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Differentiation
Exercise A, Question 1

Question:

F is the point with co-ordinates $(3, 9)$ on the curve with equation $y = x^2$.

(a) Find the gradients of the chords joining the point F to the points with coordinates:

(i) $(4, 16)$

(ii) $(3.5, 12.25)$

(iii) $(3.1, 9.61)$

(iv) $(3.01, 9.0601)$

(v) $(3 + h, (3 + h)^2)$

(b) What do you deduce about the gradient of the tangent at the point $(3, 9)$?

Solution:

a (i) Gradient $= \frac{16 - 9}{4 - 3} = \frac{7}{1} = 7$

(ii) Gradient $= \frac{12.25 - 9}{3.5 - 3} = \frac{3.25}{0.5} = 6.5$

(iii) Gradient $= \frac{9.61 - 9}{3.1 - 3} = \frac{0.61}{0.1} = 6.1$

(iv) Gradient $= \frac{9.0601 - 9}{3.01 - 3} = \frac{0.0601}{0.01} = 6.01$

(v) Gradient $= \frac{(3 + h)^2 - 9}{(3 + h) - 3} = \frac{9 + 6h + h^2 - 9}{h} = \frac{6h + h^2}{h} = \frac{h(6 + h)}{h} = 6 + h$

(b) The gradient at the point $(3, 9)$ is the value of $6 + h$ as h becomes very small, i.e. the gradient is 6.

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Differentiation
Exercise A, Question 2

Question:

G is the point with coordinates $(4 , 16)$ on the curve with equation $y = x^2$.

(a) Find the gradients of the chords joining the point G to the points with coordinates:

(i) $(5 , 25)$

(ii) $(4.5 , 20.25)$

(iii) $(4.1 , 16.81)$

(iv) $(4.01 , 16.0801)$

(v) $(4 + h , (4 + h)^2)$

(b) What do you deduce about the gradient of the tangent at the point $(4 , 16)$?

Solution:

(a) (i) Gradient $= \frac{25 - 16}{5 - 4} = \frac{9}{1} = 9$

(ii) Gradient $= \frac{20.25 - 16}{4.5 - 4} = \frac{4.25}{0.5} = 8.5$

(iii) Gradient $= \frac{16.81 - 16}{4.1 - 4} = \frac{0.81}{0.1} = 8.1$

(iv) Gradient $= \frac{16.0801 - 16}{4.01 - 4} = \frac{0.0801}{0.01} = 8.01$

(v) Gradient $= \frac{(4 + h)^2 - 16}{4 + h - 4} = \frac{16 + 8h + h^2 - 16}{h} = \frac{8h + h^2}{h} = \frac{h (8 + h)}{h} = 8 + h$

(b) When h is small the gradient of the chord is close to the gradient of the tangent, and $8 + h$ is close to the value 8.
So the gradient of the tangent at $(4 , 16)$ is 8.

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Differentiation
Exercise B, Question 1

Question:

Find the derived function, given that f(x) equals:

x^7

Solution:

$f(x) = x^7$
$f'(x) = 7x^6$

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Differentiation
Exercise B, Question 2

Question:

Find the derived function, given that f(x) equals:

x^8

Solution:

$f(x) = x^8$
$f'(x) = 8x^7$

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Differentiation
Exercise B, Question 3

Question:

Find the derived function, given that f(x) equals:

x^4

Solution:

$f(x) = x^4$
$f'(x) = 4x^3$

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Differentiation
Exercise B, Question 4

Question:

Find the derived function, given that f(x) equals:

$x^{\frac{1}{3}}$

Solution:

$f(x) = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{\frac{1}{3} - 1} = \frac{1}{3}x^{-\frac{2}{3}}$

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Differentiation
Exercise B, Question 5

Question:

Find the derived function, given that f(x) equals:

$x^{\frac{1}{4}}$

Solution:

$f(x) = x^{\frac{1}{4}}$

$f'(x) = \frac{1}{4}x^{\frac{1}{4} - 1} = \frac{1}{4}x^{-\frac{3}{4}}$

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Differentiation
Exercise B, Question 6

Question:

Find the derived function, given that f(x) equals:

$\sqrt[3]{x}$

Solution:

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{\frac{1}{3} - 1} = \frac{1}{3}x^{-\frac{2}{3}}$

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Differentiation
Exercise B, Question 7

Question:

Find the derived function, given that f(x) equals:

x^{-3}

Solution:

$f(x) = x^{-3}$
$f'(x) = -3x^{-3-1} = -3x^{-4}$

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Differentiation
Exercise B, Question 8

Question:

Find the derived function, given that f(x) equals:

x^{-4}

Solution:

$f(x) = x^{-4}$
$f'(x) = -4x^{-4-1} = -4x^{-5}$

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Differentiation
Exercise B, Question 9

Question:

Find the derived function, given that f(x) equals:

$\frac{1}{x^2}$

Solution:

$f(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-2-1} = -2x^{-3}$

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Differentiation
Exercise B, Question 10

Question:

Find the derived function, given that f(x) equals:

$\frac{1}{x^5}$

Solution:

$f(x) = \frac{1}{x^5} = x^{-5}$

$f'(x) = -5x^{-5-1} = -5x^{-6}$

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Differentiation
Exercise B, Question 11

Question:

Find the derived function, given that f(x) equals:

$$\frac{1}{\sqrt[3]{x}}$$

Solution:

$$f(x) = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$$

$$f'(x) = -\frac{1}{3}x^{-\frac{1}{3}-1} = -\frac{1}{3}x^{-\frac{4}{3}}$$

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Differentiation
Exercise B, Question 12

Question:

Find the derived function, given that f(x) equals:

$$\frac{1}{\sqrt{x}}$$

Solution:

$$f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}}$$

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Differentiation
Exercise B, Question 13

Question:

Find the derived function, given that f(x) equals:

$$\frac{x^2}{x^4}$$

Solution:

$f(x) = \frac{x^2}{x^4} = x^{2-4} = x^{-2}$

$f'(x) = -2x^{-2-1} = -2x^{-3}$

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Differentiation
Exercise B, Question 14

Question:

Find the derived function, given that f(x) equals:

$$\frac{x^3}{x^2}$$

Solution:

$f(x) = \frac{x^3}{x^2} = x^{3-2} = x^1$

$f'(x) = 1x^{1-1} = 1x^0 = 1$

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Differentiation
Exercise B, Question 15

Question:

Find the derived function, given that f(x) equals:

$\frac{x^6}{x^3}$

Solution:

$f(x) = \frac{x^6}{x^3} = x^{6-3} = x^3$

$f'(x) = 3x^2$

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Differentiation
Exercise B, Question 16

Question:

Find the derived function, given that f(x) equals:

$x^3 \times x^6$

Solution:

$f(x) = x^3 \times x^6 = x^{3 + 6} = x^9$
$f'(x) = 9x^8$

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Differentiation
Exercise B, Question 17

Question:

Find the derived function, given that f(x) equals:

$x^2 \times x^3$

Solution:

$f(x) = x^2 \times x^3 = x^{2+3} = x^5$
$f'(x) = 5x^4$

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Differentiation
Exercise B, Question 18

Question:

Find the derived function, given that f(x) equals:

$x \times x^2$

Solution:

$f(x) = x \times x^2 = x^{1+2} = x^3$
$f'(x) = 3x^2$

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Differentiation
Exercise C, Question 1

Question:

Find $\frac{dy}{dx}$ when y equals:

(a) $2x^2 - 6x + 3$

(b) $\frac{1}{2}x^2 + 12x$

(c) $4x^2 - 6$

(d) $8x^2 + 7x + 12$

(e) $5 + 4x - 5x^2$

Solution:

(a) $y = 2x^2 - 6x + 3$
$\frac{dy}{dx} = 2(2x) - 6(1) + 0 = 4x - 6$

(b) $y = \frac{1}{2}x^2 + 12x$

$\frac{dy}{dx} = \frac{1}{2}(2x) + 12(1) = x + 12$

(c) $y = 4x^2 - 6$
$\frac{dy}{dx} = 4(2x) - 0 = 8x$

(d) $y = 8x^2 + 7x + 12$
$\frac{dy}{dx} = 8(2x) + 7 + 0 = 16x + 7$

(e) $y = 5 + 4x - 5x^2$
$\frac{dy}{dx} = 0 + 4(1) - 5(2x) = 4 - 10x$

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Differentiation
Exercise C, Question 2

Question:

Find the gradient of the curve whose equation is

(a) $y = 3x^2$ at the point $(2 , 12)$

(b) $y = x^2 + 4x$ at the point $(1 , 5)$

(c) $y = 2x^2 - x - 1$ at the point $(2 , 5)$

(d) $y = \frac{1}{2}x^2 + \frac{3}{2}x$ at the point $(1 , 2)$

(e) $y = 3 - x^2$ at the point $(1 , 2)$

(f) $y = 4 - 2x^2$ at the point $(-1 , 2)$

Solution:

(a) $y = 3x^2$

$\frac{dy}{dx} = 6x$

At the point $(2 , 12)$, $x = 2$.

Substitute $x = 2$ into the gradient expression $\frac{dy}{dx} = 6x$ to give

gradient $= 6 \times 2 = 12$.

(b) $y = x^2 + 4x$

$\frac{dy}{dx} = 2x + 4$

At the point $(1 , 5)$, $x = 1$.

Substitute $x = 1$ into $\frac{dy}{dx} = 2x + 4$ to give

gradient $= 2 \times 1 + 4 = 6$

(c) $y = 2x^2 - x - 1$

$\frac{dy}{dx} = 4x - 1$

At the point $(2 , 5)$, $x = 2$.

Substitute $x = 2$ into $\frac{dy}{dx} = 4x - 1$ to give

gradient $= 4 \times 2 - 1 = 7$

(d) $y = \frac{1}{2}x^2 + \frac{3}{2}x$

$\frac{dy}{dx} = x + \frac{3}{2}$

At the point $(1 , 2)$, $x = 1$.

Substitute $x = 1$ into $\frac{dy}{dx} = x + \frac{3}{2}$ to give

gradient $= 1 + \frac{3}{2} = 2\frac{1}{2}$

(e) $y = 3 - x^2$

$\frac{dy}{dx} = -2x$

At $(1, 2)$, $x = 1$.

Substitute $x = 1$ into $\frac{dy}{dx} = -2x$ to give

gradient $= -2 \times 1 = -2$

(f) $y = 4 - 2x^2$

$\frac{dy}{dx} = -4x$

At $(-1, 2)$, $x = -1$.

Substitute $x = -1$ into $\frac{dy}{dx} = -4x$ to give

gradient $= -4 \times -1 = +4$

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Differentiation
Exercise C, Question 3

Question:

Find the y-coordinate and the value of the gradient at the point P with x-coordinate 1 on the curve with equation $y = 3 + 2x - x^2$.

Solution:

$y = 3 + 2x - x^2$
When $x = 1$, $y = 3 + 2 - 1$
$\Rightarrow \quad y = 4$ when $x = 1$
Differentiate to give
$\frac{dy}{dx} = 0 + 2 - 2x$

When $x = 1$, $\frac{dy}{dx} = 2 - 2$

$\Rightarrow \quad \frac{dy}{dx} = 0$ when $x = 1$

Therefore, the y-coordinate is 4 and the gradient is 0 when the x-coordinate is 1 on the given curve.

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Differentiation
Exercise C, Question 4

Question:

Find the coordinates of the point on the curve with equation $y = x^2 + 5x - 4$ where the gradient is 3.

Solution:

$y = x^2 + 5x - 4$

$\frac{dy}{dx} = 2x + 5$

Put $\frac{dy}{dx} = 3$

Then $2x + 5 = 3$

$\Rightarrow \quad 2x = -2$

$\Rightarrow \quad x = -1$

Substitute $x = -1$ into $y = x^2 + 5x - 4$:

$y = (-1)^2 + 5(-1) - 4 = 1 - 5 - 4 = -8$

Therefore, $(-1, -8)$ is the point where the gradient is 3.

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Differentiation
Exercise C, Question 5

Question:

Find the gradients of the curve $y = x^2 - 5x + 10$ at the points A and B where the curve meets the line $y = 4$.

Solution:

The curve $y = x^2 - 5x + 10$ meets the line $y = 4$ when
$x^2 - 5x + 10 = 4$
$x^2 - 5x + 6 = 0$
$(x - 3) (x - 2) = 0$
$x = 3$ or $x = 2$
The gradient function for the curve is given by
$\frac{dy}{dx} = 2x - 5$

when $x = 3$, $\frac{dy}{dx} = 2 \times 3 - 5 = 1$

when $x = 2$, $\frac{dy}{dx} = 2 \times 2 - 5 = -1$

So the gradients are -1 and 1 at $(2 , 4)$ and $(3 , 4)$ respectively.

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Differentiation
Exercise C, Question 6

Question:

Find the gradients of the curve $y = 2x^2$ at the points C and D where the curve meets the line $y = x + 3$.

Solution:

The curve $y = 2x^2$ meets the line $y = x + 3$ when
$2x^2 = x + 3$
$2x^2 - x - 3 = 0$
$(2x - 3)(x + 1) = 0$
$x = 1.5$ or -1

The gradient of the curve is given by the equation $\frac{dy}{dx} = 4x$.

The gradient at the point where $x = -1$ is $4 \times -1 = -4$.
The gradient at the point where $x = 1.5$ is $4 \times 1.5 = 6$.
So the gradient is -4 at $(-1, 2)$ and is 6 at $(1.5, 4.5)$.

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Differentiation
Exercise D, Question 1

Question:

Use standard results to differentiate:

(a) $x^4 + x^{-1}$

(b) $\frac{1}{2}x^{-2}$

(c) $2x^{-\frac{1}{2}}$

Solution:

(a) $f(x) = x^4 + x^{-1}$
$f'(x) = 4x^3 + (-1)x^{-2}$

(b) $f(x) = \frac{1}{2}x^{-2}$

$f'(x) = \frac{1}{2}(-2)x^{-3} = -x^{-3}$

(c) $f(x) = 2x^{-\frac{1}{2}}$

$f'(x) = 2\left(-\frac{1}{2}\right)x^{-1\frac{1}{2}} = -x^{-\frac{3}{2}}$

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Differentiation
Exercise D, Question 2

Question:

Find the gradient of the curve with equation $y = f(x)$ at the point A where:

(a) $f(x) = x^3 - 3x + 2$ and A is at $(-1, 4)$

(b) $f(x) = 3x^2 + 2x^{-1}$ and A is at $(2, 13)$

Solution:

(a) $f(x) = x^3 - 3x + 2$
$f'(x) = 3x^2 - 3$
At $(-1, 4)$, $x = -1$.
Substitute $x = -1$ to find $f'(-1) = 3(-1)^2 - 3 = 0$
Therefore, gradient $= 0$.

(b) $f(x) = 3x^2 + 2x^{-1}$
$f'(x) = 6x + 2(-1)x^{-2} = 6x - 2x^{-2}$
At $(2, 13)$, $x = 2$.
$f'(2) = 6(2) - 2(2)^{-2} = 12 - \frac{2}{4} = 11\frac{1}{2}$

Therefore, gradient $= 11\frac{1}{2}$.

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Differentiation
Exercise D, Question 3

Question:

Find the point or points on the curve with equation $y = f(x)$, where the gradient is zero:

(a) $f(x) = x^2 - 5x$

(b) $f(x) = x^3 - 9x^2 + 24x - 20$

(c) $f(x) = x^{\frac{3}{2}} - 6x + 1$

(d) $f(x) = x^{-1} + 4x$

Solution:

(a) $f(x) = x^2 - 5x$
$f'(x) = 2x - 5$
When gradient is zero, $f'(x) = 0$.
$\Rightarrow \quad 2x - 5 = 0$
$\Rightarrow \quad x = 2.5$
As $y = f(x)$, $y = f(2.5)$ when $x = 2.5$.
$\Rightarrow \quad y = (2.5)^2 - 5(2.5) = -6.25$
Therefore, $(2.5, -6.25)$ is the point on the curve where the gradient is zero.

(b) $f(x) = x^3 - 9x^2 + 24x - 20$
$f'(x) = 3x^2 - 18x + 24$
When gradient is zero, $f'(x) = 0$.
$\Rightarrow \quad 3x^2 - 18x + 24 = 0$
$\Rightarrow \quad 3(x^2 - 6x + 8) = 0$
$\Rightarrow \quad 3(x-4)(x-2) = 0$
$\Rightarrow \quad x = 4$ or $x = 2$
As $y = f(x)$, $y = f(4)$ when $x = 4$.
$\Rightarrow \quad y = 4^3 - 9 \times 4^2 + 24 \times 4 - 20 = -4$
Also $y = f(2)$ when $x = 2$.
$\Rightarrow \quad y = 2^3 - 9 \times 2^2 + 24 \times 2 - 20 = 0$.
Therefore, at $(4, -4)$ and at $(2, 0)$ the gradient is zero.

(c) $f(x) = x^{\frac{3}{2}} - 6x + 1$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 6$

When gradient is zero, $f'(x) = 0$.

$\Rightarrow \quad \frac{3}{2}x^{\frac{1}{2}} - 6 = 0$

$\Rightarrow \quad x^{\frac{1}{2}} = 4$

$\Rightarrow \quad x = 16$
As $y = f(x)$, $y = f(16)$ when $x = 16$.

$\Rightarrow \quad y = 16^{\frac{3}{2}} - 6 \times 16 + 1 = -31$

Therefore, at $(16, -31)$ the gradient is zero.

(d) $f(x) = x^{-1} + 4x$

$f'(x) = -1x^{-2} + 4$

For zero gradient, $f'(x) = 0$.

$\Rightarrow \quad -x^{-2} + 4 = 0$

$\Rightarrow \quad \frac{1}{x^2} = 4$

$\Rightarrow \quad x = \pm \frac{1}{2}$

When $x = \frac{1}{2}$, $y = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{-1} + 4\left(\frac{1}{2}\right) = 2 + 2 = 4$

When $x = -\frac{1}{2}$, $y = f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^{-1} + 4\left(-\frac{1}{2}\right) = -2 - 2 = -4$

Therefore, $\left(\frac{1}{2}, 4\right)$ and $\left(-\frac{1}{2}, -4\right)$ are points on the curve where the gradient is zero.

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Differentiation
Exercise E, Question 1

Question:

Use standard results to differentiate:

(a) $2\sqrt{x}$

(b) $\frac{3}{x^2}$

(c) $\frac{1}{3x^3}$

(d) $\frac{1}{3}x^3(x-2)$

(e) $\frac{2}{x^3}+\sqrt{x}$

(f) $\sqrt[3]{x}+\frac{1}{2x}$

(g) $\frac{2x+3}{x}$

(h) $\frac{3x^2-6}{x}$

(i) $\frac{2x^3+3x}{\sqrt{x}}$

(j) $x(x^2-x+2)$

(k) $3x^2(x^2+2x)$

(l) $(3x-2)\left(4x+\frac{1}{x}\right)$

Solution:

(a) $y=2\sqrt{x}=2x^{\frac{1}{2}}$

$\frac{dy}{dx}=2\left(\frac{1}{2}\right)x^{-\frac{1}{2}}=x^{-\frac{1}{2}}$

(b) $y = \frac{3}{x^2} = 3x^{-2}$

$\frac{dy}{dx} = 3(-2)x^{-3} = -6x^{-3}$

(c) $y = \frac{1}{3x^3} = \frac{1}{3}x^{-3}$

$\frac{dy}{dx} = \frac{1}{3}(-3)x^{-4} = -x^{-4}$

(d) $y = \frac{1}{3}x^3(x-2) = \frac{1}{3}x^4 - \frac{2}{3}x^3$

$\frac{dy}{dx} = \frac{4}{3}x^3 - \frac{2}{3} \times 3x^2 = \frac{4}{3}x^3 - 2x^2$

(e) $y = \frac{2}{x^3} + \sqrt{x} = 2x^{-3} + x^{\frac{1}{2}}$

$\frac{dy}{dx} = -6x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$

(f) $y = \sqrt[3]{x} + \frac{1}{2x} = x^{\frac{1}{3}} + \frac{1}{2}x^{-1}$

$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$

(g) $y = \frac{2x+3}{x} = \frac{2x}{x} + \frac{3}{x} = 2 + 3x^{-1}$

$\frac{dy}{dx} = 0 - 3x^{-2}$

(h) $y = \frac{3x^2-6}{x} = \frac{3x^2}{x} - \frac{6}{x} = 3x - 6x^{-1}$

$\frac{dy}{dx} = 3 + 6x^{-2}$

(i) $y = \frac{2x^3+3x}{\sqrt{x}} = \frac{2x^3}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} = 2x^{2\frac{1}{2}} + 3x^{\frac{1}{2}}$

$\frac{dy}{dx} = 5x^{1\frac{1}{2}} + 1.5x^{-\frac{1}{2}}$

(j) $y = x(x^2 - x + 2) = x^3 - x^2 + 2x$

$\frac{dy}{dx} = 3x^2 - 2x + 2$

(k) $y = 3x^2(x^2 + 2x) = 3x^4 + 6x^3$

$\frac{dy}{dx} = 12x^3 + 18x^2$

(l) $y = (3x - 2)\left(4x + \frac{1}{x}\right) = 12x^2 - 8x + 3 - \frac{2}{x} = 12x^2 - 8x + 3 - 2x^{-1}$

$\frac{dy}{dx} = 24x - 8 + 2x^{-2}$

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Differentiation
Exercise E, Question 2

Question:

Find the gradient of the curve with equation $y = f(x)$ at the point A where:

(a) $f(x) = x(x+1)$ and A is at $(0, 0)$

(b) $f(x) = \frac{2x-6}{x^2}$ and A is at $(3, 0)$

(c) $f(x) = \frac{1}{\sqrt{x}}$ and A is at $\left(\frac{1}{4}, 2\right)$

(d) $f(x) = 3x - \frac{4}{x^2}$ and A is at $(2, 5)$

Solution:

(a) $f(x) = x(x+1) = x^2 + x$
$f'(x) = 2x + 1$
At $(0, 0)$, $x = 0$.
Therefore, gradient $= f'(0) = 1$

(b) $f(x) = \frac{2x-6}{x^2} = \frac{2x}{x^2} - \frac{6}{x^2} = \frac{2}{x} - 6x^{-2} = 2x^{-1} - 6x^{-2}$
$f'(x) = -2x^{-2} + 12x^{-3}$
At $(3, 0)$, $x = 3$.
Therefore, gradient $= f'(3) = -\frac{2}{3^2} + \frac{12}{3^3} = -\frac{2}{9} + \frac{12}{27} = \frac{2}{9}$

(c) $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$

At $\left(\frac{1}{4}, 2\right)$, $x = \frac{1}{4}$.

Therefore, gradient $= f'\left(\frac{1}{4}\right) = -\frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}} = -\frac{1}{2} \times 2^3 = -4$

(d) $f(x) = 3x - \frac{4}{x^2} = 3x - 4x^{-2}$
$f'(x) = 3 + 8x^{-3}$
At $(2, 5)$, $x = 2$.
Therefore, gradient $= f'(2) = 3 + 8(2)^{-3} = 3 + \frac{8}{8} = 4$.

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Differentiation
Exercise F, Question 1

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$12x^2 + 3x + 8$

Solution:

$y = 12x^2 + 3x + 8$

$\frac{dy}{dx} = 24x + 3$

$\frac{d^2y}{dx^2} = 24$

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Differentiation
Exercise F, Question 2

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$15x + 6 + \frac{3}{x}$

Solution:

$y = 15x + 6 + \frac{3}{x} = 15x + 6 + 3x^{-1}$

$\frac{dy}{dx} = 15 - 3x^{-2}$

$\frac{d^2y}{dx^2} = 0 + 6x^{-3}$

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Differentiation
Exercise F, Question 3

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$9\sqrt{x} - \frac{3}{x^2}$

Solution:

$y = 9\sqrt{x} - \frac{3}{x^2} = 9x^{\frac{1}{2}} - 3x^{-2}$

$\frac{dy}{dx} = 4\frac{1}{2}x^{-\frac{1}{2}} + 6x^{-3}$

$\frac{d^2y}{dx^2} = -2\frac{1}{4}x^{-\frac{3}{2}} - 18x^{-4}$

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Differentiation
Exercise F, Question 4

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$(5x + 4) (3x - 2)$

Solution:

$y = (5x + 4)(3x - 2) = 15x^2 + 2x - 8$

$\frac{dy}{dx} = 30x + 2$

$\frac{d^2y}{dx^2} = 30$

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Differentiation
Exercise F, Question 5

Question:

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when y equals:

$\frac{3x+8}{x^2}$

Solution:

$y = \frac{3x+8}{x^2} = \frac{3x}{x^2} + \frac{8}{x^2} = \frac{3}{x} + 8x^{-2} = 3x^{-1} + 8x^{-2}$

$\frac{dy}{dx} = -3x^{-2} - 16x^{-3}$

$\frac{d^2y}{dx^2} = 6x^{-3} + 48x^{-4}$

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Differentiation
Exercise G, Question 1

Question:

Find $\frac{d\theta}{dt}$ where $\theta = t^2 - 3t$

Solution:

$\theta = t^2 - 3t$

$\frac{d\theta}{dt} = 2t - 3$

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Differentiation
Exercise G, Question 2

Question:

Find $\frac{\mathrm{d}A}{\mathrm{d}r}$ where $A = 2 \pi r$

Solution:

$A = 2 \pi r$

$\frac{\mathrm{d}A}{\mathrm{d}r} = 2 \pi$

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Differentiation
Exercise G, Question 3

Question:

Find $\frac{dr}{dt}$ where $r = \frac{12}{t}$

Solution:

$r = \frac{12}{t} = 12t^{-1}$

$\frac{dr}{dt} = -12t^{-2}$

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Differentiation
Exercise G, Question 4

Question:

Find $\frac{dv}{dt}$ where $v = 9.8t + 6$

Solution:

$v = 9.8t + 6$

$\frac{dv}{dt} = 9.8$

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Differentiation
Exercise G, Question 5

Question:

Find $\frac{dR}{dr}$ where $R = r + \frac{5}{r}$

Solution:

$R = r + \frac{5}{r} = r + 5r^{-1}$

$\frac{dR}{dr} = 1 - 5r^{-2}$

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Differentiation
Exercise G, Question 6

Question:

Find $\frac{dx}{dt}$ where $x = 3 - 12t + 4t^2$

Solution:

$x = 3 - 12t + 4t^2$

$\frac{dx}{dt} = 0 - 12 + 8t$

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Differentiation
Exercise G, Question 7

Question:

Find $\frac{\mathrm{d}A}{\mathrm{d}x}$ where $A = x (10 - x)$

Solution:

$A = x(10 - x) = 10x - x^2$

$\frac{\mathrm{d}A}{\mathrm{d}x} = 10 - 2x$

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Differentiation
Exercise H, Question 1

Question:

Find the equation of the tangent to the curve:

(a) $y = x^2 - 7x + 10$ at the point $(2 , 0)$

(b) $y = x + \frac{1}{x}$ at the point $\left(2 , 2 \frac{1}{2} \right)$

(c) $y = 4 \sqrt{x}$ at the point $(9 , 12)$

(d) $y = \frac{2x - 1}{x}$ at the point $(1 , 1)$

(e) $y = 2x^3 + 6x + 10$ at the point $(- 1 , 2)$

(f) $y = x^2 + \frac{- 7}{x^2}$ at the point $(1 , - 6)$

Solution:

(a) $y = x^2 - 7x + 10$

$\frac{dy}{dx} = 2x - 7$

At $(2 , 0)$, $x = 2$, gradient $= 2 \times 2 - 7 = - 3$.
Therefore, equation of tangent is
$y - 0 = - 3 (x - 2)$
$y = - 3x + 6$
$y + 3x - 6 = 0$

(b) $y = x + \frac{1}{x} = x + x^{- 1}$

$\frac{dy}{dx} = 1 - x^{- 2}$

At $\left(2 , 2 \frac{1}{2} \right)$, $x = 2$, gradient $= 1 - 2^{- 2} = \frac{3}{4}$.

Therefore, equation of tangent is

$y - 2 \frac{1}{2} = \frac{3}{4}(x - 2)$

$y = \frac{3}{4}x - 1 \frac{1}{2} + 2 \frac{1}{2}$

$y = \frac{3}{4}x + 1$

$4y - 3x - 4 = 0$

(c) $y = 4 \sqrt{x} = 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$

At $(9, 12)$, $x = 9$, gradient $= 2 \times 9^{-\frac{1}{2}} = \frac{2}{3}$.

Therefore, equation of tangent is

$y - 12 = \frac{2}{3}(x - 9)$

$y = \frac{2}{3}x - 6 + 12$

$y = \frac{2}{3}x + 6$

$3y - 2x - 18 = 0$

(d) $y = \frac{2x - 1}{x} = \frac{2x}{x} - \frac{1}{x} = 2 - x^{-1}$

$\frac{dy}{dx} = 0 + x^{-2}$

At $(1, 1)$, $x = 1$, gradient $= 1^{-2} = 1$.
Therefore, equation of tangent is
$y - 1 = 1 \times (x - 1)$
$y = x$

(e) $y = 2x^3 + 6x + 10$
$\frac{dy}{dx} = 6x^2 + 6$

At $(-1, 2)$, $x = -1$, gradient $= 6(-1)^2 + 6 = 12$.
Therefore, equation of tangent is
$y - 2 = 12[x - (-1)]$
$y - 2 = 12x + 12$
$y = 12x + 14$

(f) $y = x^2 - \frac{7}{x^2} = x^2 - 7x^{-2}$

$\frac{dy}{dx} = 2x + 14x^{-3}$

At $(1, -6)$, $x = 1$, gradient $= 2 + 14 = 16$.
Therefore, equation of tangent is
$y - (-6) = 16(x - 1)$
$y + 6 = 16x - 16$
$y = 16x - 22$

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Differentiation
Exercise H, Question 2

Question:

Find the equation of the normal to the curves:

(a) $y = x^2 - 5x$ at the point $(6 , 6)$

(b) $y = x^2 - \frac{8}{\sqrt{x}}$ at the point $(4 , 12)$

Solution:

(a) $y = x^2 - 5x$

$\frac{dy}{dx} = 2x - 5$

At $(6 , 6)$, $x = 6$, gradient of curve is $2 \times 6 - 5 = 7$.

Therefore, gradient of normal is $- \frac{1}{7}$.

The equation of the normal is

$y - 6 = - \frac{1}{7} (x - 6)$

$7y - 42 = - x + 6$
$7y + x - 48 = 0$

(b) $y = x^2 - \frac{8}{\sqrt{x}} = x^2 - 8x^{- \frac{1}{2}}$

$\frac{dy}{dx} = 2x + 4x^{- \frac{3}{2}}$

At $(4 , 12)$, $x = 4$, gradient of curve is $2 \times 4 + 4 (4)^{- \frac{3}{2}} = 8 + \frac{4}{8} = \frac{17}{2}$

Therefore, gradient of normal is $- \frac{2}{17}$.

The equation of the normal is

$y - 12 = - \frac{2}{17} (x - 4)$

$17y - 204 = - 2x + 8$
$17y + 2x - 212 = 0$

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Differentiation
Exercise H, Question 3

Question:

Find the coordinates of the point where the tangent to the curve $y = x^2 + 1$ at the point $(2 , 5)$ meets the normal to the same curve at the point $(1 , 2)$.

Solution:

$y = x^2 + 1$

$\frac{dy}{dx} = 2x$

At $(2 , 5)$, $x = 2$, $\frac{dy}{dx} = 4$.

The tangent at $(2 , 5)$ has gradient 4.
Its equation is
$y - 5 = 4 (x - 2)$
$y = 4x - 3$ ①
The curve has gradient 2 at the point $(1 , 2)$.

The normal is perpendicular to the curve. Its gradient is $-\frac{1}{2}$.

The equation of the normal is

$y - 2 = -\frac{1}{2}(x - 1)$

$y = -\frac{1}{2}x + 2\frac{1}{2}$ ②

Solve Equations ① and ② to find where the tangent and the normal meet.
Equation ① − Equation ②:

$0 = 4\frac{1}{2}x - 5\frac{1}{2}$

$x = \frac{11}{9}$

Substitute into Equation ① to give $y = \frac{44}{9} - 3 = \frac{17}{9}$.

Therefore, the tangent at $(2 , 5)$ meets the normal at $(1 , 2)$ at $\left(\frac{11}{9} , \frac{17}{9} \right)$.

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Differentiation
Exercise H, Question 4

Question:

Find the equations of the normals to the curve $y = x + x^3$ at the points $(0 , 0)$ and $(1 , 2)$, and find the coordinates of the point where these normals meet.

Solution:

$y = x + x^3$

$\frac{dy}{dx} = 1 + 3x^2$

At $(0 , 0)$ the curve has gradient $1 + 3 \times 0^2 = 1$.

The gradient of the normal at $(0 , 0)$ is $- \frac{1}{1} = - 1$.

The equation of the normal at $(0 , 0)$ is

$y - 0 = - 1 (x - 0)$

$y = - x$ ①

At $(1 , 2)$ the curve has gradient $1 + 3 \times 1^2 = 4$.

The gradient of the normal at $(1 , 2)$ is $- \frac{1}{4}$.

The equation of the normal at $(1 , 2)$ is

$y - 2 = - \frac{1}{4}(x - 1)$

$4y - 8 = - x + 1$

$4y + x - 9 = 0$ ②

Solve Equations ① and ② to find where the normals meet.

Substitute $y = - x$ into Equation ②:

$- 4x + x = 9 \quad \Rightarrow \quad x = - 3$ and $y = + 3$.

Therefore, the normals meet at $(- 3 , 3)$.

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Differentiation
Exercise H, Question 5

Question:

For $f(x) = 12 - 4x + 2x^2$, find an equation of the tangent and normal at the point where $x = -1$ on the curve with equation $y = f(x)$. **[E]**

Solution:

$y = 12 - 4x + 2x^2$

$\frac{dy}{dx} = 0 - 4 + 4x$

when $x = -1$, $\frac{dy}{dx} = -4 - 4 = -8$.

The gradient of the curve is -8 when $x = -1$.
As $y = f(x)$, when $x = -1$
$y = f(-1) = 12 + 4 + 2 = 18$
The tangent at $(-1, 18)$ has gradient -8. So its equation is
$y - 18 = -8(x + 1)$
$y - 18 = -8x - 8$
$y = 10 - 8x$

The normal at $(-1, 18)$ has gradient $\frac{-1}{-8} = \frac{1}{8}$. So its equation is

$y - 18 = \frac{1}{8}\left(x + 1\right)$

$8y - 144 = x + 1$
$8y - x - 145 = 0$

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Differentiation
Exercise I, Question 1

Question:

A curve is given by the equation $y = 3x^2 + 3 + \frac{1}{x^2}$, where $x > 0$.

At the points A, B and C on the curve, $x = 1$, 2 and 3 respectively.
Find the gradients at A, B and C. **[E]**

Solution:

$y = 3x^2 + 3 + \frac{1}{x^2} = 3x^2 + 3 + x^{-2}$

$\frac{dy}{dx} = 6x - 2x^{-3} = 6x - \frac{2}{x^3}$

When $x = 1$, $\frac{dy}{dx} = 6 \times 1 - \frac{2}{1^3} = 4$

When $x = 2$, $\frac{dy}{dx} = 6 \times 2 - \frac{2}{2^3} = 12 - \frac{2}{8} = 11\frac{3}{4}$

When $x = 3$, $\frac{dy}{dx} = 6 \times 3 - \frac{2}{3^3} = 18 - \frac{2}{27} = 17\frac{25}{27}$

The gradients at points A, B and C are 4, $11\frac{3}{4}$ and $17\frac{25}{27}$, respectively.

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Differentiation
Exercise I, Question 2

Question:

Taking $f(x) = \frac{1}{4}x^4 - 4x^2 + 25$, find the values of x for which $f'(x) = 0$. **[E]**

Solution:

$f(x) = \frac{1}{4}x^4 - 4x^2 + 25$

$f'(x) = x^3 - 8x$
When $f'(x) = 0$,
$x^3 - 8x = 0$
$x(x^2 - 8) = 0$
$x = 0$ or $x^2 = 8$
$x = 0$ or $\pm \sqrt{8}$
$x = 0$ or $\pm 2\sqrt{2}$

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Differentiation
Exercise I, Question 3

Question:

A curve is drawn with equation $y = 3 + 5x + x^2 - x^3$. Find the coordinates of the two points on the curve where the gradient of the curve is zero. **[E]**

Solution:

$y = 3 + 5x + x^2 - x^3$

$\frac{dy}{dx} = 5 + 2x - 3x^2$

Put $\frac{dy}{dx} = 0$. Then

$5 + 2x - 3x^2 = 0$

$(5 - 3x)(1 + x) = 0$

$x = -1$ or $x = \frac{5}{3}$

Substitute to obtain

$y = 3 - 5 + 1 - (-1)^3$ when $x = -1$, i.e.

$y = 0$ when $x = -1$

and

$y = 3 + 5\left(\frac{5}{3}\right) + \left(\frac{5}{3}\right)^2 - \left(\frac{5}{3}\right)^3$ when $x = \frac{5}{3}$, i.e.

$y = 3 + \frac{25}{3} + \frac{25}{9} - \frac{125}{27} = 9\frac{13}{27}$ when $x = \frac{5}{3}$

So the points have coordinates $(-1, 0)$ and $\left(1\frac{2}{3}, 9\frac{13}{27}\right)$.

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Differentiation
Exercise I, Question 4

Question:

Calculate the x-coordinates of the points on the curve with equation $y = 7x^2 - x^3$ at which the gradient is equal to 16. **[E]**

Solution:

$y = 7x^2 - x^3$

$\frac{dy}{dx} = 14x - 3x^2$

Put $\frac{dy}{dx} = 16$, i.e.

$14x - 3x^2 = 16$

$3x^2 - 14x + 16 = 0$

$(3x - 8)(x - 2) = 0$

$x = \frac{8}{3}$ or $x = 2$

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Differentiation
Exercise I, Question 5

Question:

Find the x-coordinates of the two points on the curve with equation $y = x^3 - 11x + 1$ where the gradient is 1. Find the corresponding y-coordinates. **[E]**

Solution:

$y = x^3 - 11x + 1$

$\frac{dy}{dx} = 3x^2 - 11$

As gradient is 1, put $\frac{dy}{dx} = 1$, then

$3x^2 - 11 = 1$
$3x^2 = 12$
$x^2 = 4$
$x = \pm 2$
Substitute these values into $y = x^3 - 11x + 1$:
$y = 2^3 - 11 \times 2 + 1 = -13$ when $x = 2$ and
$y = (-2)^3 - 11(-2) + 1 = 15$ when $x = -2$
The gradient is 1 at the points $(2, -13)$ and $(-2, 15)$.

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Differentiation
Exercise I, Question 6

Question:

The function f is defined by $f(x) = x + \frac{9}{x}$, $x \in \mathbb{R}$, $x \neq 0$.

(a) Find $f'(x)$.

(b) Solve $f'(x) = 0$. **[E]**

Solution:

(a) $f(x) = x + \frac{9}{x} = x + 9x^{-1}$

$f'(x) = 1 - 9x^{-2} = 1 - \frac{9}{x^2}$

(b) When $f'(x) = 0$,

$1 - \frac{9}{x^2} = 0$

$\frac{9}{x^2} = 1$

$x^2 = 9$

$x = \pm 3$

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Differentiation
Exercise I, Question 7

Question:

Given that

$y = x^{\frac{3}{2}} + \frac{48}{x}$, $x > 0$,

find the value of x and the value of y when $\frac{dy}{dx} = 0$. **[E]**

Solution:

$y = x^{\frac{3}{2}} + \frac{48}{x} = x^{\frac{3}{2}} + 48x^{-1}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 48x^{-2}$

Put $\frac{dy}{dx} = 0$, then

$\frac{3}{2}x^{\frac{1}{2}} - \frac{48}{x^2} = 0$

$\frac{3}{2}x^{\frac{1}{2}} = \frac{48}{x^2}$

Multiply both sides by x^2:

$\frac{3}{2}x^{2\frac{1}{2}} = 48$

$x^{2\frac{1}{2}} = 32$

$x = (32)^{\frac{2}{5}}$

$x = 4$

Substitute to give $y = 4^{\frac{3}{2}} + \frac{48}{4} = 8 + 12 = 20$

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Differentiation
Exercise I, Question 8

Question:

Given that

$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}, x > 0,$

find $\frac{dy}{dx}$. **[E]**

Solution:

$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + \frac{4}{2}x^{-\frac{3}{2}} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$

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Differentiation
Exercise I, Question 9

Question:

A curve has equation $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$.

(a) Show that $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$

(b) Find the coordinates of the point on the curve where the gradient is zero. **[E]**

Solution:

(a) $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$

$$\frac{dy}{dx} = 12\left(\frac{1}{2}\right)x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$$

(b) The gradient is zero when $\frac{dy}{dx} = 0$:

$\frac{3}{2}x^{-\frac{1}{2}}(4 - x) = 0$

$x = 4$

Substitute into $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}}$ to obtain

$y = 12 \times 2 - 2^3 = 16$

The gradient is zero at the point with coordinates $(4, 16)$.

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Differentiation
Exercise I, Question 10

Question:

(a) Expand $\left(x^{\frac{3}{2}} - 1 \right) \left(x^{-\frac{1}{2}} + 1 \right)$.

(b) A curve has equation $y = \left(x^{\frac{3}{2}} - 1 \right) \left(x^{-\frac{1}{2}} + 1 \right)$, $x > 0$. Find $\frac{dy}{dx}$.

(c) Use your answer to **b** to calculate the gradient of the curve at the point where $x = 4$. **[E]**

Solution:

(a) $\left(x^{\frac{3}{2}} - 1 \right) \left(x^{-\frac{1}{2}} + 1 \right) = x + x^{\frac{3}{2}} - x^{-\frac{1}{2}} - 1$

(b) $y = x + x^{\frac{3}{2}} - x^{-\frac{1}{2}} - 1$

$\frac{dy}{dx} = 1 + \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$

(c) When $x = 4$, $\frac{dy}{dx} = 1 + \frac{3}{2} \times 2 + \frac{1}{2} \times \frac{1}{4^{\frac{3}{2}}} = 1 + 3 + \frac{1}{16} = 4\frac{1}{16}$

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Differentiation
Exercise I, Question 11

Question:

Differentiate with respect to x:

$2x^3 + \sqrt{x} + \frac{x^2 + 2x}{x^2}$ [E]

Solution:

Let $y = 2x^3 + \sqrt{x} + \frac{x^2 + 2x}{x^2}$

$\Rightarrow \quad y = 2x^3 + x^{\frac{1}{2}} + \frac{x^2}{x^2} + \frac{2x}{x^2}$

$\Rightarrow \quad y = 2x^3 + x^{\frac{1}{2}} + 1 + 2x^{-1}$

$\frac{dy}{dx} = 6x^2 + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2} = 6x^2 + \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$

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Differentiation
Exercise I, Question 12

Question:

The volume, V cm^3, of a tin of radius r cm is given by the formula $V = \pi(40r - r^2 - r^3)$. Find the positive value of r for which $\frac{dV}{dr} = 0$, and find the value of V which corresponds to this value of r. [E]

Solution:

$V = \pi(40r - r^2 - r^3)$

$\frac{dV}{dr} = 40\pi - 2\pi r - 3\pi r^2$

Put $\frac{dV}{dr} = 0$, then

$\pi(40 - 2r - 3r^2) = 0$

$(4 + r)(10 - 3r) = 0$

$r = \frac{10}{3}$ or -4

As r is positive, $r = \frac{10}{3}$.

Substitute into the given expression for V:

$V = \pi\left(40 \times \frac{10}{3} - \frac{100}{9} - \frac{1000}{27}\right) = \frac{2300}{27}\pi$

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Differentiation
Exercise I, Question 13

Question:

The total surface area of a cylinder $A\text{cm}^2$ with a fixed volume of 1000 cubic cm is given by the formula $A = 2\pi x^2 + \frac{2000}{x}$, where x cm is the radius. Show that when the rate of change of the area with respect to the radius is zero, $x^3 = \frac{500}{\pi}$. **[E]**

Solution:

$A = 2\pi x^2 + \frac{2000}{x} = 2\pi x^2 + 2000x^{-1}$

$\frac{dA}{dx} = 4\pi x - 2000x^{-2} = 4\pi x - \frac{2000}{x^2}$

When $\frac{dA}{dx} = 0$,

$4\pi x = \frac{2000}{x^2}$

$x^3 = \frac{2000}{4\pi} = \frac{500}{\pi}$

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Differentiation
Exercise I, Question 14

Question:

The curve with equation $y = ax^2 + bx + c$ passes through the point (1 , 2) . The gradient of the curve is zero at the point (2 , 1) . Find the values of a, b and c. **[E]**

Solution:

The point (1 , 2) lies on the curve with equation $y = ax^2 + bx + c$.
Therefore, substitute $x = 1$, $y = 2$ into the equation to give

$2 = a + b + c$ ①

The point (2 , 1) also lies on the curve.
Therefore, substitute $x = 2$, $y = 1$ to give

$1 = 4a + 2b + c$ ②

Eliminate c by subtracting Equation ② − Equation ①:

$-1 = 3a + b$ ③

The gradient of the curve is zero at (2 , 1) so substitute $x = 2$ into the expression for $\frac{dy}{dx} = 0$.

As $y = ax^2 + bx + c$

$\frac{dy}{dx} = 2ax + b$

At (2 , 1)

$0 = 4a + b$ ④

Solve Equations ③ and ④ by subtracting ④ − ③:
$1 = a$
Substitute $a = 1$ into Equation ③ to give $b = -4$.
Then substitute a and b into Equation ① to give $c = 5$.
Therefore, $a = 1$, $b = -4$, $c = 5$.

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Differentiation
Exercise I, Question 15

Question:

A curve C has equation $y = x^3 - 5x^2 + 5x + 2$.

(a) Find $\frac{dy}{dx}$ in terms of x.

(b) The points P and Q lie on C. The gradient of C at both P and Q is 2. The x-coordinate of P is 3.

(i) Find the x-coordinate of Q.

(ii) Find an equation for the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(iii) If this tangent intersects the coordinate axes at the points R and S, find the length of RS, giving your answer as a surd. **[E]**

Solution:

$y = x^3 - 5x^2 + 5x + 2$

(a) $\frac{dy}{dx} = 3x^2 - 10x + 5$

(b) Given that the gradient is 2, $\frac{dy}{dx} = 2$

$3x^2 - 10x + 5 = 2$
$3x^2 - 10x + 3 = 0$
$(3x - 1)(x - 3) = 0$
$x = \frac{1}{3}$ or 3

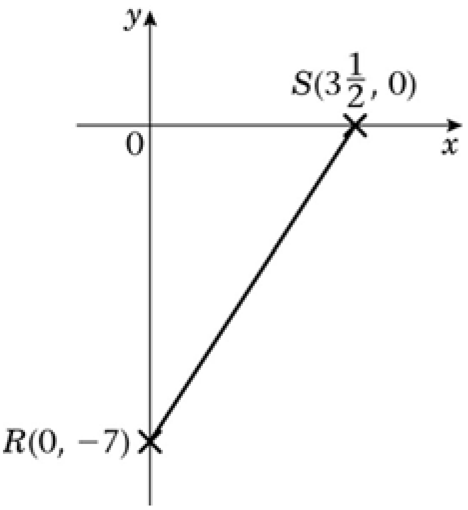
(i) At P, $x = 3$. Therefore, at Q, $x = \frac{1}{3}$.

(ii) At the point P, $x = 3$, $y = 3^3 - 5 \times 3^2 + 5 \times 3 + 2 = 27 - 45 + 15 + 2 = -1$
The gradient of the curve is 2.
The equation of the tangent at P is
$y - (-1) = 2(x - 3)$
$y + 1 = 2x - 6$
$y = 2x - 7$

(iii) This tangent meets the axes when $x = 0$ and when $y = 0$.

When $x = 0$, $y = -7$. When $y = 0$, $x = 3\frac{1}{2}$.

The tangent meets the axes at $(0, -7)$ and $\left(3\frac{1}{2}, 0\right)$.



The distance $RS = \sqrt{\left(3\frac{1}{2} - 0\right)^2 + [0 - (-7)]^2} = \sqrt{\frac{49}{4} + 49} = \frac{7}{2}\sqrt{1 + 4} = \frac{7}{2}\sqrt{5}$.

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Differentiation
Exercise I, Question 16

Question:

Find an equation of the tangent and the normal at the point where $x = 2$ on the curve with equation $y = \frac{8}{x} - x + 3x^2$, $x > 0$. **[E]**

Solution:

$y = \frac{8}{x} - x + 3x^2 = 8x^{-1} - x + 3x^2$

$\frac{dy}{dx} = -8x^{-2} - 1 + 6x = -\frac{8}{x^2} - 1 + 6x$

when $x = 2$, $\frac{dy}{dx} = -\frac{8}{4} - 1 + 12 = 9$

At $x = 2$, $y = \frac{8}{2} - 2 + 3 \times 2^2 = 14$

So the equation of the tangent through the point $(2 , 14)$ with gradient 9 is
$y - 14 = 9(x - 2)$
$y = 9x - 18 + 14$
$y = 9x - 4$

The gradient of the normal is $-\frac{1}{9}$, as the normal is at right angles to the tangent.

So the equation of the normal is

$y - 14 = -\frac{1}{9}(x - 2)$

$9y + x = 128$

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Differentiation
Exercise I, Question 17

Question:

The normals to the curve $2y = 3x^3 - 7x^2 + 4x$, at the points $O(0, 0)$ and $A(1, 0)$, meet at the point N.

(a) Find the coordinates of N.

(b) Calculate the area of triangle OAN. **[E]**

Solution:

(a) $2y = 3x^3 - 7x^2 + 4x$

$y = \frac{3}{2}x^3 - \frac{7}{2}x^2 + 2x$

$\frac{dy}{dx} = \frac{9}{2}x^2 - 7x + 2$

At $(0, 0)$, $x = 0$, gradient of curve is $0 - 0 + 2 = 2$.

The gradient of the normal at $(0, 0)$ is $-\frac{1}{2}$.

The equation of the normal at $(0, 0)$ is $y = -\frac{1}{2}x$.

At $(1, 0)$, $x = 1$, gradient of curve is $\frac{9}{2} - 7 + 2 = -\frac{1}{2}$.

The gradient of the normal at $(1, 0)$ is 2.
The equation of the normal at $(1, 0)$ is $y = 2(x - 1)$.

The normals meet when $y = 2x - 2$ and $y = -\frac{1}{2}x$:

$2x - 2 = -\frac{1}{2}x$

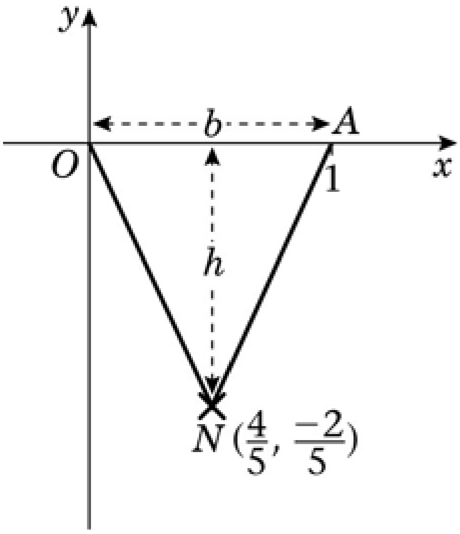
$2\frac{1}{2}x = 2$

$x = 2 \div 2\frac{1}{2} = \frac{4}{5}$

Substitute into $y = 2x - 2$ to obtain $y = -\frac{2}{5}$ and check in $y = -\frac{1}{2}x$.

N has coordinates $\left(\frac{4}{5}, -\frac{2}{5}\right)$.

(b)



The area of $\triangle$ OAN = $\frac{1}{2}$ base × height

base (b) = 1

height(h) = $\frac{2}{5}$

Area = $\frac{1}{2} \times 1 \times \frac{2}{5} = \frac{1}{5}$