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Edexcel AS and A Level Modular Mathematics

Review Exercise 2
Exercise A, Question 1

Question:

The random variable X has an F distribution with 10 and 12 degrees of freedom. Find a and b such that $P(a < X < b) = 0.90$. **[E]**

Solution:

$F_{10,12}(5\%) = 2.75 \therefore b = 2.75$

$$a = \frac{1}{F_{12,10}(5\%)} = \frac{1}{2.91} = 0.344$$

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Review Exercise 2
Exercise A, Question 2

Question:

A doctor believes that the span of a person's dominant hand is greater than that of the weaker hand. To test this theory, the doctor measures the spans of the dominant and weaker hands of a random sample of 8 people. He subtracts the span of the weaker hand from that of the dominant hand. The spans, in millimetres, are summarised in the table below.

	Dominant hand	Weaker hand
A	202	195
B	251	249
C	215	218
D	235	234
E	210	211
F	195	197
G	191	181
H	230	225

Test, at the 5% significance level, the doctor's belief. **[E]**

Solution:

d: 7 2 −3 1 −1 −2 10 5

$\Sigma d = 19; \Sigma d^2 = 193$

$\therefore \bar{d} = \frac{19}{8} = 2.375; S_d^2 = \frac{1}{7}(193 - \frac{19^2}{8}) = 21.125$

$H_0 : \mu_D = 0; H_1 : \mu_D > 0$

$t = \frac{2.375 - 0}{\sqrt{\frac{21.125}{8}}} = 1.4615\ldots$

$\nu = 7 \Rightarrow$ critical region : $t > 1.895$

Since 1.4915 … is *not* in the critical region there is insufficient evidence to reject H_0 and we conclude that there is insufficient evidence to support the doctors' belief.

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Review Exercise 2
Exercise A, Question 3

Question:

The times, x seconds, taken by the competitors in the 100 m freestyle events at a school swimming gala are recorded. The following statistics are obtained from the data.

	Number of competitors	Sample mean $\bar{x}$	$\sum x^2$
Girls	8	83.10	55 746
Boys	7	88.90	56 130

Following the gala a proud parent claims that girls are faster swimmers than boys. Assuming that the times taken by the competitors are two independent random samples from normal distributions,

a test, at the 10% level of significance, whether or not the variances of the two distributions are the same. State your hypotheses clearly.

b Stating your hypotheses clearly, test the parent's claim. Use a 5% level of significance. **[E]**

Solution:

a $H_0 : \sigma_G^2 = \sigma_B^2, H_1 : \sigma_G^2 \neq \sigma_B^2,$

$$s_B^2 = \frac{1}{6}(56\ 130 - 7 \times 88.9^2) = \frac{807.53}{6} = 134.6$$

$$s_G^2 = \frac{1}{7}(55\ 746 - 8 \times 83.1^2) = \frac{501.12}{7} = 71.58$$

$$\frac{s_B^2}{s_G^2} = 1.880\ldots$$

critical value $F_{6,7} = 3.87$

not significant, variances are the same

b $H_0 : \mu_B = \mu_G, H_1 : \mu_B > \mu_G$

pooled estimate of variance $s^2 = \dfrac{6 \times 134.6 + 7 \times 71.58}{13} = 100.666153\ldots$

test statistic $t = \dfrac{88.9 - 83.1}{s\sqrt{\dfrac{1}{7} + \dfrac{1}{8}}} = 1.1169$

critical value $t_{13}(5\%) = 1.771$

Insufficient evidence to support parent's claim

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Review Exercise 2
Exercise A, Question 4

Question:

Two methods of extracting juice from an orange are to be compared. Eight oranges are halved. One half of each orange is chosen at random and allocated to Method A and the other half is allocated to Method B. The amounts of juice extracted, in ml, are given in the table.

	Orange							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Method A	29	30	26	25	26	22	23	28
Method B	27	25	28	24	23	26	22	25

One statistician suggests performing a two-sample t-test to investigate whether or not there is a difference between the mean amounts of juice extracted by the two methods.

a Stating your hypotheses clearly and using a 5% significance level, carry out this test. (You may assume $\bar{x}_A = 26.125, s_A^2 = 7.84$, $\bar{x}_B = 25, s_B^2 = 4$ and $\sigma_A^2 = \sigma_B^2$)

Another statistician suggests analysing these data using a paired t-test.

b Using a 5% significance level, carry out this test.

c State which of these two tests you consider to be more appropriate. Give a reason for your choice. **[E]**

Solution:

a $s_p^2 = \dfrac{7\times 7.84 + 7\times 4}{7+7} = 5.92$

$s_p = 2.433105$

$H_0 : \mu_A = \mu_B, H_1 : \mu_A \neq \mu_B$

$t = \dfrac{26.125 - 25}{2.43\sqrt{\frac{1}{8}+\frac{1}{8}}} = 0.92474$

$t_{14}(2.5\%) = 2.145$

Insufficient evidence to reject H_0

Conclude that there is no difference in the means.

b $d = 2, 5, -2, 1, 3, -4, 1, 3$

$\overline{d} = \dfrac{9}{8} = 1.125$

$s_d^2 = \dfrac{69 - 8\times 1.125^2}{7} = 8.410714$

$H_0 : \delta = 0, H_1 : \delta \neq 0$

$t = \dfrac{1.125}{\sqrt{\frac{8.41}{8}}} = 1.0972$

$t_7(2.5\%) = 2.365$

There is no significant evidence of a difference between method A and method B.

c Paired sample as they are two measurements on the same orange

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Review Exercise 2
Exercise A, Question 5

Question:

The random variable X has an F-distribution with 8 and 12 degrees of freedom.

Find $\mathrm{P}\left(\frac{1}{5.67} < X < 2.85\right)$. **[E]**

Solution:

$$\mathrm{P}(X > 2.85) = 0.05$$

$$\mathrm{P}\left(X < \frac{1}{5.67}\right) = 0.01$$

$$\therefore \mathrm{P}\left(\frac{1}{5.67} < X < 2.85\right) = 1 - 0.05 - 0.01$$

$$= 0.94$$

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Review Exercise 2
Exercise A, Question 6

Question:

A grocer receives deliveries of cauliflowers from two different growers, A and B. The grocer takes random samples of cauliflowers from those supplied by each grower. He measures the weight x, in grams, of each cauliflower. The results are summarised in the table below.

	Sample size	$\sum x$	$\sum x^2$
A	11	6600	3 960 540
B	13	9815	7410 579

a Show, at the 10% significance level, that the variances of the populations from which the samples are drawn can be assumed to be equal by testing the hypothesis $H_0: \sigma_A^2 = \sigma_B^2$ against hypothesis $H_1: \sigma_A^2 \neq \sigma_B^2$.
(You may assume that the two samples come from normal populations.)
The grocer believes that the mean weight of cauliflowers provided by B is at least 150 g more than the mean weight of cauliflowers provided by A.

b Use a 5% significance level to test the grocer's belief.

c Justify your choice of test. **[E]**

Solution:

a $S_A^2 = \frac{1}{10}\{3\,960\,540 - \frac{6600^2}{11}\} = 54.0$

$S_B^2 = \frac{1}{12}\{7\,410\,579 - \frac{9815^2}{13}\} = 21.1\dot{6}$

$H_0 : \sigma_A^2 = \sigma_B^2; H_1 : \sigma_A^2 \neq \sigma_B^2$

critical region: $F_{10,12} > 2.75$

$\frac{S_A^2}{S_B^2} = \frac{54.0}{21.1\dot{6}} = 2.55118\ldots$

Since $2.55118\ldots$ is not in the critical region we can assume that the variances are equal.

b $H_0 : \mu_B = \mu_A + 150; H_1 : \mu_B > \mu_A + 150$

CR: $t_{22}(0.05) > 1.717$

$S_p^2 = \frac{10 \times 54.0 + 12 \times 21.1\dot{6}}{22} = 36.0909$

$t = \frac{755 - 600 - 150}{\sqrt{36.0909\ldots\left(\frac{1}{11} + \frac{1}{13}\right)}} = 2.03157$

Since $2.03\ldots$ is in the critical region we reject H_0 and conclude that the mean weight of cauliflowers from B exceeds that from A by at least 50g.

c Samples from normal populations
Equal variances
Independent samples

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Review Exercise 2
Exercise A, Question 7

Question:

The random variable X has a χ^2-distribution with 9 degrees of freedom.

a Find $P(2.088 < X < 19.023)$.

The random variable Y follows an F-distribution with 12 and 5 degrees of freedom.

b Find the upper and lower 5% critical values for Y. **[E]**

Solution:

a

$$P(X > 19.023) = 0.025$$
$$P(X > 2.088) = 0.990$$
$$P(2.088 < X < 19.023) = 0.990 - 0.025$$
$$= 0.965$$

b

Upper critical value of $F_{12,5} = 4.68$

Lower critical value of $F_{12,5} = \frac{1}{F_{5,12}}$

$$= \frac{1}{3.11}$$
$$= 0.3215\ldots$$

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Review Exercise 2
Exercise A, Question 8

Question:

A group of 10 technology students is assessed by coursework and a written examination. The marks, given as percentages, are given in the table below.

Student	Coursework	Written exam.
1	65	61
2	73	76
3	62	65
4	81	77
5	78	72
6	74	71
7	68	72
8	59	42
9	76	69
10	70	63

a Use a suitable t-test to determine whether or not the coursework marks are significantly higher than the written examination marks. Use a 5% level of significance.

b State an assumption about the distribution of marks that is needed to make the above test valid. **[E]**

Solution:

a $d =$ coursework $-$ written: $4, -3, -3, 4, 6, 3, -4, 17, 7, 7$

$$\bar{d} = \frac{38}{10} = 3.8,\ s_d^2 = \frac{498 - 10\bar{d}^2}{9} = 39.2\dot{8}$$

test statistic: $t = \dfrac{3.8}{\frac{s_d}{\sqrt{10}}} = 1.917\ldots$

$H_0: \mu_d = 0 \quad H_1: \mu_d > 0$

$t_9(5\%)$ c.v. is 1.833;

$\therefore$ significant $-$ there is evidence coursework marks are higher

b The difference between the marks follows a normal distribution.

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Review Exercise 2
Exercise A, Question 9

Question:

An engineer decided to investigate whether or not the strength of rope was affected by water. A random sample of 9 pieces of rope was taken and each piece was cut in half. One half of each piece was soaked in water over night, and then each piece of rope was tested to find its strength. The results, in coded units, are given in the table below

Rope number	1	2	3	4	5	6	7	8	9
Dry rope	9.7	8.5	6.3	8.3	7.2	5.4	6.8	8.1	5.9
Wet rope	9.1	9.5	8.2	9.7	8.5	4.9	8.4	8.7	7.7

Assuming that the strength of rope follows a normal distribution, test whether or not there is any difference between the mean strengths of dry and wet rope. State your hypotheses clearly and use a 1% level of significance. **[E]**

Solution:

$D = \text{dry} - \text{wet} \qquad H_0 : \mu_D = 0, H_1 : \mu_D \neq 0$

$d : 0.6, -1, -1.9, -1.4, -1.3, 0.5, -1.6, -0.6, -1.8$

$\bar{d} : -\frac{8.5}{9} = -0.9\dot{4}, \quad s_d^2 = \frac{15.03 - 9 \times (\bar{d})^2}{8} = 0.87527\ldots$

$t = \frac{-0.9\dot{4}}{\frac{s_d}{\sqrt{9}}} = \text{awrt} -3.03$

t_8 2-tail 1% critical value $= 3.355$

Not significant – insufficient evidence of a difference between mean strength

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Review Exercise 2
Exercise A, Question 10

Question:

An educational researcher is testing the effectiveness of a new method of teaching a topic in Mathematics. A random sample of 10 children were taught by the new method and a second random sample of 9 children, of similar age and ability, were taught by the conventional method. At the end of the teaching, the same test was given to both groups of children.

The marks obtained by the two groups are summarised in the table below.

	New method	**Conventional method**
Mean ($\bar{x}$)	82.3	78.2
Standard deviation (s)	3.5	5.7
Number of students (n)	10	9

a Stating your hypotheses clearly and using a 5% level of significance, investigate whether or not

i the variance of the marks of children taught by the conventional method is greater than that of children taught by the new method,

ii the mean score of children taught by the conventional method is lower than the mean score of those taught by the new method.

[In each case you should give full details of the calculation of the test statistics.]

b State any assumptions you made in order to carry out these tests.

c Find a 95% confidence interval for the common variance of the marks of the two groups. **[E]**

Solution:

a **i** $H_0 : \sigma_C^2 = \sigma_N^2, H_1 : \sigma_C^2 > \sigma_N^2$

$\frac{S_C^2}{s_N^2} = \frac{5.7^2}{3.5^2} = 2.652\ldots$; $F_{8,9}$ (5%) critical value $= 3.23$

Not significant so do not reject H_0.

There is insufficient evidence that variance using conventional method is greater

ii $H_0 : \mu_N = \mu_C, H_1 : \mu_N > \mu_C$

$s^2 = \frac{8 \times 5.7^2 + 9 \times 3.5^2}{17} = \frac{370.17}{17} = 21.774\ldots$

Test statistic $t = \frac{82.3 - 78.2}{\sqrt{21.774\ldots\left(\frac{1}{9} + \frac{1}{10}\right)}} = 1.9122\ldots$

t_{17} (5%) 1-tail critical value $= 1.740$

Significant $-$ reject H_0.

There is evidence that new style leads to an increase in mean

b Assumed population of marks obtained were normally distributed

c Unbiased estimate of common variance is s^2 in **ii**

$7.564 < \frac{17s^2}{\sigma^2} < 30.191$

$\sigma^2 > \frac{17 \times 21.774\ldots}{30.191} = 12.3$ (1 d.p.)

$\sigma^2 < \frac{17 \times 21.774\ldots}{7.564} = 48.9$ (1 d.p.)

Confidence interval on σ^2 is (12.3, 48.9)

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Review Exercise 2
Exercise A, Question 11

Question:

Brickland and Goodbrick are two manufacturers of bricks. The lengths of the bricks produced by each manufacturer can be assumed to be normally distributed. A random sample of 20 bricks is taken from Brickland and the length, x mm, of each brick is recorded. The mean of this sample is 207.1 mm and the variance is 3.2 mm^2.

a Calculate the 98% confidence interval for the mean length of brick from Brickland.

A random sample of 10 bricks is selected from those manufactured by Goodbrick. The length of each brick, y mm, is recorded. The results are summarised as follows.

$\sum y = 2046.2 \quad \sum y^2 = 418\,785.4$

The variances of the length of brick for each manufacturer are assumed to be the same.

b Find a 90% confidence interval for the value by which the mean length of brick made by Brickland exceeds the mean length of brick made by Goodbrick. **[E]**

Solution:

a Confidence interval is given by

$$\bar{x} \pm t_{19} \times \frac{s}{\sqrt{n}}$$

i.e. $207.1 \pm 2.539 \times \sqrt{\frac{3.2}{20}}$

i.e. 207.1 ± 1.0156

i.e. $(206.08\ldots, 208.1156)$

b $\bar{x}_G = \frac{2046.2}{10} = 204.62$

$$S_p^2 = \frac{19 \times 3.2 + 9 \times 10.217\dot{3}}{28}$$

$= 5.45557\ldots$

Confidence interval is given by

$$\bar{x}_B - \bar{x}_G \pm t_{21} \times \sqrt{5.45557\left(\frac{1}{20} + \frac{1}{10}\right)}$$

i.e. $(207.1 - 204.62) \pm 1.701\sqrt{5.45557\left(\frac{1}{20} + \frac{1}{10}\right)}$

i.e. 2.48 ± 1.53875

i.e. $(0.94125, 4.0187)$

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Review Exercise 2
Exercise A, Question 12

Question:

The weights, in grams, of apples are assumed to follow a normal distribution.
The weights of apples sold by a supermarket have variance σ_s^2. A random sample of 4 apples from the supermarket had weights

114, 110, 119, 123.

a Find a 95% confidence interval for σ_s^2.

The weights of apples sold on a market stall have variance σ_M^2. A second random sample of 7 apples was taken from the market stall. The sample variance s_M^2 of the apples was 318.8.

b Stating your hypotheses clearly test, at the 1% level of significance, whether or not there is evidence that $\sigma_M^2 > \sigma_s^2$.

Solution:

a $\left(\bar{x} = \frac{466}{4} = 116.5\right)$ $s_x^2 = \frac{54\,386 - 4\bar{x}^2}{3} = 32.\dot{3}$ or $\frac{97}{3}$

$0.216 < \frac{3s_x^2}{\sigma^2} < 9.348$

$10.376\ldots < \sigma^2 < 449.07\ldots$ so confidence interval is (10.376, 449.07)

b $H_0 : \sigma_M^2 = \sigma_s^2$ $H_1 : \sigma_M^2 > \sigma_s^2$

$\frac{S_M^2}{S_S^2} = \frac{318.8}{32.\dot{3}} = 9.859\ldots$

$F_{6,3}$(1% c.v.) = 27.91

$9.15 < 27.91$, insufficient evidence of an increase in variance

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Edexcel AS and A Level Modular Mathematics

Review Exercise 2
Exercise A, Question 13

Question:

As part of an investigation into the effectiveness of solar heating, a pair of houses was identified where the mean weekly fuel consumption was the same. One of the houses was then fitted with solar heating and the other was not. Following the fitting of the solar heating, a random sample of 9 weeks was taken and the table below shows the weekly fuel consumption for each house.

Week	1	2	3	4	5	6	7	8	9
Without solar heating	19	19	18	14	6	7	5	31	43
With solar heating	13	22	11	16	14	1	0	20	38

Units of fuel used per week

a Stating your hypotheses clearly, test, at the 5% level of significance, whether or not there is evidence that the solar heating reduces the mean weekly fuel consumption.

b State an assumption about weekly fuel consumption that is required to carry out this test. **[E]**

Solution:

a $H_0 : \mu_d = 0$

$H_1 : \mu_d > 0$

where $d =$ without solar heating − with solar heating

$d = 6 \quad -3 \quad 7 \quad -2 \quad -8 \quad 6 \quad 5 \quad 11 \quad 5$

$\bar{d} = 3$

$s_d = 6$

$n_d = 9$

$$\therefore \text{ test statistic} = \frac{(3-0)}{\left(\frac{6}{\sqrt{9}}\right)}$$

t.s. $= 1.5$

critical value $= t_8(5\%) = 1.860$

so critical region: $t > 1.860$

Test statistic not in critical region so accept H_0. Conclude there is insufficient evidence that solar heating reduces mean weekly fuel consumption.

b The differences are normally distributed.

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Review Exercise 2
Exercise A, Question 14

Question:

A large number of students are split into two groups A and B. The students sit the same test but under different conditions. Group A has music playing in the room during the test, and group B has no music playing during the test. Small samples are then taken from each group and their marks recorded. The marks are normally distributed.
The marks are as follows:

Sample from Group A	42	40	35	37	34	43	42	44	49
Sample from Group B	40	44	38	47	38	37	33		

a Stating your hypotheses clearly, and using a 10% level of significance, test whether or not there is evidence of a difference between the variances of the marks of the two groups.

b State clearly an assumption you have made to enable you to carry out the test in part **a**.

c Use a two-tailed test, with a 5% level of significance, to determine if the playing of music during the test has made any difference in the mean marks of the two groups. State your hypotheses clearly.

d Write down what you can conclude about the effect of music on a student's performance during the test. **[E]**

Solution:

a $H_1 : \sigma_A^2 = \sigma_B^2 \quad H_0 : \sigma_A^2 \neq \sigma_B^2$

$s_A^2 = 22.5 \quad s_B^2 = 21.6$

$\frac{s_A^2}{s_B^2} = 1.04$

$F_{(8,6)} = 4.15$

$1.04 < 4.15$ do not reject H_0. The variances are the same.

b Assume the samples are selected at random (independent)

c $s^2{}_p = \frac{8(22.5)+6(21.62)}{14} = 22.12$

$H_0 : \mu_A = \mu_B \quad H_1 : \mu_A \neq \mu_B$

$t = \frac{40.667 - 39.57}{\sqrt{22.12}\sqrt{\frac{1}{9}+\frac{1}{7}}} = 0.462$

Critical value $= t_{14}(2.5\%) = 2.145$

$0.462 < 2.145$ No evidence to reject H_0

The means are the same.

d Music has no effect on performance

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Review Exercise 2
Exercise A, Question 15

Question:

A company undertakes investigations to compare fuel consumption x, in miles per gallon, of two different cars the *Relaxant* and the *Elegane*, with a view to purchasing a number of cars. A random sample of 13 *Relaxants* and an independent random sample of 7 *Eleganes* were taken and the following statistics calculated.

Car	Sample size n	Sample mean $\bar{x}$	Sample variance s^2
Relaxant	13	32.31	14.48
Elegane	7	28.43	35.79

The company assumes that fuel consumption for each make of car follows a normal distribution.

a Stating your hypotheses clearly test, at the 10% level of significance, whether or not the two distributions have the same variance.

b Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is a difference in mean fuel consumption between the two types of car.

c Explain the importance of the conclusion to the test in part **a** in justifying the use of the test in part **b**.

d State two factors which might be considered when undertaking an investigation into fuel consumption of two models of car to ensure that a fair comparison is made. **[E]**

Solution:

a $H_0: \sigma_R^2 = \sigma_E^2 \quad H_1: \sigma_R^2 \neq \sigma_E^2$

$F_{6,12}(5\%)_{1\text{tail}}\text{cv} = 3.00, \quad \frac{s_E^2}{s_R^2} = \frac{35.79}{14.48} = 2.4716\ldots$

Not significant so do not reject H_0

Insufficient evidence to suspect $\sigma_R^2 \neq \sigma_E^2$

b $H_0: \mu_R = \mu_E \quad H_1: \mu_R \neq \mu_E$

$$s^2 = \frac{6 \times 35.79 + 12 \times 14.48}{18} = 21.58\dot{3}$$

$$t = \frac{32.31 - 28.43}{s\sqrt{\frac{1}{13} + \frac{1}{7}}} = 1.78146\ldots$$

$t_{18}(5\%)_{2\text{tail}}\text{cv} = 2.101$

$\therefore$ Not significant

Insufficient evidence of difference in mean performance

c Test in **b** requires $\sigma_1^2 = \sigma_2^2$

d for example, same type of driving
same roads and journey length
same weather
same driver

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Review Exercise 2
Exercise A, Question 16

Question:

A beach is divided into two areas A and B. A random sample of pebbles is taken from each of the two areas and the length of each pebble is measured. A sample of size 26 is taken from area A and the unbiased estimate for the population variance is $s_A^2 = 0.495\,\text{mm}^2$. A sample of size 25 is taken from area B and the unbiased estimate for the population variance is $s_B^2 = 1.04\,\text{mm}^2$.

a Stating your hypotheses clearly test, at the 10% significance level, whether or not there is a difference in variability of pebble length between area A and area B.

b State the assumption you have made about the populations of pebble lengths in order to carry out the test. **[E]**

Solution:

a $H_0 : \sigma_A^2 = \sigma_B^2, H_1 : \sigma_A^2 \neq \sigma_B^2$

critical value $F_{24,25} = 1.96$

$$\frac{s_B^2}{s_A^2} = 2.10$$

Since 2.10 is in the critical region we reject H_0 and conclude there is evidence that the two variances are different.

b The populations of pebble lengths are normal.