Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

a Write down a brief description of a census.
b Write down two advantages of using a census rather than a sample.
c Write down two disadvantages of using a census rather than a sample.

Solution:

a A census is when **every member of a population** is used.
b ANY TWO FROM:
It is unbiased.
It gives an accurate, reliable answer.
It looks at every single member of the population.
c ANY TWO FROM:
It can take a long time to do.
It is often costly.
It is not easy to ensure that every member of the population is taken into account.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

Write down which of the following are finite populations and which are infinite populations.

a Stars in the sky.

b Workers in a supermarket.

c The number of cows in Farmer Jacob's herd of cows.

Solution:

a is an infinite population.

b and **c** are finite populations.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 3

Question:

a Write down a brief description of a sample.

b Write down one disadvantage of taking a sample rather than a census.

c Write down two advantages of taking a sample rather than a census.

Solution:

a EITHER: A sample is a subset of the population.
OR: A sample consists of a selected group of the members of the population.

b ANY ONE FROM:
It may be biased.
It may be subject to natural variation.

c ANY TWO FROM:
It is generally cheaper.
Data is often easier to get.
It generally takes less time.
It avoids testing to destruction.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 4

Question:

A city council wants to know what people think about its recycling centre.
The council decides to carry out a sample survey to get the opinion of resident's views.

a Write down one reason why the council should not take a census.

b Suggest a suitable sampling frame.

c Identify the sampling units.

Solution:

a ANY ONE FROM:
It would be expensive.
It would be time consuming.
It would be difficult.

b A list of residents.

c A resident.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 5

Question:

A factory manufactures climbing ropes. The manager of the factory decides to investigate the breaking point of the ropes.
Write down a reason, other than easier and cheaper, why he would not use a census.

Solution:

The climbing ropes would all be destroyed.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 6

Question:

A supermarket manager wants to find out whether customers are satisfied with the range of products in the supermarket. He decides to do a survey.

a Write down a reason why the manager decides to use a sample rather than a census.

He decides to do a sample survey.

b Describe the sampling units for the sample survey.

c Give one advantage and one disadvantage of using a sample survey.

Solution:

a ANY ONE FROM:
It will be easier.
It will be quicker.
It will be cheaper.

b Customer.

c Advantages:
ANY ONE FROM:
It will be quick to do.
It will be easy to do.
It will not cost too much.
PLUS
Disadvantages:
ANY ONE FROM:
Not everyone's views will be known.
It might be biased.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 7

Question:

A manager of a garage wants to know what his mechanics think about a new pension scheme designed for them. He decides to ask all the mechanics in the garage.

a Describe the population he will use.

b Write down the main advantage there will be in asking all his mechanics.

Solution:

a All the mechanics in the garage.

b Everyone's views will be known.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 8

Question:

Each computer produced by a manufacturer is stamped with a unique serial number. ITPro Limited make their computers in batches of 1000. Before selling the computers, they test a random sample of 5 to see what electrical overload they will take before breaking down.

a Give one reason, other than to save time and cost, why a sample is taken rather than a census.

b Suggest a suitable sampling frame.

c Identify the sampling units.

Solution:

a If a census were used all the computers would be destroyed.

b The list of unique serial numbers.

c A computer.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 1

Question:

A forester wants to estimate the height of the trees in a forest. He measures the heights of 50 randomly selected trees and works out the mean height. State with a reason whether or not this mean is a statistic.

Solution:

This mean is from the values of a sample so it is a statistic.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 2

Question:

A random sample $M_1, M_2, M_3, \ldots, M_n$ is taken from a population with unknown mean μ. For each of the following state whether or not it is a statistic.

a $\dfrac{M_3 + M_8}{2}$

b $\dfrac{\Sigma M}{n}$

c $\dfrac{\Sigma M}{n} - \mu^2$

Solution:

i) and ii) are statistics.
iii) is not a statistic since it uses μ.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 3

Question:

The owners of a chain of hairdressing shops want to introduce the use of overalls in all the shops. The random variable Y is defined as

$Y = 0$ if the staff are happy to wear the overalls and

$Y = 1$ if the staff are unhappy about wearing the overalls.

a Suggest a suitable population and identify any parameter of interest.

A random sample of 20 of the hairdressers are asked whether they are happy or unhappy about wearing the overalls.

b Write down the name of the sampling distribution of the statistic $X = \sum_{1}^{20} Y$.

Solution:

a All the hairdressers who work for the chain of hairdressing shops.
The proportion p of the staff happy to wear overalls.

b This is a binomial distribution since we are only interested in two options – whether or not the hairdressers are happy to wear the overalls.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 4

Question:

A secretary makes spelling mistakes at the rate of 5 for every 10 pages. He has just finished typing a six-page document.

a Write down a suitable sampling distribution for the number of spelling mistakes in his document.

b Find the probability that there has been fewer than 2 spelling mistakes in the document.

Solution:

a) Po(3)

b) $P(X < 2) = P(X \le 1) = 0.1991$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 5

Question:

A bag contains a large number of coins. 50% are 50 pence coins.
25% are 20 pence coins. 25% are 10 pence coins.

a Find the mean, μ, and the variance, σ, for the value of this population of coins.

A random sample of 2 coins is chosen from the bag.

b List all the possible samples that can be chosen.

c Find the sampling distribution for the mean.

$$\overline{X} = \frac{X_1 + X_2}{2}$$

Solution:

a

X	50	20	10
X^2	2500	400	100
p	0.5	0.25	0.25

Mean $= (50 \times 0.5) + (20 \times 0.25) + (10 \times 0.25) = 25 + 5 + 2.5 = \mathbf{32.5}$

Variance $= ((2500 \times 0.5) + (400 \times 0.25) + (100 \times 0.25)) - 32.5^2$
$= (1250 + 100 + 25) - 1056.25 = 1375 - 1056.25 = 318.75$

b (50, 50)
(50, 20) (20, 50)
(50, 10) (10, 50)
(20, 20)
(20, 10) (10, 20)
(10, 10)

c $P(\overline{X} = 50) = 0.5 \times 0.5 = 0.25$
$P(\overline{X} = 35) = (0.5 \times 0.25) \times 2 = 0.25$
$P(\overline{X} = 30) = (0.5 \times 0.25) \times 2 = 0.25$
$P(\overline{X} = 20) = (0.25 \times 0.25) = 0.0625$
$P(\overline{X} = 15) = (0.25 \times 0.25) \times 2 = 0.125$
$P(\overline{X} = 10) = (0.25 \times 0.25) = 0.0625$

So the sampling distribution for the mean is:

$\overline{X}$	50	35	30	20	15	10
$P(\overline{X})$	0.25	0.25	0.25	0.0625	0.125	0.0625

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 6

Question:

A manufacturer makes three sizes of toaster. 40% of the toasters sell for £16, 50% sell for £20 and 10% sell for £30.

a Find the mean and variance of the value of the toasters.

A sample of 2 toasters is sent to a shop.

b List all the possible prices of the samples that could be sent.

c Find the sampling distribution for the mean price $\bar{X}$ of these samples.

Solution:

a

X	16	20	30
X^2	256	400	900
p	0.4	0.5	0.1

$$\text{Mean} = (16\times 0.4) + (20\times 0.5) + (30\times 0.1) = 6.4 + 10 + 3 = 19.4$$
$$\text{Variance} = ((256\times 0.4) + (400\times 0.5) + (900\times 0.1)) - 19.4^2$$
$$= (102.4 + 200 + 90) - 376.36 = 392.4 - 376.36 = 16.04$$

b

(16, 16)
(16, 20) (20, 16)
(16, 30) (30, 16)
(30, 30)
(30, 20) (20, 30)
(20, 20)

c

$\mathrm{P}(\bar{X} = 16) = 0.4\times 0.4 = 0.16$
$\mathrm{P}(\bar{X} = 18) = (0.4\times 0.5)\times 2 = 0.4$
$\mathrm{P}(\bar{X} = 23) = (0.4\times 0.1)\times 2 = 0.08$
$\mathrm{P}(\bar{X} = 30) = (0.1\times 0.1) = 0.01$
$\mathrm{P}(\bar{X} = 25) = (0.1\times 0.5)\times 2 = 0.1$
$\mathrm{P}(\bar{X} = 20) = (0.5\times 0.5) = 0.25$

So the sampling distribution for the mean is: [use UC X throughout]

$\bar{X}$	16	18	20	23	25	30
$\mathrm{P}(\bar{X})$	0.16	0.4	0.25	0.08	0.1	0.01

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 7

Question:

A supermarket sells a large number of 3-litre and 2-litre cartons of milk. They are sold in the ratio 3:2

a Find the mean and variance of the milk content in this population of cartons.

A random sample of 3 cartons is taken from the shelves (X_1, X_2 and X_3).

b List all the possible samples.

c Find the sampling distribution of the mean $\bar{X}$.

d Find the sampling distribution of the mode M.

e Find the sampling distribution of the median N of these samples.

Solution:

a

X	3	2
X^2	9	4
p	0.6	0.4

$$\text{Mean} = (3\times0.6) + (2\times0.4) = 1.8 + 0.8 = 2.6$$
$$\text{Variance} = ((9\times0.6) + (4\times0.4)) - 2.6^2 = (5.4 + 1.6) - 6.76 = 0.24$$

b (3, 3, 3)
(3, 3, 2) (3, 2, 3) (2, 3, 3)
(3, 2, 2) (2, 3, 2) (2, 2, 3)
(2, 2, 2)

c

$P(\bar{X} = 3) = 0.6^3 = 0.216$

$P(\bar{X} = 2\frac{2}{3}) = (0.6\times0.6\times0.4)\times3 = 0.432$

$P(\bar{X} = 2\frac{1}{3}) = (0.6\times0.4\times0.4)\times3 = 0.288$

$P(\bar{X} = 2) = 0.4^3 = 0.064$

So the sampling distribution for $\bar{X}$ is:

$\bar{X}$	3	$2\frac{2}{3}$	$2\frac{1}{3}$	2
$P(\bar{X})$	0.216	0.432	0.288	0.064

d The mode can be 3 or 2
$P(M = 3) = 0.216 + 0.432 = 0.648$
$P(M = 2) = 0.288 + 0.064 = 0.352$
So the sampling distribution for the mode M is:

M	3	2
$P(M)$	0.648	0.352

e

The median can be 3 (i.e. the cases (3, 3, 3) (3, 3, 2) (3, 2, 3) (2, 3, 3))
or 2 (i.e. the cases (3, 2, 2) (2, 3, 2) (2, 2, 3) (2, 2, 2))
$P(N = 3) = 0.6^3 + 3(0.6\times0.6\times0.4) = 0.216 + 0.432 = 0.648$
$P(N = 2) = 0.4^3 + 3(0.6\times0.4\times0.4) = 0.064 + 0.288 = 0.352$
So the sampling distribution for the median N is:

N	3	2
$P(N)$	0.648	0.352

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

A doctor's surgery is to offer health checks to all its patients over 65. In order to estimate the amount of time needed to do these health checks the doctor decides to do the health check for a random sample of 20 patients over 65.

a Write down a suitable sampling frame that the doctor might use.

b Identify the sampling units.

Solution:

a A list of all the patients on the surgery books.

b A patient.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 2

Question:

The owners of a large gym wish to change the opening hours. They want to find out whether the members will be happy with the new hours. They ask a random sample of 30 members.

a Write two likely reasons why the owners did not ask all the members.

b Suggest a suitable sampling frame.

c Identify the sampling units.

Solution:

a ANY TWO FROM:
It would take too long.
It could cost too much.
It could be difficult to get hold of all members.

b
A list of all members of the gym.

c
A member of the gym.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

a Write down a reason why a sampling frame and a population may not be the same.

b Explain briefly why a sample is often used rather than a census.

Solution:

a A sampling frame has to be some sort of list – it may not be possible to list a population.

b A sample is usually easier to do, quicker to do and not as costly as a census.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 4

Question:

a Explain what a statistic is.

A random sample $Y_1, Y_2, \ldots, Y_n$ is taken from a population with unknown mean μ.

b For each of the following state with a reason whether or not it is a statistic.

i $\dfrac{Y_1 + Y_2 + Y_3}{4}$

ii $\dfrac{\Sigma Y}{n} - \mu$

Solution:

a A statistic is a quantity calculated solely from the observations of a sample.

b i) is a statistic ii) is not a statistic as it depends on the value μ.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 5

Question:

A company manufactures electric light bulbs. They wish to see how many hours the light bulbs will work before failing. The company decides to test every 200th light bulb coming off the assembly line.

a Write down why the company does not test every light bulb.

b Identify the sampling units.

Solution:

a The light bulbs would all be destroyed.

b A light bulb.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 6

Question:

A call centre has 400 people operating the telephones. The manager decides that he needs to know how long the operatives are spending on each call. He times a random sample of 30 operators over one day and works out the mean time per call.

a Write down two advantages of using a sample rather than a census in this case.

b Write down one disadvantage of using a sample in this case.

A sample is to be taken.

c Suggest a sampling frame.

d Identify the sampling units

e Is the mean time the manager works out from the sample a statistic? Give a reason for your answer.

Solution:

a ANY TWO FROM:
It is quicker to do.
It is cheaper to do.
It is easier to do.

b It can be biased. OR it is subject to natural variations.

c A numbered list of all 400 call-centre operatives.

d A call-centre operative.

e Yes, because he is using only the values from a sample. There are no parameters.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 7

Question:

A flower shop has ten florists. The owner wants to know whether the florists are happy with the quality of the flowers being delivered to the shop. The owner asks all the florists their views. Write down two reasons why the owner of the florist shop uses a census.

Solution:

ANY TWO FROM:
It takes into account everyone's views.
It is unbiased
To take a sample when the population is only 10 would be silly.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 8

Question:

The weights of tomatoes in a greenhouse are assumed to have mean μ and standard deviation σ.
A sample of 20 tomatoes were each weighed and their weights were recorded. If the sample is represented by $X_1, X_2, \ldots, X_{20}$ state whether or not the following are statistics.

a $\dfrac{X_1 + X_{20}}{3}$

b $\dfrac{\Sigma X}{20}$

c $\Sigma X^2 + \mu$

d $\dfrac{\Sigma X^2}{20} - \sigma^2$

Solution:

a) and b) are statistics c) and d) are not statistics since they involve a population parameter.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 9

Question:

A large box of coins contains 5p, 10p, and 20p coins in the ratio 3:2:1.

a Find the mean μ and the variance σ^2 of the value of the coins.

A random sample of 2 coins is taken from the box and their values Y_1 and Y_2 are recorded.

b List all the possible samples that can be taken.

c Find the sampling distribution for the mean $(\bar{Y})$.

Solution:

a

$$\text{Mean} = (5\times\frac{3}{6}) + (10\times\frac{2}{6}) + (20\times\frac{1}{6}) = \frac{15}{6} + \frac{20}{6} + \frac{20}{6} = \frac{55}{6} = 9\frac{1}{6}$$

$$\text{Variance} = (25\times\frac{3}{6}) + (100\times\frac{2}{6}) + (400\times\frac{1}{6}) - (9\frac{1}{6})^2$$

$$= \frac{75}{6} + \frac{200}{6} + \frac{400}{6} - \frac{3025}{36} = \frac{675}{6} - \frac{3025}{36} = 28.47$$

b (5, 5) (10, 10) (20, 20)
(5, 10) (10, 5) (5, 20) (20, 5)
(10, 20) (20, 10)

c Possible means are:
5, 10, 20,
7.5, 12.5, 15 These all occur twice

$P(\bar{Y} = 5) = \frac{3}{6}\times\frac{3}{6} = \frac{9}{36} = \frac{1}{4}$

$P(\bar{Y} = 10) = \frac{2}{6}\times\frac{2}{6} = \frac{4}{36} = \frac{1}{9}$

$P(\bar{Y} = 20) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$

$P(\bar{Y} = 7.5) = \frac{3}{6}\times\frac{2}{6}\times 2 = \frac{12}{36} = \frac{1}{3}$

$P(\bar{Y} = 12.5) = \frac{3}{6}\times\frac{1}{6}\times 2 = \frac{6}{36} = \frac{1}{6}$

$P(\bar{Y} = 15) = \frac{2}{6}\times\frac{1}{6}\times 2 = \frac{4}{36} = \frac{1}{9}$

So the sampling distribution for the means is:

$\bar{Y}$	5	7.5	10	12.5	15	20
$P(\bar{Y})$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{36}$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 10

Question:

A bag contains a large number of counters
60% have a value of 6
40% have a value of 10.
A random sample of 3 counters is drawn from the bag.

a Write down all the possible samples.
b Find the sampling distribution for the median N.
c Find the sampling distribution for the mode M.

Solution:

a (6, 6, 6)
(6, 6, 10) (6, 10, 6) (10, 6, 6)
(6, 10, 10) (10, 6, 10) (10, 10, 6)
(10, 10, 10)

b Medians can be 6 or 10. If put in order 4 give a median of 10 and 4 give a median of 6

$$P(N=6)=\left(\frac{6}{10}\times\frac{6}{10}\times\frac{6}{10}\right)+3\left(\frac{6}{10}\times\frac{6}{10}\times\frac{4}{10}\right)=\frac{216}{1000}+\frac{432}{1000}=\frac{648}{1000}=0.648$$

$$P(N=10)=3\left(\frac{6}{10}\times\frac{4}{10}\times\frac{4}{10}\right)+\left(\frac{4}{10}\times\frac{4}{10}\times\frac{4}{10}\right)=\frac{288}{1000}+\frac{64}{1000}=\frac{352}{1000}=0.352$$

So distribution of median is:

N	6	10
P(M)	0.648	0.352

c The mode is either 6 or 10

$$P(M=6)=\left(\frac{6}{10}\times\frac{6}{10}\times\frac{6}{10}\right)+3\left(\frac{6}{10}\times\frac{6}{10}\times\frac{4}{10}\right)=\frac{216}{1000}+\frac{432}{1000}=\frac{648}{1000}=0.648$$

$$P(M=10)=3\left(\frac{6}{10}\times\frac{4}{10}\times\frac{4}{10}\right)+\left(\frac{4}{10}\times\frac{4}{10}\times\frac{4}{10}\right)=\frac{288}{1000}+\frac{64}{1000}=\frac{352}{1000}=0.352$$

So distribution of mode is:

M	6	10
P(M)	0.648	0.352