

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise A, Question 1

Question:

Work out the gradients of these lines:

(a) $y = -2x + 5$

(b) $y = -x + 7$

(c) $y = 4 + 3x$

(d) $y = \frac{1}{3}x - 2$

(e) $y = -\frac{2}{3}x$

(f) $y = \frac{5}{4}x + \frac{2}{3}$

(g) $2x - 4y + 5 = 0$

(h) $10x - 5y + 1 = 0$

(i) $-x + 2y - 4 = 0$

(j) $-3x + 6y + 7 = 0$

(k) $4x + 2y - 9 = 0$

(l) $9x + 6y + 2 = 0$

Solution:

(a) Gradient = -2

(b) Gradient = -1

(c) Gradient = 3

(d) Gradient = $\frac{1}{3}$

(e) Gradient = $-\frac{2}{3}$

(f) Gradient = $\frac{5}{4}$

(g) $2x - 4y + 5 = 0$
 $2x + 5 = 4y$

$$4y = 2x + 5$$

$$y = \frac{2}{4}x + \frac{5}{4}$$

$$y = \frac{1}{2}x + \frac{5}{4}$$

$$\text{Gradient} = \frac{1}{2}$$

$$(h) 10x - 5y + 1 = 0$$

$$10x + 1 = 5y$$

$$5y = 10x + 1$$

$$y = \frac{10}{5}x + \frac{1}{5}$$

$$y = 2x + \frac{1}{5}$$

$$\text{Gradient} = 2$$

$$(i) -x + 2y - 4 = 0$$

$$2y - 4 = x$$

$$2y = x + 4$$

$$y = \frac{1}{2}x + 2$$

$$\text{Gradient} = \frac{1}{2}$$

$$(j) -3x + 6y + 7 = 0$$

$$6y + 7 = 3x$$

$$6y = 3x - 7$$

$$y = \frac{3}{6}x - \frac{7}{6}$$

$$y = \frac{1}{2}x - \frac{7}{6}$$

$$\text{Gradient} = \frac{1}{2}$$

$$(k) 4x + 2y - 9 = 0$$

$$2y - 9 = -4x$$

$$2y = -4x + 9$$

$$y = -\frac{4}{2}x + \frac{9}{2}$$

$$y = -2x + \frac{9}{2}$$

$$\text{Gradient} = -2$$

$$(l) 9x + 6y + 2 = 0$$

$$6y + 2 = -9x$$

$$6y = -9x - 2$$

$$y = -\frac{9}{6}x - \frac{2}{6}$$

$$y = -\frac{3}{2}x - \frac{1}{3}$$

$$\text{Gradient} = -\frac{3}{2}$$

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Exercise A, Question 2

Question:

These lines intercept the y-axis at $(0, c)$. Work out the value of c in each case.

(a) $y = -x + 4$

(b) $y = 2x - 5$

(c) $y = \frac{1}{2}x - \frac{2}{3}$

(d) $y = -3x$

(e) $y = \frac{6}{7}x + \frac{7}{5}$

(f) $y = 2 - 7x$

(g) $3x - 4y + 8 = 0$

(h) $4x - 5y - 10 = 0$

(i) $-2x + y - 9 = 0$

(j) $7x + 4y + 12 = 0$

(k) $7x - 2y + 3 = 0$

(l) $-5x + 4y + 2 = 0$

Solution:

(a) $c = 4$

(b) $c = -5$

(c) $c = -\frac{2}{3}$

(d) $y = -3x$
 $y = -3x + 0$
 $c = 0$

(e) $c = \frac{7}{5}$

(f) $y = 2 - 7x$
 $y = -7x + 2$
 $c = 2$

(g) $3x - 4y + 8 = 0$

$$3x + 8 = 4y$$

$$4y = 3x + 8$$

$$y = \frac{3}{4}x + \frac{8}{4}$$

$$y = \frac{3}{4}x + 2$$

$$c = 2$$

$$(h) 4x - 5y - 10 = 0$$

$$4x - 10 = 5y$$

$$5y = 4x - 10$$

$$y = \frac{4}{5}x - \frac{10}{5}$$

$$y = \frac{4}{5}x - 2$$

$$c = -2$$

$$(i) -2x + y - 9 = 0$$

$$y - 9 = 2x$$

$$y = 2x + 9$$

$$c = 9$$

$$(j) 7x + 4y + 12 = 0$$

$$4y + 12 = -7x$$

$$4y = -7x - 12$$

$$y = -\frac{7}{4}x - \frac{12}{4}$$

$$y = -\frac{7}{4}x - 3$$

$$c = -3$$

$$(k) 7x - 2y + 3 = 0$$

$$7x + 3 = 2y$$

$$2y = 7x + 3$$

$$y = \frac{7}{2}x + \frac{3}{2}$$

$$c = \frac{3}{2}$$

$$(l) -5x + 4y + 2 = 0$$

$$4y + 2 = 5x$$

$$4y = 5x - 2$$

$$y = \frac{5}{4}x - \frac{2}{4}$$

$$y = \frac{5}{4}x - \frac{1}{2}$$

$$c = -\frac{1}{2}$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise A, Question 3

Question:

Write these lines in the form $ax + by + c = 0$.

(a) $y = 4x + 3$

(b) $y = 3x - 2$

(c) $y = -6x + 7$

(d) $y = \frac{4}{5}x - 6$

(e) $y = \frac{5}{3}x + 2$

(f) $y = \frac{7}{3}x$

(g) $y = 2x - \frac{4}{7}$

(h) $y = -3x + \frac{2}{9}$

(i) $y = -6x - \frac{2}{3}$

(j) $y = -\frac{1}{3}x + \frac{1}{2}$

(k) $y = \frac{2}{3}x + \frac{5}{6}$

(l) $y = \frac{3}{5}x + \frac{1}{2}$

Solution:

$$\begin{aligned} \text{(a)} \quad y &= 4x + 3 \\ 0 &= 4x + 3 - y \\ 4x + 3 - y &= 0 \\ 4x - y + 3 &= 0 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad y &= 3x - 2 \\ 0 &= 3x - 2 - y \\ 3x - 2 - y &= 0 \\ 3x - y - 2 &= 0 \end{aligned}$$

$$\begin{aligned} \text{(c) } y &= -6x + 7 \\ 6x + y &= 7 \\ 6x + y - 7 &= 0 \end{aligned}$$

$$\text{(d) } y = \frac{4}{5}x - 6$$

Multiply each term by 5:

$$\begin{aligned} 5y &= 4x - 30 \\ 0 &= 4x - 30 - 5y \\ 4x - 30 - 5y &= 0 \\ 4x - 5y - 30 &= 0 \end{aligned}$$

$$\text{(e) } y = \frac{5}{3}x + 2$$

Multiply each term by 3:

$$\begin{aligned} 3y &= 5x + 6 \\ 0 &= 5x + 6 - 3y \\ 5x + 6 - 3y &= 0 \\ 5x - 3y + 6 &= 0 \end{aligned}$$

$$\text{(f) } y = \frac{7}{3}x$$

Multiply each term by 3:

$$\begin{aligned} 3y &= 7x \\ 0 &= 7x - 3y \\ 7x - 3y &= 0 \end{aligned}$$

$$\text{(g) } y = 2x - \frac{4}{7}$$

Multiply each term by 7:

$$\begin{aligned} 7y &= 14x - 4 \\ 0 &= 14x - 4 - 7y \\ 14x - 4 - 7y &= 0 \\ 14x - 7y - 4 &= 0 \end{aligned}$$

$$\text{(h) } y = -3x + \frac{2}{9}$$

Multiply each term by 9:

$$\begin{aligned} 9y &= -27x + 2 \\ 27x + 9y &= 2 \\ 27x + 9y - 2 &= 0 \end{aligned}$$

$$\text{(i) } y = -6x - \frac{2}{3}$$

Multiply each term by 3:

$$\begin{aligned} 3y &= -18x - 2 \\ 18x + 3y &= -2 \\ 18x + 3y + 2 &= 0 \end{aligned}$$

$$\text{(j) } y = -\frac{1}{3}x + \frac{1}{2}$$

Multiply each term by 6 (6 is divisible by both 3 and 2):

$$\begin{aligned} 6y &= -2x + 3 \\ 2x + 6y &= 3 \\ 2x + 6y - 3 &= 0 \end{aligned}$$

$$\text{(k) } y = \frac{2}{3}x + \frac{5}{6}$$

Multiply each term by 6 (6 is divisible by both 3 and 6):

$$6y = 4x + 5$$

$$0 = 4x + 5 - 6y$$

$$4x + 5 - 6y = 0$$

$$4x - 6y + 5 = 0$$

$$(1) y = \frac{3}{5}x + \frac{1}{2}$$

Multiply each term by 10 (10 is divisible by both 5 and 2):

$$10y = 6x + 5$$

$$0 = 6x + 5 - 10y$$

$$6x + 5 - 10y = 0$$

$$6x - 10y + 5 = 0$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane
Exercise A, Question 4

Question:

A line is parallel to the line $y = 5x + 8$ and its intercept on the y -axis is $(0, 3)$. Write down the equation of the line.

Solution:

The line is parallel to $y = 5x + 8$, so $m = 5$.

The line intercepts the y -axis at $(0, 3)$, so $c = 3$.

Using $y = mx + c$, the equation of the line is $y = 5x + 3$.

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Coordinate geometry in the (x, y) plane

Exercise A, Question 5

Question:

A line is parallel to the line $y = -\frac{2}{5}x + 1$ and its intercept on the y-axis is $(0, -4)$. Work out the equation of the line. Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Solution:

The line is parallel to $y = -\frac{2}{5}x + 1$, so $m = -\frac{2}{5}$.

The line intercepts the y-axis at $(0, -4)$, so $c = -4$.
Using $y = mx + c$, the equation of the line is

$$y = -\frac{2}{5}x - 4$$

Multiply each term by 5:

$$5y = -2x - 20$$

$$2x + 5y = -20$$

$$2x + 5y + 20 = 0$$

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Coordinate geometry in the (x, y) plane

Exercise A, Question 6

Question:

A line is parallel to the line $3x + 6y + 11 = 0$ and its intercept on the y-axis is $(0, 7)$. Write down the equation of the line.

Solution:

$$3x + 6y + 11 = 0$$

$$6y + 11 = -3x$$

$$6y = -3x - 11$$

$$y = -\frac{3}{6}x - \frac{11}{6}$$

$$y = -\frac{1}{2}x - \frac{11}{6}$$

The line is parallel to $y = -\frac{1}{2}x - \frac{11}{6}$, so $m = -\frac{1}{2}$.

The line intercepts the y-axis at $(0, 7)$, so $c = 7$.

Using $y = mx + c$, the equation of the line is $y = -\frac{1}{2}x + 7$

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Coordinate geometry in the (x, y) plane

Exercise A, Question 7

Question:

A line is parallel to the line $2x - 3y - 1 = 0$ and it passes through the point $(0, 0)$. Write down the equation of the line.

Solution:

$$2x - 3y - 1 = 0$$

$$2x - 1 = 3y$$

$$3y = 2x - 1$$

$$y = \frac{2}{3}x - \frac{1}{3}$$

The line is parallel to $y = \frac{2}{3}x - \frac{1}{3}$, so $m = \frac{2}{3}$.

The intercept on the y-axis is $(0, 0)$, so $c = 0$.

Using $y = mx + c$:

$$y = \frac{2}{3}x + 0$$

$$y = \frac{2}{3}x$$

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Coordinate geometry in the (x, y) plane
Exercise A, Question 8

Question:

The line $y = 6x - 18$ meets the x -axis at the point P . Work out the coordinates of P .

Solution:

$$y = 6x - 18$$

Substitute $y = 0$:

$$6x - 18 = 0$$

$$6x = 18$$

$$x = 3$$

The line meets the x -axis at $P (3 , 0)$.

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Coordinate geometry in the (x, y) plane
Exercise A, Question 9

Question:

The line $3x + 2y - 5 = 0$ meets the x -axis at the point R . Work out the coordinates of R .

Solution:

$$3x + 2y - 5 = 0$$

Substitute $y = 0$:

$$3x + 2(0) - 5 = 0$$

$$3x - 5 = 0$$

$$3x = 5$$

$$x = \frac{5}{3}$$

The line meets the x -axis at $R \left(\frac{5}{3}, 0 \right)$.

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Coordinate geometry in the (x, y) plane

Exercise A, Question 10

Question:

The line $5x - 4y + 20 = 0$ meets the y -axis at the point A and the x -axis at the point B . Work out the coordinates of the points A and B .

Solution:

$$5x - 4y + 20 = 0$$

Substitute $x = 0$:

$$5(0) - 4y + 20 = 0$$

$$-4y + 20 = 0$$

$$20 = 4y$$

$$4y = 20$$

$$y = 5$$

The line meets the y -axis at $A(0, 5)$.

Substitute $y = 0$:

$$5x - 4(0) + 20 = 0$$

$$5x + 20 = 0$$

$$5x = -20$$

$$x = -4$$

The line meets the x -axis at $B(-4, 0)$.

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise B, Question 1

Question:

Work out the gradient of the line joining these pairs of points:

(a) $(4, 2)$, $(6, 3)$

(b) $(-1, 3)$, $(5, 4)$

(c) $(-4, 5)$, $(1, 2)$

(d) $(2, -3)$, $(6, 5)$

(e) $(-3, 4)$, $(7, -6)$

(f) $(-12, 3)$, $(-2, 8)$

(g) $(-2, -4)$, $(10, 2)$

(h) $\left(\frac{1}{2}, 2\right)$, $\left(\frac{3}{4}, 4\right)$

(i) $\left(\frac{1}{4}, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{2}{3}\right)$

(j) $(-2.4, 9.6)$, $(0, 0)$

(k) $(1.3, -2.2)$, $(8.8, -4.7)$

(l) $(0, 5a)$, $(10a, 0)$

(m) $(3b, -2b)$, $(7b, 2b)$

(n) (p, p^2) , (q, q^2)

Solution:

(a) $(x_1, y_1) = (4, 2)$, $(x_2, y_2) = (6, 3)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{6 - 4} = \frac{1}{2}$$

(b) $(x_1, y_1) = (-1, 3)$, $(x_2, y_2) = (5, 4)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{5 - (-1)} = \frac{1}{6}$$

(c) $(x_1, y_1) = (-4, 5)$, $(x_2, y_2) = (1, 2)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{1 - (-4)} = -\frac{3}{5}$$

(d) $(x_1, y_1) = (2, -3)$, $(x_2, y_2) = (6, 5)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{6 - 2} = \frac{8}{4} = 2$$

(e) $(x_1, y_1) = (-3, 4)$, $(x_2, y_2) = (7, -6)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 4}{7 - (-3)} = -\frac{10}{10} = -1$$

(f) $(x_1, y_1) = (-12, 3)$, $(x_2, y_2) = (-2, 8)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 3}{-2 - (-12)} = \frac{5}{-2 + 12} = \frac{5}{10} = \frac{1}{2}$$

(g) $(x_1, y_1) = (-2, -4)$, $(x_2, y_2) = (10, 2)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{10 - (-2)} = \frac{6}{12} = \frac{1}{2}$$

(h) $\left(x_1, y_1 \right) = \left(\frac{1}{2}, 2 \right)$, $\left(x_2, y_2 \right) = \left(\frac{3}{4}, 4 \right)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{\frac{3}{4} - \frac{1}{2}} = \frac{2}{\frac{1}{4}} = 8$$

(i) $\left(x_1, y_1 \right) = \left(\frac{1}{4}, \frac{1}{2} \right)$, $\left(x_2, y_2 \right) = \left(\frac{1}{2}, \frac{2}{3} \right)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{2}{3} - \frac{1}{2}}{\frac{1}{2} - \frac{1}{4}} = \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{2}{3}$$

(j) $(x_1, y_1) = (-2.4, 9.6)$, $(x_2, y_2) = (0, 0)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 9.6}{0 - (-2.4)} = \frac{-9.6}{2.4} = -4$$

(k) $(x_1, y_1) = (1.3, -2.2)$, $(x_2, y_2) = (8.8, -4.7)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4.7 - (-2.2)}{8.8 - 1.3} = \frac{-2.5}{7.5} = -\frac{1}{3}$$

(l) $(x_1, y_1) = (0, 5a)$, $(x_2, y_2) = (10a, 0)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 5a}{10a - 0} = \frac{-5a}{10a} = \frac{-5}{10} = -\frac{1}{2}$$

(m) $(x_1, y_1) = (3b, -2b)$, $(x_2, y_2) = (7b, 2b)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2b - (-2b)}{7b - 3b} = \frac{4b}{4b} = 1$$

$$(n) (x_1, y_1) = (p, p^2), (x_2, y_2) = (q, q^2)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{q^2 - p^2}{q - p} = \frac{(q - p)(q + p)}{q - p} = q + p$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise B, Question 2

Question:

The line joining $(3, -5)$ to $(6, a)$ has gradient 4. Work out the value of a .

Solution:

$$(x_1, y_1) = (3, -5), (x_2, y_2) = (6, a)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = 4$$

$$\text{so } \frac{a - (-5)}{6 - 3} = 4$$

$$\Rightarrow \frac{a + 5}{3} = 4$$

$$\Rightarrow a + 5 = 12$$

$$\Rightarrow a = 7$$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 3

Question:

The line joining $(5, b)$ to $(8, 3)$ has gradient -3 . Work out the value of b .

Solution:

$$(x_1, y_1) = (5, b), (x_2, y_2) = (8, 3)$$

$$\frac{3-b}{8-5} = -3$$

$$\frac{3-b}{3} = -3$$

$$3-b = -9$$

$$b = 12$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane
Exercise B, Question 4

Question:

The line joining $(c, 4)$ to $(7, 6)$ has gradient $\frac{3}{4}$. Work out the value of c .

Solution:

$$(x_1, y_1) = (c, 4), (x_2, y_2) = (7, 6)$$

$$\frac{6-4}{7-c} = \frac{3}{4}$$

$$\frac{2}{7-c} = \frac{3}{4}$$

$$2 = \frac{3}{4} (7 - c)$$

$$8 = 3(7 - c)$$

$$8 = 21 - 3c$$

$$-13 = -3c$$

$$c = \frac{-13}{-3} = \frac{13}{3} = 4\frac{1}{3}$$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 5

Question:

The line joining $(-1, 2b)$ to $(1, 4)$ has gradient $-\frac{1}{4}$. Work out the value of b .

Solution:

$$(x_1, y_1) = (-1, 2b), (x_2, y_2) = (1, 4)$$

$$\frac{4 - 2b}{1 - (-1)} = -\frac{1}{4}$$

$$\frac{4 - 2b}{2} = -\frac{1}{4}$$

$$2 - b = -\frac{1}{4}$$

$$2\frac{1}{4} - b = 0$$

$$b = 2\frac{1}{4}$$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 6

Question:

The line joining $(-3, -2)$ to $(2e, 5)$ has gradient 2. Work out the value of e .

Solution:

$$(x_1, y_1) = (-3, -2), (x_2, y_2) = (2e, 5)$$

$$\frac{5 - (-2)}{2e - (-3)} = 2$$

$$\frac{7}{2e + 3} = 2$$

$$7 = 2(2e + 3)$$

$$7 = 4e + 6$$

$$4e = 1$$

$$e = \frac{1}{4}$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane
Exercise B, Question 7

Question:

The line joining $(7, 2)$ to $(f, 3f)$ has gradient 4. Work out the value of f .

Solution:

$$(x_1, y_1) = (7, 2), (x_2, y_2) = (f, 3f)$$

$$\frac{3f-2}{f-7} = 4$$

$$3f - 2 = 4(f - 7)$$

$$3f - 2 = 4f - 28$$

$$-2 = f - 28$$

$$28 - 2 = f$$

$$f = 26$$

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Coordinate geometry in the (x, y) plane
Exercise B, Question 8

Question:

The line joining $(3, -4)$ to $(-g, 2g)$ has gradient -3 . Work out the value of g .

Solution:

$$(x_1, y_1) = (3, -4), (x_2, y_2) = (-g, 2g)$$

$$\frac{2g - (-4)}{-g - 3} = -3$$

$$\frac{2g + 4}{-g - 3} = -3$$

$$2g + 4 = -3(-g - 3)$$

$$2g + 4 = 3g + 9$$

$$4 = g + 9$$

$$g = -5$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise B, Question 9

Question:

Show that the points $A(2, 3)$, $B(4, 4)$, $C(10, 7)$ can be joined by a straight line. (Hint: Find the gradient of the lines joining the points: **i** A and B and **ii** A and C .)

Solution:

The gradient of AB is $\frac{4-3}{4-2} = \frac{1}{2}$

The gradient of AC is $\frac{7-3}{10-2} = \frac{4}{8} = \frac{1}{2}$

The gradients are equal so the points can be joined by a straight line.

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Coordinate geometry in the (x, y) plane
Exercise B, Question 10

Question:

Show that the points $(-2a, 5a)$, $(0, 4a)$, $(6a, a)$ are collinear (i.e. on the same straight line).

Solution:

The gradient of the line joining $(-2a, 5a)$ and $(0, 4a)$ is

$$\frac{4a - 5a}{0 - (-2a)} = \frac{-a}{2a} = \frac{-1}{2}$$

The gradient of the line joining $(-2a, 5a)$ and $(6a, a)$ is

$$\frac{a - 5a}{6a - (-2a)} = \frac{-4a}{8a} = \frac{-4}{8} = \frac{-1}{2}$$

The gradients are equal so the points can be joined by a straight line (i.e. they are collinear).

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 1

Question:

Find the equation of the line with gradient m that passes through the point (x_1, y_1) when:

(a) $m = 2$ and $(x_1, y_1) = (2, 5)$

(b) $m = 3$ and $(x_1, y_1) = (-2, 1)$

(c) $m = -1$ and $(x_1, y_1) = (3, -6)$

(d) $m = -4$ and $(x_1, y_1) = (-2, -3)$

(e) $m = \frac{1}{2}$ and $(x_1, y_1) = (-4, 10)$

(f) $m = -\frac{2}{3}$ and $(x_1, y_1) = (-6, -1)$

(g) $m = 2$ and $(x_1, y_1) = (a, 2a)$

(h) $m = -\frac{1}{2}$ and $(x_1, y_1) = (-2b, 3b)$

Solution:

(a) $y - y_1 = m(x - x_1)$

$$y - 5 = 2(x - 2)$$

$$y - 5 = 2x - 4$$

$$y = 2x + 1$$

(b) $y - y_1 = m(x - x_1)$

$$y - 1 = 3[x - (-2)]$$

$$y - 1 = 3(x + 2)$$

$$y - 1 = 3x + 6$$

$$y = 3x + 7$$

(c) $y - y_1 = m(x - x_1)$

$$y - (-6) = -1(x - 3)$$

$$y + 6 = -x + 3$$

$$y = -x - 3$$

(d) $y - y_1 = m(x - x_1)$

$$y - (-3) = -4[x - (-2)]$$

$$y + 3 = -4(x + 2)$$

$$y + 3 = -4x - 8$$

$$y = -4x - 11$$

(e) $y - y_1 = m(x - x_1)$

$$y - 10 = \frac{1}{2} \left[x - \left(-4 \right) \right]$$

$$y - 10 = \frac{1}{2} \left(x + 4 \right)$$

$$y - 10 = \frac{1}{2}x + 2$$

$$y = \frac{1}{2}x + 12$$

$$(f) y - y_1 = m (x - x_1)$$

$$y - \left(-1 \right) = -\frac{2}{3} \left[x - \left(-6 \right) \right]$$

$$y + 1 = -\frac{2}{3} \left(x + 6 \right)$$

$$y + 1 = -\frac{2}{3}x - 4$$

$$y = -\frac{2}{3}x - 5$$

$$(g) y - y_1 = m (x - x_1)$$

$$y - 2a = 2 (x - a)$$

$$y - 2a = 2x - 2a$$

$$y = 2x$$

$$(h) y - y_1 = m (x - x_1)$$

$$y - 3b = -\frac{1}{2} \left[x - \left(-2b \right) \right]$$

$$y - 3b = -\frac{1}{2} \left(x + 2b \right)$$

$$y - 3b = -\frac{1}{2}x - b$$

$$y = -\frac{1}{2}x - b + 3b$$

$$y = -\frac{1}{2}x + 2b$$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 2

Question:

The line $y = 4x - 8$ meets the x -axis at the point A . Find the equation of the line with gradient 3 that passes through the point A .

Solution:

$$y = 4x - 8$$

Substitute $y = 0$:

$$4x - 8 = 0$$

$$4x = 8$$

$$x = 2$$

So A has coordinates $(2, 0)$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 3(x - 2)$$

$$y = 3x - 6$$

The equation of the line is $y = 3x - 6$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 3

Question:

The line $y = -2x + 8$ meets the y-axis at the point B . Find the equation of the line with gradient 2 that passes through the point B .

Solution:

$$y = -2x + 8$$

Substitute $x = 0$:

$$y = -2(0) + 8$$

$$y = 8$$

So B has coordinates $(0, 8)$.

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 2(x - 0)$$

$$y - 8 = 2x$$

$$y = 2x + 8$$

The equation of the line is $y = 2x + 8$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 4

Question:

The line $y = \frac{1}{2}x + 6$ meets the x -axis at the point C . Find the equation of the line with gradient $\frac{2}{3}$ that passes through the point C . Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Solution:

$$y = \frac{1}{2}x + 6$$

Substitute $y = 0$:

$$\frac{1}{2}x + 6 = 0$$

$$\frac{1}{2}x = -6$$

$$x = -12$$

So C has coordinates $(-12, 0)$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{2}{3} \left[x - \left(-12 \right) \right]$$

$$y = \frac{2}{3} \left(x + 12 \right)$$

$$y = \frac{2}{3}x + 8$$

Multiply each term by 3:

$$3y = 2x + 24$$

$$0 = 2x + 24 - 3y$$

$$2x - 3y + 24 = 0$$

The equation of the line is $2x - 3y + 24 = 0$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 5

Question:

The line $y = \frac{1}{4}x + 2$ meets the y-axis at the point B . The point C has coordinates $(-5, 3)$. Find the gradient of the line joining the points B and C .

Solution:

$$y = \frac{1}{4}x + 2$$

Substitute $x = 0$:

$$y = \frac{1}{4} \left(0 \right) + 2$$

$$y = 2$$

So B has coordinates $(0, 2)$.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{-5 - 0} = \frac{1}{-5} = -\frac{1}{5}$$

The gradient of the line joining B and C is $-\frac{1}{5}$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 6

Question:

The lines $y = x$ and $y = 2x - 5$ intersect at the point A. Find the equation of the line with gradient $\frac{2}{5}$ that passes through the point A. (Hint: Solve $y = x$ and $y = 2x - 5$ simultaneously.)

Solution:

Substitute $y = x$:

$$x = 2x - 5$$

$$0 = x - 5$$

$$x = 5$$

$$y = x$$

Substitute $x = 5$:

$$y = 5$$

The coordinates of A are (5 , 5) .

$$y - y_1 = m (x - x_1)$$

$$y - 5 = \frac{2}{5} (x - 5)$$

$$y - 5 = \frac{2}{5}x - 2$$

$$y = \frac{2}{5}x + 3$$

The equation of the line is $y = \frac{2}{5}x + 3$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 7

Question:

The lines $y = 4x - 10$ and $y = x - 1$ intersect at the point T . Find the equation of the line with gradient $-\frac{2}{3}$ that passes through the point T . Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Solution:

Substitute $y = x - 1$:

$$x - 1 = 4x - 10$$

$$-1 = 3x - 10$$

$$9 = 3x$$

$$x = 3$$

$$y = x - 1$$

Substitute $x = 3$:

$$y = 3 - 1 = 2$$

The coordinates of T are $(3, 2)$.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{2}{3} \left(x - 3 \right)$$

$$y - 2 = -\frac{2}{3}x + 2$$

$$\frac{2}{3}x + y - 2 = 2$$

$$\frac{2}{3}x + y - 4 = 0$$

$$2x + 3y - 12 = 0$$

The equation of the line is $2x + 3y - 12 = 0$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 8

Question:

The line p has gradient $\frac{2}{3}$ and passes through the point $(6, -12)$. The line q has gradient -1 and passes through the point $(5, 5)$. The line p meets the y -axis at A and the line q meets the x -axis at B . Work out the gradient of the line joining the points A and B .

Solution:

The equation of p is

$$y - (-12) = \frac{2}{3}(x - 6)$$

$$y + 12 = \frac{2}{3}x - 4$$

$$y = \frac{2}{3}x - 16$$

The equation of q is

$$y - 5 = -1(x - 5)$$

$$y - 5 = -x + 5$$

$$y = -x + 10$$

For the coordinates of A substitute $x = 0$ into

$$y = \frac{2}{3}x - 16$$

$$y = \frac{2}{3}(0) - 16$$

$$y = -16$$

Coordinates are $A(0, -16)$

For the coordinates of B substitute $y = 0$ into

$$y = -x + 10$$

$$0 = -x + 10$$

$$x = 10$$

Coordinates are $B(10, 0)$

Gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-16 - 0}{0 - 10} = \frac{-16}{-10} = \frac{8}{5}$$

The gradient of the line joining A and B is $\frac{8}{5}$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 9

Question:

The line $y = -2x + 6$ meets the x -axis at the point P . The line $y = \frac{3}{2}x - 4$ meets the y -axis at the point Q . Find the equation of the line joining the points P and Q . (Hint: First work out the gradient of the line joining the points P and Q .)

Solution:

$$y = -2x + 6$$

Substitute $y = 0$:

$$0 = -2x + 6$$

$$2x = 6$$

$$x = 3$$

P has coordinates $(3, 0)$.

$$y = \frac{3}{2}x - 4$$

Substitute $x = 0$:

$$y = \frac{3}{2} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 4$$

$$y = -4$$

Q has coordinates $(0, -4)$

Gradient of PQ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-4)}{3 - 0} = \frac{4}{3}$$

Equation of PQ is

$$y - y_1 = m(x - x_1)$$

Substitute $(3, 0)$:

$$y - 0 = \frac{4}{3} \begin{pmatrix} x - 3 \end{pmatrix}$$

$$y = \frac{4}{3}x - 4$$

The equation of the line through P and Q is $y = \frac{4}{3}x - 4$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise C, Question 10

Question:

The line $y = 3x - 5$ meets the x -axis at the point M . The line $y = -\frac{2}{3}x + \frac{2}{3}$ meets the y -axis at the point N . Find the equation of the line joining the points M and N . Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Solution:

$$y = 3x - 5$$

Substitute $y = 0$:

$$3x - 5 = 0$$

$$3x = 5$$

$$x = \frac{5}{3}$$

M has coordinates $\left(\frac{5}{3}, 0\right)$.

$$y = -\frac{2}{3}x + \frac{2}{3}$$

Substitute $x = 0$:

$$y = -\frac{2}{3}\left(0\right) + \frac{2}{3} = \frac{2}{3}$$

N has coordinates $\left(0, \frac{2}{3}\right)$.

Gradient of MN is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - \frac{2}{3}}{\frac{5}{3} - 0} = \frac{-\frac{2}{3}}{\frac{5}{3}} = -\frac{2}{5}$$

Equation of MN is

$$y - y_1 = m(x - x_1)$$

Substitute $\left(\frac{5}{3}, 0\right)$:

$$y - 0 = -\frac{2}{5}\left(x - \frac{5}{3}\right)$$

$$y = -\frac{2}{5}x + \frac{2}{3}$$

Multiply each term by 15:

$$15y = -6x + 10$$

$$6x + 15y = 10$$

$$6x + 15y - 10 = 0$$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 1

Question:

Find the equation of the line that passes through these pairs of points:

(a) (2, 4) and (3, 8)

(b) (0, 2) and (3, 5)

(c) (-2, 0) and (2, 8)

(d) (5, -3) and (7, 5)

(e) (3, -1) and (7, 3)

(f) (-4, -1) and (6, 4)

(g) (-1, -5) and (-3, 3)

(h) (-4, -1) and (-3, -9)

(i) $\left(\frac{1}{3}, \frac{2}{5}\right)$ and $\left(\frac{2}{3}, \frac{4}{5}\right)$

(j) $\left(-\frac{3}{4}, \frac{1}{7}\right)$ and $\left(\frac{1}{4}, \frac{3}{7}\right)$

Solution:

(a) $(x_1, y_1) = (2, 4)$, $(x_2, y_2) = (3, 8)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 4}{8 - 4} = \frac{x - 2}{3 - 2}$$

$$\frac{y - 4}{4} = \frac{x - 2}{1}$$

$$\frac{y - 4}{4} = x - 2$$

Multiply each side by 4:

$$4 \times \frac{y - 4}{4} = 4 \left(x - 2 \right)$$

$$y - 4 = 4(x - 2)$$

$$y - 4 = 4x - 8$$

$$y = 4x - 4$$

(b) $(x_1, y_1) = (0, 2)$, $(x_2, y_2) = (3, 5)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{5 - 2} = \frac{x - 0}{3 - 0}$$

$$\frac{y - 2}{3} = \frac{x}{3}$$

Multiply each side by 3:

$$3 \times \frac{y - 2}{3} = 3 \times \frac{x}{3}$$

$$y - 2 = x$$

$$y = x + 2$$

(c) $(x_1, y_1) = (-2, 0)$, $(x_2, y_2) = (2, 8)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{8 - 0} = \frac{x - (-2)}{2 - (-2)}$$

$$\frac{y}{8} = \frac{x + 2}{4}$$

Multiply each side by 8:

$$8 \times \frac{y}{8} = 8 \times \frac{x + 2}{4}$$

$$y = 2(x + 2)$$

$$y = 2x + 4$$

(d) $(x_1, y_1) = (5, -3)$, $(x_2, y_2) = (7, 5)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-3)}{5 - (-3)} = \frac{x - 5}{7 - 5}$$

$$\frac{y + 3}{8} = \frac{x - 5}{2}$$

Multiply each side by 8:

$$8 \times \frac{y + 3}{8} = 8 \times \frac{x - 5}{2}$$

$$y + 3 = 4(x - 5)$$

$$y + 3 = 4x - 20$$

$$y = 4x - 23$$

(e) $(x_1, y_1) = (3, -1)$, $(x_2, y_2) = (7, 3)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-1)}{3 - (-1)} = \frac{x - 3}{7 - 3}$$

$$\frac{y + 1}{4} = \frac{x - 3}{4}$$

Multiply each side by 4:

$$y + 1 = x - 3$$

$$y = x - 4$$

(f) $(x_1, y_1) = (-4, -1)$, $(x_2, y_2) = (6, 4)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-1)}{4 - (-1)} = \frac{x - (-4)}{6 - (-4)}$$

$$\frac{y+1}{5} = \frac{x+4}{10}$$

Multiply each side by 10:

$$2(y+1) = x+4$$

$$2y+2 = x+4$$

$$2y = x+2$$

Divide each term by 2:

$$y = \frac{1}{2}x + 1$$

(g) $(x_1, y_1) = (-1, -5)$, $(x_2, y_2) = (-3, 3)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y - (-5)}{3 - (-5)} = \frac{x - (-1)}{-3 - (-1)}$$

$$\frac{y+5}{8} = \frac{x+1}{-2}$$

Multiply each side by 8:

$$y+5 = -4(x+1) \text{ (Note: } \frac{8}{-2} = -4)$$

$$y+5 = -4x-4$$

$$y = -4x-9$$

(h) $(x_1, y_1) = (-4, -1)$, $(x_2, y_2) = (-3, -9)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y - (-1)}{-9 - (-1)} = \frac{x - (-4)}{-3 - (-4)}$$

$$\frac{y+1}{-8} = \frac{x+4}{1}$$

Multiply each side by -8 :

$$y+1 = -8(x+4)$$

$$y+1 = -8x-32$$

$$y = -8x-33$$

(i) $\left(x_1, y_1 \right) = \left(\frac{1}{3}, \frac{2}{5} \right)$, $\left(x_2, y_2 \right) = \left(\frac{2}{3}, \frac{4}{5} \right)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y - \frac{2}{5}}{\frac{4}{5} - \frac{2}{5}} = \frac{x - \frac{1}{3}}{\frac{2}{3} - \frac{1}{3}}$$

$$\frac{y - \frac{2}{5}}{\frac{2}{5}} = \frac{x - \frac{1}{3}}{\frac{1}{3}}$$

$$\frac{5}{2} \left(y - \frac{2}{5} \right) = 3 \left(x - \frac{1}{3} \right) \quad (\text{Note: } \frac{1}{\frac{2}{5}} = \frac{5}{2} \text{ and } \frac{1}{\frac{1}{3}} = 3)$$

$$\frac{5}{2}y - 1 = 3x - 1$$

$$\frac{5}{2}y = 3x$$

$$5y = 6x$$

$$y = \frac{6}{5}x$$

$$(i) \left(x_1, y_1 \right) = \left(\frac{-3}{4}, \frac{1}{7} \right), \left(x_2, y_2 \right) = \left(\frac{1}{4}, \frac{3}{7} \right)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - \frac{1}{7}}{\frac{3}{7} - \frac{1}{7}} = \frac{x - \left(-\frac{3}{4} \right)}{\frac{1}{4} - \left(-\frac{3}{4} \right)}$$

$$\frac{y - \frac{1}{7}}{\frac{2}{7}} = \frac{x + \frac{3}{4}}{1}$$

Multiply each side by $\frac{2}{7}$:

$$y - \frac{1}{7} = \frac{2}{7} \left(x + \frac{3}{4} \right)$$

$$y - \frac{1}{7} = \frac{2}{7}x + \frac{3}{14}$$

$$y = \frac{2}{7}x + \frac{3}{14} + \frac{1}{7}$$

$$y = \frac{2}{7}x + \frac{5}{14}$$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 2

Question:

The line that passes through the points $(2, -5)$ and $(-7, 4)$ meets the x -axis at the point P . Work out the coordinates of the point P .

Solution:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-5)}{4 - (-5)} = \frac{x - 2}{-7 - 2}$$

$$\frac{y + 5}{9} = \frac{x - 2}{-9}$$

Multiply each side by 9:

$$y + 5 = -1(x - 2) \quad (\text{Note: } \frac{9}{-9} = -1)$$

$$y + 5 = -x + 2$$

$$y = -x - 3$$

Substitute $y = 0$:

$$0 = -x - 3$$

$$x = -3$$

So the line meets the x -axis at $P(-3, 0)$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 3

Question:

The line that passes through the points $(-3, -5)$ and $(4, 9)$ meets the y-axis at the point G . Work out the coordinates of the point G .

Solution:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-5)}{9 - (-5)} = \frac{x - (-3)}{4 - (-3)}$$

$$\frac{y + 5}{14} = \frac{x + 3}{7}$$

Multiply each side by 14:

$$y + 5 = 2(x + 3)$$

$$y + 5 = 2x + 6$$

$$y = 2x + 1$$

Substitute $x = 0$:

$$y = 2(0) + 1 = 1$$

The coordinates of G are $(0, 1)$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 4

Question:

The line that passes through the points $\left(3, 2\frac{1}{2}\right)$ and $\left(-1\frac{1}{2}, 4\right)$ meets the y-axis at the point J . Work out the coordinates of the point J .

Solution:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2\frac{1}{2}}{4 - 2\frac{1}{2}} = \frac{x - 3}{-1\frac{1}{2} - 3}$$

$$\frac{y - 2\frac{1}{2}}{1\frac{1}{2}} = \frac{x - 3}{-4\frac{1}{2}}$$

Multiply top and bottom of each fraction by 2:

$$\frac{2y - 5}{3} = \frac{2x - 6}{-9}$$

Multiply each side by 9:

$$3(2y - 5) = -1(2x - 6) \quad (\text{Note: } \frac{9}{-9} = -1)$$

$$6y - 15 = -2x + 6$$

$$6y = -2x + 21$$

$$y = -\frac{2}{6}x + \frac{21}{6}$$

$$y = -\frac{1}{3}x + \frac{7}{2}$$

Substitute $x = 0$:

$$y = -\frac{1}{3}\left(0\right) + \frac{7}{2} = \frac{7}{2}$$

The coordinates of J are $\left(0, \frac{7}{2}\right)$ or $\left(0, 3\frac{1}{2}\right)$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 5

Question:

The line $y = 2x - 10$ meets the x -axis at the point A . The line $y = -2x + 4$ meets the y -axis at the point B . Find the equation of the line joining the points A and B . (Hint: First work out the coordinates of the points A and B .)

Solution:

$$y = 2x - 10$$

Substitute $y = 0$:

$$2x - 10 = 0$$

$$2x = 10$$

$$x = 5$$

The coordinates of A are $(5, 0)$.

$$y = -2x + 4$$

Substitute $x = 0$:

$$y = -2(0) + 4 = 4$$

The coordinates of B are $(0, 4)$.

Equation of AB :

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{4 - 0} = \frac{x - 5}{0 - 5}$$

$$\frac{y}{4} = \frac{x - 5}{-5}$$

Multiply each side by 4:

$$y = 4 \frac{(x - 5)}{-5} = \frac{4}{-5} (x - 5) = -\frac{4}{5} (x - 5) = -\frac{4}{5}x + 4$$

The equation of the line is $y = -\frac{4}{5}x + 4$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 6

Question:

The line $y = 4x + 5$ meets the y -axis at the point C . The line $y = -3x - 15$ meets the x -axis at the point D . Find the equation of the line joining the points C and D . Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Solution:

$$y = 4x + 5$$

Substitute $x = 0$:

$$y = 4(0) + 5 = 5$$

The coordinates of C are $(0, 5)$.

$$y = -3x - 15$$

Substitute $y = 0$:

$$0 = -3x - 15$$

$$3x = -15$$

$$x = -5$$

The coordinates of D are $(-5, 0)$.

Equation of CD :

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{0 - 5} = \frac{x - 0}{-5 - 0}$$

$$\frac{y - 5}{-5} = \frac{x}{-5}$$

Multiply each side by -5 :

$$y - 5 = x$$

$$-5 = x - y$$

$$0 = x - y + 5$$

The equation of the line is $x - y + 5 = 0$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 7

Question:

The lines $y = x - 5$ and $y = 3x - 13$ intersect at the point S . The point T has coordinates $(-4, 2)$. Find the equation of the line that passes through the points S and T .

Solution:

$$y = 3x - 13$$

$$y = x - 5$$

$$\text{So } 3x - 13 = x - 5$$

$$\Rightarrow 3x = x + 8$$

$$\Rightarrow 2x = 8$$

$$\Rightarrow x = 4$$

$$\text{when } x = 4, y = 4 - 5 = -1$$

The coordinates of S are $(4, -1)$.

Equation of ST :

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-1)}{2 - (-1)} = \frac{x - 4}{-4 - 4}$$

$$\frac{y + 1}{3} = \frac{x - 4}{-8}$$

Multiply each side by 3:

$$y + 1 = 3 \times \frac{(x - 4)}{-8}$$

$$y + 1 = \frac{3}{-8} \times (x - 4)$$

$$y + 1 = -\frac{3}{8} (x - 4)$$

$$y + 1 = -\frac{3}{8}x + \frac{3}{2}$$

$$y = -\frac{3}{8}x + \frac{1}{2}$$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 8

Question:

The lines $y = -2x + 1$ and $y = x + 7$ intersect at the point L . The point M has coordinates $(-3, 1)$. Find the equation of the line that passes through the points L and M .

Solution:

$$y = x + 7$$

$$y = -2x + 1$$

$$\text{So } x + 7 = -2x + 1$$

$$\Rightarrow 3x + 7 = 1$$

$$\Rightarrow 3x = -6$$

$$\Rightarrow x = -2$$

$$\text{when } x = -2, y = (-2) + 7 = 5$$

The coordinates of L are $(-2, 5)$.

Equation of LM :

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{1 - 5} = \frac{x - (-2)}{-3 - (-2)}$$

$$\frac{y - 5}{-4} = \frac{x + 2}{-1}$$

Multiply each side by -4 :

$$y - 5 = 4(x + 2) \quad (\text{Note: } \frac{-4}{-1} = 4)$$

$$y - 5 = 4x + 8$$

$$y = 4x + 13$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 9

Question:

The vertices of the triangle ABC have coordinates $A(3, 5)$, $B(-2, 0)$ and $C(4, -1)$. Find the equations of the sides of the triangle.

Solution:

(1) Equation of AB :

$$(x_1, y_1) = (3, 5), (x_2, y_2) = (-2, 0)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{0 - 5} = \frac{x - 3}{-2 - 3}$$

$$\frac{y - 5}{-5} = \frac{x - 3}{-5}$$

Multiply each side by -5 :

$$y - 5 = x - 3$$

$$y = x + 2$$

(2) Equation of AC :

$$(x_1, y_1) = (3, 5), (x_2, y_2) = (4, -1)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{-1 - 5} = \frac{x - 3}{4 - 3}$$

$$\frac{y - 5}{-6} = \frac{x - 3}{1}$$

Multiply each side by -6 :

$$y - 5 = -6(x - 3)$$

$$y - 5 = -6x + 18$$

$$y = -6x + 23$$

(3) Equation of BC :

$$(x_1, y_1) = (-2, 0), (x_2, y_2) = (4, -1)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{-1 - 0} = \frac{x - (-2)}{4 - (-2)}$$

$$\frac{y}{-1} = \frac{x + 2}{6}$$

Multiply each side by -1 :

$$y = -1 \frac{(x + 2)}{6}$$

$$y = -\frac{1}{6} (x + 2)$$

$$y = -\frac{1}{6}x - \frac{1}{3}$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise D, Question 10

Question:

The line V passes through the points $(-5, 3)$ and $(7, -3)$ and the line W passes through the points $(2, -4)$ and $(4, 2)$. The lines V and W intersect at the point A . Work out the coordinates of the point A .

Solution:

(1) The equation of V :

$$(x_1, y_1) = (-5, 3), (x_2, y_2) = (7, -3)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{-3 - 3} = \frac{x - (-5)}{7 - (-5)}$$

$$\frac{y - 3}{-6} = \frac{x + 5}{12}$$

Multiply each side by -6 :

$$y - 3 = -\frac{1}{2} \left(x + 5 \right) \quad (\text{Note: } \frac{-6}{12} = -\frac{1}{2})$$

$$y - 3 = -\frac{1}{2}x - \frac{5}{2}$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

(2) The equation of W :

$$(x_1, y_1) = (2, -4), (x_2, y_2) = (4, 2)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-4)}{2 - (-4)} = \frac{x - 2}{4 - 2}$$

$$\frac{y + 4}{6} = \frac{x - 2}{2}$$

Multiply each side by 6:

$$y + 4 = 3(x - 2) \quad (\text{Note: } \frac{6}{2} = 3)$$

$$y + 4 = 3x - 6$$

$$y = 3x - 10$$

Solving simultaneously:

$$y = -\frac{1}{2}x + \frac{1}{2}$$

$$y = 3x - 10$$

$$\text{So } 3x - 10 = -\frac{1}{2}x + \frac{1}{2}$$

$$\Rightarrow \frac{7}{2}x - 10 = \frac{1}{2}$$

$$\Rightarrow \frac{7}{2}x = \frac{21}{2}$$

$$\Rightarrow 7x = 21$$

$$\Rightarrow x = 3$$

When $x = 3$, $y = 3(3) - 10 = 9 - 10 = -1$

The lines intersect at $A(3, -1)$.

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Coordinate geometry in the (x, y) plane

Exercise E, Question 1

Question:

Work out if these pairs of lines are parallel, perpendicular or neither:

(a) $y = 4x + 2$

$$y = -\frac{1}{4}x - 7$$

(b) $y = \frac{2}{3}x - 1$

$$y = \frac{2}{3}x - 11$$

(c) $y = \frac{1}{5}x + 9$

$$y = 5x + 9$$

(d) $y = -3x + 2$

$$y = \frac{1}{3}x - 7$$

(e) $y = \frac{3}{5}x + 4$

$$y = -\frac{5}{3}x - 1$$

(f) $y = \frac{5}{7}x$

$$y = \frac{5}{7}x - 3$$

(g) $y = 5x - 3$

$$5x - y + 4 = 0$$

(h) $5x - y - 1 = 0$

$$y = -\frac{1}{5}x$$

(i) $y = -\frac{3}{2}x + 8$

$$2x - 3y - 9 = 0$$

(j) $4x - 5y + 1 = 0$

$$8x - 10y - 2 = 0$$

(k) $3x + 2y - 12 = 0$

$$2x + 3y - 6 = 0$$

(l) $5x - y + 2 = 0$

$$2x + 10y - 4 = 0$$

Solution:

(a) The gradients of the lines are 4 and $-\frac{1}{4}$.

$$4 \times -\frac{1}{4} = -1$$

The lines are **perpendicular**.

(b) The gradients of the lines are $\frac{2}{3}$ and $\frac{2}{3}$, i.e. they have the same gradient.

The lines are **parallel**.

(c) The gradients of the lines are $\frac{1}{5}$ and 5.

$$\frac{1}{5} \times 5 = 1$$

The lines are **neither** perpendicular nor parallel.

(d) The gradients of the lines are -3 and $\frac{1}{3}$.

$$-3 \times \frac{1}{3} = -1$$

The lines are **perpendicular**.

(e) The gradients of the lines are $\frac{3}{5}$ and $-\frac{5}{3}$.

$$\frac{3}{5} \times -\frac{5}{3} = -1$$

The lines are **perpendicular**.

(f) The gradients of the lines are $\frac{5}{7}$ and $\frac{5}{7}$, i.e. they have the same gradient.

The lines are **parallel**.

(g) The gradient of $y = 5x - 3$ is 5.

$$5x - y + 4 = 0$$

$$5x + 4 = y$$

$$y = 5x + 4$$

The gradient of $5x - y + 4 = 0$ is 5.

The lines have the same gradient.

The lines are **parallel**.

(h) $5x - y - 1 = 0$

$$5x - 1 = y$$

$$y = 5x - 1$$

The gradient of $5x - y - 1 = 0$ is 5.

The gradient of $y = -\frac{1}{5}x$ is $-\frac{1}{5}$.

The product of the gradients is $5 \times -\frac{1}{5} = -1$

So the lines are **perpendicular**.

(i) The gradient of $y = -\frac{3}{2}x + 8$ is $-\frac{3}{2}$.

$$2x - 3y - 9 = 0$$

$$2x - 9 = 3y$$

$$3y = 2x - 9$$

$$y = \frac{2}{3}x - 3$$

The gradient of $2x - 3y - 9 = 0$ is $\frac{2}{3}$.

The product of the gradients is $\frac{2}{3} \times -\frac{3}{2} = -1$

So the lines are **perpendicular**.

(j) $4x - 5y + 1 = 0$

$$4x + 1 = 5y$$

$$5y = 4x + 1$$

$$y = \frac{4}{5}x + \frac{1}{5}$$

The gradient of $4x - 5y + 1 = 0$ is $\frac{4}{5}$.

$$8x - 10y - 2 = 0$$

$$8x - 2 = 10y$$

$$10y = 8x - 2$$

$$y = \frac{8}{10}x - \frac{2}{10}$$

$$y = \frac{4}{5}x - \frac{1}{5}$$

The gradient of $8x - 10y - 2 = 0$ is $\frac{4}{5}$.

The lines have the same gradient, they are **parallel**.

(k) $3x + 2y - 12 = 0$

$$3x + 2y = 12$$

$$2y = -3x + 12$$

$$y = -\frac{3}{2}x + 6$$

The gradient of $3x + 2y - 12 = 0$ is $-\frac{3}{2}$.

$$2x + 3y - 6 = 0$$

$$2x + 3y = 6$$

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + 2$$

The gradient of $2x + 3y - 6 = 0$ is $-\frac{2}{3}$.

The product of the gradient is

$$-\frac{3}{2} \times -\frac{2}{3} = 1$$

So the lines are **neither** parallel nor perpendicular.

(l) $5x - y + 2 = 0$

$$5x + 2 = y$$

$$y = 5x + 2$$

The gradient of $5x - y + 2 = 0$ is 5.

$$2x + 10y - 4 = 0$$

$$2x + 10y = 4$$

$$10y = -2x + 4$$

$$y = -\frac{2}{10}x + \frac{4}{10}$$

$$y = -\frac{1}{5}x + \frac{2}{5}$$

The gradient of $2x + 10y - 4 = 0$ is $-\frac{1}{5}$.

The product of the gradients is

$$5 \times -\frac{1}{5} = -1$$

So the lines are **perpendicular**.

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise E, Question 2

Question:

Find an equation of the line that passes through the point $(6, -2)$ and is perpendicular to the line $y = 3x + 5$.

Solution:

The gradient of $y = 3x + 5$ is 3.

The gradient of a line perpendicular to $y = 3x + 5$ is $-\frac{1}{3}$.

$$y - y_1 = m(x - x_1)$$

$$y - \begin{pmatrix} -2 \end{pmatrix} = -\frac{1}{3} \begin{pmatrix} x - 6 \end{pmatrix}$$

$$y + 2 = -\frac{1}{3}x + 2$$

$$y = -\frac{1}{3}x$$

The equation of the line is $y = -\frac{1}{3}x$.

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise E, Question 3

Question:

Find an equation of the line that passes through the point $(-2, 7)$ and is parallel to the line $y = 4x + 1$. Write your answer in the form $ax + by + c = 0$.

Solution:

The gradient of a line parallel to $y = 4x + 1$ is 4.

$$y - y_1 = m (x - x_1)$$

$$y - 7 = 4 [x - (-2)]$$

$$y - 7 = 4 (x + 2)$$

$$y - 7 = 4x + 8$$

$$y = 4x + 15$$

$$0 = 4x + 15 - y$$

$$4x - y + 15 = 0$$

The equation of the line is $4x - y + 15 = 0$.

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Coordinate geometry in the (x, y) plane

Exercise E, Question 4

Question:

Find an equation of the line:

(a) parallel to the line $y = -2x - 5$, passing through $\left(-\frac{1}{2}, \frac{3}{2}\right)$.

(b) parallel to the line $x - 2y - 1 = 0$, passing through $(0, 0)$.

(c) perpendicular to the line $y = x - 4$, passing through $(-1, -2)$.

(d) perpendicular to the line $2x + y - 9 = 0$, passing through $(4, -6)$.

Solution:

(a) The gradient of a line parallel to $y = -2x - 5$ is -2 .

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{2} = -2 \left[x - \left(-\frac{1}{2}\right) \right]$$

$$y - \frac{3}{2} = -2 \left(x + \frac{1}{2} \right)$$

$$y - \frac{3}{2} = -2x - 1$$

$$y = -2x + \frac{1}{2}$$

(b) $x - 2y - 1 = 0$

$$x - 1 = 2y$$

$$2y = x - 1$$

$$y = \frac{1}{2}x - \frac{1}{2}$$

The gradient of $x - 2y - 1 = 0$ is $\frac{1}{2}$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{2} \left(x - 0 \right)$$

$$y = \frac{1}{2}x$$

(c) The gradient of $y = x - 4$ is 1.

The gradient of a line perpendicular to $y = x - 4$ is $-\frac{1}{1} = -1$.

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -1 [x - (-1)]$$

$$y + 2 = -1(x + 1)$$

$$y + 2 = -x - 1$$

$$y = -x - 3$$

$$(d) 2x + y - 9 = 0$$

$$2x + y = 9$$

$$y = -2x + 9$$

The gradient of $2x + y - 9 = 0$ is -2 .

The gradient of a line perpendicular to $2x + y - 9 = 0$ is $-\frac{1}{-2} = \frac{1}{2}$.

$$y - y_1 = m(x - x_1)$$

$$y - \begin{pmatrix} -6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x - 4 \end{pmatrix}$$

$$y + 6 = \frac{1}{2} \begin{pmatrix} x - 4 \end{pmatrix}$$

$$y + 6 = \frac{1}{2}x - 2$$

$$y = \frac{1}{2}x - 8$$

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Coordinate geometry in the (x, y) plane

Exercise E, Question 5

Question:

Find an equation of the line:

- (a) parallel to the line $y = 3x + 6$, passing through $(-2, 5)$.
- (b) perpendicular to the line $y = 3x + 6$, passing through $(-2, 5)$.
- (c) parallel to the line $4x - 6y + 7 = 0$, passing through $(3, 4)$.
- (d) perpendicular to the line $4x - 6y + 7 = 0$, passing through $(3, 4)$.

Solution:

- (a) The gradient of a line parallel to $y = 3x + 6$ is 3.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 3[x - (-2)]$$

$$y - 5 = 3(x + 2)$$

$$y - 5 = 3x + 6$$

$$y = 3x + 11$$

- (b) The gradient of a line perpendicular to $y = 3x + 6$ is $-\frac{1}{3}$.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{1}{3}\left[x - (-2)\right]$$

$$y - 5 = -\frac{1}{3}(x + 2)$$

$$y - 5 = -\frac{1}{3}x - \frac{2}{3}$$

$$y = -\frac{1}{3}x + \frac{13}{3}$$

- (c) $4x - 6y + 7 = 0$

$$4x + 7 = 6y$$

$$6y = 4x + 7$$

$$y = \frac{4}{6}x + \frac{7}{6}$$

$$y = \frac{2}{3}x + \frac{7}{6}$$

The gradient of a line parallel to $4x - 6y + 7 = 0$ is $\frac{2}{3}$.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{2}{3}(x - 3)$$

$$y - 4 = \frac{2}{3}x - 2$$

$$y = \frac{2}{3}x + 2$$

(d) The gradient of the line $4x - 6y + 7 = 0$ is $\frac{2}{3}$ [see part (c)].

The gradient of a line perpendicular to $4x - 6y + 7 = 0$ is $-\frac{1}{\frac{2}{3}} = -\frac{3}{2}$.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{3}{2}(x - 3)$$

$$y - 4 = -\frac{3}{2}x + \frac{9}{2}$$

$$y = -\frac{3}{2}x + \frac{17}{2}$$

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Coordinate geometry in the (x, y) plane

Exercise E, Question 6

Question:

Find an equation of the line that passes through the point $(5, -5)$ and is perpendicular to the line $y = \frac{2}{3}x + 5$. Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Solution:

The gradient of a line perpendicular to $y = \frac{2}{3}x + 5$ is $-\frac{1}{\frac{2}{3}} = -\frac{3}{2}$.

$$y - y_1 = m(x - x_1)$$

$$y - \begin{pmatrix} -5 \end{pmatrix} = -\frac{3}{2} \begin{pmatrix} x - 5 \end{pmatrix}$$

$$y + 5 = -\frac{3}{2} \begin{pmatrix} x - 5 \end{pmatrix}$$

Multiply each term by 2:

$$2y + 10 = -3(x - 5)$$

$$2y + 10 = -3x + 15$$

$$3x + 2y + 10 = 15$$

$$3x + 2y - 5 = 0$$

The equation of the line is $3x + 2y - 5 = 0$.

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise E, Question 7

Question:

Find an equation of the line that passes through the point $(-2, -3)$ and is perpendicular to the line $y = -\frac{4}{7}x + 5$.

Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Solution:

The gradient of a line perpendicular to $y = -\frac{4}{7}x + 5$ is $-\frac{1}{-\frac{4}{7}} = \frac{7}{4}$.

$$y - y_1 = m(x - x_1)$$

$$y - \begin{pmatrix} -3 \end{pmatrix} = \frac{7}{4} \left[x - \begin{pmatrix} -2 \end{pmatrix} \right]$$

$$y + 3 = \frac{7}{4} \begin{pmatrix} x + 2 \end{pmatrix}$$

Multiply each term by 4:

$$4y + 12 = 7(x + 2)$$

$$4y + 12 = 7x + 14$$

$$4y = 7x + 2$$

$$0 = 7x + 2 - 4y$$

$$7x - 4y + 2 = 0$$

The equation of the line is $7x - 4y + 2 = 0$.

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Coordinate geometry in the (x, y) plane

Exercise E, Question 8

Question:

The line r passes through the points $(1, 4)$ and $(6, 8)$ and the line s passes through the points $(5, -3)$ and $(20, 9)$. Show that the lines r and s are parallel.

Solution:

The gradient of r is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{6 - 1} = \frac{4}{5}$$

The gradient of s is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-3)}{20 - 5} = \frac{12}{15} = \frac{4}{5}$$

The gradients are equal, so the lines are **parallel**.

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Coordinate geometry in the (x, y) plane

Exercise E, Question 9

Question:

The line l passes through the points $(-3, 0)$ and $(3, -2)$ and the line n passes through the points $(1, 8)$ and $(-1, 2)$. Show that the lines l and n are perpendicular.

Solution:

The gradient of l is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{3 - (-3)} = -\frac{2}{6} = -\frac{1}{3}$$

The gradient of n is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{-1 - 1} = \frac{-6}{-2} = 3$$

The product of the gradients is

$$-\frac{1}{3} \times 3 = -1$$

So the lines are **perpendicular**.

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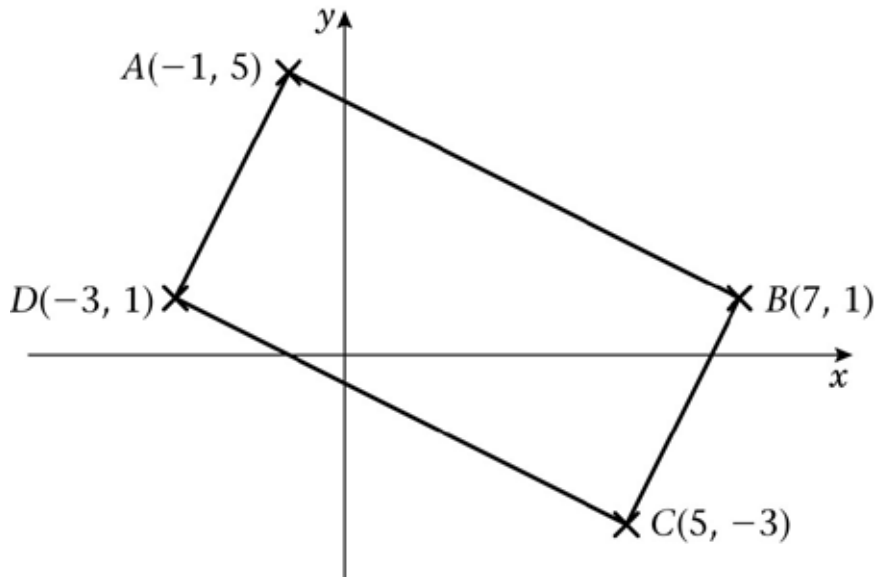
Coordinate geometry in the (x, y) plane

Exercise E, Question 10

Question:

The vertices of a quadrilateral $ABCD$ has coordinates $A(-1, 5)$, $B(7, 1)$, $C(5, -3)$, $D(-3, 1)$. Show that the quadrilateral is a rectangle.

Solution:



(1) The gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-1 - 7} = \frac{4}{-8} = -\frac{1}{2}$$

(2) The gradient of DC is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{5 - (-3)} = -\frac{4}{8} = -\frac{1}{2}$$

The gradient of AB is the same as the gradient of DC , so the lines are parallel.

(3) The gradient of AD is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-1 - (-3)} = \frac{4}{-1 + 3} = \frac{4}{2} = 2$$

(4) The gradient of BC is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{5 - 7} = \frac{-4}{-2} = 2$$

The gradient of AD is the same as the gradient of BC , so the lines are parallel.

The line AD is perpendicular to the line AB as

$$2 \times -\frac{1}{2} = -1$$

So $ABCD$ is a rectangle.

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Coordinate geometry in the (x, y) plane

Exercise F, Question 1

Question:

The points A and B have coordinates $(-4, 6)$ and $(2, 8)$ respectively. A line p is drawn through B perpendicular to AB to meet the y -axis at the point C .

- (a) Find an equation of the line p .
- (b) Determine the coordinates of C . **[E]**

Solution:

(a) The gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 6}{2 - (-4)} = \frac{2}{6} = \frac{1}{3}$$

The gradient of a line perpendicular to AB is

$$-\frac{1}{\frac{1}{3}} = -3$$

The equation of p is

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -3(x - 2)$$

$$y - 8 = -3x + 6$$

$$y = -3x + 14$$

(b) Substitute $x = 0$:

$$y = -3(0) + 14 = 14$$

The coordinates of C are $(0, 14)$.

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Coordinate geometry in the (x, y) plane

Exercise F, Question 2

Question:

The line l has equation $2x - y - 1 = 0$.

The line m passes through the point $A(0, 4)$ and is perpendicular to the line l .

(a) Find an equation of m and show that the lines l and m intersect at the point $P(2, 3)$.

The line n passes through the point $B(3, 0)$ and is parallel to the line m .

(b) Find an equation of n and hence find the coordinates of the point Q where the lines l and n intersect. **[E]**

Solution:

$$(a) \quad 2x - y - 1 = 0$$

$$2x - 1 = y$$

$$y = 2x - 1$$

The gradient of $2x - y - 1 = 0$ is 2.

The gradient of a line perpendicular to $2x - y - 1 = 0$ is $-\frac{1}{2}$.

The equation of the line m is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{2} \left(x - 0 \right)$$

$$y - 4 = -\frac{1}{2}x$$

$$y = -\frac{1}{2}x + 4$$

To find P solve $y = -\frac{1}{2}x + 4$ and $2x - y - 1 = 0$ simultaneously.

Substitute:

$$2x - \left(-\frac{1}{2}x + 4 \right) - 1 = 0$$

$$2x + \frac{1}{2}x - 4 - 1 = 0$$

$$\frac{5}{2}x - 5 = 0$$

$$\frac{5}{2}x = 5$$

$$5x = 10$$

$$x = 2$$

Substitute $x = 2$ into $y = -\frac{1}{2}x + 4$:

$$y = -\frac{1}{2} \left(2 \right) + 4 = -1 + 4 = 3$$

The lines intersect at $P(2, 3)$, as required.

(b) A line parallel to the line m has gradient $-\frac{1}{2}$.

The equation of the line n is

$$y - y_1 = m (x - x_1)$$

$$y - 0 = - \frac{1}{2} \left(x - 3 \right)$$

$$y = - \frac{1}{2}x + \frac{3}{2}$$

To find Q solve $2x - y - 1 = 0$ and $y = - \frac{1}{2}x + \frac{3}{2}$ simultaneously.

Substitute:

$$2x - \left(- \frac{1}{2}x + \frac{3}{2} \right) - 1 = 0$$

$$2x + \frac{1}{2}x - \frac{3}{2} - 1 = 0$$

$$\frac{5}{2}x - \frac{5}{2} = 0$$

$$\frac{5}{2}x = \frac{5}{2}$$

$$x = 1$$

Substitute $x = 1$ into $y = - \frac{1}{2}x + \frac{3}{2}$:

$$y = - \frac{1}{2} \left(1 \right) + \frac{3}{2} = - \frac{1}{2} + \frac{3}{2} = 1$$

The lines intersect at $Q (1 , 1)$.

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Coordinate geometry in the (x, y) plane

Exercise F, Question 3

Question:

The line L_1 has gradient $\frac{1}{7}$ and passes through the point $A(2, 2)$. The line L_2 has gradient -1 and passes through the point $B(4, 8)$. The lines L_1 and L_2 intersect at the point C .

(a) Find an equation for L_1 and an equation for L_2 .

(b) Determine the coordinates of C . **[E]**

Solution:

(a) The equation of L_1 is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{7}(x - 2)$$

$$y - 2 = \frac{1}{7}x - \frac{2}{7}$$

$$y = \frac{1}{7}x + \frac{12}{7}$$

The equation of L_2 is

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -1(x - 4)$$

$$y - 8 = -x + 4$$

$$y = -x + 12$$

(b) Solve $y = \frac{1}{7}x + \frac{12}{7}$ and $y = -x + 12$ simultaneously.

Substitute:

$$-x + 12 = \frac{1}{7}x + \frac{12}{7}$$

$$12 = \frac{8}{7}x + \frac{12}{7}$$

$$10 \frac{2}{7} = \frac{8}{7}x$$

$$x = \frac{10 \frac{2}{7}}{\frac{8}{7}} = 9$$

Substitute $x = 9$ into $y = -x + 12$:

$$y = -9 + 12 = 3$$

The lines intersect at $C(9, 3)$.

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise F, Question 4

Question:

The straight line passing through the point $P(2, 1)$ and the point $Q(k, 11)$ has gradient $-\frac{5}{12}$.

(a) Find the equation of the line in terms of x and y only.

(b) Determine the value of k . **[E]**

Solution:

$$(a) m = -\frac{5}{12}, (x_1, y_1) = (2, 1)$$

The equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{5}{12}(x - 2)$$

$$y - 1 = -\frac{5}{12}x + \frac{5}{6}$$

$$y = -\frac{5}{12}x + \frac{11}{6}$$

(b) Substitute $(k, 11)$ into $y = -\frac{5}{12}x + \frac{11}{6}$:

$$11 = -\frac{5}{12}k + \frac{11}{6}$$

$$11 - \frac{11}{6} = -\frac{5}{12}k$$

$$\frac{55}{6} = -\frac{5}{12}k$$

Multiply each side by 12:

$$110 = -5k$$

$$k = -22$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise F, Question 5

Question:

(a) Find an equation of the line l which passes through the points $A(1, 0)$ and $B(5, 6)$.
The line m with equation $2x + 3y = 15$ meets l at the point C .

(b) Determine the coordinates of the point C . **[E]**

Solution:

(a) The equation of l is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{6 - 0} = \frac{x - 1}{5 - 1}$$

$$\frac{y}{6} = \frac{x - 1}{4}$$

Multiply each side by 6:

$$y = 6 \frac{(x - 1)}{4}$$

$$y = \frac{3}{2} (x - 1)$$

$$y = \frac{3}{2}x - \frac{3}{2}$$

(b) Solve $2x + 3y = 15$ and $y = \frac{3}{2}x - \frac{3}{2}$ simultaneously.

Substitute:

$$2x + 3 \left(\frac{3}{2}x - \frac{3}{2} \right) = 15$$

$$2x + \frac{9}{2}x - \frac{9}{2} = 15$$

$$\frac{13}{2}x - \frac{9}{2} = 15$$

$$\frac{13}{2}x = \frac{39}{2}$$

$$13x = 39$$

$$x = 3$$

Substitute $x = 3$ into $y = \frac{3}{2}x - \frac{3}{2}$:

$$y = \frac{3}{2} \left(3 \right) - \frac{3}{2} = \frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3$$

The coordinates of C are $(3, 3)$.

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Coordinate geometry in the (x, y) plane

Exercise F, Question 6

Question:

The line L passes through the points $A (1 , 3)$ and $B (- 19 , - 19)$.

Find an equation of L in the form $ax + by + c = 0$, where a , b and c are integers. **[E]**

Solution:

$$(x_1, y_1) = (1, 3), (x_2, y_2) = (-19, -19)$$

The equation of L is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{-19 - 3} = \frac{x - 1}{-19 - 1}$$

$$\frac{y - 3}{-22} = \frac{x - 1}{-20}$$

Multiply each side by -22 :

$$y - 3 = \frac{-22}{-20} (x - 1)$$

$$y - 3 = \frac{11}{10} (x - 1)$$

Multiply each term by 10:

$$10y - 30 = 11(x - 1)$$

$$10y - 30 = 11x - 11$$

$$10y = 11x + 19$$

$$0 = 11x - 10y + 19$$

The equation of L is $11x - 10y + 19 = 0$.

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise F, Question 7

Question:

The straight line l_1 passes through the points A and B with coordinates $(2, 2)$ and $(6, 0)$ respectively.

(a) Find an equation of l_1 .

The straight line l_2 passes through the point C with coordinates $(-9, 0)$ and has gradient $\frac{1}{4}$.

(b) Find an equation of l_2 . **[E]**

Solution:

(a) The equation of l_1 is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{0 - 2} = \frac{x - 2}{6 - 2}$$

$$\frac{y - 2}{-2} = \frac{x - 2}{4}$$

Multiply each side by -2 :

$$y - 2 = -\frac{1}{2} \left(x - 2 \right) \quad (\text{Note: } -\frac{2}{4} = -\frac{1}{2})$$

$$y - 2 = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 3$$

(b) The equation of l_2 is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{4} \left[x - \left(-9 \right) \right]$$

$$y = \frac{1}{4} \left(x + 9 \right)$$

$$y = \frac{1}{4}x + \frac{9}{4}$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise F, Question 8

Question:

The straight line l_1 passes through the points A and B with coordinates $(0, -2)$ and $(6, 7)$ respectively.

(a) Find the equation of l_1 in the form $y = mx + c$.

The straight line l_2 with equation $x + y = 8$ cuts the y -axis at the point C . The lines l_1 and l_2 intersect at the point D .

(b) Calculate the coordinates of the point D .

(c) Calculate the area of $\triangle ACD$. **[E]**

Solution:

(a) The equation of l_1 is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - (-2)}{7 - (-2)} = \frac{x - 0}{6 - 0}$$

$$\frac{y + 2}{9} = \frac{x}{6}$$

Multiply each term by 9:

$$y + 2 = \frac{9}{6}x$$

$$y + 2 = \frac{3}{2}x$$

$$y = \frac{3}{2}x - 2$$

(b) Solve $x + y = 8$ and $y = \frac{3}{2}x - 2$ simultaneously.

Substitute:

$$x + \left(\frac{3}{2}x - 2 \right) = 8$$

$$x + \frac{3}{2}x - 2 = 8$$

$$\frac{5}{2}x - 2 = 8$$

$$\frac{5}{2}x = 10$$

$$5x = 20$$

$$x = 4$$

Substitute $x = 4$ into $x + y = 8$:

$$(4) + y = 8$$

$$y = 4$$

The coordinates of D are $(4, 4)$.

(c) $x + y = 8$ cuts the y -axis when $x = 0$.

Substitute $x = 0$:

$$0 + y = 8$$

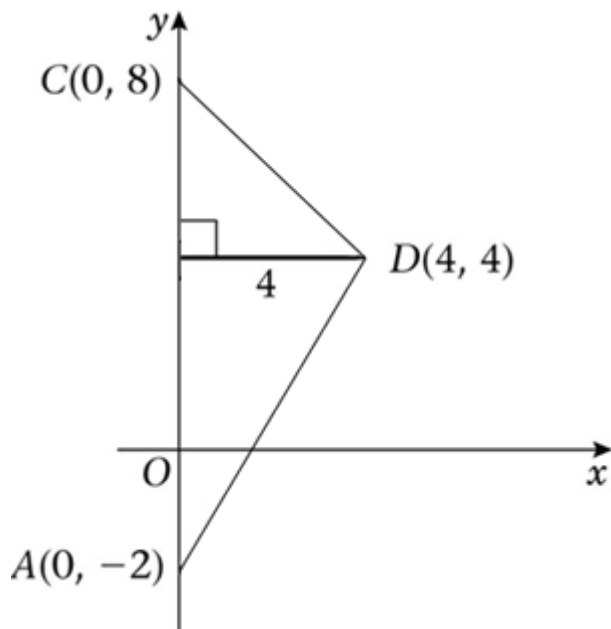
$$y = 8$$

The coordinates of C are $(0, 8)$

$$AC = 10$$

$$h = 4$$

$$\text{Area} = \frac{1}{2} \times 10 \times 4 = 20$$



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Coordinate geometry in the (x, y) plane

Exercise F, Question 9

Question:

The points A and B have coordinates $(2, 16)$ and $(12, -4)$ respectively. A straight line l_1 passes through A and B .

(a) Find an equation for l_1 in the form $ax + by = c$.

The line l_2 passes through the point C with coordinates $(-1, 1)$ and has gradient $\frac{1}{3}$.

(b) Find an equation for l_2 . **[E]**

Solution:

(a) The equation of l_1 is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 16}{-4 - 16} = \frac{x - 2}{12 - 2}$$

$$\frac{y - 16}{-20} = \frac{x - 2}{10}$$

Multiply each side by -20 :

$$y - 16 = -2(x - 2) \quad (\text{Note: } -\frac{20}{10} = -2)$$

$$y - 16 = -2x + 4$$

$$y = -2x + 20$$

$$2x + y = 20$$

(b) The equation of l_2 is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{3} \left[x - \left(-1 \right) \right]$$

$$y - 1 = \frac{1}{3} (x + 1)$$

$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

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Coordinate geometry in the (x, y) plane

Exercise F, Question 10

Question:

The points $A(-1, -2)$, $B(7, 2)$ and $C(k, 4)$, where k is a constant, are the vertices of $\triangle ABC$. Angle ABC is a right angle.

(a) Find the gradient of AB .

(b) Calculate the value of k .

(c) Find an equation of the straight line passing through B and C . Give your answer in the form $ax + by + c = 0$, where a , b and c are integers. **[E]**

Solution:

(a) The gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{7 - (-1)} = \frac{4}{8} = \frac{1}{2}$$

(b) The gradient of BC is

$$\frac{-1}{\frac{1}{2}} = -2$$

$$\text{So } \frac{y_2 - y_1}{x_2 - x_1} = -2$$

$$\Rightarrow \frac{4 - 2}{k - 7} = -2$$

$$\Rightarrow \frac{2}{k - 7} = -2$$

Multiply each side by $(k - 7)$:

$$2 = -2(k - 7)$$

$$2 = -2k + 14$$

$$-12 = -2k$$

$$k = 6$$

(c) The equation of the line passing through B and C is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 2}{4 - 2} = \frac{x - 7}{6 - 7}$$

$$\frac{y - 2}{2} = \frac{x - 7}{-1}$$

Multiply each side by 2:

$$y - 2 = -2(x - 7) \quad (\text{Note: } \frac{2}{-1} = -2)$$

$$y - 2 = -2x + 14$$

$$y = -2x + 16$$

$$2x + y = 16$$

$$2x + y - 16 = 0$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x, y) plane

Exercise F, Question 11

Question:

The straight line l passes through $A (1 , 3 \sqrt{3})$ and $B (2 + \sqrt{3} , 3 + 4 \sqrt{3})$.

- (a) Calculate the gradient of l giving your answer as a surd in its simplest form.
- (b) Give the equation of l in the form $y = mx + c$, where constants m and c are surds given in their simplest form.
- (c) Show that l meets the x -axis at the point $C (- 2 , 0)$. **[E]**

Solution:

(a) The gradient of l is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(3 + 4\sqrt{3}) - 3\sqrt{3}}{(2 + \sqrt{3}) - 1} = \frac{3 + \sqrt{3}}{1 + \sqrt{3}}$$

Rationalise the denominator:

$$\frac{3 + \sqrt{3}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{3 - 3\sqrt{3} + \sqrt{3} - 3}{1 - 3} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

(b) The equation of l is

$$\begin{aligned} y - y_1 &= m (x - x_1) \\ y - 3\sqrt{3} &= \sqrt{3} (x - 1) \\ y - 3\sqrt{3} &= \sqrt{3}x - \sqrt{3} \\ y &= \sqrt{3}x + 2\sqrt{3} \end{aligned}$$

(c) Substitute $y = 0$:

$$\begin{aligned} 0 &= \sqrt{3}x + 2\sqrt{3} \\ \sqrt{3}x &= -2\sqrt{3} \\ x &= \frac{-2\sqrt{3}}{\sqrt{3}} = -2 \end{aligned}$$

The coordinates of C are $(- 2 , 0)$.

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Coordinate geometry in the (x, y) plane

Exercise F, Question 12

Question:

(a) Find an equation of the straight line passing through the points with coordinates $(-1, 5)$ and $(4, -2)$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.
The line crosses the x -axis at the point A and the y -axis at the point B , and O is the origin.

(b) Find the area of $\triangle OAB$. **[E]**

Solution:

(a) The equation of the line is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{-2 - 5} = \frac{x - (-1)}{4 - (-1)}$$

$$\frac{y - 5}{-7} = \frac{x + 1}{5}$$

Multiply each side by -35 :

$$5(y - 5) = -7(x + 1) \quad (\text{Note: } \frac{-35}{-7} = 5 \text{ and } \frac{-35}{5} = -7)$$

$$5y - 25 = -7x - 7$$

$$7x + 5y - 25 = -7$$

$$7x + 5y - 18 = 0$$

(b) For the coordinates of A substitute $y = 0$:

$$7x + 5(0) - 18 = 0$$

$$7x - 18 = 0$$

$$7x = 18$$

$$x = \frac{18}{7}$$

The coordinates of A are $\left(\frac{18}{7}, 0\right)$.

For the coordinates of B substitute $x = 0$:

$$7(0) + 5y - 18 = 0$$

$$5y - 18 = 0$$

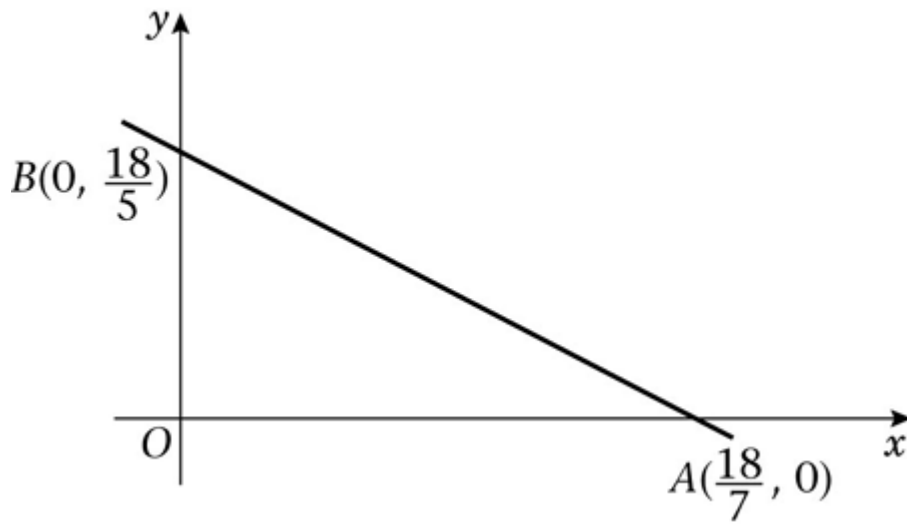
$$5y = 18$$

$$y = \frac{18}{5}$$

The coordinates of B are $\left(0, \frac{18}{5}\right)$.

The area of $\triangle OAB$ is

$$\frac{1}{2} \times \frac{18}{7} \times \frac{18}{5} = \frac{162}{35}$$



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Coordinate geometry in the (x, y) plane

Exercise F, Question 13

Question:

The points A and B have coordinates $(k, 1)$ and $(8, 2k - 1)$ respectively, where k is a constant. Given that the gradient of AB is $\frac{1}{3}$,

(a) Show that $k = 2$.

(b) Find an equation for the line through A and B . **[E]**

Solution:

(a) The gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{3}$$

$$\frac{(2k - 1) - 1}{8 - k} = \frac{1}{3}$$

$$\frac{2k - 1 - 1}{8 - k} = \frac{1}{3}$$

$$\frac{2k - 2}{8 - k} = \frac{1}{3}$$

Multiply each side by $(8 - k)$:

$$2k - 2 = \frac{1}{3} (8 - k)$$

Multiply each term by 3:

$$6k - 6 = 8 - k$$

$$7k - 6 = 8$$

$$7k = 14$$

$$k = 2$$

(b) $k = 2$

So A and B have coordinates $(2, 1)$ and $(8, 3)$.

The equation of the line is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 1}{3 - 1} = \frac{x - 2}{8 - 2}$$

$$\frac{y - 1}{2} = \frac{x - 2}{6}$$

Multiply each side by 2:

$$y - 1 = \frac{1}{3} (x - 2)$$

$$y - 1 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

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Coordinate geometry in the (x, y) plane

Exercise F, Question 14

Question:

The straight line l_1 has equation $4y + x = 0$.

The straight line l_2 has equation $y = 2x - 3$.

(a) On the same axes, sketch the graphs of l_1 and l_2 . Show clearly the coordinates of all points at which the graphs meet the coordinate axes.

The lines l_1 and l_2 intersect at the point A.

(b) Calculate, as exact fractions, the coordinates of A.

(c) Find an equation of the line through A which is perpendicular to l_1 . Give your answer in the form $ax + by + c = 0$, where a , b and c are integers. **[E]**

Solution:

(a) (1) Rearrange $4y + x = 0$ into the form $y = mx + c$:

$$4y = -x$$

$$y = -\frac{1}{4}x$$

l_1 has gradient $-\frac{1}{4}$ and it meets the coordinate axes at $(0, 0)$.

(2) l_2 has gradient 2 and it meets the y-axis at $(0, -3)$.

l_2 meets the x-axis when $y = 0$.

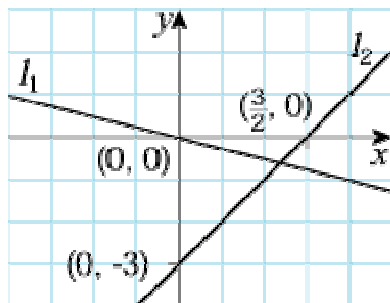
Substitute $y = 0$:

$$0 = 2x - 3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

l_2 meets the x-axis at $\left(\frac{3}{2}, 0\right)$.



(b) Solve $4y + x = 0$ and $y = 2x - 3$ simultaneously.

Substitute:

$$4(2x - 3) + x = 0$$

$$8x - 12 + x = 0$$

$$9x - 12 = 0$$

$$9x = 12$$

$$x = \frac{12}{9}$$

$$x = \frac{4}{3}$$

Substitute $x = \frac{4}{3}$ into $y = 2x - 3$:

$$y = 2 \left(\frac{4}{3} \right) - 3 = \frac{8}{3} - 3 = -\frac{1}{3}$$

The coordinates of A are $\left(\frac{4}{3}, -\frac{1}{3} \right)$.

(c) The gradient of l_1 is $-\frac{1}{4}$.

The gradient of a line perpendicular to l_1 is $-\frac{1}{-\frac{1}{4}} = 4$.

The equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - \left(-\frac{1}{3} \right) = 4 \left(x - \frac{4}{3} \right)$$

$$y + \frac{1}{3} = 4x - \frac{16}{3}$$

$$y = 4x - \frac{17}{3}$$

Multiply each term by 3:

$$3y = 12x - 17$$

$$0 = 12x - 3y - 17$$

The equation of the line is $12x - 3y - 17 = 0$.

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Coordinate geometry in the (x, y) plane

Exercise F, Question 15

Question:

The points A and B have coordinates $(4, 6)$ and $(12, 2)$ respectively.

The straight line l_1 passes through A and B .

(a) Find an equation for l_1 in the form $ax + by + c = 0$, where a , b and c are integers.

The straight line l_2 passes through the origin and has gradient -4 .

(b) Write down an equation for l_2 .

The lines l_1 and l_2 intersect at the point C .

(c) Find the coordinates of C . **[E]**

Solution:

(a) The equation of l_1 is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 6}{2 - 6} = \frac{x - 4}{12 - 4}$$

$$\frac{y - 6}{-4} = \frac{x - 4}{8}$$

Multiply each side by 8:

$$-2(y - 6) = x - 4 \quad (\text{Note: } \frac{8}{-4} = -2)$$

$$-2y + 12 = x - 4$$

$$-2y + 16 = x$$

$$16 = x + 2y$$

$$0 = x + 2y - 16$$

The equation of the line is $x + 2y - 16 = 0$

(b) The equation of l_2 is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -4(x - 0)$$

$$y = -4x$$

(c) Solve $y = -4x$ and $x + 2y = 16$ simultaneously.

Substitute:

$$x + 2(-4x) = 16$$

$$x - 8x = 16$$

$$-7x = 16$$

$$x = \frac{16}{-7}$$

$$x = -\frac{16}{7}$$

Substitute $x = -\frac{16}{7}$ in $y = -4x$:

$$y = -4 \left(-\frac{16}{7} \right) = \frac{64}{7}$$

The coordinates of C are $\left(-\frac{16}{7}, \frac{64}{7} \right)$.

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