Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise A, Question 1

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

4, 9, 14, 19, …

Solution:

$$4 \xrightarrow{+5} 9 \xrightarrow{+5} 14 \xrightarrow{+5} 19 \xrightarrow{+5} 24 \xrightarrow{+5} 29 \xrightarrow{+5} 34$$

“Add 5 to previous term”

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise A, Question 2

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

$2, \ -2, 2, \ -2, \quad \ldots$

Solution:

$$2 \xrightarrow{\times -1} -2 \xrightarrow{\times -1} 2 \xrightarrow{\times -1} -2 \xrightarrow{\times -1} 2 \xrightarrow{\times -1} -2 \xrightarrow{\times -1} 2$$

"Multiply previous term by -1"

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise A, Question 3

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

30, 27, 24, 21, …

Solution:

$$30 \xrightarrow{-3} 27 \xrightarrow{-3} 24 \xrightarrow{-3} 21 \xrightarrow{-3} 18 \xrightarrow{-3} 15 \xrightarrow{-3} 12$$

"Subtract 3 from previous term"

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise A, Question 4

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

2, 6, 18, 54, …

Solution:

$$2 \xrightarrow{\times 3} 6 \xrightarrow{\times 3} 18 \xrightarrow{\times 3} 54 \xrightarrow{\times 3} 162 \xrightarrow{\times 3} 486 \xrightarrow{\times 3} 1458$$

"Multiply previous term by 3"

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise A, Question 5

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

$4, \ -2, 1, \ -\frac{1}{2}, \quad \ldots$

Solution:

$$4 \xrightarrow{\times -\frac{1}{2}} -2 \xrightarrow{\times -\frac{1}{2}} 1 \xrightarrow{\times -\frac{1}{2}} -\frac{1}{2} \xrightarrow{\times -\frac{1}{2}} +\frac{1}{4} \xrightarrow{\times -\frac{1}{2}} -\frac{1}{8} \xrightarrow{\times -\frac{1}{2}} +\frac{1}{16}$$

"Multiply previous term by $-\frac{1}{2}$" (or "divide by -2")

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise A, Question 6

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

1, 2, 5, 14, …

Solution:

$$1 \xrightarrow{\times 3\ -1} 2 \xrightarrow{\times 3\ -1} 5 \xrightarrow{\times 3\ -1} 14 \xrightarrow{\times 3\ -1} 41 \xrightarrow{\times 3\ -1} 122 \xrightarrow{\times 3\ -1} 365$$

"Multiply previous term by 3 then subtract 1"

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise A, Question 7

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

1, 1, 2, 3, 5, …

Solution:

$$1 \qquad 1 \qquad 2 \qquad 3 \qquad 5 \qquad 8 \qquad 13 \qquad 21$$

$$1 + 1 = \nearrow \qquad 3 + 5 = \nearrow \qquad 8 + 13 = \nearrow$$

"Add together the two previous terms"

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise A, Question 8

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

$1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \quad \ldots$

Solution:

$1 \, , \, \frac{2}{3} \, , \, \frac{3}{5} \, , \, \frac{4}{7} \, , \, \frac{5}{9} \, , \, \frac{6}{11} \, , \, \frac{7}{13}$

"Add 1 to previous numerator, 2 to previous denominator"

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise A, Question 9

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

4, 3, 2.5, 2.25, 2.125, …

Solution:

$$4 \xrightarrow{\div 2 + 1} 3 \xrightarrow{\div 2 + 1} 2.5 \xrightarrow{\div 2 + 1} 2.25 \xrightarrow{\div 2 + 1} 2.125 \xrightarrow{\div 2 + 1} 2.0625 \xrightarrow{\div 2 + 1} 2.03125 \xrightarrow{\div 2 + 1} 2.015625$$

"Divide previous term by 2 then add 1"

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

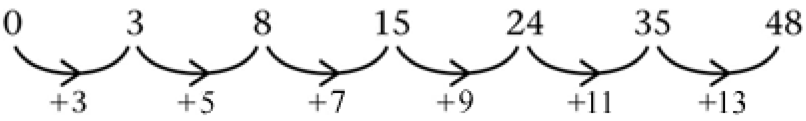
Sequences and series
Exercise A, Question 10

Question:

Work out the next three terms of the following sequence. State the rule to find the next term:

0, 3, 8, 15, …

Solution:



"Add consecutive odd numbers to previous term"

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise B, Question 1

Question:

Find the U_1, U_2, U_3 and U_{10} of the following sequences, where:

(a) $U_n = 3n + 2$

(b) $U_n = 10 - 3n$

(c) $U_n = n^2 + 5$

(d) $U_n = (n - 3)^2$

(e) $U_n = (-2)^n$

(f) $U_n = \frac{n}{n+2}$

(g) $U_n = (-1)^n \frac{n}{n+2}$

(h) $U_n = (n - 2)^3$

Solution:

(a) $U_1 = 3 \times 1 + 2 = 5$, $U_2 = 3 \times 2 + 2 = 8$, $U_3 = 3 \times 3 + 2 = 11$, $U_{10} = 3 \times 10 + 2 = 32$

(b) $U_1 = 10 - 3 \times 1 = 7$, $U_2 = 10 - 3 \times 2 = 4$, $U_3 = 10 - 3 \times 3 = 1$, $U_{10} = 10 - 3 \times 10 = -20$

(c) $U_1 = 1^2 + 5 = 6$, $U_2 = 2^2 + 5 = 9$, $U_3 = 3^2 + 5 = 14$, $U_{10} = 10^2 + 5 = 105$

(d) $U_1 = (1 - 3)^2 = 4$, $U_2 = (2 - 3)^2 = 1$, $U_3 = (3 - 3)^2 = 0$, $U_{10} = (10 - 3)^2 = 49$

(e) $U_1 = (-2)^1 = -2$, $U_2 = (-2)^2 = 4$, $U_3 = (-2)^3 = -8$, $U_{10} = (-2)^{10} = 1024$

(f) $U_1 = \frac{1}{1+2} = \frac{1}{3}$, $U_2 = \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$, $U_3 = \frac{3}{3+2} = \frac{3}{5}$, $U_{10} = \frac{10}{10+2} = \frac{10}{12} = \frac{5}{6}$

(g) $U_1 = (-1)^1 \frac{1}{1+2} = -\frac{1}{3}$, $U_2 = (-1)^2 \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$, $U_3 = (-1)^3 \frac{3}{3+2} = -\frac{3}{5}$, $U_{10} = (-1)^{10} \frac{10}{10+2} = \frac{10}{12} = \frac{5}{6}$

(h) $U_1 = (1 - 2)^3 = (-1)^3 = -1$, $U_2 = (2 - 2)^3 = 0$, $U_3 = (3 - 2)^3 = 1$, $U_{10} = (10 - 2)^3 = 8^3 = 512$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise B, Question 2

Question:

Find the value of n for which U_n has the given value:

(a) $U_n = 2n - 4$, $U_n = 24$

(b) $U_n = \left(n - 4 \right)^2$, $U_n = 25$

(c) $U_n = n^2 - 9$, $U_n = 112$

(d) $U_n = \frac{2n+1}{n-3}$, $U_n = \frac{19}{6}$

(e) $U_n = n^2 + 5n - 6$, $U_n = 60$

(f) $U_n = n^2 - 4n + 11$, $U_n = 56$

(g) $U_n = n^2 + 4n - 5$, $U_n = 91$

(h) $U_n = \left(-1 \right)^n \frac{n}{n+4}$, $U_n = \frac{7}{9}$

(i) $U_n = \frac{n^3+3}{5}$, $U_n = 13.4$

(j) $U_n = \frac{n^3}{5} + 3$, $U_n = 28$

Solution:

(a) $24 = 2n - 4$
$28 = 2n \quad \left(+4 \right)$
$14 = n \quad \left(\div 2 \right)$
$n = 14$

(b) $25 = \left(n - 4 \right)^2$
$\pm 5 = \left(n - 4 \right) \quad \left(\sqrt{\ } \right)$
$9, -1 = n \quad \left(+4 \right)$
$n = 9$ (it must be positive)

(c) $112 = n^2 - 9$
$121 = n^2 \quad \left(+9 \right)$
$\pm 11 = n \quad \left(\sqrt{\ } \right)$
$n = 11$

(d) $\frac{19}{6} = \frac{2n+1}{n-3}$ (cross multiply)

$19 \left(n - 3 \right) = 6 \left(2n + 1 \right)$

$19n - 57 = 12n + 6 \quad (-12n)$
$7n - 57 = 6 \quad (+57)$
$7n = 63$
$n = 9$

(e) $60 = n^2 + 5n - 6 \quad (-60)$
$0 = n^2 + 5n - 66$ (factorise)
$0 = (n + 11) (n - 6)$
$n = -11, 6$
$n = 6$

(f) $56 = n^2 - 4n + 11 \quad (-56)$
$0 = n^2 - 4n - 45$ (factorise)
$0 = (n - 9) (n + 5)$
$n = 9, -5$
$n = 9$

(g) $91 = n^2 + 4n - 5 \quad (-91)$
$0 = n^2 + 4n - 96$ (factorise)
$0 = (n + 12) (n - 8)$
$n = -12, 8$
$n = 8$

(h) $\frac{7}{9} = (-1)^n \frac{n}{n + 4}$

n must be even

$\frac{7}{9} = \frac{n}{n + 4}$

$7 (n + 4) = 9n$
$7n + 28 = 9n$
$28 = 2n$
$n = 14$

(i) $13.4 = \frac{n^3 + 3}{5} \quad (\times 5)$

$67 = n^3 + 3 \quad (-3)$
$64 = n^3 \quad (\sqrt[3]{\ })$
$n = 4$

(j) $28 = \frac{n^3}{5} + 3 \quad (-3)$

$25 = \frac{n^3}{5} \quad (\times 5)$

$125 = n^3 \quad (\sqrt[3]{\ })$
$n = 5$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise B, Question 3

Question:

Prove that the $(2n+1)$ th term of the sequence $U_n = n^2 - 1$ is a multiple of 4.

Solution:

$(2n+1)$ th term
$= (2n+1)^2 - 1$
$= (2n+1)(2n+1) - 1$
$= 4n^2 + 4n + 1 - 1$
$= 4n^2 + 4n$
$= 4n(n+1)$
$= 4 \times n(n+1)$
= multiple of 4 because it is 4 × whole number.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise B, Question 4

Question:

Prove that the terms of the sequence $U_n = n^2 - 10n + 27$ are all positive. For what value of n is U_n smallest?

Solution:

$U_n = n^2 - 10n + 27 = \left(n - 5 \right)^2 - 25 + 27 = \left(n - 5 \right)^2 + 2$
$\left(n - 5 \right)^2$ is always positive (or zero) because it is a square.
$\therefore U_n \geq 0 + 2$
Smallest value of U_n is 2.
(It occurs when $n = 5$.)

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise B, Question 5

Question:

A sequence is generated according to the formula $U_n = an + b$, where a and b are constants. Given that $U_3 = 14$ and $U_5 = 38$, find the values of a and b.

Solution:

$U_n = an + b$

when $n = 3$, $U_3 = 14 \quad \Rightarrow \quad 14 = 3a + b$ ①

when $n = 5$, $U_5 = 38 \quad \Rightarrow \quad 38 = 5a + b$ ②

② − ①: $24 = 2a \quad \Rightarrow \quad a = 12$

substitute $a = 12$ in ①: $14 = 3 \times 12 + b \quad \Rightarrow \quad 14 = 36 + b \quad \Rightarrow \quad b = -22$

$\therefore U_n = 12n - 22$

(check: when $n = 3$, $U_3 = 12 \times 3 - 22 = 36 - 22 = 14$ ✓)

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise B, Question 6

Question:

A sequence is generated according to the formula $U_n = an^2 + bn + c$, where a, b and c are constants. If $U_1 = 4$, $U_2 = 10$ and $U_3 = 18$, find the values of a, b and c.

Solution:

$U_n = an^2 + bn + c$

when $n = 1$, $U_n = 4 \quad \Rightarrow \quad 4 = a \times 1^2 + b \times 1 + c \quad \Rightarrow \quad 4 = a + b + c$

when $n = 2$, $U_2 = 10 \quad \Rightarrow \quad 10 = a \times 2^2 + b \times 2 + c \quad \Rightarrow \quad 10 = 4a + 2b + c$

when $n = 3$, $U_3 = 18 \quad \Rightarrow \quad 18 = a \times 3^2 + b \times 3 + c \quad \Rightarrow \quad 18 = 9a + 3b + c$

we need to solve simultaneously

$a + b + c = 4$ ①

$4a + 2b + c = 10$ ②

$9a + 3b + c = 18$ ③

② − ①: $3a + b = 6$ ④

③ − ②: $5a + b = 8$ ⑤

⑤ − ④: $2a = 2 \quad \Rightarrow \quad a = 1$

Substitute $a = 1$ in ④: $3 + b = 6 \quad \Rightarrow \quad b = 3$

Substitute $a = 1$, $b = 3$ in ①: $1 + 3 + c = 4 \quad \Rightarrow \quad c = 0$

$\therefore U_n = 1n^2 + 3n + 0 = n^2 + 3n$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise B, Question 7

Question:

A sequence is generated from the formula $U_n = pn^3 + q$, where p and q are constants. Given that $U_1 = 6$ and $U_3 = 19$, find the values of the constants p and q.

Solution:

$U_n = pn^3 + q$

when $n = 1$, $U_1 = 6 \quad \Rightarrow \quad 6 = p \times 1^3 + q \quad \Rightarrow \quad 6 = p + q$

when $n = 3$, $U_3 = 19 \quad \Rightarrow \quad 19 = p \times 3^3 + q \quad \Rightarrow \quad 19 = 27p + q$

Solve simultaneously:

$p + q = 6$ ①

$27p + q = 19$ ②

② − ①: $26p = 13 \quad \Rightarrow \quad p = \frac{1}{2}$

substitute $p = \frac{1}{2}$ in ①: $\frac{1}{2} + q = 6 \quad \Rightarrow \quad q = 5\frac{1}{2}$

$\therefore U_n = \frac{1}{2}n^3 + 5\frac{1}{2}$ or $\frac{1}{2}n^3 + \frac{11}{2}$ or $\frac{n^3 + 11}{2}$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise C, Question 1

Question:

Find the first four terms of the following recurrence relationships:

(a) $U_{n+1} = U_n + 3, U_1 = 1$

(b) $U_{n+1} = U_n - 5, U_1 = 9$

(c) $U_{n+1} = 2U_n , U_1 = 3$

(d) $U_{n+1} = 2U_n + 1, U_1 = 2$

(e) $U_{n+1} = \frac{U_n}{2}, U_1 = 10$

(f) $U_{n+1} = \left(U_n \right)^2 - 1, U_1 = 2$

(g) $U_{n+2} = 2U_{n+1} + U_n, U_1 = 3, U_2 = 5$

Solution:

(a) $U_{n+1} = U_n + 3, U_1 = 1$
$n = 1 \quad \Rightarrow \quad U_2 = U_1 + 3 = 1 + 3 = 4$
$n = 2 \quad \Rightarrow \quad U_3 = U_2 + 3 = 4 + 3 = 7$
$n = 3 \quad \Rightarrow \quad U_4 = U_3 + 3 = 7 + 3 = 10$
Terms are 1, 4, 7, 10, ...

(b) $U_{n+1} = U_n - 5, U_1 = 9$
$n = 1 \quad \Rightarrow \quad U_2 = U_1 - 5 = 9 - 5 = 4$
$n = 2 \quad \Rightarrow \quad U_3 = U_2 - 5 = 4 - 5 = -1$
$n = 3 \quad \Rightarrow \quad U_4 = U_3 - 5 = -1 - 5 = -6$
Terms are 9, 4, -1, -6, ...

(c) $U_{n+1} = 2U_n, U_1 = 3$
$n = 1 \quad \Rightarrow \quad U_2 = 2U_1 = 2 \times 3 = 6$
$n = 2 \quad \Rightarrow \quad U_3 = 2U_2 = 2 \times 6 = 12$
$n = 3 \quad \Rightarrow \quad U_4 = 2U_3 = 2 \times 12 = 24$
Terms are 3, 6, 12, 24, ...

(d) $U_{n+1} = 2U_n + 1, U_1 = 2$
$n = 1 \quad \Rightarrow \quad U_2 = 2U_1 + 1 = 2 \times 2 + 1 = 5$
$n = 2 \quad \Rightarrow \quad U_3 = 2U_2 + 1 = 2 \times 5 + 1 = 11$
$n = 3 \quad \Rightarrow \quad U_4 = 2U_3 + 1 = 2 \times 11 + 1 = 23$
Terms are 2, 5, 11, 23, ...

(e) $U_{n+1} = \frac{U_n}{2}$, $U_1 = 10$

$n = 1 \quad \Rightarrow \quad U_2 = \frac{U_1}{2} = \frac{10}{2} = 5$

$n = 2 \quad \Rightarrow \quad U_3 = \frac{U_2}{2} = \frac{5}{2} = 2.5$

$n = 3 \quad \Rightarrow \quad U_4 = \frac{U_3}{2} = \frac{2.5}{2} = 1.25$

Terms are 10, 5, 2.5, 1.25, ...

(f) $U_{n+1} = (U_n)^2 - 1$, $U_1 = 2$
$n = 1 \quad \Rightarrow \quad U_2 = (U_1)^2 - 1 = 2^2 - 1 = 4 - 1 = 3$
$n = 2 \quad \Rightarrow \quad U_3 = (U_2)^2 - 1 = 3^2 - 1 = 9 - 1 = 8$
$n = 3 \quad \Rightarrow \quad U_4 = (U_3)^2 - 1 = 8^2 - 1 = 64 - 1 = 63$
Terms are 2, 3, 8, 63, ...

(g) $U_{n+2} = 2U_{n+1} + U_n$, $U_1 = 3$, $U_2 = 5$
$n = 1 \quad \Rightarrow \quad U_3 = 2U_2 + U_1 = 2 \times 5 + 3 = 13$
$n = 2 \quad \Rightarrow \quad U_4 = 2U_3 + U_2 = 2 \times 13 + 5 = 31$
Terms are 3, 5, 13, 31, ...

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise C, Question 2

Question:

Suggest possible recurrence relationships for the following sequences (remember to state the first term):

(a) 3, 5, 7, 9, …

(b) 20, 17, 14, 11, …

(c) 1, 2, 4, 8, …

(d) 100, 25, 6.25, 1.5625, …

(e) 1, − 1, 1, − 1, 1, …

(f) 3, 7, 15, 31, …

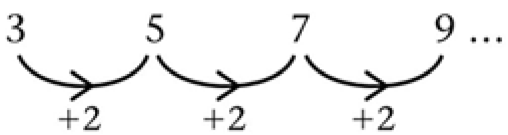
(g) 0, 1, 2, 5, 26, …

(h) 26, 14, 8, 5, 3.5, …

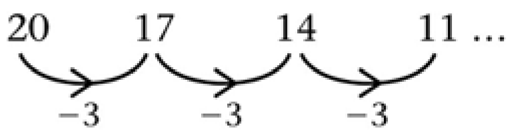
(i) 1, 1, 2, 3, 5, 8, 13, …

(j) 4, 10, 18, 38, 74, …

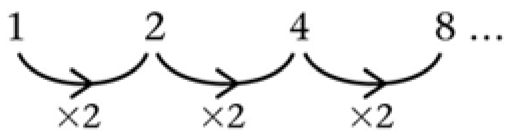
Solution:

(a) 

$U_{n+1} = U_n + 2$, $U_1 = 3$

(b) 

$U_{n+1} = U_n - 3$, $U_1 = 20$

(c) 

$U_{n+1} = 2 \times U_n$, $U_1 = 1$

(d) $100 \xrightarrow{\div 4} 25 \xrightarrow{\div 4} 6.25 \xrightarrow{\div 4} 1.5625 \ldots$

$U_{n+1} = \frac{U_n}{4}, U_1 = 100$

(e) $1 \xrightarrow{\times -1} -1 \xrightarrow{\times -1} 1 \xrightarrow{\times -1} -1 \ldots$

$U_{n+1} = (-1) \times U_n, U_1 = 1$

(f) $3 \xrightarrow{\times 2 + 1} 7 \xrightarrow{\times 2 + 1} 15 \xrightarrow{\times 2 + 1} 31 \ldots$

$U_{n+1} = 2U_n + 1, U_1 = 3$

(g) $0 \xrightarrow{0^2 + 1} 1 \xrightarrow{1^2 + 1} 2 \xrightarrow{2^2 + 1} 5 \xrightarrow{5^2 + 1} 26 \ldots$

$U_{n+1} = (U_n)^2 + 1, U_1 = 0$

(h) $26 \xrightarrow{+2 \div 2} 14 \xrightarrow{+2 \div 2} 8 \xrightarrow{+2 \div 2} 5 \xrightarrow{+2 \div 2} 3.5 \ldots$

$U_{n+1} = \frac{U_n + 2}{2}, U_1 = 26$

(i) $1,\quad 1,\quad \underset{1+1}{\overset{\uparrow}{2,}}\quad \underset{2+1}{\overset{\uparrow}{3,}}\quad \underset{3+2}{\overset{\uparrow}{5,}}\quad \underset{5+3}{\overset{\uparrow}{8,}}\quad \underset{8+5}{\overset{\uparrow}{13,}} \ldots$

$U_{n+2} = U_{n+1} + U_n, U_1 = 1, U_2 = 1$

(j) $4,\quad 10,\quad \underset{2 \times 4 + 10}{\overset{\nearrow}{18,}}\quad \underset{2 \times 10 + 18}{\overset{\nearrow}{38,}}\quad \underset{2 \times 18 + 38}{\overset{\nearrow}{74,}} \ldots$

$U_{n+2} = U_{n+1} + 2U_n, U_1 = 4, U_2 = 10$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise C, Question 3

Question:

By writing down the first four terms or otherwise, find the recurrence formula that defines the following sequences:

(a) $U_n = 2n - 1$

(b) $U_n = 3n + 2$

(c) $U_n = n + 2$

(d) $U_n = \frac{n+1}{2}$

(e) $U_n = n^2$

(f) $U_n = (-1)^n n$

Solution:

(a) $U_n = 2n - 1$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 1 \xrightarrow{+2} U_2 = 3 \xrightarrow{+2} U_3 = 5 \xrightarrow{+2} U_4 = 7$$

Recurrence formula is $U_{n+1} = U_n + 2$, $U_1 = 1$.

(b) $U_n = 3n + 2$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 5 \xrightarrow{+3} U_2 = 8 \xrightarrow{+3} U_3 = 11 \xrightarrow{+3} U_4 = 14$$

Recurrence formula is $U_{n+1} = U_n + 3$, $U_1 = 5$.

(c) $U_n = n + 2$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 3 \xrightarrow{+1} U_2 = 4 \xrightarrow{+1} U_3 = 5 \xrightarrow{+1} U_4 = 6$$

Recurrence formula is $U_{n+1} = U_n + 1$, $U_1 = 3$.

(d) $U_n = \frac{n+1}{2}$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 1 \xrightarrow{+\frac{1}{2}} U_2 = 1\tfrac{1}{2} \xrightarrow{+\frac{1}{2}} U_3 = 2 \xrightarrow{+\frac{1}{2}} U_4 = 2\tfrac{1}{2}$$

Recurrence formula is $U_{n+1} = U_n + \frac{1}{2}, U_1 = 1$.

(e) $U_n = n^2$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = 1 \xrightarrow[= 2 \times 1 + 1]{+3} U_2 = 4 \xrightarrow[= 2 \times 2 + 1]{+5} U_3 = 9 \xrightarrow[= 2 \times 3 + 1]{+7} U_4 = 16$$

$U_{n+1} = U_n + 2n + 1, U_1 = 1$.

(f) $U_n = (-1)^n n$. Substituting $n = 1, 2, 3$ and 4 gives

$$U_1 = -1 \xrightarrow[= 2 \times 1 + 1]{+3} U_2 = 2 \xrightarrow[= -(2 \times 2 + 1)]{-5} U_3 = -3 \xrightarrow[= 2 \times 3 + 1]{+7} U_4 = 4$$

$U_{n+1} = U_n - (-1)^n(2n + 1), U_1 = 1$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise C, Question 4

Question:

A sequence of terms $\{ U_n \{$ is defined $n \geq 1$ by the recurrence relation $U_{n+1} = kU_n + 2$, where k is a constant. Given that $U_1 = 3$:

(a) Find an expression in terms of k for U_2.

(b) Hence find an expression for U_3.

Given that $U_3 = 42$:

(c) Find possible values of k.

Solution:

$U_{n+1} = kU_n + 2$
(a) Substitute $n = 1 \Rightarrow U_2 = kU_1 + 2$
As $U_1 = 3 \Rightarrow U_2 = 3k + 2$

(b) Substitute $n = 2 \Rightarrow U_3 = kU_2 + 2$
As $U_2 = 3k + 2 \Rightarrow U_3 = k(3k + 2) + 2$
$\Rightarrow U_3 = 3k^2 + 2k + 2$

(c) We are given $U_3 = 42$
$\Rightarrow 3k^2 + 2k + 2 = 42 \quad (-42)$
$\Rightarrow 3k^2 + 2k - 40 = 0$
$\Rightarrow (3k - 10)(k + 4) = 0$
$\Rightarrow k = \frac{10}{3}, -4$

Possible values of k are $\frac{10}{3}, -4$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Sequences and series

Exercise C, Question 5

Question:

A sequence of terms $\{ U_k \{$ is defined $k \geq 1$ by the recurrence relation $U_{k+2} = U_{k+1} - pU_k$, where p is a constant.
Given that $U_1 = 2$ and $U_2 = 4$:

(a) Find an expression in terms of p for U_3.

(b) Hence find an expression in terms of p for U_4.

Given also that U_4 is twice the value of U_3:

(c) Find the value of p.

Solution:

(a) $U_{k+2} = U_{k+1} - pU_k$
Let $k = 1$, then $U_3 = U_2 - pU_1$
Substitute $U_1 = 2$, $U_2 = 4$: $U_3 = 4 - p \times 2 \Rightarrow U_3 = 4 - 2p$

(b) $U_{k+2} = U_{k+1} - pU_k$
Let $k = 2$, then $U_4 = U_3 - pU_2$
Substitute $U_2 = 4$, $U_3 = 4 - 2p$: $U_4 = (4 - 2p) - p \times 4 = 4 - 2p - 4p = 4 - 6p$

(c) We are told U_4 is twice U_3, so
$U_4 = 2 \times U_3$
$4 - 6p = 2 (4 - 2p)$
$4 - 6p = 8 - 4p$
$-4 = 2p$
$-2 = p$
Hence $p = -2$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise D, Question 1

Question:

Which of the following sequences are arithmetic?

(a) 3, 5, 7, 9, 11, …

(b) 10, 7, 4, 1, …

(c) $y, 2y, 3y, 4y, \ldots$

(d) 1, 4, 9, 16, 25, …

(e) 16, 8, 4, 2, 1, …

(f) $1, -1, 1, -1, 1, \ldots$

(g) $y, y^2, y^3, y^4, \ldots$

(h) $U_{n+1} = U_n + 2, U_1 = 3$

(i) $U_{n+1} = 3U_n - 2, U_1 = 4$

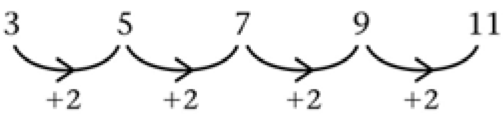
(j) $U_{n+1} = (U_n)^2, U_1 = 2$

(k) $U_n = n(n+1)$

(l) $U_n = 2n + 3$

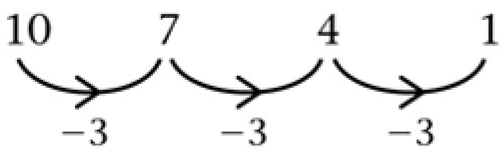
Solution:

(a)



Arithmetic $(+2)$

(b)



Arithmetic (-3)

(c)

$$y \xrightarrow{+y} 2y \xrightarrow{+y} 3y \xrightarrow{+y} 4y$$

Arithmetic ($+ y$)

(d)

$$1 \xrightarrow{+3} 4 \xrightarrow{+5} 9 \xrightarrow{+7} 16$$

Not arithmetic

(e)

$$16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4 \xrightarrow{\div 2} 2 \xrightarrow{\div 2} 1$$

Not arithmetic

(f)

$$1 \xrightarrow{\times -1} -1 \xrightarrow{\times -1} 1 \xrightarrow{\times -1} -1 \xrightarrow{\times -1} 1$$

Not arithmetic

(g)

$$y \xrightarrow{\times y} y^2 \xrightarrow{\times y} y^3 \xrightarrow{\times y} y^4$$

Not arithmetic

(h) $U_{n+1} = U_n + 2$

$$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11$$

Arithmetic ($+ 2$)

(i) $U_{n+1} = 3U_n - 2$

$$4 \xrightarrow{\times 3 - 2} 10 \xrightarrow{\times 3 - 2} 28$$

Not arithmetic

(j) $U_{n+1} = (U_n)^2$, $U_1 = 2$
2, 4, 16, 256
Not arithmetic

(k) $U_n = n(n+1)$
2, 6, 12, 20
Not arithmetic

(l) $U_n = 2n + 3$

$$5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11$$

Arithmetic (+ 2)

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise D, Question 2

Question:

Find the 10th and *n*th terms in the following arithmetic progressions:

(a) 5, 7, 9, 11, …

(b) 5, 8, 11, 14, …

(c) 24, 21, 18, 15, …

(d) -1, 3, 7, 11, …

(e) x, $2x$, $3x$, $4x$, …

(f) a, $a+d$, $a+2d$, $a+3d$, …

Solution:

(a)

$$5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11$$

10th term $= 5 + 9 \times 2 = 5 + 18 = 23$
nth term $= 5 + (n-1) \times 2 = 5 + 2n - 2 = 2n + 3$

(b)

$$5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \xrightarrow{+3} 14$$

10th term $= 5 + 9 \times 3 = 5 + 27 = 32$
nth term $= 5 + (n-1) \times 3 = 5 + 3n - 3 = 3n + 2$

(c)

$$24 \xrightarrow{-3} 21 \xrightarrow{-3} 18 \xrightarrow{-3} 15$$

10th term $= 24 + 9 \times -3 = 24 - 27 = -3$
nth term $= 24 + (n-1) \times -3 = 24 - 3n + 3 = 27 - 3n$

(d)

$$-1 \xrightarrow{+4} 3 \xrightarrow{+4} 7 \xrightarrow{+4} 11$$

10th term = $-1 + 9 \times 4 = -1 + 36 = 35$
nth term = $-1 + (n - 1) \times 4 = -1 + 4n - 4 = 4n - 5$

(e)

$$x \xrightarrow{+x} 2x \xrightarrow{+x} 3x \xrightarrow{+x} 4x$$

10th term = $x + 9 \times x = 10x$
nth term = $x + (n - 1)x = nx$

(f)

$$a \xrightarrow{+d} a + d \xrightarrow{+d} a + 2d \xrightarrow{+d} a + 3d$$

10th term = $a + 9d$
nth term = $a + (n - 1)d$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Sequences and series

Exercise D, Question 3

Question:

An investor puts £4000 in an account. Every month thereafter she deposits another £200. How much money in total will she have invested at the start of **a** the 10th month and **b** the *m*th month? (Note that at the start of the 6th month she will have made only 5 deposits of £200.)

Solution:

(a) Initial amount $= £\ 4000$ (start of month 1)
Start of month 2 $= £\ (4000 + 200)$
Start of month 3 $= £\ (4000 + 200 + 200) = £\ (4000 + 2 \times 200)$
$\vdots$
Start of month 10 $= £\ (4000 + 9 \times 200) = £\ (4000 + 1800) = £\ 5800$

(b) Start of *m*th month
$= £\ [4000 + (m - 1) \times 200]$
$= £\ (4000 + 200m - 200)$
$= £\ (3800 + 200m)$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise D, Question 4

Question:

Calculate the number of terms in the following arithmetic sequences:

(a) 3, 7, 11, … , 83, 87

(b) 5, 8, 11, … , 119, 122

(c) 90, 88, 86, … , 16, 14

(d) 4, 9, 14, … , 224, 229

(e) x, $3x$, $5x$, … , $35x$

(f) a, $a+d$, $a+2d$, … , $a+(n-1)d$

Solution:

(a) $3 \xrightarrow{+4} 7 \xrightarrow{+4} 11 \ \ldots \ 83 \xrightarrow{+4} 87$

number of jumps $= \frac{87-3}{4} = 21$

therefore number of terms $= 21 + 1 = 22$.

(b) $5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \ \ldots \ 119 \xrightarrow{+3} 122$

number of jumps $= \frac{122-5}{3} = 39$

therefore number of terms $= 40$

(c) $90 \xrightarrow{-2} 88 \xrightarrow{-2} 86 \ \ldots \ 16 \xrightarrow{-2} 14$

number of jumps $= \frac{90-14}{2} = 38$

therefore number of terms $= 39$

(d) $4 \xrightarrow{+5} 9 \xrightarrow{+5} 14 \ \ldots \ 224 \xrightarrow{+5} 229$

number of jumps $= \frac{229-4}{5} = 45$

therefore number of terms $= 46$

(e) $x \xrightarrow{+2x} 3x \xrightarrow{+2x} 5x \quad \ldots \quad 35x$

number of jumps $= \frac{35x - x}{2x} = 17$

number of terms $= 18$

(f) $a \xrightarrow{+d} a + d \xrightarrow{+d} a + 2d \quad \ldots \quad a + (n - 1)d$

number of jumps $= \frac{a + (n - 1)d - a}{d} = \frac{(n - 1)d}{d} = n - 1$

number of terms $= n$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise E, Question 1

Question:

Find **i** the 20th and **ii** the nth terms of the following arithmetic series:

(a) $2 + 6 + 10 + 14 + 18 \quad \ldots$

(b) $4 + 6 + 8 + 10 + 12 + \quad \ldots$

(c) $80 + 77 + 74 + 71 + \quad \ldots$

(d) $1 + 3 + 5 + 7 + 9 + \quad \ldots$

(e) $30 + 27 + 24 + 21 + \quad \ldots$

(f) $2 + 5 + 8 + 11 + \quad \ldots$

(g) $p + 3p + 5p + 7p + \quad \ldots$

(h) $5x + x + (- 3x) + (- 7x) + \quad \ldots$

Solution:

(a) $2 + 6 + 10 + 14 + 18$
$a = 2, d = 4$
(i) 20th term $= a + 19d = 2 + 19 \times 4 = 78$
(ii) nth term $= a + (n - 1) d = 2 + (n - 1) \times 4 = 4n - 2$

(b) $4 + 6 + 8 + 10 + 12$
$a = 4, d = 2$
(i) 20th term $= a + 19d = 4 + 19 \times 2 = 42$
(ii) nth term $= a + (n - 1) d = 4 + (n - 1) \times 2 = 2n + 2$

(c) $80 + 77 + 74 + 71 +$
$a = 80, d = - 3$
(i) 20th term $= a + 19d = 80 + 19 \times - 3 = 23$
(ii) nth term $= a + (n - 1) d = 80 + (n - 1) \times - 3 = 83 - 3n$

(d) $1 + 3 + 5 + 7 + 9$
$a = 1, d = 2$
(i) 20th term $= a + 19d = 1 + 19 \times 2 = 39$
(ii) nth term $= a + (n - 1) d = 1 + (n - 1) \times 2 = 2n - 1$

(e) $30 + 27 + 24 + 21$
$a = 30, d = - 3$
(i) 20th term $= a + 19d = 30 + 19 \times - 3 = - 27$
(ii) nth term $= a + (n - 1) d = 30 + (n - 1) \times - 3 = 33 - 3n$

(f) $2 + 5 + 8 + 11$
$a = 2, d = 3$
(i) 20th term $= a + 19d = 2 + 19 \times 3 = 59$
(ii) nth term $= a + (n - 1) d = 2 + (n - 1) \times 3 = 3n - 1$

(g) $p + 3p + 5p + 7p$
$a = p, d = 2p$

(i) 20th term $= a + 19d = p + 19 \times 2p = 39p$
(ii) nth term $= a + (n - 1)d = p + (n - 1) \times 2p = 2pn - p = (2n - 1)p$

(h) $5x + x + (-3x) + (-7x)$
$a = 5x, d = -4x$
(i) 20th term $= a + 19d = 5x + 19 \times -4x = -71x$
(ii) nth term $= a + (n - 1)d = 5x + (n - 1) \times -4x = 9x - 4nx = (9 - 4n)x$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise E, Question 2

Question:

Find the number of terms in the following arithmetic series:

(a) $5 + 9 + 13 + 17 + \ldots + 121$

(b) $1 + 1.25 + 1.5 + 1.75 \ldots + 8$

(c) $-4 + -1 + 2 + 5 \ldots + 89$

(d) $70 + 61 + 52 + 43 \ldots + (-200)$

(e) $100 + 95 + 90 + \ldots + (-1000)$

(f) $x + 3x + 5x \ldots + 153x$

Solution:

(a) $5 + 9 + 13 + 17 + \ldots + 121$
nth term $= a + (n-1)d$
$121 = 5 + (n-1) \times 4$
$116 = (n-1) \times 4$
$29 = (n-1)$
$30 = n$
$n = 30$ (30 terms)

(b) $1 + 1.25 + 1.5 + 1.75 + \ldots + 8$
nth term $= a + (n-1)d$
$8 = 1 + (n-1) \times 0.25$
$7 = (n-1) \times 0.25$
$28 = (n-1)$
$29 = n$
$n = 29$ (29 terms)

(c) $-4 + -1 + 2 + 5 + \ldots + 89$
nth term $= a + (n-1)d$
$89 = -4 + (n-1) \times 3$
$93 = (n-1) \times 3$
$31 = (n-1)$
$32 = n$
$n = 32$ (32 terms)

(d) $70 + 61 + 52 + 43 + \ldots + (-200)$
nth term $= a + (n-1)d$
$-200 = 70 + (n-1) \times -9$
$-270 = (n-1) \times -9$
$+30 = (n-1)$
$31 = n$
$n = 31$ (31 terms)

(e) $100 + 95 + 90 + \ldots + (-1000)$
nth term $= a + (n-1)d$
$-1000 = 100 + (n-1) \times -5$
$-1100 = (n-1) \times -5$
$+220 = (n-1)$

$221 = n$
$n = 221$ (221 terms)

(f) $x + 3x + 5x + \quad \ldots \quad + 153x$
nth term $= a + (n - 1) d$
$153x = x + (n - 1) \times 2x$
$152x = (n - 1) \times 2x$
$76 = (n - 1)$
$77 = n$
$n = 77$ (77 terms)

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise E, Question 3

Question:

The first term of an arithmetic series is 14. If the fourth term is 32, find the common difference.

Solution:

Let the common difference be d.
4th term $= a + 3d = 14 + 3d$ (first term = 14)
we are told the 4th term is 32

$\Rightarrow \quad 14 + 3d = 32$

$\Rightarrow \quad 3d = 18$

$\Rightarrow \quad d = 6$

Common difference is 6.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise E, Question 4

Question:

Given that the 3rd term of an arithmetic series is 30 and the 10th term is 9 find a and d.
Hence find which term is the first one to become negative.

Solution:

Let $a =$ first term and $d =$ common difference in the arithmetic series.

If 3rd term $= 30 \quad \Rightarrow \quad a + 2d = 30$ ①

If 10th term $= 9 \quad \Rightarrow \quad a + 9d = 9$ ②

② − ①: $7d = -21 \quad \Rightarrow \quad d = -3$

Substitute $d = -3$ into equation ①:

$a + 2 \times -3 = 30 \quad \Rightarrow \quad a = 36$

nth term in series $= 36 + (n - 1) \times -3 = 36 - 3n + 3 = 39 - 3n$
when $n = 13$, nth term $= 39 - 39 = 0$
when $n = 14$, nth term $= 39 - 42 = -3$
The 14th term is the first to be negative.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise E, Question 5

Question:

In an arithmetic series the 20th term is 14 and the 40th term is -6. Find the 10th term.

Solution:

Let $a =$ first term in the series and $d =$ common difference in the series.

20th term in series is 14 $\Rightarrow$ $a + 19d = 14$ ①

40th term in series is -6 $\Rightarrow$ $a + 39d = -6$ ②

Equation ② − ①: $20d = -20$ $\Rightarrow$ $d = -1$

Substitute $d = -1$ into equation ①:

$a + 19 \times -1 = 14$ $\Rightarrow$ $a = 33$

10th term $= a + 9d = 33 + 9 \times -1 = 33 - 9 = 24$

The 10th term in the series is 24.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise E, Question 6

Question:

The first three terms of an arithmetic series are $5x$, 20 and $3x$. Find the value of x and hence the values of the three terms.

Solution:

$5x, 20, 3x, \quad \ldots$
Term2 − Term1 = Term3 − Term2
$20 - 5x = 3x - 20$
$40 = 8x$
$5 = x$
Substituting $x = 5$ into the expressions gives
$5 \times 5, 20, 3 \times 5$
25, 20, 15
1st, 2nd, 3rd term

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise E, Question 7

Question:

For which values of x would the expression -8, x^2 and $17x$ form the first three terms of an arithmetic series?

Solution:

$-8, x^2, 17x$
Term2 − Term1 = Term3 − Term2
$x^2 - (-8) = 17x - x^2$
$x^2 + 8 = 17x - x^2$
$2x^2 - 17x + 8 = 0$
$(2x - 1)(x - 8) = 0$
$x = +\frac{1}{2}, +8$

Values of x are $+\frac{1}{2}$ or $+8$

Check:
$x = \frac{1}{2}$ gives terms

$$-8 \xrightarrow{+8\frac{1}{4}} \frac{1}{4} \xrightarrow{+8\frac{1}{4}} 8\frac{1}{2}$$

$x = 8$ gives terms

$$-8 \xrightarrow{+72} 64 \xrightarrow{+72} 136$$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise F, Question 1

Question:

Find the sums of the following series:

(a) $3 + 7 + 11 + 14 + \ldots$ (20 terms)

(b) $2 + 6 + 10 + 14 + \ldots$ (15 terms)

(c) $30 + 27 + 24 + 21 + \ldots$ (40 terms)

(d) $5 + 1 + -3 + -7 + \ldots$ (14 terms)

(e) $5 + 7 + 9 + \ldots + 75$

(f) $4 + 7 + 10 + \ldots + 91$

(g) $34 + 29 + 24 + 19 + \ldots + -111$

(h) $(x + 1) + (2x + 1) + (3x + 1) + \ldots + (21x + 1)$

Solution:

(a) $3 + 7 + 11 + 14 + \ldots$ (for 20 terms)
Substitute $a = 3$, $d = 4$ and $n = 20$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1 \right) d \right] = \frac{20}{2}\left(6 + 19 \times 4 \right) = 10 \times 82 = 820$$

(b) $2 + 6 + 10 + 14 + \ldots$ (for 15 terms)
Substitute $a = 2$, $d = 4$ and $n = 15$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1 \right) d \right] = \frac{15}{2}\left(4 + 14 \times 4 \right) = \frac{15}{2} \times 60 = 450$$

(c) $30 + 27 + 24 + 21 + \ldots$ (for 40 terms)
Substitute $a = 30$, $d = -3$ and $n = 40$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1 \right) d \right] = \frac{40}{2}\left(60 + 39 \times -3 \right) = 20 \times -57 = -1140$$

(d) $5 + 1 + -3 + -7 + \ldots$ (for 14 terms)
Substitute $a = 5$, $d = -4$ and $n = 14$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1 \right) d \right] = \frac{14}{2}\left(10 + 13 \times -4 \right) = 7 \times -42 = -294$$

(e) $5 + 7 + 9 + \ldots + 75$
Here $a = 5$, $d = 2$ and $L = 75$.
Use $L = a + (n - 1) d$ to find the number of terms n.
$75 = 5 + (n - 1) \times 2$
$70 = (n - 1) \times 2$
$35 = n - 1$
$n = 36$ (36 terms)

Substitute $a = 5$, $d = 2$, $n = 36$ and $L = 75$ into

$$S_n = \frac{n}{2}\left(a + L\right) = \frac{36}{2}\left(5 + 75\right) = 18 \times 80 = 1440$$

(f) $4 + 7 + 10 + \dots + 91$
Here $a = 4$, $d = 3$ and $L = 91$.
Use $L = a + \left(n - 1\right)d$ to find the number of terms n.
$91 = 4 + \left(n - 1\right) \times 3$
$87 = \left(n - 1\right) \times 3$
$29 = \left(n - 1\right)$
$n = 30$ (30 terms)
Substitute $a = 4$, $d = 3$, $L = 91$ and $n = 30$ into

$$S_n = \frac{n}{2}\left(a + L\right) = \frac{30}{2}\left(4 + 91\right) = 15 \times 95 = 1425$$

(g) $34 + 29 + 24 + 19 + \dots + -111$
Here $a = 34$, $d = -5$ and $L = -111$.
Use $L = a + \left(n - 1\right)d$ to find the number of terms n.
$-111 = 34 + \left(n - 1\right) \times -5$
$-145 = \left(n - 1\right) \times -5$
$29 = \left(n - 1\right)$
$30 = n$ (30 terms)
Substitute $a = 34$, $d = -5$, $L = -111$ and $n = 30$ into

$$S_n = \frac{n}{2}\left(a + L\right) = \frac{30}{2}\left(34 + -111\right) = 15 \times -77 = -1155$$

(h) $\left(x + 1\right) + \left(2x + 1\right) + \left(3x + 1\right) + \dots + \left(21x + 1\right)$
Here $a = x + 1$, $d = x$ and $L = 21x + 1$.
Use $L = a + \left(n - 1\right)d$ to find the number of terms n.
$21x + 1 = x + 1 + \left(n - 1\right) \times x$
$20x = \left(n - 1\right) \times x$
$20 = \left(n - 1\right)$
$21 = n$ (21 terms)
Substitute $a = x + 1$, $d = x$, $L = 21x + 1$ and $n = 21$ into

$$S_n = \frac{n}{2}\left(a + L\right) = \frac{21}{2}\left(x + 1 + 21x + 1\right) = \frac{21}{2} \times \left(22x + 2\right) = 21\left(11x + 1\right)$$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise F, Question 2

Question:

Find how many terms of the following series are needed to make the given sum:

(a) $5 + 8 + 11 + 14 + \ldots = 670$

(b) $3 + 8 + 13 + 18 + \ldots = 1575$

(c) $64 + 62 + 60 + \ldots = 0$

(d) $34 + 30 + 26 + 22 + \ldots = 112$

Solution:

(a) $5 + 8 + 11 + 14 + \ldots = 670$
Substitute $a = 5$, $d = 3$, $S_n = 670$ into

$$S_n = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$$

$$670 = \frac{n}{2} \left[10 + \left(n - 1 \right) \times 3 \right]$$

$$670 = \frac{n}{2} \left(3n + 7 \right)$$

$1340 = n (3n + 7)$
$0 = 3n^2 + 7n - 1340$
$0 = (n - 20) (3n + 67)$

$n = 20 \text{ or } - \frac{67}{3}$

Number of terms is 20

(b) $3 + 8 + 13 + 18 + \ldots = 1575$
Substitute $a = 3$, $d = 5$, $S_n = 1575$ into

$$S_n = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$$

$$1575 = \frac{n}{2} \left[6 + \left(n - 1 \right) \times 5 \right]$$

$$1575 = \frac{n}{2} \left(5n + 1 \right)$$

$3150 = n (5n + 1)$
$0 = 5n^2 + n - 3150$
$0 = (5n + 126) (n - 25)$

$n = - \frac{126}{5}, 25$

Number of terms is 25

(c) $64 + 62 + 60 + \ldots = 0$

Substitute $a = 64$, $d = -2$ and $S_n = 0$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$$

$$0 = \frac{n}{2}\left[128 + \left(n - 1\right) \times -2\right]$$

$$0 = \frac{n}{2}\left(130 - 2n\right)$$

$0 = n(65 - n)$
$n = 0$ or 65
Number of terms is 65

(d) $34 + 30 + 26 + 22 + \ldots = 112$
Substitute $a = 34$, $d = -4$ and $S_n = 112$ into

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$$

$$112 = \frac{n}{2}\left[68 + \left(n - 1\right) \times -4\right]$$

$$112 = \frac{n}{2}\left(72 - 4n\right)$$

$112 = n(36 - 2n)$
$2n^2 - 36n + 112 = 0$
$n^2 - 18n + 56 = 0$
$(n - 4)(n - 14) = 0$
$n = 4$ or 14
Number of terms is 4 or 14

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise F, Question 3

Question:

Find the sum of the first 50 even numbers.

Solution:

$$S = \underbrace{2 + 4 + 6 + 8 + \cdots}_{50 \text{ terms}}$$

This is an arithmetic series with $a = 2$, $d = 2$ and $n = 50$.

Use $S_n = \frac{n}{2}\left[2a + (n-1)d \right]$

So $S = \frac{50}{2}(4 + 49 \times 2) = 25 \times 102 = 2550$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise F, Question 4

Question:

Carol starts a new job on a salary of £20000. She is given an annual wage rise of £500 at the end of every year until she reaches her maximum salary of £25000. Find the total amount she earns (assuming no other rises),

(a) in the first 10 years and

(b) over 15 years.

Solution:

Total salary

$$= \underset{\text{Year 1}}{\underset{\uparrow}{20\,000}} + \underset{\text{Year 2}}{\underset{\uparrow}{20\,500}} + \underset{\text{Year 3}}{\underset{\uparrow}{21\,000}} + \underset{\text{Year 4}}{\underset{\uparrow}{21\,500}} + \ldots$$

1st increment 2nd increment 3rd increment

Carol will reach her maximum salary after

$\frac{25000 - 20000}{500} = 10$ increments

This will be after 11 years.

(a) Total amount after 10 years

$$= \underbrace{20\,000 + 20\,500 + 21\,000 + \ldots}$$

This is an arithmetic series with $a = 20000$, $d = 500$ and $n = 10$. Use $S_n = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$.

$= \frac{10}{2} \left(40000 + 9 \times 500 \right)$

$= 5 \times 44500$

$=$ £ 222 500

(b) From year 11 to year 15 she will continue to earn £ 25 000.

Total in this time $= 5 \times 25000 =$ £ 125000.

Total amount in the first 15 years is

£ 222 500 + £ 125000 = £ 347 500

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise F, Question 5

Question:

Find the sum of the multiples of 3 less than 100. Hence or otherwise find the sum of the numbers less than 100 which are not multiples of 3.

Solution:

Sum of multiples of 3 less than 100

$= \underbrace{3 + 6 + 9 + 12 \ldots + 96 + 99}$

This is an arithmetic series with $a = 3$, $d = 3$ and $n = \frac{99-3}{3} + 1 = 33$ terms.

Use $S_n = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$

$= \frac{33}{2} \left[2 \times 3 + \left(33 - 1 \right) \times 3 \right]$

$= \frac{33}{2} (6 + 96)$

$= 33 \times 51$

$= 1683$

Sum of numbers less than 100 that are not multiples of 3

$= 1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 + \ldots + 97 + 98$

$= \left(1 + 2 + 3 + \ldots + 97 + 98 + 99 \right) - \left(3 + 6 + \ldots 96 + 99 \right)$

$= \frac{99}{2} \left[2 + \left(99 - 1 \right) \times 1 \right] - 1683$

$= \frac{99}{2} \times 100 - 1683$

$= 4950 - 1683$

$= 3267$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise F, Question 6

Question:

James decides to save some money during the six-week holiday. He saves 1p on the first day, 2p on the second, 3p on the third and so on. How much will he have at the end of the holiday (42 days)? If he carried on, how long would it be before he has saved £100?

Solution:

Amount saved by James

$= \underbrace{1 + 2 + 3 + \ldots 42}$

This is an arithmetic series with $a = 1$, $d = 1$, $n = 42$ and $L = 42$.

Use $S_n = \frac{n}{2} \left(a + L \right)$

$= \frac{42}{2} \left(1 + 42 \right)$

$= 21 \times 43$

$= 903\text{p}$

$= £ 9.03$

To save £100 we need

$\underbrace{1 + 2 + 3 + \ldots}_{\text{Sum to } n \text{ terms}} = 10000$

$\frac{n}{2} \left[2 \times 1 + \left(n - 1 \right) \times 1 \right] = 10000$

$\frac{n}{2} \left(n + 1 \right) = 10000$

$n \left(n + 1 \right) = 20000$

$n^2 + n - 20000 = 0$

$n = \frac{-1 \pm \sqrt{\left(1 \right)^2 - 4 \times 1 \times \left(-20000 \right)}}{2}$

$n = 140.9$ or -141.9

It takes James 141 days to save £100.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Sequences and series

Exercise F, Question 7

Question:

The first term of an arithmetic series is 4. The sum to 20 terms is -15. Find, in any order, the common difference and the 20th term.

Solution:

Let common difference = d.
Substitute $a=4$, $n=20$, and $S_{20}=-15$ into

$$S_n=\frac{n}{2}\left[2a+\left(n-1\right)d\right]$$

$$-15=\frac{20}{2}\left[8+\left(20-1\right)d\right]$$

$-15=10\left(8+19d\right)$
$-1.5=8+19d$
$19d=-9.5$
$d=-0.5$
The common difference is -0.5.
Use nth term $=a+\left(n-1\right)d$ to find
20th term $=a+19d=4+19\times -0.5=4-9.5=-5.5$
20th term is -5.5.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise F, Question 8

Question:

The sum of the first three numbers of an arithmetic series is 12. If the 20th term is -32, find the first term and the common difference.

Solution:

Let the first term be a and the common difference d.
Sum of first three terms is 12, so
$a + (a + d) + (a + 2d) = 12$
$3a + 3d = 12$
$a + d = 4$ ①
20th term is -32, so
$a + 19d = -32$ ②
Equation ② − equation ①:
$18d = -36$
$d = -2$
Substitute $d = -2$ into equation ①:
$a + -2 = 4$
$a = 6$
Therefore, first term is 6 and common difference is -2.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise F, Question 9

Question:

Show that the sum of the first $2n$ natural numbers is $n\left(2n+1\right)$.

Solution:

Sum required

$$= \underbrace{1 + 2 + 3 + \ldots 2n}$$

Arithmetic series with $a = 1$, $d = 1$ and $n = 2n$.

Use $S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

$$= \frac{2n}{2}\left[2 \times 1 + \left(2n-1\right) \times 1\right]$$

$$= \frac{\cancel{2}n}{\cancel{2}}\left(2n+1\right)$$

$$= n\left(2n+1\right)$$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise F, Question 10

Question:

Prove that the sum of the first n odd numbers is n^2.

Solution:

Required sum

$$= \underbrace{1 + 3 + 5 + 7 + \ldots}_{n \text{ terms}}$$

This is an arithmetic series with $a = 1$, $d = 2$ and $n = n$.

Use $S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

$= \frac{n}{2}\left[2 \times 1 + \left(n-1\right) \times 2\right]$

$= \frac{n}{2}\left(2 + 2n - 2\right)$

$= \frac{n \times \cancel{2}n}{\cancel{2}}$

$= n \times n$

$= n^2$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise G, Question 1

Question:

Rewrite the following sums using Σ notation:

(a) $4 + 7 + 10 + \ldots + 31$

(b) $2 + 5 + 8 + 11 + \ldots + 89$

(c) $40 + 36 + 32 + \ldots + 0$

(d) The multiples of 6 less than 100

Solution:

(a) $4 + 7 + 10 + \ldots + 31$
Here $a = 4$ and $d = 3$,
nth term $= 4 + (n - 1) \times 3 = 3n + 1$
4 is the 1st term $(3 \times 1 + 1)$
31 is the 10th term $(3 \times 10 + 1)$

Hence series is $\sum_{r=1}^{10} (3r + 1)$.

(b) $2 + 5 + 8 + 11 + \ldots + 89$
Here $a = 2$ and $d = 3$,
nth term $= 2 + (n - 1) \times 3 = 3n - 1$
2 is the 1st term $(3 \times 1 - 1)$
89 is the 30th term $(3 \times 30 - 1)$

Hence series is $\sum_{r=1}^{30} (3r - 1)$.

(c) $40 + 36 + 32 + \ldots + 0$
Here $a = 40$ and $d = -4$,
nth term $= 40 + (n - 1) \times -4 = 44 - 4n$
40 is the 1st term $(44 - 4 \times 1)$
0 is the 11th term $(44 - 4 \times 11)$

Hence series is $\sum_{r=1}^{11} (44 - 4r)$.

(d) Multiples of 6 less than 100 $= 6 + 12 + 18 + \ldots + 96$
6 is the 1st multiple
96 is the 16th multiple

Hence series is $\sum_{r=1}^{16} 6r$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise G, Question 2

Question:

Calculate the following:

(a) <semantics> $\sum_{r=1}^{5} 3r$ </semantics>

(b) <semantics> $\sum_{r=1}^{10} (4r-1)$ </semantics>

(c) <semantics> $\sum_{r=1}^{20} (5r-2)$ </semantics>

(d) <semantics> $\sum_{r=0}^{5} r(r+1)$ </semantics>

Solution:

(a) <semantics> $\sum_{r=1}^{5} 3r = 3 + 6 + \ldots + 15$ </semantics>

Arithmetic series with $a = 3$, $d = 3$, $n = 5$, $L = 15$

Use $S_n = \frac{n}{2}\left(a + L \right)$

$= \frac{5}{2}\left(3 + 15 \right)$

$= 45$

(b) <semantics> $\sum_{r=1}^{10} (4r-1) = 3 + 7 + 11 + \ldots + 39$ </semantics>

Arithmetic series with $a = 3$, $d = 4$, $n = 10$, $L = 39$

Use $S_n = \frac{n}{2}\left(a + L \right)$

$= \frac{10}{2}\left(3 + 39 \right)$

$= 5 \text{ x } 42$

$= 210$

(c) <semantics> $\sum_{r=1}^{20} (5r - 2) = (5 \times 1 - 2) + (5 \times 2 - 2) + (5 \times 3 - 2) + \ldots + (5 \times 20 - 2)$
</semantics>

$= 3 + 8 + 13 + \ldots + 98$

Arithmetic series with $a = 3$, $d = 5$, $n = 20$, $L = 98$

Use $S_n = \frac{n}{2}\left(a + L\right)$

$= \frac{20}{2}\left(3 + 98\right)$

$= 10 \times 101$

$= 1010$

(d) <semantics> $\sum_{r=0}^{5} r(r+1)$ </semantics> is not an arithmetic series, so simply add the terms

<semantics> $\sum_{r=0}^{5} r(r+1) = 0 + 2 + 6 + 12 + 20 + 30$ </semantics>

$= 70$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise G, Question 3

Question:

For what value of n does $\sum_{r=1}^{n} (5r + 3)$ first exceed 1000?

Solution:

$$\sum_{r=1}^{n} (5r+3)$$
$$= (5\times 1+3) + (5\times 2+3) + (5\times 3+3) + \ldots + (5\times n+3)$$
$$= \underbrace{8 + 13 + 18 + \ldots + 5n+3}$$

Arithmetic series with $a = 8$, $d = 5$ and $n = n$.

Use $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{n}{2}\left[16 + (n-1)\times 5\right]$

$= \frac{n}{2}(5n + 11)$

If sum exceeds 1000 then

$\frac{n}{2}\left(5n + 11\right) > 1000$

$n(5n + 11) > 2000$

$5n^2 + 11n - 2000 > 0$

Solve equality $5n^2 + 11n - 2000 = 0$

$n = \frac{-11 \pm \sqrt{(11)^2 - 4\times 5\times -2000}}{2\times 5} = \frac{-11 \pm 200.30\ldots}{10} = 18.93 \text{ or } -21.13$

The sum has to be bigger than 1000

$\Rightarrow \quad n = 19$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise G, Question 4

Question:

For what value of n would $\sum_{r=1}^{n} (100 - 4r) = 0$?

Solution:

$$\sum_{r=1}^{n} (100 - 4r)$$

$= (100 - 4 \times 1) + (100 - 4 \times 2) + (100 - 4 \times 3) + \ldots + (100 - 4n)$

$= \underbrace{96 + 92 + 88 + \ldots + (100 - 4n)}$

Arithmetic series with $a = 96$, $d = -4$ and $n = n$.

Use the sum formula $S_n = \frac{n}{2}\left[2a + (n-1)d \right]$

$= \frac{n}{2}\left[192 + (n-1) \times -4 \right]$

$= \frac{n}{2}(196 - 4n)$

$= n(98 - 2n)$

we require the sum to be zero, so

$n(98 - 2n) = 0 \quad \Rightarrow \quad n = 0$ or $\frac{98}{2}$

Hence the value of n is 49.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 1

Question:

The rth term in a sequence is $2 + 3r$. Find the first three terms of the sequence.

Solution:

Substitute $r = 1$ in $2 + 3r = 2 + 3 \times 1 = 5$
1st term $= 5$
Substitute $r = 2$ in $2 + 3r = 2 + 3 \times 2 = 2 + 6 = 8$
2nd term $= 8$
Substitute $r = 3$ in $2 + 3r = 2 + 3 \times 3 = 2 + 9 = 11$
3rd term $= 11$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 2

Question:

The rth term in a sequence is $(r+3)(r-4)$. Find the value of r for the term that has the value 78.

Solution:

rth term $= (r+3)(r-4)$
when rth term $= 78$
$78 = (r+3)(r-4)$
$78 = r^2 - 1r - 12$
$0 = r^2 - 1r - 90$
$0 = (r-10)(r+9)$
$r = 10, -9$
r must be 10.
[Check: Substitute $r = 10$ in $(r+3)(r-4)$
$\Rightarrow \quad (10+3)(10-4) = 13 \times 6 = 78$ ✓]

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 3

Question:

A sequence is formed from an inductive relationship:

$U_{n+1} = 2U_n + 5$

Given that $U_1 = 2$, find the first four terms of the sequence.

Solution:

$U_{n+1} = 2U_n + 5$
Substitute $n = 1$ $\Rightarrow$ $U_2 = 2U_1 + 5$
$U_1 = 2$ $\Rightarrow$ $U_2 = 2 \times 2 + 5 = 9$
Substitute $n = 2$ $\Rightarrow$ $U_3 = 2U_2 + 5$
$U_2 = 9$ $\Rightarrow$ $U_3 = 2 \times 9 + 5 = 23$
Substitute $n = 3$ $\Rightarrow$ $U_4 = 2U_3 + 5$
$U_3 = 23$ $\Rightarrow$ $U_4 = 2 \times 23 + 5 = 51$

The first four terms of the sequence are 2, 9, 23 and 51.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 4

Question:

Find a rule that describes the following sequences:

(a) 5, 11, 17, 23, …

(b) 3, 6, 9, 12, …

(c) 1, 3, 9, 27, …

(d) 10, 5, 0, − 5, …

(e) 1, 4, 9, 16, …

(f) 1, 1.2, 1.44, 1.728

Which of the above are arithmetic sequences?

For the ones that are, state the values of *a* and *d*.

Solution:

(a) $5 \xrightarrow{+6} 11 \xrightarrow{+6} 17 \xrightarrow{+6} 23 \ldots$

"Add 6 to the previous term."

(b) $3 \xrightarrow{+3} 6 \xrightarrow{+3} 9 \xrightarrow{+3} 12 \ldots$

"Add 3 to the previous term."

(c) $1 \xrightarrow{\times 3} 3 \xrightarrow{\times 3} 9 \xrightarrow{\times 3} 27 \ldots$

"Multiply the previous term by 3."

(d) $10 \xrightarrow{-5} 5 \xrightarrow{-5} 0 \xrightarrow{-5} -5 \ldots$

"Subtract 5 from the previous term."

(e) $1 \xrightarrow{+3} 4 \xrightarrow{+5} 9 \xrightarrow{+7} 16 \ldots$

“Add consecutive odd numbers to each term.” or “They are the square numbers.”

(f) $1 \xrightarrow{\times 1.2} 1.2 \xrightarrow{\times 1.2} 1.44 \xrightarrow{\times 1.2} 1.728 \ldots$

“Multiply the previous term by 1.2.”

The arithmetic sequences are (a) where $a = 5$, $d = 6$, (b) where $a = 3$, $d = 3$,
(d) where $a = 10$, $d = -5$.
Alternatively you could give the nth terms of the series as (a) $6n - 1$ (b) $3n$ (c) 3^{n-1} (d) $15 - 5n$ (e) n^2 (f) 1.2^{n-1}

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Sequences and series

Exercise H, Question 5

Question:

For the arithmetic series $5 + 9 + 13 + 17 + \ldots$

Find **a** the 20th term, and **b** the sum of the first 20 terms.

Solution:

$$5 \xrightarrow{+4} 9 \xrightarrow{+4} 13 \xrightarrow{+4} 17 \ldots$$

The above sequence is arithmetic with $a = 5$ and $d = 4$.

(a) As nth term $= a + (n - 1)d$

20th term $= a + (20 - 1)d = a + 19d$

Substitute $a = 5$, $d = 4$ $\Rightarrow$ 20th term $= 5 + 19 \times 4 = 5 + 76 = 81$

(b) As sum to n terms $S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$

$$S_{20} = \frac{20}{2}\left[2a + \left(20 - 1\right)d\right] = 10\left(2a + 19d\right)$$

Substitute $a = 5$, $d = 4$ $\Rightarrow$ $S_{20} = 10(2 \times 5 + 19 \times 4) = 10 \times (10 + 76) = 10 \times 86 = 860$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 6

Question:

(a) Prove that the sum of the first n terms in an arithmetic series is

$S = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$

where a = first term and d = common difference.

(b) Use this to find the sum of the first 100 natural numbers.

Solution:

(a) $S = a + (a + d) + (a + 2d) + \ldots [a + (n - 2)d] + [a + (n - 1)d]$
Turning series around:
$S = [a + (n - 1)d] + [a + (n - 2)d] + \ldots (a + d) + a$
Adding the two sums:
$2S = [2a + (n - 1)d] + [2a + (n - 1)d] + \ldots [2a + (n - 1)d] + [2a + (n - 1)d]$
There are n lots of $[2a + (n - 1)d]$:
$2S = n \times [2a + (n - 1)d]$
$(\div 2)\ S = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$

(b) The first 100 natural numbers are 1,2,3, ... 100.
We need to find $S = 1 + 2 + 3 + \ldots 99 + 100$.
This series is arithmetic with $a = 1$, $d = 1$, $n = 100$.
Using $S = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$ with $a = 1$, $d = 1$ and $n = 100$ gives

$S = \frac{100}{2}\left[2 \times 1 + \left(100 - 1\right) \times 1\right] = \frac{100}{2}\left(2 + 99 \times 1\right) = 50 \times 101 = 5050$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 7

Question:

Find the least value of n for which $\sum_{r=1}^{n} (4r - 3) > 2000$.

Solution:

$$\sum_{r=1}^{n} (4r - 3) = (4 \times 1 - 3) + (4 \times 2 - 3) + (4 \times 3 - 3) \ldots .(4 \times n - 3)$$

$$= \underbrace{1 + 5 + 9 + \ldots + 4n - 3}$$

Arithmetic series with $a = 1$, $d = 4$.

Using $S_n = \frac{n}{2} \left[2a + (n - 1)d \right]$ with $a = 1$, $d = 4$ gives

$$S_n = \frac{n}{2} \left[2 \times 1 + (n - 1) \times 4 \right] = \frac{n}{2}(2 + 4n - 4) = \frac{n}{2}(4n - 2) = n(2n - 1)$$

Solve $S_n = 2000$:

$n (2n - 1) = 2000$

$2n^2 - n = 2000$

$2n^2 - n - 2000 = 0$

$n = \frac{1 \pm \sqrt{1 - 4 \times 2 \times - 2000}}{2 \times 2} = 31.87$ or $- 31.37$

n must be positive, so $n = 31.87$.

If the sum has to be greater than 2000 then $n = 32$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 8

Question:

A salesman is paid commission of £10 per week for each life insurance policy that he has sold. Each week he sells one new policy so that he is paid £10 commission in the first week, £20 commission in the second week, £30 commission in the third week and so on.

(a) Find his total commission in the first year of 52 weeks.

(b) In the second year the commission increases to £11 per week on new policies sold, although it remains at £10 per week for policies sold in the first year. He continues to sell one policy per week. Show that he is paid £542 in the second week of his second year.

(c) Find the total commission paid to him in the second year. **[E]**

Solution:

(a) Total commission

$$= \underbrace{10 + 20 + 30 + \dots + 520}$$

Arithmetic series with $a = 10$, $d = 10$, $n = 52$.

$= \frac{52}{2}\left[2 \times 10 + (52 - 1) \times 10\right]$ using $S_n = \frac{n}{2}\left[2a + (n - 1)d\right]$

$= 26\,(20 + 51 \times 10)$

$= 26\,(20 + 510)$

$= 26 \times 530$

$=$ £ 13780

(b) Commission = policies for year 1 + policies for 2nd week of year 2 $= 520 + 22 =$ £ 542

(c) Total commission for year 2

= Commission for year 1 policies + Commission for year 2 policies

$= 520 \times 52 + (11 + 22 + 33 + \dots \; 52 \times 11)$

Use $S_n = \frac{n}{2} = \left[2a + (n - 1)d\right]$ with $n = 52$, $a = 11$, $d = 11$

$= 27040 + \frac{52}{2}\left[2 \times 11 + (52 - 1) \times 11\right]$

$=$ £ $27040 + 26 \times (22 + 51 \times 11)$

$=$ £ 27 040 + £ 15 158

$=$ £ 42 198

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 9

Question:

The sum of the first two terms of an arithmetic series is 47.
The thirtieth term of this series is -62. Find:

(a) The first term of the series and the common difference.

(b) The sum of the first 60 terms of the series. **[E]**

Solution:

Let $a =$ first term and $d =$ common difference.
Sum of the first two terms $= 47$

$\Rightarrow \quad a + a + d = 47$

$\Rightarrow \quad 2a + d = 47$

30th term $= -62$
Using nth term $= a + (n-1)d$

$\Rightarrow \quad a + 29d = -62$ (Note: $a + 12d$ is a common error here)

Our two simultaneous equations are

$2a + d = 47$ ①

$a + 29d = -62$ ②

$2a + 58d = -124$ ③ (② × 2)

$57d = -171$ (③ − ①)

$d = -3$ (÷ 57)

Substitute $d = -3$ into ①: $2a - 3 = 47 \quad \Rightarrow \quad 2a = 50 \quad \Rightarrow \quad a = 25$

Therefore, (a) first term $= 25$ and common difference $= -3$

(b) using $S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

$S_{60} = \frac{60}{2}\left[2a + \left(60-1\right)d\right] = 30\left(2a + 59d\right)$

Substituting $a = 25$, $d = -3$ gives
$S_{60} = 30(2 \times 25 + 59 \times -3) = 30(50 - 177) = 30 \times -127 = -3810$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 10

Question:

(a) Find the sum of the integers which are divisible by 3 and lie between 1 and 400.

(b) Hence, or otherwise, find the sum of the integers, from 1 to 400 inclusive, which are **not** divisible by 3. **[E]**

Solution:

(a) Sum of integers divisible by 3 which lie between 1 and 400
$= 3 + 6 + 9 + 12 + \ldots + 399$
This is an arithmetic series with $a = 3$, $d = 3$ and $L = 399$.
Using $L = a + (n - 1) d$
$399 = 3 + (n - 1) \times 3$
$399 = 3 + 3n - 3$
$399 = 3n$
$n = 133$
Therefore, there are 133 of these integers up to 400.

$$S_n = \frac{n}{2} \left(a + L \right) = \frac{133}{2} \left(3 + 399 \right) = \frac{133}{2} \times 402 = 26\ 733$$

(b) Sum of integers not divisible by 3
$= 1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 \ldots 400$

$$= \underbrace{(1 + 2 + 3 + 4 \ldots + 399 + 400)}_{\text{Arithmetic series with } a = 1,\ d = 1,\ L = 400,\ n = 400} - \underbrace{(3 + 6 + 9 + \ldots + 399)}_{\text{From part (a). This equals 26733}}$$

$$Sn = \frac{400}{2}(1 + 400)$$
$$= 200 \times 401$$
$$= 80200$$

$= 80200 - 26733$
$= 53467$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 11

Question:

A polygon has 10 sides. The lengths of the sides, starting with the smallest, form an arithmetic series. The perimeter of the polygon is 675 cm and the length of the longest side is twice that of the shortest side. Find, for this series:

(a) The common difference.

(b) The first term. **[E]**

Solution:

If we let the smallest side be a, the other sides would be $a + d$, a + 2d, ... The longest side would be $a + 9d$.
If perimeter = 675, then
$a + (a + d) + (a + 2d) + \ldots + (a + 9d) = 675$

$\frac{10}{2} \left[2a + \left(10 - 1 \right) d \right] = 675$ (Sum to 10 terms of an arithmetic series)

$5 (2a + 9d) = 675 \quad (\div 5)$
$2a + 9d = 135$
The longest side is double the shortest side
$\Rightarrow \quad a + 9d = 2 \times a \qquad (- a)$
$\Rightarrow \quad 9d = a$
The simultaneous equations we need to solve are
$2a + 9d = 135$ ①
$9d = a$ ②
Substitute $9d = a$ into ①:
$2a + a = 135$
$3a = 135$
$a = 45$
Substitute back into ②:
$9d = 45$
$d = 5$
Therefore (a) the common difference = 5 and (b) the first term = 45.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Sequences and series

Exercise H, Question 12

Question:

A sequence of terms $\{ U_n \}$ is defined for $n \geq 1$, by the recurrence relation $U_{n+2} = 2kU_{n+1} + 15U_n$, where k is a constant. Given that $U_1 = 1$ and $U_2 = -2$:

(a) Find an expression, in terms of k, for U_3.

(b) Hence find an expression, in terms of k, for U_4.

(c) Given also that $U_4 = -38$, find the possible values of k. **[E]**

Solution:

$U_{n+2} = 2kU_{n+1} + 15U_n$

(a) Replacing n by 1 gives
$U_3 = 2kU_2 + 15U_1$
We know $U_1 = 1$ and $U_2 = -2$, therefore
$U_3 = 2k \times -2 + 15 \times 1$
$U_3 = -4k + 15$

(b) Replacing n by 2 gives
$U_4 = 2kU_3 + 15U_2$
We know $U_2 = -2$ and $U_3 = -4k + 15$, therefore
$U_4 = 2k(-4k + 15) + 15 \times -2$
$U_4 = -8k^2 + 30k - 30$

(c) We are told that $U_4 = -38$, therefore
$-8k^2 + 30k - 30 = -38 \quad (+38)$
$-8k^2 + 30k + 8 = 0 \quad (\div -2)$
$4k^2 - 15k - 4 = 0$ (factorise)
$(4k + 1)(k - 4) = 0$
$k = -\frac{1}{4}, 4$

Possible values of k are $-\frac{1}{4}, 4$.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 13

Question:

Prospectors are drilling for oil. The cost of drilling to a depth of 50 m is £500. To drill a further 50 m costs £640 and, hence, the total cost of drilling to a depth of 100 m is £1140. Each subsequent extra depth of 50 m costs £140 more to drill than the previous 50 m.
(a) Show that the cost of drilling to a depth of 500 m is £11300.
(b) The total sum of money available for drilling is £76000. Find, to the earnest 50 m, the greatest depth that can be drilled. **[E]**

Solution:

(a) Cost of drilling to 500 m

$$= \underset{\substack{\uparrow \\ \text{1st} \\ \text{50 m}}}{500} + \underset{\substack{\uparrow \\ \text{2nd} \\ \text{50 m}}}{640} + \underset{\substack{\uparrow \\ \text{3rd} \\ \text{50 m}}}{780} + \ldots$$

There would be 10 terms because there are 10 lots of 50 m in 500 m.
Arithmetic series with $a = 500$, $d = 140$ and $n = 10$.

Using $S_n = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$

$= \frac{10}{2} \left[2 \times 500 + \left(10 - 1 \right) \times 140 \right]$
$= 5 \left(1000 + 9 \times 140 \right)$
$= 5 \times 2260$
$= £\ 11300$

(b) This time we are given $S = 76\ 000$. The first term will still be 500 and d remains 140.

Use $S = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$ with $S = 76000$, $a = 500$, $d = 140$ and solve for n.

$76000 = \frac{n}{2} \left[2 \times 500 + \left(n - 1 \right) \times 140 \right]$

$76000 = \frac{n}{2} \left[1000 + 140 \left(n - 1 \right) \right]$

$76000 = n \left[500 + 70 \left(n - 1 \right) \right]$
$76000 = n \left(500 + 70n - 70 \right)$
$76000 = n \left(70n + 430 \right)$ (multiply out)
$76000 = 70n^2 + 430n \quad \left(\div 10 \right)$
$7600 = 7n^2 + 43n$
$0 = 7n^2 + 43n - 7600$
$n = \frac{-43 \pm \sqrt{\left(43 \right)^2 - 4 \times 7 \times \left(-7600 \right)}}{2 \times 7}$ (using $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$n = 30.02, \left(-36.16 \right)$
only accept the positive answer.
There are 30 terms (to the nearest term).
So the greatest depth that can be drilled is $30 \times 50 = 1500$ m (to the nearest 50 m)

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 14

Question:

Prove that the sum of the first $2n$ multiples of 4 is $4n(2n+1)$. **[E]**

Solution:

$$\text{Sum} = \underset{\text{1st}}{\underset{\uparrow}{4}} + \underset{\text{2nd}}{\underset{\uparrow}{8}} + \underset{\text{3rd}}{\underset{\uparrow}{12}} + \ldots + \underset{2n\text{th}}{\underset{\uparrow}{8n}}$$

This is an arithmetic series with $a = 4$, $d = 4$ and $n = 2n$.

Using $S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

$$S_{2n} = \frac{\cancel{2}n}{\cancel{2}}[2\times4+(2n-1)\times4]$$

$= n(8+8n-4)$
$= n(8n+4)$
$= n\times4(2n+1)$
$= 4n(2n+1)$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 15

Question:

A sequence of numbers { U_n { is defined, for $n \geq 1$, by the recurrence relation $U_{n+1} = kU_n - 4$, where k is a constant. Given that $U_1 = 2$:

(a) Find expressions, in terms of k, for U_2 and U_3.

(b) Given also that $U_3 = 26$, use algebra to find the possible values of k. **[E]**

Solution:

(a) Replacing n with 1 $\Rightarrow$ $U_2 = kU_1 - 4$
$U_1 = 2 \Rightarrow U_2 = 2k - 4$
Replacing n with 2 $\Rightarrow$ $U_3 = kU_2 - 4$
$U_2 = 2k - 4 \Rightarrow U_3 = k(2k - 4) - 4 \Rightarrow U_3 = 2k^2 - 4k - 4$

(b) Substitute $U_3 = 26$
$\Rightarrow 2k^2 - 4k - 4 = 26$
$\Rightarrow 2k^2 - 4k - 30 = 0$ ($\div 2$)
$\Rightarrow k^2 - 2k - 15 = 0$ (factorise)
$\Rightarrow (k - 5)(k + 3) = 0$
$\Rightarrow k = 5, -3$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Sequences and series

Exercise H, Question 16

Question:

Each year, for 40 years, Anne will pay money into a savings scheme. In the first year she pays in £500. Her payments then increase by £50 each year, so that she pays in £550 in the second year, £600 in the third year, and so on.

(a) Find the amount that Anne will pay in the 40th year.

(b) Find the total amount that Anne will pay in over the 40 years.

(c) Over the same 40 years, Brian will also pay money into the savings scheme. In the first year he pays in £890 and his payments then increase by £*d* each year. Given that Brian and Anne will pay in exactly the same amount over the 40 years, find the value of *d*.
[E]

Solution:

(a) 1^{st} year = £ 500
2^{nd} year = £ 550 = £ (500 + 1 × 50)
3^{rd} year = £ 600 = £ (500 + 2 × 50)
⋮
40^{th} year = £ 500 + 39 × 50 = £ 2450

(b) Total amount paid in

$$= \underbrace{£500 + £550 + £600 + \ldots + £2450}$$

This is an arithmetic series with $a = 500$, $d = 50$, $L = 2450$ and $n = 40$.

$$= \frac{n}{2} \left(a + L \right)$$

$$= \frac{40}{2} \left(500 + 2450 \right)$$

$= 20 \times 2950$
$= £\ 59000$

(c) Brian's amount

$$= \underbrace{890 + (890 + d) + (890 + 2d) + \ldots}_{40 \text{ years}}$$

Use $S_n = \frac{n}{2} \left[2a + \left(n - 1 \right) d \right]$ with $n = 40$, $a = 890$ and d.

$$= \frac{40}{2} \left[2 \times 890 + \left(40 - 1 \right) d \right]$$

$= 20 (1780 + 39d)$
Use the fact that
Brian's savings = Anne's savings
$20 (1780 + 39d) = 59000$ (÷ 20)
$1780 + 39d = 2950$ (− 1780)
$39d = 1170$ (÷ 39)
$d = 30$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 17

Question:

The fifth term of an arithmetic series is 14 and the sum of the first three terms of the series is -3.

(a) Use algebra to show that the first term of the series is -6 and calculate the common difference of the series.

(b) Given that the nth term of the series is greater than 282, find the least possible value of n. **[E]**

Solution:

(a) Use nth term $= a + (n-1)d$:
5th term is 14 $\Rightarrow$ $a + 4d = 14$
Use 1st term $= a$, 2nd term $= a + d$, 3rd term $= a + 2d$:
sum of 1st three terms $= -3$
$\Rightarrow a + a + d + a + 2d = -3$
$\Rightarrow 3a + 3d = -3 \quad (\div 3)$
$\Rightarrow a + d = -1$
Our simultaneous equations are
$a + 4d = 14$ ①
$a + d = -1$ ②
① − ②: $3d = 15 \quad (\div 3)$
$d = 5$
Common difference $= 5$
Substitute $d = 5$ back in ②:
$a + 5 = -1$
$a = -6$
First term $= -6$

(b) nth term must be greater than 282
$\Rightarrow a + (n-1)d > 282$
$\Rightarrow -6 + 5(n-1) > 282 \quad (+6)$
$\Rightarrow 5(n-1) > 288 \quad (\div 5)$
$\Rightarrow (n-1) > 57.6 \quad (+1)$
$n > 58.6$
$\therefore$ least value of $n = 59$

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Sequences and series
Exercise H, Question 18

Question:

The fourth term of an arithmetic series is $3k$, where k is a constant, and the sum of the first six terms of the series is $7k+9$.

(a) Show that the first term of the series is $9-8k$.

(b) Find an expression for the common difference of the series in terms of k.
Given that the seventh term of the series is 12, calculate:

(c) The value of k.

(d) The sum of the first 20 terms of the series. **[E]**

Solution:

(a) We know nth term $=a+(n-1)d$
4th term is $3k \Rightarrow a+(4-1)d=3k \Rightarrow a+3d=3k$

We know $S_n=\frac{n}{2}\left[2a+\left(n-1\right)d\right]$

Sum to 6 terms is $7k+9$, therefore

$\frac{6}{2}\left[2a+\left(6-1\right)d\right]=7k+9$

$3(2a+5d)=7k+9$
$6a+15d=7k+9$
The simultaneous equations are
$a+3d=3k$ ①
$6a+15d=7k+9$ ②
① × 5: $5a+15d=15k$ ③
② − ③: $1a=-8k+9 \Rightarrow a=9-8k$
First term is $9-8k$

(b) Substituting this is ① gives
$9-8k+3d=3k$
$3d=11k-9$
$d=\frac{11k-9}{3}$

Common difference is $\frac{11k-9}{3}$.

(c) If the 7th term is 12, then
$a+6d=12$
Substitute values of a and d:

$-8k+9+6\times\left(\frac{11k-9}{3}\right)=12$

$-8k+9+2(11k-9)=12$
$-8k+9+22k-18=12$
$14k-9=12$

$14k = 21$

$k = \frac{21}{14} = 1.5$

(d) Calculate values of *a* and *d* first:

$a = 9 - 8k = 9 - 8 \times 1.5 = 9 - 12 = -3$

$d = \frac{11k - 9}{3} = \frac{11 \times 1.5 - 9}{3} = \frac{16.5 - 9}{3} = \frac{7.5}{3} = 2.5$

$S_{20} = \frac{20}{2} \left[2a + \left(20 - 1 \right) d \right]$

$= 10 \left(2a + 19d \right)$

$= 10 \left(2 \times -3 + 19 \times 2.5 \right)$

$= 10 \left(-6 + 47.5 \right)$

$= 10 \times 41.5$

$= 415$

Sum to 20 terms is 415.