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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

a Describe what is meant by the expression 'a statistical hypothesis'.

b Describe the difference between the null hypothesis and the alternative hypothesis.

c What symbols do we use to denote the null and alternative hypotheses?

Solution:

a This is an assumption made about a population parameter that we test using evidence from a sample.

b The null hypothesis is what we assume to be correct and the alternative hypothesis is what we conclude if our assumption is wrong.

c Null Hypothesis = H_0 Alternative hypothesis = H_1

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

Dmitri wants to see whether a die is biased towards the value 6.
He throws the die 60 times and counts the number of sixes he gets.

a Describe the test statistic.

b Write down a suitable null hypothesis to test this die.

c Write down a suitable alternative hypothesis to test this die.

Solution:

a The test statistic is N – the number of sixes.

b $H_0: p = \frac{1}{6}$

c $H_1: p > \frac{1}{6}$

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 3

Question:

Shell wants to test to see whether a coin is biased. She tosses the coin 100 times and counts the number of times she gets a head.

a Describe the test statistic.

b Write down a suitable null hypothesis to test this coin.

c Write down a suitable alternative hypothesis to test this coin.

Solution:

a The test statistic is N – the number of times you get a head.

b $H_0 : p = \frac{1}{2}$

c $H_1 : p \neq \frac{1}{2}$

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 4

Question:

Over a long period of time it is found that the mean number of accidents, λ, occurring at a particular crossroads is 4 per month. New traffic lights are installed. Jess decides to test to see whether the proportion of accidents has increased, decreased or changed in any way.

a Describe the test statistic.

b Write down a suitable null hypothesis to test Jess' theory.

c Write down three possible alternative hypotheses to test Jess' theory.

Solution:

a The test statistic is the number of accidents (in a given month or other specified time period).

b H_0: $\lambda = 4$

c Change H_1: $\lambda \neq 4$ (2 tail); or Decrease H_1: $\lambda < 4$ or Increase H_1: $\lambda > 4$ (both one tail).

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Exercise A, Question 5

Question:

In a survey it was found that 4 out of 10 people supported a certain particular political party. Chang wishes to test whether or not there has been a change in the proportion (p) of people supporting the party.

a Write down whether it would be best to use a one-tail test or a two-tail test.
Give a reason for your answer.

b Suggest suitable hypotheses.

Solution:

a A two tail test would be best. The support could get better or could get worse.

b $H_0: p = 0.4$
$H_1: p \neq 0.4$

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 6

Question:

In a manufacturing process the proportion (p) of faulty articles has been found, from long experience, to be 0.1.
The proportion of faulty articles in the first batch produced by a new process is measured.
The proportion of faulty articles in this batch is 0.09.
The manufacturers wish to test at the 5% level of significance whether or not there has been a reduction in the proportion of faulty articles.

a Suggest a suitable test statistic.

b Suggest suitable hypotheses.

c Explain the condition under which the null hypothesis is rejected.

Solution:

a A suitable test statistic is p – the proportion of faulty articles in a batch.

b $H_0: p = 0.1$ $\quad$ $H_1: p < 0.1$

c If the probability of the proportion being 0.09 or less is 5% or less the null hypothesis is rejected.

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Exercise A, Question 7

Question:

A spinner has 4 sides numbered 1, 2, 3 and 4. Hajdra thinks it is biased to give a one when spun. She spins 5 times and counts the number of times, M, that she gets a 1.

a Describe the test statistics M.

She decides to do a test with a level of significance of 5%.

b What values of M would cause the null hypothesis to be rejected.

Solution:

a The test statistic is M – the number of times Hajdra gets a 1.

b $B(5, 0.25)$

N	0	1	2	3	4	5
$P(N = n)$	0.237	0.395	0.264	0.088	0.015	0.001

There is a $0.015 + 0.001 = 0.016 = 1.6\%$ chance of getting one 4 or 5 times.
There is a $0.088 + 0.001 + 0.015 = 0.104 = 10.4\%$ chance of getting one 3, 4 or 5 times.
If N is 4 or 5 then the null hypothesis would be rejected, since $P(4 \text{ or more}) = 1.6\% < 5\%$.

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Exercise B, Question 1

Question:

For each of the questions 1 to 7 carry out the following tests using the binomial distribution where the random variable, X, represents the number of successes.

$H_0: p = 0.25$; $H_1: p > 0.25$; $n = 10, x = 5$ and using a 5% level of significance.

Solution:

Distribution B(10, 0.25)

$H_0: p = 0.25 \quad H_1: p > 0.25$

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - 0.9219 \\ &= 0.0781 \end{aligned}$$

$0.0781 > 0.05$

There is insufficient evidence to reject H_0.

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Exercise B, Question 2

Question:

$H_0 : p = 0.40$; $H_1 : p < 0.40$; $n = 10, x = 1$ and using a 5% level of significance.

Solution:

Distribution B(10, 0.40)

$H_0 : p = 0.40 \quad H_1 : p < 0.40$

$P(X \leq 1) = 0.0464$

$0.0464 < 0.05$

There is sufficient evidence to reject H_0 so $p < 0.04$.

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Exercise B, Question 3

Question:

$H_0 : p = 0.30$; $H_1 : p > 0.30$; $n = 20, x = 10$ and using a 5% level of significance.

Solution:

Distribution B(20, 0.30)
$H_0 : p = 0.30 \quad H_1 : p > 0.30$
$P(X \geq 10) = 1 - P(X \leq 9)$
$= 1 - 0.9520$
$= 0.0480$
$0.0480 < 0.05$
There is sufficient evidence to reject H_0 so $p > 0.30$.

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Exercise B, Question 4

Question:

$H_0 : p = 0.45$; $H_1 : p < 0.45$; $n = 20, x = 3$ and using a 1% level of significance.

Solution:

Distribution B(20, 0.45)
$H_0 : p = 0.45 \quad H_1 : p < 0.45$
$P(X \leq 3) = 0.0049$
$0.0049 < 0.01$
There is sufficient evidence to reject H_0 so $p < 0.45$.

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Exercise B, Question 5

Question:

$H_0 : p = 0.50$; $H_1 : p \neq 0.50$; $n = 30, x = 10$ and using a 5% level of significance.

Solution:

Distribution B(30, 0.50)
$H_0 : p = 0.50 \quad H_1 : p \neq 0.50$
$P(X \leq 10) = 0.0494$
0.0494 > 0.025 (two-tailed)
There is insufficient evidence to reject H_0
so there is no reason to doubt $p = 0.5$.

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Exercise B, Question 6

Question:

$H_0 : p = 0.28$; $H_1 : p < 0.28$; $n = 20, x = 2$ and using a 5% level of significance.

Solution:

Distribution B(20, 0.28)
$H_0 : p = 0.28 \quad H_1 : p < 0.28$
$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$
$= 0.72^{20} + 20\times0.72^{19}\times0.28 + 190\times0.72^{18}\times0.28^2$
$= 0.0014 + 0.0109 + 0.0403$
$= 0.0526$
$0.0526 > 0.05$
There is insufficient evidence to reject H_0
so there is no reason to doubt $p = 0.28$.

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Exercise B, Question 7

Question:

$H_0 : p = 0.32$; $H_1 : p > 0.32$; $n = 8, x = 7$ and using a 5% level of significance.

Solution:

Distribution B(8, 0.32)

$H_0 : p = 0.32 \quad H_1 : p > 0.32$

$$\begin{aligned} P(X \geq 7) &= P(X = 7) + P(X = 8) \\ &= 8 \times 0.32^7 \times 0.68 + 0.32^8 \\ &= 0.0019 + 0.0001 \\ &= 0.0020 \end{aligned}$$

$0.0020 < 0.05$

There is sufficient evidence to reject H_0 so $p > 0.32$.

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Exercise B, Question 8

Question:

For each of the questions 8 to 10 carry out the following tests using the Poisson distribution where λ represents its mean.

$H_0 : \lambda = 8$; $H_1 : \lambda < 8$; $x = 3$ and using a 5% level of significance.

Solution:

Distribution Po(8)
$H_0 : \lambda = 8 \quad H_1 : \lambda < 8$
$P(X \le 3) = 0.0424$
$0.0424 < 0.05$
There is sufficient evidence to reject H_0 so $\lambda < 8$.

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Exercise B, Question 9

Question:

$H_0 : \lambda = 6.5$; $H_1 : \lambda < 6.5$; $x = 2$ and using a 1% level of significance.

Solution:

Distribution Po(6.5)
$H_0 : \lambda = 6.5 \quad H_1 : \lambda < 6.5$
$P(X \leq 2) = 0.0430$
$0.0430 > 0.01$ (1% sig. level)
There is insufficient evidence to reject H_0 so there is no reason to doubt $\lambda < 6.5$

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Exercise B, Question 10

Question:

$H_0 : \lambda = 5.5$; $H_1 : \lambda > 5.5$; $x = 8$ and using a 5% level of significance.

Solution:

Distribution Po(5.5)
$H_0 : \lambda = 5.5 \quad H_1 : \lambda > 5.5$
$$\begin{aligned} P(X \geq 8) &= 1 - P(X \leq 7) \\ &= 1 - 0.8095 \\ &= 0.1905 \end{aligned}$$
$0.1905 > 0.05$
There is insufficient evidence to reject H_0 so there is no reason to doubt $\lambda > 5.5$

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Exercise B, Question 11

Question:

The manufacturer of 'Supergold' margarine claims that people prefer this to butter. As part of an advertising campaign he asked 5 people to taste a sample of 'Supergold' and a sample of butter and say which they prefer. Four people chose 'Supergold'. Assess the manufacturer's claim in the light of this evidence. Use a 5% level of significance.

Solution:

Distribution B(5, 0.5)

$H_0 : p = 0.5 \quad H_1 : p > 0.5$

$P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.8125$

$= 0.1875$

$0.1875 > 0.05$

There is insufficient evidence to reject H_0 (not significant).

There is insufficient evidence to suggest that people prefer 'Supergold' to butter.

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Exercise B, Question 12

Question:

I tossed a coin 20 times and obtained a head on 6 occasions. Is there evidence that the coin is biased? Use a 5% two-tailed test.

Solution:

Distribution B(20, 0.50)
$H_0 : p = 0.50 \quad H_1 : p \neq 0.50$
$P(X \leq 6) = 0.0577$
$0.0577 > 0.025$ (two tailed)
There is insufficient evidence to reject H_0 so we conclude there is no evidence the coin is biased.

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Exercise B, Question 13

Question:

A die used in playing a board game is suspected of not giving the number 6 often enough. During a particular game it was rolled 12 times and only one 6 appeared. Does this represent significant evidence, at the 5% level of significance, that the probability of a 6 on this die is less than $\frac{1}{6}$?

Solution:

Distribution $B(12, \frac{1}{6})$

$H_0 : p = \frac{1}{6}$ $H_1 : p < \frac{1}{6}$

$$\begin{aligned} P(X \le 1) &= P(X = 0) + P(X = 1) \\ &= \frac{5}{6}^{12} + 12\left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right) \\ &= 0.3813 \end{aligned}$$

$0.3813 > 0.05$

There is insufficient evidence to reject H_0 (not significant).

There is no evidence that the probability is less than $\frac{1}{6}$.

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Exercise B, Question 14

Question:

The success rate of the standard treatment for patients suffering from a particular skin disease is claimed to be 68%.

a In a sample of n patients, X is the number for which the treatment is successful. Write down a suitable distribution to model X. Give reasons for your choice of model.

A random sample of 10 patients receives the standard treatment and in only 3 cases was the treatment successful. It is thought that the standard treatment was not as effective as it is claimed.

b Test the claim at the 5% level of significance.

Solution:

a
Distribution $B(n, 0.68)$
Fixed number of trials.
Outcomes of trials are independent.
There are two outcomes success and failure.
The probability of success is constant.

b
Distribution $B(10, 0.68)$
$H_0 : p = 0.68 \quad H_1 : p < 0.68$
$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$
$= 0.32^{10} + 10(0.32)^9(0.68) + 45(0.32)^8(0.68)^2 + 120(0.32)^7(0.68)^3$
$= 0.0000 + 0.0002 + 0.0023 + 0.0130$
$= 0.0155$
$0.0155 < 0.05$
There is sufficient evidence to reject H_0 so $p < 0.68$.
The treatment is not as effective as is claimed.

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Exercise B, Question 15

Question:

Every year a statistics teacher takes her class out to observe the traffic passing the school gates during a Tuesday lunch hour. Over the years she has established that the average number of lorries passing the gates in a lunch hour is 7.5. During the last 12 months a new bypass has been built and the number of lorries passing the school gates in this year's experiment was 4. Test, at the 5% level of significance, whether or not the mean number of lorries passing the gates during a Tuesday lunch hour has been reduced.

Solution:

Distribution Po(7.5)
$H_0 : \lambda = 7.5 \quad H_1 : \lambda < 7.5$
$P(X \leq 4) = 0.1321$
$0.1321 > 0.05$
There is insufficient evidence to reject H_0 (not significant).
There is no evidence of a decrease in the number of lorries passing the gates in a lunch hour.

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Exercise B, Question 16

Question:

Over a long period, John has found that the bus taking him to school arrives late on average 9 times per month. In the month following the start of the new summer schedule the bus arrives late 13 times. Assuming that the number of times the bus is late has a Poisson distribution, test, at the 5% level of significance, whether the new schedules have in fact increased the number of times on which the bus is late. State clearly your null and alternative hypotheses.

Solution:

Distribution Po(9)

$H_0 : \lambda = 9 \quad H_1 : \lambda > 9$

$$\begin{aligned} P(X \geq 13) &= 1 - P(X \leq 12) \\ &= 1 - 0.8758 \\ &= 0.1242 \end{aligned}$$

$0.1242 > 0.05$

There is insufficient evidence to reject H_0 (not significant).
There is no evidence that the new schedules have increased the number of times the bus is late.

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Exercise C, Question 1

Question:

For each of the questions 1 to 6 find the critical region for the test statistic X representing the number of successes. Assume a binomial distribution.

$H_0: p = 0.20$; $H_1: p > 0.20$; $n = 10$, using a 5% level of significance.

Solution:

B(10, 0.2)
$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.8791 = 0.1209 > 0.05$
$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9672 = 0.0328 < 0.05$
The critical value is $x = 5$ and the critical region is $X \geq 5$
since $P(X \geq 5) = 0.0328 < 0.05$.

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Exercise C, Question 2

Question:

$H_0 : p = 0.15$; $H_1 : p < 0.15$; $n = 20$, using a 5% level of significance.

Solution:

B(20, 0.15)
$P(X \leq 1) = 0.1756 > 0.05$
$P(X = 0) = 0.0388 < 0.05$
The critical value is $x = 0$ and the critical region is $X = 0$.

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Exercise C, Question 3

Question:

$H_0 : p = 0.40$; $H_1 : p \neq 0.40$; $n = 20$, using a 5% level of significance (2.5% at each tail).

Solution:

B(20, 0.4)
$P(X \leq 4) = 0.0510 > 0.025$
$P(X \leq 3) = 0.0160 < 0.025$
The critical value is $x = 3$
$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9790 = 0.0210 < 0.025$
$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9435 = 0.0565 > 0.025$
The critical value is $x = 13$
The critical region is $X \geq 13$ and $X \leq 3$.

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Exercise C, Question 4

Question:

$H_0 : p = 0.18$; $H_1 : p < 0.18$; $n = 20$, using a 1% level of significance.

Solution:

B(20, 0.18)
$P(X = 0) = 0.82^{20} = 0.0189 < 0.05$
$P(X \leq 1) = 0.0189 + 20(0.82)^{19}(0.18) = 0.0189 + 0.0829 = 0.1018 > 0.05$
The critical value is $x = 0$. The critical region is $X = 0$.

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Exercise C, Question 5

Question:

$H_0: p = 0.63$; $H_1: p > 0.36$; $n = 10$, using a 5% level of significance.

Solution:

B(10, 0.63)
$P(X = 10) = 0.63^{10} = 0.0098 < 0.05$
$P(X \geqslant 9) = 0.0098 + 10(0.63)^9\ (0.37) = 0.0675 > 0.05$
The critical value is $x = 10$ and the critical region is $X = 10$.

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Exercise C, Question 6

Question:

$H_0: p = 0.22$; $H_1: p \neq 0.22$; $n = 10$, using a 1% level of significance (0.005 at each tail).

Solution:

$B(10, 0.22)$
$P(X = 0) = 0.78^{10} = 0.0834 > 0.005$
$P(X \geq 6) = 0.010 > 0.005$
$P(X \geq 7) = 0.0016 < 0.005$
Critical region is $X \geq 7$.

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Exercise C, Question 7

Question:

For each of the questions 7 to 9 find the critical region for the test statistic X given that X has a $\text{Po}(\lambda)$ distribution.

$\text{H}_0 : \lambda = 4$; $\text{H}_1 : \lambda > 4$; using a 5% level of significance.

Solution:

Po(4)
$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9489 = 0.0511 > 0.05$
$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9786 = 0.0214 < 0.05$
Critical region is $X \geq 9$.

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Exercise C, Question 8

Question:

$H_0: \lambda = 9$; $H_1: \lambda < 9$; using a 1% level of significance.

Solution:

Po(9)
$P(X \leq 2) = 0.0062 < 0.01$
$P(X \leq 3) = 0.0212 > 0.01$
Critical region is $X \leq 2$.

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Exercise C, Question 9

Question:

$H_0: \lambda = 3.5$; $H_1: \lambda < 3.5$; using a 5% level of significance.

Solution:

Po(3.5)
$P(X = 0) = 0.0302 < 0.05$
$P(X \leq 1) = 0.1359 > 0.05$
Critical region is $X = 0$.

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Exercise C, Question 10

Question:

A seed merchant usually kept her stock in carefully monitored conditions. After the Christmas holidays one year she discovered that the monitoring system had broken down and there was a danger that the seed might have been damaged by frost. She decided to check a sample of 10 seeds to see if the proportion p that germinates had been reduced from the usual value of 0.85. Find the critical region for a one-tailed test using a 5% level of significance.

Solution:

$X \sim \mathrm{B}(10, 0.85)$ let $Y \sim \mathrm{B}(10, 0.15)$
$\mathrm{H_0} : p = 0.85 \quad \mathrm{H_1} : p < 0.85$
$\mathrm{P}(X \leq 6) = \mathrm{P}(Y \geq 4) = 1 - \mathrm{P}(Y \leq 3) = 1 - 0.9500 = 0.0500 = 0.05$
Critical region is $X \leq 6$.

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Exercise C, Question 11

Question:

The national proportion of people experiencing complications after having a particular operation in hospitals is 20%. A particular hospital decides to take a sample of size 20 from their records.

a State all the possible numbers of patients with complications that would cause them to decide that their proportion of complications differs from the national figure at the 5% level of significance ensuring that the probability in each tail is as near to 2.5% as possible.

The hospital finds that out of 20 such operations, 8 of their patients experienced complications.

b Find critical regions, at the 5% level of significance, to test whether or not their proportion of complications differs from the national proportion. The probability in each tail should be as near 2.5% as possible.

c State the actual significance level of the above test.

Solution:

$X \sim \mathrm{B}(20, 0.20)$

a

$\mathrm{H}_0 : p = 0.20 \quad \mathrm{H}_1 : p \neq 0.20$

$\mathrm{P}(X \leq 1) = 0.0692$

$\mathrm{P}(X \leq 0) = 0.0115$

(0.015 nearest to 0.025)

critical value = 0

$\mathrm{P}(X \geq 9) = 1 - \mathrm{P}(X \leq 8) = 1 - 0.9900 = 0.0100$

$\mathrm{P}(X \geq 8) = 1 - \mathrm{P}(X \leq 7) = 1 - 0.9679 = 0.0321$

critical value = 8 (0.0321 nearer to 0.025)

Critical region $X = 0$ and $X \geq 8$.

b $X = 8$ is in the critical region. There is enough evidence to reject H_0.
The hospital's proportion of complications differs from the national figure.

c Actual significance level is $0.0115 + 0.0321 = 0.0436$

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Exercise C, Question 12

Question:

Over a number of years the mean number of hurricanes experienced in a certain area during the month of August is 4. A scientist suggests that, due to global warming, the number of hurricanes will have increased, and proposes to do a hypothesis test based on the number of hurricanes this year.

a Suggest suitable hypotheses for this test.

b Find to what level the number of hurricanes must increase for the null hypothesis to be rejected at the 5% level of significance.

c The actual number of hurricanes this year was 8. What conclusion did the scientist come to?

Solution:

Po(4)

a $H_0: \lambda = 4$ $H_1: \lambda > 4$

b $P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9489 = 0.0511 > 0.05$

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9786 = 0.0214 < 0.05$

Critical region is $X \geq 9$.

c 8 is not in the critical region. The scientist concluded there was not enough evidence to suggest an increase in the number of hurricanes.

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Exercise C, Question 13

Question:

An estate agent usually sells properties at the rate of 10 per week.
During a recession, when money was less available, over an eight-week period he sold 55 properties.
Using a suitable approximation test, at the 5% level of significance, whether or not there is evidence that the weekly rate of sales decreased.

Solution:

$H_0 : \lambda = 10 \quad H_1 : \lambda < 10$
Let Y = properties sold in 8 weeks
Under H_0 $Y \sim Po(80)$
$P(Y \leq 55) \approx P(W < 55.5)$ where $W \sim N(80, 80)$
$\approx P\left(Z < \frac{55.5 - 80}{\sqrt{80}}\right)$
$= 1 - P(Z < 2.74)$
$= 1 - 0.9970$
$= 0.0030$
$0.0030 < 0.05$ Reject H_0.
There is evidence that the rate of weekly sales has decreased.

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Exercise C, Question 14

Question:

A manager thinks that 20% of his workforce are absent for at least one day each month. He chooses 100 workers at random and finds that in the last month 2 had been absent for at least one day.
Using a suitable approximation test, at the 5% level of significance, whether or not this provides evidence that the percentage of workers that are absent for at least 1 day per month is less than 20%.

Solution:

B(100, 0.2)
$H_0 : p = 0.2 \quad H_1 : p < 0.2$
$P(X \le 2) = P(Y < 2.5)$ where $Y \sim N(20, 4^2)$
$$P(Y < 2.5) = P\left(Z < \frac{2.5 - 20}{\sqrt{16}}\right)$$
$= 1 - P(Z > 4.375)$
$= 0.000 < 0.05$
Reject H_0
There is evidence that the percentage of workers
who are absent for at least 1 day per month is less than 20%.

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Exercise D, Question 1

Question:

Mai commutes to work five days a week on a train. She does two journeys a day.
Over a long period of time she finds that the train is late 20% of the time.
A new company takes over the train service Mai uses. Mai thinks that the service will be late more often. In the first week of the new service the train is late 3 times.
You may assume that the number of times the train is late in a week has a binomial distribution.
Test, at the 5% level of significance, whether or not there is evidence that there is an increase in the number of times the train is late. State your hypothesis clearly.

Solution:

$X \sim B(10, 0.20)$
$H_0 : p = 0.20 \quad H_1 : p > 0.20$
$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.6778 = 0.3222$
$0.3222 > 0.05$
There is insufficient evidence to reject H_0.
There is no evidence that the trains are late more often.

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Edexcel AS and A Level Modular Mathematics

Exercise D, Question 2

Question:

Over a long period of time it was observed that the mean number of lorries passing a hospital was 7.5 every 10 minutes.
A new by-pass was built that avoided the hospital. In a survey after the by-pass was opened, it was found that in one particular week the mean number of lorries passing the hospital was 4 every 10 minutes. It is decided that a significance test will be done to test whether or not the mean number of lorries passing the hospital has changed.

a State whether a one- or two-tailed test will be needed. Give a reason for your answer.

b Write down the name of the distribution that will be tested. Give a reason for your choice.

c Carry out the significance test at the 5% level of significance.

Solution:

a A two-tail test will be needed. We are looking to see if the number of lorries has changed.

b $X \sim \text{Po}(7.5)$ Lorries arrive independent of each other, singly and at a constant rate.

c $\text{H}_0 : \lambda = 7.5 \quad \text{H}_1 : \lambda \neq 7.5$

$\text{P}(X \leq 4) = 0.1321$

$0.1321 > 0.025$

There is insufficient evidence to reject H_0.

There is no evidence that the rate at which lorries pass the hospital has changed.

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Exercise D, Question 3

Question:

A marketing company claims that Chestly cheddar cheese tastes better than Cumnauld cheddar cheese.
Five people chosen at random as they entered a supermarket were asked to say which they preferred. Four people preferred Chestly cheddar cheese.
Test, at the 5% level of significance, whether or not the company's claim is true. State your hypothesis clearly.

Solution:

$X \sim B(5, 0.50)$
$H_0 : p = 0.50 \quad H_1 : p > 0.50$
$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.8125 = 0.1875$
$0.1875 > 0.05$
There is insufficient evidence to reject H_0. There is insufficient evidence that the company's claim is true.

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Exercise D, Question 4

Question:

In 2006 and 2007 much of Greebe suffered earth tremors at a rate of 5 per month.
A survey was done in the first two months of 2008 and 13 tremors were recorded.
Stating your hypothesis clearly test, at the 10% level of significance, whether or not there is evidence to suggest the rate of earth tremors has increased.

Solution:

$X \sim \text{Po}(10)$

$\text{H}_0 : \lambda = 10 \quad \text{H}_1 : \lambda > 10$

$\text{P}(X \geq 13) = 1 - \text{P}(X \leq 12) = 1 - 0.7916 = 0.2084$

$0.2084 > 0.10$

There is insufficient evidence to reject H_0.

There is no evidence that the rate of tremors has increased.

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Exercise D, Question 5

Question:

Historical information finds that nationally 30% of cars fail a brake test.

a Give a reason to support the use of a binomial distribution as a suitable model for the number of cars failing a brake test.

b Find the probability that, of 5 cars taking the test, all of them pass the brake test.

A garage decides to conduct a survey of their cars. A randomly selected sample of 10 of their cars is tested. Two of them fail the test.

c Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that cars in this garage fail less than the national average.

Solution:

a Fixed number; independent trials; two outcomes (pass or fail); p constant for each car.

b $X \sim B(5, 0.30)$

$P(\text{all pass}) = 0.70^5 = 0.16807$

c $X \sim B(10, 0.30)$

$H_0 : p = 0.30 \quad H_1 : p < 0.30$

$P(X \leq 2) = 0.3828$

$0.3828 > 0.05$

There is insufficient evidence to reject H_0. There is no evidence that the garage fails fewer than the national average.

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Exercise D, Question 6

Question:

a Explain what you understand by an hypothesis test.

During a garden fete cups of tea are thought to be sold at a rate of 2 every minute. To test this, the number of cups of tea sold during a random 30-minute interval is recorded.

b State one reason why the sale of cups of tea can be modelled by a Poisson distribution.

c Find the critical region for a two-tailed hypothesis that the number of cups of tea sold occurs at a rate of 2 every minute. The probability in each tail should be as close to 2.5% as possible.

d Write down the actual significance level of the above test.

Solution:

a A hypothesis test about a population parameter θ tests a null hypothesis H_0, which specifies a particular value for θ, against an alternative hypothesis H_1 which is that θ has increased, decreased or changed.
H_1 will indicate if the test is one-or two-tailed.

b You can count the number of cups of tea that were served in a given time interval (30 minutes). (You cannot count the number of of cups that were *not* served.

c $X \sim \text{Po}(2)$
$H_0 : \lambda = 2 \quad H_1 : \lambda \neq 2$
$P(X = 0) = 0.1353 > 0.025$
No lower critical value
$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9473 = 0.0527$
$P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9834 = 0.0166$
Critical value $x = 6$ since 0.0166 is closer to 0.025.
Critical region is $X \geq 6$.

d Actual level of significance = 0.0166

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Exercise D, Question 7

Question:

The probability that Jacinth manages to hit a coconut on the coconut shy at a fair is 0.4. She decides to practise at home. After practising she thinks that the practising has helped her to improve. After practising Jacinth is going to the fair and will have 20 throws.

a Find the critical region for an hypothesis test at the 5% level of significance.
After practising, Jacinth hits the coconut 11 times.

b Determine whether or not there is evidence that practising has helped Jacinth improve. State your hypothesis clearly.

Solution:

a $X \sim B(20, 0.4)$
$H_0 : p = 0.40 \quad H_1 : p > 0.40$
$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9435 = 0.0565 > 0.05$
$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9790 = 0.0210 < 0.05$
Critical region is $X \geq 13$.

b $H_0 : p = 0.40 \quad H_1 : p > 0.40$
11 is not in the critical region.
There is insufficient evidence to reject H_0. There is no evidence that practice has improved Jacinth's throwing.

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Exercise D, Question 8

Question:

The proportion of defective articles in a certain manufacturing process has been found from long experience to be 0.1.
A random sample of 50 articles was taken in order to monitor the production. The number of defective articles was recorded.

a Using a 5% level of significance, find the critical regions for a two-tailed test of the hypothesis that 1 in 10 articles has a defect. The probability in each tail should be as near 2.5% as possible.

b State the actual significance level of the above test.

Another sample of 20 articles was taken at a later date. Four articles were found to be defective.

c Test at the 10% significance level, whether or not there is evidence that the proportion of defective articles has increased. State your hypothesis clearly.

Solution:

a $X \sim B(50, 0.1)$
$H_0 : p = 0.10 \quad H_1 : p \neq 0.10$
$P(X \leq 1) = 0.0338$
$P(X = 0) = 0.0052$
Critical value $x = 1$ (0.0338 nearer to 0.025)
$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9421 = 0.0579$
$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9755 = 0.0245$
Critical value $x = 10$
Critical region $X \leq 1$ and $X \geq 10$.

b Actual significance level $= 0.0338 + 0.0245 = 0.0583$

c $B(20, 0.1)$
$H_0 : p = 0.1 \quad H_1 : p > 0.1$
$P(X \geq 4) = 1 - P(X \leq 3)$
$= 1 - 0.8670$
$= 0.133$
$0.133 > 0.1$
Accept H_0. There is no evidence that the proportion of defective articles has increased.

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Exercise D, Question 9

Question:

It is claimed that 50% of women use Oriels powder. In a random survey of 20 women 12 said they did not use Oriels powder.
Test at the 5% significance level, whether or not there is evidence that the proportion of women using Oriels powder is 0.5. State your hypothesis carefully.

Solution:

$X \sim B(20, 0.5)$
$H_0 : p = 0.50 \quad H_1 : p \neq 0.50$
8 used Oriels powder.
$P(X \leq 8) = 0.2517$
$0.2517 > 0.025$
There is insufficient evidence to reject H_0.
There is no evidence that the claim is wrong.

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Exercise D, Question 10

Question:

A large caravan company hires caravans out for a week at a time. During winter the mean number of caravans hired is 6 per week.

a Calculate the probability that in one particular week in winter the company will hire out exactly 4 caravans.

The company decides to reduce prices in winter and do extra advertising. This results in the mean number of caravans being hired out rising to 11 per week.

b Test, at the 5% significance level, whether or not the proportion of caravans hired out has increased. State your hypothesis clearly.

Solution:

a $X \sim \text{Po}(6)$

$\text{P}(X = 4) = \text{P}(X \le 4) - \text{P}(X \le 3) = 0.2851 - 0.1512 = 0.1339$

b $\text{H}_0 : \lambda = 6 \quad \text{H}_1 : \lambda > 6$

$\text{P}(X \ge 11) = 1 - \text{P}(X \le 10) = 1 - 0.9574 = 0.0426$

$0.0426 < 0.05$

There is sufficient evidence to reject H_0.

There is evidence that the rate of hiring caravans has increased.

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Exercise D, Question 11

Question:

The manager of a superstore thinks that the probability of a person buying a certain make of computer is only 0.2.

To test whether this hypothesis is true the manager decides to record the make of computer bought by a random sample of 50 people who bought a computer.

a Find the critical region that would enable the manager to test whether or not there is evidence that the probability is different from 0.2. The probability of each tail should be as close to 2.5% as possible.

b Write down the significance level of this test.

15 people buy that certain make.

c Carry out the significance test. State your hypothesis clearly.

Solution:

$X \sim \text{B}(50, 0.2)$

a $\text{P}(X \le 4) = 0.0185$ (closer to 0.025)

$\text{P}(X \le 5) = 0.0480$

$c_1 = 4$

$\text{P}(X \ge 16) = 1 - \text{P}(X \le 15) = 1 - 0.9692 = 0.0308$

$\text{P}(X \ge 17) = 1 - \text{P}(X \le 16) = 1 - 0.9856 = 0.0144$

$c_2 = 16$ (0.0308 nearer to 0.0250)

Critical region is $X \le 4$ and $X \ge 16$.

b Actual significance level $= 0.0185 + 0.0308 = 0.0493$

c $\text{H}_0 : p = 0.2 \quad \text{H}_1 : p \ne 0.2$

15 is not in the critical region.

There is insufficient evidence to reject H_0.

There is no evidence to suggest that the proportion of people buying that certain make of computer differs from 0.2.

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Exercise D, Question 12

Question:

At one stage of a water treatment process the number of particles of foreign matter per litre present in the water has a Poisson distribution with mean 10. The water then enters a filtration bed which should extract 75% of foreign matter.
The manager of the treatment works orders a study into the effectiveness of this filtration bed. Twenty samples, each of 1 litre, are taken from the water and 64 particles of foreign matter are found.
Using a suitable approximation test, at the 5% level of significance, whether or not there is evidence that the filter bed is failing to work properly.

Solution:

Po(50) since if 75% removed then $\lambda = 20 \times (0.25 \times 10)$

$H_0 : \lambda = 50 \quad H_1 : \lambda > 50$

$P(X \geq 64) \approx P(Y > 63.5)$ where $Y \sim N(50, 50)$

$\approx P\left(Z > \frac{63.5 - 50}{\sqrt{50}}\right)$

$= 1 - 0.9718$

$= 0.0282$

$0.0281 < 0.05$ Reject H_0.

There is evidence that the filter bed is failing to work properly.

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Exercise D, Question 13

Question:

A shop finds that it sells jars of onion marmalade at the rate of 10 per week. During a television cookery program, onion marmalade is used in a recipe.
Over the next six weeks the shop sells 84 jars of onion marmalade. Using a suitable approximation test at the 5% significance level whether or not there is evidence that the rate of sales after the television program has increased as a result of the television program.

Solution:

Po (60)
$H_0 : \lambda = 60 \quad H_1 : \lambda > 60$
$P(X \geq 84) \approx P(Y > 83.5)$ where $Y \sim N(60, 60)$
$\approx P\left(Z > \dfrac{83.5 - 60}{\sqrt{60}}\right)$
$= 1 - 0.9989$
$= 0.0011$
$0.0011 < 0.05$ Reject H_0.
There is evidence that the rate of sales of onion marmalade has increased after the program.

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Exercise D, Question 14

Question:

A manufacturer produces large quantities of patterned plates. It is known from previous records that 6% of the plates will be seconds because of flaws in the patterns.
To verify that the production process is not getting worse the manager takes a sample of 150 plates and finds that 15 have flaws in their patterns. Use a suitable approximation to test, at the 5% significance level, whether or not the process is getting worse.

Solution:

B(150, 0.06)
$H_0 : p = 0.06 \quad H_1 : p > 0.06$

n is large (150), p is small (0.06)
so a Poisson approximation should be used.
$\lambda = np = 150 \times 0.06 = 9$
i.e. Po (9)
Upper tail $P(X \geq 15) = 1 - P(X \leq 14)$
$= 1 - 0.9585$
$= 0.0415 < 0.05$
Reject H_0.
There is evidence that the process is getting worse.

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Exercise D, Question 15

Question:

Jack grows apples. Over a period of time he finds that the probability of an apple being below the size required by a supermarket is 0.45.
He has recently set another orchard using a different variety of apple. A sample of 200 of this new type of apple had 60 rejected as being undersize.
Use a suitable approximation to test, at the 5% significance level, whether or not the new variety of apple is better than the old type of apple.

Solution:

B(200, 0.45)
$H_0 : p = 0.45 \quad H_1 : p < 0.45$
$P(X \le 60) = P(Y \le 60.5)$ where $Y \sim N(90, 49.5)$
$= P\left(Z < \frac{60.5 - 90}{\sqrt{49.5}}\right)$
$= P(Z < -4.19)$
$= 0.0000 < 0.05$
Reject H_0.
The new variety is better.