

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

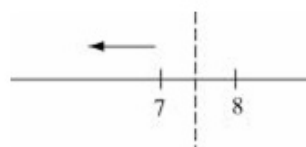
Question:

The discrete random variable X takes integer values and is to be approximated by a normal distribution. Apply a continuity correction to the following probabilities.

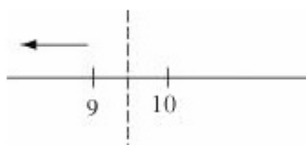
- a $P(X \leq 7)$
- b $P(X < 10)$
- c $P(X > 5)$
- d $P(X \geq 3)$
- e $P(17 \leq X \leq 20)$
- f $P(18 < X < 30)$
- g $P(28 < X \leq 40)$
- h $P(23 \leq X < 35)$

Solution:

a $P(X \leq 7) = P(X \leq 7.5)$



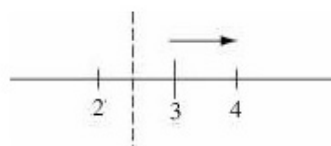
b $P(X < 10) = P(X \leq 9.5)$



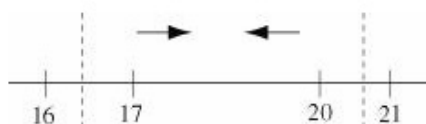
c $P(X > 5) = P(X \geq 5.5)$



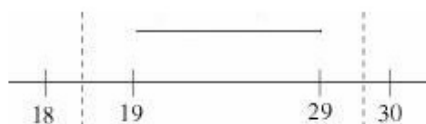
d $P(X \geq 3) = P(X \geq 2.5)$



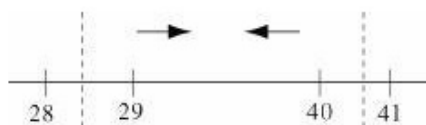
e $P(17 \leq X \leq 20) = P(16.5 \leq X < 20.5)$



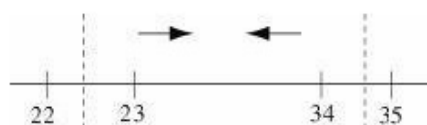
f $P(18 < X < 30) = P(18.5 \leq X < 29.5)$



g $P(28 < X \leq 40) = P(28.5 \leq X < 40.5)$



h $P(23 \leq X < 35) = P(22.5 \leq X < 34.5)$



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Exercise B, Question 1

Question:

The random variable $X \sim B(150, \frac{1}{3})$. Use a suitable approximation to estimate

- $P(X \leq 40)$,
- $P(X > 60)$,
- $P(45 \leq X \leq 60)$.

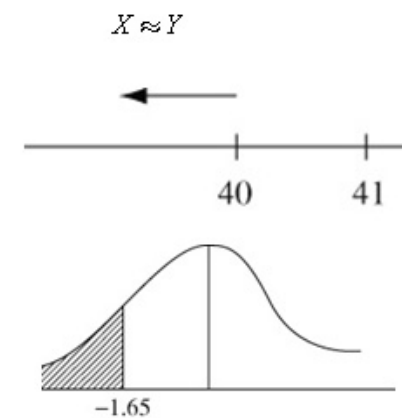
Solution:

$$X \sim B(150, \frac{1}{3})$$

$$Y \sim N(50, \sqrt{\frac{100}{3}})$$

a

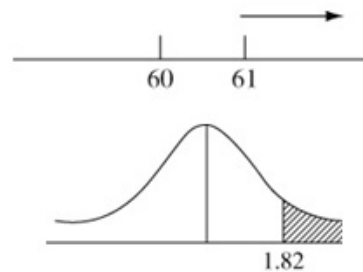
$$\begin{aligned} P(X \leq 40) &\approx P(Y < 40.5) \\ &= P\left(Z < \frac{40.5 - 50}{\sqrt{\frac{100}{3}}}\right) \\ &= P(Z < -1.645\dots) \\ &= 1 - 0.9505 \\ &= 0.0495 \end{aligned}$$



(calc = 0.0499...)

b

$$\begin{aligned} P(X > 60) &\approx P(Y > 60.5) \\ &= P(Z > 1.818\dots) \\ &= 1 - 0.9656 \\ &= 0.0344 \end{aligned}$$



(calc = 0.03448...)

So accept awrt 0.0340 (3 s.f.)

c

$$\begin{aligned} P(45 \leq X \leq 60) &\approx P(44.5 \leq Y < 60.5) \\ &= P(-0.95 \leq Z < 1.82) \\ &= 0.9656 - (1 - 0.8289) \\ &= 0.7945 \end{aligned}$$

(calc = 0.79512...)

accept awrt 0.795

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Exercise B, Question 2

Question:

The random variable $X \sim B(200, 0.2)$. Use a suitable approximation to estimate

- $P(X < 45)$,
- $P(25 \leq X < 35)$,
- $P(X = 42)$.

Solution:

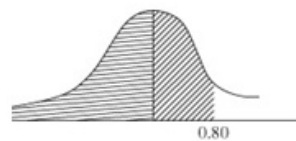
$$X \sim B(200, 0.2)$$

$$Y \sim N(40, \sqrt{32}^2)$$

$$X \approx Y$$

a

$$\begin{aligned} P(X < 45) &\approx P(Y \leq 44.5) \\ &= P\left(Z < \frac{44.5 - 40}{\sqrt{32}}\right) \\ &= P(Z < 0.7954\dots) \\ &= 0.7881 \end{aligned}$$

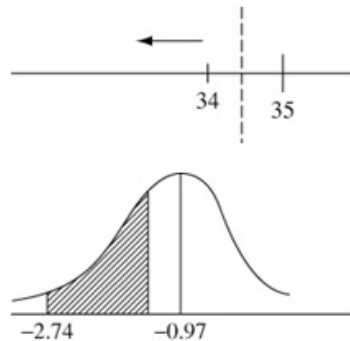


(calc = 0.7868...)

So accept awrt 0.788 ~ 0.787

b

$$\begin{aligned} P(25 \leq X < 35) &= P(24.5 \leq Y < 34.5) \\ &= P(-2.74 \leq Z < -0.97\dots) \\ &= [1 - 0.8340] - [1 - 0.9970] \\ &= 0.163 \end{aligned}$$

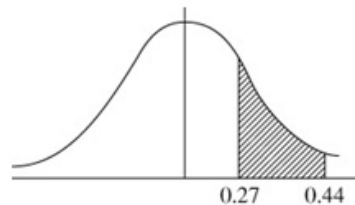


(calc = 0.162386...)

so accept awrt 0.162 ~ 0.163

c

$$\begin{aligned} P(X = 42) &= P(41.5 \leq Y < 42.5) \\ &= P(0.265\dots \leq Z < 0.4419\dots) \\ &= 0.6700 - 0.6064 \\ &= 0.0636 \end{aligned}$$



(calc = 0.066175...)

so accept awrt 0.0640 ~ 0.0660

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Exercise B, Question 3

Question:

The random variable $X \sim B(100, 0.65)$. Use a suitable approximation to estimate

- a $P(X > 58)$,
- b $P(60 < X \leq 72)$,
- c $P(X = 70)$.

Solution:

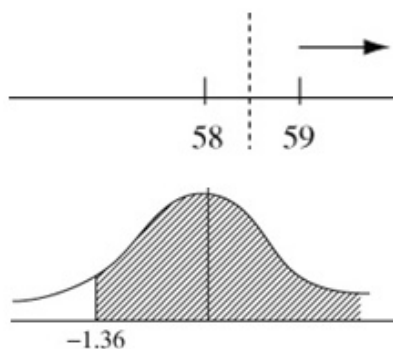
$$X \sim B(100, 0.65)$$

$$Y \sim N(65, \sqrt{22.75^2})$$

$$X \approx Y$$

a

$$\begin{aligned} P(X > 58) &\approx P(Y > 58.5) \\ &= P(Z > -1.36\dots) \\ &= 0.9131 \end{aligned}$$

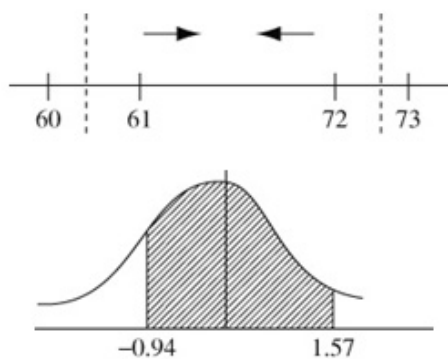


(calc = 0.9135...)

So accept awrt 0.913 ~ 0.914

b

$$\begin{aligned} P(60 < X \leq 72) &\approx P(60.5 \leq Y < 72.5) \\ &= P(-0.94 \leq Z < 1.57\dots) \\ &= 0.9418 - (1 - 0.8264) \\ &= 0.7682 \end{aligned}$$

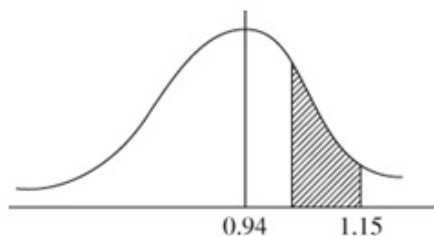


(calc = 0.76935...)

So accept awrt 0.768 ~ 0.769

c

$$\begin{aligned} P(X = 70) &\approx P(69.5 \leq Y < 70.5) \\ &= P(0.943 \leq Z < 1.153\dots) \\ &= 0.8749 - 0.8264 \\ &= 0.0485 \end{aligned}$$



(calc = 0.04829...)

So accept awrt 0.0483 ~ 0.0485

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Exercise B, Question 4

Question:

Sarah rolls a fair die 90 times. Use a suitable approximation to estimate the probability that the number of sixes she obtains is over 20.

Solution:

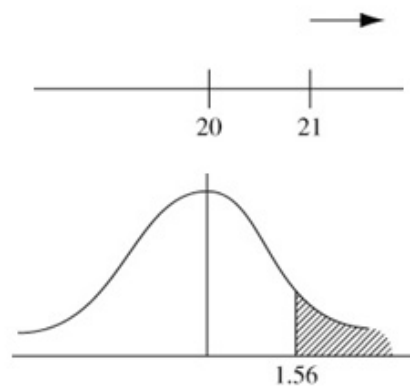
X = number of sixes in 90 rolls

$$X \sim B(90, \frac{1}{6})$$

$$Y \sim N(15, \sqrt{12.5^2})$$

$$X \approx Y$$

$$\begin{aligned} P(X > 20) &\approx P(Y \geq 20.5) \\ &= P\left(Z \geq \frac{20.5 - 15}{\sqrt{12.5}}\right) \\ &= P(Z \geq 1.555\dots) \\ &= 1 - 0.9406 \\ &= 0.0594 \end{aligned}$$



(calc = 0.059897...)

So accept awrt 0.059 ~ 0.060

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Exercise B, Question 5

Question:

In a multiple choice test there are 4 possible answers to each question. Given that there are 60 questions on the paper, use a suitable approximation to estimate the probability of getting more than 20 questions correct if the answer to each question is chosen at random from the 4 available choices for each question.

Solution:

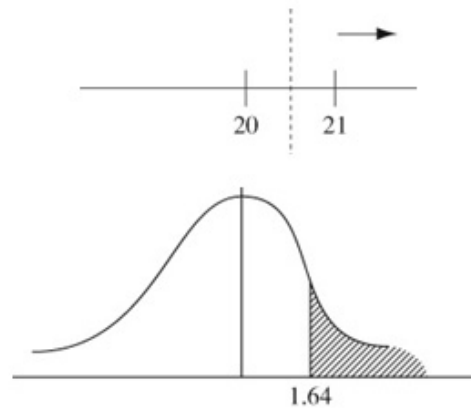
X = number of correct answers

$$X \sim B(60, \frac{1}{4})$$

$$Y \sim N(15, \sqrt{11.25}^2)$$

$$X \approx Y$$

$$\begin{aligned} P(X > 20) &\approx P(Y \geq 20.5) \\ &= P\left(Z \geq \frac{20.5 - 15}{\sqrt{11.25}}\right) \\ &= P(Z \geq 1.639\dots) \\ &= 1 - 0.9495 \\ &= 0.0505 \end{aligned}$$



(calc gives 0.050525...)
So accept awrt 0.0505

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Exercise B, Question 6

Question:

A fair coin is tossed 70 times. Use a suitable approximation to estimate the probability of obtaining more than 45 heads.

Solution:

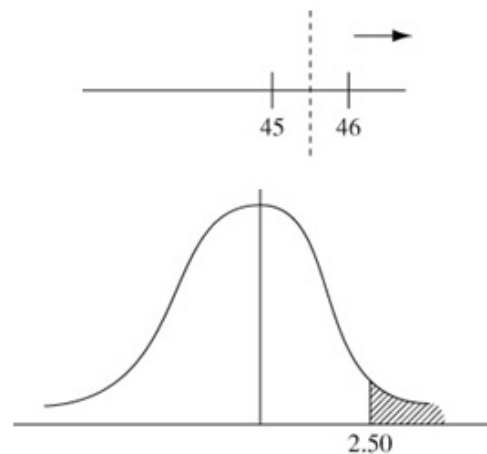
X = number of heads in 70 tosses of a fair coin

$$X \sim B(70, 0.5)$$

$$Y \sim N(35, \sqrt{17.5}^2)$$

$$X \approx Y$$

$$\begin{aligned} P(X > 45) &= P(Y \geq 45.5) \\ &= P\left(Z > \frac{45.5 - 35}{\sqrt{17.5}}\right) \\ &= P(Z > 2.5099\dots) \\ &= 1 - 0.9940 \\ &= 0.0060 \end{aligned}$$



(calc gives 0.006036...)

So accept awrt 0.006

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Exercise C, Question 1

Question:

The random variable $X \sim \text{Po}(30)$. Use a suitable approximation to estimate

- a $P(X \leq 20)$,
- b $P(X > 43)$,
- c $P(25 \leq X \leq 35)$.

Solution:

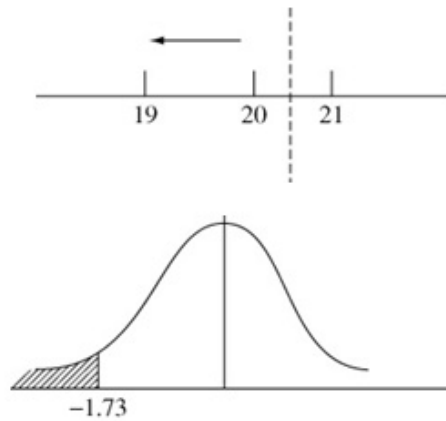
$$X \sim \text{Po}(30)$$

$$Y \sim \text{N}(30, \sqrt{30})$$

$$X \approx Y$$

a

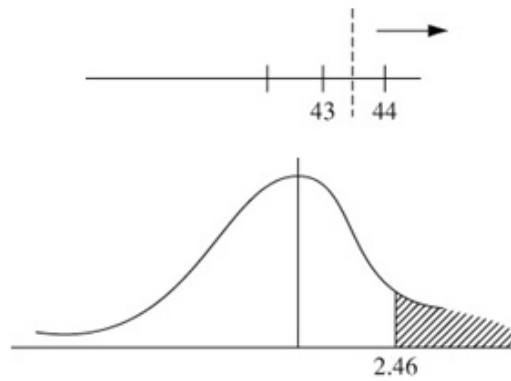
$$\begin{aligned} P(X \leq 20) &\approx P(Y \leq 20.5) \\ &= P\left(Z < \frac{20.5 - 30}{\sqrt{30}}\right) \\ &= P(Z < -1.7344\dots) \\ &= 1 - 0.9582 \\ &= 0.0418 \end{aligned}$$



(calc gives 0.041418...)
So accept awrt 0.0410 ~ 0.0420

b

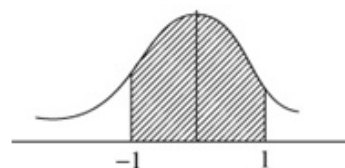
$$\begin{aligned} P(X > 43) &\approx P(Y > 43.5) \\ &= P\left(Z > \frac{43.5 - 30}{\sqrt{30}}\right) \\ &= P(Z > 2.46\dots) \\ &= 1 - 0.9931 \\ &= 0.0069 \end{aligned}$$



(calc gives 0.006855...)
accept awrt 0.0069

c

$$\begin{aligned} P(25 \leq X \leq 35) &\approx P(24.5 \leq Y < 35.5) \\ &= P(-1.00 < Z < 1.00\dots) \\ &= 2 \times 0.3413 \\ &= 0.6826 \end{aligned}$$



(calc gives 0.68469...)
accept awrt 0.683 ~ 0.685

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 2

Question:

The random variable $X \sim \text{Po}(45)$. Use a suitable approximation to estimate

- a $P(X < 40)$,
- b $P(X \geq 50)$,
- c $P(43 < X \leq 52)$.

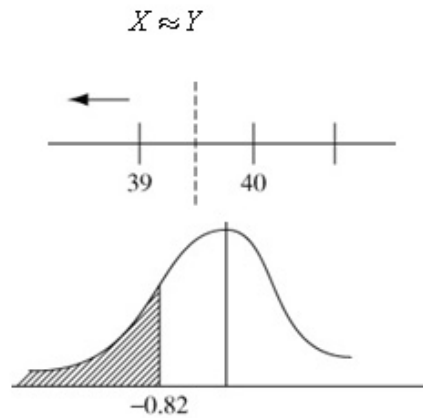
Solution:

$$X \sim \text{Po}(45)$$

$$Y \sim N(45, \sqrt{45})$$

a

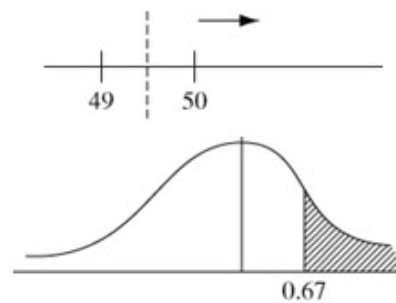
$$\begin{aligned} P(X < 40) &\approx P(Y \leq 39.5) \\ &= P\left(Z < \frac{39.5 - 45}{\sqrt{45}}\right) \\ &= P(Z < -0.819\dots) \\ &= 1 - 0.7939 \\ &= 0.2061 \end{aligned}$$



(calc gives 0.20613...)
So accept awrt 0.206

b

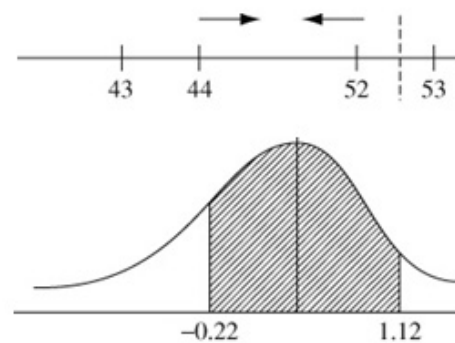
$$\begin{aligned} P(X \geq 50) &\approx P(Y \geq 49.5) \\ &= P(Z \geq 0.6708\dots) \\ &= 1 - 0.7486 \\ &= 0.2514 \end{aligned}$$



(calc gives 0.25116...)
So accept awrt 0.251

c

$$\begin{aligned} P(43 < X \leq 52) &\approx P(43.5 \leq Y < 52.5) \\ &= P(-0.22\dots \leq Z < 1.12\dots) \\ &= 0.8686 - (1 - 0.5871) \\ &= 0.4557 \end{aligned}$$



(calc gives 0.45669...)
So accept awrt 0.456 ~ 0.457

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

The random variable $X \sim \text{Po}(60)$. Use a suitable approximation to estimate

- a $P(X \leq 62)$,
- b $P(X = 63)$,
- c $P(55 \leq X < 65)$.

Solution:

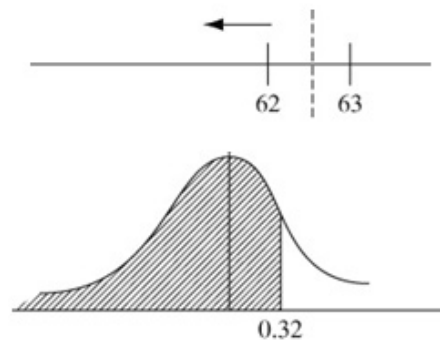
$$X \sim \text{Po}(60)$$

$$Y \sim \text{N}(60, \sqrt{60}^2)$$

$$X \approx Y$$

a

$$\begin{aligned} P(X \leq 62) &\approx P(Y < 62.5) \\ &= P\left(Z < \frac{62.5 - 60}{\sqrt{60}}\right) \\ &= P(Z < 0.3227\dots) \\ &= 0.6255 \end{aligned}$$

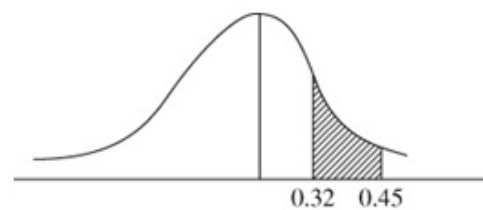


(calc gives 0.62655...)

So accept awrt 0.626 ~ 0.627

b

$$\begin{aligned} P(X = 63) &\approx P(62.5 \leq Y < 63.5) \\ &= P(0.32\dots \leq Z < 0.45\dots) \\ &= 0.6736 - 0.6255 \\ &= 0.0481 \end{aligned}$$

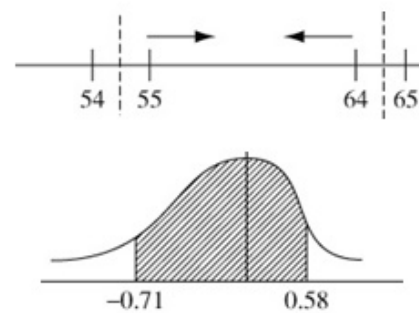


(calc gives 0.04775...)

So accept awrt 0.0480

c

$$\begin{aligned} P(55 \leq X < 65) &= P(54.5 \leq Y < 64.5) \\ &= P(-0.71\dots \leq Z < 0.58\dots) \\ &= 0.7190 - (1 - 0.7611) \\ &= 0.4801 \end{aligned}$$



(Calc gives 0.48052...)

So accept awrt 0.480 ~ 0.481

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 4

Question:

The disintegration of a radioactive specimen is known to be at the rate of 14 counts per second. Using a normal approximation for a Poisson distribution, determine the probability that in any given second the counts will be

- a 20, 21 or 22,
- b greater than 10,
- c above 12 but less than 16.

Solution:

X = number of counts in one second

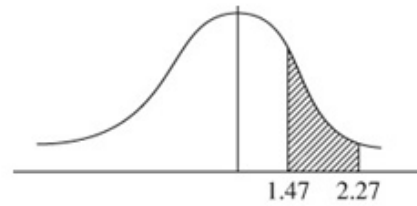
$$X \sim \text{Po}(14)$$

$$Y \sim \text{N}(14, \sqrt{14}^2)$$

$$X \approx Y$$

a

$$\begin{aligned} P(20 \leq X \leq 22) &\approx P(19.5 \leq Y < 22.5) \\ &= P(1.47 \leq Z < 2.27 \dots) \\ &= 0.9884 - 0.9292 \\ &= 0.0592 \end{aligned}$$

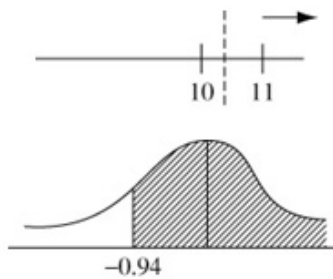


(calc gives 0.059237...)

So accept awrt 0.0590 ~ 0.0600

b

$$\begin{aligned} P(X > 10) &\approx P(Y \geq 10.5) \\ &= P(Z \geq -0.935 \dots) \\ &= 0.8264 \end{aligned}$$

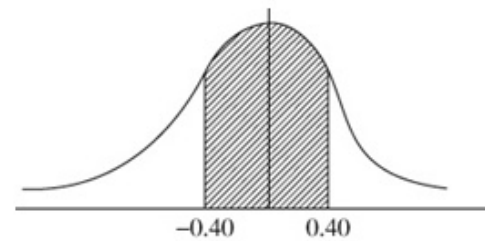


(calc gives 0.8252...)

So accept awrt 0.825 ~ 0.826

c

$$\begin{aligned} P(12 < X < 16) &\approx P(12.5 \leq Y \leq 15.5) \\ &= P(-0.40 \dots \leq Z < 0.40 \dots) \\ &= 2 \times 0.1554 \\ &= 0.3108 \end{aligned}$$



(calc gives 0.311500...)

So accept awrt 0.311 ~ 0.312

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 5

Question:

A marina hires out boats on a daily basis. The mean number of boats hired per day is 15. Using the normal approximation for a Poisson distribution, find, for a period of 100 days

- a how often 5 or fewer boats are hired,
- b how often exactly 10 boats are hired,
- c on how many days they will have to turn customers away if the marina owns 20 boats.

Solution:

X = number of boats hired per day

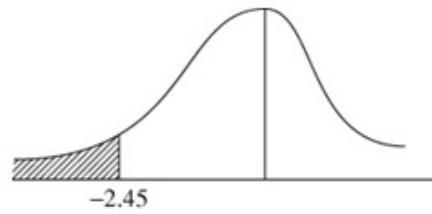
$$X \sim \text{Po}(15)$$

$$Y \sim \text{N}(15, \sqrt{15}^2)$$

$$X \approx Y$$

a

$$\begin{aligned} P(X \leq 5) &\approx P(Y < 5.5) \\ &= P(Z < -2.452\dots) \\ &= 1 - 0.9929 \\ &= 0.0071 \end{aligned}$$



(calc gives 0.00708...)

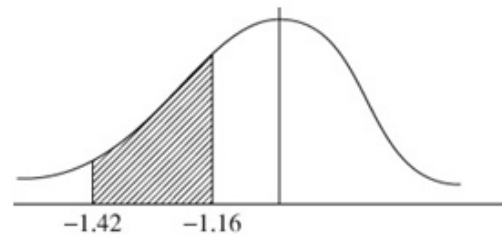
So accept awrt 0.0070 to 0.0071

i.e. in 100 days ≈ 0.7 times

i.e. 1 day

b

$$\begin{aligned} P(X = 10) &\approx P(9.5 \leq Y < 10.5) \\ &= P(-1.42 \leq Z < -1.16) \\ &= [1 - 0.8770] - [1 - 0.9222] \\ &= 0.0452 \end{aligned}$$



(calc gives 0.04484...)

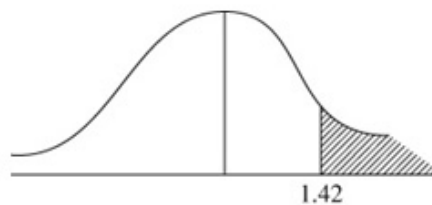
So accept awrt 0.045

i.e. in 100 days ≈ 4.5 days

i.e. 4 or 5 days

c

$$\begin{aligned} P(X > 20) &\approx P(Y > 20.5) \\ &= P(Z > 1.42\dots) \\ &= 1 - 0.9222 \\ &= 0.0778 \end{aligned}$$



(calc gives 0.07779...)

So accept 0.078

i.e. in 100 days ≈ 7.8 days

i.e. 8 days

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Edexcel AS and A Level Modular Mathematics

Exercise D, Question 1

Question:

A fair die is rolled and the number of sixes obtained is recorded.

Using suitable approximations, find the probability of

- no more than 10 sixes in 48 rolls of the die,
- at least 25 sixes in 120 rolls of the die.

Solution:

a X = number of sixes in 48 rolls of a die

$$X \sim B(48, \frac{1}{6})$$

$$Y \sim \text{Po}(8) \quad \mu = 8$$

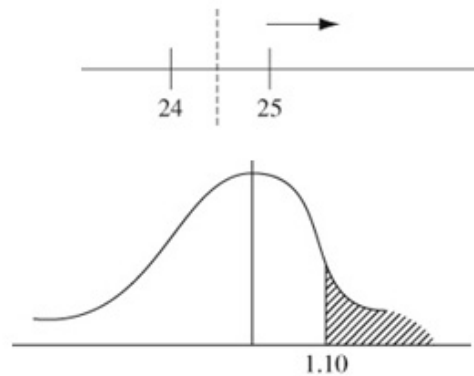
$$\begin{aligned} P(X \leq 10) &\approx P(Y \leq 10) \\ &= 0.8159 \quad (\text{Poisson tables}) \end{aligned}$$

b X = number of sixes in 120 rolls of a die

$$X \sim B(120, \frac{1}{6})$$

$$Y \sim N(20, \sqrt{\frac{100}{6}}) \text{ or } Y \sim N(20, \sqrt{\frac{50}{3}}) \quad \mu = 20$$

$$\begin{aligned} P(X \geq 25) &\approx P(Y \geq 24.5) \\ &= P(Z \geq 1.10\dots) \\ &= 1 - 0.8643 \\ &= 0.1357 \end{aligned}$$



(calc gives 0.135172...)

So accept awrt 0.135 ~ 0.136

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Exercise D, Question 2

Question:

A fair coin is spun 60 times.

Use a suitable approximation to estimate the probability of obtaining fewer than 25 heads.

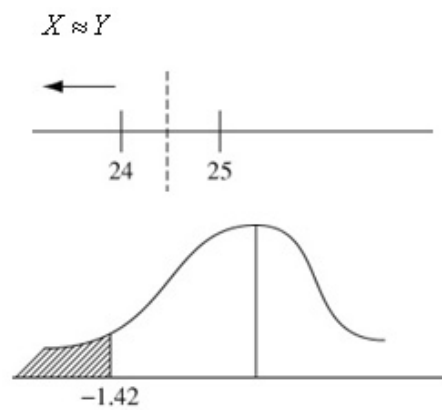
Solution:

X = number of heads in 60 spins of a coin

$$X \sim B(60, 0.5)$$

$$Y \sim N(30, \sqrt{15}^2)$$

$$\begin{aligned} P(X < 25) &\approx P(Y < 24.5) \\ &= P(Z < -1.42\dots) \\ &= 1 - 0.9222 \\ &= 0.0778 \end{aligned}$$



(calc gives 0.07779022...)

So accept awrt 0.0778

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Exercise D, Question 3

Question:

The owner of a local corner shop calculates that the probability of a customer buying a newspaper is 0.40 but the proportion of customers who spend over £10 is 0.04.

A random sample of 100 customer's shopping is recorded. Use suitable approximations to estimate the probability that in this sample

- at least half of the customers bought a newspaper,
- more than 5 of them spent over £10.

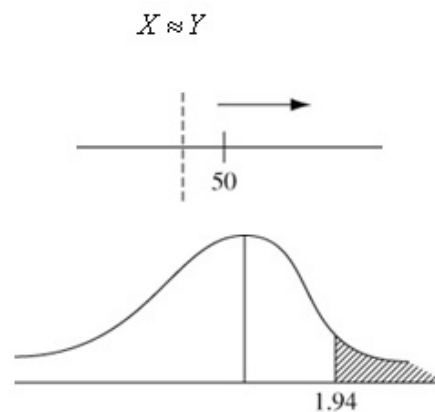
Solution:

- a X = number of customers who bought a newspaper

$$X \sim B(100, 0.40)$$

$$Y \sim N(40, \sqrt{24}^2)$$

$$\begin{aligned} P(X \geq 50) &\approx P(Y \geq 49.5) \\ &= P\left(Z \geq \frac{49.5 - 40}{\sqrt{24}}\right) \\ &= P(Z \geq 1.939\dots) \\ &= 1 - 0.9738 \\ &= 0.0262 \end{aligned}$$



calc gives 0.026239...

So accept awrt 0.0262

- b T = number of customers who spent over £10

$$T \sim B(100, 0.04)$$

$$\mu = 4$$

$$S \approx \text{Po}(4)$$

$$T \approx S$$

$$\begin{aligned} P(T > 5) &\approx P(S \geq 6) && \text{(No continuity correction)} \\ &= 1 - P(S \leq 5) \\ &= 1 - 0.7851 && \text{(Poisson tables)} \\ &= 0.2149 \end{aligned}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 4

Question:

Street light failures in a town occur at a rate of one every two days. Assuming that X , the number of street light failures per week, has a Poisson distribution, find the probabilities that the number of street lights that will fail in a given week is

- a exactly 2,
- b less than 6.

Using a suitable approximation estimate the probability that

- c there will be fewer than 45 street light failures in a 10-week period.

Solution:

$$X \sim \text{Po}(3.5)$$

a

$$\begin{aligned} P(X=2) &= P(X \leq 2) - P(X \leq 1) \\ &= 0.3208 - 0.1359 \\ &= 0.1849 \end{aligned}$$

b

$$\begin{aligned} P(X < 6) &= P(X \leq 5) \\ &= 0.8576 \end{aligned}$$

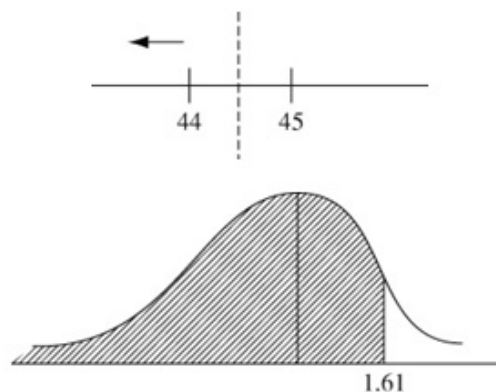
Let Y = number of failures in 10-week period

$$Y \sim \text{Po}(35)$$

$$W \sim N(35, \sqrt{35}^2)$$

c

$$\begin{aligned} P(Y < 45) &\approx P(W \leq 44.5) \\ &= P\left(Z \leq \frac{44.5 - 35}{\sqrt{35}}\right) \\ &= P(Z \leq 1.605\dots) \\ &= 0.9463 \end{aligned}$$



calc gives 0.945840...

So accept awrt 0.946

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 5

Question:

Past records from a supermarket show that 20% of people who buy chocolate bars buy the family size bar. A random sample of 80 people is taken from those who had bought chocolate bars.

- a Use a suitable approximation to estimate the probability that more than 20 of these 80 bought family size bars.

The probability of a customer buying a gigantic chocolate bar is 0.02.

- b Using a suitable approximation estimate the probability that fewer than 5 customers in a sample of 150 buy a gigantic chocolate bar.

Solution:

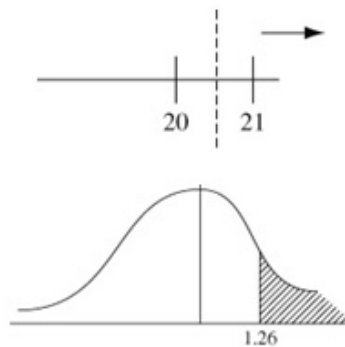
X = number of people out of 80 who buy family size chocolate bars

$$X \sim B(80, 0.20)$$

$$Y \sim N(16, \sqrt{12.8}^2)$$

a

$$\begin{aligned} P(X > 20) &\approx P(Y \geq 20.5) \\ &= P\left(Z \geq \frac{20.5 - 16}{\sqrt{12.8}}\right) \\ &= P(Z \geq 1.2577\dots) \\ &= 1 - 0.8962 \\ &= 0.1038 \end{aligned}$$



calc gives 0.104234...

So accept awrt 0.104

- b G = number who buy a gigantic bar of chocolate

$$G \sim B(150, 0.02)$$

$$\mu = 150 \times 0.02 = 3$$

$$H \sim \text{Po}(3)$$

$$G \approx H$$

$$\begin{aligned} P(G < 5) &\approx P(H < 5) && \text{(No continuity correction)} \\ &= P(H \leq 4) \\ &= 0.8153 && \text{(Poisson tables)} \end{aligned}$$