Solutionbank S2
Edexcel AS and A Level Modular Mathematics

2 Review Exercise
Exercise A, Question 1

Question:

The random variable X is uniformly distributed over the interval $[-1, 5]$.

a Sketch the probability density function, $f(x)$, of X.

Find

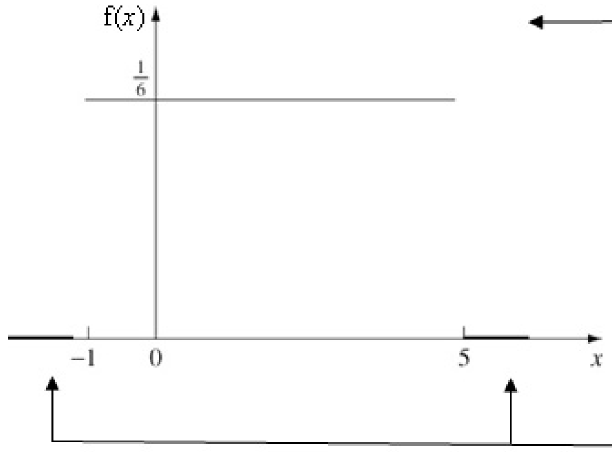
b $E(X)$,

c $Var(X)$,

d $P(-0.3 < X < 3.3)$. ***E***

Solution:

a



$f(x) = \frac{1}{5+1} = \frac{1}{6}$ for $-1 \le x \le 5$

Label the key values $x = -1, x = 5$ and $f(x) = \frac{1}{6}$.

Don't forget $x > 5$ and $x < -1$.

b

$$E(X) = -1 + 3 = 2 \text{ or}$$
$$E(X) = \frac{-1+5}{2} = 2$$

Use symmetry rather than integration.

c

$$Var(X) = \frac{1}{12}(5+1)^2 = 3$$

Use the formula for a uniform distribution rather than integration.

d

$$P(-0.3 < X < 3.3) = 3.6 \times \frac{1}{6} = 0.6$$

Use *area under graph* from $x = -0.3$ to $x = 3.3$ rather than definite integration. It is easier!

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2 Review Exercise

Exercise A, Question 2

Question:

A bag contains a large number of coins. Half of them are 1p coins, one third are 2p coins and the remainder are 5p coins.

a Find the mean and variance of the value of the coins.

A random sample of 2 coins is chosen from the bag.

b List all the possible samples that can be drawn.

c Find the sampling distribution of the mean value of these samples. ***E***

Solution:

a

X	1	2	5
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Draw a probability distribution table to help with calculations.

$\text{Mean} = 1\times\frac{1}{2}+2\times\frac{1}{3}+5\times\frac{1}{6}=2$

Use $\Sigma x P(X=x)$

Mean value is 2p or £0.02

$$\begin{aligned}\text{Variance} &= 1^2\times\frac{1}{2}+2^2\times\frac{1}{3}+5^2\times\frac{1}{6}-2^2\\ &= 2\end{aligned}$$

Use $\Sigma x^2 P(X=x)-\mu^2$

Variance has units of $(\text{pence})^2$.

b (1, 1)
(1, 2) and (2, 1)
(1, 5) and (5, 1)
(2, 2)
(2, 5) and (5, 2)
(5, 5)

2 coins drawn so easy to list as (first coin, second coin)

Write down the list carefully and don't miss any possible samples.

c

$\bar{x}$	1	1.5	2	3	3.5	5
$P(X=x)$	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$	$2\times\frac{1}{2}\times\frac{1}{3}=\frac{1}{3}$	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$	$\frac{1}{6}$	$2\times\frac{1}{3}\times\frac{1}{6}=\frac{1}{9}$	$\frac{1}{36}$

Distribution means set out in a table.

Possible means are

$$\frac{1+1}{2} = 1,\ \frac{1+2}{2}=1.5$$
$$\frac{1+5}{2} = 3,\ \frac{2+2}{2}=2$$
$$\frac{2+5}{2} = 3.5,\ \frac{5+5}{2}=5$$

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2 Review Exercise

Exercise A, Question 3

Question:

A teacher thinks that 20% of the pupils in a school read the Deano comic regularly.
He chooses 20 pupils at random and finds 9 of them read Deano.

a **i** Test, at the 5% level of significance, whether or not there is evidence that the percentage of pupils that read Deano is different from 20%. State your hypotheses clearly.

ii State all the possible numbers of pupils that read Deano from a sample of size 20 that will make the test in part **a i** significant at the 5% level.

The teacher takes another 4 random samples of size 20 and they contain 1, 3, 1 and 4 pupils that read Deano.

b By combining all 5 samples and using a suitable approximation test, at the 5% level of significance, whether or not this provides evidence that the percentage of pupils in the school that read Deano is not 20%.

c Comment on your results for the tests in part **a** and part **b**. ***E***

Solution:

a **i**

$H_0 : p = 0.2$
$H_1 : p \neq 0.2$

Two-tailed test as 'is different from'

$P(X \geq 9) = 1 - P(x \leq 8)$
$= 1 - 0.9900$
$= 0.0100 < 0.025$

$X \sim B(20, 0.2)$ so use tables to look up $P(X \leq 8)$.

Compare with 0.025 at 5%, two-tailed test.

Reject H_0.
Evidence that the percentage of pupils that read Deano is not 20%, it is more than 20%.

State conclusion in context from question.

ii From tables

$P(X = 0) = 0.0115 < 0.025$
$P(X \leq 1) = 0.0692 > 0.025$
$P(X \geq 8) = 1 - 0.9679 = 0.032 > 0.025$
$P(X \geq 9) = 0.0100 < 0.025$
All possible values are 0 or [9, 20] or 0 and 9 or more.

Compare with 0.025 so include 0.

8 not included as > 0.025.

Upper limit is sample size of 20.

b

$H_0 : p = 0.2$
$H_1 : p \neq 0.2$

Two-tailed test as question says 'is not 20%'.

$W \sim B(100, 0.2)$
$W \sim N(20, 16)$
Total $= 9 + 1 + 3 + 1 + 4 = 18$

Use normal approximation to binomial, N(*np*, *npq*).

$$P(X \leq 18) = P\left(Z \leq \frac{18.5 - 20}{4}\right)$$
$$= P(Z \leq -0.375)$$
$$= 0.352$$

Include 18 so use continuity correction +0.5 then standardise.

Use tables: you do not need to interpolate.

$0.352 > 0.025$ so insufficient evidence to reject H_0.
Combined numbers of Deano readers suggests there is no reason to doubt 20% of pupils read Deano.

Write conclusion in context.

c

In part **a** we *rejected* H_0
In part **b** we had *insufficient evidence* to reject H_0.
The results are *different*.
Either sample size matters and
larger samples give more reliable results
or not all pupils are drawn from the same population.

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2 Review Exercise

Exercise A, Question 4

Question:

The continuous random variable X is uniformly distributed over the interval [2, 6].

a Write down the probability density function f(x).

Find

b E(X),

c Var(X),

d the cumulative distribution function of X, for all x,

e $P(2.3 < X < 3.4)$. ***E***

Solution:

a $f(x) = \begin{cases} \frac{1}{4}, & 2 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}$

$\frac{1}{6-2} = \frac{1}{4}$

b $E(x) = 4$

By symmetry half way between 2 and 6.

c

$$\text{Var}(X) = \frac{(6-2)^2}{12} = \frac{4}{3}$$

Using variance formula for a uniform distribution.

d

$$F(x) = \int_2^x \frac{1}{4}\,dt = \left[\frac{1}{4}t\right]_2^x = \frac{1}{4}(x-2)$$

Remember $F(x) = \int_{-\infty}^{x} f(x)\,dx$ but use t with a variable upper limit of x.

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{4}(x-2), & 2 \le x \le 6 \\ 1, & x > 6 \end{cases}$$

Remember the ends.

e

$$P(2.3 < X < 3.4) = \frac{1}{4}(3.4 - 2.3) = 0.275$$

Area of rectangle under f(x), height $\frac{1}{4}$.

Alternative method

$$P(2.3 < X < 3.4) = F(3.4) - F(2.3) = \frac{1}{4}(3.4-2) - \frac{1}{4}(2.3-2) = 0.275$$

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2 Review Exercise
Exercise A, Question 5

Question:

The random variable X is the number of misprints per page in the first draft of a novel.

a State two conditions under which a Poisson distribution is a suitable model for X.

The number of misprints per page has a Poisson distribution with mean 2.5. Find the probability that

b a randomly chosen page has no misprints,

c the total number of misprints on 2 randomly chosen pages is more than 7.

The first chapter contains 20 pages.

d Using a suitable approximation find, to 2 decimal places, the probability that the chapter will contain fewer than 40 misprints. ***E***

Solution:

a Misprints occur randomly and independently.
Misprints occur singly in space.
Misprints occur at a constant rate.

Use the context of misprints.

b

$$P(X=0) = e^{-2.5} = 0.0821$$

Question gives Po(2.5).

c For 2 pages, $Y \sim Po(5)$

$\lambda = 2 \times 2.5 = 5$

$$P(Y > 7) = 1 - P(Y \leq 7) = 1 - 0.8666 = 0.1334 = 0.133 (3 s.f.)$$

Question says 'more than 7' so 7 not included.

Use tables of Po(5).

d For 20 pages, $Y \sim Po(50)$
$Y \sim N(50, 50)$ approx

$\lambda = 20 \times 2.5 = 50$

Satisfies normal approximation to Poisson.

$$P(Y < 40) = P(Y \leq 39.5) = P\left(Z \leq \frac{39.5 - 50}{\sqrt{50}}\right) = P(Z \leq -1.4849) = 1 - 0.93 = 0.07$$

'Less than 40' so 40 not included so include continuity correction by $40 - 0.5 = 39.5$.

Standardise then use normal distribution tables to look up 2 values.

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2 Review Exercise
Exercise A, Question 6

Question:

Explain what you understand by

a a sampling unit,

b a sampling frame,

c a sampling distribution. ***E***

Solution:

a *Element* of the *population*.

b A *list of all* the sampling units.

A sampling frame is a list, register or database of sampling units.

c *All possible samples* are chosen from a population, the *values* of a statistic and the associated *probabilities* is a sampling distribution.

Usually displayed in a table.

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2 Review Exercise
Exercise A, Question 7

Question:

A drugs company claims that 75% of patients suffering from depression recover when treated with a new drug.
A random sample of 10 patients with depression is taken from a doctor's records.

a Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new drug.

Given that the claim is correct,

b find the probability that the treatment will be successful for exactly 6 patients.

The doctor believes that the claim is incorrect and the percentage who will recover is lower. From her records she took a random sample of 20 patients who had been treated with the new drug. She found that 13 had recovered.

c Stating your hypotheses clearly, test, at the 5% level of significance, the doctor's belief.

d From a sample of size 20, find the greatest number of patients who need to recover for the test in part **c** to be significant at the 1% level. *E*

Solution:

a $X \sim B(10, 0.75)$
where X is the random variable 'number of patients who recover when treated'

p is the probability that a patient recovers in the random sample we are told it is claimed $p = 0.75$.

b

$$P(X = 6) = P(X \leq 6) - P(X \leq 5)$$
$$= 0.9219 - 0.7759$$
$$= 0.146$$

'Claim is correct' means take $p = 0.75$.

Alternative method

$$P(X = 6) = \frac{10!}{6!4!} \times 0.75^6 \times 0.25^4$$
$$= 0.146$$

Use the binominal formula.

c

$H_0 : p = 0.75$
$H_1 : p < 0.75$
$X \sim B(20, 0.75)$

One-tailed test as 'lower' in question.

$$P(X \leq 13) = 1 - 0.7858$$
$$= 0.2142 > 0.05$$

Looking at 'left hand tail'.

One-tailed test so compare with 5%.

Insufficient evidence to reject H_0.
Doctor's belief is not supported.

Remember the context of 'doctor's belief'.

d

$$P(X \leq 9) = 1 - 0.9961 = 0.0039 < 0.01$$
$$P(X \leq 10) = 1 - 0.9861 = 0.0139 > 0.01$$

So greatest number of patients is 9.

Look at values in B(20, 0.75) table.
Compare with 0.01, i.e. 1% significance.

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2 Review Exercise
Exercise A, Question 8

Question:

a Explain what you understand by a census.

Each cooker produced at GT Engineering is stamped with a unique serial number. GT Engineering produces cookers in batches of 2000. Before selling them, they test a random sample of 5 to see what electric current overload they will take before breaking down.

b Give one reason, other than to save time and cost, why a sample is taken rather than a census.

c Suggest a suitable sampling frame from which to obtain this sample.

d Identify the sampling units. ***E***

Solution:

a A census is when *every member* of a *population* is investigated.

b This is destructive testing, so there would be no cookers left to sell if a census were taken.

'Cheap' or 'quick' is not enough for this part.

c A *list* of the serial numbers of the cookers.

Register/database is also OK for list.

d A cooker.

Serial number of a cooker is OK too.

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2 Review Exercise
Exercise A, Question 9

Question:

Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. She decides to try a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm. Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.
Test Dhriti's claim at the 5% level of significance. State your hypotheses clearly. ***E***

Solution:

$H_0: p = 0.3 \quad H_1: p > 0.3$ ← One-tailed test as question says 'increased'.

X is the random variable
'number of tomatoes greater than 4 cm'
$X \sim B(40, 0.3)$ ← Samples size n, $n = 40, p = 0.3$

$$\begin{aligned} P(X \geq 18) &= 1 - P(X \leq 17) \\ &= 1 - 0.9680 \\ &= 0.0320 < 0.05 \end{aligned}$$

← One-tailed test at 5% significance level.

Reject H_0
Dhiriti's claim is supported by sample. ← Remember context, you could write out the claim from the question too.

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2 Review Exercise
Exercise A, Question 10

Question:

The probability that a sunflower plant grows over 1.5 metres high is 0.25. A random sample of 40 sunflower plants is taken and each sunflower plant is measured and its height recorded.

a Find the probability that the number of sunflower plants over 1.5 m high is between 8 and 13 (inclusive) using

i a Poisson approximation,

ii a Normal approximation.

b Write down which of the approximations used in part **a** is a more accurate estimate of the probability.
You must give a reason for your answer. ***E***

Solution:

a Let X be the random variable 'the number of sunflower plants more than 1.5 m high'

i $X \sim \text{Po}(10)$ — $\lambda = 40 \times 0.25 = 10$

$$\begin{aligned} P(8 \le X \le 13) &= P(X \le 13) - P(X \le 7) \\ &= 0.8645 - 0.2202 \\ &= 0.6443 \\ &= 0.644 \text{ (3 s.f.)} \end{aligned}$$

Inclusive, so subtract $P(X \le 7)$.

Use tables for these values.

ii $X \sim N(10, 7.5)$

$np = 40 \times 0.25 = 10$
$npq = 10 \times 0.75 = 7.5$

$$P(7.5 \le X \le 13.5) = P\left(\frac{7.5 - 10}{\sqrt{7.5}} \le Z \le \frac{13.5 - 10}{\sqrt{7.5}}\right)$$

Inclusive so
$8 - 0.5 = 7.5$
$13 + 0.5 = 13.5$
for continuity correction.

Don't forget to square root 7.5.

$$\begin{aligned} &= P(-0.913 \le Z \le 1.278) \\ &= 0.8997 - (1 - 0.8186) \\ &= 0.7183 \\ &= 0.718 \text{ (3 s.f.)} \end{aligned}$$

Use normal tables.

-0.913 so use '$1-$'

b Normal approximation as n large,
p close to $\frac{1}{2}$ and $np = 10 > 5$.
(Exact binomial is 0.7148 $(0.8968 - 0.1820)$.)
The Poisson approximation shouldn't be used because p isn't small, it is bigger than 0.1.

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2 Review Exercise
Exercise A, Question 11

Question:

a Explain what you understand by

i an hypothesis test,

ii a critical region.

During term time, incoming calls to a school are thought to occur at a rate of 0.45 per minute. To test this, the number of calls during a random 20-minute interval, is recorded.

b Find the critical region for a two-tailed test of the hypothesis that the number of incoming calls occurs at a rate of 0.45 per 1-minute interval. The probability in each tail should be as close to 2.5% as possible.

c Write down the actual significance level of the above test.

In the school holidays, 1 call occurs in a 10-minute interval.

d Test, at the 5% level of significance, whether or not there is evidence that the rate of incoming calls is less during the school holidays than in term time. ***E***

Solution:

a

i A hypothesis test is where the value of a population parameter (whose assumed value is given in H_0) is tested against what value it takes if H_0 is rejected (this could be an increase, a decrease or a change).

ii A *range of values of a test stanstic* that would lead to the *rejection of the null hypothesis*.

b Let X be the random variable 'the number of incoming calls'

$X \sim \text{Po}(9)$

From table

$\lambda = 20 \times 0.45 = 9$

$$\begin{aligned} P(X \geq 16) &= 1 - P(X \leq 15) \\ &= 1 - 0.9780 \\ &= 0.0220 \text{ which is closest to } 0.025 \end{aligned}$$

$$\begin{aligned} P(X \geq 15) &= 1 - (X \leq 14) \\ &= 0.0415 > 2.5\% \end{aligned}$$

$P(X \leq 3) = 0.212$ which is closest to 0.025

$P(X \leq 4) = 0.0550 > 2.5\%$

Critical region $X \leq 3$ or $X \geq 16$

c Actual significance level

$0.0220 + 0.0212 = 0.0432$ or 4.32%

d $H_0 : \lambda = 0.45 \quad H_1 : \lambda < 0.45$

$\lambda = 4.5$

Testing 0.45 per 1-minute interval, but using $10 \times 0.45 = 4.5$ as the parameter λ to look up in the table.

$X \sim \text{Po}(4.5)$

$P(X \leq 1) = 0.0611 > 0.05$

Insufficient evidence to reject H_0.

The rate of incoming calls is *less* during the school holidays is *not* supported.

'is less' in question so one-tailed H_1 and conclusion clear.

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2 Review Exercise

Exercise A, Question 12

Question:

A string AB of length 5 cm is cut, in a random place C, into two pieces. The random variable X is the length of AC.

a Write down the name of the probability distribution of X and sketch the graph of its probability density function.

b Find the values of $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

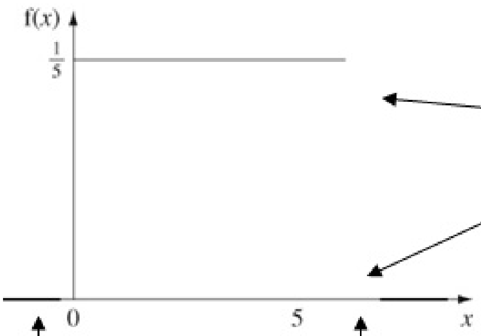
c Find $\mathrm{P}(X > 3)$.

d Write down the probability that AC is 3 cm long ***E***

Solution:

a Continuous uniform distribution

Alternatively, a rectangular distribution.



All your lines should be parallel to the x-axis.

Don't forget $x < 0$ and $x > 5$.

b

$$\mathrm{E}(X) = 2.5$$

By symmetry of sketch.

$$\mathrm{Var}(X) = \frac{(5-0)^2}{12} = \frac{25}{12}$$

Use the formula rather than integration.

c

$$\mathrm{P}(X > 3) = (5-3) \times \frac{1}{5} = \frac{2}{5}$$

Area of rectangle.

d $\mathrm{P}(X = 3) = 0$

It is a continuous distribution so an exact value has probability 0.

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2 Review Exercise
Exercise A, Question 13

Question:

Bacteria are randomly distributed in a river at a rate of 5 per litre of water. A new factory opens and a scientist claims it is polluting the river with bacteria. He takes a sample of 0.5 litres of water from the river near the factory and it contains 7 bacteria. Stating your hypotheses clearly, test his claim at the 5% level of significance. ***E***

Solution:

$H_0 : \lambda = 5$ ← '5 per litre of water'

$H_1 : \lambda > 5$ ← 'Polluting the river' so bacteria being added, so one-tailed test.

$X \sim \text{Po}(2.5)$

$P(X \geq 7) = 1 - P(X \leq 6)$ ← Use $\lambda = 5 \times 0.5 = 2.5$ table.

$= 1 - 0.9858$

$= 0.0142 < 0.05$

Reject H_0

There is significant evidence that near the factory the river is polluted with bacteria at the 5% level. ← You can also say 'scientist's claim is justified'.

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2 Review Exercise
Exercise A, Question 14

Question:

A bag contains a large number of coins

75% are 10p coins,

25% are 5p coins.

A random sample of 3 coins is drawn from the bag.

Find the sampling distribution for the median of the values of the 3 selected coins. ***E***

Solution:

Possible samples of 3 coins are

(5, 5, 5)

(5, 5, 10) in any order (3 cases) ← (5, 10, 5) or (10, 5, 5)

(10, 10, 5) in any order (3 cases) ← (10, 5, 10) or (5, 10, 10)

(10, 10, 10)

8 cases altogether with median of 5 or 10.

Median 5 ← (5, 5, 5) and (5, 5, 10) have median of 5.

$$P(M=5) = \left(\frac{1}{4}\right)^3 + 3\times\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)$$

$$= \frac{10}{64} \text{ or } \frac{5}{32}$$

Remember the 3 cases.

Median 10

$$P(M=10) = \left(\frac{3}{4}\right)^3 + 3\times\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)$$

$$= \frac{54}{64} \text{ or } \frac{27}{32}$$

(5, 10, 10) and (10, 10, 10) have median of 10.

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2 Review Exercise
Exercise A, Question 15

Question:

Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week with the new driver, the taxi is late 3 times.
You may assume that the number of times a taxi is late in a week has a binomial distribution.
Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly. ***E***

Solution:

$H_0 : p = 0.2$ ← $p = \frac{1}{5} = 0.2$

$H_1 : p > 0.2$ ← 'late more often' so one-tail test

$X \sim B(5, 0.2)$ ← 5 days, $P(\text{late}) = 0.2$

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.9421$ ← Use tables.

$= 0.0579 > 0.05$

Insufficient evidence to reject H_0.
No evidence of an increase in the number of times the taxi driver is late. ← Remember context of 'taxi driver is late' or you could say 'Linda's claim is not justified'.

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2 Review Exercise
Exercise A, Question 16

Question:

a **i** Write down two conditions for $X \sim B(n, p)$ to be approximated by a normal distribution $Y \sim N(\mu, \sigma)$.

ii Write down the mean and variance of this normal approximation in terms of n and p.

A factory manufactures 2000 DVDs every day. It is known that 3% of DVDs are faulty.

b Using a normal approximation, estimate the probability that at least 40 faulty DVDs are produced in one day.

The quality control system in the factory identifies and destroys every faulty DVD at the end of the manufacturing process. It costs £0.70 to manufacture a DVD and the factory sells non-faulty DVDs for £11.

c Find the expected profit made by the factory per day. ***E***

Solution:

a **i** If $X \sim B(n, p)$
n is large
p is close to 0.5
and $np > 5$
then X can be approximated by
$N(np, np(1-p))$

ii

$$\text{mean} = np$$
$$\text{variance} = npq = np(1-p)$$

'In terms of n and p'

b $X \sim N(60, 58.2)$

$$np = 2000 \times 0.03 = 60$$
$$npq = 2000 \times 0.03 \times 0.97 = 58.2$$

$$\begin{aligned} P(X \geq 40) &= P(X > 39.5) \\ &= 1 - P\left(Z \leq \frac{39.5 - 60}{\sqrt{58.2}}\right) \\ &= 1 - P(Z < -2.687\ldots) \\ &= 0.9964 \\ &= 0.996 \text{ (3 s.f.)} \end{aligned}$$

40 included so continuity correction $40 - 0.5 = 39.5$

Don't forget the square root of the variance.

c $E(X) = 60$

$np = 60$ above.

Expected profit

$$= (2000 - 60) \times 11 - 2000 \times 0.70$$
$$= £19\,940$$

Profit from sales.

Manufacturing costs

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2 Review Exercise
Exercise A, Question 17

Question:

a Define a statistic.

A random sample $X_1, X_2, \ldots, X_n$ is taken from a population with unknown mean μ.

b For each of the following state whether or not it is a statistic.

i $\frac{X_1 + X_4}{2}$,

ii $\frac{\Sigma X^2}{n} - \mu^2$. ***E***

Solution:

a A random variable that is a function of known observations from a population.
or A statistic is a numerical property of a sample.

b i Yes, it is a statistic.

X_1 and X_2 are known.

ii No, it is not a statistic.

μ is unknown.

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2 Review Exercise

Exercise A, Question 18

Question:

For a particular type of plant 45% have white flowers and the remainder have coloured flowers. Gardenmania sells plants in batches of 12. A batch is selected at random.

Calculate the probability this batch contains

a exactly 5 plants with white flowers,

b more plants with white flowers than coloured ones.

Gardenmania takes a random sample of 10 batches of plants.

c Find the probability that exactly 3 of these batches contain more plants with white flowers than coloured ones.

Due to an increasing demand for these plants by large companies, Gardenmania decides to sell them in batches of 50.

d Use a suitable approximation to calculate the probability that a batch of 50 plants contains more than 25 plants with white flowers. ***E***

Solution:

a Let W be the random variable 'the number of white plants'

$W \sim B(12, 0.45)$

'batches of 12': $n = 12$
'45% have white flowers'

$P(W = 5) = C_5^{12} 0.45^5 0.55^7$
$= 0.2225$

You can also use tables: $P(W \leq 5) - P(W \leq 4)$

b

$P(W \geq 7) = 1 - P(W \leq 6)$
$= 1 - 0.7393$
$= 0.2607$

Batches of 12, so
7 white, 5 coloured
8 white, 4 coloured, etc.

c

$P(\text{exactly}3) = C_3^{10}(0.2607)^3(1-0.2607)^7$
$= 0.2567$

Use your answer to **b**: $p = 0.2607, n = 10$

d

mean $= np = 22.5$

variance $= npq = 12.375$

$n = 50$
$p = 0.45$
$q = 0.55$

$W \sim N(22.5, 12.375)$

$P(W > 25) = P\left(Z > \frac{25.5 - 22.5}{\sqrt{12.375}}\right)$
$= P(Z > 0.8528\ldots)$
$= 1 - 0.8023$
$= 0.1977$

25 not included so continuity correction is $25 + 0.5 = 25.5$

Look up 0.85 in table.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

2 Review Exercise

Exercise A, Question 19

Question:

a State the condition under which the normal distribution may be used as an approximation to the Poisson distribution.

b Explain why a continuity correction must be incorporated when using the normal distribution as an approximation to the Poisson distribution.

A company has yachts that can only be hired for a week at a time. All hiring starts on a Saturday. During the winter the mean number of yachts hired per week is 5.

c Calculate the probability that fewer than 3 yachts are hired on a particular Saturday in winter.

During the summer the mean number of yachts hired per week increases to 25. The company has only 30 yachts for hire.

d Using a suitable approximation find the probability that the demand for yachts cannot be met on a particular Saturday in summer.

In the summer there are 16 Saturdays on which a yacht can be hired.

e Estimate the number of Saturdays in the summer that the company will not be able to meet the demand for yachts. ***E***

Solution:

a $\lambda > 10$ or large

b The Poisson distribution is *discrete* and the normal distribution is *continuous*.

c Let Y be the random variable 'the number of yachts hired in winter'.

$$Y \sim \text{Po}(5), \text{P}(Y < 3) = \text{P}(Y \leq 2)$$

'Yachts hired per week is 5'.

$$= 0.1247$$

Use table.

d Y is approximately N(25, 25)

Mean number of yachts hired per week is 25.

$$\text{P}(X > 30) = \text{P}\left(Z > \frac{30.5 - 25}{5}\right)$$

'30 yachts for hire' so 'cannot be met' is *greater than* 30.

30 not included, so use continuity correction $30 + 0.5 = 30.5$.

$$= \text{P}(Z > 1.1)$$
$$= 1 - 0.8643$$
$$= 0.1357$$

e

$$\text{Number of weeks} = 0.1357 \times 16 = 2.17$$

So 2 (or 3) Saturdays.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

2 Review Exercise
Exercise A, Question 20

Question:

The continuous random variable X is uniformly distributed over the interval $\alpha < x < \beta$.

a Write down the probability density function of X, for all x.

b Given that $\mathrm{E}(X) = 2$ and $\mathrm{P}(X < 3) = \frac{5}{8}$, find the value of α and the value of β.

A gardener has wire cutters and a piece of wire 150 cm long which has a ring attached at one end. The gardener cuts the wire, at a randomly chosen point, into 2 pieces. The length, in cm, of the piece of wire with the ring on it is represented by the random variable X. Find

c E(X),

d the standard deviation of X,

e the probability that the shorter piece of wire is at most 30 cm long. ***E***

Solution:

a $$\mathrm{f}(x) = \begin{cases} \frac{1}{\beta - \alpha}, & \alpha < x < \beta \\ 0, & \text{otherwise} \end{cases}$$

b $\mathrm{E}(X) = 2, \quad \mathrm{P}(X < 3) = \frac{5}{8}$

i.e. $\frac{\alpha + \beta}{2} = 2 \quad \frac{3 - \alpha}{\beta - \alpha} = \frac{5}{8}$

Use properties of uniform distribution to form 2 equations.

$$\begin{aligned} \alpha + \beta &= 4 \\ 3\alpha + 5\beta &= 24 \\ \beta &= 6 \\ \alpha &= -2 \end{aligned}$$

Solve simultaneously.

c $\mathrm{E(X)} = \frac{150 + 0}{2} = 75 \text{ cm}$

Use formula.

d $$\text{Standard deviation} = \sqrt{\frac{(150 - 0)^2}{12}} = 43.3 \text{ (3 s.f.)}$$

Use formula.

e $\mathrm{P}(X \le 30) + \mathrm{P}(X \ge 120)$

There are 2 ends!

$= \frac{30}{150} + \frac{30}{150}$

$= \frac{60}{150} = \frac{2}{5}$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

2 Review Exercise
Exercise A, Question 21

Question:

Past records from a large supermarket show that 20% of people who buy chocolate bars buy the family size bar. On one particular day a random sample of 30 people was taken from those that had bought chocolate bars and 2 of them were found to have bought a family size bar.

a Test, at the 5% significance level, whether or not the proportion p of people who bought a family size bar of chocolate that day had decreased. State your hypotheses clearly.

The manager of the supermarket thinks that the probability of a person buying a gigantic chocolate bar is only 0.02. To test whether this hypothesis is true the manager decides to take a random sample of 200 people who bought chocolate bars.

b Find the critical region that would enable the manager to test whether or not there is evidence that the probability is different from 0.02. The probability of each tail should be as close to 2.5% as possible.

c Write down the significance level of this test. ***E***

Solution:

a $H_0: p = 0.2, \quad H_1: p < 0.2$ ← 'Lower' so one-tailed test.

Let X be the random variable 'number of people buying a family size bar'
$X \sim B(30, 0.2)$ ← Sample of 30 people; $n = 30$; 20% of people : $p = 0.2$

$P(X \leq 2) = 0.0442 < 0.05$ ← Table value.

Sufficient evidence to reject H_0.
Conclude that the proportion of family size sold is lower than usual ← Conclusion in context.

b $H_0: p = 0.02, H_1: p \neq 0.02$ ← 'is different from' so two-tailed test.
Let Y be the random variable 'number of gigantic bars sold'

$Y \sim B(200, 0.02)$ which is $Po(4)$ approximately ← $np = 200 \times 0.02 = 4$
$P(Y = 0) = 0.0183$ is closest to 0.025
$P(Y \geq 9) = 1 - 0.9786 = 0.0214$ is closest to 0.025 ← From table values look for 'as close to 2.5% as possible'
Critical region $Y = 0$ and $Y \geq 9$

c

$$\text{Significance level} = 0.0183 + 0.0214 = 0.0397$$

i.e. 3.97%