Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise A, Question 1

Question:

The random variable $H \sim \mathrm{N}(\mu, \sigma^2)$ represents the height of a variety of flower where μ, σ^2 are unknown population parameters.

A random sample of 5 flowers of this variety are measured and their height, in cm, is given below.

$h_1 = 35.1, h_2 = 32.3, h_3 = 34.5, h_4 = 37.4, h_5 = 32.8$

Determine which of the following are statistics.

a $\sum_{i=1}^{5}(X_i - \mu)$

b $\sum_{i=1}^{5}\frac{(X_i - \bar{X})^2}{4}$

c $\sum\left|\frac{X_i - \mu}{\sigma}\right|$

d $X_1 - X_5$

Solution:

a Not a statistic since μ not known

b Is a statistic – no unknown parameters

c Not a statistic since μ, σ not known

d Is a statistic – no unknown parameters

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise A, Question 2

Question:

A random sample of 6 apples are weighed and their weights, x_i g, are recorded

$x_1 = 168, x_2 = 185, x_3 = 161, x_4 = 172, x_5 = 187, x_6 = 176$

Calculate the values of the following statistics.

a $\dfrac{X_6 + X_1}{2}$

b $\displaystyle\sum_{i=1}^{6} \frac{\left(X_i - \bar{X}\right)^2}{6}$

c $\dfrac{\displaystyle\sum_{i=1}^{6} X_i^2}{\displaystyle\sum_{i=1}^{6} X_i}$

Solution:

a $\dfrac{X_6 + X_1}{2} = \dfrac{176+168}{2} = 172$

b $\bar{X} = \dfrac{168+185+\cdots+176}{6} = \dfrac{1049}{6} = 174.83$

$$\sum \frac{\left(X_i - \bar{X}\right)^2}{6} = 83.138\cdots \quad \left(\sigma x^2 \text{ on a calculator}\right)$$
$$= 83.1 \text{ (3 s.f.)}$$

c

$$\frac{\sum X_i^2}{\sum X_i} = \frac{183\,899}{1049}$$
$$= 175.308\cdots$$
$$= 175 \text{ (3 s.f.)}$$

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Estimation, confidence intervals and tests
Exercise A, Question 3

Question:

The lengths of nails produced by a certain machine are normally distributed with a mean μ and standard deviation σ. A random sample of 10 nails is taken and their lengths $\{X_1, X_2, X_3, \ldots, X_{10}\}$ are measured.

i Write down the distributions of the following:

a $\sum_1^{10} X_i$

b $\dfrac{2X_1 + 3X_{10}}{5}$

c $\sum_1^{10}(X_i - \mu)$

d $\bar{X}$

e $\sum_1^{5} X_i - \sum_6^{10} X_i$

f $\sum_1^{10}\left(\dfrac{X_i - \mu}{\sigma}\right)$

ii State which of the above are statistics.

Solution:

i **a** $\sum X_i \sim \mathrm{N}(10\mu, 10\sigma^2)$

b $\dfrac{2X_1 + 3X_{10}}{5} \sim \mathrm{N}\left(\mu, \dfrac{13}{25}\sigma^2\right), \quad \dfrac{13}{25} = \dfrac{2^2 + 3^2}{5^2}$

c $\mathrm{E}(X_i - \mu) = 0 \quad \mathrm{Var}(X_i - \mu) = \mathrm{Var}(X_i) = \sigma^2$

$\therefore \sum(X_i - \mu) \sim \mathrm{N}(0, 10\sigma^2)$

d $\bar{X} = \dfrac{\sum X_i}{n} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{10}\right) \quad (n = 10)$

e $\sum_{i=1}^{5} X_i - \sum_{i=6}^{10} X_i \sim \mathrm{N}(5\mu, 5\sigma^2) - \mathrm{N}(5\mu, 5\sigma^2)$

$\therefore$ combined distribution $\sim \mathrm{N}(0, 10\sigma^2)$

[Remember $\mathrm{Var}(X - Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$]

f $\dfrac{X_i - \mu}{\sigma} \sim \mathrm{N}(0, 1^2) \quad \therefore \sum\left(\dfrac{X_i - \mu}{\sigma}\right) \sim \mathrm{N}(0, 10)$

ii **a**, **b**, **d**, **e** are statistics since they do not contain μ or σ, the unknown population parameters.

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Estimation, confidence intervals and tests
Exercise A, Question 4

Question:

A large bag of coins contains 1p, 5p and 10p coins in the ratio 2:2:1.

a Find the mean μ and the variance σ^2 for the value of coins in this population.

A random sample of two coins is taken and their values X_1 and X_2 are recorded.

b List all possible samples.

c Find the sampling distribution for the mean $\bar{X} = \dfrac{X_1 + X_2}{2}$.

d Hence show that $\mathrm{E}(\bar{X}) = \mu$ and $\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{2}/2$

Solution:

a $X =$ value of a coin.

x	1	5	10
$\mathrm{P}(X = x)$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{5}$

$$\therefore \mu = \mathrm{E}(X) = \frac{2}{5} + \frac{10}{5} + \frac{10}{5} = \frac{22}{5} \text{ or } 4.4$$

$$\mathrm{E}(X^2) = 1^2 \times \frac{2}{5} + 25 \times \frac{2}{5} + 100 \times \frac{1}{5} = \frac{152}{5}$$

$$\therefore \sigma^2 = \mathrm{E}(X^2) - \mu^2 = \frac{152}{5} - \frac{22^2}{25} = 11.04 \text{ or } \frac{276}{25}$$

b $\{1, 1\}$ $\{1, 5\}^{\times 2}$ $\{1, 10\}^{\times 2}$

$\{5, 5\}$ $\{5, 10\}^{\times 2}$

$\{10, 10\}$

c

$\bar{x}$	1	3	5	5.5	7.5	10
$\mathrm{P}(\bar{X} = \bar{x})$	$\frac{4}{25}$	$\frac{8}{25}$	$\frac{4}{25}$	$\frac{4}{25}$	$\frac{4}{25}$	$\frac{1}{25}$

$$\left[\begin{aligned} \text{e.g. } \mathrm{P}(\bar{X} = 5.5) &= \mathrm{P}(X_i = (nX_2 = 10) + \mathrm{P}(X_i = 10n\ X_2 = 1) \\ &= \frac{2}{5} \times \frac{1}{5} + \frac{1}{5} \times \frac{2}{5} \quad = \frac{4}{25} \end{aligned}\right]$$

d $\mathrm{E}(\bar{X}) = 1 \times \dfrac{4}{25} + 3 \times \dfrac{8}{25} + \cdots + 10 \times \dfrac{1}{25} = 4.4 = \mu$

$$\mathrm{Var}(\bar{X}) = 1^2 \times \frac{4}{25} + 3^2 \times \frac{8}{25} + \cdots + 10^2 \times \frac{1}{25} - 4.4^2 = 5.52 = \frac{\sigma^2}{2}$$

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise B, Question 1

Question:

Find unbiased estimates of the mean and variance of the populations from which the following random samples have been taken:

a 21.3; 19.6; 18.5; 22.3; 17.4; 16.3; 18.9; 17.6; 18.7; 16.5; 19.3; 21.8; 20.1; 22.0

b 1; 2; 5; 1; 6; 4; 1; 3; 2; 8; 5; 6; 2; 4; 3; 1

c 120.4; 230.6; 356.1; 129.8; 185.6; 147.6; 258.3; 329.7; 249.3

d 0.862; 0.754; 0.459; 0.473; 0.493; 0.681; 0.743; 0.469; 0.538; 0.361.

Solution:

Unbiased estimate of mean $= \bar{x} = \dfrac{\sum x}{n}$

Unbiased estimate of variance $= S^2 = \dfrac{\sum x^2 - n\bar{x}^2}{n-1}$

a $\sum x = 270.3, \sum x^2 = 5270.49, n = 14$

$\therefore\ \bar{x} = 19.3,\ S^2 = 3.98$

b $\sum x = 54, \sum x^2 = 252, n = 16$

$\therefore\ \bar{x} = 3.375,\ S^2 = 4.65$

c $\sum x = 2007.4, \sum x^2 = 505\,132.36, n = 9$

$\therefore\ \bar{x} = 223,\ S^2 = 7174$

d $\sum x = 5.833, \sum x^2 = 3.644555, n = 10$

$\therefore\ \bar{x} = 0.5833,\ S^2 = 0.0269$

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Estimation, confidence intervals and tests

Exercise B, Question 2

Question:

Find unbiased estimates of the mean and the variance of the populations from which random samples with the following summaries have been taken.

a $n = 120$ $\Sigma x = 4368$ $\Sigma x^2 = 162\,466$

b $n = 30$ $\Sigma x = 270$ $\Sigma x^2 = 2546$

c $n = 1037$ $\Sigma x = 1140.7$ $\Sigma x^2 = 1278.08$

d $n = 15$ $\Sigma x = 168$ $\Sigma x^2 = 1913$

Solution:

a $\bar{x} = \frac{\sum x}{n} = \frac{4368}{120} = 36.4$

$$S^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{162\,466 - 120 \times 36.4^2}{119} = 29.166\ldots$$

$$= 29.2 \text{ (3 s.f.)}$$

b $\bar{x} = \frac{\sum x}{n} = \frac{270}{30} = 9$

$$S^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{2546 - 30 \times 9^2}{29} = 4$$

c $\bar{x} = \frac{\sum x}{n} = \frac{1140.7}{1037} = 1.1$

$$S^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{1278.08 - 1037 \times 1.1^2}{1036} = 0.0225$$

d $\bar{x} = \frac{\sum x}{n} = \frac{168}{15} = 11.2$

$$S^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{1913 - 15 \times 11.2^2}{14} = 2.24285\ldots$$

$$= 2.24 \text{ (3 s.f.)}$$

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Estimation, confidence intervals and tests
Exercise B, Question 3

Question:

The concentrations, in mg per litre, of a trace element in 7 randomly chosen samples of water from a spring were:
240.8 237.3 236.7 236.6 234.2 233.9 232.5.
Determine unbiased estimates of the mean and the variance of the concentration of the trace element per litre of water from the spring.

Solution:

$$\sum x = 1652, \sum x^2 = 389\,917.48, n = 7$$

$$\therefore \bar{x} = \frac{1652}{7} = 236$$

$$S^2 = \frac{389\,917.48 - 7 \times 236^2}{6}$$

$$= 7.58$$

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Estimation, confidence intervals and tests
Exercise B, Question 4

Question:

Cartons of orange are filled by a machine. A sample of 10 cartons selected at random from the production contained the following quantities of orange (in ml).

201.2	205.0	209.1	202.3	204.6
206.4	210.1	201.9	203.7	207.3

Calculate unbiased estimates of the mean and variance of the population from which this sample was taken.

Solution:

$$\begin{aligned}
\sum x &= 2051.6, \sum x^2 = 420\,989.26, n = 10 \\
\bar{x} &= 205.16 = 205 \text{ (3 s.f.)} \\
S^2 &= \frac{420\,989.26 - 10 \times \bar{x}^2}{9} \\
&= 9.22266\ldots = 9.22 \text{ (3 s.f.)}
\end{aligned}$$

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Estimation, confidence intervals and tests
Exercise B, Question 5

Question:

A manufacturer of self-assembly furniture required bolts of two lengths, 5 cm and 10 cm, in the ratio 2 : 1 respectively.

a Find the mean μ and the variance σ^2 for the lengths of bolts in this population.

A random sample of three bolts is selected from a large box containing bolts in the required ratio.

b List all possible samples.

c Find the sampling distribution for the mean $\bar{X}$.

d Hence find $\mathrm{E}(\bar{X})$ and $\mathrm{Var}(\bar{X})$.

e Find the sampling distribution for the mode M.

f Hence find $\mathrm{E}(M)$ and $\mathrm{Var}(M)$.

g Find the bias when M is used as an estimator of the population mode.

Solution:

$X = $ length of a bolt

x	5	10
$P(X = x)$	$\frac{2}{3}$	$\frac{1}{3}$

a $\mu = 5 \times \frac{2}{3} + 10 \times \frac{1}{3} = \frac{20}{3}$

$\sigma^2 = 25 \times \frac{2}{3} + 100 \times \frac{1}{3} - \left(\frac{20}{3}\right)^2 = \frac{50}{9}$

b $\{5, 5, 5\}$

$\{5, 5, 10\}^{\times 3}$ $\{5, 10, 10\}^{\times 3}$

$\{10, 10, 10\}$

c

$\bar{x}$	5	$\frac{20}{3}$	$\frac{25}{3}$	10
$P(\bar{X} = \bar{x})$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$

d $E(\bar{X}) = 5 \times \frac{8}{27} + \frac{20}{3} \times \frac{12}{27} + \cdots + 10 \times \frac{1}{27} = \frac{20}{3} = \mu$

$\text{Var}(\bar{X}) = 5^2 \times \frac{8}{27} + \cdots + 10^2 \times \frac{1}{27} - \left(\frac{20}{3}\right)^2 = \frac{50}{27} = \frac{\sigma^2}{3}$

e

m	5	10
$P(M = m)$	$\frac{20}{27}$	$\frac{7}{27}$

$\boxed{P(M = 10) \text{ is cases } \{5, 10, 10\} \text{ and } \{10, 10, 10\}}$

f $E(M) = 5 \times \frac{20}{27} + 10 \times \frac{7}{27} = \frac{170}{27} = 6.296\ldots$

$\text{Var}(M) = 25 \times \frac{20}{27} + 100 \times \frac{7}{27} - \left(\frac{170}{27}\right)^2 = \frac{3500}{729} = 4.80\ldots$

g $\text{Bias} = E(M) - 5 = 1.296\ldots = 1.30 \text{ (3 s.f.)}$

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Estimation, confidence intervals and tests
Exercise B, Question 6

Question:

A biased six-sided die has probability p of landing on a six.
Every day, for a period of 25 days, the die is rolled 10 times and the number of sixes X is recorded giving rise to a sample $X_1, X_2, \ldots, X_{25}$.

a Write down $\mathrm{E}(X)$ in terms of p.

b Show that the sample mean $\bar{X}$ is a biased estimator of p and find the bias.

c Suggest a suitable unbiased estimator of p.

Solution:

$X \sim B(10, p)$

a $\mathrm{E}(X) = np = 10p$

b $\bar{X} = \dfrac{X_1 + \ldots + X_{25}}{25}$

$\mathrm{E}(\bar{X}) = \dfrac{\mathrm{E}(X_1) + \mathrm{E}(X_2) + \ldots + \mathrm{E}(X_{25})}{25} = \dfrac{\mu + \mu + \ldots + \mu}{25} = \dfrac{25\mu}{25} = \mu$

$\therefore \bar{X}$ is an unbiased estimator of μ

But $\mathrm{E}(\bar{X}) = 10p \quad \therefore \bar{X}$ is a *biased* estimator of p.

so bias $= 10p - p = 9p$

c $\mathrm{E}\left(\dfrac{\bar{X}}{10}\right) = \dfrac{1}{10}\mathrm{E}(\bar{X}) = p$

$\therefore \dfrac{\bar{X}}{10}$ is an unbiased estimator of p.

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Estimation, confidence intervals and tests
Exercise B, Question 7

Question:

The random variable $X \sim \mathrm{U}[-\alpha, \alpha]$

a Find $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$.

A random sample X_1, X_2, X_3 is taken and the statistic $Y = X_1^2 + X_2^2 + X_3^2$ is calculated.

b Show that Y is an unbiased estimator of α^2.

Solution:

$X \sim \mathrm{U}[-\alpha, \alpha]$

a $\mathrm{E}(X) = \dfrac{-\alpha + \alpha}{2} = 0$

$$\mathrm{Var}(X) = \frac{(\alpha - (-\alpha))^2}{12} = \frac{4\alpha^2}{12} = \frac{\alpha^2}{3}$$

μ and σ^2 for $\mathrm{U}[a, b]$ are given in formula booklet under S2.

$$\therefore \mathrm{E}(X^2) = \mathrm{Var}(X) + [\mathrm{E}(X)]^2$$

$$\therefore \mathrm{E}(X^2) = \frac{\alpha^2}{3} + 0 = \frac{\alpha^2}{3}$$

b $Y = X_1^2 + X_2^2 + X_3^2$

$$\mathrm{E}(Y) = \mathrm{E}(X_1^2) + \mathrm{E}(X_2^2) + \mathrm{E}(X_3^2)$$

$$= \frac{\alpha^2}{3} \times 3 = \alpha^2$$

$\therefore$ Y is an unbiased estimator of α^2.

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Estimation, confidence intervals and tests
Exercise C, Question 1

Question:

John and Mary each independently took a random sample of sixth-formers in their college and asked them how much money, in pounds, they earned last week. John used his sample of size 20 to obtain unbiased estimates of the mean and variance of the amount earned by a sixth-former at their college last week. He obtained values of $\bar{x} = 15.5$ and $S_x^2 = 8.0$.

Mary's sample of size 30 can be summarised as $\Sigma y = 486$ and $\Sigma y^2 = 8222$.

a Use Mary's sample to find unbiased estimates of μ and σ^2.

b Combine the samples and use all 50 observations to obtain further unbiased estimates of μ and σ^2.

c Find the standard error of the mean for each of these estimates of μ.

d Comment on which estimate of μ you would prefer to use.

Solution:

a $\bar{y} = \frac{486}{30} = 16.2$

$$S_y^2 = \frac{8222 - 30 \times 16.2^2}{29} = 12.0275\ldots = 12.0 \text{ (3 s.f.)}$$

b Let $\sum w = \sum x + \sum y$

$\bar{x} = 15.5 \Rightarrow \sum x = 15.5 \times 20 = 310$

$\therefore \sum w = 796$

$S_x^2 = 8.0 \Rightarrow \sum x^2 = 8 \times 19 + 20 \times 15.5^2 = 4957$

$\therefore \sum w^2 = 13179$

$\therefore \bar{w} = \frac{796}{50} = 15.92$

$$S_w^2 = \frac{13179 - 50 \times 15.92^2}{49} = 10.340\ldots = 10.34$$

c Standard error of the mean is $\frac{S}{\sqrt{n}}$

$$\frac{S_x}{\sqrt{20}} = 0.632 \text{ (3 s.f.)}, \frac{S_y}{\sqrt{30}} = 0.633 \text{ (3 s.f.)}, \frac{S_w}{\sqrt{50}} = 0.455 \text{ (3 s.f.)}$$

d Prefer to use $\bar{w}$ since it is based on a larger sample size and has smallest standard error.

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Estimation, confidence intervals and tests
Exercise C, Question 2

Question:

A machine operator checks a random sample of 20 bottles from a production line in order to estimate the mean volume of bottles (in cm^3) from this production run. The 20 values can be summarised as $\Sigma x = 1300$ and $\Sigma x^2 = 84\,685$.

a Use this sample to find unbiased estimates of μ and σ^2

A supervisor knows from experience that the standard deviation of volumes on this process, σ, should be $3\,\text{cm}^3$ and he wishes to have an estimate of μ that has a standard error of less than $0.5\,\text{cm}^3$.

b What size sample will he need to achieve this?

The supervisor takes a further sample of size 16 and finds $\Sigma x = 1060$.

c Combine the two samples to obtain a revised estimate of μ.

Solution:

a $\bar{x} = \dfrac{1300}{20} = 65$

$$S_x^2 = \frac{84\,685 - 20 \times 65^2}{19} = 9.7368\cdots = 9.74 \text{ (3 s.f.)}$$

b $\dfrac{\sigma}{\sqrt{n}} < 0.5 \Rightarrow \dfrac{3}{\sqrt{n}} < 0.5$

$6 < \sqrt{n}$

$n > 36$

$\therefore$ need a sample of 37 or more

c Let y = combined sample

$\sum y = 1300 + 1060$

$\sum y = 2360 \qquad n = 36$

$\therefore \bar{y} = \dfrac{2360}{36} = 65.555\ldots$ or 65.6 (3 s.f.)

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise C, Question 3

Question:

The heights of certain seedlings after growing for 10 weeks in a greenhouse have a standard deviation of 2.6 cm. Find the smallest sample that must be taken for the standard error of the mean to be less than 0.5 cm.

Solution:

$$\frac{\sigma}{\sqrt{n}} < 0.5$$

$\sigma = 2.6 \Rightarrow \quad \sqrt{n} > 2.6 \times 2 = 5.2$

i.e. $n > 27.04$

So need a sample of 28 (or more)

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise C, Question 4

Question:

The hardness of a plastic compound was determined by measuring the indentation produced by a heavy pointed device.
The following observations in tenths of a millimetre were obtained:
4.7, 5.2, 5.4, 4.8, 4.5, 4.9, 4.5, 5.1, 5.0, 4.8.

a Estimate the mean indentation for this compound.

b Estimate the standard error of the mean.

c Estimate the size of sample required in order that in future the standard error of the mean should be just less than 0.05.

Solution:

Let $x =$ indentation

$\sum x = 48.9, \quad \sum x^2 = 239.89, n = 10$

a $\hat{\mu} = \bar{x} = \dfrac{48.9}{10} = 4.89$

b $\hat{\sigma}^2 = S^2 = \dfrac{239.89 - 10 \times 4.89^2}{9} = 0.08544...$

$\dfrac{S}{\sqrt{n}} \quad = \quad 0.092436... = 0.0924 \text{ (3 s.f.)}$

c Require $\dfrac{S}{\sqrt{n}} = \dfrac{0.2923...}{\sqrt{n}} < 0.05$

$\Rightarrow \quad \sqrt{n} > 5.846...$

$n > 34.17...$

$\therefore$ need $n = 35$ (or more)

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise C, Question 5

Question:

Prospective army recruits receive a medical test. The probability of each recruit passing the test is p, independent of any other recruit. The medicals are carried out over two days and on the first day n recruits are seen and on the next day $2n$ are seen. Let X_1 be the number of recruits who pass the test on the first day and let X_2 be the number passing on the second day.

a Write down $\mathrm{E}(X_1), \mathrm{E}(X_2), \mathrm{Var}(X_1)$ and $\mathrm{Var}(X_2)$.

b Show that $\frac{X_1}{n}$ and $\frac{X_2}{2n}$ are both unbiased estimates of p and state, giving a reason, which you would prefer to use.

c Show that $X = \frac{1}{2}\left(\frac{X_1}{n} + \frac{X_2}{2n}\right)$ is an unbiased estimator of p.

d Show that $Y = \left(\frac{X_1 + X_2}{3n}\right)$ is an unbiased estimator of p.

e Which of the statistics $\frac{X_1}{n}$, $\frac{X_2}{2n}$, X or Y is the best estimator of p?

The statistic $T = \left(\frac{2X_1 + X_2}{3n}\right)$ is proposed as an estimator of p.

f Find the bias.

Solution:

$X_1 \sim \mathrm{B}(n, p) \qquad X_2 \sim \mathrm{B}(2n, p)$

a $\mathrm{E}(X_1) = np, \mathrm{E}(X_2) = 2np, \mathrm{Var}(X_1) = np(1-p), \mathrm{Var}(X_2) = 2np(1-p)$

b $\mathrm{E}\left(\dfrac{X_1}{n}\right) = \dfrac{\mathrm{E}(X_1)}{n} = \dfrac{np}{n} = p \therefore \dfrac{X_1}{n}$ is unbiased estimator of p

$\mathrm{E}\left(\dfrac{X_2}{2n}\right) = \dfrac{\mathrm{E}(X_2)}{2n} = \dfrac{2np}{2n} = p \therefore \dfrac{X_2}{2n}$ is unbiased estimator of p

Prefer $\dfrac{X_2}{2n}$ since based on a larger sample (and therefore will have smaller variance)

c
$$X = \frac{1}{2}\left(\frac{X_1}{n} + \frac{X_2}{2n}\right) \Rightarrow \mathrm{E}(X) = \frac{1}{2}\left[\frac{\mathrm{E}(X_1)}{n} + \frac{\mathrm{E}(X_2)}{2n}\right]$$
$$= \frac{1}{2}\left[\frac{np}{n} + \frac{2np}{2n}\right]$$
$$= \frac{1}{2}[p + p] = p$$

$\therefore X$ is an unbiased estimator of p

d $Y = \left(\dfrac{X_1 + X_2}{3n}\right) \Rightarrow \mathrm{E}(Y) = \dfrac{\mathrm{E}(X_1) + \mathrm{E}(X_2)}{3n} = \dfrac{np + 2np}{3n} = p$

$\therefore Y$ is an unbiased estimator of p

e $\mathrm{Var}\left(\dfrac{X_1}{n}\right) = \dfrac{1}{n^2} \quad \mathrm{Var}\,(X_1) = \dfrac{np(1-p)}{n^2} = \dfrac{p(1-p)}{n}$

$\mathrm{Var}\left(\dfrac{X_2}{2n}\right) = \dfrac{1}{4n^2} \quad \mathrm{Var}\,(X_2) = \dfrac{2np(1-p)}{4n^2} = \dfrac{p(1-p)}{2n}$

$$\mathrm{Var}(X) = \frac{1}{4}\left[\mathrm{Var}\left(\frac{X_1}{n}\right) + \mathrm{Var}\left(\frac{X_2}{2n}\right)\right] = \frac{1}{4}\left[\frac{p(1-p)}{n} + \frac{p(1-p)}{2n}\right] = \frac{3p(1-p)}{8n}$$

$$\mathrm{Var}(Y) = \frac{1}{9n^2}\left[\mathrm{Var}\,(X_1) + \mathrm{Var}\,(X_2)\right] = \frac{1}{9n^2}[np(1-p) + 2np(1-p)]$$

$$\mathrm{Var}(Y) = \frac{3np(1-p)}{9n^2} = \frac{p(1-p)}{3n}$$

$\therefore$ Var (Y) is smallest so Y is the best estimator.

f $T = \left(\dfrac{2X_1 + X_2}{3n}\right)$

$$\mathrm{E}(T) = \frac{2\mathrm{E}(X_1) + \mathrm{E}(X_2)}{3n} = \frac{2np + 2np}{3n} = \frac{4p}{3}$$

$$\text{bias} = \mathrm{E}(T) - p$$
$$= \frac{p}{3}$$

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Estimation, confidence intervals and tests
Exercise C, Question 6

Question:

Two independent random samples $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ are taken from a population with mean μ and variance σ^2. The unbiased estimators $\bar{X}$ and $\bar{Y}$ of μ are calculated. A new unbiased estimator T of μ is sought of the form $T = r\bar{X} + s\bar{Y}$.

a Show that, since T is unbiased, $r + s = 1$.

b By writing $T = r\bar{X} + (1-r)\bar{Y}$, show that

$$\text{Var}(T) = \sigma^2\left[\frac{r^2}{n} + \frac{(1-r)^2}{m}\right]$$

c Show that the minimum variance of T is when $r = \dfrac{n}{n+m}$.

d Find the best (in the sense of minimum variance) estimator of μ of the form $r\bar{X} + s\bar{Y}$.

Solution:

$$\text{E}(\bar{X}) = \mu \quad \text{Var}(\bar{X}) = \frac{\sigma^2}{n}$$

$$\text{E}(\bar{Y}) = \mu \quad \text{Var}(\bar{Y}) = \frac{\sigma^2}{m}$$

a $T = r\bar{X} + s\bar{Y}$

$\text{E}(T) = r\mu + s\mu = (r+s)\mu$

So if T is unbiased $r + s = 1$

b $r + s = 1 \Rightarrow s = 1 - r$

$\therefore T = r\bar{X} + (1-r)\bar{Y}$

$$\text{Var}(T) = r^2\text{Var}(\bar{X}) + (1-r)^2\text{Var}(\bar{Y}) = r^2\frac{\sigma^2}{n} + (1-r)^2\frac{\sigma^2}{m}$$

$$= \sigma^2\left[\frac{r^2}{n} + \frac{(1-r)^2}{m}\right]$$

c $$\frac{\text{d}}{\text{d}r}\text{Var}(T) = \sigma^2\left[\frac{2r}{n} + \frac{2(1-r)\times(-1)}{m}\right]$$

$\therefore$ Var (T) is a quadratic function of r with positive r^2 term $\therefore$ min

$$\frac{\text{d}}{\text{d}r}\text{Var}(T) = 0 \Rightarrow rm = (1-r)n \quad \text{or} \quad r(m+n) = n \text{ or } r = \frac{n}{m+n}$$

d Best estimator of form T is

$$T = \frac{n}{m+n}\bar{X} + \frac{m}{m+n}\bar{Y} \quad \text{or} \quad \frac{n\bar{X} + m\bar{Y}}{m+n}$$

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise C, Question 7

Question:

A large bag of counters has 40% with the number 0 on, 40% with the number 2 on and 20% with the number 1.

a Find the mean μ, and the variance σ^2, for this population of counters.

A random sample of size 3 is taken from the bag.

b List all possible samples.

c Find the sampling distribution for the mean $\bar{X}$.

d Find $\mathrm{E}(\bar{X})$ and $\mathrm{Var}(\bar{X})$.

e Find the sampling distribution for the median N.

f Hence find $\mathrm{E}(N)$ and $\mathrm{Var}(N)$.

g Show that N is an unbiased estimator of μ.

h Explain which estimator, $\bar{X}$ or N, you would choose as an estimator of μ.

Solution:

Let X = number on a counter.

x	0	1	2
$\mathrm{P}(X=x)$	0.4	0.2	0.4

a $\mu = 1$ (by symmetry)

$\sigma^2 = 0 + 1\times 0.2 + 2^2 \times 0.4 - 1 = 0.8$ or $\frac{4}{5}$

b $\{0,0,0\}$ $\{0,0,1\}^{\times 3}$ $\{0,0,2\}^{\times 3}$

$\{1,1,1\}$ $\{1,1,0\}^{\times 3}$ $\{1,1,2\}^{\times 3}$

$\{2,2,2\}$ $\{2,2,0\}^{\times 3}$ $\{2,2,1\}^{\times 3}$ $\{0,1,2\}^{\times 3!=6}$

c

$\bar{x}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$\mathrm{P}(\bar{X}=\bar{x})$	$\frac{8}{125}$	$\frac{12}{125}$	$\frac{30}{125}$	$\frac{25}{125}$	$\frac{30}{125}$	$\frac{12}{125}$	$\frac{8}{125}$

d $\mathrm{E}(\bar{X}) = 1$ (by symmetry) $(= \mu)$

$$\mathrm{Var}(\bar{X}) = 0 + \frac{1}{9}\times\frac{12}{125} + \frac{4}{9}\times\frac{30}{125} + \cdots + 4\times\frac{8}{125} - 1^2$$

$$= \frac{4}{15} \qquad \left(= \frac{\sigma^2}{3}\right)$$

e

n	0	1	2
$\mathrm{P}(N=n)$	$\frac{44}{125}$	$\frac{37}{125}$	$\frac{44}{125}$

e.g. $\mathrm{P}(N=2)$ is cases $\{2,2,2\};\{2,2,0\};\{2,2,1\}$

f $\mathrm{E}(N) = 1$ (by symmetry)

$$\mathrm{Var}(N) = 0 + 1^2\times\frac{37}{125} + 2^2\times\frac{44}{125} - 1^2 = \frac{88}{125} \quad (= \sigma^2)$$

g $\because \mathrm{E}(N) = 1 = \mu \therefore N$ is an unbiased estimator of μ.

h $\because \mathrm{Var}(\bar{X}) < \mathrm{Var}(N)$ choose $\bar{X}$

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise D, Question 1

Question:

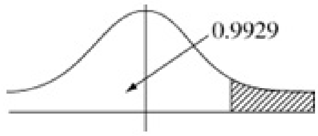
A sample of size 6 is taken from a normal distribution $N(10, 2^2)$. What is the probability that the sample mean exceeds 12?

Solution:

$X \sim N(10, 2^2)$

$\overline{X} \sim N(10, \frac{4}{6})$

$$P(\overline{X} > 12) = P\left(Z > \frac{12-10}{\sqrt{\frac{4}{6}}}\right)$$
$$= P(Z > 2.449...)$$



$\boxed{\text{Use } 2.44}$

$= 1 - 0.9927$

$= 0.0073$ (tables)

or $= 0.00715...$ (from calculator)

So accept awrt 0.007

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Estimation, confidence intervals and tests
Exercise D, Question 2

Question:

A machine fills cartons in such a way that the amount of drink in each carton is distributed normally with a mean of 40 cm^3 and a standard deviation of 1.5 cm^3.

a A sample of four cartons is examined. Find the probability that the mean amount of drink is more than 40.5 cm^3

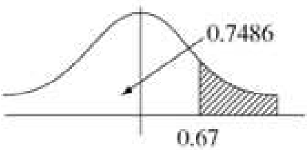
b A sample of 49 cartons is examined. Find the probability that the mean amount of drink is more than 40.5 cm^3 on this occasion.

Solution:

$D \sim N(40, 1.5^2)$

$\bar{D} \sim N(40, \frac{1.5^2}{4})$

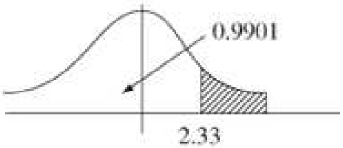
a



$$\begin{aligned} P(\bar{D} > 40.5) &= P\left(Z > \frac{40.5-40}{\frac{1.5}{2}}\right) \\ &= P(Z > \tfrac{2}{3}) \quad \text{Use 0.67} \\ &= 1 - 0.7486 \\ &= 0.2514 \quad \text{(tables)} \\ \text{or} &= 0.25249... \quad \text{(calculator)} \end{aligned}$$

So accept awrt 0.251 or 0.252

b $\bar{X} \sim N(40, \frac{1.5^2}{49})$



$$\begin{aligned} P(\bar{X} > 40.5) &= P\left(Z > \frac{40.5-40}{\frac{1.5}{7}}\right) \\ &= P(Z > 2.33...) \quad \text{Use 2.34} \\ &= 1 - 0.9904 \\ &= 0.0096 \quad \text{(tables)} \\ \text{or} &= 0.009815... \end{aligned}$$

So accept awrt $0.0098 \sim 0.0096$

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Estimation, confidence intervals and tests
Exercise D, Question 3

Question:

The lengths of bolts produced by a machine have an unknown distribution with mean 3.03 cm and standard deviation 0.20 cm. A sample of 100 bolts is taken.

a Estimate the probability that the mean length of this sample is less than 3 cm.

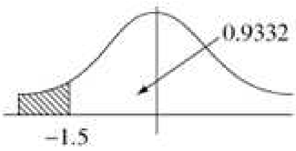
b What size sample is required if the probability that the mean is less than 3 cm is to be less than 1%?

Solution:

$$L \sim N(3.03, 0.20^2)$$

$$\bar{L} \simeq\sim N(3.03, \frac{0.20^2}{100})$$

a



$$\begin{aligned} P(\bar{L} < 3) &= P\left(Z < \frac{3-3.03}{0.02}\right) \\ &= P(Z < -1.5) \\ &= 1 - 0.9332 \\ &= 0.0668 \qquad \text{(tables)} \\ \text{or} &= 0.066807... \qquad \text{(calculator)} \end{aligned}$$

So accept awrt 0.0668

b $P(\bar{L} < 3) = P\left(Z < \frac{3-3.03}{\frac{0.2}{\sqrt{n}}}\right) < 0.01$

$$\begin{aligned} \Rightarrow \frac{3-3.03}{\frac{0.2}{\sqrt{n}}} &< -2.3263 \\ \frac{0.03}{2.3263} &> \frac{0.2}{\sqrt{n}} \\ \sqrt{n} &> 15.50... \\ n &> 240.51... \end{aligned}$$

$\therefore n = 241$ (or more)

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Estimation, confidence intervals and tests
Exercise D, Question 4

Question:

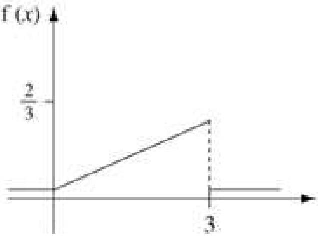
Forty observations are taken from a population with distribution given by the probability density function

$$f(x)=\begin{cases}\frac{2}{9}x, & 0\le x\le 3,\\ 0, & \text{otherwise.}\end{cases}$$

a Find the mean and variance of this population.

b Find an estimate of the probability that the mean of the 40 observations is more than 2.10.

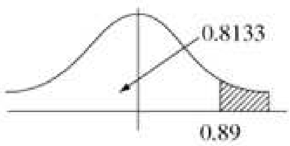
Solution:



a $\mu = 2$ (Centroid of Δ) or from $\int_0^3 x\times\frac{2}{9}x\,dx$

$$\sigma^2 = \int_0^3 \frac{2}{9}x^3\,dx - 2^2$$
$$= \left[\frac{2}{9}\times\frac{x^4}{4}\right]_0^3 - 4$$
$$= \frac{9}{2} - 0 - 4 = 0.5$$

b $\bar{X} \simeq\sim N\left(2, \frac{0.5}{40}\right)$



$$P(\bar{X} > 2.10) = P\left(Z > \frac{2.10-2}{\sqrt{\frac{0.5}{40}}}\right)$$
$$= P(Z > 0.8944...)$$
$$= 1 - 0.8133 \quad \text{Use 0.89}$$
$$= 0.1867 \quad \text{(tables)}$$
$$\text{or} = 0.1855466... \quad \text{(calculator)}$$

so accept awrt $0.186 \sim 0.187$

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Estimation, confidence intervals and tests

Exercise D, Question 5

Question:

A fair die is rolled 35 times.

a Find the approximate probability that the mean of the 35 scores is more than 4.

b Find the approximate probability that the total of the 35 scores is less than 100.

Solution:

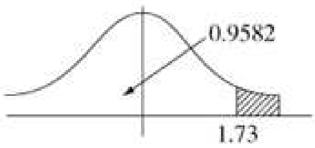
$S =$ score on die

$\boxed{\text{See S1}}$

$\mathrm{E}(S) = 3.5 \quad \mathrm{Var}(S) = \dfrac{35}{12}$

$\bar{S} \simeq \sim \mathrm{N}\left(3.5, \dfrac{\left(\frac{35}{12}\right)}{35}\right)$

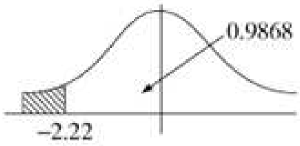
a



$$\begin{aligned} \mathrm{P}(\bar{S} > 4) &= \mathrm{P}\left(Z > \frac{4-3.5}{\sqrt{\frac{1}{12}}}\right) \quad \text{and } \frac{\frac{35}{12}}{35} = \frac{1}{12} \\ &= \mathrm{P}(Z > 1.732\ldots) \quad \text{Use } 1.73 \\ &= 1 - 0.9582 \\ &= 0.0418 \quad \text{(tables)} \\ \text{or} &= 0.041632\ldots \quad \text{(calculator)} \end{aligned}$$

so accept awrt 0.042

b $T =$ total score, $T = 35\bar{S}$



$$\begin{aligned} \mathrm{P}(T < 100) &= \mathrm{P}\left(\bar{S} < \frac{100}{35}\right) \\ &= \mathrm{P}\left(Z < \frac{\frac{100}{35} - 3.5}{\sqrt{\frac{1}{12}}}\right) \\ &= \mathrm{P}(Z < -2.2269\ldots) \quad \text{Use } -2.22 \\ &= 1 - 0.9868 \\ &= 0.0132 \quad \text{(tables)} \\ \text{or} &= 0.01297618\ldots \end{aligned}$$

so accept awrt 0.013

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Estimation, confidence intervals and tests
Exercise D, Question 6

Question:

The 25 children in a class each roll a fair die 30 times and record the number of sixes they obtain. Find an estimate of the probability that the mean number of sixes recorded for the class is less than 4.5.

Solution:

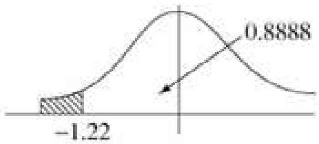
Let S = number of sixes in 30 rolls of a die

$S \sim B(30, \frac{1}{6})$

$\mu = 5 \quad \sigma^2 = \frac{25}{6}$

Using formula in S2. mean $= np$
variance $= np(1-p)$

$\bar{S} \simeq\sim N\left(5, \frac{\left(\frac{25}{6}\right)}{25}\right)$



$$\begin{aligned} P(\bar{S} < 4.5) &= P\left(Z < \frac{4.5-5}{\sqrt{\frac{1}{6}}}\right) \\ &= P(Z < -1.2247...) \quad \text{Use } -1.22. \\ &= 1 - 0.8888 \\ &= 0.1112 \quad \text{(tables)} \\ \text{or} &= 0.110335... \quad \text{(calculator)} \end{aligned}$$

$\therefore$ accept awrt 0.11

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Estimation, confidence intervals and tests
Exercise D, Question 7

Question:

The error in mm made in measuring the length of a table has a uniform distribution over the range $[-5,5]$. The table is measured 20 times. Find an estimate of the probability that the mean error is less than -1 mm.

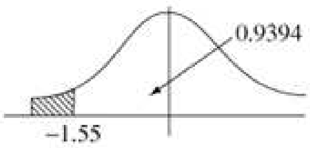
Solution:

$E=$ error

$E \sim \mathrm{U}[-5,5]$

$\mu=0 \quad \sigma^2=\frac{(5-(-5))^2}{12}=\frac{100}{12}=\frac{25}{3}$ $\boxed{\text{See formula in S2.}}$

$$\bar{E} \simeq \sim \mathrm{N}\left(0, \frac{\left(\frac{25}{3}\right)}{20}\right)$$



$$\begin{aligned}
\mathrm{P}(\bar{E}<-1) &= \mathrm{P}\left(Z<\frac{-1-0}{\sqrt{\frac{5}{12}}}\right) \text{ since } \frac{\left(\frac{25}{3}\right)}{20}=\frac{5}{12} \\
&= \mathrm{P}(Z<-1.5491\ldots) \quad \text{Use } -1.55 \\
&= 1-0.9394 \quad \text{(tables)} \\
&= 0.0606 \quad \text{(calculator)} \\
\text{or} &= 0.060667\ldots
\end{aligned}$$

So accept awrt 0.061

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise D, Question 8

Question:

Telephone calls arrive at an exchange at an average, rate of two per minute. Over a period of 30 days a telephonist records the number of calls that arrive in the five-minute period before her break.

a Find an approximation for the probability that the total number of calls recorded is more than 350.

b Estimate the probability that the mean number of calls in the five-minute interval is less than 9.0.

Solution:

C = number of calls that arrive in 5-minute period

$C \sim \text{Po}(10)$

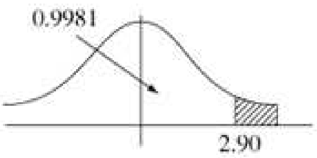
$n = 30$

$\bar{C} \simeq\sim \text{N}(10, \frac{10}{30})$

T = total number of calls in 30 days

$T = 30\bar{C}$

a



$$\therefore \text{P}(T > 350) = \text{P}\left(\bar{C} > \frac{350}{30}\right)$$

$$= \text{P}\left(Z > \frac{\frac{35}{3} - 10}{\sqrt{\frac{1}{3}}}\right)$$

$= \text{P}(Z > 2.8867...)$ Use 2.90

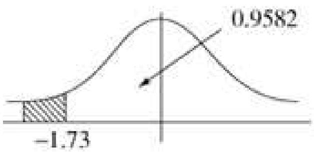
$= 1 - 0.9981$

$= 0.0019$ (tables)

or $= 0.001946...$ (calculator)

So accept awrt 0.0019

b



$$\text{P}(\bar{C} < 9) = \text{P}\left(Z < \frac{9 - 10}{\sqrt{\frac{1}{3}}}\right)$$

$= \text{P}(Z < -1.732...)$ Use −1.73

$= 1 - 0.9582$

$= 0.0418$ (tables)

or $= 0.041632...$ (calculator)

So accept awrt 0.042

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise D, Question 9

Question:

How many times must a fair die be rolled in order for there to be a less than 1% chance that the mean of all the scores differs from 3.5 by more than 0.1?

Solution:

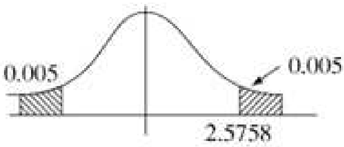
$S =$ score on a die

$\mu = 3.5 \quad \sigma^2 = \frac{35}{12}$

$\bar{S} \simeq\sim \mathrm{N}\left(3.5, \frac{35}{12n}\right)$

$\mathrm{P}(|\bar{S} - 3.5| > 0.1) < 0.01$

$= \mathrm{P}\left(|Z| > \frac{0.1}{\sqrt{\frac{35}{12n}}}\right) < 0.01$



$\Rightarrow \frac{0.1}{\sqrt{\frac{35}{12n}}} > 2.5758$

$\left(\frac{0.1}{2.5758}\right)^2 > \frac{35}{12n}$

$\therefore n > \frac{35}{12\left(\frac{0.1}{2.5758}\right)^2} = 1935.12\ldots$

$\therefore$ need n at least 1936

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Estimation, confidence intervals and tests
Exercise D, Question 10

Question:

The heights of women in a certain area have a mean of 175 cm and a standard deviation of 2.5 cm. The heights of men in the same area have a mean of 177 cm and a standard deviation of 2.0 cm. Samples of 40 women and 50 men are taken and their heights are recorded. Find the probability that the mean height of the men is more than 3 cm greater than the mean height of the women.

Solution:

$W =$ height of a woman

$W \sim \mathrm{N}(175, 2.5^2)$

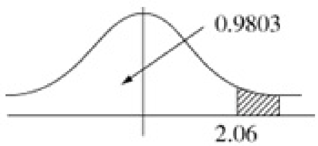
$M =$ height of a man

$M \sim \mathrm{N}(177, 2^2)$

$\overline{W} \sim \mathrm{N}\left(175, \frac{2.5^2}{40}\right) \quad \overline{M} \sim \mathrm{N}\left(177, \frac{2^2}{50}\right)$

$\mathrm{P}(\overline{M} - \overline{W} > 3)$ requires $X = \overline{M} - \overline{W}$

$$X \sim \mathrm{N}\left(2, \frac{2^2}{50} + \frac{2.5^2}{40}\right) \quad \frac{2^2}{50} + \frac{2.5^2}{40} = 0.23625$$



$$\begin{aligned} \therefore \mathrm{P}(X > 3) &= \mathrm{P}\left(Z > \frac{3-2}{\sqrt{0.23625}}\right) \\ &= \mathrm{P}(Z > 2.0573\cdots) \quad \text{Use } 2.06 \\ &= 1 - 0.9803 \\ &= 0.0197 \quad \text{(tables)} \\ \text{or} &= 0.0198248\ldots \quad \text{(calculator)} \end{aligned}$$

So accept awrt $0.0197 \sim 0.198$

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests

Exercise D, Question 11

Question:

A computer, in adding numbers, rounds each number off to the nearest integer. All the rounding errors are independent and come from a uniform distribution over the range $[-0.5, 0.5]$.

a Given that 1000 numbers are added, find the probability that the total error is greater than $+10$.

b Find how many numbers can be added together so that the probability that the magnitude of the total error is less than 10 is at least 0.95.

Solution:

$E =$ rounding error

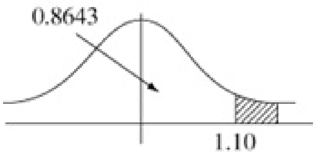
$E \sim \mathrm{U}[-0.5, 0.5]$

$\mu = 0, \sigma^2 = \dfrac{1}{12}$

$n = 1000 \quad \overline{E} \sim \mathrm{N}\left(0, \dfrac{1}{12\,000}\right)$

$T =$ total error $= 1000\overline{E}$

a



$$\begin{aligned}
\mathrm{P}(T > 10) &= \mathrm{P}\left(\overline{E} > \frac{10}{1000}\right) \\
&= \mathrm{P}\left(Z > \frac{0.01 - 0}{\sqrt{\frac{1}{12\,000}}}\right) \\
&= \mathrm{P}(Z > 1.09544...) && \text{Use } 1.10 \\
&= 1 - 0.8643 \\
&= 0.1357 && \text{(tables)} \\
\text{or } &= 0.136660... && \text{(calculator)}
\end{aligned}$$

So accept awrt $0.136 \sim 0.137$

b $n = N, T' = N\overline{E}' \quad$ where $\overline{E}' \sim \mathrm{N}\left(0, \dfrac{1}{12N}\right)$

$$\mathrm{P}(|T'| < 10) = \mathrm{P}\left(|\overline{E}| < \frac{10}{N}\right) \geq 0.95$$

$$= \mathrm{P}\left(|Z| < \frac{\frac{10}{N}}{\sqrt{\frac{1}{12N}}}\right) \geq 0.95$$

$$\Rightarrow \frac{10}{N\sqrt{\frac{1}{12N}}} \geq 1.96 \Rightarrow 10\sqrt{12} > 1.96N$$

$$\sqrt{N} < \frac{10\sqrt{12}}{1.96}$$

$$\sqrt{N} < 17.673$$

$$N < 312.3$$

so need 312

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Estimation, confidence intervals and tests
Exercise D, Question 12

Question:

An electrical company repairs very large numbers of television sets and wishes to estimate the mean time taken to repair a particular fault. It is known from previous research that the standard deviation of the time taken to repair this particular fault is 2.5 minutes.
The manager wishes to ensure that the probability that the estimate differs from the true mean by less than 30 seconds is 0.95.
Find how large a sample is required.

Solution:

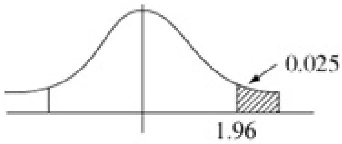
T = time to repair fault

$T \sim N(\mu, 2.5^2)$

$\bar{T} \simeq\sim N(\mu, \frac{2.5^2}{n})$

$$P(|\bar{T}-\mu| < 0.5) = 0.95$$

$$\Rightarrow P\left(|Z| < \frac{0.5}{\left(\frac{2.5}{\sqrt{n}}\right)}\right) = 0.95$$



$$\therefore \frac{0.5}{\left(\frac{2.5}{\sqrt{n}}\right)} = 1.96$$

$$\sqrt{n} = \frac{1.96 \times 2.5}{0.5}$$

$$n = 9.8^2 = 96.04$$

So need 97 (accept 96)

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise E, Question 1

Question:

A random sample of size 9 is taken from a normal distribution with variance 36. The sample mean is 128.

a Find a 95% confidence interval for the mean μ of the distribution.

b Find a 99% confidence interval for the mean μ of the distribution.

Solution:

$n = 9,\ \sigma^2 = 36,\ \bar{x} = 128$

a 95% C.I. for μ is $128 \pm 1.96 \times \frac{6}{\sqrt{9}} = (124.08, 131.92\ldots)$

$= (124, 132)$ (3 s.f.)

b 99% C.I. for μ is $128 \pm 2.5758 \times \frac{6}{\sqrt{9}} = (122.84\ldots, 133.15\ldots)$

$= (123, 133)$ (3 s.f.)

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise E, Question 2

Question:

A random sample of size 25 is taken from a normal distribution with standard deviation 4. The sample mean is 85.

a Find a 90% confidence interval for the mean μ of the distribution.

b Find a 95% confidence interval for the mean μ of the distribution.

Solution:

$n = 25,\ \sigma = 4,\ \bar{x} = 85$

a 90% C.I. for μ is $85 \pm 1.6449 \times \frac{4}{\sqrt{25}} = (83.684..., 86.315...)$

$= (83.7, 86.3)$

b 95% C.I. for μ is $85 \pm 1.96 \times \frac{4}{\sqrt{25}} = (83.432, 86.568)$

$= (83.4, 86.6)$

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise E, Question 3

Question:

A normal distribution has mean μ and variance 4.41. A random sample has the following values:
23.1, 21.8, 24.6, 22.5
Use this sample to find 98% confidence limits for the mean μ.

Solution:

$n = 4$, $\sigma^2 = 4.41$, $\bar{x} = 23$

98% C.I. for μ is $23 \pm 2.3263 \times \sqrt{\dfrac{4.41}{4}}$

$= (20.557, 25.443)$

$= (20.6, 25.4)$ (3 s.f.)

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise E, Question 4

Question:

A normal distribution has standard deviation 15. Estimate the sample size required if the following confidence intervals for the mean should have width of less than 2.

a 90%
b 95%
c 99%

Solution:

$\sigma = 15$

C.I. is $\bar{x} \pm z \times \dfrac{\sigma}{\sqrt{n}}$

width $= \dfrac{2z\sigma}{\sqrt{n}}$

a $90\% \Rightarrow z = 1.6449 \quad \therefore \dfrac{2 \times 1.6449 \times 15}{\sqrt{n}} < 2$

$\Rightarrow \sqrt{n} > 24.67\ldots \quad \therefore n > 608.78\ldots$

So $n = 609$

b $95\% \Rightarrow z = 1.96 \therefore \dfrac{2 \times 1.96 \times 15}{\sqrt{n}} < 2$

$\Rightarrow \sqrt{n} > 1.96 \times 15 \therefore n > 864.36\ldots$

So $n = 865$

c $99\% \Rightarrow z = 2.5758 \quad \therefore \dfrac{2 \times 2.5758 \times 15}{\sqrt{n}} < 2$

$\Rightarrow \sqrt{n} > 2.5758 \times 15 \quad \therefore n > 1492.817\ldots$

So $n = 1493$

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Estimation, confidence intervals and tests
Exercise E, Question 5

Question:

Repeat Question 4 for a normal distribution with standard deviation 2.4 and a desired width of less than 0.8.

Solution:

$\sigma = 2.4$

$\text{width} = \frac{2z\sigma}{\sqrt{n}} < 0.8$

$\therefore \sqrt{n} > \frac{4.8z}{0.8} = 6z$

a $z = 1.6449 \quad \therefore n > 97.405\ldots \quad \text{So } n = 98$

b $z = 1.96 \quad \therefore n > 138.29\ldots \quad \text{So } n = 139$

c $z = 2.5758 \quad \therefore n > 238.85\ldots \quad \text{So } n = 239$

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Estimation, confidence intervals and tests
Exercise E, Question 6

Question:

An experienced poultry farmer knows that the mean weight μ kg for a large population of chickens will vary from season to season but the standard deviation of the weights should remain at 0.70 kg. A random sample of 100 chickens is taken from the population and the weight x kg of each chicken in the sample is recorded, giving $\Sigma x = 190.2$. Find a 95% confidence interval for μ.

Solution:

$\sigma = 0.70$, $n = 100$, $\bar{x} = \frac{190.2}{100} = 1.902$

95% C.I. for μ is $\bar{x} \pm 1.96 \times \frac{\sigma}{\sqrt{100}}$

$= \quad 1.902 \pm 1.96 \times \frac{0.7}{10}$

$= \quad (1.7648, 2.0392)$

$= \quad (1.76, 2.04) \quad$ (3 s.f.)

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Estimation, confidence intervals and tests
Exercise E, Question 7

Question:

A railway watchdog is studying the number of seconds that express trains are late in arriving. Previous surveys have shown that the standard deviation is 50. A random sample of 200 trains was selected and gave rise to a mean of 310 seconds late. Find a 90% confidence interval for the mean number of seconds that express trains are late.

Solution:

$\sigma = 50 \quad n = 200 \quad \bar{x} = 310$

90% C.I. is $\bar{x} \pm 1.6449 \times \dfrac{\sigma}{\sqrt{200}}$

$= \left(310 \pm 1.6449 \times \dfrac{50}{\sqrt{200}}\right)$

$= (304.184\ldots, 315.815\ldots)$

$= (304, 316) \quad (3\text{ s.f.})$

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests

Exercise E, Question 8

Question:

An investigation was carried out into the total distance travelled by lorries in current use. The standard deviation can be assumed to be 15 000 km. A random sample of 80 lorries were stopped and their mean distance travelled was found to be 75 872 km. Find a 90% confidence interval for the mean distance travelled by lorries in current use.

Solution:

$\sigma = 15\,000 \quad n = 80 \quad \bar{x} = 75\,872$

90% C.I. is $\bar{x} \pm 1.6449 \times \dfrac{\sigma}{\sqrt{80}}$

$= 75\,872 \pm 1.6449 \times \dfrac{15\,000}{\sqrt{80}}$

$= (73\,113.41\ldots, 78\,630.58\ldots)$

$= (73\,113, 78\,631)$ (nearest integer)

or $(73\,100, 78\,600)$ (3 s.f.)

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise E, Question 9

Question:

It is known that each year the standard deviation of the marks in a certain examination is 13.5 but the mean mark μ will fluctuate. An examiner wishes to estimate the mean mark of all the candidates on the examination but he only has the marks of a sample of 250 candidates which give a sample mean of 68.4.

a What assumption about these candidates must the examiner make in order to use this sample mean to calculate a confidence interval for μ?

b Assuming that the above assumption is justified, calculate a 95% confidence interval for μ.

Later the examiner discovers that the actual value of μ was 65.3.

c What conclusions might the examiner draw about his sample?

Solution:

$\sigma = 13.5 \quad n = 250 \quad \bar{x} = 68.4$

a Must assume that these students form a random sample or that they are representative of the population.

b 95% C.I. is $68.4 \pm 1.96 \times \dfrac{13.5}{\sqrt{250}}$

$= \; (66.726\ldots, 70.073\ldots)$

$= \; (66.7, 70.1)$ (3 s.f.)

c If $\mu = 65.3$ that is outside the C.I. so the examiner's sample was not representative. The examiner marked more better than average candidates.

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Estimation, confidence intervals and tests
Exercise E, Question 10

Question:

The number of hours for which an electronic device can retain information has a uniform distribution over the range $[\mu-10, \mu+10]$ but the value of μ is not known.

a Show that the variance of the number of hours the device can retain the information for is $\frac{100}{3}$.

A random sample of 120 devices were tested and the mean number of hours they were retained information for was 78.7.

b Find a 95% confidence interval for μ.

Solution:

$H \sim U[\mu-10, \mu+10]$

a $E(H)=\mu \quad \text{Var}(H)=\frac{(20)^2}{12}=\frac{400}{12}=\frac{100}{3}$

Use formula in S2.

b $n=120 \quad \bar{h}=78.7$

$$95\% \text{ C.I. is } \bar{h} \pm 1.96 \times \frac{\sqrt{\frac{100}{3}}}{\sqrt{120}}$$

$$= \left(78.7 \pm 1.96 \times \sqrt{\frac{100}{360}}\right)$$

$$= (77.666\ldots, 79.733\ldots)$$

$$= (77.7, 79.7) \quad (3 \text{ s.f.})$$

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Estimation, confidence intervals and tests
Exercise E, Question 11

Question:

A statistics student calculated a 95% and a 99% confidence interval for the mean μ of a certain population but failed to label them. The two intervals were (22.7, 27.3) and (23.2, 26.8).

a State, with a reason, which interval is the 95% one.

b Estimate the standard error of the mean in this case.

c What was the student's unbiased estimate of the mean μ in this case?

Solution:

a (23.2, 26.8) is 95% C.I. since it is the narrower interval.

b $\bar{x} = \frac{1}{2}(23.2 + 26.8) = 25$

$\therefore 1.96 \frac{\sigma}{\sqrt{n}} = 25 - 23.2 = 1.8$

$\therefore \frac{\sigma}{\sqrt{n}} = 0.9183\ldots = 0.918 \quad (3 \text{ s.f.})$

c $\hat{\mu} = \bar{x} = 25$ (mid-point of the intervals)

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Estimation, confidence intervals and tests
Exercise E, Question 12

Question:

A 95% confidence interval for a mean μ is 85.3 ± 2.35. Find the following confidence intervals for μ.

a 90%
b 98%
c 99%

Solution:

$$\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}} = 85.3 \pm 2.35$$

$$\therefore \bar{x} = 85.3 \quad \text{and} \quad \frac{\sigma}{\sqrt{n}} = \frac{2.35}{1.96} = 1.1989.$$

a 90% C.I. is $85.3 \pm 1.6449 \times \frac{2.35}{1.96}$

$= (83.327\ldots, 87.272\ldots)$

$= (83.3, 87.3)$ (3 s.f.)

b 98% C.I. is $85.3 \pm 2.3263 \times \frac{2.35}{1.96}$

$= (82.510\ldots, 88.089\ldots)$

$= (82.5, 88.1)$ (3 s.f.)

c 99% C.I. is $85.3 \pm 2.5758 \times \frac{2.35}{1.96}$

$= (82.211\ldots, 88.388\ldots)$

$= (82.2, 88.4)$ (3 s.f.)

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Estimation, confidence intervals and tests
Exercise E, Question 13

Question:

The managing director of a certain firm has commissioned a survey to estimate the mean expenditure of customers on electrical appliances. A random sample of 100 people were questioned and the research team presented the managing director with a 95% confidence interval of (£128.14, £141.86). The director says that this interval is too wide and wants a confidence interval of total width £10.

a Using the same value of $\bar{x}$, find the confidence limits in this case.

b Find the level of confidence for the interval in part **a**.

The managing director is still not happy and now wishes to know how large a sample would be required to obtain a 95% confidence interval of total width no more than £10.

c Find the smallest size of sample that will satisfy this request.

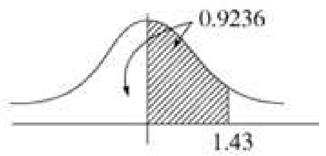
Solution:

a $\bar{x} = \frac{1}{2}(128.14 + 141.86) = \frac{270}{2} = 135$

$\therefore$ C.I. will be (130, 140)

b $z \times \frac{\sigma}{\sqrt{n}} = 5$ but $1.96\frac{\sigma}{\sqrt{n}} = 6.86$

$\therefore z = \frac{5}{\left(\frac{6.86}{1.96}\right)} = 1.4285\ldots$ Use 1.43



$\therefore$ C.I. is $2 \times (0.9236 - 0.5)$

$= 0.8472$ (tables)

or $= 0.846872\ldots$ (calculator)

$\therefore$ C.I. is 85%

c Now we know $1.96\frac{\sigma}{\sqrt{100}} = 6.86$

$\therefore \sigma = \frac{6.86 \times 10}{1.96} = 35$

and require $z \times \frac{\sigma}{\sqrt{n}} = 5$ where $z = 1.96$

$\therefore \frac{1.96 \times 35}{5} = \sqrt{n}$

$\Rightarrow n = 188.23\ldots$

$\therefore$ Need $n = 189$ or more

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Estimation, confidence intervals and tests
Exercise E, Question 14

Question:

A plant produces steel sheets whose weights are known to be normally distributed with a standard deviation of 2.4 kg. A random sample of 36 sheets had a mean weight of 31.4 kg. Find 99% confidence limits for the population mean.

Solution:

$W \sim \mathrm{N}(\mu, 2.4^2) \quad n = 36 \quad \bar{w} = 31.4$

$$99\% \text{ C.I. is } 31.4 \pm 2.5758 \times \frac{2.4}{\sqrt{36}}$$

$$= (30.369\ldots, 32.430\ldots)$$

$$= (30.4, 32.4) \quad (3 \text{ s.f.})$$

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Estimation, confidence intervals and tests
Exercise E, Question 15

Question:

A machine is regulated to dispense liquid into cartons in such a way that the amount of liquid dispensed on each occasion is normally distributed with a standard deviation of 20 ml. Find 99% confidence limits for the mean amount of liquid dispensed if a random sample of 40 cartons had an average content of 266 ml.

Solution:

$\sigma = 20,\ n = 40,\ \bar{x} = 266$

$$99\%\text{ C.I. is } 266 \pm 2.5758 \times \frac{20}{\sqrt{40}}$$

$$= (257.854\ldots, 274.145\ldots)$$

$$= (258, 274) \quad (3\text{ s.f.})$$

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Estimation, confidence intervals and tests
Exercise E, Question 16

Question:

a The error made when a certain instrument is used to measure the body length of a butterfly of a particular species is known to be normally distributed with mean 0 and standard deviation 1 mm. Calculate, to 3 decimal places, the probability that the error made when the instrument is used once is numerically less than 0.4mm.

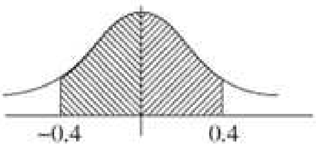
b Given that the body length of a butterfly is measured 9 times with the instrument, calculate, to 3 decimal places, the probability that the mean of the 9 readings will be within 0.5 mm of the true length.

c Given that the mean of the 9 readings was 22.53 mm, determine a 98% confidence interval for the true body length of the butterfly.

Solution:

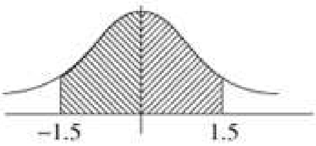
$E \sim \mathrm{N}(0, 1^2)$

a $\mathrm{P}(|E| < 0.4) = (0.6554 - 0.5) \times 2$
$= 0.3108$



b $\bar{E} \sim \mathrm{N}(0, \frac{1}{9})$

$\mathrm{P}(|\bar{E}| < 0.5) = \mathrm{P}(|Z| < \frac{0.5}{\sqrt{\frac{1}{9}}})$
$= (0.9332 - 0.5) \times 2$
$= 0.8664$



c 98% C.I. is $22.53 \pm 2.3263 \times \frac{1}{\sqrt{9}}$
$= (21.754\ldots, 23.305\ldots)$
$= (21.8, 23.3)$ (3 s.f.)

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Estimation, confidence intervals and tests
Exercise F, Question 1

Question:

A random sample of size n is taken from a population having a normal distribution with mean μ and variance σ^2. Test the hypotheses at the stated levels of significance.

$\mathrm{H}_0; \mu = 21, \mathrm{H}_1 : \mu \neq 21, n = 20, \bar{x} = 21.2, \sigma = 1.5$, at the 5% level

Solution:

$\mathrm{H}_0 : \mu = 21 \quad \mathrm{H}_1 : \mu \neq 21$ 5% c.v. is $z = \pm 1.96$

$$\text{t.s.} = z = \frac{(21.2 - 21)}{\left(\frac{1.5}{\sqrt{20}}\right)} = 0.596\ldots < 1.96$$

Not significant

Accept H_0

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Estimation, confidence intervals and tests
Exercise F, Question 2

Question:

A random sample of size n is taken from a population having a normal distribution with mean μ and variance σ^2. Test the hypotheses at the stated levels of significance.

$\mathrm{H_0}; \mu = 100, \mathrm{H_1} : \mu < 100, n = 36, \bar{x} = 98.5, \ \sigma = 5.0$, at the 5% level

Solution:

$\mathrm{H_0} : \mu = 100 \quad \mathrm{H_1} : \mu < 100$ 5% c.v. is $z = -1.6449$

$$\text{t.s.} = z = \frac{(98.5 - 100)}{\left(\frac{5}{\sqrt{36}}\right)} = -1.8 < -1.6449$$ Significant

Reject $\mathrm{H_0}$

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Estimation, confidence intervals and tests
Exercise F, Question 3

Question:

A random sample of size n is taken from a population having a normal distribution with mean μ and variance σ^2. Test the hypotheses at the stated levels of significance.

$\mathrm{H_0}: \mu = 5, \mathrm{H_1}: \mu \neq 5, n = 25, \bar{x} = 6.1, \sigma = 3.0$, at the 5% level

Solution:

$\mathrm{H_0}: \mu = 5 \quad \mathrm{H_1}: \mu \neq 5$ 5% c.v. is $z = \pm 1.96$

$$\text{t.s.} = z = \frac{(6.1 - 5)}{\left(\frac{3}{\sqrt{25}}\right)} = 1.83\ldots < 1.96$$

Not significant

Accept $\mathrm{H_0}$

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Estimation, confidence intervals and tests
Exercise F, Question 4

Question:

A random sample of size n is taken from a population having a normal distribution with mean μ and variance σ^2. Test the hypotheses at the stated levels of significance.

$H_0: \mu = 15$, $H_1: \mu > 15$, $n = 40$, $\bar{x} = 16.5$, $\sigma = 3.5$, at the 1% level

Solution:

$H_0: \mu = 15 \quad H_1: \mu > 15$ 1% c.v. is $z = 2.3263$

$$\text{t.s.} = z = \frac{(16.5 - 15)}{\left(\frac{3.5}{\sqrt{40}}\right)} = 2.710\ldots > 2.3263$$

Significant

Reject H_0

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Estimation, confidence intervals and tests
Exercise F, Question 5

Question:

A random sample of size n is taken from a population having a normal distribution with mean μ and variance σ^2. Test the hypotheses at the stated levels of significance.

$H_0; \mu = 50, H_1 : \mu \neq 50, n = 60, \bar{x} = 48.9, \ \sigma = 4.0$, at the 1% level

Solution:

$H_0 : \mu = 50 \quad H_1 : \mu \neq 50$ 1% c.v. is $z = \pm 2.5758$

$$\text{t.s.} = z = \frac{(48.9 - 50)}{\left(\frac{4}{\sqrt{60}}\right)} = -2.130\ldots > -2.5758$$

Not significant

Accept H_0

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Estimation, confidence intervals and tests
Exercise F, Question 6

Question:

A sample of size n is taken from a population having a $N(\mu, \sigma^2)$ distribution. Find the critical regions for the test statistic $\bar{X}$ in the following test.

$H_0 : \mu = 120, H_1 : \mu < 120, n = 30,\ \sigma = 2.0,$ at the 5% level

Solution:

$H_0 : \mu = 120 \quad H_1 : \mu < 120 \quad$ c.v. is $Z = -1.6449$

$$Z = \frac{(\bar{X} - 120)}{\left(\frac{2}{\sqrt{30}}\right)}$$

Reject H_0 for $Z < -1.6449$

$$\Rightarrow \quad \bar{X} < 120 - 1.6449 \times \frac{2}{\sqrt{30}}$$

i.e. $\bar{X} < 119.39\ldots$ or 119 (3 s.f.)

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Estimation, confidence intervals and tests
Exercise F, Question 7

Question:

A sample of size n is taken from a population having a $N(\mu, \sigma^2)$ distribution. Find the critical regions for the test statistic $\bar{X}$ in the following test.

$H_0 : \mu = 12.5, H_1 : \mu > 12.5, n = 25, \sigma = 1.5,$ at the 1% level

Solution:

$H_0 : \mu = 12.5 \quad H_1 : \mu > 12.5 \quad$ c.v. is $Z = 2.3263$

$$Z = \frac{(\bar{X} - 12.5)}{\left(\frac{1.5}{\sqrt{25}}\right)}$$

Reject H_0 for $Z > 2.3263$

$$\Rightarrow \quad \bar{X} > 12.5 + 2.3263 \times \frac{1.5}{\sqrt{25}}$$

i.e. $\bar{X} > 13.19789$

$\bar{X} > 13.2$ (3 s.f.)

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Estimation, confidence intervals and tests
Exercise F, Question 8

Question:

A sample of size n is taken from a population having a $N(\mu, \sigma^2)$ distribution. Find the critical regions for the test statistic $\bar{X}$ in the following test.

$H_0 : \mu = 85, H_1 : \mu < 85, n = 50, \ \sigma = 4.0,$, at the 10% level

Solution:

$H_0 : \mu = 85 \quad H_1 : \mu < 85 \quad$ c.v. is $Z = -1.2816$

$$Z = \frac{(\bar{X} - 85)}{\left(\frac{4}{\sqrt{50}}\right)}$$

Reject H_0 for $Z < -1.2816$

$$\Rightarrow \quad \bar{X} < 85 - 1.2816 \times \frac{4}{\sqrt{50}}$$

i.e. $\bar{X} < 84.2750\ldots$ or $\bar{X} < 84.3$

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests

Exercise F, Question 9

Question:

A sample of size n is taken from a population having a $N(\mu, \sigma^2)$ distribution. Find the critical regions for the test statistic $\bar{X}$ in the following test.

$H_0 : \mu = 0, H_1 : \mu \neq 0, n = 45, \ \sigma = 3.0,$ at the 5% level

Solution:

$H_0 : \mu = 0 \quad H_1 : \mu \neq 0 \quad$ c.v. is $Z = \pm 1.96$

$$Z = \frac{(\bar{X} - 0)}{\left(\frac{3}{\sqrt{45}}\right)}$$

Reject H_0 for $|Z| > 1.96$

$\Rightarrow \quad \bar{X} > 0 + 1.96 \times \frac{3}{\sqrt{45}}$ or $\bar{X} < 0 - 1.96 \times \frac{3}{\sqrt{45}}$

i.e. $\bar{X} > 0.87653\ldots$ or $\bar{X} < -0.87653\ldots$

i.e. $\bar{X} > 0.877$ or $\bar{X} < -0.877 \quad$ (3 s.f.)

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Estimation, confidence intervals and tests

Exercise F, Question 10

Question:

A sample of size n is taken from a population having a $\mathrm{N}(\mu, \sigma^2)$ distribution. Find the critical regions for the test statistic $\bar{X}$ in the following test.

$\mathrm{H}_0 : \mu = -8, \mathrm{H}_1 : \mu \neq -8, n = 20, \ \sigma = 1.2,$ at the 1% level

Solution:

$\mathrm{H}_0 : \mu = -8 \quad \mathrm{H}_1 : \mu \neq -8 \quad$ c.v. is $Z = \pm 2.5758$

$$Z = \frac{\left(\bar{X} - (-8)\right)}{\left(\frac{1.2}{\sqrt{20}}\right)}$$

Reject H_0 for $|Z| > 2.5758$

$$\Rightarrow \bar{X} > -8 + 2.5758 \times \frac{1.2}{\sqrt{20}} \text{ or } \bar{X} < -8 - 2.5758 \times \frac{1.2}{\sqrt{20}}$$

$$\bar{X} > -7.3088\ldots \text{ or } \bar{X} < -8.6911\ldots$$

$$\text{i.e. } \bar{X} > -7.31 \text{ or } \bar{X} < -8.69 \quad (3\text{ s.f.})$$

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Estimation, confidence intervals and tests
Exercise F, Question 11

Question:

The times taken for a capful of stain remover to remove a standard chocolate stain from a baby's bib are normally distributed with a mean of 185s and a standard deviation of 15s. The manufacturers of the stain remover claim to have developed a new formula which will shorten the time taken for a stain to be removed. A random sample of 25 capfuls of the new formula are tested and the mean time for the sample is 179s.
Test, at the 5% level, whether or not there is evidence that the new formula is an improvement.

Solution:

$\sigma = 15 \quad n = 25 \quad \bar{x} = 179$

$H_0 : \mu = 185$ (no improvement) $\qquad H_1 : \mu < 185$ (shorter time)

$$\text{t.s. is } z = \frac{(179 - 185)}{\left(\frac{15}{\sqrt{25}}\right)} = -2 < -1.6449, \quad 5\% \text{ c.v. is } z = -1.6449$$

Result is significant, so reject H_0

There is evidence that the new formula is an improvement.

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Estimation, confidence intervals and tests
Exercise F, Question 12

Question:

The IQ scores of a population are normally distributed with a mean of 100 and standard deviation of 15. A psychologist wishes to test the theory that eating chocolate before sitting an IQ test improves your score. A random sample of 100 people are selected and they are each given a 100 g bar of chocolate to eat before taking a standard IQ test. Their mean score on the test was 102.5. Test the psychologist's theory at the 5% level.

Solution:

$\sigma = 15, n = 100, \bar{x} = 102.5$

$\mathrm{H_0}: \mu = 100 \quad \mathrm{H_1}: \mu > 100$ (improved score)

t.s. is $z = \dfrac{(102.5 - 100)}{\left(\dfrac{15}{\sqrt{100}}\right)} = 1.66 > 1.6449$, 5% c.v. is $z = 1.6449$

Result is significant so reject $\mathrm{H_0}$.

There is evidence to support the psychologist's theory.

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise F, Question 13

Question:

The diameters of circular cardboard drinks mats produced by a certain machine are normally distributed with a mean of 9 cm and a standard deviation of 0.15 cm. After the machine is serviced a random sample of 30 mats is selected and their diameters are measured to see if the mean diameter has altered. The mean of the sample was 8.95 cm. Test, at the 5% level, whether there is significant evidence of a change in the mean diameter of mats produced by the machine.

Solution:

$\sigma = 0.15, n = 30, \bar{x} = 8.95$

$H_0 : \mu = 9$ (no change) $H_1 : \mu \neq 9$ (change in area diameter)

t.s. is $z = \dfrac{(8.95 - 9)}{\left(\dfrac{0.15}{\sqrt{30}}\right)} = -1.8257,\quad$ 5% c.v. is $z = \pm 1.96$

$-1.8257\ldots > -1.96$ so result is not significant.

There is insufficient evidence of a change in mean diameter.

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise F, Question 14

Question:

a Research workers measured the body lengths, in mm, of 10 specimens of fish spawn of a certain species off the coast of Eastern Scotland and found these lengths to be
12.5 10.2 11.1 9.6 12.1 9.3 10.7 11.4 14.7 10.4
Obtain unbiased estimates for the mean and variance of the lengths of all such fish spawn off Eastern Scotland.

b Research shows that, for a very large number of specimens of spawn of this species off the coast of Wales, the mean body length is 10.2 mm. Assuming that the variance of the lengths of spawn off Eastern Scotland is 2.56, perform a significance test at the 5% level to decide whether the mean body length of fish spawn off the coast of Eastern Scotland is larger than that of fish spawn off the coast of Wales.

Solution:

$n = 10, \sum x = 112, \sum x^2 = 1277.26$

a $\bar{x} = \dfrac{112}{10} = 11.2$

$s^2 = \dfrac{1277.26 - 10 \times 11.2^2}{9}$

$s^2 = 2.54$

$\sigma^2 = 2.56$

b $H_0 : \mu = 10.2$ $\quad$ $H_1 : \mu > 10.2$ (Scottish fish are longer)

t.s. is $z = \dfrac{(11.2 - 10.2)}{\sqrt{\dfrac{2.56}{10}}} = 1.976\ldots$ $\quad$ 5% c.v. is $z = 1.6449$

$1.976\ldots > 1.6449$ so the result is significant.

There is evidence that the fish off the coast of Eastern Scotland are longer than those off the coast of Wales.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise F, Question 15

Question:

a Explain what you understand by the Central Limit Theorem.

b An electrical firm claims that the average lifetime of the bulbs it produces is 800 hours with a standard deviation of 42 hours. To test this claim a random sample of 120 bulbs was taken and these bulbs were found to have an average lifetime of 789 hours. Stating your hypotheses clearly and using a 5% level of significance, test the claim made by the electrical firm.

Solution:

a The Central Limit Theorem enables you to assume that $\bar{X}$ has a normal distribution no matter what the distribution of X since it states that no matter what the distribution of the parent population, provided the size of randomly chosen samples is sufficiently large, then the distribution of the mean of such samples will be approximately normally distributed.

b $n = 120$, $\bar{x} = 789$, $\sigma = 42$

$\mathrm{H}_0 : \mu = 800$ (claim is true) $\mathrm{H}_1 : \mu \neq 800$ (claim is false)

$$\text{t.s. is } z = \frac{(789 - 800)}{\left(\frac{42}{\sqrt{120}}\right)} = -2.869\ldots, \quad \text{c.v. is } z = \pm 1.96$$

$-2.869 < -1.96$ so the result is significant

There is evidence that the mean length of lifetime of the bulbs is not 800 hours.

Suspect the claim made by the firm.

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Estimation, confidence intervals and tests
Exercise G, Question 1

Question:

Carry out a test on the given hypotheses at the given level of significance. The population from which the random sample is drawn is normally distributed.
$H_0 : \mu_1 = \mu_2; H_1 : \mu_1 > \mu_2; n_1 = 15, \sigma_1 = 5.0, n_2 = 20, \sigma_2 = 4.8, \bar{x}_1 = 23.8$ and $\bar{x}_2 = 21.5$
using a 5% level

Solution:

$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_1 > \mu_2$ 5% c.v. is $z = 1.6449$

$$\text{t.s. is } z = \frac{(23.8 - 21.5) - 0}{\sqrt{\frac{5^2}{15} + \frac{4.8^2}{20}}} = 1.3699\ldots$$

$1.3699 < 1.6449$ so result is not significant, accept H_0.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise G, Question 2

Question:

Carry out a test on the given hypotheses at the given level of significance.
The population from which the random sample is drawn is normally distributed.
$H_0 : \mu_1 = \mu_2$; $H_1 : \mu_1 \neq \mu_2$; $n_1 = 30$, $\sigma_1 = 4.2$, $n_2 = 25$, $\sigma_2 = 3.6$, $\bar{x}_1 = 49.6$ and $\bar{x}_2 = 51.7$
using a 5% level

Solution:

$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_1 \neq \mu_2$ 5% c.v. is $z = \pm 1.96$

t.s. is $z = \dfrac{(51.7 - 49.6) - 0}{\sqrt{\dfrac{4.2^2}{30} + \dfrac{3.6^2}{25}}}$ Choose $\bar{x}_2 - \bar{x}_1$ to get $z > 0$

t.s. is $z = 1.996\ldots$ > 1.96 so result is significant, reject H_0.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise G, Question 3

Question:

Carry out a test on the given hypotheses at the given level of significance. The population from which the random sample is drawn is normally distributed.
$H_0: \mu_1 = \mu_2$; $H_1: \mu_1 < \mu_2$; $n_1 = 25$, $\sigma_1 = 0.81$, $n_2 = 36$, $\sigma_2 = 0.75$, $\bar{x}_1 = 3.62$ and $\bar{x}_2 = 4.11$ using a 1% level

Solution:

$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 < \mu_2$ 1% c.v. is $z = -2.3263$

$$\text{t.s. is } z = \frac{(3.62 - 4.11) - 0}{\sqrt{\frac{0.81^2}{25} + \frac{0.75^2}{36}}} = -2.3946\ldots$$

t.s. is $-2.3946\ldots < -2.3263$ so result is significant, reject H_0.

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Estimation, confidence intervals and tests
Exercise G, Question 4

Question:

Carry out a test on the given hypothesis at the given level of significance. What is the significance of the Central Limit Theorem in these three questions?
$H_0 : \mu_1 = \mu_2$; $H_1 : \mu_1 \neq \mu_2$; $n_1 = 85$, $\sigma_1 = 8.2$, $n_2 = 100$, $\sigma_2 = 11.3$, $\bar{x}_1 = 112.0$ and $\bar{x}_2 = 108.1$ using a 1% level

Solution:

$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_1 \neq \mu_2$ 1% c.v. is $z = \pm 2.5758$

$$\text{t.s. is } z = \frac{(112.0 - 108.1) - 0}{\sqrt{\frac{8.2^2}{85} + \frac{11.3^2}{100}}} = 2.712\ldots > 2.5758$$

Significant result so reject H_0.

Central Limit Theorem applies since n_1, n_2 are large and enables you to assume $\bar{X}_1$ and $\bar{X}_2$ are both normally distributed.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise G, Question 5

Question:

Carry out a test on the given hypothesis at the given level of significance. What is the significance of the Central Limit Theorem in these three questions?
$H_0 : \mu_1 = \mu_2$; $H_1 : \mu_1 > \mu_2$; $n_1 = 100$, $\sigma_1 = 18.3$, $n_2 = 150$, $\sigma_2 = 15.4$, $\bar{x}_1 = 72.6$ and $\bar{x}_2 = 69.5$ using a 5% level

Solution:

$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_1 > \mu_2$ 5% c.v. is $z = 1.96$

$$\text{t.s. is } z = \frac{(72.6 - 69.5) - 0}{\sqrt{\frac{18.3^2}{100} + \frac{15.4^2}{150}}} = 1.396\ldots < 1.96$$

Result is not significant so accept H_0.

Central Limit Theorem applies since n_1, n_2 are both large and enables you to assume $\bar{X}_1$ and $\bar{X}_2$ are normally distributed.

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Estimation, confidence intervals and tests
Exercise G, Question 6

Question:

Carry out a test on the given hypothesis at the given level of significance. What is the significance of the Central Limit Theorem in these three questions?
$H_0: \mu_1 = \mu_2$; $H_1: \mu_1 < \mu_2$; $n_1 = 120$, $\sigma_1 = 0.013$, $n_2 = 90$, $\sigma_2 = 0.015$, $\bar{x}_1 = 0.863$ and $\bar{x}_2 = 0.868$ using a 1% level

Solution:

$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 < \mu_2$ 1% c.v. is $z = -2.3263$

$$\text{t.s. is } z = \frac{(0.863 - 0.868) - 0}{\sqrt{\frac{0.013^2}{120} + \frac{0.015^2}{90}}} = -2.5291\ldots < -2.3263$$

Result is significant so reject H_0.
Central Limit Theorem is used to assume $\bar{X}_1$ and $\bar{X}_2$ are normally distributed since both samples are large.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise G, Question 7

Question:

A certain factory has two machines designed to cut piping. The first machine works to a standard deviation of 0.011 cm and the second machine has a standard deviation of 0.015 cm. A random sample of 10 pieces of piping from the first machine has a mean length of 6.531 cm and a random sample of 15 pieces from the second machine has a length of 6.524 cm. Assuming that the lengths of piping follow a normal distribution, test, at the 5% level, whether or not the machines are producing piping of the same mean length.

Solution:

$\sigma_1 = 0.011 \quad n_1 = 10 \quad \bar{x}_1 = 6.531$
$\sigma_2 = 0.015 \quad n_2 = 15 \quad \bar{x}_2 = 6.524$

$\mathrm{H}_0 : \mu_1 = \mu_2 \quad \mathrm{H}_1 : \mu_1 \neq \mu_2, \quad 5\%$ c.v. is $z = \pm 1.96$

$$\text{t.s. is } z = \frac{(6.531 - 6.524) - 0}{\sqrt{\frac{0.011^2}{10} + \frac{0.015^2}{15}}} = 1.34466\ldots < 1.96$$

Not significant.
Accept H_0.
There is insufficient evidence to suggest that the machines are producing pipes of different lengths.

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Estimation, confidence intervals and tests
Exercise G, Question 8

Question:

A certain health authority set up an investigation to examine the ages of mothers when they give birth to their first children.
A random sample of 250 first-time mothers from a certain year had a mean age of 22.45 years with a standard deviation of 2.9 years. A further random sample of 280 first-time mothers taken 10 years later had a mean age of 22.96 years with a standard deviation of 2.8 years.

a Test whether or not these figures suggest that there is a difference in the mean age of first-time mothers between these two dates.

b State any assumptions you have made about the distribution of ages of first-time mothers.

Solution:

$n_1 = 250 \quad \bar{x}_1 = 22.45 \quad S_1 = 2.9$

$n_2 = 280 \quad \bar{x}_2 = 22.96 \quad S_2 = 2.8$

Assume $S_i = \sigma_i$ since samples are large.

$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_1 \neq \mu_2$

Use 5% significance level
c.v. is $Z = \pm 1.96$

a $$\text{t.s.} = Z = \frac{(22.96 - 22.45) - 0}{\sqrt{\frac{2.9^2}{250} + \frac{2.8^2}{280}}}$$

Use $\bar{x}_2 - \bar{x}_1$ to make

t.s. $= Z = 2.054\ldots > 1.96$

Result is significant.

There is evidence of a difference in mean age of first-time mothers between these two dates.

b There is no need to have to assume that both populations were normally distributed since both samples were large so the Central Limit Theorem allows you to assume both sample means are normally distributed.
We have assumed that $S_1 = \sigma_1$ and $S_2 = \sigma_2$

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Estimation, confidence intervals and tests
Exercise H, Question 1

Question:

An experiment was conducted to compare the drying properties of two paints, Quickdry and Speedicover. In the experiment, 200 similar pieces of metal were painted, 100 randomly allocated to Quickdry and the rest to Speedicover.
The table below summarises the times, in minutes, taken for these pieces of metal to become touch-dry.

	Quickdry	Speedicover
Mean	28.7	30.6
Standard deviation	7.32	3.51

Using a 5% significance level, test whether or not the mean time for Quickdry to become touch-dry is less than that for Speedicover. State your hypotheses clearly. ***E***

Solution:

$n_Q = 100 \quad \bar{x}_Q = 28.7 \quad s_Q = 7.32$

$n_S = 100 \quad \bar{x}_S = 30.6 \quad s_S = 3.51$

$\mathrm{H}_0 : \mu_Q = \mu_S \quad \mathrm{H}_1 : \mu_Q < \mu_S$ (i.e. Quickdry dries in a shorter time than Speedicover.)

$$\text{t.s. is } z = \frac{(30.6 - 28.7) - 0}{\sqrt{\frac{7.32^2}{100} + \frac{3.51^2}{100}}} = 2.34$$

Test $\mu_S > \mu_Q$ to get $z > 0$.

5% c.v. is $z = 1.6449$

t.s. is $2.34 > 1.6449$ so the result is significant.
There is evidence that Quickdry dries faster than Speedicover.

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Estimation, confidence intervals and tests
Exercise H, Question 2

Question:

A supermarket examined a random sample of 80 weekend shoppers' purchases and an independent random sample of 120 weekday shoppers' purchases. The results are summarised in the table below.

	n	$\bar{x}$	s
Weekend	80	38.64	6.59
Weekday	120	40.13	8.23

a Stating your hypotheses clearly test, at the 5% level of significance, whether there is evidence that the mean expenditure in the week is more than at weekends.

b State an assumption you have made in carrying out this test.

Solution:

a $n_1 = 80 \quad \bar{x}_1 = 38.64 \quad s_1 = 6.59$

$n_2 = 120 \quad \bar{x}_2 = 40.13 \quad s_2 = 8.23$

$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_2 > \mu_1$ 5% c.v. is $z = 1.6449$

$$\text{t.s. is } z = \frac{(40.13 - 38.64) - 0}{\sqrt{\frac{6.59^2}{80} + \frac{8.23^2}{120}}} = 1.4159 < 1.6449$$

Not significant
There is insufficient evidence to confirm that mean expenditure in the week is more than at week ends.

b We have assumed that $s_1 = \sigma_1$ and $s_2 = \sigma_2$.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise H, Question 3

Question:

It is claimed that the masses of components produced in a small factory have a mean mass of 10 g. A random sample of 250 of these components is tested and the sample mean, $\bar{x}$, is 9.88 g and the standard deviation, s, is 1.12 g.

a Test, at the 5% level, whether or not there has been change in the mean mass of a component.

b State any assumptions you would make to carry out this test.

Solution:

$s = \sigma = 1.12, n = 250, \bar{x} = 9.88$

a $H_0 : \mu = 10 \quad H_1 : \mu \neq 10$ 5% c.v. is $z = \pm 1.96$

$$\text{t.s. is } z = \frac{9.88 - 10}{\sqrt{\frac{1.12^2}{250}}} = -1.694\ldots > -1.96$$

Not significant
Insufficient evidence to support a change in mean mass.

b We have assumed that $s = \sigma$ since n is large.

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise H, Question 4

Question:

Two independent samples are taken from population A and population B. Carry out the following tests using the information given.

a $H_0: \mu_A = \mu_B \quad H_1: \mu_A > \mu_B$ using a 1% level of significance

$n_A = 90, \quad n_B = 110, \quad \bar{x}_A = 84.1, \quad \bar{x}_B = 87.9, \quad s_A = 12.5, \quad s_B = 14.6$

b $H_0: \mu_A - \mu_B = 2 \quad H_1: \mu_A - \mu_B > 2$ using a 5% level of significance

$n_A = 150, \quad n_B = 200, \quad \bar{x}_A = 125.1, \quad \bar{x}_B = 119.3, \quad s_A = 23.2, \quad s_B = 18.4$

c State an assumption that you have made in carrying out these tests.

Solution:

a $H_0: \mu_A = \mu_B \quad H_1: \mu_A < \mu_B \quad$ c.v. is $z = -2.3263$

$$\text{t.s. is } z = \frac{(84.1 - 87.9) - 0}{\sqrt{\frac{12.5^2}{90} + \frac{14.6^2}{110}}} = -1.9825\ldots > -2.3263$$

Not significant so accept H_0.

b $H_0: \mu_A - \mu_B = 2 \quad H_1: \mu_A - \mu_B > 2 \quad$ c.v. is $z = 1.6449$

$$\text{t.s. is } z = \frac{(125.1 - 119.3) - 2}{\sqrt{\frac{23.2^2}{150} + \frac{18.4^2}{200}}} = 1.6535\ldots > 1.6449$$

Significant so reject H_0.

c We have assumed $s_A = \sigma_A$ and $s_B = \sigma_B$ since the samples are both large.

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise H, Question 5

Question:

A shopkeeper complains that the average weight of chocolate bars of a certain type that he is buying from a wholesaler is less than the stated value of 85.0 g. The shopkeeper weighed 100 bars from a large delivery and found that their weights had a mean of 83.6 g and a standard deviation of 7.2 g. Using a 5% significance level, determine whether or not the shopkeeper is justified in his complaint. State clearly the null and alternative hypotheses that you are using, and express your conclusion in words. ***E***

Solution:

$n = 100,\ \bar{x} = 83.6,\ s = 7.2$

$\mathrm{H_0}: \mu = 85 \quad \mathrm{H_1}: \mu < 85 \quad$ c.v. is $z = -1.6449$

$$\text{t.s. is } z = \frac{(83.6 - 85)}{\left(\frac{7.2}{\sqrt{100}}\right)} = -1.944\ldots < -1.6449$$

Significant

There is evidence that the weights of chocolate bars are less than the stated value.

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 1

Question:

The breaking stresses of rubber bands are normally distributed. A company uses bands with a mean breaking stress of 46.50 N. A new supplier claims that they can supply bands that are stronger and provides a sample of 100 bands for the company to test. The company checked the breaking stress, x, for each of these 100 bands and the results are summarised as follows:

$n = 100 \quad \Sigma x = 4715 \quad \Sigma x^2 = 222\,910$

a Test, at the 5% level, whether or not there is evidence that the new bands are better.

b Find an approximate 95% confidence interval for the mean breaking stress of these new rubber bands.

Solution:

a $\bar{x} = \dfrac{4715}{100} = 47.15$

$$s = \sqrt{\frac{222\,910 - 100 \times 47.15^2}{99}} = 2.4572\ldots$$

$H_0 : \mu = 46.50$ (no better) $\qquad H_1 : \mu > 46.50$

$$\text{t.s. is } z = \frac{(47.15 - 46.50)}{\left(\dfrac{2.4572\ldots}{\sqrt{100}}\right)} = 2.645\ldots$$

5% c.v. is $z = 1.6449$

t.s. is $2.645 > 1.6449$

Result is significant so reject H_0.

There is evidence that the new bands are better.

b 95% C.I. is $\bar{x} \pm 1.96 \dfrac{\sigma}{\sqrt{n}}$

$$= 47.15 \pm 1.96 \times \frac{2.4572\ldots}{\sqrt{100}}$$

$= (46.6683\ldots, 47.6316\ldots)$

$= (46.7, 47.6) \quad$ (3 s.f.)

N.B. Since n is large, we have assumed $s = \sigma$.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 2

Question:

On each of 100 days a conservationist took a sample of 1 litre of water from a particular place along a river, and measured the amount, x mg, of chlorine in the sample. The results she obtained are shown in the table.

x	1	2	3	4	5	6	7	8	9
Number of days	4	8	20	22	16	13	10	6	1

a Calculate the mean amount of chlorine present per litre of water, and estimate, to 3 decimal places, the standard error of this mean.

b Obtain approximate 98% confidence limits for the mean amount of chlorine present per litre of water.

Given that measurements at the same point under the same conditions are taken for a further 100 days,

c estimate, to 3 decimal places, the probability that the mean of these measurements will be greater than 4.6 mg per litre of water. ***E***

Solution:

$n = 100, \quad \sum x = 453, \quad \sum x^2 = 2391$

a $\bar{x} = \frac{453}{100} = 4.53$

$s = \sqrt{\frac{2391 - 100 \times 4.53^2}{99}} = 1.85022\ldots$

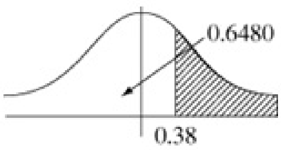
Standard error $= \frac{s}{\sqrt{n}} = 0.185$ (3 d.p.)

b 98% C.I. is $\bar{x} \pm 2.3263 \frac{\sigma}{\sqrt{n}}$ Use $\frac{s}{\sqrt{n}}$

$= (4.0995\ldots, 4.9604)$

$= (4.10, 4.96)$ (3 s.f.)

c



$P(\bar{x} > 4.6) = P\left(Z > \frac{4.6 - 4.53}{0.185\ldots}\right)$

$= P(Z > 0.378\ldots)$ Use 0.38

$= 1 - 0.6480$

$= 0.3520$ (tables)

or $= 0.35259\ldots$ (calculator)

So accept awrt $0.352 \sim 0.353$

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 3

Question:

The amount, to the nearest mg, of a certain chemical in particles in the atmosphere at a meterological station was measured each day for 300 days. The results are shown in the table.

Amount of chemical (mg)	12	13	14	15	16
Number of days	5	42	210	31	12

Find the mean daily amount of chemical over the 300 days and estimate, to 2 decimal places, its standard error.

Solution:

$n = 300, \sum x = 4203, \sum x^2 = 59\,025$

a $\bar{x} = \frac{4203}{300} = 14.01$

$s = \sqrt{\frac{59\,025 - 300 \times 14.01^2}{299}} = 0.6866\ldots$

$\text{Standard error} = \frac{s}{\sqrt{n}} = 0.039643\ldots = 0.04 \text{ (2 d.p.)}$

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Estimation, confidence intervals and tests
Exercise I, Question 4

Question:

From time to time a firm manufacturing pre-packed furniture needs to check the mean distance between pairs of holes drilled by machine in pieces of chipboard to ensure that no change has occurred. It is known from experience that the standard deviation of the distance is 0.43 mm. The firm intends to take a random sample of size n, and to calculate a 99% confidence interval for the mean of the population. The width of this interval must be no more than 0.60 mm.
Calculate the minimum value of n. ***E***

Solution:

$\sigma = 0.43$

Width of 99% C.I. is $2 \times 2.5758 \frac{\sigma}{\sqrt{n}}$

Require $\frac{2 \times 2.5758 \times 0.43}{\sqrt{n}} < 0.60$

$\therefore \sqrt{n} > \frac{2 \times 2.5758 \times 0.43}{0.6} = 3.691\ldots$

$n > 13.63\ldots$

So the smallest value of n is 14

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 5

Question:

The times taken by five-year-old children to complete a certain task are normally distributed with a standard deviation of 8.0s. A random sample of 25 five-year-old children from school A were given this task and their mean time was 44.2s.

a Find 95% confidence limits for the mean time taken by five-year-old children from school A to complete this task.

The mean time for a random sample of 20 five-year-old children from school B was 40.9 s. The headteacher of school B concluded that the overall mean for school B must be less than that of school A. Given that the two samples were independent,

b test the headteacher's conclusion using a 5% significance level. State your hypotheses clearly. ***E***

Solution:

$\sigma = 8.0$
$n_A = 25 \quad \bar{x}_A = 44.2$

a 95% C.I. is $44.2 \pm 1.96 \times \frac{8.0}{\sqrt{25}}$

$= (41.064, 47.336)$

$= (41.1, 47.3) \quad (3 \text{ s.f.})$

b $n_B = 20 \quad \bar{x}_B = 40.9$

$\mathrm{H}_0 : \mu_A = \mu_B \quad \mathrm{H}_1 : \mu_B < \mu_A$ 5% c.v. is $z = -1.6449$

$$\text{t.s. is } z = \frac{(40.9 - 44.2) - 0}{\sqrt{\frac{8^2}{20} + \frac{8^2}{25}}} = -1.375 > -1.6449$$

Not significant so accept H_0.

There is insufficient evidence to support the headteacher's claim.

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 6

Question:

The random variable X has a normal distribution with mean μ and standard deviation 2.
A random sample of 25 observations is taken and the sample mean $\bar{X}$ is calculated in order to test the null hypothesis $\mu = 7$ against the alternative hypothesis $\mu > 7$ using a 5% level of significance.
Find the critical region for $\bar{X}$. ***E***

Solution:

$X \sim \mathrm{N}(\mu, 2^2)$

$n = 25 \quad \bar{X} \sim \mathrm{N}\left(\mu, \left(\frac{2}{5}\right)^2\right)$

$\mathrm{H}_0 : \mu = 7 \quad \mathrm{H}_1 : \mu > 7$ c.v. is $z = 1.6449$

Reject H_0 for $Z > 1.6449$

$$Z = \frac{(\bar{X} - 7)}{\left(\frac{2}{5}\right)} \Rightarrow \bar{X} > 7 + 1.6449 \times \frac{2}{5}$$

i.e. $\bar{X} > 7.65796$

i.e. $\bar{X} > 7.66$ (3 s.f.)

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 7

Question:

A certain brand of mineral water comes in bottles. The amount of water in a bottle, in millilitres, follows a normal distribution of mean μ and standard deviation 2. The manufacturer claims that μ is 125. In order to maintain standards the manufacturer takes a sample of 15 bottles and calculates the mean amount of water per bottle to be 124.2 millilitres.
Test, at the 5% level, whether or not there is evidence that the value of μ is lower than the manufacturer's claim. State your hypotheses clearly. ***E***

Solution:

$B \sim \mathrm{N}(\mu, 2^2)$

$n = 15 \qquad \bar{B} \sim \mathrm{N}(\mu, \frac{4}{15}) \quad \bar{b} = 124.2$

$\mathrm{H}_0 : \mu = 125 \quad \mathrm{H}_1 : \mu < 125 \quad$ c.v. is $z = -1.6449$

$$\text{t.s. is } z = \frac{(124.2 - 125) - 0}{\sqrt{\frac{4}{15}}} = -1.5491\ldots > -1.6449$$

Not significant, so accept H_0.

There is insufficient evidence to suggest that the mean contents of a bottle is lower than the manufacturer's claim.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 8

Question:

The random variable X is normally distributed with mean μ and variance σ^2.

a Write down the distribution of the sample mean $\bar{X}$ of a random sample of size n.

An efficiency expert wishes to determine the mean time taken to drill a fixed number of holes in a metal sheet.

b Determine how large a random sample is needed so that the expert can be 95% certain that the sample mean time will differ from the true mean time by less than 15 seconds. Assume that it is known from previous studies that $\sigma = 40$ seconds. ***E***

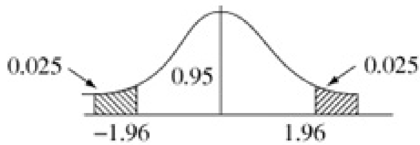
Solution:

$X \sim N(\mu, \sigma^2)$

a $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

b $P(|\bar{X} - \mu| < 15) = P\left(|Z| < \frac{15}{\left(\frac{\sigma}{\sqrt{n}}\right)}\right)$

Require $P\left(|Z| < \frac{15\sqrt{n}}{\sigma}\right) > 0.95$



i.e. $\frac{15\sqrt{n}}{\sigma} > 1.96$

$\sigma = 40 \Rightarrow \sqrt{n} > \frac{40 \times 1.96}{15} = 5.2266\ldots$

$\therefore n > 27.318\ldots$

So need $n = 28$ or more

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests

Exercise I, Question 9

Question:

A commuter regularly uses a train service which should arrive in London at 0931. He decided to test this stated arrival time. Each working day for a period of 4 weeks he recorded the number of minutes x that the train was late on arrival in London. If the train arrived early then the value of x was negative. His results are summarised as follows:

$n = 20$, $\Sigma x = 15.0$, $\Sigma x^2 = 103.21$.

a Calculate unbiased estimates of the mean and variance of the number of minutes late of his train service.

The random variable X represents the number of minutes that the train is late on arriving in London. Records kept by the railway company show that over fairly short periods, the standard deviation of X is 2.5 minutes. The commuter made 2 assumptions about the distribution of X and the values obtained in the sample and went on to calculate a 95% confidence interval for the mean arrival time of this train service.

b State the two assumptions.

c Find the confidence interval.

d Given that the assumptions are reasonable, comment on the stated arrival time of the service. ***E***

Solution:

a $\bar{x} = \frac{\sum x}{n} = \frac{15.0}{20} = 0.75$

$$s^2 = \frac{103.21 - 20 \times 0.75^2}{19} = 4.84$$

b $\sigma = 2.5$

i assume that X has a normal distribution

ii assume that the sample was random.

c 95% C.I. is $\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$

$$= 0.75 \pm 1.96 \times \frac{2.5}{\sqrt{20}}$$

$$= (-0.34567\ldots, 1.8456\ldots)$$

$$= (-0.346, 1.85) \quad (3\text{ s.f.})$$

d Since 0 is in the interval it is reasonable to assume that trains do arrive on time.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 10

Question:

The random variable X is normally distributed with mean μ and variance σ^2.

a Write down the distribution of the sample mean $\bar{X}$ of a random sample of size n.

b Explain what you understand by a 95% confidence interval.

A garage sells both leaded and unleaded petrol. The distribution of the values of sales for each type is normal. During 1990 the standard deviation of individual sales of each type of petrol is £3.25. The mean of the individual sales of leaded petrol during this time is £8.72. A random sample of 100 individual sales of unleaded petrol gave a mean of £9.71.

Calculate

c an interval within which 90% of the sales of leaded petrol will lie,

d a 95% confidence interval for the mean sales of unleaded petrol.

The mean of the sales of unleaded petrol for 1989 was £9.10.

e Using a 5% significance level, investigate whether there is sufficient evidence to conclude that the mean of all the 1990 unleaded sales was *greater* than the mean of the 1989 sales.

f Find the size of the sample that should be taken so that the garage proprietor can be 95% certain that the sample mean of sales of unleaded petrol during 1990 will differ from the true mean by less than 50p. ***E***

Solution:

a $\bar{X} \sim \mathrm{N}(\mu, \frac{\sigma^2}{n})$

b 95% C.I. is an interval within which we are 95% confident μ lies.

$L =$ sales of leaded petrol $\quad U =$ sales of unleaded petrol

$L \sim N(8.72, 3.25^2) \quad U \sim N(9.71, 3.25^2)$

c 90% of L between $8.72 \pm 1.6449 \times 3.25$

$= (3.3740\ldots, 14.0659\ldots)$

$= (3.37, 14.1) \quad (3\text{ s.f.})$

d $n = 100 \quad \bar{u} = 9.71$

95% C.I. for μ_u is: $9.71 \pm 1.96 \times \frac{3.25}{\sqrt{100}}$

$= (9.073, 10.347)$

$= (9.07, 10.35)$ (nearest penny)

e $\mathrm{H}_0 : \mu_u = 9.10$ (i.e. same as 1989) $\quad \mathrm{H}_1 : \mu_u > 9.10$ (1990 sales > 1989 sales)

5% c.v. is $z = 1.6449$

t.s. is $z = \frac{(9.71 - 9.10)}{\left(\frac{3.25}{\sqrt{100}}\right)} = 1.8769\ldots > 1.6449$ Significant so reject H_0.

There is evidence that the mean sales of unleaded petrol in 1990 were greater than in 1989.

f $\mathrm{P}(|\bar{U} - \mu_u| < 0.50) > 0.95 \Rightarrow \frac{0.5\sqrt{n}}{3.25} > 1.96$

i.e. $\sqrt{n} > 12.74$ or $n > 162.30\ldots \therefore n = 163$

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 11

Question:

a Explain what is meant by a 98% confidence interval for a population mean.

The lengths, in cm, of the leaves of willow trees are known to be normally distributed with variance 1.33 cm^2.

A sample of 40 willow tree leaves is found to have a mean of 10.20 cm.

b Estimate, giving your answer to 3 decimal places, the standard error of the mean.

c Use this value to estimate symmetrical 95% confidence limits for the mean length of the population of willow tree leaves, giving your answer to 2 decimal places.

d Find the minimum size of the sample of leaves which must be taken if the width of the symmetrical 98% confidence interval for the population mean is at most 1.50 cm. ***E***

Solution:

a A 98% C.I. is an interval within which we are 98% sure the population mean will lie.

b $L =$ length of willow tree leaves $\quad L \sim \mathrm{N}(\mu, 1.33)$

$n = 40, \bar{L} = 10.20$

Standard error of the mean $= \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{1.33}}{\sqrt{40}} = 0.18234\ldots$

$= 0.182$ (3 d.p.)

c 95% C.I. is $10.20 \pm 1.96 \times 0.182\cdots$

$= (9.8426\ldots, 10.5573\ldots)$

$= (9.84, 10.56)$ (2 d.p.)

d Width of 98% C.I. is $2 \times 2.3263 \times \frac{\sigma}{\sqrt{n}}$

$\therefore$ Require $\frac{2 \times 2.3263 \times \sqrt{1.33}}{\sqrt{n}} < 1.50$

or $\sqrt{n} > 3.57\ldots$

i.e. $n > 12.79\ldots$

$\therefore$ need $n = 13$

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 12

Question:

The distance driven by a long distance lorry driver in a week is a normally distributed variable having mean 1130 km and standard deviation 106 km.

a Find, to 3 decimal places, the probability that in a given week he will drive less than 1000 km.

b Find, to 3 decimal places, the probability that in 20 weeks his average distance driven per week is more than 1200 km.

New driving regulations are introduced and, in the first 20 weeks after their introduction, he drives a total of 21 900 km.

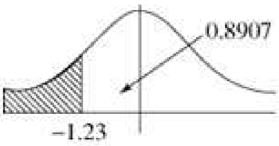
Assuming that the standard deviation of the weekly distances he drives is unchanged,

c test, at the 10% level of significance, if his mean weekly driving distance has been reduced. State clearly your null and alternative hypotheses. ***E***

Solution:

$D \sim N(1130, 106^2)$

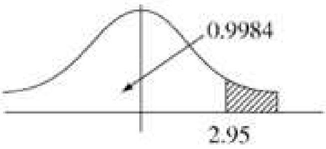
a



$$\begin{aligned} P(D < 1000) &= P\left(Z < \frac{1000-1130}{106}\right) \\ &= P(Z < -1.2264\ldots) \quad \text{Use } -1.23 \\ &= 1 - 0.8907 \\ &= 0.1093 \quad \text{(tables)} \\ \text{or} \quad &= 0.11002\ldots \quad \text{(calculator)} \end{aligned}$$

So accept 0.109 or 0.110 (3 d.p.)

b $n = 20 \quad \bar{D} \sim N\left(1130, \frac{106^2}{20}\right)$



$$\begin{aligned} P(\bar{D} > 1200) &= P\left(Z > \frac{(1200-1130)}{\left(\frac{106}{\sqrt{20}}\right)}\right) \\ &= P(Z > 2.953\ldots) \quad \text{Use } 2.95 \\ &= 1 - 0.9984 \\ &= 0.0016 \quad \text{(tables)} \\ \text{or} \quad &= 0.001572\ldots \quad \text{(calculator)} \end{aligned}$$

So accept 0.002 (3 d.p.)

c $H_0 : \mu = 1130$ (same) $\qquad H_1 : \mu < 1130$ (reduced)

10% c.v. is $z = -1.2816$

$$\bar{d} = \frac{21\,900}{20} = 1095$$

$$\therefore \text{ t.s. is } z = \frac{(1095-1130)}{\left(\frac{106}{\sqrt{20}}\right)} = -1.4766\ldots < -1.2816$$

Significant so reject H_0

There is evidence that his mean weekly driving distance has been reduced.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 13

Question:

Climbing rope produced by a manufacturer is known to be such that one-metre lengths have breaking strengths that are normally distributed with mean 170.2 kg and standard deviation 10.5 kg. Find, to 3 decimal places, the probability that

a a one-metre length of rope chosen at random from those produced by the manufacturer will have a breaking strength of 175 kg to the nearest kg,

b a random sample of 50 one-metre lengths will have a mean breaking strength of more than 172.4 kg.

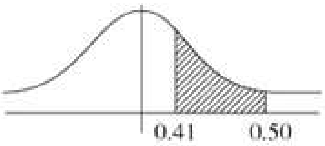
A new component material is added to the ropes being produced. The manufacturer believes that this will increase the mean breaking strength without changing the standard deviation. A random sample of 50 one-metre lengths of the new rope is found to have a mean breaking strength of 172.4 kg.

c Perform a significance test at the 5% level to decide whether this result provides sufficient evidence to confirm the manufacturer's belief that the mean breaking strength is increased. State clearly the null and alternative hypotheses that you are using. [E]

Solution:

B = breaking strength $\sim N(170.2, 10.5^2)$

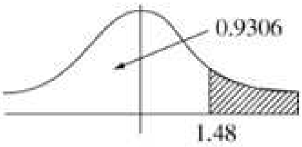
a



$$\begin{aligned}P(174.5 < B < 175.5) &= P(0.4095 < Z < 0.5047\cdots)\\ &= 0.6915 - 0.6591\\ &= 0.0324 \qquad \text{(tables)}\\ \text{or} &= 0.034214\ldots \qquad \text{(calculator)}\end{aligned}$$

So accept $0.032 \sim 0.034$

b $n = 50$ $\quad \bar{B} \sim N\left(170.2, \frac{10.5^2}{50}\right)$



$$P(\bar{B} > 172.4) = P\left(Z > \frac{172.4 - 170.2}{\frac{10.5}{\sqrt{50}}}\right)$$

$= P(Z > 1.4815\ldots)$ Use 1.48

$= 1 - 0.9306$

$= 0.0694$ (tables)

or $= 0.069229\ldots$ (calculator)

i.e. accept awrt 0.069

c $H_0 : \mu = 170.2$ $\quad H_1 : \mu > 170.2$ $\qquad$ 5% c.v. is $z = 1.6449$

t.s. is $z = \dfrac{(172.4 - 170.2)}{\left(\dfrac{10.5}{\sqrt{50}}\right)} = 1.4815\ldots < 1.6449$ $\quad \therefore$ Not significant so accept H_0

Insufficient evidence of an increase in the mean breaking strength of climbing rope.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests

Exercise I, Question 14

Question:

A machine fills 1 kg packets of sugar. The actual weight of sugar delivered to each packet can be assumed to be normally distributed. The manufacturer requires that,

i the mean weight of the contents of a packet is 1010 g, and

ii 95% of all packets filled by the machine contain between 1000 g and 1020 g of sugar.

a Show that this is equivalent to demanding that the variance of the sampling distribution, to 2 decimal places, is equal to 26.03 g^2.

A sample of 8 packets was selected at random from those filled by the machine.
The weights, in grams, of the contents of these packets were,
1012.6 1017.7 1015.2 1015.7 1020.9 1005.7 1009.9 1011.4.

Assuming that the variance of the actual weights is 26.03 g^2,

b test at the 2% significance level, (stating clearly the null and alternative hypotheses that you are using) to decide whether this sample provides sufficient evidence to conclude that the machine is not fulfilling condition **i**. ***E***

Solution:

W = weight of sugar

$W \sim \text{N}(1010, \sigma^2)$

a $\text{P}(1000 < W < 1020) = 0.95$

$\Rightarrow \quad 1.96\sigma = 10$

$\sigma = 5.102\ldots \quad \sigma^2 = 26.0308\ldots \;\therefore\; \sigma^2 = 26.03 \text{ (2 d.p.)}$

b $n = 8, \sum x = 8109.1, \; \left(\sum x^2 = 8\,219\,846.85\right)$

$\bar{x} = 1013.6375$

$\text{H}_0: \mu = 1010 \quad \text{H}_1: \mu \neq 1010$ 2% c.v. is $z = \pm 2.3263$

$$\text{t.s. is } z = \frac{(1013.6375 - 1010)}{\left(\frac{\sqrt{26.03}}{\sqrt{8}}\right)} = 2.0165\ldots < 2.3263$$

Not significant so accept H_0.

There is insufficient evidence of a deviation in mean from 1010, so we can assume that condition **i** is being met.

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Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 15

Question:

a Write down the mean and the variance of the distribution of the means of all possible samples of size n taken from an infinite population having mean μ and variance σ^2.

b Describe the form of this distribution of sample means when

i n is large,

ii the distribution of the population is normal.

The standard deviation of all the till receipts of a supermarket during 1984 was £4.25.

c Given that the mean of a random sample of 100 of the till receipts is £18.50, obtain an approximate 95% confidence interval for the mean of all the till receipts during 1984.

d Find the size of sample that should be taken so that the management can be 95% confident that the sample mean will not differ from the true mean by more than 50p.

e The mean of all the till receipts of the supermarket during 1983 was £19.40. Using a 5% significance level, investigate whether the sample in **a** above provides sufficient evidence to conclude that the mean of all the 1984 till receipts is different from that in 1983. ***E***

Solution:

a $\mathrm{E}(\bar{X}) = \mu \quad \mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$

b **i** By Central Limit Theorem $\bar{X} \simeq \sim$ Normal i.e. $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$

ii $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$

$\sigma = 4.25$

c $n = 100 \quad \bar{x} = 18.50$

95% C.I. is $18.50 \pm 1.96 \times \dfrac{4.25}{\sqrt{100}}$

$= \ (17.667, 19.333)$

$= \ (17.7, 19.3) \quad (3\text{ s.f.})$

d $\mathrm{P}(|\bar{X} - \mu| < 0.50) > 0.95$

i.e. $\dfrac{0.50 \times \sqrt{n}}{4.25} > 1.96$

i.e. $\sqrt{n} > 16.66\ldots$ i.e. $n > 277.55\ldots$

i.e. $n = 278$ or more

e $\mathrm{H}_0 : \mu = 19.4 \quad \mathrm{H}_1 : \mu \neq 19.4$ 5% c.v. is $z = \pm 1.96$

t.s. is $z = \dfrac{(18.50 - 19.4)}{\left(\dfrac{4.25}{\sqrt{100}}\right)} = -2.1176\ldots \quad < -1.96$ Significant so reject H_0.

There is evidence that the mean of till receipts in 1984 is different from the mean value in 1983.

Solutionbank S3

Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests

Exercise I, Question 16

Question:

The diameters of eggs of the little gull are approximately normally distributed with mean 4.11 cm and standard deviation 0.19 cm.

a Calculate the probability that an egg chosen at random has a diameter between 3.9 cm and 4.5 cm.

A sample of 8 little gull eggs was collected from a particular island and their diameters, in cm, were

4.4, 4.5, 4.1, 3.9, 4.4, 4.6, 4.5, 4.1

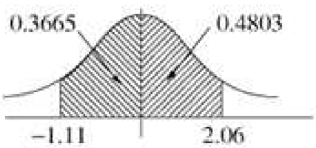
Assuming that the standard deviation of the diameters of eggs from the island is also 0.19 cm,

b test, at the 1% level, whether the results indicate that the mean diameter of little gull eggs on this island is different from elsewhere. ***E***

Solution:

$D =$ diameter $\sim N(4.11, 0.19^2)$

a



$P(3.9 < D < 4.5) = P(-1.105\ldots < Z < 2.052\ldots)$

$= 0.4803 + 0.3665$

$= 0.8468$ (tables)

or $= 0.845423\ldots$ (calculator)

so accept awrt $0.845 \sim 0.847$

b $\sigma = 0.19, n = 8, \sum x = 34.5, \bar{x} = 4.3125$

$H_0 : \mu = 4.11 \quad H_1 : \mu \neq 4.11$ 1% significance level c.v. is $z = \pm 2.5758$

t.s. is $z = \dfrac{(4.3125 - 4.11)}{\left(\dfrac{0.19}{\sqrt{8}}\right)} = 3.0145\ldots > 2.5758$ Significant so reject H_0.

There is evidence that the mean length of eggs from this island is different from elsewhere.

Solutionbank S3
Edexcel AS and A Level Modular Mathematics

Estimation, confidence intervals and tests
Exercise I, Question 17

Question:

Records of the diameters of spherical ball bearings produced on a certain machine indicate that the diameters are normally distributed with mean 0.824 cm and standard deviation 0.046 cm. Two hundred samples, each consisting of 100 ball bearings, are chosen.

a Calculate the expected number of the 200 samples having a mean diameter less than 0.823 cm.

On a certain day it was suspected that the machine was malfunctioning. It may be assumed that if the machine is malfunctioning it will change the mean of the diameters without changing their standard deviation. On that day a random sample of 100 ball bearings had mean diameter of 0.834 cm.

b Determine a 98% confidence interval for the mean diameter of the ball bearings being produced that day.

c Hence state whether or not you would conclude that the machine is malfunctioning on that day given that the significance level is 2%. ***E***

Solution:

$D = \text{diameter} \sim \text{N}(0.824, 0.046^2)$
$n = 100$

a $\text{P}(\bar{D} < 0.823) = \text{P}\left(Z < \frac{(0.823 - 0.824)}{\left(\frac{0.046}{\sqrt{100}}\right)}\right)$

$= \text{P}(Z < -0.2173\ldots)$ Use -0.22

$= 1 - 0.5871$

$= 0.4129$ (tables)

or $= 0.41395\ldots$ (calculator)

i.e. accept awrt $0.413 \sim 0.414$

So out of $200 \simeq 83$ samples will have mean < 0.823

b $n = 100, \bar{d} = 0.834$

98% C.I. is $0.834 \pm 2.3263 \times \frac{0.046}{\sqrt{100}}$

$= (0.82329\ldots, 0.84470\ldots)$

$= (0.823, 0.845)$ (3 s.f.)

c Since 0.824 is *in* the C.I. we can conclude that there is insufficient evidence of a malfunction.