# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 1**

**Question:**

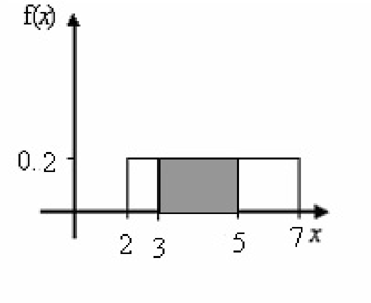
The continuous random variable $X \sim \text{U}[2, 7]$.
Find

**a** $\text{P}(3 < X < 5)$,

**b** $\text{P}(X > 4)$.

**Solution:**

**a** $\frac{1}{b-a} = \frac{1}{7-2} = 0.2$




$$\begin{aligned}\text{P}(3 < X < 5) &= (5-3) \times 0.2 \\ &= 0.4\end{aligned}$$

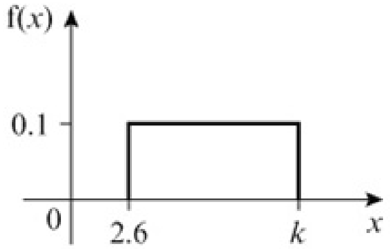
**b** $$\begin{aligned}\text{P}(X > 4) &= (7-4) \times 0.2 \\ &= 0.6\end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 2**

**Question:**

The continuous random variable $X$ has p.d.f. as shown in the diagram.




Find

**a** the value of $k$,

**b** $P(4 < X < 7.9)$ .

**Solution:**

**a**
$$\begin{aligned} \text{Area} &= 1 \\ (k-2.6)\times 0.1 &= 1 \\ (k-2.6) &= 10 \\ k &= 12.6 \end{aligned}$$

**b**
$$\begin{aligned} P(4 < X < 7.9) &= (7.9-4) \times 0.1 \\ &= 0.39 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 3**

**Question:**

The continuous random variable $X$ has p.d.f.

$$f(x) = \begin{cases} k, & -2 \le x \le 6, \\ 0, & \text{otherwise.} \end{cases}$$

Find

**a** the value of $k$,

**b** $P(-1.3 < X < 4.2)$ .

**Solution:**

**a**
$$\begin{aligned} \text{Area} &= 1 \\ k \times (6 - (-2)) &= 1 \\ 8k &= 1 \\ k &= \frac{1}{8} \end{aligned}$$

**b**
$$\begin{aligned} P(-1.3 < X < 4.2) &= \frac{1}{8} \times (4.2 - (-1.3)) \\ &= 0.6875 \end{aligned}$$

# Solutionbank S2

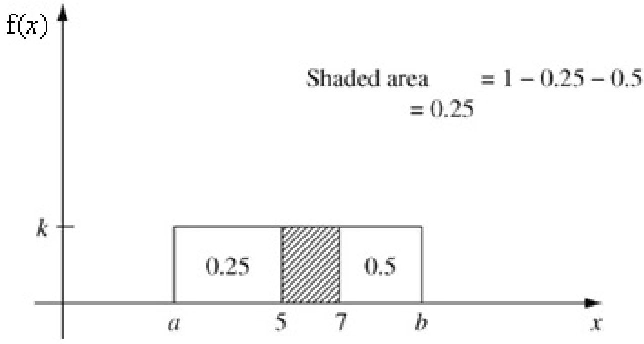
## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 4**

**Question:**

The continuous random variable $Y \sim \mathrm{U}[a,\ b]$. Given that $\mathrm{P}(Y < 5) = \frac{1}{4}$ and $\mathrm{P}(Y > 7) = \frac{1}{2}$, find the value of $a$ and the value of $b$.

**Solution:**




$$\text{shaded area} = \frac{1}{4}$$
$$2 \times k = \frac{1}{4}$$
$$k = \frac{1}{8}$$
$$(b-7)\times\frac{1}{8} = 0.5$$
$$(b-7) = 4$$
$$b = 11$$
$$(5-a)\times\frac{1}{8} = 0.25$$
$$(5-a) = 2$$
$$a = 3$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 5**

**Question:**

The continuous random variable $X \sim U[2, 8]$.

**a** Write down the distribution of $Y = 2X + 5$.

**b** Find $P(12 < Y < 20)$.

**Solution:**

**a**

$2 \times 2 + 5 = 9$

$2 \times 8 + 5 = 21$

$Y \sim U[9, 21]$

**b** For $Y$, $\frac{1}{b-a} = \frac{1}{21-9} = \frac{1}{12}$

$$P(12 < Y < 20) = (20 - 12) \times \frac{1}{12}$$

$$= \frac{2}{3}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 1**

**Question:**

The continuous variable $Y$ is uniformly distributed over the interval $[-3, 5]$.

Find:

**a** $\mathrm{E}(X)$,

**b** $\mathrm{Var}(X)$,

**c** $\mathrm{E}(X^2)$,

**d** the cumulative distribution function of $X$, for all $x$.

**Solution:**

**a** $\mathrm{E}(X) = \dfrac{5+(-3)}{2}$

$= 1$

**b** $\mathrm{Var}(X) = \dfrac{(5-(-3))^2}{12}$

$= 5\frac{1}{3}$

**c** $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

$5\frac{1}{3} = \mathrm{E}(X^2) - 1$

$\mathrm{E}(X^2) = 6\frac{1}{3}$

**d** $\mathrm{F}(x) = \displaystyle\int_{-3}^{x} \frac{1}{5-(-3)}\, \mathrm{d}t$

$= \left[\dfrac{t}{8}\right]_{-3}^{x}$

$= \dfrac{x+3}{8}$

$$\mathrm{F}(x) = \begin{cases} 0 & x < -3 \\ \dfrac{x+3}{8} & -3 \le x \le 5 \\ 1 & x > 5 \end{cases}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 2**

**Question:**

Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ for the following probability density functions.

**a** $\mathrm{f}(x)=\begin{cases}\frac{1}{4}, & 1\le x\le 5,\\ 0, & \text{otherwise.}\end{cases}$

**b** $\mathrm{f}(x)=\begin{cases}\frac{1}{8}, & -2\le x\le 6,\\ 0, & \text{otherwise.}\end{cases}$

**Solution:**

**a** $\mathrm{E}(X)=\frac{5+1}{2}$

$= 3$

$\mathrm{Var}(X)=\frac{(5-1)^2}{12}$

$= 1\frac{1}{3}$

**b** $\mathrm{E}(X)=\frac{6+(-2)}{2}$

$= 2$

$\mathrm{Var}(X)=\frac{(6-(-2))^2}{12}$

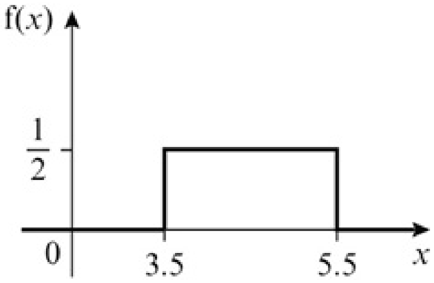
$= 5\frac{1}{3}$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 3**

**Question:**

The continuous random variable $X$ has p.d.f as shown in the diagram.




Find:

**a** $E(X)$,

**b** $Var(X)$,

**c** $E(X^2)$,

**d** the cumulative distribution function of $X$, for all $x$.

**Solution:**

**a** $\mathrm{E}(X) = \dfrac{5.5+3.5}{2}$

$= 4.5$

**b** $\mathrm{Var}(X) = \dfrac{(5.5-3.5)^2}{12}$

$= \dfrac{1}{3}$

**c** $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

$\dfrac{1}{3} = \mathrm{E}(X^2) - 20.25$

$\mathrm{E}(X^2) = 20\dfrac{7}{12} = 20.6 \text{ (3 s.f.)}$

**d** $\mathrm{F}(x) = \displaystyle\int_{3.5}^{x} \frac{1}{5.5-3.5}\,\mathrm{d}t$

$= \left[\dfrac{t}{2}\right]_{3.5}^{x}$

$= \dfrac{x}{2} - \dfrac{3.5}{2}$

$= \dfrac{x}{2} - 1.75$

$$\mathrm{F}(x) = \begin{cases} 0 & x < 3.5 \\ \dfrac{x}{2} - 1.75 & 3.5 \le x \le 5.5 \\ 1 & x > 5.5 \end{cases}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 4**

**Question:**

The continuous random variable $Y \sim \mathrm{U}[a, b]$. Given $\mathrm{E}(Y) = 1$ and $\mathrm{Var}(Y) = \frac{4}{3}$, find the value of $a$ and the value of $b$.

**Solution:**

$\mathrm{E}(Y)$

$$\frac{a+b}{2} = 1$$

$$a + b = 2 \qquad (1)$$

$\mathrm{Var}(Y)$

$$\frac{(b-a)^2}{12} = \frac{4}{3}$$

$$(b-a)^2 = 16 \qquad (2)$$

Solving equations (**1**) and (**2**) simultaneously

$$b = 2 - a$$

$$(2 - a - a)^2 = 16$$

$$(2 - 2a) = \pm 4$$

$2 - 2a = 4$, $a = -1$, $b = 2 - (-1) = 3$

$2 - 2a = -4$, $a = 3$, $b = 2 - 3 = -1$

Since $a < b$ $a = -1$ and $b = 3$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 5**

**Question:**

The continuous random variable $X$ has probability density function

$$f(x) = \begin{cases} \frac{1}{6}, & -1 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

Given that $Y = 4X - 6$, find E($Y$) and Var($Y$).

**Solution:**

$$\begin{aligned} E(X) &= \frac{5+(-1)}{2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= \frac{(5-(-1))^2}{12} \\ &= 3 \end{aligned}$$

$$\begin{aligned} E(Y) &= 4E(X) - 6 \\ &= 8 - 6 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{Var}(Y) &= 16\,\text{Var}(X) \\ &= 48 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 6**

**Question:**

The random variable $X$ is the length of a side of a square. $X \sim \mathrm{U}[4.5, 5.5]$.
The random variable $Y$ is the area of the square.
Find $\mathrm{E}(Y)$.

**Solution:**

$\mathrm{E}(Y) = \mathrm{E}(X^2)$ or $\displaystyle\int_{4.5}^{5.5} x^2\,\mathrm{d}x = \left[\frac{x^3}{3}\right]_{4.5}^{5.5}$

$= \left[\dfrac{5.5^3}{3}\right] - \left[\dfrac{4.5^3}{3}\right]$

$= 25\frac{1}{12}$

$\mathrm{E}(X) = \dfrac{4.5+5.5}{2}$

$= 5$

$\mathrm{Var}(X) = \dfrac{(5.5-4.5)^2}{12}$

$= \frac{1}{12}$

$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

$\frac{1}{12} = \mathrm{E}(X^2) - 25$

$\mathrm{E}(X^2) = 25\frac{1}{12}$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 7**

**Question:**

In a computer game an alien appears every 2 seconds. The player stops the alien by pressing a key. The object of the game is to stop the alien as soon as it appears. Given that the player actually presses the key $T$ s after the alien first appears, a simple model of the game assumes that $T$ is a continuous uniform random variable defined over the interval [0, 1].

**a** Write down $P(T < 0.2)$.

**b** Write down $E(T)$.

**c** Use integration to find Var ($T$).

**Solution:**

$$\frac{1}{b-a} = \frac{1}{1-0} = 1$$

**a** $P(T < 0.2) = (0.2 - 0) \times 1$
$= 0.2$

**b** $E(T) = 0.5$

**c** $$\begin{aligned}\text{Var}(T) &= \int_0^1 t^2 \, dt - 0.5^2 \\ &= \left[\frac{t^3}{3}\right]_0^1 - 0.25 \\ &= \frac{1}{12}\end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 1**

**Question:**

The continuous random variable $X$ is uniformly distributed over the interval $[-2, 5]$.

**a** Sketch the probability density function $f(x)$ of $X$.

Find

**b** $E(X)$,

**c** $Var(X)$,

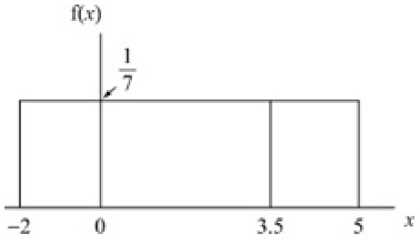
**d** the cumulative distribution function of $X$, for all $x$,

**e** $P(3.5 < X < 5.5)$,

**f** $P(X = 4)$.

**Solution:**

**a**




**b** $E(X) = \dfrac{5+(-2)}{2} = 1.5$

**c** $\text{Var}(X) = \dfrac{(5-(-2))^2}{12} = 4\frac{1}{12}$

**d** $F(x) = \displaystyle\int_{-2}^{x} \frac{1}{5-(-2)}\, dt$

$= \left[\dfrac{t}{7}\right]_{-2}^{x}$

$= \dfrac{x+2}{7}$

$$F(x) = \begin{cases} 0 & x < -2 \\ \dfrac{x+2}{7} & -2 \le x \le 5 \\ 1 & x > 5 \end{cases}$$

**e** $P(3.5 < X < 5.5) = P(3.5 < X < 5)$

$= ((5 - 3.5) \times \frac{1}{7})$

$= \frac{3}{14}$

**f** $P(X = 4) = 0$

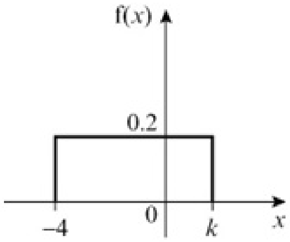
# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 2**

**Question:**

The continuous random variable $X$ has p.d.f. as shown in the diagram.




Find

**a** the value of $k$,

**b** $P(-2 < X < -1)$,

**c** $E(X)$,

**d** $Var(X)$,

**e** the cumulative distribution function of $X$, for all $x$.

**Solution:**

**a** Area = 1 so

$$(k+4)\times 0.2 = 1$$

$$0.2k + 0.8 = 1$$

$$k = 1$$

**b** $P(-2 < X < -1) = 1 \times 0.2 = 0.2$

**c** $E(X) = \frac{-4+1}{2} = -1.5$

**d** $Var(X) = \frac{(b-a)^2}{12} = \frac{(1-(-4))^2}{12} = 2\frac{1}{12}$

**e**

$$F(x) = \int_{-4}^{x} \frac{1}{1-(-4)}\,dt$$

$$= \left[\frac{t}{5}\right]_{-4}^{x}$$

$$= \frac{x+4}{5}$$

$$F(x) = \begin{cases} 0 & x < -4 \\ \frac{x+4}{5} & -4 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 3**

**Question:**

The continuous random variable $Y$ is uniformly distributed on the interval $a \le Y \le b$.
Given $\mathrm{E}(Y) = 2$ and $\mathrm{Var}(Y) = 3$.
Find

**a** the value of $a$ and the value of $b$,

**b** $\mathrm{P}(X > 1.8)$.

**Solution:**

**a** $\mathrm{E}(Y) = \dfrac{a+b}{2} = 2$ so $a + b = 4$ so $a = 4 - b$

$\mathrm{Var}\,(Y) = \dfrac{(b-a)^2}{12} = 3$

Substituting for $a$ gives $(2b-4)^2 = 36$

$(2b-4) = \pm 6$

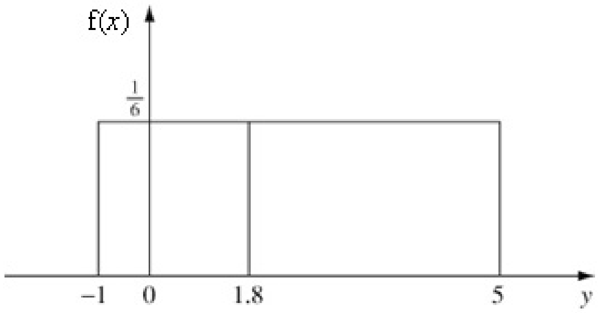
$b = 5$ or $b = -1$

$a = -1$ $\quad a = 5$

but $b > a$

$b = 5 \quad a = -1$

**b**




$\mathrm{P}(X > 1.8) = (5 - 1.8) \times \frac{1}{6} = 0.533$ (3 s.f.)

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 4**

**Question:**

A child has a pair of scissors and a piece of string 20 cm long, which has a mark on one end. The child cuts the string, at a randomly chosen point, into two pieces. Let $X$ represent the length of the piece of string with the mark on it.

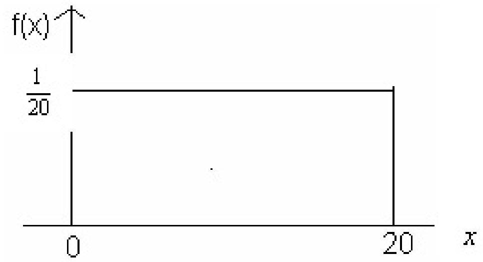
**a** Write down the name of the probability distribution of $X$ and sketch the graph of its probability density function.

**b** Find the values of E($X$) and Var($X$).

**c** Using your model, calculate the probability that the shorter piece of string is at least 8 cm long.

**Solution:**

**a** $X \sim \text{U}(0, 20)$




**b**

$$\text{E}(X) = \frac{20+0}{2} = 10$$

$$\text{Var}(X) = \frac{(20-0)^2}{12} = \frac{400}{12} = 33\frac{1}{3}$$

**c** $\text{P}(8 \leq X \leq 12) = (12-8) \times \frac{1}{20} = 0.2$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 5**

**Question:**

Joan records the temperature every day. The highest temperature she recorded was 29 °C to the nearest degree. Let $X$ represent the error in the measured temperature.

**a** Suggest a suitable model for the distribution of $X$.

**b** Using your model calculate the probability that the error will be less than 0.2 °C.

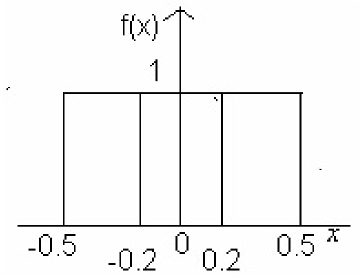
**c** Find the variance of the error in the measured temperature.

**Solution:**

**a**

$X \sim U(-0.5, 0.5)$

**b**




$P(-0.2 < X < 0.2) = 0.4 \times 1 = 0.4$

**c** $\text{Var}(X) = \frac{(b-a)^2}{12} = \frac{(0.5-(-0.5))^2}{12} = \frac{1}{12}$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 6**

**Question:**

Jameil catches a bus to work every morning. According to the timetable the bus is due at 9 a.m., but Jameil knows that the bus can arrive at a random time between three minutes early and ten minutes late. The random variable $X$ represents the time, in minutes, after 9 a.m. when the bus arrives.

**a** Suggest a suitable model for the distribution of $X$ and specify it fully.

**b** Calculate the mean value of $X$.

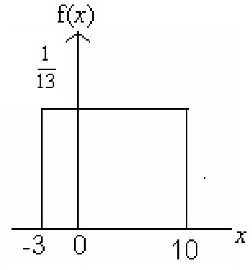
**c** Find the cumulative distribution function of $X$.

Jameil will be late for work if the bus arrives after 9.05 a.m.

**d** Find the probability that Jameil is late for work.

**Solution:**

**a**




$X \sim U(-3, 10)$

$$f(x) = \begin{cases} \frac{1}{13} & -3 \leq x \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

**b** Mean $= E(X) = \frac{-3+10}{2} = 3.5$ minutes

**c**

$$F(x) = \int_{-3}^{x} \frac{1}{13}\, dt$$

$$= \left[\frac{t}{13}\right]_{-3}^{x}$$

$$= \frac{x+3}{13}$$

$$F(x) = \begin{cases} 0 & x < -3 \\ \frac{x+3}{13} & -3 \leq x \leq 10 \\ 1 & x > 10 \end{cases}$$

**d** $P(5 < X < 10) = (10 - 5) \times \frac{1}{13} = \frac{5}{13}$

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 7**

**Question:**

A plumber measures, to the nearest cm, the lengths of pipes.

**a** Suggest a suitable model to represent the difference between the true lengths and the measured lengths.

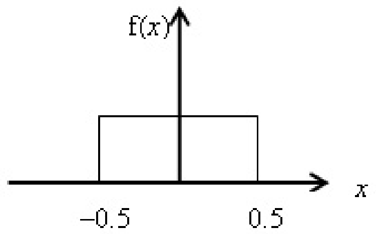
**b** Find the probability that for a randomly chosen rod the measured length will be within 0.2 cm of the true length.

**c** Three pipes are selected at random. Find the probability that all three pipes will be within 0.2 cm of the true length.

**Solution:**

**a** U(−0.5, 0.5)

**b**




$P(-0.2 \le X \le 0.2) = (0.2 - (-0.2)) \times 1 = 0.4$

**c** P(3 pipes between −0.2 and 0.2) $= 0.4^3 = 0.064$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 8**

**Question:**

A coffee machine dispenses coffee into cups. It is electronically controlled to cut off the flow of coffee randomly between 190 ml and 210 ml. The random variable $X$ is the volume of coffee dispensed into a cup.

**a** Specify the probability density function of $X$ and sketch its graph.

**b** Find the probability that the machine dispenses

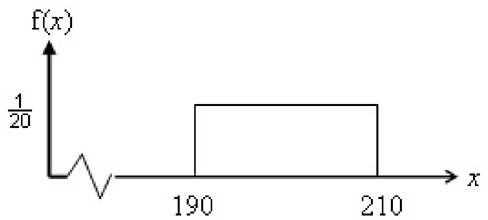
**i** less than 198 ml,

**ii** exactly 198 ml.

**c** Calculate the inter-quartile range of $X$.

**Solution:**

**a**




$$f(x) = \begin{cases} \frac{1}{20} & 190 \le x \le 210 \\ 0 & \text{otherwise} \end{cases}$$

**b i** $P(X < 198) = (198 - 190) \times \frac{1}{20} = \frac{2}{5}$

**ii** 0

**c** $\frac{210 - 190}{4} = 5$ is one quarter of the range

$Q_1 = 190 + 5 = 195$ $\quad Q_3 = 210 - 5 = 205$

$IQR = 205 - 195 = 10$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 9**

**Question:**

Write down the name of the distribution you would recommend as a suitable model for each of the following situations.

**a** the difference between the true height and the height measured, to the nearest cm, of randomly chosen people.

**b** the heights of randomly selected 18-year-old females.

**Solution:**

**a** Uniform

**b** Normal