Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Give reasons why the following are not valid probability density functions.

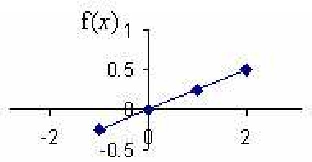
a $f(x)=\begin{cases}\frac{1}{4}x, & -1\le x\le 2,\\ 0, & \text{otherwise.}\end{cases}$

b $f(x)=\begin{cases}x^2, & 1\le x\le 3,\\ 0, & \text{otherwise.}\end{cases}$

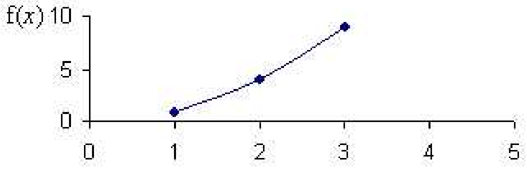
c $f(x)=\begin{cases}(x^3-2), & -1\le x\le 3,\\ 0, & \text{otherwise.}\end{cases}$

Solution:

a There are negative values for f(x) when x < 0 so this is not a probability density function.



b



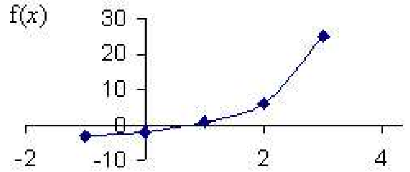
No negative values of f(x)

Area $=\int_1^3 x^2\,dx$

$=\left[\frac{x^3}{3}\right]_1^3$

$=8\frac{2}{3}$ not equal to 1 therefore it is not a valid probability density function.

c There are negative values for f(x) so this is not a probability density function.



Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

For what value of k is the following a valid probability density function?

$$f(x) = \begin{cases} k(1 - x^2), & -4 \le x \le -2, \\ 0, & \text{otherwise.} \end{cases}$$

Solution:

$$\int_{-4}^{-2} k - kx^2 \, dx = 1$$

$$\left[kx - \frac{kx^3}{3}\right]_{-4}^{-2} = 1$$

$$\left[-2k + \frac{8k}{3}\right] - \left[-4k + \frac{64k}{3}\right] = 1$$

$$-\frac{50}{3}k = 1$$

$$k = -\frac{3}{50}$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 3

Question:

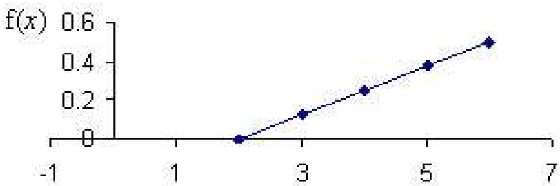
Sketch the following probability density functions.

a $f(x) = \begin{cases} \frac{1}{8}(x-2), & 2 \le x \le 6, \\ 0, & \text{otherwise.} \end{cases}$

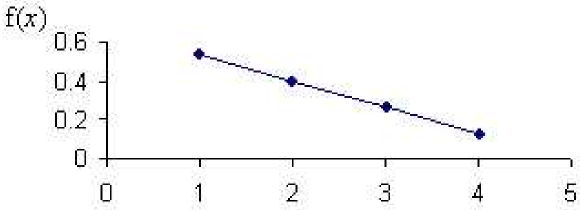
b $f(x) = \begin{cases} \frac{2}{15}(5-x), & 1 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$

Solution:

a



b



Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 4

Question:

Find the value of k so that each of the following are valid probability density functions.

a $\mathrm{f}(x)=\begin{cases} kx, & 1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$

b $\mathrm{f}(x)=\begin{cases} kx^2, & 0 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$

c $\mathrm{f}(x)=\begin{cases} k(1+x^2), & -1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$

Solution:

a

$$\int_1^3 kx \, dx = 1$$

$$\left[\frac{kx^2}{2}\right]_1^3 = 1$$

$$\frac{9k}{2} - \frac{k}{2} = 1$$

$$4k = 1$$

$$k = \frac{1}{4}$$

b

$$\int_0^3 kx^2 dx = 1$$

$$\left[\frac{kx^3}{3}\right]_0^3 = 1$$

$$\frac{27k}{3} = 1$$

$$27k = 3$$

$$k = \frac{3}{27} = \frac{1}{9}$$

c

$$\int_{-1}^2 k(1 + x^2) \, dx = 1$$

$$\left[kx + \frac{kx^3}{3}\right]_{-1}^2 = 1$$

$$(2k + \frac{8k}{3}) - (-k - \frac{k}{3}) = 1$$

$$\frac{14k}{3} - (-\frac{4k}{3}) = 1$$

$$\frac{14k}{3} + \frac{4k}{3} = 1$$

$$6k = 1$$

$$k = \frac{1}{6}$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 5

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k(4-x), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the value of k.

b Sketch the probability density function for all values of x.

Solution:

a

$$\int_0^2 k(4-x)\,dx = 1$$

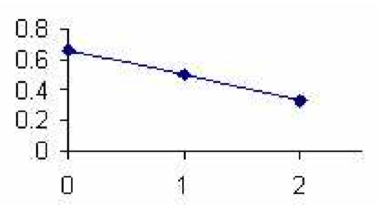
$$\left[4kx - \frac{kx^2}{2}\right]_0^2 = 1$$

$$8k - 2k = 1$$

$$6k = 1$$

$$k = \frac{1}{6}$$

b



Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 6

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} kx^2(2-x), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of k.

Solution:

$$\int_0^2 kx^2(2-x)\,dx = 1$$

$$\left[\frac{2kx^3}{3} - \frac{kx^4}{4}\right]_0^2 = 1$$

$$\left(\frac{16k}{3} - \frac{16k}{4}\right) - 0 = 1$$

$$\frac{16k}{12} = 1$$

$$16k = 12$$

$$k = \frac{3}{4} \text{ or } 0.75$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 7

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} kx^3, & 1 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of k.

Solution:

$$\int_1^4 kx^3 \, dx = 1$$

$$\left[\frac{kx^4}{4}\right]_1^4 = 1$$

$$\frac{256k}{4} - \frac{k}{4} = 1$$

$$\frac{255k}{4} = 1$$

$$k = \frac{4}{255}$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 8

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k, & 0 < x < 2, \\ k(2x-3), & 2 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the value of k.

b Sketch the probability density function for all values of x.

Solution:

a

$$\int_0^2 k\,dx + \int_2^3 k(2x-3)\,dx = 1$$

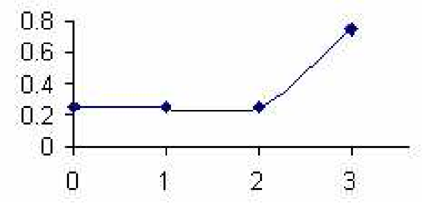
$$[kx]_0^2 + \left[\frac{2kx^2}{2} - 3kx\right]_2^3 = 1$$

$$2k + [(9k - 9k) - (4k - 6k)] = 1$$

$$2k + 2k = 1$$

$$k = \frac{1}{4} \text{ or } 0.25$$

b



Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 1

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \dfrac{3x^2}{8}, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find $F(x)$.

Solution:

1 Method 1: $F(x) = \int_0^x \frac{3t^2}{8}\,dt$ $= \left[\frac{3t^3}{24}\right]_0^x$ $= \frac{3x^3}{24} - 0$ $= \frac{3x^3}{24}$	**Method 2:** $F(x) = \int \frac{3x^2}{8}\,dx$ $= \frac{3x^3}{24} + C$ $F(2) = 1$ $1 + C = 1$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3x^3}{24} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$	

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 2

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{1}{4}(4-x), & 1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

Find $F(x)$.

Solution:

2 Method 1: $F(x) = \int_1^x \frac{1}{4}(4-t)\,dt$ $= \left[t - \frac{t^2}{8}\right]_1^x$ $= \left(x - \frac{x^2}{8}\right) - \left(1 - \frac{1}{8}\right)$ $= x - \frac{x^2}{8} - \frac{7}{8}$	**Method 2:** $F(x) = \int \frac{1}{4}(4-x)\,dx$ $= x - \frac{x^2}{8} + C$ $F(3) = 1$ $3 - \frac{9}{8} + C = 1$ $C = -\frac{7}{8}$
$F(x) = \begin{cases} 0 & x < 1 \\ x - \frac{x^2}{8} - \frac{7}{8} & 1 \le x \le 3 \\ 1 & x > 3 \end{cases}$	

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 3

Question:

The continuous random variable X has probability density function given by:

$$F(x) = \begin{cases} \frac{x}{9}, & 0 < x < 3, \\ \frac{1}{9}(6-x) & 3 \le x \le 6, \\ 0, & \text{otherwise.} \end{cases}$$

Find $F(x)$.

Solution:

Method 1:	Method 2:
$F(x) = \int_0^x \frac{t}{9}\,dt$ $= \left[\frac{t^2}{18}\right]_0^x$ $= \frac{x^2}{18}$ $F(x) = \int_3^x \frac{1}{9}(6-t)\,dt + \int_0^3 \frac{x}{9}\,dx$ $= \left[\frac{2t}{3} - \frac{t^2}{18}\right]_3^x + \left[\frac{x^2}{18}\right]_0^3$ $= \left(\frac{2x}{3} - \frac{x^2}{18}\right) - \left(2 - \frac{9}{18}\right) + 0.5$ $= \frac{2x}{3} - \frac{x^2}{18} - 1$	$F(x) = \int \frac{x}{9}\,dx$ $= \frac{x^2}{18} + C$ $F(0) = 0$ $0 + C = 0$ $C = 0$ $F(x) = \int \frac{1}{9}(6-x)\,dx$ $= \frac{2x}{3} - \frac{x^2}{18} + C$ $F(6) = 1$ $4 - 2 + C = 1$ $C = -1$
$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{x^2}{18} & 0 < x < 3 \\ \frac{2x}{3} - \frac{x^2}{18} - 1 & 3 \le x \le 6 \\ 1 & x > 6 \end{cases}$	

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 4

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k, & 0 \le x \le 3, \\ k(2x-5), & 3 \le x \le 5 \\ 0, & \text{otherwise.} \end{cases}$$

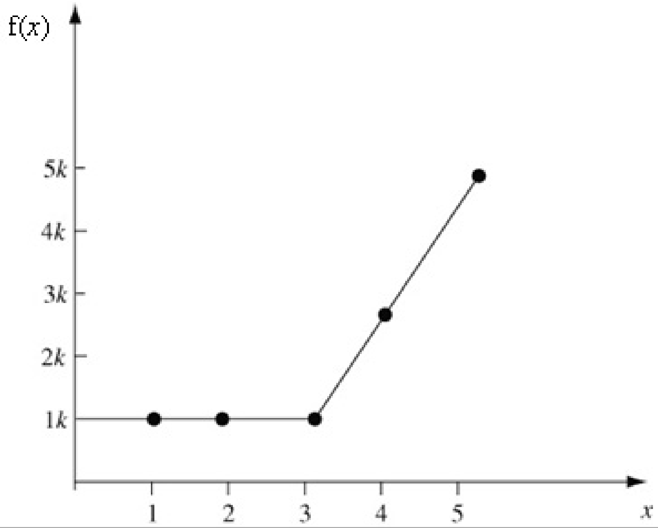
a Sketch $f(x)$.

b Find the value of k.

c Find $F(x)$.

Solution:

a



b

$$\int_0^3 k\,dx + \int_3^5 k(2x-5)\,dx = 1$$

$$[kx]_0^3 + \left[k\left(x^2-5x\right)\right]_3^5 = 1$$

$$3k + \left[k(25-25) - k(9-15)\right] = 1$$

$$9k = 1$$

$$k = \frac{1}{9}$$

c Method 1

$$\int_0^x \frac{1}{9}\,dt = \left[\frac{1}{9}t\right]_0^x$$

$$= \frac{1}{9}x$$

$$\int_3^x \frac{1}{9}(2t-5)\,dt + \int_0^3 \frac{1}{9}\,dx = \left[\frac{1}{9}\left(t^2-5t\right)\right]_3^x + \left[\frac{x}{9}\right]_0^3$$

$$= \left[\frac{1}{9}\left(x^2-5x\right) - \frac{1}{9}(9-15)\right] + \left[\frac{3}{9}\right]$$

$$= \frac{1}{9}x^2 - \frac{5}{9}x + 1$$

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{x}{9} & 0 < x < 3 \\ \frac{x^2}{9} - \frac{5x}{9} + 1 & 3 \le x \le 5 \\ 1 & x > 5 \end{cases}$$

Method 2

$$\int \frac{1}{9}\,dx = \frac{1}{9}x + C$$

$$F(0) = 0$$

$$C = 0$$

$$\int \frac{1}{9}(2x-5)\,dx = \frac{1}{9}\left(x^2-5x\right) + C$$

$$F(5) = 1$$

$$\frac{1}{9}(25-25) + C = 1$$

$$C = 1$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 5

Question:

The continuous random variable X has cumulative distribution function given by:

$$F(x) = \begin{cases} 0, & x < 2, \\ \frac{1}{5}(x^2 - 4), & 2 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

Find the probability density function, f(x).

Solution:

$\frac{d}{dx}F(x) = \frac{2x}{5}$ $f(x) = \begin{cases} \frac{2x}{5} & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$	

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 6

Question:

The continuous random variable X has cumulative distribution function given by:

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{1}{2}(x-1), & 1 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

a Find $P(X \le 2.5)$.

b Find $P(X > 1.5)$.

c Find $P(1.5 \le X \le 2.5)$.

Solution:

a $P(X \le 2.5) = F(2.5)$ $= \frac{1}{2}(2.5-1)$ $= 0.75$ **b** $P(X > 1.5) = 1 - F(1.5)$ $= 1 - \frac{1}{2}(1.5-1)$ $= 0.75$ **c** $P(1.5 \le X \le 2.5) = F(2.5) - F(1.5)$ $= 0.75 - 0.25$ $= 0.5$	

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 7

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{3x^2}{8} & 0 \le x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the cumulative distribution function of X.

b Find $P(X \le 1)$.

Solution:

a **Method 1:** $F(X) = \int_0^x \frac{3x^2}{8}\,dx$ $= \left[\frac{x^3}{8}\right]_0^x$ $= \frac{x^3}{8}$	**Method 2:** $F(X) = \int \frac{3x^2}{8}\,dx$ $= \frac{x^3}{8} + C$ $F(2) = 1$ $1 + C = 1$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{8} & 0 \le x < 2 \\ 1 & x \ge 2 \end{cases}$	
b $P(x \le 1) = F(1) = \frac{1}{8}$	

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 8

Question:

The continuous random variable X has cumulative distribution function given by:

$$F(x)=\begin{cases}0, & x<1,\\ \frac{1}{2}(x^3-2x^2+x), & 1\le x\le 2,\\ 1, & x>2.\end{cases}$$

a Find the probability density function $f(x)$.
b Sketch the probability density function.
c Find $P(X<1.5)$.

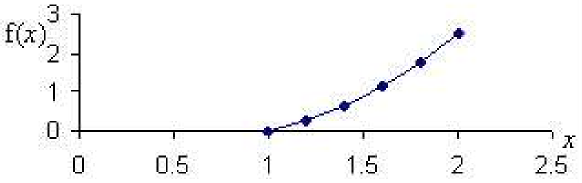
Solution:

a

$$\frac{d}{dx}F(x)=\frac{3x^2}{2}-2x+\frac{1}{2}$$

$$f(x)=\begin{cases}\frac{3}{2}x^2-2x+\frac{1}{2} & 1\le x\le 2\\ 0 & \text{otherwise}\end{cases}$$

b



c

$$P(X<1.5)=F(1.5)$$
$$=\frac{1}{2}(1.5^3-2\times1.5^2+1.5)$$
$$=0.1875$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 9

Question:

The continuous random variable X has probability density function given by:

$$f(x) = \begin{cases} k(4 - x^2), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Show that $k = \frac{3}{16}$.

b Find the cumulative distribution function of X.

c Find $P(0.69 < X < 0.70)$. Give your answer correct to one significant figure.

Solution:

a $\int_0^2 k(4 - x^2)\,dx = 1$ $\left[k\left(4x - \frac{x^3}{3}\right)\right]_0^2 = 1$ $k\left(8 - \frac{8}{3}\right) = 1$ $\frac{16k}{3} = 1$ $k = \frac{3}{16}$	
b **Method 1:** $F(x) = \int_0^x \frac{3}{16}(4 - t^2)\,dt$ $= \left[\frac{3}{16}\left(4t - \frac{t^3}{3}\right)\right]_0^x$ $= \frac{3}{16}\left(4x - \frac{x^3}{3}\right)$ $F(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{16}\left(4x - \frac{x^3}{3}\right) & 0 \le x < 2 \\ 1 & x \ge 2 \end{cases}$ **c** $P(0.69 < X < 0.70) = F(0.70) - F(0.69)$ $= \frac{3}{16}\left(2.8 - \frac{0.343}{3}\right) - \frac{3}{16}\left(2.76 - \frac{0.328509}{3}\right)$ $= 0.00659$ $= 0.007$ (1 s.f.)	**Method 2:** $F(x) = \int \frac{3}{16}(4 - x^2)\,dx$ $= \frac{3}{16}\left(4x - \frac{x^3}{3}\right) + C$ $F(2) = 1$ $\frac{3}{16}\left(8 - \frac{8}{3}\right) + C = 1$ $C = 0$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} kx^2, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find

a k,

b $E(X)$,

c $Var(X)$.

Solution:

a

$$\int_0^2 kx^2 = 1$$

$$\left[\frac{kx^3}{3}\right]_0^2 = 1$$

$$\frac{8k}{3} - 0 = 1$$

$$8k = 3$$

$$k = \frac{3}{8}$$

b

$$E(X) = \int_0^2 \frac{3x^3}{8}\,dx$$

$$= \left[\frac{3x^4}{32}\right]_0^2$$

$$= \frac{48}{32} - 0$$

$$= 1.5$$

c

$$Var(X) = \int_0^2 \frac{3x^4}{8}\,dx - 1.5^2$$

$$= \left[\frac{3x^5}{40}\right]_0^2 - 1.5^2$$

$$= \left(\frac{96}{40} - 0\right) - 2.25$$

$$= 2.4 - 2.25$$

$$= 0.15$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 2

Question:

The continuous random variable Y has a probability density function given by

$$f(y) = \begin{cases} \frac{y^2}{9}, & 0 \le y \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

a Find $E(Y)$.

b Find $Var(Y)$.

c Find the standard deviation of Y.

Solution:

a

$$E(Y) = \int_0^3 \frac{y^3}{9}\, dy$$
$$= \left[\frac{y^4}{36}\right]_0^3$$
$$= \frac{81}{36} - 0$$
$$= 2.25$$

b

$$Var(Y) = \int_0^3 \frac{y^4}{9}\, dy - 2.25^2$$
$$= \left[\frac{y^5}{45}\right]_0^3 - 2.25^2$$
$$= \left(\frac{243}{45} - 0\right) - 5.0625$$
$$= 5.4 - 5.0625$$
$$= 0.3375$$

c $\sigma = \sqrt{0.3375} = 0.581$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

The continuous random variable Y has a probability density function given by

$$f(y) = \begin{cases} \frac{y}{8}, & 0 \le y \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

a Find $E(Y)$.

b Find $Var(Y)$.

c Find the standard deviation of Y.

d Find $P(Y > \mu)$.

e Find $Var(3Y + 2)$.

f Find $E(Y + 2)$.

Solution:

a

$$\mathrm{E}(Y) = \int_0^4 \frac{y^2}{8}\,\mathrm{d}y$$
$$= \left[\frac{y^3}{24}\right]_0^4$$
$$= \frac{64}{24} - 0$$
$$= \frac{8}{3}$$

b

$$\mathrm{Var}(Y) = \int_0^4 \frac{y^3}{8}\mathrm{d}y - \left(\frac{8}{3}\right)^2$$
$$= \left[\frac{y^4}{32}\right]_0^4 - \left(\frac{64}{9}\right)$$
$$= \left(\frac{256}{32} - 0\right) - \left(\frac{64}{9}\right)$$
$$= 8 - \left(\frac{64}{9}\right)$$
$$= \frac{8}{9}$$

c $\sigma = \sqrt{\frac{8}{9}} = 0.943$

d

$$\mathrm{P}(Y > \mu) = \mathrm{P}(Y > \tfrac{8}{3})$$
$$= \int_{\frac{8}{3}}^4 \frac{y}{8}\mathrm{d}y$$
$$= \left[\frac{y^2}{16}\right]_{\frac{8}{3}}^4$$
$$= 1 - 0.4444$$
$$= 0.556$$

e

$$\mathrm{Var}\,(3Y + 2) = 9\,\mathrm{Var}(Y)$$
$$= 9 \times \frac{8}{9}$$
$$= 8$$

f

$$\mathrm{E}\,(Y + 2) = \mathrm{E}(Y) + 2$$
$$= \frac{8}{3} + 2$$
$$= 4\tfrac{2}{3}$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 4

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k(1-x), & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Find k.

b Find $E(X)$.

c Show that $\text{Var}(X) = \frac{1}{18}$.

d Find $P(X > \mu)$.

Solution:

a

$$\int_0^1 k(1-x)\,\mathrm{d}x = 1$$

$$\left[kx - \frac{kx^2}{2}\right]_0^1 = 1$$

$$k - \tfrac{1}{2}k = 1$$

$$k = 2$$

b

$$\mathrm{E}(X) = \int_0^1 (2x - 2x^2)\,\mathrm{d}x$$

$$= \left[\frac{2x^2}{2} - \frac{2x^3}{3}\right]_0^1$$

$$= 1 - \tfrac{2}{3}$$

$$= \tfrac{1}{3}$$

c

$$\mathrm{Var}(X) = \int_0^1 (2x^2 - 2x^3)\,\mathrm{d}x \;-\; \left(\tfrac{1}{3}\right)^2 \mathrm{d}x$$

$$= \left[\frac{2x^3}{3} - \frac{2x^4}{4}\right]_0^1 - \tfrac{1}{9}$$

$$= \left(\tfrac{2}{3} - \tfrac{1}{2}\right) - 0 - \tfrac{1}{9}$$

$$= \tfrac{1}{18}$$

d

$$\mathrm{P}(X > \tfrac{1}{3}) = \int_{\frac{1}{3}}^1 2(1-x)\,\mathrm{d}x$$

$$= \left[2x - x^2\right]_{\frac{1}{3}}^1$$

$$= (2-1) - \left(\tfrac{2}{3} - \tfrac{1}{9}\right)$$

$$= \tfrac{4}{9}$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 5

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} 12x^2(1-x), & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Find $P(X < 0.5)$.

b Find $E(X)$.

Solution:

a

$$\begin{aligned} P(X < 0.5) &= \int_0^{0.5} 12x^2 - 12x^3 \ dx \\ &= \left[4x^3 - 3x^4\right]_0^{0.5} \\ &= \frac{1}{2} - \frac{3}{16} \\ &= \frac{5}{16} \text{ or } 0.3125 \end{aligned}$$

b

$$\begin{aligned} E(X) &= \int_0^1 12x^3 - 12x^4 \ dx \\ &= \left[3x^4 - \frac{12x^5}{5}\right]_0^1 \\ &= (3 - 2.4) - 0 \\ &= 0.6 \text{ or } \frac{3}{5} \end{aligned}$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 6

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{3}{8}(1+x^2), & -1 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

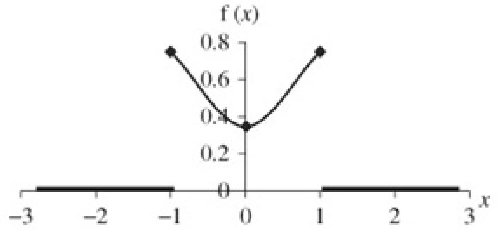
b Write down E(X).

c Show that $\sigma^2 = 0.4$.

d Find $P(-\sigma < X < \sigma)$.

Solution:

a



b

E(X) = 0 (by symmetry)

c

$$\sigma^2 = \text{Var}(X)$$
$$= \int_{-1}^{1} \frac{3x^2}{8} + \frac{3x^4}{8}\, dx - 0^2$$
$$= \left[\frac{3x^3}{24} + \frac{3x^5}{40}\right]_{-1}^{1}$$
$$= \left(\frac{3}{24} + \frac{3}{40}\right) - \left(-\frac{3}{24} - \frac{3}{40}\right)$$
$$= 0.4$$

d

$$P(-\sqrt{0.4} < X < \sqrt{0.4}) = \int_{-\sqrt{0.4}}^{\sqrt{0.4}} \frac{3}{8} + \frac{3x^2}{8}\, dx$$
$$= \left[\frac{3x}{8} + \frac{3x^3}{24}\right]_{-\sqrt{0.4}}^{\sqrt{0.4}}$$
$$= \left(\frac{3}{8} \times \sqrt{0.4} + \frac{3}{24} \times \left(\sqrt{0.4}\right)^3\right) - \left(\frac{3}{8} \times \left(-\sqrt{0.4}\right) + \frac{3}{24} \times \left(-\sqrt{0.4}\right)^3\right)$$
$$= 0.538$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 7

Question:

The continuous random variable T has c.d.f. given by

$$f(t) = \begin{cases} kt^3, & 0 \le t \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

where k is a positive constant.

a Find k.

b Show that $E(T)$ is 1.6.

c Find $E(2T+3)$.

d Find $\text{Var}(T)$.

e Find $\text{Var}(2T+3)$.

f Find $P(T < 1)$.

Solution:

a

$$\int_0^2 kt^3 \, dt = 1$$

$$\left[\frac{kt^4}{4}\right]_0^2 = 1$$

$$4k - 0 = 1$$

$$4k = 1$$

$$k = \frac{1}{4}$$

b

$$E(T) = \int_0^2 \frac{t^4}{4} \, dx$$

$$= \left[\frac{t^5}{20}\right]_0^2$$

$$= \frac{32}{20} - 0$$

$$= 1.6$$

c

$$E(2T+3) = 2E(T) + 3$$

$$= 2 \times 1.6 + 3$$

$$= 6.2$$

d

$$Var(T) = \int_0^2 \frac{t^5}{4} \, dt - \left(\frac{8}{5}\right)^2$$

$$= \left[\frac{t^6}{24}\right]_0^2 - \left(\frac{8}{5}\right)^2$$

$$= \left(\frac{64}{24} - 0\right) - \left(\frac{64}{25}\right)$$

$$= \frac{8}{75}$$

e

$$Var(2T+3) = 4 \, Var(T)$$

$$= \frac{32}{75}$$

f

$$P(T < 1) = \int_0^1 \frac{t^3}{4} \, dt$$

$$= \left[\frac{t^4}{16}\right]_0^1$$

$$= \frac{1}{16}$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 8

Question:

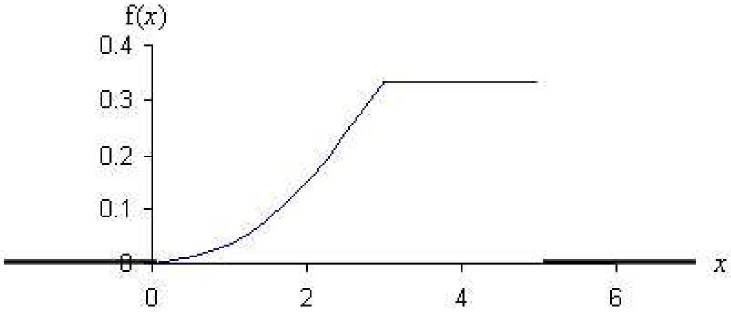
The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{x^2}{27}, & 0 \le x < 3, \\ \frac{1}{3}, & 3 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

a Draw a rough sketch of $f(x)$.
b Find $E(X)$.
c Find $Var(X)$
d Find the standard deviation, σ, of X.

Solution:

a



b

$$E(X) = \int_0^3 \frac{x^3}{27}\,dx + \int_3^5 \frac{x}{3}\,dx$$
$$= \left[\frac{x^4}{108}\right]_0^3 + \left[\frac{x^2}{6}\right]_3^5$$
$$= \left(\frac{81}{108} - 0\right) + \left(\frac{25}{6} - \frac{9}{6}\right)$$
$$= \frac{41}{12}$$
$$= 3.417$$

c

$$\text{Var}(X) = \left(\int_0^3 \frac{x^4}{27}\,dx + \int_3^5 \frac{x^2}{3}\,dx\right) - \left(\frac{41}{12}\right)^2$$
$$= \left(\left[\frac{x^5}{135}\right]_0^3 + \left[\frac{x^3}{9}\right]_3^5\right) - \left(\frac{1681}{144}\right)$$
$$= \left(\frac{243}{135} - 0\right) + \left(\frac{125}{9} - \frac{27}{9}\right) - \left(\frac{1681}{144}\right)$$
$$= 1.0152$$

d

$$\sigma = \sqrt{1.0152} = 1.01$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 9

Question:

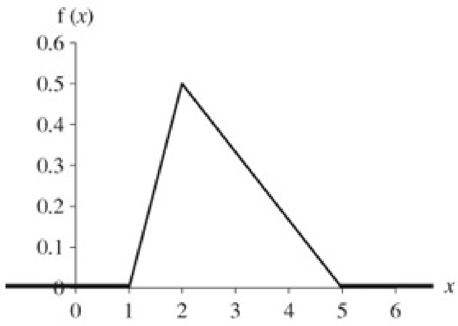
The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}(x-1), & 1 \le x < 2, \\ \frac{1}{6}(5-x), & 2 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch f(x).
b Find E(X).
c Find Var(X).

Solution:

a



b

$$E(X) = \int_1^2 \left(\frac{x^2}{2} - \frac{x}{2}\right) dx + \int_2^5 \left(\frac{5x}{6} - \frac{x^2}{6}\right) dx$$

$$= \left[\frac{x^3}{6} - \frac{x^2}{4}\right]_1^2 + \left[\frac{5x^2}{12} - \frac{x^3}{18}\right]_2^5$$

$$= \left[\left(\frac{8}{6} - 1\right) - \left(\frac{1}{6} - \frac{1}{4}\right)\right] + \left[\left(\frac{125}{12} - \frac{125}{18}\right) - \left(\frac{20}{12} - \frac{8}{18}\right)\right]$$

$$= 2\frac{2}{3}$$

c

$$\text{Var}(X) = \int_1^2 \left(\frac{x^3}{2} - \frac{x^2}{2}\right) dx + \int_2^5 \left(\frac{5x^2}{6} - \frac{x^3}{6}\right) dx - \left(2\frac{2}{3}\right)^2$$

$$= \left[\frac{x^4}{8} - \frac{x^3}{6}\right]_1^2 + \left[\frac{5x^3}{18} - \frac{x^4}{24}\right]_2^5 - \left(2\frac{2}{3}\right)^2$$

$$= \left[\left(\frac{16}{8} - \frac{8}{6}\right) - \left(\frac{1}{8} - \frac{1}{6}\right)\right] + \left[\left(\frac{625}{18} - \frac{625}{24}\right) - \left(\frac{40}{18} - \frac{16}{24}\right)\right] - \left(2\frac{2}{3}\right)^2$$

$$= \frac{13}{18}$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 10

Question:

Telephone calls arriving at a company are referred immediately by the telephonist to other people working in the company. The time a call takes, in minutes, is modelled by a continuous random variable T, having a p.d.f. given by

$$f(t) = \begin{cases} kt^2, & 0 \le t \le 10, \\ 0, & \text{otherwise.} \end{cases}$$

a Show that $k = 0.003$.

b Find $E(T)$.

c Find $Var(T)$.

d Find the probability of a call lasting between 7 and 9 minutes.

e Sketch the p.d.f.

Solution:

a

$$\int_0^{10} kt^2\,dt = 1$$

$$\left[\frac{kt^3}{3}\right]_0^{10} = 1$$

$$\frac{1000k}{3} - 0 = 1$$

$$1000k = 3$$

$$k = 0.003$$

b

$$E(T) = \int_0^{10} 0.003t^3\,dt$$

$$= \left[\frac{0.003t^4}{4}\right]_0^{10}$$

$$= \frac{30}{4} - 0$$

$$= 7.5$$

c

$$\text{Var}(x) = \int_0^{10} 0.003t^4\,dt - 7.5^2$$

$$= \left[\frac{0.003t^5}{5}\right]_0^{10} - 7.5^2$$

$$= (60 - 0) - 56.25$$

$$= 3.75$$

d

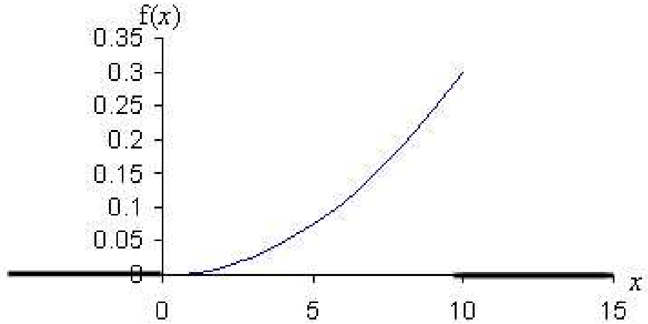
$$P(7 < T < 9) = \int_7^9 0.003t^2\,dt$$

$$= \left[\frac{0.003t^3}{3}\right]_7^9$$

$$= 0.729 - 0.343$$

$$= 0.386$$

e



Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 1

Question:

The continuous random variable X has p.d.f. given by

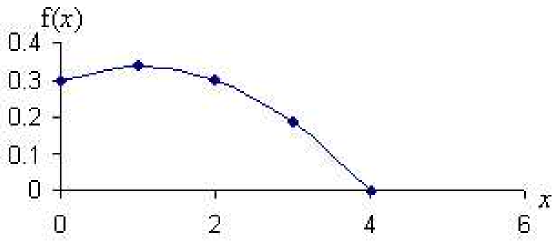
$$f(x) = \begin{cases} \frac{3}{80}(8+2x-x^2), & 0 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

b Find the mode of X.

Solution:

a



b Differentiating $\frac{d}{dx}\frac{3}{80}(8+2x-x^2) = 0$

$$\frac{3}{80}(2-2x) = 0$$

This $= 0$ when $(2-2x) = 0$

$$x = 1$$

The mode is 1.

(Note: To check this is a maximum you could differentiate again and see if f'' is negative for all values of x.)

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 2

Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{1}{8}x, & 0 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

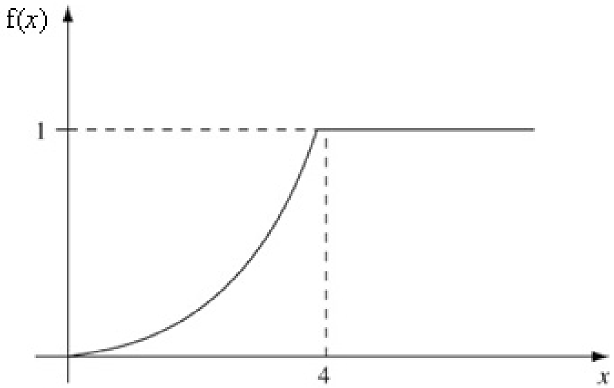
a Sketch the c.d.f. of X.

b Find the median of X.

Solution:

a

Method 1	Method 2
$\int_0^x \frac{1}{8}t \, dx = \left[\frac{t^2}{16}\right]_0^x$ $= \frac{x^2}{16}$	$F(x) = \int \frac{1}{8}x \, dx$ $= \frac{x^2}{16} + C$ $F(4) = 1 \quad 1 = 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{16} & 0 \le x \le 4 \\ 1 & x > 4 \end{cases}$	



b $\frac{m^2}{16} = 0.5$ so $m^2 = 8$

$m = \sqrt{8} = 2.83$ or -2.83

Median $= 2.83$ since -2.83 is not in the range.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 3

Question:

The continuous random variable X has c.d.f. given by

$$F(x)=\begin{cases} 0, & x<0 \\ \frac{x^2}{6}, & 0\le x\le 2, \\ -\frac{x^2}{3}+2x-2, & 2\le x\le 3 \\ 1, & x\ge 3. \end{cases}$$

a Find the median value of X. Give your answer to 3 decimal places.

b Find the quartiles and the inter-quartile range of X. Give your answer to 3 decimal places.

Solution:

a $F(m)=0.5$ where m is the median.

Since $F(2)=\frac{2}{3}$ the median must lie in the range $0\le x\le 2$

So $F(m)=\frac{m^2}{6}=0.5$

$m^2=2$

$m=1.73$ or -1.73

Median $=1.73$ since -1.73 is not in the range.

b Lower quartile lies in the range $0\le x\le 2$

$\frac{Q_1^2}{6}=0.25$

$Q_1=\sqrt{1.5}=1.225$

Upper quartile lies in the range $2\le x\le 3$

$-\frac{Q_3^2}{3}+2Q_3-2=0.75$

$-Q_3^2+6Q_3-6=2.25$

$-Q_3^2+6Q_3-8.25=0$

$Q_3=\frac{-6\pm\sqrt{36-33}}{-2}$

$=2.134$ or 3.87

$Q_3=2.134$ as 3.87 does not lie in the range

IQR $=2.134-1.225=0.909$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 4

Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} 1 - \frac{1}{2}x, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

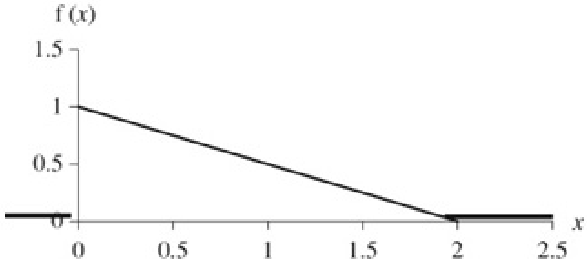
b Write down the mode of X.

c Find the c.d.f. of X.

d Find the median value of X.

Solution:

a



b 0

c

	Method 1	Method 2	
	$\int_0^x \left(1 - \frac{1}{2}t\right) dt = \left[t - \frac{1}{4}t^2\right]_0^x$ $= x - \frac{1}{4}x^2$	$F(x) = \int 1 - \frac{1}{2}x \, dx$ $= x - \frac{1}{4}x^2 + C$ $F(2) = 1 \quad 1 = 2 - 1 + C$ $C = 0$	
	$F(x) = \begin{cases} 0 & x < 0 \\ x - \frac{1}{4}x^2 & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$		

d $m - \frac{1}{4}m^2 = 0.5$

$m^2 - 4m + 2 = 0$

$m = \frac{4 \pm \sqrt{16 - 8}}{2}$

$m = 2 - \sqrt{2}$ or $2 + \sqrt{2}$ therefore median $= 2 - \sqrt{2}$ as $2 + \sqrt{2}$ is not in range.

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 5

Question:

The continuous random variable Y has p.d.f. given by

$$f(x) = \begin{cases} \frac{1}{2} - \frac{1}{9}y, & 0 \le y \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of Y.

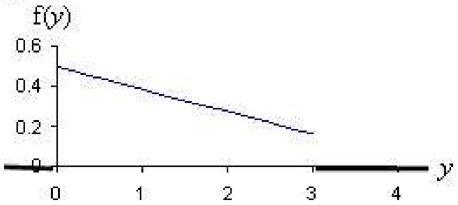
b Write down the mode of Y.

c Find the c.d.f. of Y.

d Find the median value of Y.

Solution:

a



b 0

c

Method 1	Method 2
$\int_0^y \frac{1}{2} - \frac{1}{9}t \, dt = \left[\frac{t}{2} - \frac{1}{18}t^2\right]_0^y$ $= \frac{y}{2} - \frac{1}{18}y^2$	$F(x) = \int \frac{y}{2} - \frac{1}{9}y \, dy$ $= \frac{y}{2} - \frac{1}{18}y^2 + C$ $F(3) = 1 \quad 1 = \frac{3}{2} - \frac{9}{18} + C$ $C = 0$
$F(y) = \begin{cases} 0 & y < 0 \\ \frac{y}{2} - \frac{1}{18}y^2 & 0 \le y \le 3 \\ 1 & y > 3 \end{cases}$	

d $\frac{m}{2} - \frac{1}{18}m^2 = 0.5$

$m^2 - 9m + 9 = 0$

$m = \frac{9 \pm \sqrt{81 - 36}}{2}$

$\text{median} = \frac{9 - 3\sqrt{5}}{2} = 1.15$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 6

Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{1}{4}x^3, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

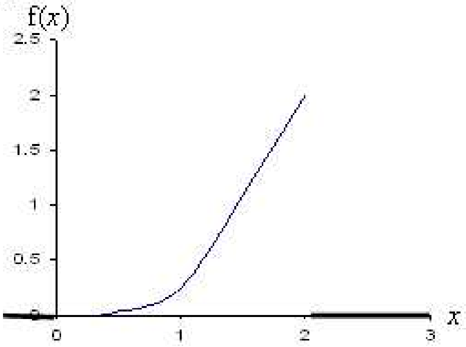
b Write down the mode of X.

c Find the c.d.f. of X.

d Find the median value of X.

Solution:

a



b 2

c

Method 1	Method 2
$\int_0^x \frac{1}{4}t^3 \, dt = \left[\frac{1}{16}t^4\right]_0^x$ $= \frac{1}{16}x^4$	$F(x) = \int \frac{1}{4}x^3 \, dx$ $= \frac{1}{16}x^4 + C$ $F(2) = 1 \quad 1 = 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{16}x^4 & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$	

d $\frac{1}{16}m^4 = 0.5$

$m^4 = 8$

$m = \pm\sqrt[4]{8}$

median $= 1.68$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 7

Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{3}{8}(x^2+1), & -1 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

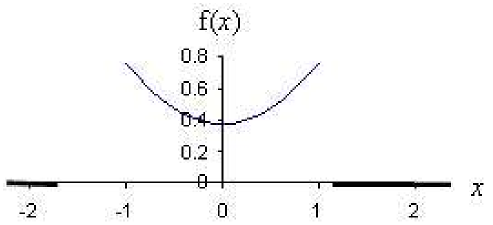
b What can you say about the mode of X?

c Write down the median value of X.

d Find the c.d.f. of X.

Solution:

a



b bimodal -1 and 1

c median $= 0$

d

Method 1	Method 2
$\int_{-1}^{x} \frac{3}{8}x^2 + \frac{3}{8}\mathrm{d}x = \left[\frac{1}{8}x^3 + \frac{3}{8}x\right]_{-1}^{x}$ $= \left[\frac{1}{8}x^3 + \frac{3}{8}x\right] - \left[-\frac{1}{8} - \frac{3}{8}\right]$ $= \frac{1}{8}x^3 + \frac{3}{8}x + \frac{1}{2}$	$\mathrm{F}(x) = \int \frac{3}{8}x^2 + \frac{3}{8}\,\mathrm{d}x$ $= \frac{1}{8}x^3 + \frac{3}{8}x + C$ $\mathrm{F}(1) = 1 \qquad 1 = \frac{1}{8} + \frac{3}{8} + C$ $C = \frac{1}{2}$
$\mathrm{F}(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{8}x^3 + \frac{3}{8}x + \frac{1}{2} & -1 \le x \le 1 \\ 1 & x > 1 \end{cases}$	

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 8

Question:

The continuous random variable X has p.d.f. given by

$$f(x) = \begin{cases} \frac{3}{10}(3x - x^2), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the p.d.f. of X.

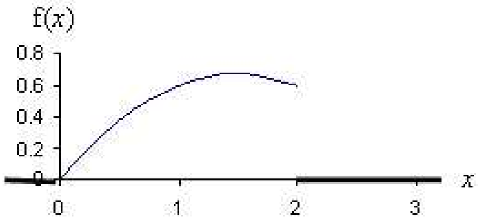
b Find the mode of X.

c Find the c.d.f. of X.

d Show that the median value of X lies between 1.23 and 1.24.

Solution:

a



b Find maximum by differentiating

$$\frac{d}{dx}\left(\frac{9}{10}x-\frac{3}{10}x^2\right)=\frac{9}{10}-\frac{6}{10}x$$

$$\frac{9}{10}-\frac{6}{10}x=0$$

$$x=\frac{3}{2} \quad \text{mode} = 1.5$$

c

	Method 1	Method 2
	$\int_0^x\left(\frac{9}{10}t-\frac{3}{10}t^2\right)dt=\left[\frac{9}{20}t^2-\frac{1}{10}t^3\right]_0^x$ $=\frac{9}{20}x^2-\frac{1}{10}x^3$	$F(x)=\int\frac{9}{10}x-\frac{3}{10}x^2\,dx$ $=\frac{9}{20}x^2-\frac{1}{10}x^3+C$ $F(2)=1 \quad 1=\frac{36}{20}-\frac{8}{10}+C$ $C=0$
	$F(x)\begin{cases}0 & x<0\\ \frac{9}{20}x^2-\frac{1}{10}x^3 & 0\le x\le 2\\ 1 & x>2\end{cases}$	

d $F(1.23)=\frac{9}{20}\times1.23^2-\frac{1}{10}\times1.23^3=0.495$

$F(1.24)=\frac{9}{20}\times1.24^2-\frac{1}{10}\times1.24^3=0.501$

Since 0.5 is in between the median lies between 1.23 and 1.24.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 9

Question:

The continuous random variable X has c.d.f. given by

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{1}{8}(x^2 - 1), & 1 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

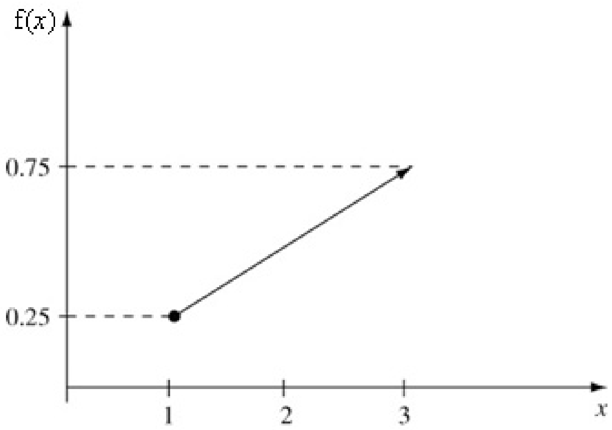
a Find the p.d.f. of the random variable X.
b Find the mode of X.
c Find the median of X.
d Find the quartiles of X.

Solution:

a Differentiating $\frac{d}{dx}\left(\frac{1}{8}x^2 - \frac{1}{8}\right) = \frac{1}{4}x$

$$f(x) = \begin{cases} \frac{1}{4}x, & 1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

b mode = 3



c $\frac{1}{8}m^2 - \frac{1}{8} = 0.5$

$\frac{1}{8}m^2 = \frac{5}{8}$

$m = \sqrt{5}$

median = $\sqrt{5}$

d $\frac{1}{8}Q_1{}^2 - \frac{1}{8} = 0.25$

$\frac{1}{8}Q_1{}^2 = \frac{3}{8}$

$Q_1 = \sqrt{3}$

lower quartile = $\sqrt{3}$

$\frac{1}{8}Q_3{}^2 - \frac{1}{8} = 0.75$

$\frac{1}{8}Q_3{}^2 = \frac{7}{8}$

$Q_3 = \sqrt{7}$

upper quartile = $\sqrt{7}$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 10

Question:

The continuous random variable X has c.d.f. given by

$$F(x)=\begin{cases}0, & x<0,\\ 4x^3-3x^4, & 0\le x\le 1,\\ 1, & x>1.\end{cases}$$

a Find the p.d.f. of the random variable X.

b Find the mode of X.

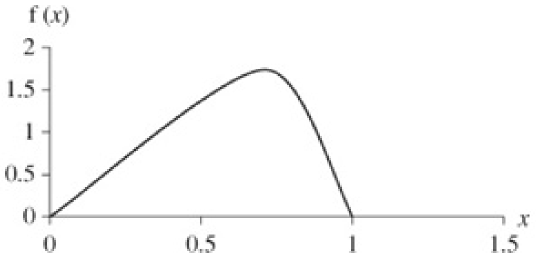
c Find $P(0.2 < X < 0.5)$.

Solution:

a $\frac{d}{dx}\left(4x^3-3x^4\right)=12x^2-12x^3$

$$f(x)=\begin{cases}12x^2(1-x), & 0\le x\le 1,\\ 0, & \text{otherwise.}\end{cases}$$

b



maximum $\frac{d}{dx}\left(12x^2-12x^3\right)=24x-36x^2$

$$24x-36x^2=0$$
$$12x\ (2-3x)\ =0$$
$$x=0 \text{ or } \frac{2}{3} \text{ mode} = \frac{2}{3}$$

c $P(0.2 < X < 0.5) = F(0.5) - F(0.2)$

$$=\left(4\times 0.5^3-3\times 0.5^4\right)-\left(4\times 0.2^3-3\times 0.2^4\right)$$
$$=0.2853$$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise D, Question 11

Question:

The amount of vegetables eaten by a family in a week is a continuous random variable W kg. The continuous random variable W has p.d.f. given by

$$f(w) = \begin{cases} \frac{20}{5^5} w^3(5-w), & 0 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the c.d.f. of the random variable W.

b Find, to 3 decimal places, the probability that the family eat between 2 kg and 4 kg of vegetables in one week. ***E***

Solution:

a

Method 1	Method 2
$\int_0^w \frac{20}{5^5} t^3(5-t)\,dt = \left[\frac{100}{4\times5^5}t^4 - \frac{20}{5\times5^5}t^5\right]_0^w$ $= \frac{25}{5^5}w^4 - \frac{4}{5^5}w^5$ $= \frac{w^4}{5^5}(25-4w)$	$F(x) = \int \frac{20}{5^5} w^3(5-w)\,dx$ $= \frac{25}{5^5}w^4 - \frac{4}{5^5}w^5 + C$ $F(5) = 1 \quad 1 = 5-4+C$ $C = 0$
$F(x)\begin{cases} 0 & x < 0 \\ \frac{w^4}{5^5}(25-4w) & 0 \le x \le 5 \\ 1 & x > 5 \end{cases}$	

b $P(2 < w < 4) = F(4) - F(2)$

$$= \left[\frac{4^4}{5^5}(25-16)\right] - \left[\frac{2^4}{5^5}(25-8)\right]$$

$$= 0.650$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise D, Question 12

Question:

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}, & 0 \le x < 1, \\ \frac{x^3}{5}, & 1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Find the cumulative distribution function.

b Find, to 3 decimal places, the median and the inter-quartile range of the distribution. ***E***

Solution:

a **Method 1**

$$F(x) = \int_0^x \frac{1}{4}\,dt$$
$$= \left[\frac{t}{4}\right]_0^x$$
$$= \frac{x}{4}$$

$$\int_0^1 \frac{1}{4}\,dx + \int_1^x \frac{t^3}{5}\,dt = \left[\frac{x}{4}\right]_0^1 + \left[\frac{t^4}{20}\right]_1^x$$
$$= \frac{1}{4} + \frac{x^4}{20} - \frac{1}{20}$$
$$= \frac{x^4}{20} + \frac{1}{5}$$

Method 2

$$F(x) = \int \frac{1}{4}\,dx$$
$$= \frac{x}{4} + C$$

$F(0) = 0$ therefore $C = 0$

$$\int \frac{x^3}{5}dx = \frac{x^4}{20} + C$$

$F(2) = 1$ therefore

$$1 = \frac{16}{20} + C$$

$$C = \frac{1}{5}$$

$$F(x)\begin{cases} 0, & x < 0, \\ \frac{x}{4}, & 0 \le x < 1, \\ \frac{x^4}{20} + \frac{1}{5}, & 1 \le x \le 2, \\ 1, & x > 2 \end{cases}$$

b $\frac{Q_1{}^4}{20} + \frac{1}{5} = 0.25$

$Q_1{}^4 + 4 - 5 = 0$

$Q_1 = 1$

$\frac{Q_3{}^4}{20} + \frac{1}{5} = 0.75$

$Q_3{}^4 + 4 = 15$

$Q_3{}^4 = 11$

$Q_3 = 1.821$ or -1.821

Therefore upper quartile $= 1.821$ as -1.821 is not in range

$\text{IQR} = 1.82 - 1 = 0.821$

$\frac{m^4}{20} + \frac{1}{5} = 0.5$

$m^4 + 4 - 10 = 0$

$m^4 = 6$

$m = 1.57$

Therefore median $= 1.57$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 1

Question:

The random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{1}{3}\left(1+\frac{x}{2}\right), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find

a $E(X)$ and $E(3X+2)$,

b $Var(X)$ and $Var(3X+2)$,

c $P(X < 1)$,

d $P(X > \mu)$,

e $P(0.5 < X < 1.5)$.

Solution:

a $\mathrm{E}(X) = \int_0^2 \frac{x}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x$

$= \int_0^2 \frac{x}{3}+\frac{x^2}{6} \mathrm{d}x$

$= \left[\frac{x^2}{6}+\frac{x^3}{18}\right]_0^2$

$= \left[\frac{2^2}{6}+\frac{2^3}{18}\right]$

$= \frac{10}{9}$

$\mathrm{E}(3X+2) = 3\,\mathrm{E}(X)+2$

$= 3\times\frac{10}{9}+2$

$= 5\frac{1}{3}$

b $\mathrm{Var}(X) = \int_0^2 \frac{x^2}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x - \left(\frac{10}{9}\right)^2$

$= \int_0^2 \frac{x^2}{3}+\frac{x^3}{6} \mathrm{d}x - \left(\frac{100}{81}\right)$

$= \left[\frac{x^3}{9}+\frac{x^4}{24}\right]_0^2 - \left(\frac{100}{81}\right)$

$= \left[\frac{2^3}{9}+\frac{2^4}{24}\right] - \left(\frac{100}{81}\right)$

$= 0.321$

$\mathrm{Var}(3X+2) = 9\mathrm{Var}(X)$

$= 2.89$

c $\mathrm{P}(X<1) = \int_0^1 \frac{1}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x$

$= \int_0^1 \frac{1}{3}+\frac{x}{6} \mathrm{d}x$

$= \left[\frac{x}{3}+\frac{x^2}{12}\right]_0^1$

$= \left[\frac{1}{3}+\frac{1}{12}\right]$

$= \frac{5}{12}$

d $\mathrm{P}(X>\mu) = \mathrm{P}(X>\frac{10}{9})$

$= 1 - \int_0^{\frac{10}{9}} \frac{1}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x$ or $\int_{\frac{10}{9}}^2 \frac{1}{3}\left(1+\frac{x}{2}\right) \mathrm{d}x = \left[\frac{x}{3}+\frac{x^2}{12}\right]_{\frac{10}{9}}^2$

$= 1 - \int_0^{\frac{10}{9}} \frac{1}{3}+\frac{x}{6} \mathrm{d}x$ $\qquad = \left[\frac{2}{3}+\frac{4}{12}\right] - \left[\frac{10}{27}+\frac{100}{972}\right]$

$= 1 - \left[\frac{x}{3}+\frac{x^2}{12}\right]_0^{\frac{10}{9}}$ $\qquad = \frac{128}{243}$

$= 1 - \left[\frac{10}{27}+\frac{100}{972}\right]$

$= 1 - \frac{115}{243}$

$= \frac{128}{243}$

e $\mathrm{P}(0.5<X<1.5) = \mathrm{P}(X<1.5) - \mathrm{P}(X<0.5)$

$= \int_{0.5}^{1.5} \left(\frac{1}{3}+\frac{x}{6}\right) \mathrm{d}x$

$= \left[\frac{x}{3}+\frac{x^2}{12}\right]_{0.5}^{1.5}$

$= \left[\frac{1.5}{3}+\frac{1.5^2}{12}\right] - \left[\frac{0.5}{3}+\frac{0.5^2}{12}\right]$

$= 0.5$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 2

Question:

The random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} 2 - 2x, & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Evaluate E(X).

b Evaluate Var(X).

c Write down the values of $E(2X+1)$ and $Var(2X+1)$.

d Specify fully the cumulative distribution function of X.

e Work out the median value of X.

Solution:

a $E(X) = \int_0^1 2x - 2x^2 dx$

$= \left[x^2 - \frac{2}{3}x^3\right]_0^1$

$= \frac{1}{3}$

b $Var(X) = \int_0^1 2x^2 - 2x^3 dx - \left(\frac{1}{3}\right)^2$

$= \left[\frac{2}{3}x^3 - \frac{1}{2}x^4\right]_0^1 - \left(\frac{1}{9}\right)$

$= \frac{1}{18}$

c $E(2X + 1) = 2E(X) + 1$

$= 2 \times \frac{1}{3} + 1$

$= \frac{5}{3}$

$Var(2X + 1) = 4Var(X)$

$= \frac{4}{18} = \frac{2}{9}$

d

Method 1	Method 2
$\int_0^x (2-2t)\,dt = \left[2x - x^2\right]_0^x$ $= 2x - x^2$	$\int 2 - 2x dx = 2x - x^2 + C$ $F(2) = 1 \quad 1 = 2 - 1 + C$ $C = 0$
$F(x) = \begin{cases} 0 & x < 0 \\ 2x - x^2 & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$	

e $2x - x^2 = 0.5$

$x^2 - 2x + 0.5 = 0$

$x = \frac{2 \pm \sqrt{4-2}}{2}$

$x = 1.71$ or 0.293

median $= 0.293$ as 1.71 is not in the range

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 3

Question:

The continuous random variable Y has cumulative distribution function given by

$$F(y) = \begin{cases} 0, & y < 1, \\ k(y^2 - y), & 1 \le y \le 2, \\ 1, & y > 2, \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{1}{2}$.

b Find $P(Y < 1.5)$.

c Find the value of the median.

d Specify fully the probability density function f(y).

Solution:

a $F(2) = 1$

$F(y) = k(y^2 - y)$

$k\ (4 - 2) = 1$

$k = \frac{1}{2}$

b $P(Y < 1.5) = F(1.5)$

$= \frac{1}{2} \times (1.5^2 - 1.5)$

$= 0.375$

c $\frac{1}{2}(y^2 - y) = 0.5$

$y^2 - y - 1 = 0$

$y = \dfrac{1 \pm \sqrt{1+4}}{2}$

$y = 1.62$ or -0.618

median $= 1.62$ as -0.618 is not in the range

d $\dfrac{d}{dx}\left[\frac{1}{2}(y^2 - y)\right] = y - \frac{1}{2}$

$$f(x) = \begin{cases} y - \frac{1}{2} & 1 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 4

Question:

The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 2, \\ \frac{1}{5}(x^2 - 4), & 2 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

a Find $P(X > 2.4)$.

b Find the median.

c Find the probability density function, $f(x)$.

d Evaluate $E(X)$.

e Find the mode of X.

Solution:

a $P(X > 2.4) = F(3) - F(2.4)$

$= \frac{1}{5}(3^2 - 4) - \frac{1}{5}(2.4^2 - 4)$

$= 0.648$

or

$P(X > 2.4) = 1 - F(2.4)$

$= 1 - \frac{1}{5}(2.4^2 - 4)$

$= 0.648$

b $\frac{1}{5}(x^2 - 4) = 0.5$

$2x^2 - 8 = 5$

$2x^2 = 13$

$x^2 = 6.5$

$x = 2.55$ or -2.55

median $= 2.55$ as -2.55 is not in the range

c $\frac{d}{dx}\left[\frac{1}{5}(x^2 - 4)\right] = \frac{2x}{5}$

$$f(x) = \begin{cases} \frac{2x}{5} & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

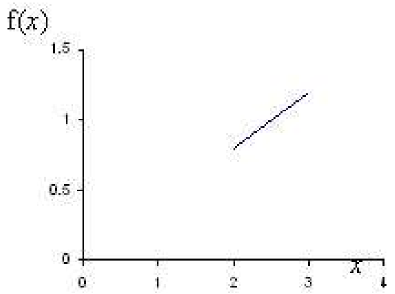
d $E(X) = \int_2^3 \frac{2x^2}{5} dx$

$= \left[\frac{2x^3}{15}\right]_2^3$

$= \frac{54}{15} - \frac{16}{15}$

$= \frac{38}{15}$

e



mode $= 3$

Solutionbank S2

Edexcel AS and A Level Modular Mathematics

Exercise E, Question 5

Question:

The random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{3}{8}$.

b Calculate $E(X)$.

c Specify fully the cumulative distribution function of X.

d Find the value of the median.

e Find the value of the mode.

Solution:

a $\int_0^2 kx^2\,dx = 1$

$\left[\frac{kx^3}{3}\right]_0^2 = 1$

$\frac{8k}{3} = 1$

$k = \frac{3}{8}$

b $E(X) = \int_0^2 \frac{3x^3}{8}dx$

$= \left[\frac{3x^4}{32}\right]_0^2$

$= 1.5$

c Method 1

$F(x) = \int_0^x \frac{3t^2}{8}dt$

$= \left[\frac{t^3}{8}\right]_0^x$

$= \frac{x^3}{8}$

Method 2

$F(x) = \int \frac{3x^2}{8}dx$

$= \frac{x^3}{8} + C$

$F(2) = 1$ therefore $\frac{8}{8} + C = 1$

$C = 0$

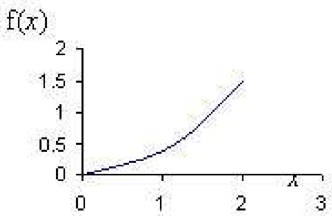
$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{8} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

d $\frac{m^3}{8} = 0.5$

$m^3 = 4$

$m = 1.59$

e



mode = 2

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 6

Question:

The random variable Y has probability density function f(y) given by

$$f(x) = \begin{cases} k(y^2 + 2y + 2), & 1 \le y \le 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{3}{62}$.

b Specify fully the cumulative distribution function of Y.

c Evaluate $P(Y \le 2)$.

Solution:

a $\int_1^3 k(y^2+2y+2)\,dy = 1$

$\left[k\left(\frac{y^3}{3}+y^2+2y\right)\right]_1^3 = 1$

$k\left(\frac{3^3}{3}+3^2+6\right)-k\left(\frac{1}{3}+1+2\right) = 1$

$\frac{62}{3}k = 1$

$k = \frac{3}{62}$

b **Method 1**

$F(y) = \int_1^y \frac{3}{62}(t^2+2t+2)\,dt$

$= \left[\frac{3}{62}\left(\frac{t^3}{3}+t^2+2t\right)\right]_1^y$

$= \frac{3}{62}\left(\frac{y^3}{3}+y^2+2y\right)-\frac{3}{62}\left(\frac{1}{3}+1+2\right)$

$= \frac{y^3}{62}+\frac{3y^2}{62}+\frac{3y}{31}-\frac{5}{31}$

Method 2

$F(x) = \int \frac{3}{62}(y^2+2y+2)\,dx$

$= \frac{3}{62}\left(\frac{y^3}{3}+y^2+2y\right)+C$

$F(3) = 1$ therefore

$\frac{3}{62}\left(\frac{3^3}{3}+3^2+6\right)+C = 1$

$C = -\frac{5}{31}$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{y^3}{62}+\frac{3y^2}{62}+\frac{3y}{31}-\frac{5}{31} & 1 \le x \le 3 \\ 1 & x > 3 \end{cases}$$

c $P(Y < 2) = \int_1^2 \frac{3}{62}(y^2+2y+2)\,dy$

$= \left[\frac{3}{62}\left(\frac{y^3}{3}+y^2+2y\right)\right]_1^2$

$= \frac{3}{62}\left(\frac{2^3}{3}+2^2+4\right)-\frac{3}{62}\left(\frac{1}{3}+1+2\right)$

$= \frac{11}{31}$

or $F(2) = \frac{2^3}{62}+\frac{3\times 2^2}{62}+\frac{6}{31}-\frac{5}{31}$

$= \frac{11}{31}$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 7

Question:

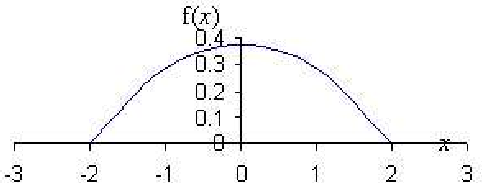
A random variable X has probability density function $f(x)$ given by

$$f(x)=\begin{cases}\frac{3}{32}(4-x^2), & -2\le x\le 2,\\ 0, & \text{otherwise.}\end{cases}$$

a Sketch the probability density function of X.
b Write down the mode of X.
c Specify fully the cumulative distribution function of X.
d Find $P(0.5 < X < 1.5)$.

Solution:

a



b mode = 0

c Method 1

$$F(x)=\int_{-2}^{x}\frac{3}{32}\left(4-t^2\right)dt$$

$$=\left[\frac{12t}{32}-\frac{t^3}{32}\right]_{-2}^{x}$$

$$=\left(\frac{12x}{32}-\frac{x^3}{32}\right)-\left(-\frac{24}{32}+\frac{8}{32}\right)$$

$$=\frac{12x}{32}-\frac{x^3}{32}+\frac{1}{2}$$

Method 2

$$F(x)=\int\frac{3}{32}\left(4-x^2\right)dx$$

$$=\frac{12x}{32}-\frac{x^3}{32}+C$$

$F(2)=1$ therefore $\frac{24}{32}-\frac{8}{32}+C$

=1

$C=0.5$

$$F(x)=\begin{cases}0 & x<-2\\ \frac{12x}{32}-\frac{x^3}{32}+\frac{1}{2} & -2\le x\le 2\\ 1 & x>2\end{cases}$$

d $P(0.5 < X < 1.5) = F(1.5) - F(0.5)$

$$=\left(\frac{18}{32}-\frac{1.5^3}{32}+\frac{1}{2}\right)-\left[\frac{6}{32}-\frac{0.5^3}{32}+\frac{1}{2}\right]$$

$$=\frac{35}{128}\text{ or }0.273$$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 8

Question:

A random variable X has probability density function f(x) given by

$$f(x)=\begin{cases}\frac{1}{3}, & 0\le x<1,\\ \frac{2}{7}x^2, & 1\le x\le 2,\\ 0, & \text{otherwise.}\end{cases}$$

a Find E(X).
b Specify fully the cumulative distribution function of X.
c Find the median of X.

Solution:

a $\mathrm{E}(X)=\int_0^1 \frac{x}{3}\mathrm{d}x+\int_1^2 \frac{2x^3}{7}\mathrm{d}x$

$=\left[\frac{x^2}{6}\right]_0^1+\left[\frac{2x^4}{28}\right]_1^2$

$=\frac{1}{6}+\left(\frac{32}{28}-\frac{2}{28}\right)$

$=\frac{26}{21}$

b Method 1

$\mathrm{F}(x)=\int_0^x \frac{1}{3}\,\mathrm{d}t$

$=\left[\frac{t}{3}\right]_0^x$

$=\frac{x}{3}$

$\int_0^1 \frac{1}{3}\mathrm{d}x+\int_1^x \frac{2t^2}{7}\mathrm{d}t=\left[\frac{x}{3}\right]_0^1+\left[\frac{2t^3}{21}\right]_1^x$

$=\frac{1}{3}+\frac{2x^3}{21}-\frac{2}{21}$

$=\frac{2x^3}{21}+\frac{5}{21}$

Method 2

$\mathrm{F}(x)=\int \frac{1}{3}\mathrm{d}x$

$=\frac{x}{3}+C$

$\mathrm{F}(0)=0$ therefore $C=0$

$\int \frac{2x^2}{7}\mathrm{d}x=\frac{2x^3}{21}+C$

$\mathrm{F}(2)=1$ therefore

$1=\frac{16}{21}+C$

$C=\frac{5}{21}$

$$\mathrm{F}(x)=\begin{cases}0 & x<0\\ \frac{x}{3} & 0\le x<1\\ \frac{2x^3}{21}+\frac{5}{21} & 1\le x\le 2\\ 1 & x>2\end{cases}$$

c $\mathrm{F}(1)=\frac{1}{3}$ therefore median lies in interval $1\le x\le 2$

$\frac{2x^3}{21}+\frac{5}{21}=0.5$

$2x^3+5=10.5$

$2x^3=5.5$

$x^3=2.75$

$x=1.40$

median $=1.40$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 9

Question:

A continuous random variable X has probability density function $\mathrm{f}(x)$ given by

$$\mathrm{f}(x)=\begin{cases} kx-k, & 1\le x\le 3, \\ 0, & \text{otherwise,}\end{cases}$$

where k is a positive constant.

a Show that $k=\frac{1}{2}$.

b Find $\mathrm{E}(X)$.

c Work out the cumulative distribution function, $\mathrm{F}(x)$.

d Show that the median value lies between 2.4 and 2.5.

Solution:

a $\int_1^3 kx - k\,dx = 1$

$\left[\frac{kx^2}{2} - kx\right]_1^3 = 1$

$\left(\frac{9k}{2} - 3k\right) - \left(\frac{k}{2} - k\right) = 1$

$2k = 1$

$k = \frac{1}{2}$

b $E(X) = \int_1^3 \frac{x^2}{2} - \frac{x}{2}dx$

$= \left[\frac{x^3}{6} - \frac{x^2}{4}\right]_1^3$

$= \left(\frac{9}{4}\right) - \left(-\frac{1}{12}\right)$

$= \frac{7}{3}$

c Method 1

$F(x) = \int_1^x \frac{t}{2} - \frac{1}{2}dt$

$= \left[\frac{t^2}{4} - \frac{t}{2}\right]_1^x$

$= \left(\frac{x^2}{4} - \frac{x}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)$

$= \frac{x^2}{4} - \frac{x}{2} + \frac{1}{4}$

Method 2

$F(x) = \int \frac{x}{2} - \frac{1}{2}dx$

$= \frac{x^2}{4} - \frac{x}{2} + C$

$F(3) = 1$ therefore $\frac{9}{4} - \frac{3}{2} + C = 1$

$C = 0.25$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{x^2}{4} - \frac{x}{2} + \frac{1}{4} & 1 \le x \le 3 \\ 1 & x > 3 \end{cases}$$

d $F(2.4) = \frac{2.4^2}{4} - \frac{2.4}{2} + \frac{1}{4} = 0.5$

$F(2.5) = \frac{2.5^2}{4} - \frac{2.5}{2} + \frac{1}{4} = 0.5$

Since 0.5 lies in between, the median is between 2.4 and 2.5.

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 10

Question:

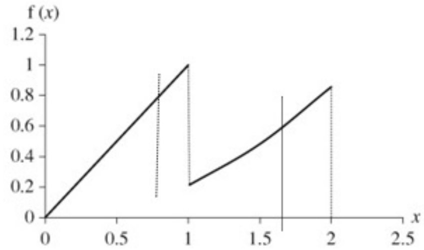
The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} x, & 0 \le x < 1, \\ \frac{3x^2}{14}, & 1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the probability density function of X.
b Find the mode of X.
c Find $E(2X)$.
d Find $\text{Var}(2X+1)$
e Specify fully the cumulative distribution function of X.
f Using your answer to part **e** find the median of X.

Solution:

a



b mode = 1

c
$$E(X) = \int_0^1 x^2 dx + \int_1^2 \frac{3x^3}{14} dx$$
$$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{3x^4}{56}\right]_1^2$$
$$= \frac{1}{3} + \left(\frac{48}{56} - \frac{3}{56}\right)$$
$$= \frac{191}{168}$$
$$E(2X) = 2 \times \frac{191}{168}$$
$$= \frac{191}{84}$$

d
$$Var(X) = \int_0^1 x^3 dx + \int_1^2 \frac{3x^4}{14} dx - \left(\frac{191}{168}\right)^2$$
$$= \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{3x^5}{70}\right]_1^2 - \left(\frac{191}{168}\right)^2$$
$$= \frac{1}{4} + \left(\frac{48}{35} - \frac{3}{70}\right) - \left(\frac{191}{168}\right)^2$$
$$= 0.28601$$
$$Var(2X+1) = 4 \times 0.286$$
$$= 1.14$$

e Method 1
$$F(x) = \int_0^x t\, dt$$
$$= \left[\frac{t^2}{2}\right]_0^x$$
$$= \frac{x^2}{2}$$
$$\int_0^1 t\, dt + \int_1^x \frac{3t^2}{14} dt = \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{t^3}{14}\right]_1^x$$
$$= \frac{1}{2} + \frac{x^3}{14} - \frac{1}{14}$$
$$= \frac{x^3}{14} + \frac{3}{7}$$

Method 2
$$F(x) = \int x dx$$
$$= \frac{x^2}{2} + C$$
$F(0) = 0$ therefore $C = 0$
$$\int \frac{3x^2}{14} dx = \frac{x^3}{14} + C$$
$F(2) = 1$ therefore
$$1 = \frac{8}{14} + C$$
$$C = \frac{3}{7}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \le x < 1 \\ \frac{x^3}{14} + \frac{3}{7} & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

f $\frac{x^2}{2} = 0.5$ median = 1