Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

The discrete random variable $X \sim \text{Po}(2.3)$. Find

a $\text{P}(X = 4)$,

b $\text{P}(X \geq 1)$,

c $\text{P}(4 < X < 6)$.

Solution:

$X \sim \text{P}_0(2.3)$

a $\text{P}(X = 4) = \dfrac{e^{-2.3}(2.3)^4}{4!} = 0.11690\ldots = 0.117 \text{ (3 s.f.)}$

b
$$\begin{aligned}\text{P}(X \geq 1) &= 1 - \text{P}(X = 0)\\ &= 1 - e^{-2.3}\\ &= 0.89974\ldots = 0.900 \text{ (3 s.f.)}\end{aligned}$$

c $\text{P}(4 < X < 6)$

$= \text{P}(X = 5)$

$= 0.05377 \quad = 0.0538 \text{ (3 s.f.)}$

Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

The discrete random variable $X \sim \text{Po}(5.7)$. Find

a $\text{P}(X = 7)$,

b $\text{P}(X \leq 1)$,

c $\text{P}(X > 2)$.

Solution:

$X \sim \text{P}_0(5.7)$

a
$$\begin{aligned}\text{P}(X=7) &= \frac{e^{-5.7}(5.7)^7}{7!}\\ &= 0.12978\ldots = 0.130 \text{ (3 s.f.)}\end{aligned}$$

b
$$\begin{aligned}\text{P}(X \leq 1) &= \text{P}(X=0)+\text{P}(X=1)\\ &= e^{-5.7}+e^{-5.7}\times\frac{5.7}{1!}\\ &= 6.7\times e^{-5.7}\\ &= 0.022417\ldots = 0.0224 \text{ (3 s.f.)}\end{aligned}$$

c
$$\begin{aligned}\text{P}(X > 2) &= 1-\text{P}(X \leq 2)\\ &= 1-\left[\text{P}(X=0)+\text{P}(X=1)+\text{P}(X=2)\right]\\ &= 1-0.022417-\frac{e^{-5.7}\times 5.7^2}{2!}\\ &= 0.923226\ldots \quad = 0.923 \text{ (3 s.f.)}\end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 3

Question:

The random variable $Y \sim \text{Po}(0.35)$. Find

a $\text{P}(Y = 1)$,

b $\text{P}(Y \geq 1)$,

c $\text{P}(1 \leq Y < 3)$.

Solution:

$Y \sim \text{P}_0(0.35)$

a
$$\text{P}(Y=1) = \frac{e^{-0.35} \times 0.35}{1!}$$
$$= 0.24664\ldots = 0.247$$

b
$$\text{P}(Y \geq 1) = 1 - \text{P}(Y=0)$$
$$= 0.29531\ldots = 0.295$$

c
$$\text{P}(1 \leq Y < 3) = \text{P}(Y=1) + \text{P}(Y=2)$$
$$= 0.24664\ldots + \frac{e^{-0.35} \times (0.35)^2}{2!}$$
$$= 0.289802\ldots = 0.290$$

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Exercise A, Question 4

Question:

The random variable $X \sim \text{Po}(3.6)$. Find

a $\text{P}(X = 5)$,

b $\text{P}(3 < X \leq 6)$,

c $\text{P}(X < 2)$.

Solution:

$X \sim \text{P}_0(3.6)$

a

$$\text{P}(X = 5) = \frac{e^{-3.6}(3.6)^5}{5!} = 0.13768\ldots = 0.138$$

b

$$\begin{aligned}\text{P}(3 < X \leq 6) &= \text{P}(X = 4) + \text{P}(X = 5) + \text{P}(X = 6) \\ &= \frac{e^{-3.6}(3.6)^4}{4!} + \frac{e^{-3.6}(3.6)^5}{5!} + \frac{e^{-3.6}(3.6)^6}{6!} \\ &= 0.41151\ldots = 0.412\end{aligned}$$

c

$$\begin{aligned}\text{P}(X < 2) &= \text{P}(X = 0) + \text{P}(X = 1) \\ &= e^{-3.6} + e^{-3.6} \times 3.6 \\ &= 4.6 \times e^{-3.6} \\ &= 0.12568\ldots = 0.126\end{aligned}$$

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Exercise B, Question 1

Question:

The random variable $X \sim \text{Po}(2.5)$. Find

a $\text{P}(X = 1)$,

b $\text{P}(X > 2)$,

c $\text{P}(X \le 5)$,

d $\text{P}(3 \le X \le 5)$.

Solution:

$X \sim \text{P}_0(2.5)$

a $\text{P}(X = 1) = e^{-2.5} \times 2.5 \quad = 0.2052$ (4 d.p.)

b
$$\begin{aligned} \text{P}(X > 2) &= 1 - \text{P}(X \le 2) \\ &= 1 - 0.5438 \quad \text{(tables)} \\ &= 0.4562 \end{aligned}$$

c $\text{P}(X \le 5) = 0.9580$ (tables)

d
$$\begin{aligned} \text{P}(3 \le X \le 5) &= \text{P}(X \le 5) - \text{P}(X \le 2) \\ &= 0.9580 - 0.5438 \quad \text{(tables)} \\ &= 0.4142 \end{aligned}$$

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Exercise B, Question 2

Question:

The random variable $X \sim \text{Po}(6)$. Find

a $P(X \leq 3)$,

b $P(X > 4)$,

c $P(X = 5)$,

d $P(2 < X \leq 7)$.

Solution:

$X \sim P_0(6)$

a $P(X \leq 3) = 0.1512$ (tables)

b $P(X > 4) = 1 - P(X \leq 4)$
$= 1 - 0.2851$ (tables)
$= 0.7149$

c $P(X = 5) = P(X \leq 5) - P(X \leq 4)$
$= 0.4457 - 0.2851$ (tables)
$= 0.1606$

d $P(2 < X \leq 7) = P(X \leq 7) - P(X \leq 2)$
$= 0.7440 - 0.0620$ (tables)
$= 0.6820$

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Exercise B, Question 3

Question:

The random variable Y has a Poisson distribution with mean 4.5. Find

a $P(Y = 2)$,

b $P(Y \leq 1)$,

c $P(Y > 4)$,

d $P(2 \leq Y \leq 6)$.

Solution:

$Y \sim P_0(4.5)$

a
$$\begin{aligned} P(Y = 2) &= P(Y \leq 2) - P(Y \leq 1) \\ &= 0.1736 - 0.0611 \quad \text{(tables)} \\ &= 0.1125 \end{aligned}$$

b $P(Y \leq 1) = 0.0611$ (tables)

c
$$\begin{aligned} P(Y > 4) &= 1 - P(Y \leq 4) \\ &= 1 - 0.5321 \quad \text{(tables)} \\ &= 0.4679 \end{aligned}$$

d
$$\begin{aligned} P(2 \leq Y \leq 6) &= P(Y \leq 6) - P(Y \leq 1) \\ &= 0.8311 - 0.0611 \quad \text{(tables)} \\ &= 0.7700 \end{aligned}$$

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Exercise B, Question 4

Question:

The random variable $X \sim \text{Po}(8)$. Find the values of a, b, c and d such that

a $\text{P}(X \leq a) = 0.3134$,

b $\text{P}(X \leq b) = 0.7166$,

c $\text{P}(X < c) = 0.0996$,

d $\text{P}(X > d) = 0.8088$.

Solution:

$X \sim \text{P}_0(8)$

a $\text{P}(X \leq 6) = 0.3134$ (tables)

$\therefore a = 6$

b $\text{P}(X \leq 9) = 0.7166$ (tables)

$\therefore b = 9$

c $\text{P}(X \leq 4) = 0.0996$

$\text{P}(X < 5) = 0.0996$ (tables)

$\therefore c = 5$

d $\text{P}(X \leq 5) = 0.1912$

$\therefore \text{P}(X > 5) = 1 - 0.1912$ (tables)

$= 0.8088$

$\therefore d = 5$

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Exercise B, Question 5

Question:

The random variable $X \sim \text{Po}(3.5)$. Find the values of a, b, c and d such that

a $\text{P}(X \leq a) = 0.8576$,

b $\text{P}(X > b) = 0.6792$,

c $\text{P}(X \leq c) \geq 0.95$,

d $\text{P}(X > d) \leq 0.005$.

Solution:

$X \sim \text{P}_0(3.5)$

a
$$\begin{aligned} \text{P}(X \leq 5) &= 0.8576 \quad \text{(tables)} \\ \therefore a &= 5 \end{aligned}$$

b
$$\begin{aligned} 1 - 0.6792 &= 0.3208 \\ \text{P}(X \leq 2) &= 0.3208 \quad \text{(tables)} \\ \text{P}(X > 2) &= 0.6792 \\ \therefore b &= 2 \end{aligned}$$

c
$$\begin{aligned} \text{P}(X \leq 6) &= 0.9347 \\ \text{P}(X \leq 7) &= 0.9733 \quad \text{(tables)} \\ \therefore c &\geq 7 \end{aligned}$$

d
$$\begin{aligned} \text{P}(X \leq 8) &= 0.9901 \Rightarrow \text{P}(X > 8) = 0.0099 > 0.005 \\ \text{P}(X \leq 9) &= 0.9967 \Rightarrow \text{P}(X > 9) = 0.0033 < 0.005 \quad \text{(tables)} \\ \therefore d &\geq 9 \end{aligned}$$

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Exercise B, Question 6

Question:

The number of telephone calls received at an exchange during a weekday morning follows a Poisson distribution with a mean of 6 calls per 5-minute period. Find the probability that

a there are no calls in the next 5 minutes,

b 3 or fewer calls are received in the next 5 minutes,

c fewer than 2 calls are received between 11:00 and 11:05,

d no more than 2 calls are received between 11:30 and 11:35.

Solution:

Let X = number of telephone calls in a week day morning in 5 minutes

$X \sim \mathrm{P_o}(6)$

a $\mathrm{P}(X = 0) = \mathrm{e}^{-6} = 0.002478$

$= 0.00248 \quad (3 \text{ s.f.})$

or 0.0025 (tables)

b $\mathrm{P}(X \leq 3) = 0.1512$ (tables)

c $\mathrm{P}(X < 2) = \mathrm{P}(X \leq 1)$ (tables)

$= 0.0174$

d $\mathrm{P}(X \leq 2) = 0.0620$ (tables)

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Exercise B, Question 7

Question:

The random variable $X \sim \text{Po}(9)$. Find

a $\mu = \text{E}(X)$,

b $\sigma =$ standard deviation of X,

c $\text{P}(\mu \le X < \mu + \sigma)$,

d $\text{P}(X \le \mu - \sigma)$.

Solution:

$X \sim \text{P}_0(9)$

a $\mu = \text{E}(X) = 9$

b $\sigma^2 = \text{Var}(X) = 9 \quad \sigma = 3$

c
$$\begin{aligned} \text{P}(9 \le X < 12) &= \text{P}(X \le 11) - \text{P}(X \le 8) \\ &= 0.8030 - 0.4557 \\ &= 0.3473 \end{aligned}$$

d
$$\begin{aligned} \text{P}(X \le 9 - 3) &= \text{P}(X \le 6) \qquad \text{(tables)} \\ &= 0.2068 \end{aligned}$$

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Exercise B, Question 8

Question:

The mean number of faults in 2 m^2 of cloth produced by a factory is 1.5.

a Find the probability of a 2 m^2 piece of cloth containing no faults.

b Find the probability that a 2 m^2 piece of cloth contains no more than 2 faults.

Solution:

Let X = number of faults in 2 m^2 of cloth

$X \sim \mathrm{P_o}(1.5)$

a $\mathrm{P}(X = 0) = 0.2231$ (tables)

b $\mathrm{P}(X \leq 2) = 0.8088$ (tables)

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Exercise C, Question 1

Question:

A technician is responsible for a large number of machines. Minor adjustments have to be made to these machines and these occur at random and at a constant rate of 7 per hour. Find the probability that

a in a particular hour the technician makes 4 or fewer adjustments,

b during a half-hour break no adjustments will be required.

Solution:

Let X = number of adjustments in an hour

$X \sim P_0(7)$

a $P(X \leq 4) = 0.1730$ (tables)

b Y = number of adjustments in a half-hour

$Y \sim P_0(3.5)$

$P(Y = 0) = 0.0302$ (tables)

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Exercise C, Question 2

Question:

A textile firm produces rolls of cloth but slight defects sometimes occur. The average number of defects per square metre is 2.5. Use a Poisson distribution to calculate the probability that

a a $1.5\,\text{m}^2$ portion of cloth bought to make a skirt contains no defects,

b a $4\,\text{m}^2$ portion of cloth contains fewer than 5 defects.

c State briefly what assumptions have to be made before a Poisson distribution can be accepted as a suitable model in this situation.

Solution:

a Let

$$\begin{aligned} X &= \text{number of defects in } 1.5\,\text{m}^2 \\ \lambda &= 1.5\times 2.5 = 3.75 \\ \therefore X &\sim \text{P}_0(3.75) \\ \text{P}(X=0) &= e^{-3.75} = 0.0235177\ldots \\ &= 0.0235\ (3\text{ s.f.}) \end{aligned}$$

b Let

$$\begin{aligned} Y &= \text{number of defects in } 4\,\text{m}^2 \\ \lambda &= 4\times 2.5 = 10 \\ \therefore Y &\sim \text{P}_0(10) \qquad \text{(tables)} \\ \text{P}(Y<5) &= \text{P}(Y\le 4) = 0.0293 \end{aligned}$$

c Assume that defects occur independently and at random in the cloth and defects occur at a constant rate.

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Exercise C, Question 3

Question:

State which of the following could be modelled by a Poisson distribution and which can not. Give reasons for your answers.

a The number of misprints on this page in the first draft of this book.

b The number of pigs in a particular 5 m square of their field 1 hour after their food was placed in a central trough.

c The number of pigs in a particular 5 m square of their field 1 minute after their food was placed in a central trough.

d The amount of salt, in mg, contained in $1\,\text{cm}^3$ of water taken from a bucket immediately after a teaspoon of salt was added to the bucket.

e The number of marathon runners passing the finishing post between 20 and 21 minutes after the winner of the race.

Solution:

a If the misprints occur independently and at random and at a constant average rate then this could be Poisson.

b Yes because after 1 hour the pigs are probably dispersed fairly randomly and independently around the field.

c No because after 1 minute the pigs will probably be clustered around the feeding trough and so will not be randomly and independently scattered.

d No because the salt needs to diffuse so that it is randomly dissolved at a constant rate throughout the contents of the bucket.

e Yes, this may be Poisson provided that the runners are not in groups, since they need to pass the post independently and at random.

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Exercise C, Question 4

Question:

The number of accidents per week at a certain road intersection has a Poisson distribution with parameter 2.5. Find the probability that

a exactly 5 accidents will occur in a particular week,

b more than 14 accidents will occur in 4 weeks. ***E***

Solution:

a X = number of accidents in a week

$X \sim P_0(2.5)$

$$
\begin{aligned}
P(X=5) &= P(X \le 5) - P(X \le 4) \\
&= 0.9580 - 0.8912 \qquad \text{(tables)} \\
&= 0.0668
\end{aligned}
$$

b Y = number of accidents in a 4-week period

$Y \sim P_0(4 \times 2.5 = 10)$

$$
\begin{aligned}
P(Y > 14) &= 1 - P(Y \le 14) \\
&= 1 - 0.9165 \qquad \text{(tables)} \\
&= 0.0835
\end{aligned}
$$

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Exercise C, Question 5

Question:

In a particular district it has been found, over a long period, that the number, X, of cases of measles reported per month has a Poisson distribution with parameter 1.5. Find the probability that in this district

a in any given month, exactly 2 cases of measles will be reported,

b in a period of 6 months, fewer than 10 cases of measles will be reported.

Solution:

$X \sim \mathrm{P_0}(1.5)$

a $\mathrm{P}(X = 2) = \mathrm{P}(X \le 2) - \mathrm{P}(X \le 1)$

$= 0.8088 - 0.5578$

$= 0.2510$

b Y = number of reported cases of measles in 6 months

$Y \sim \mathrm{P_0}(9)$ ← $9 = 6 \times 1.5$

$\mathrm{P}(Y < 10) = \mathrm{P}(Y \le 9)$

$= 0.5874$ (tables)

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Exercise C, Question 6

Question:

A biologist is studying the behaviour of sheep in a large field. The field is divided into a number of equally sized squares and the average number of sheep per square is 2.5. The sheep are randomly scattered throughout the field.

a Suggest a suitable model for the number of sheep in a square and give a value for any parameter or parameters required.

b Calculate the probability that a randomly selected square contains more than 3 sheep.

A sheep dog has been sent into the field to round up the sheep.

c Explain why the model may no longer be applicable.

Solution:

a X = number of sheep per square

$X \sim \text{P}_0(2.5)$

b

$$\begin{aligned} \text{P}(X > 3) &= 1 - \text{P}(X \leq 3) \\ &= 1 - 0.7576 \quad \text{(tables)} \\ &= 0.2424 \end{aligned}$$

c The sheep will no longer be randomly scattered.

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Exercise C, Question 7

Question:

During office hours, telephone calls to a single telephone in an office come in at an average rate of 18 calls per hour. Assuming that a Poisson distribution can be applied, find the probability that in a 5-minute period there will be

a fewer than 2 calls,

b more than 3 calls.

c Find the probability of no calls during a 20-minute coffee break.

Solution:

Let X = number of calls in a 5-minute period

$\lambda = \frac{18}{12} = 1.5$

$\therefore X \sim P_0(1.5)$

a $P(X < 2) = P(X \leq 1)$

$= 0.5578$ (tables)

b $P(X > 3) = 1 - P(X \leq 3)$

$= 1 - 0.9344$ (tables)

$= 0.0656$

c Y = number of calls in 20-minute period

$\lambda = 4 \times 1.5 = 6$

$\therefore Y \sim P_0(6)$

$P(Y = 0) = 0.0025$ (tables)

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Exercise C, Question 8

Question:

A shop sells large birthday cakes at a rate of 2 every 3 days.

a Find the probability of selling no large birthday cakes on a randomly selected day.

Fresh cakes are baked every 3 days and any cakes older than 3 days can not be sold.

b Find how many large birthday cakes should be baked so that the probability of running out of large birthday cakes to sell is less than 1%.

Solution:

$X =$ number of large cakes sold in a day

$X \sim \mathrm{P_0}(\frac{2}{3})$

a

$$\mathrm{P}(X=0)=\mathrm{e}^{-\frac{2}{3}} = 0.51341$$
$$= 0.513 \text{ (3 s.f.)}$$

b $Y =$ number of large cakes sold in 3 days

$Y \sim \mathrm{P_0}(2)$

Let $n =$ number of cakes baked

To run out of cakes you require $Y > n$

Require $\mathrm{P}(Y > n) < 0.01$

i.e. $\mathrm{P}(Y \le n) > 0.99$

$\mathrm{P}(Y \le 5) = 0.9834 < 0.99$ (tables)

$\mathrm{P}(Y \le 6) = 0.9955 > 0.99$

$\therefore$ need $n = 6$

($n \ge 6$ but baking more is likely to create more waste)

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Exercise C, Question 9

Question:

On a typical summer's day a boat company hires rowing boats at a rate of 9 per hour.

a Find the probability of hiring out at least 6 boats in a randomly selected 30-minute period.

The company has 8 boats to hire and decides to hire them out for 20-minute periods.

b Show that the probability of running out of boats is less than 1%.

c Find how many boats the company should have to be 99% sure of meeting all demands if the hire period is extended to 30 minutes.

Solution:

a Let X = number of boats hired in a 30-minute period

$\lambda = \frac{1}{2} \times 9 = 4.5$

$\therefore X \sim P_0(4.5)$ (tables)

$P(X \geq 6) = 1 - P(X \leq 5)$

$= 1 - 0.7029$

$= 0.2971$

b Let Y = number of requests for hire in 20-minute period

$\lambda = \frac{1}{3} \times 9 = 3$

$Y \sim P_0(3)$

$P(Y > 8) = 1 - P(Y \leq 8)$

$= 1 - 0.9962$ (tables)

$= 0.0038 < 0.01$

c Let n = number of boats

Require $P(X > n) < 0.01$

or $P(X \leq n) > 0.99$

Use $P_0(4.5)$.

$P(X \leq 9) = 0.9829 < 0.99$ (tables)

$P(X \leq 10) = 0.9933 > 0.99$

$\therefore$ they would need 10 boats

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Exercise C, Question 10

Question:

Breakdowns on a particular machine occur at random at a rate of 1.5 per week.

a Find the probability that no more than 2 breakdowns occur in a randomly chosen week.

b Find the probability of at least 5 breakdowns in a randomly chosen two-week period.

A maintenance firm offers a contract for repairing breakdowns over a six-week period. The firm will give a full refund if there are more than n breakdowns in a six-week period. The firm want the probability of having to pay a refund to be 5% or less.

c find the smallest value of n.

Solution:

Let $X =$ number of breakdowns in a week

$X \sim P_0(1.5)$

a $P(X \leq 2) = 0.8088$ (tables)

b Let $Y =$ no of breakdowns in a 2-week period

$Y \sim P_0(3)$

$$\begin{aligned} P(Y \geq 5) &= 1 - P(Y \leq 4) \\ &= 1 - 0.8153 \quad \text{(tables)} \\ &= 0.1847 \end{aligned}$$

c Let $B =$ number of breakdowns in a 6-week period

$B \sim P_0(9)$

Firm requires $P(B > n) \leq 0.05$

i.e. $P(B \leq n) \geq 0.95$

$P(B \leq 13) = 0.9261 < 0.95$ (tables)

$P(B \leq 14) = 0.9585 > 0.95$

$\therefore n = 14$

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Exercise D, Question 1

Question:

The random variable $X \sim B(80, 0.10)$. Using a suitable approximation, find

a $P(X \geq 1)$,

b $P(X \leq 6)$.

Solution:

$X \sim B(80, 0.10)$

$X \approx\sim P_0(8)$

a

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &\approx 1 - 0.0003 \qquad \text{(Poisson tables)} \\ &= 0.9997 \end{aligned}$$

b $P(X \leq 6) \approx 0.3134$ (Poisson tables)

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Exercise D, Question 2

Question:

The random variable $X \sim B(120, 0.02)$. Using a suitable approximation, find

a $P(X = 1)$,

b $P(X \geq 3)$.

Solution:

$X \sim B(120, 0.02)$

$X \approx\sim P_0(2.4)$

a
$$\begin{aligned} P(X = 1) &= e^{-2.4}(2.4)^1 \\ &= 0.21772\ldots = 0.218\ (3\text{ s.f.}) \end{aligned}$$

b
$$\begin{aligned} P(X \geq 3) &= 1 - P(X \leq 2) \\ &= 1 - e^{-2.4}\left[1 + 2.4 + \frac{2.4^2}{2!}\right] \\ &= 1 - 0.56970\ldots \\ &= 0.430(3\text{ s.f.}) \end{aligned}$$

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Exercise D, Question 3

Question:

The random variable $X \sim \mathrm{B}(50, 0.05)$. Find the percentage error in $\mathrm{P}(X \leq 4)$ when X is approximated by a Poisson distribution.

Solution:

$X \sim \mathrm{B}(50, 0.05)$

$\mathrm{P}(X \leq 4) = 0.8964$ (Binomial tables)

$X \approx\sim \mathrm{P_0}(2.5)$

$\mathrm{P}(X \leq 4) \approx 0.8912$ (Poisson tables)

$$\text{Percentage error} = \frac{(0.8964 - 0.8912)}{0.8964} \times 100 = 0.58\%$$

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Exercise D, Question 4

Question:

In a certain manufacturing process the proportion of defective articles produced is 2%. In a batch of 300 articles, use a suitable approximation to find the probability that

a there are fewer than 2 defectives,

b there are exactly 4 defectives.

Solution:

X = number of defectives in a batch of 300

$X \sim B(300, 0.02)$

$X \approx\sim P_0(6)$

a $P(X < 2) = P(X \leq 1)$ (Poisson tables)

≈ 0.0174

b $P(X = 4) = P(X \leq 4) - P(X \leq 3)$

$\approx 0.2851 - 0.1512$ (Poisson tables)

$= 0.1339$

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Exercise D, Question 5

Question:

A medical practice screens a random sample of 250 of its patients for a certain condition which is present in 1.5% of the population. Use a suitable approximation to find the probability that they obtain

a no patients with the condition,

b at least two patients with the condition.

Solution:

$X =$ number of people with the condition in sample of 250

$X \sim \mathrm{B}(250, 0.015)$

$X \approx\sim \mathrm{P_0}(3.75)$

Poisson is suitable approximation.

a $\mathrm{P}(X = 0) \approx e^{-3.75} = 0.0235177\ldots$ (using Poisson)

So using Poisson approximation probability ≈ 0.0235 (3 s.f.)

b
$$\begin{aligned} \mathrm{P}(X \geq 2) &= 1 - \mathrm{P}(X \leq 1) \\ &\approx 1 - e^{-3.75}[1 + 3.75] \\ &= 1 - 4.75 \times e^{-3.75} \\ &= 1 - 0.111709\ldots \\ &= 0.8883 \end{aligned}$$

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Exercise D, Question 6

Question:

An experiment involving 2 fair dice is carried out 180 times. The dice are placed in a container, shaken and the number of times a double six is obtained recorded. Use a suitable approximation to find the probability that a double six is obtained

a once,

b twice,

c at least three times.

Solution:

$X =$ number of double sixes in 180 throws

$X \sim \mathrm{B}(180, \frac{1}{36})$

$X \approx\sim \mathrm{P_o}(5)$

a $\mathrm{P}(X = 1) \approx \frac{e^{-5} \times 5^1}{1!} = 0.03368$

$= 0.0337 \quad (3 \text{ s.f.})$

b $\mathrm{P}(X = 2) \approx \frac{e^{-5} \times 5^2}{2!} = 0.084224\ldots$

$= 0.0842 \ (3 \text{ s.f.})$

c $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2)$

$\approx 1 - 0.1247$ (Poisson tables)

$= 0.8753$

$= 0.875 \ (3 \text{ s.f.})$

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Exercise D, Question 7

Question:

It is claimed that 95% of the population in a certain village are right-handed. A random sample of 80 villagers is tested to see whether or not they are right-handed. Use a Poisson approximation to estimate the probability that the number who are right-handed is

a 80,
b 79,
c at least 78.

Solution:

X = number of villagers who are left-handed
$X \sim B(80, 0.05)$
(Need P small to use Poisson approximation)
$X \approx P_0(4)$

a $P(80 \text{ are right-handed}) = P(X=0) \approx e^{-4}$ (Poisson Tables)
$= 0.0183$

b $P(79 \text{ are right-handed}) = P(X=1)$
$= P(X \le 1) - P(X=0)$
$\approx 0.0916 - 0.0183$ (Poisson Tables)
$= 0.0733$

c $P(\text{at least 78 are right-handed}) = P(X \le 2)$
≈ 0.2381 (Poisson tables)

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Exercise D, Question 8

Question:

In a computer simulation 500 dots were fired at a target and the probability of a dot hitting the target was 0.98. Find the probability that

a all the dots hit the target,

b at least 495 hit the target.

Solution:

X = number of hits out of 500

$X \sim \text{B}(500, 0.98)$

Y = number of misses out of 500 ($\because p$ is small and n large)

$Y \sim \text{B}(500, 0.02)$

$Y \approx \text{P}_0(10)$

a

$$\text{P}(X = 500) = \text{P}(Y = 0) \approx e^{-10} = 0.000045399 \approx 0.0000454 \text{ (3 s.f.)}$$

b

$$\text{P}(X \geq 495) = \text{P}(Y \leq 5) \approx 0.0671 \quad \text{(Poisson tables)}$$

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Exercise D, Question 9

Question:

a State the conditions under which the Poisson distribution may be used as an approximation to the binomial distribution.

Independently for each call into the telephone exchange of a large organisation, there is a probability of 0.002 that the call will be connected to a wrong extension.

b Find, to 3 significant figures, the probability that, on a given day, exactly one of the first 5 incoming calls will be wrongly connected.

c Use a Poisson approximation to find, to 3 decimal places, the probability that, on a day when there are 1000 incoming calls, at least 3 of them are wrongly connected during that day. ***E***

Solution:

a For large n
and small p
$B(n, p) \approx P_0(np)$

b $X =$ number of wrongly connected calls in sample of 5
$X \sim B(5, 0.002)$

$$\begin{aligned} P(X = 1) &= 5 \times (0.002)^1 (0.998)^4 \\ &= 0.00992 \end{aligned}$$

c $Y =$ number of wrongly connected calls in sample of 1000
$Y \sim B(1000, 0.002)$
So $Y \approx\sim P_0(2)$

$$\begin{aligned} P(Y \geq 3) &= 1 - P(Y \leq 2) \\ &\approx 1 - 0.6767 \qquad \text{(Poisson Tables)} \\ &= 0.3233 \end{aligned}$$

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Exercise E, Question 1

Question:

a State conditions under which the Poisson distribution is a suitable model to use in statistical work.

Flaws in a certain brand of tape occur at random and at a rate of 0.75 per 100 metres.
Assuming a Poisson distribution for the number of flaws in a 400 metre roll of tape,

b find the probability that there will be at least one flaw.

c Show that the probability that there will be at most 2 flaws is 0.423 (to 3 decimal places).

In a batch of 5 rolls, each of length 400 metres,

d find the probability that at least 2 rolls will contain fewer than 3 flaws.

Solution:

a If the outcomes occur:

1 singly

2 at a constant rate

3 independently and at random then a Poisson distribution can be suitable.

b Let $F =$ number of flows in 400 m of tape

$F \sim \text{P}_0(3)$

$$\begin{aligned} \text{P}(F \geq 1) &= 1 - \text{P}(F = 0) \quad \text{(tables)} \\ &= 1 - 0.0498 \\ &= 0.9502 \end{aligned}$$

c

$$\begin{aligned} \text{P}(F \leq 2) &= 0.4232 \quad \text{(tables)} \\ &= 0.423 \text{ (3 d.p.)} \end{aligned}$$

d Let $R =$ number of rolls (out of 5) with fewer than 3 flaws.

$\text{P(fewer than 3 flows)} = \text{P}(F < 3) = \text{P}(F \leq 2) = 0.423.$

$R \sim \text{B}(5, 0.423)$

$$\begin{aligned} \text{P}(R \geq 2) &= 1 - \text{P}(R \leq 1) \\ &= 1 - \left[5 \times (0.423)^1 \times (0.577)^4 + (0.577)^5\right] \\ &= 1 - 0.29838\ldots \\ &= 0.702 \end{aligned}$$

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Exercise E, Question 2

Question:

An archer fires arrows at a target and for each arrow, independently of all others, the probability that it hits the bull's eye is $\frac{1}{8}$.

a Given that the archer fires 5 arrows, find the probability that fewer than 2 arrows hit the bull's eye.

The archer fires 5 arrows, collects them and then fires all 5 again.

b Find the probability that on both occasions fewer than 2 hit the bull's eye.

The archer now fires 60 arrows at the target. Using a suitable approximation find

c the probability that fewer than 10 hit the bull's eye,

d the greatest value of m such that the probability that the archer hits the bull's eye with at least m arrows is greater than 0.5.

Solution:

$X =$ number of arrows that hit bull's eye

$X \sim \mathrm{B}(5, \frac{1}{8})$

a $\mathrm{P}(X < 2) = \mathrm{P}(X \le 1)$

$= 5 \times \left(\frac{1}{8}\right) \times \left(\frac{7}{8}\right)^4 + \left(\frac{7}{8}\right)^5$

$= 0.87927\ldots = 0.879$ (3 s.f.)

b $[\mathrm{P}(X < 2)]^2 = (0.87927)^2$

$= 0.773$ (3 s.f.)

$Y =$ number of arrows that hit bull's eye out of 60

c $Y \sim \mathrm{B}(60, \frac{1}{8})$

$Y \sim \mathrm{P_0}(7.5)$

$\mathrm{P}(Y < 10) = \mathrm{P}(Y \le 9)$ (Poisson tables)

≈ 0.7764

d Require $\mathrm{P}(Y \ge m) > 0.5$

i.e. $\mathrm{P}(Y \le m - 1) < 0.5$

$\mathrm{P}(Y \le 6) = 0.3782 < 0.5$

$\mathrm{P}(Y \le 7) = 0.5246 > 0.5$ (Using Poisson tables)

$\therefore m - 1 = 6$

so $m = 7$

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Exercise E, Question 3

Question:

In Joe's roadside café $\frac{2}{5}$ of the customers buy a cup of tea.

a Find the probability that at least 4 of the next 10 customers will buy a cup of tea.

Joe has calculated that, on a typical morning, customers arrive in the café at a rate of 0.5 per minute.

b Find the probability that at least 10 customers arrive in the next 15 minutes.

c Find the probability that exactly 10 customers arrive in the next 20 minutes.

d Find the probability that in the next 20 minutes exactly 10 customers arrive and at least 4 of them buy a cup of tea.

Solution:

X = number of customers out of 10 who buy a cup of tea

$X \sim \mathrm{B}(10, 0.4)$

a
$$\begin{aligned} \mathrm{P}(X \geq 4) &= 1 - \mathrm{P}(X \leq 3) \\ &= 1 - 0.3823 \quad \text{(binomial tables)} \\ &= 0.6177 \end{aligned}$$

C = number of customers who arrive in next 15 minutes

$C \sim \mathrm{P_o}(7.5)$ ← $\lambda = 7.5 = 0.5 \times 15$

b
$$\begin{aligned} \mathrm{P}(C \geq 10) &= 1 - \mathrm{P}(C \leq 9) \\ &= 1 - 0.7764 \\ &= 0.2236 \end{aligned}$$

T = number of customers who arrive in next 20 minutes

$T \sim \mathrm{P_o}(10)$ ← $\lambda = 20 \times 0.5 = 10$

c
$$\begin{aligned} \mathrm{P}(T = 10) &= \mathrm{P}(T \leq 10) - \mathrm{P}(T \leq 9) \\ &= 0.5830 - 0.4579 \\ &= 0.1251 \end{aligned}$$

d
$$\begin{aligned} \mathrm{P}(T = 10) \times \mathrm{P}(X \geq 4) &= 0.1251 \times 0.6177 \\ &= 0.07727\ldots \\ &= 0.0773 \end{aligned}$$

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Exercise E, Question 4

Question:

The number, X, of breakdowns per week of the lifts in a large block of flats has a Poisson distribution with mean 0.25. Find, to 3 decimal places, the probability that in a particular week

a there will be at least one breakdown,

b there will be at most 2 breakdowns.

c Show that the probability that during a 12-week period there will be no lift breakdowns is 0.050 (to 3 decimal places).

The residents in the flats have a maintenance contract with *Liftserve*. The contract is for a set of 20, 12-week periods. For every 12-week period with no breakdowns the residents pay *Liftserve* £500. If there is at least 1 breakdown in a 12-week period then *Liftserve* will mend the lift free of charge and the residents pay nothing for that period of 12 weeks.

d Find the probability that over the course of the contract the residents pay no more than £1000.

Solution:

$X \sim \text{P}_0(0.25)$

a
$$\begin{aligned} \text{P}(X \geq 1) &= 1 - \text{P}(X = 0) \\ &= 1 - e^{-0.25} \\ &= 0.221199\ldots \quad = 0.221 \text{ (3 d.p.)} \end{aligned}$$

b
$$\begin{aligned} \text{P}(X \leq 2) &= \text{P}(X = 0) + \text{P}(X = 1) + \text{P}(X = 2) \\ &= e^{-0.25}\left[1 + 0.25 + \frac{0.25^2}{2!}\right] \\ &= 0.99783\ldots \quad = 0.998 \text{ (3 d.p.)} \end{aligned}$$

c $T =$ number of breakdowns in 12 weeks

$T \sim \text{P}_0(3)$ ← $\lambda = 3 = 12 \times 0.25$

$$\begin{aligned} \therefore \text{P}(T = 0) &= e^{-3} = 0.04978\ldots \\ &= 0.050 \text{ (3 d.p.)} \end{aligned}$$

d $Y =$ number of 12-week periods with no breakdowns out of 20

$Y \sim \text{B}(20, 0.050)$

Residents pay $\leq$ £1000 if $Y \leq 2$

$$\begin{aligned} \text{P}(Y \leq 2) &= 0.9245 \\ &= 0.925 \quad \text{(3 d.p.)} \end{aligned}$$

(binomial tables)

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Exercise E, Question 5

Question:

Accidents occur in a school playground at the rate of 3 per year.

a Suggest a suitable model for the number of accidents in the playground next month.

b Using this model calculate the probability of 1 or more accidents in the playground next month.

Solution:

a $X =$ number of accidents in a month

$\lambda = 3 \times \frac{1}{12} = 0.25$

$X \sim \text{P}_0(0.25)$

b
$$\begin{aligned} \text{P}(X \geq 1) &= 1 - \text{P}(X = 0) \\ &= 1 - e^{-0.25} \\ &= 0.221 \ (3 \text{ s.f.}) \end{aligned}$$

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Exercise F, Question 1

Question:

During working hours an office switchboard receives telephone calls at random and at a rate of one call every 40 seconds.

a Find, to 3 decimal places, the probability that during a given one-minute period

i no call is received,

ii at least 2 calls are received.

b Find, to 3 decimal places, the probability that no call is received between 10:30 a.m. and 10:31 a.m. and that at least two calls are received between 10:31 a.m. and 10:32 a.m. ***E***

Solution:

X = number of calls received during one minute

$X \sim \mathrm{P_0}(1.5)$ since $\frac{1}{\left(\frac{2}{3}\right)} 1.5$

i

$$\begin{aligned} \mathrm{P}(X=0) &= 0.2231 \quad \text{(tables)} \\ &= 0.223 \text{ (3 d.p.)} \end{aligned}$$

ii

$$\begin{aligned} \mathrm{P}(X \geq 2) &= 1-\mathrm{P}(X \leq 1) \\ &= 1-0.5578 \quad \text{(tables)} \\ &= 0.4422 \\ &= 0.442 \text{ (3 d.p.)} \end{aligned}$$

b

$$\begin{aligned} \mathrm{P}(X=0) &\times \mathrm{P}(X \geq 2) \\ &= 0.2231 \times 0.4422 \\ &= 0.09865\ldots \\ &= 0.0987 \text{ (3 s.f.)} \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Exercise F, Question 2

Question:

State conditions under which the Poisson distribution is a suitable model to use in statistical work.

The number of typing errors per 1000 words made by a typist has a Poisson distribution with mean 2.5.

a Find, to 3 decimal places, the probability that in an essay of 4000 words there will be at least 12 typing errors.

The typist types 3 essays, each of length 4000 words.

b Find the probability that each contains at least 12 typing errors. ***E***

Solution:

Items occur in continuous space or time:

1 singly

2 at a constant rate

3 independently of one another and at random.

a X = number of errors in 1,000 words

$X \sim \mathrm{P_o}(10)$ ← $\lambda = 4 \times 2.5$

$$\begin{aligned} \mathrm{P}(X \geq 12) &= 1 - \mathrm{P}(X \leq 11) \\ &= 1 - 0.6968 \\ &= 0.3032 \end{aligned}$$

b Y = number of 4000 word essays with at least 12 errors

$Y \sim \mathrm{B}(3, 0.3032)$

$$\begin{aligned} \mathrm{P}(Y = 3) &= (0.3032)^3 \\ &= 0.027873\ldots \\ &= 0.0279 \text{ (3 s.f.)} \end{aligned}$$

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Exercise F, Question 3

Question:

a State conditions under which the binomial distribution $B(n, p)$ may be approximated by a Poisson distribution and write down the mean of this Poisson distribution.

Samples of blood were taken from 250 children in a region of India. Of these children, 4 had blood type $A2B$.

b Write down an estimate of p, the proportion of children in this region having blood type $A2B$.

Consider a group of n children from this region and let X be the number having blood type $A2B$. Assuming that X is distributed $B(n, p)$ and that p has the value estimated above, calculate, to 3 decimal places, the probability that the number of children with blood type $A2B$ in a group of 6 children from this region will be

i zero,

ii more than 1.

c Use a Poisson approximation to calculate, to 4 decimal places, the probability that, in a group of 800 children from this region, there will be fewer than 3 children of blood type $A2B$.

Solution:

a $B(n, p)$ can be approximated to $P_0(np)$

If n is large and p is small

Then mean $= np$

b $\hat{p} = \frac{4}{250} = 0.016$

$X \sim B(6, 0.016)$

i

$$P(X = 0) = (0.984)^6 = 0.907759\ldots = 0.908 \text{ (3 d.p.)}$$

ii

$$\begin{aligned} P(X > 1) &= 1 - P(X \le 1) \\ &= 1 - \left[6 \times 0.016 \times (0.984)^5 + (0.984)^6\right] \\ &= 0.003679\ldots \\ &= 0.00370 \text{ (3 s.f.)} \end{aligned}$$

c $Y =$ number of children out of 800 with the blood group $A2B$

$Y \sim B(800, 0.016)$

$Y \approx\sim P_0(12.8)$

$$\begin{aligned} P(Y < 3) &= P(Y \le 2) \\ &= e^{-12.8}\left[1 + 12.8 + \frac{12.8^2}{2!}\right] \\ &= 0.00026426\ldots \\ &= 0.000264 \text{ (3 s.f.)} \end{aligned}$$

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Exercise F, Question 4

Question:

Which of the following variables is best modelled by a Poisson distribution and which is best modelled by a binomial distribution?

a The number of hits by an arrow on a target, when 20 arrows are fired.

b The number of earth tremors that take place in a village over a given period of time.

c The number of particles emitted per minute by a radioactive isotope.

d The number of heads you get when tossing 2 coins 100 times.

e The number of accidents in a city in a year.

f The number of flying bomb hits in specified areas of London during World War 2.

Solution:

a Binomial — a fixed number of arrows $(n = 20)$
need to assume that $p =$ probability of an arrow hitting is constant

b Poisson — no fixed number of trials
need to assume earth tremors occur at random with a constant rate

c Poisson — no fixed number of particles
need to assume particles emitted at a constant average rate

d Binomial — $n = 200$ the number of tosses of coins $p = \frac{1}{2}$

e Poisson — no fixed number
need to assume accidents occur at a constant rate

f Poisson — no fixed number
need to assume flying bomb hits occur at a constant rate

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Exercise F, Question 5

Question:

Loaves of bread on a production line pass a monitoring point at a constant rate of 300 loaves per hour.

a Find how many loaves you would expect to pass the monitoring point in 2 minutes.

b Find the probability that no loaves pass the monitoring point in a given 1-minute period.

Solution:

$X =$ number of loaves passing in 2 minutes

a $X = \frac{300}{60} \times 2 = 10 = \mathrm{E}(X)$

b $Y =$ number of loaves passing in 1 minute

$Y \sim \mathrm{Po}(5)$

$\mathrm{P}(Y = 0) = \mathrm{e}^{-5} = 0.0067$ (tables)

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Exercise F, Question 6

Question:

Accidents occur at a certain road junction at a rate of 3 per year.

a Suggest a suitable model for the number of accidents at this road junction in the next month.

b Show that, under this model, the probability of 2 or more accidents at this road junction in the next month is 0.0265 to 4 decimal places.

The local residents have applied for a crossing to be installed.

The planning committee agree to monitor the situation for the next 12 months. If there is at least one month with 2 or more accidents in it they will install a crossing.

c Find the probability that the crossing is installed. ***E***

Solution:

a $\lambda = \frac{3}{12} = 0.25$

X = number of accidents in a month

$X \sim \mathrm{P_o}(0.25)$

b
$$\begin{aligned} \mathrm{P}(X \geq 2) &= 1 - \mathrm{P}(X \leq 1) \\ &= 1 - [\mathrm{P}(X = 1) + \mathrm{P}(X = 0)] \\ &= 1 - e^{-0.25}[0.25 + 1] \\ &= 1 - 0.97350\ldots \\ &= 0.026499\ldots \\ &= 0.0265 \end{aligned}$$

Y = number of months with 2 or more accidents

$Y \sim \mathrm{B}(12, 0.0265)$

c
$$\begin{aligned} \mathrm{P}(Y \geq 1) &= 1 - \mathrm{P}(Y = 0) \\ &= 1 - (0.9735)^{12} \\ &= 1 - 0.724488\ldots \\ &= 0.275511\ldots \\ &= 0.276 \text{ (3 s.f.)} \end{aligned}$$

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Exercise F, Question 7

Question:

Breakdowns occur on a particular machine at a rate of 2.5 per month. Assuming that the number of breakdowns can be modelled by a Poisson distribution, find the probability that

a exactly 3 occur in a particular month,

b more than 10 occur in a three-month period,

c exactly 3 occur in each of 2 successive months. ***E***

Solution:

$X =$ number of breakdowns per month

$X \sim P_0(2.5)$

a $P(X=3) = P(X \le 3) - P(X \le 2)$ (tables)

$= 0.7576 - 0.5438$

$= 0.2138$

$Y =$ number of breakdowns in 3 months

$Y \sim P_0(7.5)$

b $P(Y > 10) = 1 - P(Y \le 10)$ (tables)

$= 1 - 0.8622$

$= 0.1378$

c $P(X=3) \times P(X=3)$

$= (0.2138)^2$

$= 0.04571$

$= 0.0457$

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Exercise F, Question 8

Question:

A geography student is studying the distribution of telephone boxes in a large rural area where there is an average of 300 boxes per 500 km^2. A map of part of the area is divided into 50 squares, each of area 1 km^2 and the student wishes to model the number of telephone boxes per square.

a Suggest a suitable model the student could use and specify any parameters required.

One of the squares is picked at random.

b Find the probability that this square does not contain any telephone boxes.

c Find the probability that this square contains at least 3 telephone boxes.

The student suggests using this model on another map of a large city and surrounding villages.

d Comment, giving your reason briefly, on the suitability of the model in this situation. ***E***

Solution:

X = number of telephone boxes per square

a $X \sim \text{P}_0(0.6)$ ← $\lambda = \frac{300}{500} = 0.6$

b $\text{P}(X = 0) = e^{-0.6} = 0.5488\ldots$

$= 0.549 \quad (3\text{ s.f.})$

c $\text{P}(X \geq 3) = 1 - \text{P}(X \leq 2)$

$= 1 - e^{-0.6}\left[1 + 0.6 + \frac{0.6^2}{2}\right]$

$= 1 - 0.97688$

$= 0.02312$

$= 0.0231 \quad (3\text{ s.f.})$

d Not suitable

The rate of telephone boxes will be different in cities and they are more likely to occur in clusters.

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Exercise F, Question 9

Question:

All the letters in a particular office are typed either by Pat, a trainee typist, or by Lyn, who is a fully-trained typist. The probability that a letter typed by Pat will contain one or more errors is 0.3.

a Find the probability that a random sample of 4 letters typed by Pat will include exactly one letter free from error.

The probability that a letter typed by Lyn will contain one or more errors is 0.05.

b Use tables, or otherwise, to find, to 3 decimal places, the probability that in a random sample of 20 letters typed by Lyn, not more than 2 letters will contain one or more errors.

On any one day, 6% of the letters typed in the office are typed by Pat. One letter is chosen at random from those typed on that day.

c Show that the probability that it will contain one or more errors is 0.065.

Given that each of 2 letters chosen at random from the day's typing contains one or more errors,

d find, to 4 decimal places, the probability that one was typed by Pat and the other by Lyn. ***E***

Solution:

$X =$ number of letters out of 4 with at least one error

$X \sim B(4, 0.3)$

a $P(X = 3) = 4 \times 0.3^3 \times 0.7^1$

$= 0.0756$

$L =$ number of letters out of 20 containing one or more errors

$L \sim B(20, 0.05)$

b $P(L \leq 2) = 0.9245$ (tables)

$= 0.925$ (3 d.p.)

c

Pat Lyn

$P(\text{letter has errors}) = 0.06 \times 0.3 + 0.94 \times 0.05$

$= 0.065$

d

Pat Lyn

$$\text{Probability} = \frac{2 \times (0.06 \times 0.3) \times (0.94 \times 0.05)}{(0.065)^2}$$

$= 0.4004733\ldots$

$= 0.4005$ (4 d.p.)

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Exercise F, Question 10

Question:

The number of breakdowns per day of the lifts in a large block of flats is modelled by a Poisson distribution with mean 0.2.

a Find, to 3 decimal places, the probability that on a particular day there will be at least one breakdown.

b Find the probability that there are fewer than 2 days in a 30-day month with at least one breakdown.

Solution:

$X =$ number of breakdowns per day

$X \sim \mathrm{P_0}(0.2)$

a
$$\begin{aligned} \mathrm{P}(X \geq 1) &= 1 - \mathrm{P}(X = 0) \\ &= 1 - \mathrm{e}^{-0.2} \\ &= 1 - 0.8187\ldots \\ &= 0.181 (3\text{ d.p.}) \end{aligned}$$

$Y =$ number of days in 30 day months with at least one breakdown

$Y \sim \mathrm{B}(30, 0.181)$

b
$$\begin{aligned} \mathrm{P}(Y < 2) &= \mathrm{P}(Y \leq 1) \\ &= (0.819)^{30} + 30(0.819)^{29} \times (0.181) \\ &= 0.0191 \end{aligned}$$