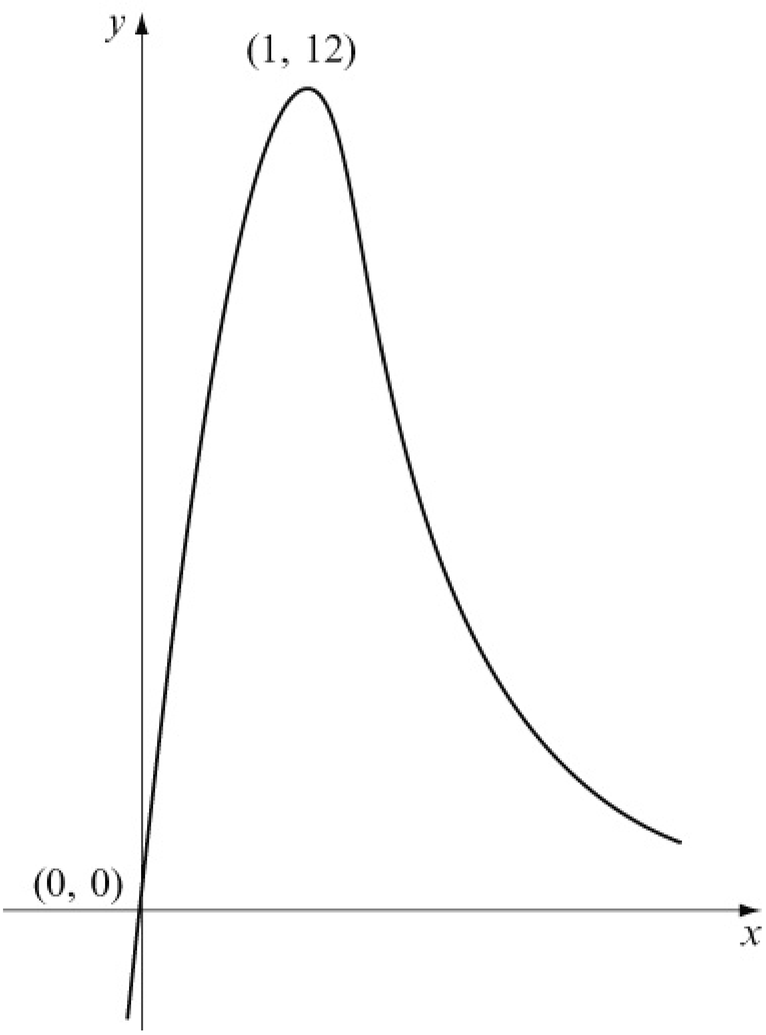
For each graph, find, where possible, the coordinates of the maximum or minimum and the coordinates of the intersection points with the axes.

Solution:

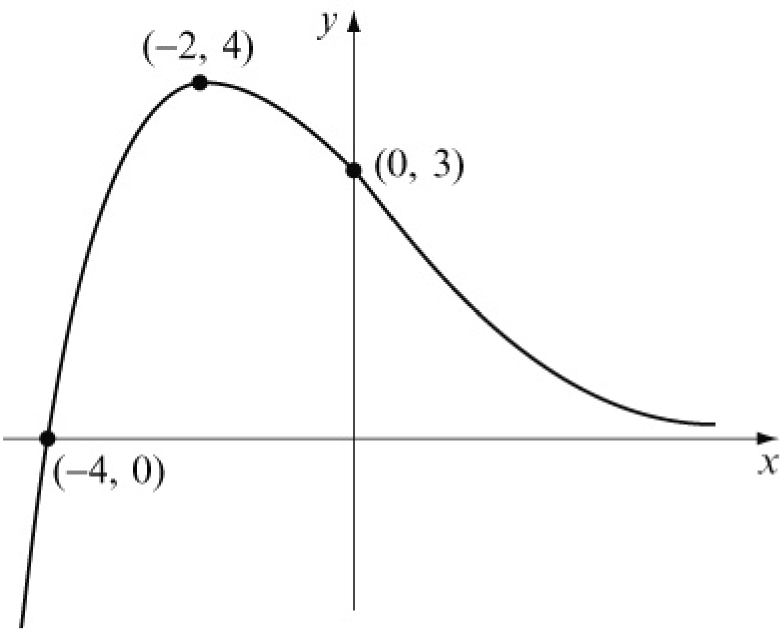
(a) $y = f(x-2)$. Horizontal translation of $+2$.



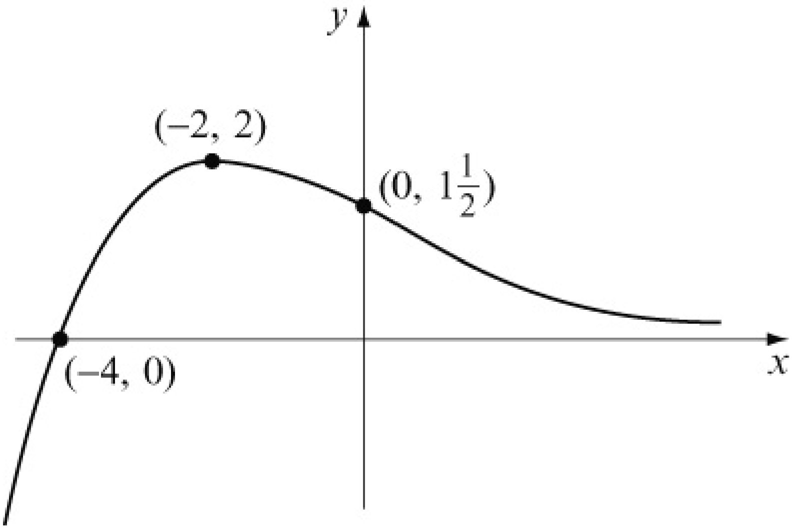
$y = 3\mathrm{f}(x - 2)$. Vertical stretch, scale factor 3.



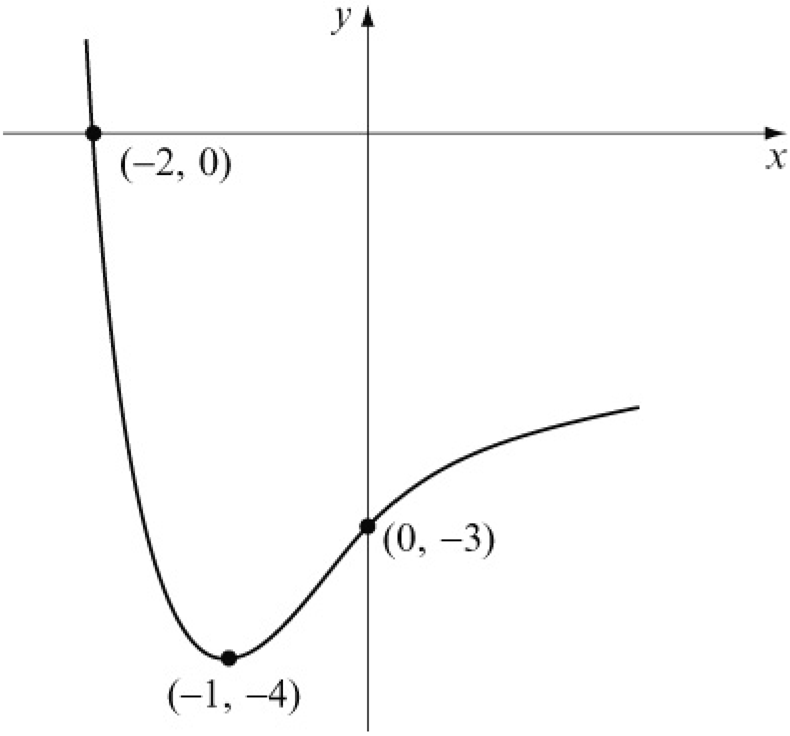
(b) $y = \mathrm{f}\left(\frac{1}{2}x\right)$. Horizontal stretch, scale factor 2.



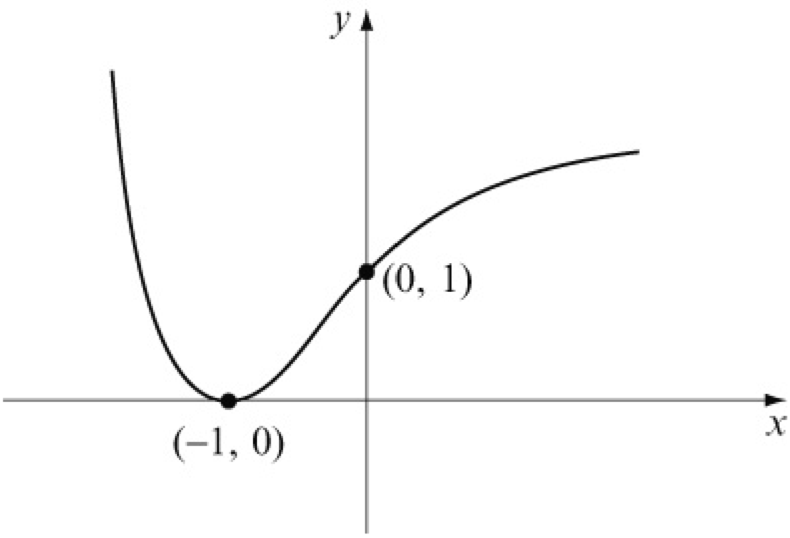
$y = \frac{1}{2}f\left(\frac{1}{2}x\right)$. Vertical stretch, scale factor $\frac{1}{2}$.



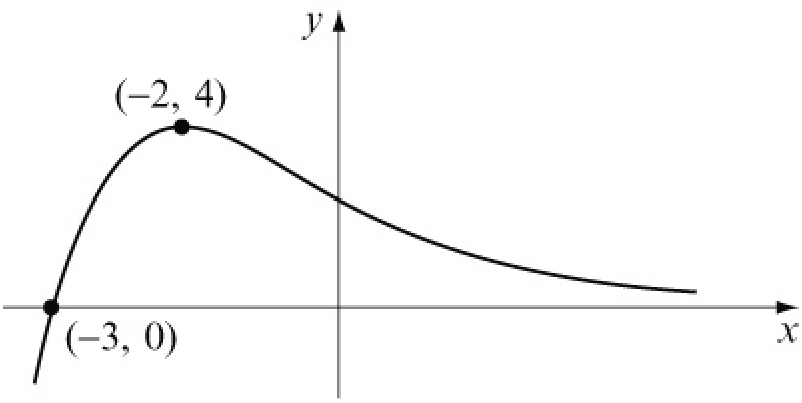
(c) $y = -f(x)$. Reflection in the x-axis. (Vertical stretch, scale factor -1).



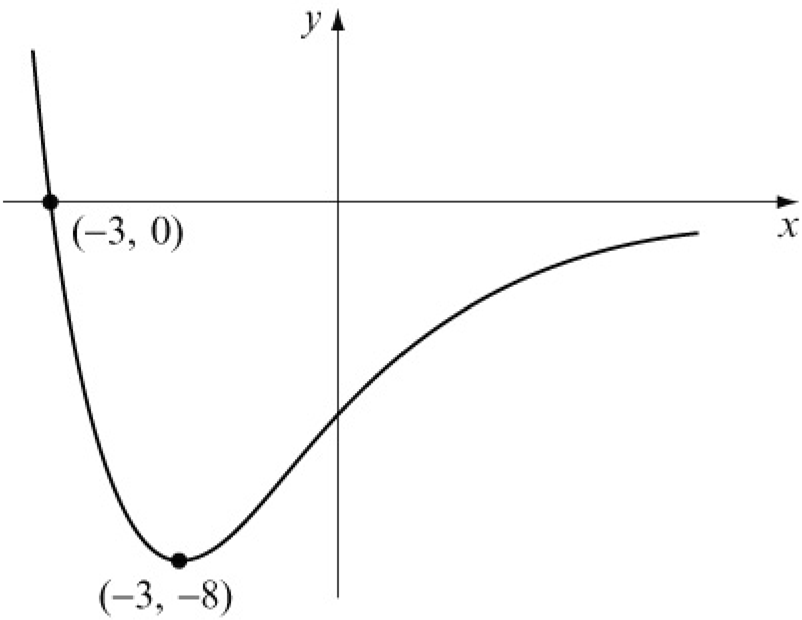
$y = -f(x) + 4$. Vertical translation of $+4$.



(d) $y = f(x + 1)$. Horizontal translation of -1.



$y = -2\text{f}(x+1)$
Reflection in the x-axis, and vertical stretch, scale factor 2.



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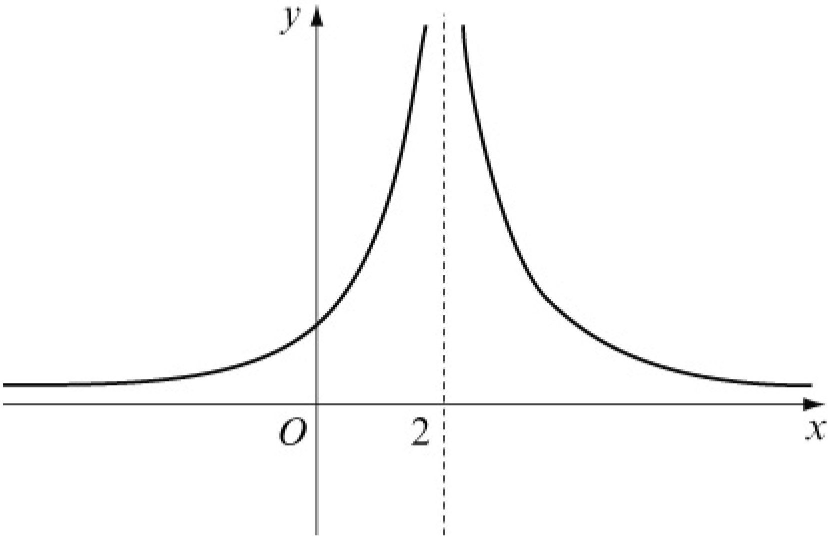
Exercise E, Question 3

Question:

The diagram shows a sketch of the graph of $y = \mathrm{f}(x)$. The lines $x = 2$ and $y = 0$ (the x-axis) are asymptotes to the curve.



Sketch the graph of:

(a) $y = 3\mathrm{f}(x) - 1$

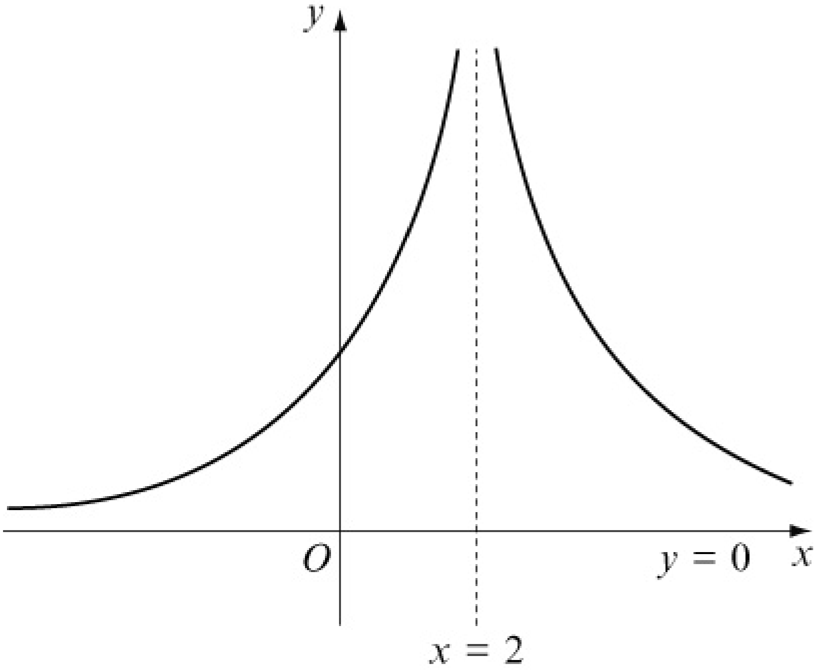
(b) $y = \mathrm{f}(x + 2) + 4$

(c) $y = -\mathrm{f}(2x)$

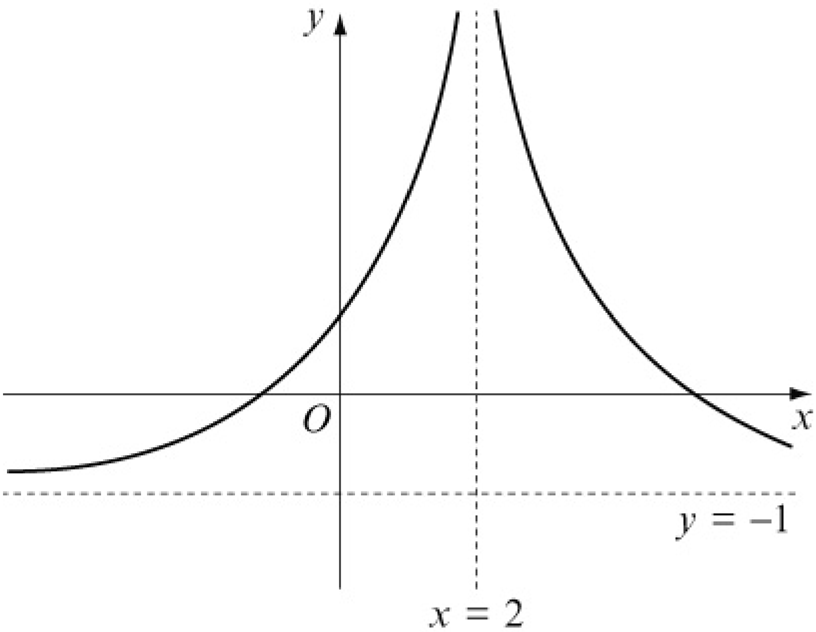
For each part, state the equations of the asymptotes.

Solution:

(a) $y = 3\mathrm{f}(x)$. Vertical stretch, scale factor 3.

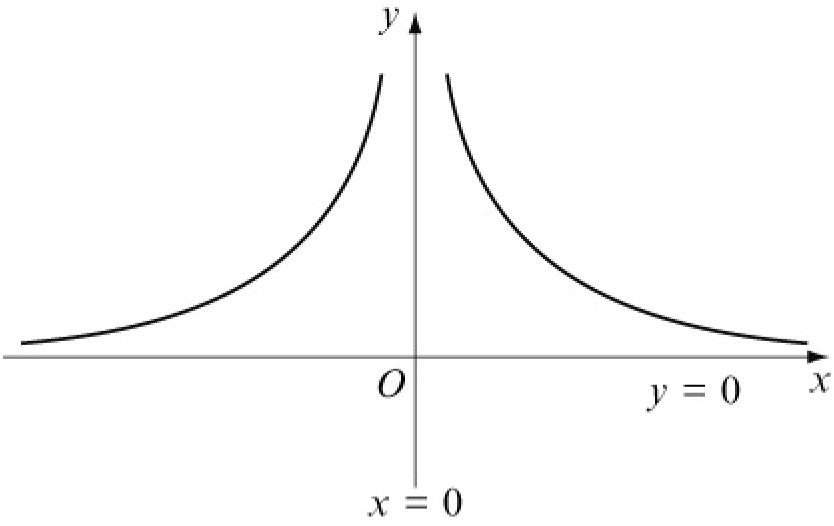


$y = 3f(x) - 1$. Vertical translation of -1.

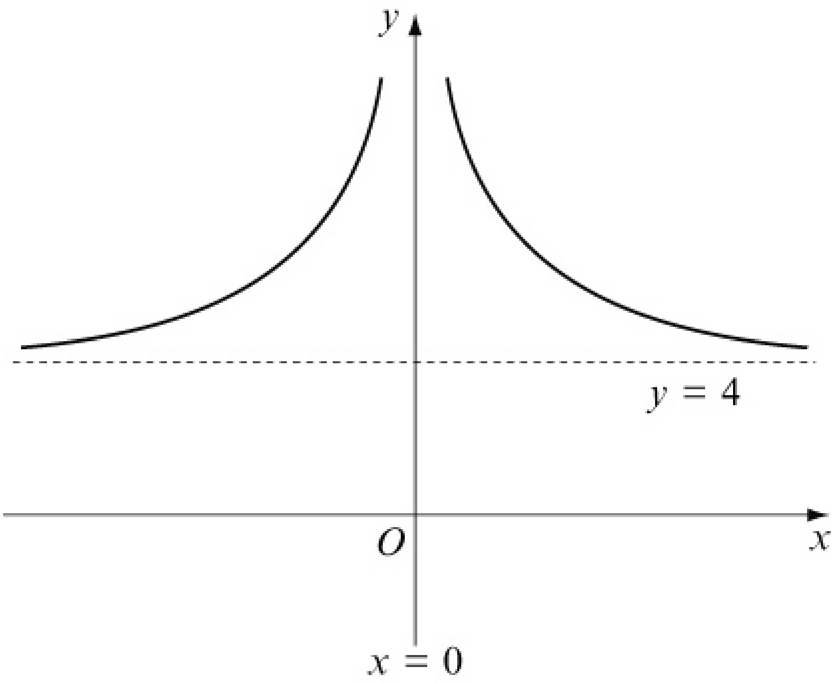


Asymptotes: $x = 2$, $y = -1$

(b) $y = f(x + 2)$. Horizontal translation of -2.

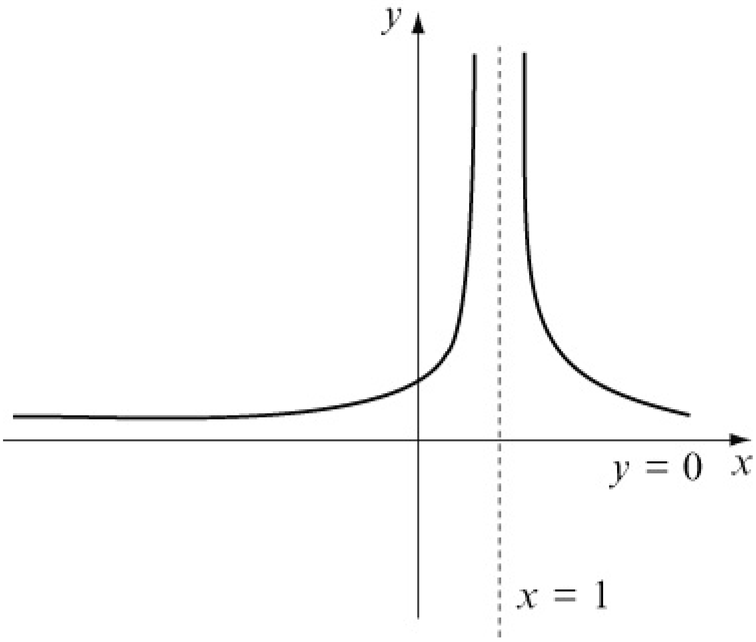


$y = f(x + 2) + 4$. Vertical translation of +4.

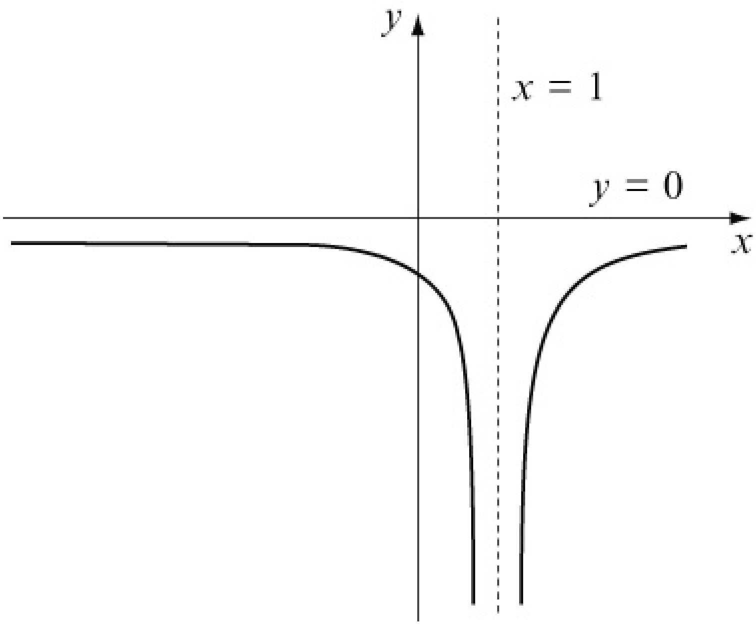


Asymptotes: $x = 0$, $y = 4$

(c) $y = f(2x)$. Horizontal stretch, scale factor $\frac{1}{2}$.



$y = -f(2x)$. Reflection in the x-axis.



Asymptotes: $x = 1$, $y = 0$

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Exercise F, Question 1

Question:

(a) Using the same scales and the same axes, sketch the graphs of $y = |2x|$ and $y = |x - a|$, where $a > 0$.

(b) Write down the coordinates of the points where the graph of $y = |x - a|$ meets the axes.

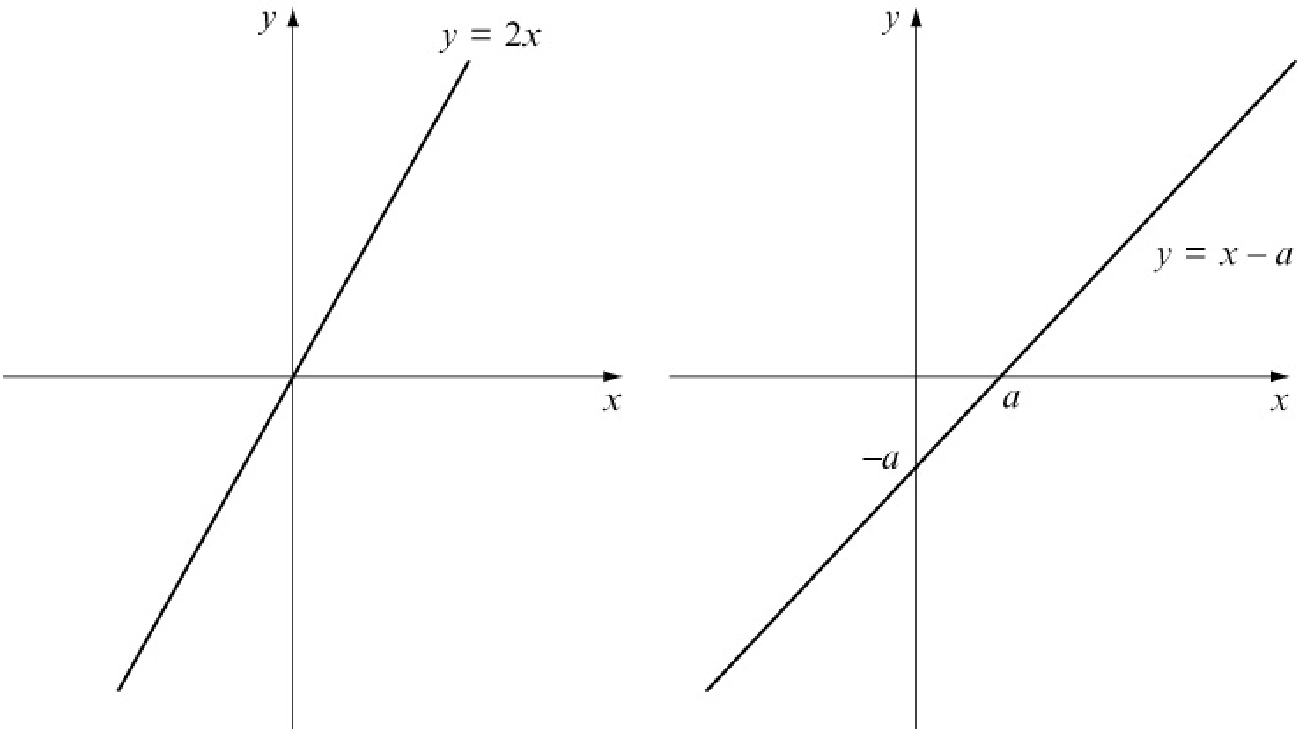
(c) Show that the point with coordinates $(-a, 2a)$ lies on both graphs.

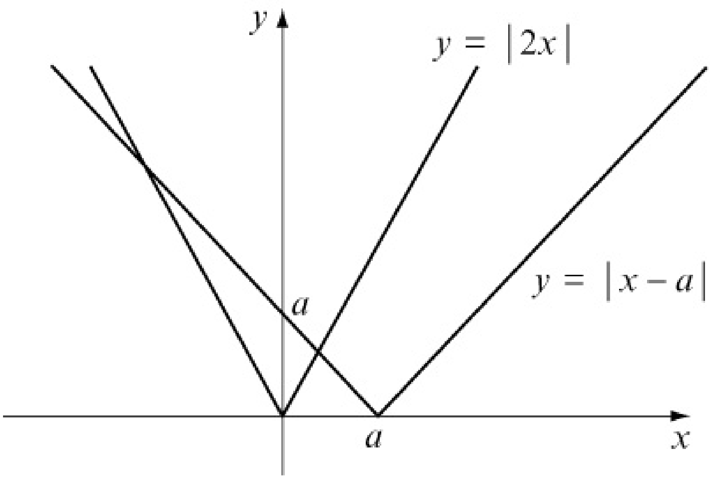
(d) Find the coordinates, in terms of a, of a second point which lies on both graphs.

[E]

Solution:

(a)





(b) For $y = | x - a |$:
When $x = 0$, $y = | - a | = a$ $(0 , a)$
When $y = 0$, $x - a = 0 \Rightarrow x = a$ $(a , 0)$

(c) For $y = | 2x |$:
When $x = - a$, $y = | - 2a | = 2a$
So $(- a , 2a)$ lies on $y = | 2x |$.
For $y = | x - a |$:
When $x = - a$, $y = | - a - a | = | - 2a | = 2a$
So $(- a , 2a)$ lies on $y = | x - a |$.

(d) The other intersection point is on the reflected part of $y = x - a$.
$2x = - (x - a)$
$2x = - x + a$
$3x = a$
$x = \frac{a}{3}$

When $x = \frac{a}{3}$, $y = \left| \frac{2a}{3} \right| = \frac{2a}{3}$

$\left(\frac{a}{3}, \frac{2a}{3} \right)$ lies on both graphs.

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Exercise F, Question 2

Question:

(a) Sketch, on a single diagram, the graphs of $y = a^2 - x^2$ and $y = |x + a|$, where a is a constant and $a > 1$.

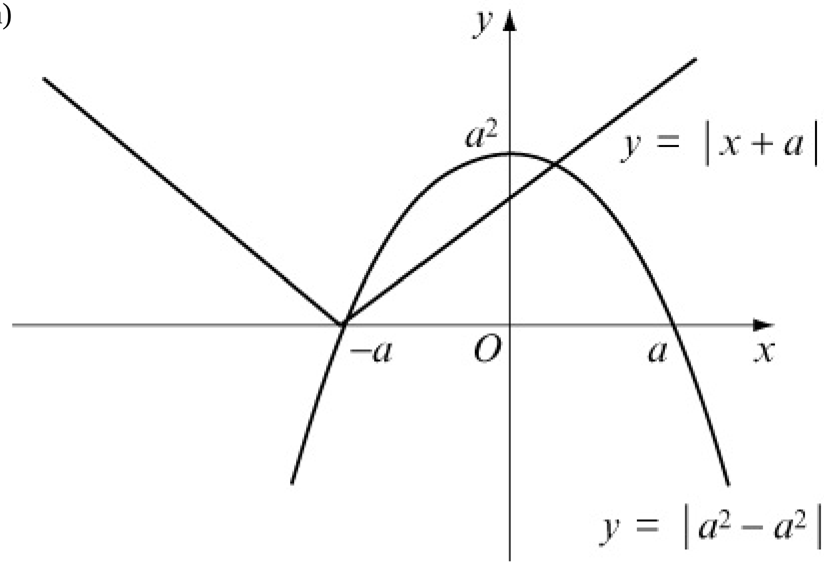
(b) Write down the coordinates of the points where the graph of $y = a^2 - x^2$ cuts the coordinate axes.

(c) Given that the two graphs intersect at $x = 4$, calculate the value of a.

[E]

Solution:

(a)



(b) For $y = a^2 - x^2$:
When $x = 0$, $y = a^2$ $(0, a^2)$
When $y = 0$, $a^2 - x^2 = 0$
$\Rightarrow \quad x^2 = a^2$
$\Rightarrow \quad x = \pm a \qquad (-a, 0)$ and $(a, 0)$

(c) The graphs intersect on the non-reflected part of $y = x + a$.
$a^2 - x^2 = x + a$
Given that $x = 4$:
$a^2 - 4^2 = 4 + a$
$a^2 - a - 20 = 0$

$(a-5)(a+4)=0$
Since $a>1$, $a=5$

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Exercise F, Question 3

Question:

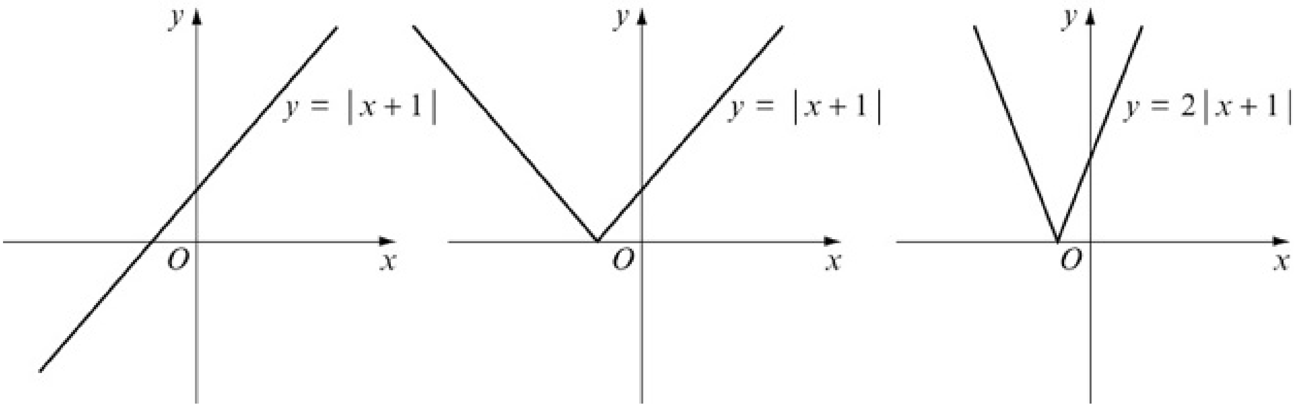
(a) On the same axes, sketch the graphs of $y = 2 - x$ and $y = 2 \mid x + 1 \mid$.

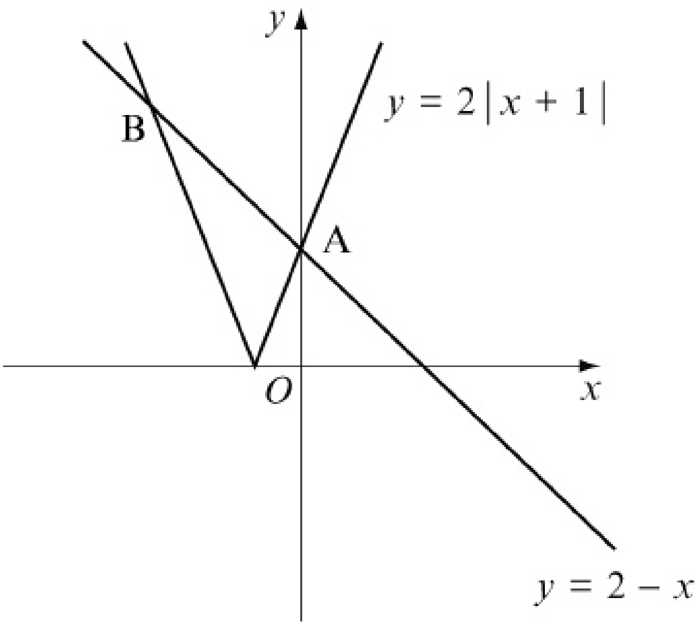
(b) Hence, or otherwise, find the values of x for which $2 - x = 2 \mid x + 1 \mid$.

[E]

Solution:

(a)





(b) Intersection point A:

$2(x + 1) = 2 - x$

$2x + 2 = 2 - x$

$3x = 0$

$x = 0$

Intersection point B is on the reflected part of the modulus graph.

$-2(x+1) = 2 - x$
$-2x - 2 = 2 - x$
$-x = 4$
$x = -4$

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Exercise F, Question 4

Question:

Functions f and g are defined by

$f : x \rightarrow 4 - x \quad \{ x \in \mathbb{R} \{$

$g : x \rightarrow 3x^2 \quad \{ x \in \mathbb{R} \{$

(a) Find the range of g.

(b) Solve $gf(x) = 48$.

(c) Sketch the graph of $y = | f(x) |$ and hence find the values of x for which $| f(x) | = 2$.

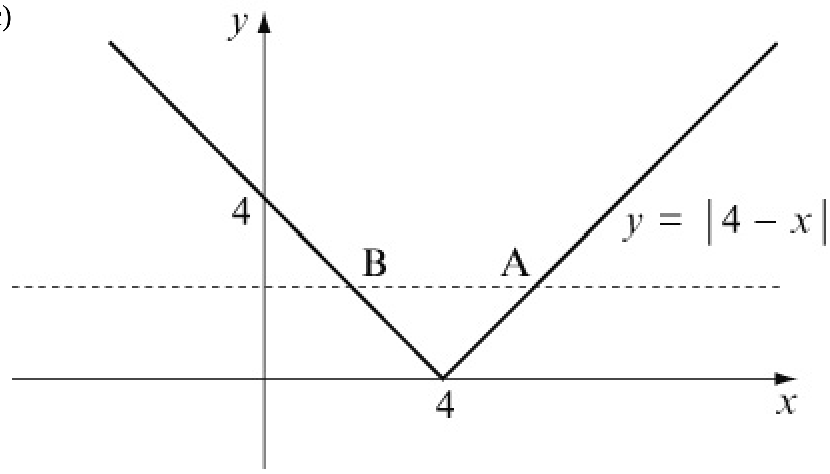
[E]

Solution:

(a) $g(x) = 3x^2$
Since $x^2 \geq 0$ for all $x \in \mathbb{R}$, the range of $g(x)$ is $g(x) \geq 0$

(b) $gf(x) = 48$
$gf(x) = g(4 - x) = 3(4 - x)^2$
$3(4 - x)^2 = 48$
$(4 - x)^2 = 16$
Either $4 - x = 4$ or $4 - x = -4$
So $x = 0$ or $x = 8$

(c)



When $| f(x) | = 2$, $x = 2$ or $x = 6$ (from symmetry of graph).

[**Or:**

At A: $-(4-x)=2 \Rightarrow -4+x=2 \Rightarrow x=6$

At B: $4-x=2 \Rightarrow x=2$]

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Exercise F, Question 5

Question:

The function f is defined by $f : x \to |2x - a| \quad \{ x \in \mathbb{R} \}$, where a is a positive constant.

(a) Sketch the graph of $y = f(x)$, showing the coordinates of the points where the graph cuts the axes.

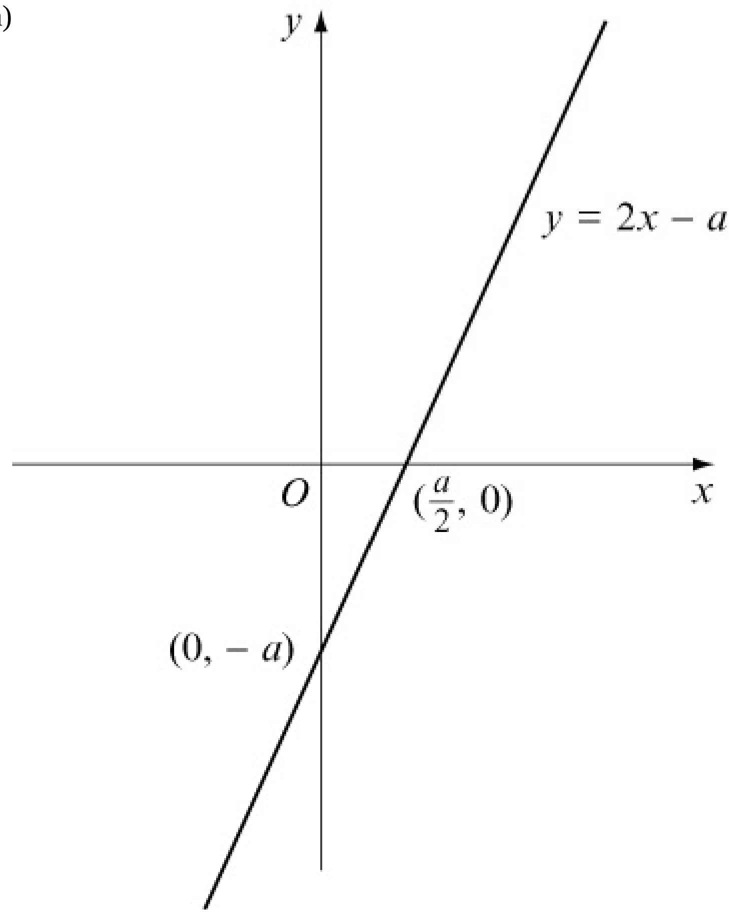
(b) On a separate diagram, sketch the graph of $y = f(2x)$, showing the coordinates of the points where the graph cuts the axes.

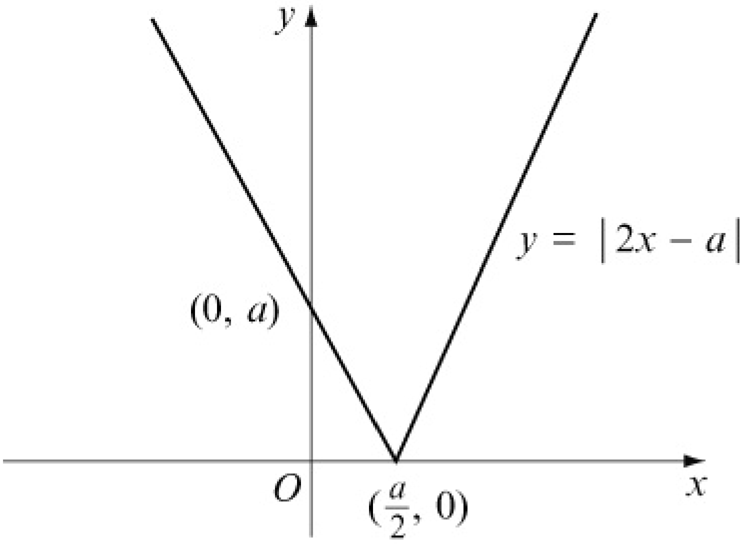
(c) Given that a solution of the equation $f(x) = \frac{1}{2}x$ is $x = 4$, find the two possible values of a.

[E]

Solution:

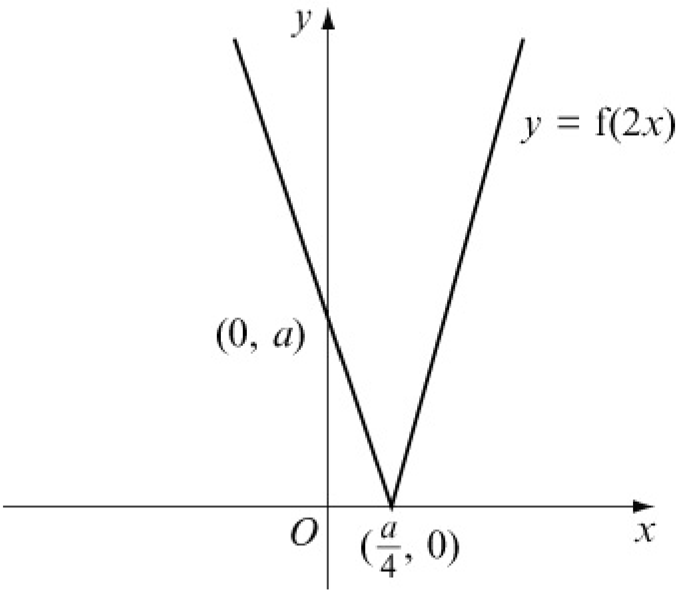
(a)





For $y = |2x - a|$:
When $x = 0$, $y = |-a| = a$ $(0, a)$
When $y = 0$, $2x - a = 0 \Rightarrow x = \frac{a}{2}$ $\left(\frac{a}{2}, 0\right)$

(b) $y = f(2x)$. Horizontal stretch, scale factor $\frac{1}{2}$.



(c) $f(x) = \frac{1}{2}x$: $|2x - a| = \frac{1}{2}x$

Either $(2x - a) = \frac{1}{2}x$ or $-(2x - a) = \frac{1}{2}x$

$2x - a = \frac{1}{2}x \Rightarrow a = \frac{3}{2}x$

Given that $x = 4$, $a = 6$

$$-(2x-a) = \frac{1}{2}x \quad \Rightarrow \quad -2x+a = \frac{1}{2}x \quad \Rightarrow \quad a = \frac{5}{2}x$$

Given that $x = 4$, $a = 10$

Either $a = 6$ or $a = 10$

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Exercise F, Question 6

Question:

(a) Sketch the graph of $y = | x - 2a |$, where a is a positive constant. Show the coordinates of the points where the graph meets the axes.

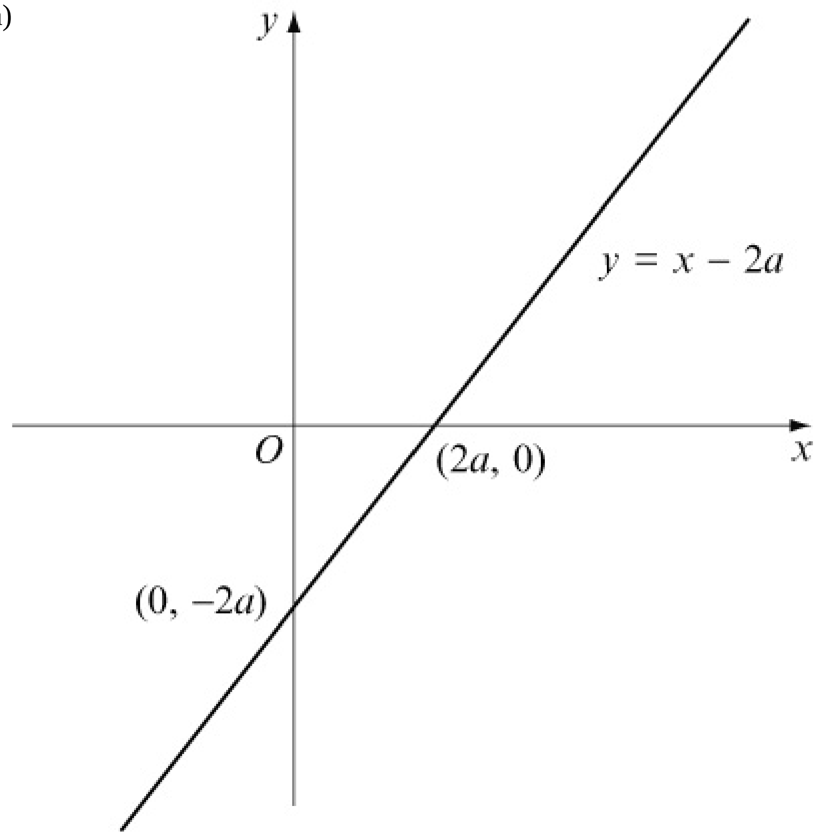
(b) Using algebra solve, for x in terms of a, $| x - 2a | = \frac{1}{3}x$.

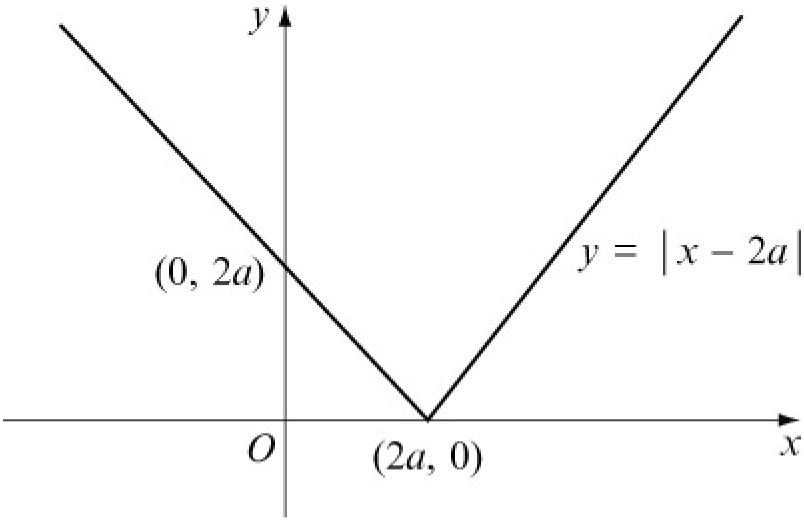
(c) On a separate diagram, sketch the graph of $y = a - | x - 2a |$, where a is a positive constant. Show the coordinates of the points where the graph cuts the axes.

[E]

Solution:

(a)





For $y = |x - 2a|$:
When $x = 0$, $y = |-2a| = 2a$ $(0, 2a)$
When $y = 0$, $x - 2a = 0 \Rightarrow x = 2a$ $(2a, 0)$

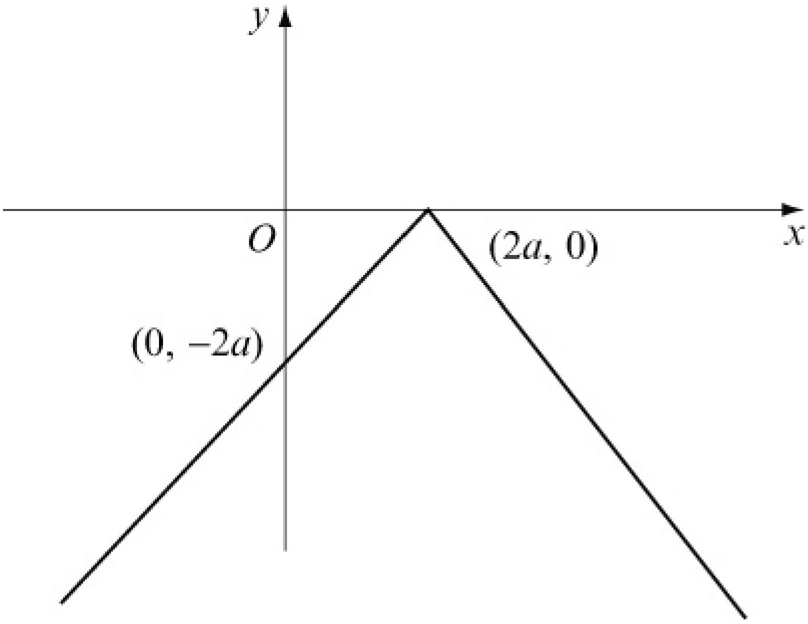
(b) $|x - 2a| = \frac{1}{3}x$

Either $(x - 2a) = \frac{1}{3}x$ or $-(x - 2a) = \frac{1}{3}x$

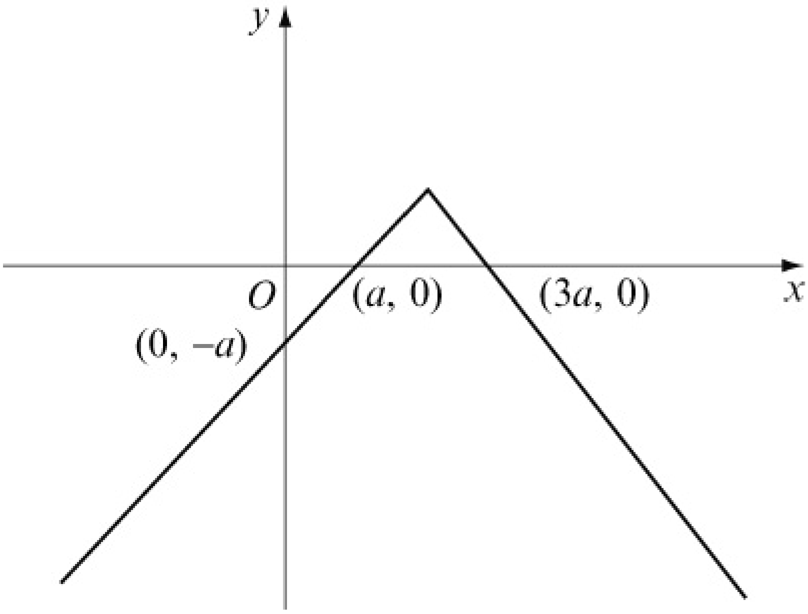
$x - 2a = \frac{1}{3}x \Rightarrow x - \frac{1}{3}x = 2a \Rightarrow \frac{2}{3}x = 2a \Rightarrow x = 3a$

$-x + 2a = \frac{1}{3}x \Rightarrow \frac{4}{3}x = 2a \Rightarrow x = \frac{3}{2}a$

(c) $y = -|x - 2a|$. Reflection in x-axis of $y = |x - 2a|$.



$y = a - | x - 2a |$. Vertical translation of $+ a$.



For $y = a - | x - 2a |$:
When $x = 0$, $y = a - | -2a | = a - 2a = -a$ $(0 , -a)$
When $y = 0$, $a - | x - 2a | = 0$
$| x - 2a | = a$
Either $x - 2a = a \Rightarrow x = 3a$ $(3a , 0)$
or $-(x - 2a) = a \Rightarrow -x + 2a = a \Rightarrow x = a$ $(a , 0)$

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Exercise F, Question 7

Question:

(a) Sketch the graph of $y = |2x + a|$, $a > 0$, showing the coordinates of the points where the graph meets the coordinate axes.

(b) On the same axes, sketch the graph of $y = \frac{1}{x}$.

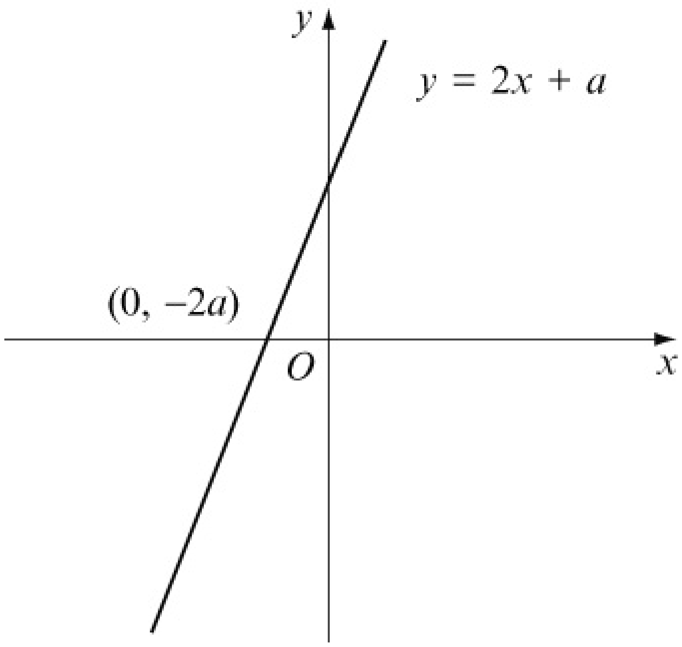
(c) Explain how your graphs show that there is only one solution of the equation $x|2x + a| - 1 = 0$.

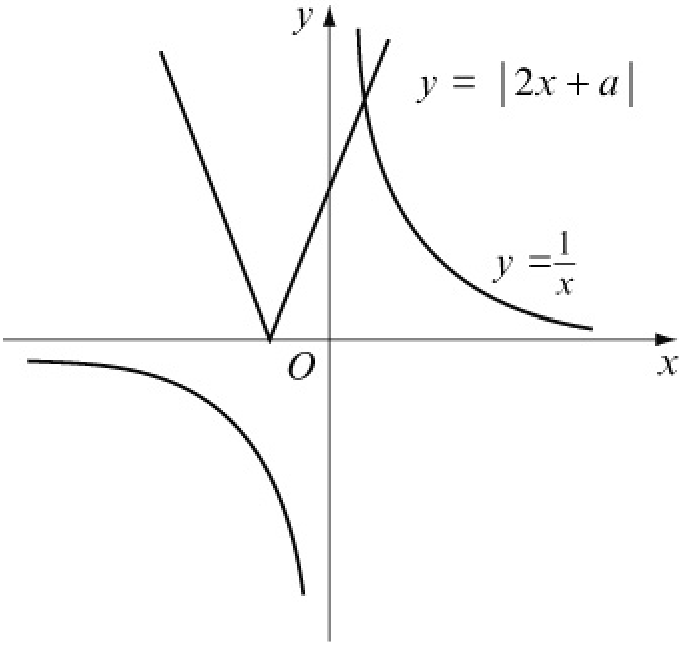
(d) Find, using algebra, the value of x for which $x|2x + a| - 1 = 0$.

[E]

Solution:

(a)(b)





For $y = |2x + a|$:
When $x = 0$, $y = |a| = a$ $(0, a)$
When $y = 0$, $2x + a = 0$ $\Rightarrow$ $x = -\frac{a}{2}$ $\left(-\frac{a}{2}, 0\right)$

(c) Intersection of graphs is given by
$|2x + a| = \frac{1}{x}$
$x|2x + a| = 1$
$x|2x + a| - 1 = 0$
There is only one intersection point, so only one solution.

(d) The intersection point is on the non-reflected part of the modulus graph, so use $(2x + a)$ rather than $-(2x + a)$.
$x(2x + a) - 1 = 0$
$2x^2 + ax - 1 = 0$
$x = \frac{-a \pm \sqrt{a^2 + 8}}{4}$

As shown on the graph, x is positive, so
$x = \frac{-a + \sqrt{a^2 + 8}}{4}$

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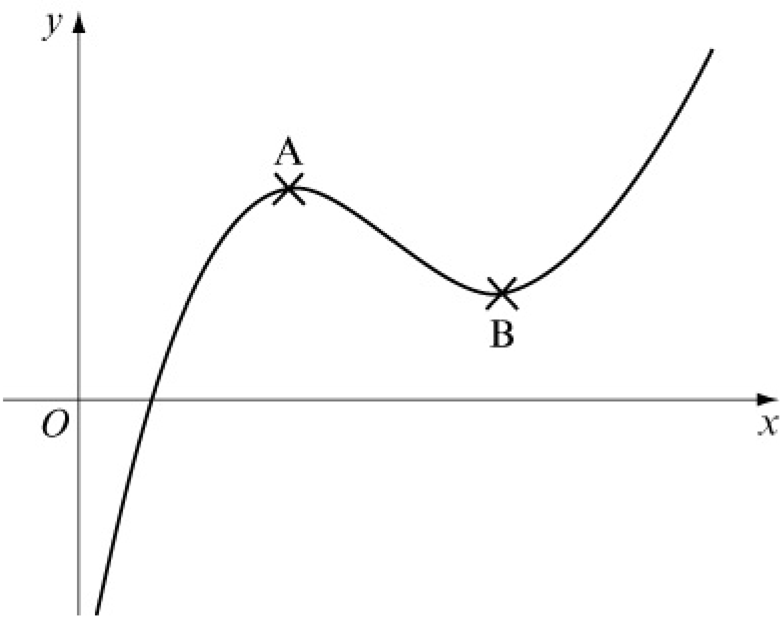
Exercise F, Question 8

Question:

The diagram shows part of the curve with equation $y = f(x)$, where

$f(x) = x^2 - 7x + 5\ln x + 8 \quad x > 0$

The points A and B are the stationary points of the curve.



(a) Using calculus and showing your working, find the coordinates of the points A and B.

(b) Sketch the curve with equation $y = -3f(x-2)$.

(c) Find the coordinates of the stationary points of the curve with equation $y = -3f(x-2)$. State, without proof, which point is a maximum and which point is a minimum.

[E]

Solution:

(a) $f(x) = x^2 - 7x + 5\ln x + 8$

$f'(x) = 2x - 7 + \frac{5}{x}$

At stationary points, $f'(x) = 0$:

$2x - 7 + \frac{5}{x} = 0$

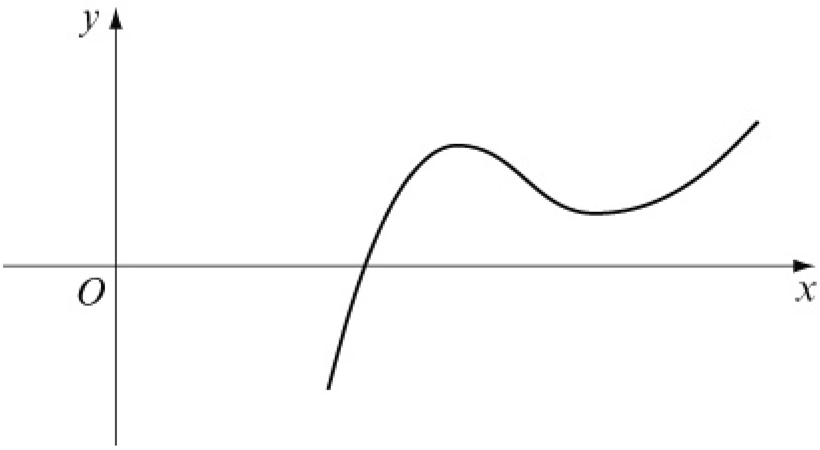
$2x^2 - 7x + 5 = 0$
$(2x - 5)(x - 1) = 0$
$x = \frac{5}{2}, x = 1$

Point A: $x = 1$, $f(x) = 1 - 7 + 5\ln 1 + 8 = 2$
A is (1, 2)

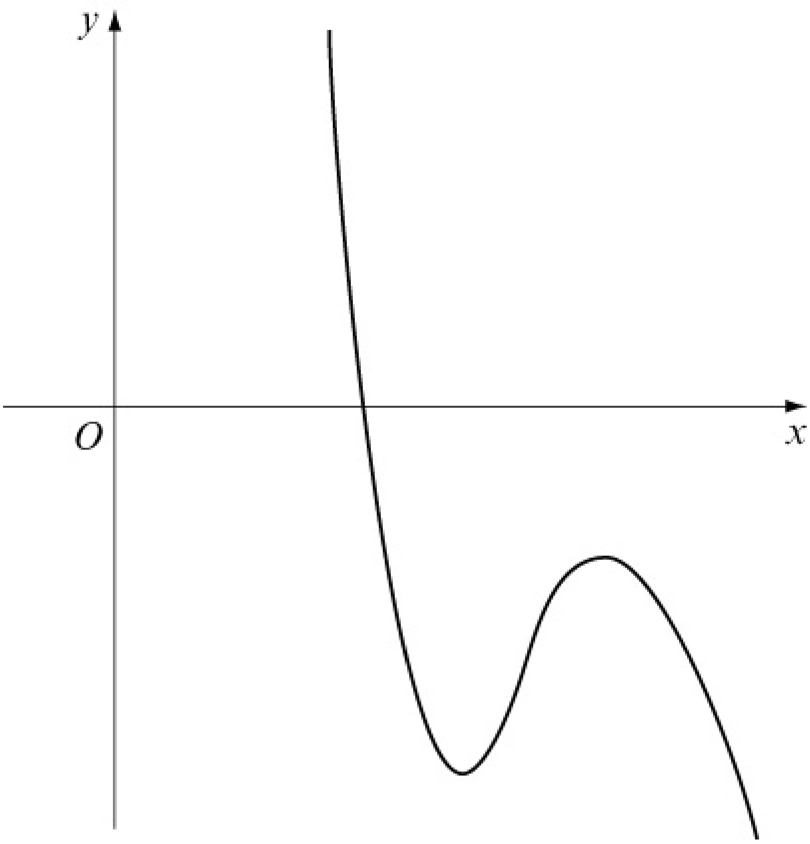
Point B: $x = \frac{5}{2}$, $f(x) = \frac{25}{4} - \frac{35}{2} + 5\ln\frac{5}{2} + 8 = 5\ln\frac{5}{2} - \frac{13}{4}$

B is $\left(\frac{5}{2}, 5\ln\frac{5}{2} - \frac{13}{4}\right)$

(b) $y = f(x - 2)$. Horizontal translation of +2.



$y = -3f(x - 2)$. Reflection in the x-axis, and vertical stretch, scale factor 3.



(c) Using the transformations, point (X, Y) becomes $(X + 2, -3Y)$

$(1, 2) \rightarrow (3, -6)$ Minimum

$\left(\frac{5}{2}, 5\ln\frac{5}{2} - \frac{13}{4}\right) \rightarrow \left(\frac{9}{2}, \frac{39}{4} - 15\ln\frac{5}{2}\right)$ Maximum