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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Given that the random variable X has a t_{12}-distribution, find values of t such that,

a $\mathrm{P}(X < t) = 0.025$,

b $\mathrm{P}(X > t) = 0.05$,

c $\mathrm{P}(|X| > t) = 0.95$.

Solution:

a $\mathrm{P}(X > t) = 0.025$ when $t = 2.179$ so $\mathrm{P}(X < t) = 0.025$ when $t = -2.179$

b $\mathrm{P}(X > t) = 0.05$ when $t = 1.782$

c $\mathrm{P}(X > t) = 0.025$ when $t = 2.179$

$\mathrm{P}(|X| > t) = 0.95$ when $|t| = 2.179$

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Exercise A, Question 2

Question:

Given that the random variable X has a t_{26}-distribution. Find

a $t_\nu(0.01)$,

b $t_\nu(0.05)$.

Solution:

a 2.479 (from tables)

b 1.706 (from tables)

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Exercise A, Question 3

Question:

The random variable Y has a t_ν-distribution. Find a value (or values) of t in each of the following.

a $\nu = 10, \mathrm{P}(Y < t) = 0.95$

b $\nu = 32, \mathrm{P}(Y < t) = 0.005$

c $\nu = 5, \mathrm{P}(Y < t) = 0.025$

d $\nu = 16, \mathrm{P}(|Y| < t) = 0.98$

e $\nu = 18, \mathrm{P}(|Y| > t) = 0.10$

Solution:

a $\mathrm{P}(Y > t) = 0.05$ when $t = 1.812$ so $\mathrm{P}(Y < t) = 0.95$ when $t = 1.812$

b $\mathrm{P}(Y > t) = 0.005$ when $t = 2.738$ so $\mathrm{P}(Y < t) = 0.005$ when $t = -2.738$

c $\mathrm{P}(Y > t) = 0.025$ when $t = 2.571$ so $\mathrm{P}(Y < t) = 0.025$ when $t = -2.571$

d $\mathrm{P}(Y > t) = 0.01$, when $t = 2.583$, and $\mathrm{P}(Y < t) = 0.01$ when $t = -2.583$ so
$\mathrm{P}(|Y| < t) = 0.98$ when $|t| = 2.583$

e $\mathrm{P}(Y > t) = 0.05$ when $t = 1.734$ and $\mathrm{P}(Y < t) = 0.05$ when $t = -1.734$ so
$\mathrm{P}(|Y| > t) = 0.10$ when $|t| = 1.734$

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Exercise B, Question 1

Question:

A test on the life (in hours) of a certain make of torch batteries gave the following results:

20.3 17.3 25.0 18.4 16.3 24.8 24.3 21.2

Assuming that the lifetime of batteries is normally distributed, find a 90% confidence interval for the mean.

Solution:

$\bar{x} = 20.95 \quad s = 3.4719\ldots n = 8 \quad \nu = 7$

confidence limits $= \bar{x} \pm t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}} = 20.95 \pm 1.895 \times \frac{3.4719\ldots}{\sqrt{8}} = 18.624$

and 23.276

Confidence interval $= (18.624, 23.276)$

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Exercise B, Question 2

Question:

A sample of size 16 taken from a normal population with unknown variance gave the following sample values $\bar{x} = 12.4, s^2 = 21.0$. Find a 95% confidence interval on the population mean.

Solution:

$\bar{x} = 12.4 \quad s = \sqrt{21} \quad n = 16 \quad \nu = 15$

confidence limits $= \bar{x} \pm t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}} = 12.4 \pm 2.131 \times \frac{\sqrt{21}}{\sqrt{16}} = 9.9586\ldots$ and 14.8413 …

Confidence interval $= (9.959, 14.841)$

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Exercise B, Question 3

Question:

The mean heights (measured in centimetres) of six male students at a college were as follows:

182 178 183 180 169 184

Calculate,

a a 90% confidence interval and

b a 95% confidence interval for the mean height of male students at the college.
You may assume that the heights are normally distributed.

Solution:

a $\bar{x} = 179.333333 \quad s = 5.5015\ldots n = 6 \quad \nu = 5$

confidence limits $= \bar{x} \pm t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}} = 179.333.. \pm 2.015 \times \frac{5.5015\ldots}{\sqrt{6}} = 174.808$ and 183.859

Confidence interval $= (174.808, 183.859)$

b confidence limits $= \bar{x} \pm t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}} = 179.333.. \pm 2.571 \times \frac{5.5015\ldots}{\sqrt{6}} = 173.559$ and 185.108

Confidence interval $= (173.559, 185.108)$

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Exercise B, Question 4

Question:

The masses (in grams) of 10 nails selected at random from a bin of 90 cm long nails were:

9.7 10.2 11.2 9.4 11.0 11.2 9.8 9.8 10.0 11.3

Calculate a 98% confidence interval for the mean mass of the nails, assuming that their mass is normally distributed.

Solution:

$\bar{x} = 10.36 \quad s = 0.73363\ldots n = 10 \quad v = 9$

confidence limits $= \bar{x} \pm t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}} = 10.36 \pm 2.821 \times \frac{0.73363\ldots}{\sqrt{10}} = 9.706$ and 11.014

Confidence interval $= (9.706, 11.014)$

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Exercise B, Question 5

Question:

It is known that the length of men's feet is normally distributed. A random sample of the feet of 8 adult males gave the following summary statistics of length x (in cm):

$\sum x = 224.1 \quad \sum x^2 = 6337.39$

Calculate a 99% confidence interval for the mean lengths of men's feet based upon these results.

Solution:

$\bar{x} = \frac{224.1}{8} = 28.0125 \quad s^2 = \frac{1}{7}\left\{6337.39 - \frac{224.1^2}{8}\right\} = 8.54125$

$s = 2.92254\ldots n = 8 \quad \nu = 7$

confidence limits $= \bar{x} \pm t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}} = 28.0125 \pm 3.499 \times \frac{2.92254\ldots}{\sqrt{8}} = 24.397$ and 31.628

Confidence interval $= (24.397, 31.628)$

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Exercise B, Question 6

Question:

A random sample of 26 students from the sixth form of a school sat an intelligence test that measured their IQs. The result are summarised below

$\bar{x} = 122 \quad s^2 = 225$

Assuming that the IQ is normally distributed, calculate a 95% confidence interval for the mean IQ of the students.

Solution:

$\bar{x} = 122 \quad s = \sqrt{225} = 15 \quad \nu = 25$

confidence limits $= \bar{x} \pm t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}} = 122 \pm 2.060 \times \frac{\sqrt{225}}{\sqrt{26}} = 115.940$ and 128.060

Confidence interval $= (115.94, 128.06)$

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Exercise C, Question 1

Question:

Given that the observations 9, 11, 11, 12, 14, have been drawn from a normal distribution, test $H_0 : \mu = 11$ against $H_1 : \mu > 11$. Use a 5% significance level.

Solution:

$\bar{x} = 11.4 \quad s = 1.816\ldots$

$H_0 : \mu = 11 \quad H_1 : \mu > 11$

Critical region $t > 2.132$

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{11.4 - 11.0}{\frac{1.816\ldots}{\sqrt{5}}} = 0.492$

The result is not in critical region.

No evidence that μ is not 11

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Exercise C, Question 2

Question:

A random sample of size 28 taken from a normally distributed variable gave the following sample values $\bar{x} = 17.1$ and $s^2 = 4$. Test $H_0 : \mu = 19$ against $H_1 : \mu < 19$.
Use a 1% level of significance.

Solution:

$\bar{x} = 17.1 \quad s = 2$

$H_0 : \mu = 19 \quad H_1 : \mu < 19$

Critical region $t < -2.473$

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{17.1 - 19}{\frac{2}{\sqrt{28}}} = -5.027$

The result is in the critical region.
There is evidence that μ is less than 19

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Exercise C, Question 3

Question:

A random sample of size 13 taken from a normally distributed variable gave the following sample values $\bar{x} = 3.26, s^2 = 0.64$. Test $\mathrm{H}_0 : \mu = 3$ against $\mathrm{H}_1 : \mu \neq 3$. Use a 5% significance level.

Solution:

$\bar{x} = 3.26 \quad s = 0.8$

$\mathrm{H}_0 : \mu = 3 \quad \mathrm{H}_1 : \mu \neq 3$

Critical values ± 2.179

Critical region $t < -2.179$ or $t > 2.179$

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{3.26 - 3}{\frac{0.8}{\sqrt{13}}} = 1.172$

The result is not in the critical region

There is no evidence that μ is not 3

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Exercise C, Question 4

Question:

A certain brand of blanched hazelnuts for use in cooking is sold in packets. The weights of the packets of hazelnuts follow a normal distribution with mean μ. The manufacturer claims that $\mu = 100$ g. A sample of 15 packets was taken and the weight x of each was measured. The results are summarised by the following statistics $\sum x = 1473$, $\sum x^2 = 148119$.

Test at the 5% significance level whether or not there is evidence to justify the manufacturer's claim.

Solution:

$\bar{x} = 98.2 \quad s = 15.744\ldots$

$H_0 : \mu = 100 \quad H_1 : \mu \neq 100$

Critical region < -2.145 or > 2.145

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{98.2 - 100}{\frac{15.74\ldots}{\sqrt{15}}} = -0.443$

The result is not in the critical region

There is no evidence that μ is not 100

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Exercise C, Question 5

Question:

A manufacturer claims that the lifetimes of its 100 watt bulbs are normally distributed with a mean of 1000 hours. A laboratory tests 8 bulbs and finds their lifetimes to be 985, 920, 1110, 1040, 945, 1165, 1170, and 1055 hours.
Stating your hypotheses clearly, examine whether or not the bulbs have a longer mean life than that claimed. Use a 5% level of significance.

Solution:

$\bar{x} = 1048.75 \quad s = 95.2346\ldots$

$H_0 : \mu = 1000 \quad H_1 : \mu > 1000$

Critical region $t > 1.895$

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{1048.75 - 1000}{\frac{95.234\ldots}{\sqrt{8}}} = 1.448$

The result is not in the critical region

There is no evidence that μ is not 1000

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Exercise C, Question 6

Question:

A fertiliser manufacturer claims that by using brand F fertiliser the yield of fruit bushes will be increased. A random sample of 14 fruit bushes was fertilised with brand F and the resulting yields, x, were summarised by $\sum x = 90.8, \sum x^2 = 600$. The yield of bushes fertilised by the usual fertiliser was normally distributed with a mean of 6 kg per bush.

Test, at the 2.5% significance level, the manufacturer's claim.

Solution:

$\bar{x} = 6.4857\ldots \quad s^2 = 0.853626\ldots \quad s = 0.923919\ldots$

$H_0 : \mu = 6 \quad H_1 : \mu > 6$

Critical region $t > 2.160$

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{6.4857142 - 6}{\frac{0.923919\ldots}{\sqrt{14}}} = 1.967$

The result is not in the critical region

There is no evidence supporting manufacturer's claim.

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Exercise C, Question 7

Question:

A nuclear reprocessing company claims that the amount of radiation within a reprocessing building in which there had been an accident had been reduced to an acceptable level by their clean up team. The amount of radiation, x, at 20 sites within the building in suitable units are summarised by $\sum x = 21.7$, $\sum x^2 = 28.4$. Before the accident the level of radiation in the building was normally distributed with a mean of 1.00. Test, at the 0.10 level whether or not the claim is justified.

Solution:

$\bar{x} = 1.085 \quad s^2 = \frac{28.4 - 20 \times 1.085^2}{19} = 0.2555\ldots \quad s = 0.5055\ldots$

$H_0 : \mu = 1.00 \quad H_1 : \mu > 1.00$

Critical values $t < 1.328$

Test statistic $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{1.085 - 1}{\frac{0.5055..}{\sqrt{20}}} = 0.752$

The result is not in the critical region

There is no evidence that μ is not 1.00 so the claim is not justified

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Exercise D, Question 1

Question:

A random sample of 15 observations of a normal population gave an unbiased estimate for the variance of the population of $s^2 = 4.8$. Calculate a 95% confidence interval for the population variance.

Solution:

$$\text{Confidence interval} = \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)} \right) = \left(\frac{14\times4.8}{26.119}, \frac{14\times4.8}{5.629} \right)$$
$$= (2.573, 11.938)$$

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Exercise D, Question 2

Question:

A random sample of 20 observations of a normally distributed variable X is summarised by $\sum x = 132.4$ and $\sum x^2 = 884.3$. Calculate a 90% confidence interval for the variance of X.

Solution:

$$\bar{x} = 6.62 \quad s^2 = \frac{1}{19}\left(884.3 - \frac{132^2}{20}\right) = 0.4111\ldots$$

$$\text{Confidence interval} = \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)}\right) = \left(\frac{19\times 0.411\ldots}{30.144}, \frac{19\times 0.411..}{10.117}\right)$$

$$= (0.259, 0.772)$$

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Exercise D, Question 3

Question:

A random sample of 14 observations is taken from a population that is assumed to be normally distributed. The resulting values were:

2.3, 3.9, 3.5, 2.2, 2.6, 2.5, 2.3, 3.9, 2.1, 3.6, 2.1, 2.7, 3.2, 3.4

Calculate a 95% confidence interval for the population variance.

Solution:

$\bar{x} = 2.8785 \quad \ldots. \quad s^2 = 0.45873\ldots$

$$\text{Confidence interval} = \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)} \right) = \left(\frac{13\times 0.458\ldots}{24.736}, \frac{13\times 0.458\ldots}{5.009} \right)$$

$$= (0.241, 1.191)$$

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Exercise D, Question 4

Question:

A random sample of female voles was trapped in a wood. Their lengths, in centimetres (excluding tails) were 7.5, 8.4, 10.1, 6.2, and 8.4 cm.
Assuming that this is a sample from a normal distribution, calculate 95% confidence intervals for:

a the mean length,

b the variance of the lengths of female voles.

Solution:

$\bar{x} = 8.12 \quad s^2 = 2.037\ldots\ldots s = 1.427\ldots$

a Confidence interval $= \bar{x} \pm t_{n-1}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}} =$

$$\left(8.12 - 2.776 \times \frac{1.427..}{\sqrt{5}}, 8.12 + 2.776 \times \frac{1.427..}{\sqrt{5}}\right) = (6.348, 9.892)$$

b Confidence interval $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)}\right) = \left(\frac{4 \times 2.037}{11.143}, \frac{4 \times 2.037}{0.484}\right)$

$$= (0.731, 16.835)$$

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Exercise D, Question 5

Question:

a A random sample of 10 is taken from the annual rainfall figures, x cm, in a certain district. The result is summarised by $\sum x = 621$ and $\sum x^2 = 38\,938$.

Calculate 90% confidence limits for,

i the mean annual rainfall,

ii the variance of the annual rainfall.

b What assumption have you made about the distribution of the annual rainfall in part **a**?

Solution:

a $\bar{x} = 62.1 \quad s^2 = \frac{1}{9}\left(38\,938 - \frac{621^2}{10}\right) = 41.544\ldots \quad s = 6.445\ldots$

i $$\text{Confidence interval} = \bar{x} \pm t_{n-1}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}$$
$$= \left(62.1 - 1.833 \times \frac{6.445\ldots}{\sqrt{10}}, 62.1 + 1.833 \times \frac{6.445\ldots}{\sqrt{10}}\right)$$
$$= (58.364, 65.836)$$

ii $$\text{Confidence interval} = \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)}\right) = \left(\frac{9 \times 41.544\ldots}{16.919}, \frac{9 \times 41.544}{3.325}\right)$$
$$= (22.099, 112.450)$$

b Normal distribution

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Exercise D, Question 6

Question:

A new variety of small daffodil is grown in the trial ground of a nursery. During the flowering period a random sample of 10 flowers was taken and the lengths, in millimetres, of their stalks were measured. The results were as follows:

266, 254, 215, 220, 253, 230, 216, 248, 234, 244 mm.

Assuming that the lengths are normally distributed, calculate 95% confidence intervals for the mean and variance of the lengths.

Solution:

$\bar{x} = 238 \, s = 17.694\ldots \quad s^2 = 313.111\ldots$

$$\text{Confidence interval mean} = \left(\bar{x} - t_{n-1}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}},\ \bar{x} + t_{n-1}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}\right)$$

$$= \left(238 - 2.262 \times \frac{17.694\ldots}{\sqrt{10}}, 238 + 2.262 \times \frac{17.694\ldots}{\sqrt{10}}\right)$$

$$= (225.343, 250.657)$$

$$\text{Confidence interval variance} = \left(\frac{(n-1)s^2}{\chi_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1 - \frac{\alpha}{2}\right)}\right)$$

$$= \left(\frac{9 \times 313.111\ldots}{19.023}, \frac{9 \times 313.11\ldots}{2.700}\right)$$

$$= (148.136, 1043.704)$$

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Exercise E, Question 1

Question:

Twenty random observations (x) are taken from a normal distribution with variance σ^2. The results are summarised as follows:

$\sum x = 332.1, \sum x^2 = 5583.63$

a Calculate an unbiased estimate for the population variance.

b Test, at the 5% significance level, the hypothesis $\mathrm{H}_0 : \sigma^2 = 1.5$ against the hypothesis $\mathrm{H}_0 : \sigma^2 > 1.5$.

Solution:

a $\bar{x} = 16.605 \quad s^2 = \frac{5583.63 - 20(16.605)^2}{19} = 3.637\ldots$

b $\mathrm{H}_0 : \sigma^2 = 1.5 \quad \mathrm{H}_1 : \sigma^2 > 1.5$

Critical region > 30.144

Test statistic $= \frac{(n-1)s^2}{\sigma^2} = \frac{19 \times 3.637..}{1.5} = 46.072$

The test statistic is in the critical region

There is evidence to suggest $\sigma^2 > 1.5$

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Exercise E, Question 2

Question:

A random sample of 10 observations is taken from a normal distribution with variance σ^2 which is thought to be equal to 0.09. The results were as follows:

0.35, 0.42, 0.30, 0.26, 0.31, 0.30, 0.40, 0.33, 0.30, 0.40

Test, at the 0.025% level of significance, the hypothesis $H_0 : \sigma^2 = 0.09$ against the hypothesis $H_0 : \sigma^2 < 0.09$.

Solution:

$\bar{x} = 0.337 \qquad s^2 = 0.0028677\ldots$

$H_0 : \sigma^2 = 0.09 \qquad H_1 : \sigma^2 < 0.09$

Critical region < 2.700

$$\text{Test statistic} = \frac{(n-1)s^2}{\sigma^2} = \frac{9 \times 0.0028677\ldots}{0.09} = 0.287$$

The test statistic is in the critical region.

There is evidence to suggest that variance is less than 0.09.

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Exercise E, Question 3

Question:

The following random observations are taken from a normal distribution which is thought to have a variance of 4.1:

2.1, 2.3, 3.5, 4.6, 5.0, 6.4, 7.1, 8.6, 8.7, 9.1

Test, at the 5% significance level, the hypothesis $H_0 : \sigma^2 = 4.1$ against the hypothesis $H_1 : \sigma^2 \neq 4.1$

Solution:

$H_0 : \sigma^2 = 4.1 \quad H_1 : \sigma^2 \neq 4.1$

$\bar{x} = 5.74 \quad s^2 = 6.940\ldots$

Critical region < 2.7 and > 19.023

Test statistic $= \frac{(n-1)s^2}{\sigma^2} = \frac{9 \times 6.940\ldots}{4.1} = 15.235$

The test statistic is not in the critical region.

There is no evidence the variance does not equal 4.1.

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Exercise E, Question 4

Question:

It is claimed that the masses of a particular component produced in a small factory are normally distributed and have a mean mass of 10 g and a standard deviation of 1.12 g. A random sample of 20 such components was found to have a variance of 1.15 g.
Test, at the 5 % significance level, the hypothesis $H_0 : \sigma^2 = 1.12^2$ against the hypothesis $H_1 : \sigma^2 \neq 1.12^2$

Solution:

$H_0 : \sigma^2 = 1.12^2 \quad H_1 : \sigma^2 \neq 1.12^2$
Critical region < 8.907 and > 32.852

Test statistic $= \frac{(n-1)s^2}{\sigma^2} = \frac{19 \times 1.15}{1.12^2} = 17.419$
The test statistic is not in the critical region
There is no evidence the variance does not equal 1.12

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Exercise E, Question 5

Question:

Rollers for use in roller bearings are produced on a certain machine. The rollers are supposed be normally distributed and to have a mean diameter (μ) of 10 mm with a variance (σ^2) of 0.04 mm^2.

A random sample of 15 rollers from the machine have their diameters, x in millimetres, measured. The results are summarised below:

$\sum x = 149.941 \quad \sum x^2 = 1498.83$

a Calculate unbiased estimates for μ and σ^2.

b Test at the 5 % significance level,

i the hypothesis $\mu = 10$ against the hypothesis $\mu \neq 10$, using your estimate for σ^2 as the true variance of the population

ii the hypothesis $\sigma^2 = 0.04$ against the hypothesis $\sigma^2 \neq 0.04$

Solution:

a $\bar{x} = \dfrac{149.941}{15} = 9.996\ldots, s^2 = \dfrac{1498.83 - 15 \times 9.996\ldots^2}{14} = 0.0006977\ldots$

$s = 0.0264$

b **i** $H_0 : \mu = 10 \quad H_1 : \mu \neq 10$

Critical region < -2.145 and > 2.145.

Test statistic $= \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{9.996.. - 10}{\frac{0.0264\ldots}{\sqrt{15}}} = -0.587$ (or -0.577 of full calculator accuracy used)

The test statistic is not in the critical region

There is no evidence that the mean does not equal 10

ii $H_0 : \sigma^2 = 0.04 \quad H_1 : \sigma^2 \neq 0.04$

Critical region < 5.629 and > 26.119

Test statistic $= \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{14 \times 0.0006977}{0.04} = 0.244$

This is in the critical region

There is evidence that the variance is not 0.04

It is less than 0.04 which is good in this context because it means that there is very little variability.

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Exercise E, Question 6

Question:

The diameters of the eggs of the little gull are approximately normally distributed with mean 4.11 cm with a variance of 0.19 cm^2.

A sample of 8 little gulls eggs from a particular island which were measured had diameters in centimetres as follows:

4.4, 4.5, 4.1, 3.9, 4.4, 4.6, 4.5, 4.1

a Calculate an unbiased estimate for the variance of the population of little gull eggs on the island.

b Calculate an unbiased estimate of the mean diameter of the eggs and, test, at the 5% level, the hypothesis $\mu = 4.11$ against the hypothesis $\mu > 4.11$

c Test, at the 10% significance level, the hypothesis $\sigma^2 = 0.19$ against the hypothesis $\sigma^2 \neq 0.19$

Solution:

a $s^2 = 0.06125$

b $\bar{x} = 4.3125$

$H_0 : \mu = 4.11 \quad H_1 : \mu > 4.11$

Critical region > 1.895

$$\text{Test statistic} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{4.3125 - 4.11}{\frac{\sqrt{0.06125}}{\sqrt{8}}} = 2.3143$$

The test statistic is in the critical region

The mean weight is greater than 4.11

c $H_0 : \sigma^2 = 0.19 \quad H_1 : \sigma^2 \neq 0.19$

Critical region < 2.167 and > 14.067

$$\text{Test statistic} = \frac{(n-1)s^2}{\sigma^2} = \frac{7 \times 0.06125}{0.19} = 2.256$$

The test statistic is not in the critical region.

There is no evidence that σ^2 does not equal 0.19

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Exercise E, Question 7

Question:

Climbing rope produced by a certain manufacturer is known to have a mean tensile breaking strength (μ) of 170.2 kg and standard deviation 10.5 kg. The breaking strength of the rope is normally distributed.

A new component is added to the material which will, it is claimed, decrease the standard deviation without altering the tensile strength. A random sample of 20 pieces of the new rope is selected and each is tested to destruction. The tensile strength of each piece is noted. The results are used to calculate unbiased estimates of the mean strength and standard deviation of the population of new rope. These were found to be 172.4 kg and 8.5 kg.

a Test at the 5% level whether or not the variance has been reduced.

b What recommendation would you make to the manufacturer?

Solution:

a $H_0 : \sigma^2 = 110.25 \quad H_1 : \sigma^2 < 110.25$ $\quad \boxed{10.5^2 = 110.25}$

Critical region < 10.117

$$\text{Test statistic} = \frac{(n-1)s^2}{\sigma^2} = \frac{19 \times 8.5^2}{110.25} = 12.451$$

The test statistic is not in the not critical region

There is no evidence that the variance has reduced.

b Take a larger sample before committing to new component.

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Exercise F, Question 1

Question:

A random sample of 14 observations is taken from a normal distribution. The sample has a mean $\bar{x} = 30.4$ and a sample variance $s^2 = 36$.
It is suggested that the population mean is 28. Test this hypothesis at the 5% level of significance.

Solution:

$H_0 : \mu = 28 \quad H_1 : \mu \neq 28$

Critical region < -2.160 or > 2.160

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{30.4 - 28}{\frac{6}{\sqrt{14}}} = 1.4967$

The test statistic is not in the critical region
There is no evidence to suggest that μ does not equal 28

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Exercise F, Question 2

Question:

A random sample of 8 observations is taken from a random variable X that is normally distributed. The sample gave the following summary statistics

$\sum x^2 = 970.25 \quad \sum x = 85$

The population mean is thought to be 10. Test this hypothesis against the alternative hypothesis that the mean is greater than 10. Use the 5% level of significance.

Solution:

$H_0 : \mu = 10 \quad H_1 : \mu > 10$

Critical Region > 1.895

$$\bar{x} = \frac{85}{8} = 10.625$$

$$s^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{970.25 - 8 \times 10.625^2}{7} = 9.589\ldots$$

$$\text{Test statistic} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{10.625 - 10}{\sqrt{\frac{9.589..}{8}}} = 0.571$$

not critical – no evidence to suggest that $\mu > 10$

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Exercise F, Question 3

Question:

Six eggs selected at random from the daily output of a battery of hens had the following weights in grams.

55, 50, 53, 53, 52, 54

Calculate 95% confidence intervals for

a the mean,

b the variance of the population from which these eggs were taken.

c What assumption have you made about the distribution of the weights of eggs?

Solution:

$\bar{x} = 52.833\ldots \quad s = 1.722\ldots$

a confidence interval $= \left(\bar{x} - t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}, \bar{x} + t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}\right)$

$= \left(52.833\ldots - 2.571 \times \frac{1.722\ldots}{\sqrt{6}}, 52.833\ldots + 2.571 \times \frac{1.722\ldots}{\sqrt{6}}\right)$

$= (51.025, 54.641)$

b confidence interval $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)}\right) = \left(\frac{5 \times 1.722\ldots^2}{12.832}, \frac{5 \times 1.722\ldots^2}{0.831}\right)$

$= (1.156, 17.850)$

c They are normally distributed.

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Exercise F, Question 4

Question:

A sample of size 18 was taken from a random variable X which was normally distributed, producing the following summary statistics.

$\bar{x} = 9.8 \quad s^2 = 0.49$

Calculate 95% confidence intervals for

a the mean,

b the variance of the population.

Solution:

a confidence interval $= \left(\bar{x} - t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}, \bar{x} + t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}\right)$

$= \left(9.8 - 2.110 \times \frac{0.7}{\sqrt{18}}, 9.8 + 2.110 \times \frac{0.7}{\sqrt{18}}\right)$

$= (9.451, 10.148)$

b confidence interval $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1 - \frac{\alpha}{2}\right)}\right) = \left(\frac{17 \times 0.49}{30.191}, \frac{17 \times 0.49}{7.564}\right)$

$= (0.276, 1.101)$

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Exercise F, Question 5

Question:

A random sample of 14 observations was taken of a random variable X which was normally distributed. The sample had a mean $\bar{x} = 23.8$, and a variance $s^2 = 1.8$.
Calculate,

a a 95% confidence interval for the variance of the population,

b a 90% confidence interval for the variance of the population.

Solution:

a confidence interval $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)} \right) = \left(\frac{13\times1.8}{24.736}, \frac{13\times1.8}{5.009} \right)$

$= (0.946, 4.672)$

b confidence interval $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)} \right) = \left(\frac{13\times1.8}{22.362}, \frac{13\times1.8}{5.892} \right)$

$= (1.046, 3.971)$

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Exercise F, Question 6

Question:

A manufacturer claims that the lifetime of its batteries is normally distributed with mean 21.5 hours. A laboratory tests 8 batteries and finds the lifetimes of these batteries to be as follows:

19.7 18.4 22.2 20.8 16.9 25.3 23.2 21.1

Stating clearly your hypotheses, examine whether or not these lifetimes indicate that the batteries have a shorter mean lifetime than that claimed by the company. Use a 5% level of significance. **[E]**

Solution:

$\bar{x} = 20.95 \quad s = 2.674\ldots$

$H_0 : \mu = 21.5 \quad H_1 : \mu < 21.5$

critical region < -1.895

test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{20.95 - 21.5}{\frac{2.674\ldots}{\sqrt{8}}} = -0.5817$

The test statistic is not in the critical region

There is no evidence to reject claim.

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Exercise F, Question 7

Question:

A diabetic patient monitors his blood glucose in mmol/l at random times of the day over several days. The following is a random sample of the results for this patient.

5.1 5.8 6.1 6.8 6.2 5.1 6.3 6.6 6.1 7.9 5.8 6.5

Assuming the data to be normally distributed, calculate a 95% confidence interval for

a the mean of the population of blood glucose readings,

b the standard deviation of the population of blood glucose readings.

The level of blood glucose varies throughout the day according to the consumption of food and the amount of exercise taken during the day.

c Comment on the suitability of the patient's method of data collection. **[E]**

Solution:

$\bar{x} = 6.1916\ldots$ $s = 0.7549\ldots$ $s^2 = 0.5699\ldots$

a confidence interval $= \left(\bar{x} - t_{(n-1)}\left(\frac{\alpha}{2}\right)\times\frac{s}{\sqrt{n}}, \bar{x} + t_{(n-1)}\left(\frac{\alpha}{2}\right)\times\frac{s}{\sqrt{n}}\right)$

$= \left(6.1916\ldots - 2.201\times\frac{0.7549\ldots}{\sqrt{12}}, 6.1916\ldots + 2.201\times\frac{0.7549\ldots}{\sqrt{12}}\right)$

$= (5.712, 6.671)$

b confidence interval Var. $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)}\right)$

$= \left(\frac{11\times 0.5699\ldots}{21.920}, \frac{11\times 0.5699\ldots}{3.816}\right)$

$= (0.286, 1.643)$

confidence interval s.d. $= (0.535, 1.28)$

c He should measure his blood glucose at the same time each day.

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Exercise F, Question 8

Question:

A woollen mill produces scarves. The mill has several machines each operated by a different person. Jane has recently started working at the mill and the supervisor wishes to check the lengths of the scarves Jane is producing. A random sample of 20 scarves is taken and the length, x cm, of each scarf is recorded. The results are summarised as.

$\sum x = 1428, \quad \sum x^2 = 102\,286$

Assuming that the lengths of scarves produced by any individual follow a normal distribution,

a calculate a 95% confidence interval for the variance σ^2 of the lengths of scarves produced by Jane.

The mill's owners require that 90% of scarves should be within 10 cm of the mean length.

b Find the value of σ that would satisfy this condition.

c Explain whether or not the supervisor should be concerned about the scarves Jane is producing. **[E]**

Solution:

$$\bar{x} = \frac{1428}{20} = 71.4 \quad s^2 = \frac{102\ 286 - 20 \times 71.4^2}{19} = 17.2$$

a confidence interval $= \left(\dfrac{(n-1)s^2}{\chi^2_{n-1}\left(\dfrac{\alpha}{2}\right)}, \dfrac{(n-1)s^2}{\chi^2_{n-1}\left(1-\dfrac{\alpha}{2}\right)} \right) = \left(\dfrac{19 \times 17.2}{32.852}, \dfrac{19 \times 17.2}{8.907} \right)$

$= (9.948, 36.69)$

b $10 = 1.6449 \times \sigma$ so $\sigma = \dfrac{10}{1.6449} = 6.079$

c $\sqrt{36.69} < 6.079$ so the supervisor should not be concerned.

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Exercise F, Question 9

Question:

In order to discover the possible error in using a stop-watch, a student started the watch and stopped it again as quickly as she could. The times taken in centiseconds for 6 such attempts are recorded below:

10, 13, 14, 10, 13, 9

Assuming that the times are normally distributed, find 95% confidence limits for

a the mean,

b the variance. **[E]**

Solution:

$\bar{x} = 11.5 \qquad s = 2.073\ldots$

a $$\text{confidence interval} = \left(\bar{x} - t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}},\ \bar{x} + t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}\right)$$
$$= \left(11.5 - 2.571 \times \frac{2.073\ldots}{\sqrt{6}}, 11.5 + 2.571 \times \frac{2.073\ldots}{\sqrt{6}}\right)$$
$$= (9.324, 13.675)$$

b $$\text{confidence interval} = \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1 - \frac{\alpha}{2}\right)}\right) = \left(\frac{5 \times 2.073\ldots^2}{12.832}, \frac{5 \times 2.073\ldots^2}{0.831}\right)$$
$$= (1.675, 25.872)$$

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Exercise F, Question 10

Question:

A manufacturer claims that the car batteries which it produces have a mean lifetime of 24 months, with a standard deviation of 4 months. A garage selling the batteries doubts this claim and suggests that both values are in fact higher.
The garage monitors the lifetimes of 10 randomly selected batteries and finds that they have a mean lifetime of 27.2 months and a standard deviation of 5.2 months.
Stating clearly your hypotheses and using a 5% level of significance, test the claim made by the manufacturer for

a the standard deviation,

b the mean,

c State an assumption which has to be made when carrying out these tests. **[E]**

Solution:

a $H_0: \sigma = 4 \quad H_1: \sigma > 4$

Critical region $\chi^2 > 16.919$

$$\text{Test statistic} = \frac{(n-1)s^2}{\sigma^2} = \frac{9 \times 5.2^2}{4^2} = 15.21$$

The test statistic is not in the critical region.
standard deviation could be 4 months

b $H_0: \mu = 24 \quad H_1: \mu > 24$

Critical region $t > 1.833$

$$\text{Test statistic } t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{27.2 - 24}{\frac{5.2}{\sqrt{10}}} = 1.946$$

There is evidence to reject H_0
The mean battery life is greater than 24 months

c Lifetime is normally distributed.

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Exercise F, Question 11

Question:

The distance to 'take-off' from a standing start of an aircraft was measured on twenty occasions. The results are summarised in the following table.

Distance (m)	Frequency
700–	3
710–	5
720–	9
730–	2
740–750	1

Assuming that distance to 'take-off' is normally distributed, find 95% confidence intervals for

a the mean,

b the standard deviation.

It has been hypothesised that the mean distance to 'take-off' is 725 m.

c Comment on this hypothesis in the light of your interval from part **a**. **[E]**

Solution:

$\bar{x} = 721.5 \quad s = 10.399\ldots$

a confidence interval $= \left(\bar{x} - t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}, \bar{x} + t_{(n-1)}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}\right)$

$= \left(721.5 - 2.093 \times \frac{10.399\ldots.}{\sqrt{20}}, 721.5 + 2.093 \times \frac{10.399\ldots}{\sqrt{20}}\right)$

$= (717, 726)$

b confidence interval variance $= \left(\frac{(n-1)s^2}{\chi^2_{n-1}\left(\frac{\alpha}{2}\right)}, \frac{(n-1)s^2}{\chi^2_{n-1}\left(1-\frac{\alpha}{2}\right)}\right)$

$= \left(\frac{19 \times 10.399\ldots^2}{32.852}, \frac{19 \times 10.399\ldots^2}{8.907}\right)$

$= (62.553, 230.717)$

confidence interval standard deviation $= (7.909, 15.189)$

c 725 within confidence interval,
There is no evidence to reject this hypothesis.

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Exercise F, Question 12

Question:

The maximum weight that 50 cm lengths of a certain make of string can hold before breaking (the breaking strain) has a normal distribution with mean 40 kg and standard deviation 5 kg. The manufacturer of the string has developed a new process which should increase the mean breaking strain of the string but should not alter the standard deviation. Ten randomly selected pieces of string are tested and their breaking strains, in kg, are:

51, 48, 37, 46, 36, 53, 34, 49, 47, 50

a Stating your hypotheses clearly test, at the 5% level of significance, whether or not the new process has altered the variance.

In the light of your conclusion to the test in part **a**,

b test whether or not there is evidence that the new process has increased the mean breaking strain. State your hypotheses clearly and use a 5% level of significance.

c Explain briefly your choice of test in part **b**. **[E]**

Solution:

$\bar{x} = 45.1 \quad s = 6.838\ldots$

$\mathrm{H}_0 : \sigma = 5 \quad \mathrm{H}_1 : \sigma \neq 5$

a Critical region > 19.023 and < 2.700

Test statistic $= \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{9 \times 6.838\ldots^2}{5^2} = 16.836$

There is insufficient evidence to reject H_0

$\sigma = 5$

b Critical region $z > 1.6449$

Test statistic $z = \dfrac{X - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{45.1 - 40}{\frac{5}{\sqrt{10}}} = 3.225$

The test statistic is in the critical region.

There is evidence to suggest there is an increase in breaking strain.

c In **a** there was no change in σ so assume $\sigma = 5 \therefore$ use z not t

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Exercise F, Question 13

Question:

A company knows from previous experience that the time taken by maintenance engineers to repair a particular electrical fault on a complex piece of electrical equipment is 3.5 hours on average with a standard deviation of 0.5 hours.

A new method of repair has been devised, but before converting to this new method the company took a random sample of 10 of its engineers and each engineer carried out a repair using the new method. The time, x hours, it took each of them to carry out the repair was recorded and the data are summarised below:

$\sum x = 34.2 \quad \sum x^2 = 121.6$

Assume that the data can be regarded as a random sample from a normal population.

a For the new repair method, calculate an unbiased estimate of the variance.

b Use your estimate from **a** to calculate for the new repair method a 95% confidence interval for

  i the mean,

  ii the standard deviation.

c Use your calculations and the given data to compare the two repair methods in order to advise the company as to which method to use.

d Suggest an alternative way of comparing the two methods of repair using the 10 randomly chosen engineers. **[E]**

Solution:

a $\bar{x} = \frac{34.2}{10} = 3.42 \quad s^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{121.6 - 10 \times 3.42^2}{9} = 0.5151\ldots$

b i confidence interval mean $= \left(\bar{x} - t_{n-1}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}, \bar{x} + t_{n-1}\left(\frac{\alpha}{2}\right) \times \frac{s}{\sqrt{n}}\right)$

$= (3.42 - 2.262 \times \frac{0.7177\ldots}{\sqrt{10}}, 3.42 + 2.262 \times \frac{0.7177\ldots}{\sqrt{10}})$

$= (2.906, 3.933)$

ii confidence interval variance $= \left(\frac{(n-1)s^2}{\chi_9^2(0.025)}, \frac{(n-1)s^2}{\chi_9^2(0.975)}\right)$

$= \left(\frac{9 \times 0.515\ldots}{19.023}, \frac{9 \times 0.515\ldots}{2.700}\right) = (0.244, 1.717)$

confidence interval standard deviation $= (0.4937, 1.3103)$

c 3.5 hours is inside the confidence interval on the mean, so there is no evidence of a change in the mean time.
0.5 hours is inside the confidence interval on the standard deviation so there is no evidence of a change in the variability of the time.
There is no reason to change the repair method.

d Use a 'matched pairs' experiment, getting each engineer to carry out a similar repair using the old method and the new method and use a paired t-test.