# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 1**

**Question:**

A large bag contains counters of different colours.
Find the number of arrangements for the following selections

**a** 5 counters all of different colours,
**b** 5 counters where 3 are red and 2 are blue,
**c** 7 counters where 2 are red and 5 are green,
**d** 10 counters where 4 are blue and 6 are yellow,
**e** 20 counters where 2 are yellow and 18 are black.

**Solution:**

**a** $5! = 120$

**b** $\frac{5!}{3!2!} = 10$

**c** $\frac{7!}{5!2!} = \frac{7 \times 6}{2} = 21$

**d** $\frac{10!}{4!6!} = \frac{10 \times \cancel{9}^{3} \times \cancel{8} \times 7}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 210$

**e** $\frac{20!}{18!2!} = \frac{20 \times 19}{2} = 190$

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**Exercise A, Question 2**

**Question:**

A bag contains 4 red, 3 green and 8 yellow beads.
Five beads are selected at random from the bag without replacement.
Find the probability that they are

**a** 5 yellow beads,
**b** 2 red and 3 yellow,
**c** 4 red and 1 green.

**Solution:**

**a**

$$\frac{8}{15}\times\frac{7}{14}\times\frac{6}{13}\times\frac{5}{12}\times\frac{4}{11}$$

$$= \frac{8}{429} \text{ or } 0.0186$$

**b**

$$\frac{4}{15}\times\frac{3}{14}\times\frac{8}{13}\times\frac{7}{12}\times\frac{6}{11}\times\frac{5!}{2!3!}$$

$$= \frac{16}{143} \quad \text{or} \quad 0.112$$

**c**

$$\frac{4}{15}\times\frac{3}{14}\times\frac{2}{13}\times\frac{1}{12}\times\frac{3}{11}\times\frac{5!}{4!1!}$$

$$= 0.000999 \quad \text{or} \quad \frac{1}{1001}$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 3**

**Question:**

A fair die is rolled 7 times.
Find the probability of getting

**a** no fives,

**b** only 3 fives,

**c** 4 fives and 3 sixes.

**Solution:**

**a** $\left(\frac{5}{6}\right)^7 = 0.279$

**b**

$$\left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)^4 \times \frac{7!}{4!3!}$$
$$= 0.0781$$

**c**

$$\left(\frac{1}{6}\right)^4 \times \left(\frac{1}{6}\right)^3 \times \frac{7!}{4!3!}$$
$$= 0.000125$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 1**

**Question:**

The random variable $X \sim B(8, \frac{1}{3})$. Find

**a** $P(X = 2)$,

**b** $P(X = 5)$,

**c** $P(X \leq 1)$.

**Solution:**

**a**

$$\begin{aligned} P(X = 2) &= \binom{8}{2} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^6 \\ &= 0.273 \end{aligned}$$

**b**

$$P(X = 5) = \binom{8}{5} \times \left(\frac{1}{3}\right)^5 \times \left(\frac{2}{3}\right)^3 = 0.0683$$

**c**

$$\begin{aligned} P(X \leq 1) &= P(X = 1) + P(X = 0) \\ &= 8\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^7 + \left(\frac{2}{3}\right)^8 \\ &= \left(\frac{2}{3}\right)^7 \left[\frac{8}{3} + \frac{2}{3}\right] \\ &= \left(\frac{2}{3}\right)^7 \times \frac{10}{3} \\ &= 0.195 \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 2**

**Question:**

The random variable $Y \sim B(6, \frac{1}{4})$. Find

**a** $P(Y = 3)$,

**b** $P(Y = 1)$,

**c** $P(Y \geq 5)$.

**Solution:**

$Y \sim B\left(6, \frac{1}{4}\right)$

**a**

$$\begin{aligned} P(Y=3) &= \binom{6}{3} \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^3 \\ &= 0.132 \end{aligned}$$

**b**

$$\begin{aligned} P(Y=1) &= \binom{6}{1} \times \left(\frac{1}{4}\right)^1 \times \left(\frac{3}{4}\right)^5 \\ &= 0.356 \end{aligned}$$

**c**

$$\begin{aligned} P(Y \geq 5) &= P(Y=5) + P(Y=6) \\ &= 6 \times \left(\frac{1}{4}\right)^5 \times \left(\frac{3}{4}\right) + \left(\frac{1}{4}\right)^6 \\ &= \left(\frac{1}{4}\right)^5 \left[\frac{18}{4} + \frac{1}{4}\right] \\ &= \left(\frac{1}{4}\right)^5 \times \frac{19}{4} \\ &= 0.00464 \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 4**

**Question:**

A balloon manufacturer claims that 95% of his balloons will not burst when blown up. If you have 20 of these balloons to blow up for a birthday party

**a** what is the probability that none of them burst when blown up?

**b** Find the probability that exactly 2 balloons burst.

**Solution:**

**a**

$$\begin{aligned} P(X=0) &= (0.95)^{20} \\ &= 0.358 \end{aligned}$$

**b** Let $X$ = number of balloon that *do* burst

$X \sim B(20, 0.05)$

$$\begin{aligned} P(X=2) &= \binom{20}{2}(0.95)^{18}(0.05)^2 \\ &= 0.189 \end{aligned}$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 5**

**Question:**

A student suggests using a binomial distribution to model the following situations. Give a description of the random variable, state any assumptions that must be made and give possible values for $n$ and $p$.

**a** A sample of 20 bolts is checked for defects from a large batch. The production process should produce 1% of defective bolts.

**b** Some traffic lights have three phases: stop 48% of the time, wait or get ready 4% of the time and go 48% of the time. Assuming that you only cross a traffic light when it is in the go position, model the number of times that you have to wait or stop on a journey passing through 6 sets of traffic lights.

**c** When Stephanie plays tennis with Timothy on average one in eight of her serves is an 'ace'. How many 'aces' does Stephanie serve in the next 30 serves against Timothy?

**Solution:**

**a** $X =$ number of defective bolts in a sample of 20

$X \sim \mathrm{B}(20, 0.01)$

$n = 20$
$p = 0.01$

Assume bolts being defective are independent of each other

**b** $X =$ number of times wait or stop in 6 lights

$X \sim \mathrm{B}(6, 0.52)$

$n = 6$
$p = 0.52.$

Assume the lights operate independently and the time lights are on/off is constant.

**c** $X =$ no. of aces Stephanie serves in the next 30

$X \sim \mathrm{B}(30, \frac{1}{8})$

$n = 30$
$p = \frac{1}{8}$

Assume serves being an ace are independent and probability of an ace is constant.

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**Exercise B, Question 6**

**Question:**

State which of the following can be modelled with a binomial distribution and which can not. Give reasons for your answers.

**a** Given that 15% of people have blood that is Rhesus negative $(\text{Rh}^-)$, model the number of pupils in a statistics class of 14 who are $\text{Rh}^-$.

**b** You are given a fair coin and told to keep tossing it until you obtain 4 heads in succession. Model the number of tosses you need.

**c** A certain car manufacturer produces 12% of new cars in the colour red, 8% in blue, 15% in white and the rest in other colours. You make a note of the colour of the first 15 new cars of this make. Model the number of red cars you observe.

**Solution:**

**a** $X =$ number of people in class of 14 who are $\text{Rh}^-$
$X \sim \text{B}(14, 0.15)$ is OK if we assume the children in the class being $\text{Rh}^-$ is independent from child to child (so no siblings/twins)

**b** This is not binomial since the number of tosses is not fixed.
The probability of a head at each toss is constant $(p = 0.5)$ but there is no value of $n$.

**c** Assuming the colours of the cars are independent (which should be reasonable)
$X =$ number of red cars out of 15
$X \sim \text{B}(15, 0.12)$

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## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 7**

**Question:**

A fair die is rolled repeatedly. Find the probability that

**a** the first 6 occurs on the fourth roll,

**b** there are 3 sixes in the first 10 rolls.

**Solution:**

**a** Sequence must be $\bar{6}\,\bar{6}\,\bar{6}\,6$

$$\text{Probability} = \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$$
$$= \frac{125}{1296} \quad \text{or} \quad 0.0965$$

**b** $X =$ number of sixes in 10 rolls

$X \sim \mathrm{B}(10, \frac{1}{6})$

$$\mathrm{P}(X=3) = \binom{10}{3}\times\left(\frac{1}{6}\right)^3\times\left(\frac{5}{6}\right)^7$$
$$= 0.155$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 8**

**Question:**

A coin is biased so that the probability of it landing on heads is $\frac{2}{3}$. The coin is tossed repeatedly. Find the probability that

**a** the first tail will occur on the fifth toss,

**b** in the first 7 tosses there will be exactly 2 tails.

**Solution:**

**a** Sequence must be: H H H H T

So not Binomial.

Probability $= \frac{1}{3} \times \left(\frac{2}{3}\right)^4 = \frac{16}{243}$ or $0.0658$

**b** $X =$ number of tails in 7 tosses

$X \sim \mathrm{B}(7, \frac{1}{3})$

$$\mathrm{P}(X = 2) = \binom{7}{2} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^5$$

$$= 0.307 \quad \text{or} \quad \frac{224}{729}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 1**

**Question:**

The random variable $X \sim B(9, 0.2)$. Find

**a** $P(X \leq 4)$,

**b** $P(X < 3)$,

**c** $P(X \geq 2)$,

**d** $P(X = 1)$.

**Solution:**

$X \sim B(9, 0.2)$

**a** $P(X \leq 4) = 0.9804$ (tables)

**b** $P(X < 3) = P(X \leq 2)$ (tables)
$= 0.7382$

**c** $P(X \geq 2) = 1 - P(X \leq 1)$
$= 1 - 0.4362$ (tables)
$= 0.5638$

**d** $P(X = 1) = P(X \leq 1) - P(X = 0)$
$= 0.4362 - 0.1342$ (tables)
$= 0.3020$

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## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 2**

**Question:**

The random variable $X \sim B(20, 0.35)$. Find

**a** $P(X \leq 10)$,

**b** $P(X > 6)$,

**c** $P(X = 5)$,

**d** $P(2 \leq X \leq 7)$.

**Solution:**

$X \sim B(20, 0.35)$

**a** $P(X \leq 10) = 0.9468$ (tables)

**b**
$$\begin{aligned} P(X > 6) &= 1 - P(X \leq 6) \\ &= 1 - 0.4166 \quad \text{(tables)} \\ &= 0.5834 \end{aligned}$$

**c**
$$\begin{aligned} P(X = 5) &= P(X \leq 5) - P(X \leq 4) \\ &= 0.2454 - 0.1182 \\ &= 0.1272 \end{aligned}$$

**d**
$$\begin{aligned} P(2 \leq X \leq 7) &= P(X \leq 7) - P(X \leq 1) \\ &= 0.6010 - 0.0021 \\ &= 0.5989 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 3**

**Question:**

The random variable $X \sim B(40, 0.45)$. Find

**a** $P(X < 20)$,

**b** $P(X > 16)$,

**c** $P(11 \leq X \leq 15)$,

**d** $P(10 < X < 17)$.

**Solution:**

$X \sim B(40, 0.45)$

**a**
$$\begin{aligned} P(X < 20) &= P(X \leq 19) \\ &= 0.6844 \end{aligned} \qquad \text{(tables)}$$

**b**
$$\begin{aligned} P(X > 16) &= 1 - P(X \leq 16) \\ &= 1 - 0.3185 \qquad \text{(tables)} \\ &= 0.6815 \end{aligned}$$

**c**
$$\begin{aligned} P(11 \leq X \leq 15) &= P(X \leq 15) - P(X \leq 10) \\ &= 0.2142 - 0.0074 \\ &= 0.2068 \end{aligned}$$

**d**
$$\begin{aligned} P(10 < X < 17) &= P(X \leq 16) - P(X \leq 10) \\ &= 0.3185 - 0.0074 \\ &= 0.3111 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 4**

**Question:**

The random variable $X \sim B(30, 0.15)$. Find

**a** $P(X > 8)$,

**b** $P(X \leq 4)$,

**c** $P(2 \leq X < 10)$,

**d** $P(X = 4)$.

**Solution:**

$X \sim B(30, 0.15)$

**a**
$$\begin{aligned} P(X > 8) &= 1 - P(X \leq 8) \\ &= 1 - 0.9722 \qquad \text{(tables)} \\ &= 0.0278 \end{aligned}$$

**b** $P(X \leq 4) = 0.5245$ (tables)

**c**
$$\begin{aligned} P(2 \leq X < 10) &= P(X \leq 9) - P(X \leq 1) \\ &= 0.9903 - 0.0480 \qquad \text{(tables)} \\ &= 0.9423 \end{aligned}$$

**d**
$$\begin{aligned} P(X = 4) &= P(X \leq 4) - P(X \leq 3) \\ &= 0.5245 - 0.3217 \qquad \text{(tables)} \\ &= 0.2028 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 5**

**Question:**

Eight fair coins are tossed and the total number of heads showing is recorded. Find the probability of

**a** no heads,

**b** at least 2 heads,

**c** more heads than tails.

**Solution:**

$X =$ number of heads (coins are fair so $p = 0.5$)

$X \sim \mathrm{B}(8, 0.5)$

**a** $\mathrm{P}(X = 0) = (0.5)^8 = 0.0039$ (tables)

**b**
$$\begin{aligned} \mathrm{P}(X \geq 2) &= 1 - \mathrm{P}(X \leq 1) \\ &= 1 - 0.0352 \quad \text{(tables)} \\ &= 0.9648 \end{aligned}$$

**c**
$$\begin{aligned} \mathrm{P}(X \geq 5) &= 1 - \mathrm{P}(X \leq 4) \\ &= 1 - 0.6367 \quad \text{(tables)} \\ &= 0.3633 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 6**

**Question:**

For a particular type of plant 25% have blue flowers. A garden centre sells these plants in trays of 15 plants of mixed colours. A tray is selected at random.
Find the probability that the number of blue flowers this tray contains is

**a** exactly 4,

**b** at most 3,

**c** between 3 and 6 (inclusive).

**Solution:**

$X$ = number of plants with blue flowers on tray of 15
$X \sim \mathrm{B}(15, 0.25)$

**a**
$$\begin{aligned} \mathrm{P}(X=4) &= \mathrm{P}(X \le 4) - \mathrm{P}(X \le 3) \\ &= 0.6865 - 0.4613 \qquad \text{(tables)} \\ &= 0.2252 \end{aligned}$$

**b** $\mathrm{P}(X \le 3) = 0.4613$ (tables)

**c**
$$\begin{aligned} \mathrm{P}(3 \le X \le 6) &= \mathrm{P}(X \le 6) - \mathrm{P}(X \le 2) \\ &= 0.9434 - 0.2361 \qquad \text{(tables)} \\ &= 0.7073 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 7**

**Question:**

The random variable $X \sim B(50, 0.40)$. Find

**a** the largest value of $k$ such that $P(X \leq k) < 0.05$,

**b** the smallest number $r$ such that $P(X > r) < 0.01$.

**Solution:**

$X \sim B(50, 0.40)$

**a**
$$\begin{aligned} P(X \leq 13) &= 0.0280 \\ P(X \leq 14) &= 0.0540 \\ \therefore k &= 13 \end{aligned} \qquad \text{(tables)}$$

**b**
$$\begin{aligned} P(X \leq 27) &= 0.9840 \Rightarrow P(X > 27) = 0.0160 > 0.01 \\ P(X \leq 28) &= 0.9924 \Rightarrow P(X > 28) = 0.0076 < 0.01 \\ \therefore r &= 28 \end{aligned}$$

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 8**

**Question:**

The random variable $X \sim \mathrm{B}(40, 0.10)$. Find

**a** the largest value of $k$ such that $\mathrm{P}(X < k) < 0.02$,

**b** the smallest number $r$ such that $\mathrm{P}(X > r) < 0.01$,

**c** $\mathrm{P}(k \le X \le r)$.

**Solution:**

$X \sim \mathrm{B}(40, 0.10)$

**a**
$$\begin{aligned} \mathrm{P}(X = 0) &= 0.0148 < 0.02 \\ \mathrm{P}(X \le 1) &= 0.0805 > 0.02 \qquad \text{(tables)} \\ \therefore \mathrm{P}(X < 1) &= 0.0148 < 0.02 \text{ and so } k = 1 \end{aligned}$$

**b**
$$\begin{aligned} \mathrm{P}(X \le 8) &= 0.9845 \Rightarrow \mathrm{P}(X > 8) = 0.0155 > 0.01 \quad \text{(tables)} \\ \mathrm{P}(X \le 9) &= 0.9949 \Rightarrow \mathrm{P}(X > 9) = 0.0051 < 0.01 \\ r &= 9 \end{aligned}$$

**c**
$$\begin{aligned} \mathrm{P}(k \le X \le r) &= \mathrm{P}(X \le r) - \mathrm{P}(X \le k-1) \\ &= \mathrm{P}(X \le 9) - \mathrm{P}(X = 0) \\ &= 0.9949 - 0.0148 \\ &= 0.9801 \end{aligned}$$

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 9**

**Question:**

In a town, 30% of residents listen to the local radio.
Ten residents are chosen at random.

**a** State the distribution of the random variable
$X =$ the number of these 10 residents that listen to the local radio.

**b** Find the probability that at least half of these 10 residents listen to local radio.

**c** Find the smallest value of $s$ so that $P(X \geq s) < 0.01$.

**Solution:**

$X =$ number of residents out of 10 who listen to local radio

**a** $X \sim B(10, 0.30)$

**b**
$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - 0.8497 \qquad \text{(tables)} \\ &= 0.1503 \end{aligned}$$

**c**
$$\begin{aligned} P(X \leq 6) &= 0.9894 \text{ so } P(X \geq 7) = 1 - 0.9894 = 0.0106 > 0.01 \\ P(X \leq 7) &= 0.9984 \\ P(X \geq 8) &= 1 - 0.9984 \qquad \text{(tables)} \\ &= 0.0016 \quad < 0.01 \\ s &= 8 \end{aligned}$$

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 10**

**Question:**

A factory produces a component for the motor trade and 5% of the components are defective. A quality control officer regularly inspects a random sample of 50 components. Find the probability that the next sample contains

**a** fewer than 2 defectives,

**b** more than 5 defectives.

The officer will stop production if the number of defectives in the sample is greater than a certain value $d$. Given that the officer stops production less than 5% of the time,

**c** find the smallest value of $d$.

**Solution:**

$X =$ number of defects in 50 components
$X \sim \mathrm{B}(50, 0.05)$

**a** $\mathrm{P}(X < 2) = \mathrm{P}(X \leq 1)$ (tables)
$= 0.2794$

**b** $\mathrm{P}(X > 5) = 1 - \mathrm{P}(X \leq 5)$
$= 1 - 0.9622$
$= 0.0378$

**c** Seek $d$ such that

$\mathrm{P}(X > d) < 0.05$
$\mathrm{P}(X \leq 4) = 0.8964 \Rightarrow \mathrm{P}(X > 4) = 0.1036 > 0.05$
$\mathrm{P}(X \leq 5) = 0.9622 \Rightarrow \mathrm{P}(X > 5) = 0.0378 < 0.05$
$\therefore d = 5$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 1**

**Question:**

A fair cubical die is rolled 36 times and the random variable $X$ represents the number of sixes obtained. Find the mean and variance of $X$.

**Solution:**

$X \sim \mathrm{B}(36, \frac{1}{6})$ fair die $\Rightarrow p = \frac{1}{6}$

$$\mathrm{E}(X) = 36 \times \frac{1}{6} = 6$$

$$\mathrm{Var}(X) = 36 \times \frac{1}{6} \times \frac{5}{6} = \frac{30}{6} = 5$$

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## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 2**

**Question:**

**a** Find the mean and variance of the random variable $X \sim B(12, 0.25)$.

**b** Find $P(\mu - \sigma < X < \mu + \sigma)$.

**Solution:**

**a** $X \sim B(12, 0.25)$

$$\begin{aligned} E(X) &= 12 \times 0.25 = 3 \\ Var(X) &= 12 \times 0.25 \times 0.75 = 3 \times \frac{3}{4} = \frac{9}{4} \text{ or } 2.25 \end{aligned}$$

**b** $\sigma^2 = \frac{9}{4} \Rightarrow \sigma = \frac{3}{2}$ or 1.5

$$\begin{aligned} & P(\mu - \sigma < X < \mu + \sigma) \\ = \; & P\left(3 - \frac{3}{2} < X < 3 + \frac{3}{2}\right) \\ = \; & P\left(\frac{3}{2} < X < 4\frac{1}{2}\right) \\ = \; & P(X \leq 4) - P(X \leq 1) \\ = \; & 0.8424 - 0.1584 \qquad \text{(tables)} \\ = \; & 0.6840 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 3**

**Question:**

**a** Find the mean and variance of the random variable $X \sim \mathrm{B}(30, 0.40)$.

**b** Find $\mathrm{P}(\mu - \sigma < X \leq \mu)$.

**Solution:**

$X \sim \mathrm{B}(30, 0.40)$

**a**
$$\begin{aligned} \mathrm{E}(X) &= 30 \times 0.40 = 12 \\ \mathrm{Var}(X) &= 30 \times 0.40 \times 0.6 = 12 \times 0.6 = 7.2 \end{aligned}$$

**b** $\sigma^2 = 7.2 \Rightarrow \sigma = 2.68\ldots$

$$\begin{aligned} &\mathrm{P}(12 - 2.68\ldots < X \leq 12) \\ &= \mathrm{P}(9.32\ldots < X \leq 12) \\ &= \mathrm{P}(X \leq 12) - \mathrm{P}(X \leq 9) \\ &= 0.5785 - 0.1763 \qquad \text{(tables)} \\ &= 0.4022 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 4**

**Question:**

It is estimated that 1 in 20 people are left-handed.

**a** What size sample should be taken to ensure that the expected number of left-handed people in the sample is 3?

**b** What is the standard deviation of the number of left-handed people in this case?

**Solution:**

$X$ = number of left-handed people in a sample of size $n$

$X \sim \mathrm{B}(n, \frac{1}{20})$

**a**

$$\begin{aligned} \mathrm{E}(X) = 3 \Rightarrow \frac{n}{20} &= 3 \\ \therefore n &= 60 \end{aligned}$$

**b**

$$\begin{aligned} \mathrm{Var}(X) &= 60 \times \frac{1}{20} \times \frac{19}{20} = 2.85 \\ \therefore \sigma^2 &= 2.85 \\ \sigma &= \sqrt{2.85} = 1.69 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 5**

**Question:**

An experiment is conducted with a fair die to examine the number of sixes that occur. It is required to have the standard deviation smaller than 1. What is the largest number of throws that can be made?

**Solution:**

$X =$ number of sixes in $n$ throws

$X \sim \mathrm{B}(n, \frac{1}{6})$

$\mathrm{Var}(X) = n \times \frac{1}{6} \times \frac{5}{6} = \frac{5n}{36}$

standard deviation $< 1$

$$\Rightarrow \frac{\sqrt{5n}}{6} < 1$$
$$\sqrt{5n} < 6$$
$$5n < 36$$
$$n < \frac{36}{5} = 7.2$$

$\therefore$ need $n = 7$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 6**

**Question:**

The random variable $X \sim \mathrm{B}(n, p)$ has a mean of 45 and standard deviation of 6. Find the value of $n$ and the value of $p$.

**Solution:**

$X \sim \mathrm{B}(n, p)$

$$\begin{aligned}
\mathrm{E}(X) = 45 \Rightarrow np &= 45 \\
\sigma = 6 \Rightarrow \mathrm{Var}(X) = 36 \Rightarrow np(1-p) &= 36 \\
\therefore 45(1-p) &= 36 \\
\therefore 1-p &= \frac{36}{45} \\
p &= 1-\frac{36}{45} \\
p &= \frac{9}{45} = \frac{1}{5} \\
\text{and } n &= 225
\end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 1**

**Question:**

A coin is biased so that the probability of a head is $\frac{2}{3}$. The coin is tossed repeatedly.

Find the probability that

**a** the first tail will occur on the sixth toss,

**b** in the first 8 tosses there will be exactly 2 tails.

**Solution:**

**a** Sequence is: H H H H H T

probability : $\left(\frac{2}{3}\right)^5 \times \frac{1}{3} = 0.0439$ or $\frac{32}{729}$

**b** $X =$ number of tails in 8 tosses

$X \sim \text{B}(8, \frac{1}{3})$

$$\begin{aligned} \text{P}(X = 2) &= \binom{8}{2} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^6 \\ &= 0.273 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 2**

**Question:**

Records kept in a hospital show that 3 out of every 10 patients who visit the accident and emergency department have to wait more than half an hour. Find, to 3 decimal places, the probability that of the first 12 patients who come to the accident and emergency department

**a** none,

**b** more than 2,

will have to wait more than half an hour.

**Solution:**

$X =$ number of patients waiting more than $\frac{1}{2}$ hour.

$X \sim \mathrm{B}(12, 0.3)$

**a** $\mathrm{P}(X = 0) = (0.7)^{12} = 0.01384\ldots$

$= 0.0138 \quad (3\text{ s.f.})$

**b** $\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \leq 2)$

$= 1 - 0.2528 \quad \text{(tables)}$

$= 0.7472$

$= 0.747 \quad (3\text{ d.p.})$

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 3**

**Question:**

A factory is considering two methods of checking the quality of production of the batches of items it produces.

*Method I* A random sample of 10 items is taken from a large batch and the batch is accepted if there are no defectives in this sample. If there are 2 or more defectives the batch is rejected. If there is only 1 defective then another sample of 10 is taken and the batch is accepted if there are no defectives in this second sample, otherwise the whole batch is rejected.

*Method II* A random sample of 20 items is taken from a large batch and the batch is accepted if there is at most 1 defective in this sample, otherwise the whole batch is rejected.

The factory knows that 1% of items produced are defective and wishes to use the method of checking the quality of production for which the probability of accepting the whole batch is largest.

**a** Decide which method the factory should use.

**b** Determine the expected number of items sampled using Method I.

**Solution:**

**a** $X =$ number of defective items in sample of 10

**Method I**
$X \sim B(10, 0.01)$

$$\begin{aligned} P(\text{Accept}) &= P(X=0) + P(X=1) \times P(X=0) \\ &= P(X=0)[1 + P(X=1)] \\ &= (0.99)^{10}\left[1 + 10 \times 0.01 \times (0.99)^9\right] \\ &= 0.98699\ldots \quad = 0.987 \end{aligned}$$

or from tables

$$\begin{aligned} P(\text{Accept}) &= P(X=0) + P(X=1) \times P(X=0) \\ &= 0.9044 + [0.9957 - 0.9044] \times 0.9044 \\ &= 0.98697 \\ &= 0.987 \end{aligned}$$

**Method II** $Y =$ number of defective items in sample of 20
$Y \sim B(20, 0.01)$

$$\begin{aligned} P(\text{Accept}) &= P(Y \le 1) \\ &= (0.99)^{20} + 20 \times (0.01) \times (0.99)^{19} \text{ or find direct from tables} \\ &= 0.98314\ldots \quad = 0.983 \end{aligned}$$

$\therefore$ use Method I

**b** *Method I*

| Number sampled | 10 | 20 |
|---|---|---|
| probability | $1 - P(X=1)$ | $P(X=1)$ |

$$\begin{aligned} \therefore \text{expected number} &= 10(1 - P(X=1)) + 20P(X=1) \\ &= 10 + 10 \times P(X=1) \\ &= 10 + 0.9135\ldots \\ &= 10.9 \end{aligned}$$

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 4**

**Question:**

**a** State clearly the conditions under which it is appropriate to assume that a random variable has a binomial distribution.

A door-to-door canvasser tries to persuade people to have a certain type of double glazing installed. The probability that his canvassing at a house is successful is 0.05.

**b** Find the probability that he will have at least 2 successes out of the first 10 houses he canvasses.

**c** Find the number of houses he should canvass per day in order to average 3 successes per day.

**d** Calculate the least number of houses that he must canvass in order that the probability of his getting at least one success exceeds 0.99. ***E***

**Solution:**

**a**
1. There are $n$ independent trials.
2. $n$ is a fixed number.
3. The outcome of each trial is success or failure.
4. The probability of success at each trial is constant.
5. The outcome of any trial is independent of any other trial.

**b** $X =$ number of successes

$X \sim \mathrm{B}(10, 0.05)$

$$\begin{aligned} \mathrm{P}(X \geq 2) &= 1 - \mathrm{P}(X \leq 1) \\ &= 1 - 0.9139 \qquad \text{(tables)} \\ &= 0.0861 \end{aligned}$$

**c** $Y =$ number of successes in $n$ houses

$Y \sim \mathrm{B}(n, 0.05)$

$n \times 0.05 = 3$

$\mathrm{E}(Y) = 3 \Rightarrow$ or $\dfrac{n}{20} = 3 \quad \therefore n = 60$

**d**

$$\begin{aligned} \mathrm{P}(Y \geq 1) &> 0.99 \\ \Rightarrow 1 - \mathrm{P}(Y = 0) &> 0.99 \\ 0.01 &> \mathrm{P}(Y = 0) \\ 0.01 &> (0.95)^n \\ \log(0.01) &> n \log(0.95) \\ \frac{\log 0.01}{\log 0.95} &< n \quad \longleftarrow \boxed{[\log 0.95 < 0 \text{ so change } > \text{ when dividing}]} \\ n &> 89.78\ldots \text{ so } n = 90 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 5**

**Question:**

An archer fires arrows at a target and for each arrow, independently of all the others, the probability that it hits the bull's eye is $\frac{1}{8}$.

**a** Given that the archer fires 5 arrows, find the probability that fewer than 2 arrows hit the bull's-eye.

The archer fires 5 arrows, collects them from a target and fires all 5 again.

**b** Find the probability that on both occasions fewer than 2 hit the bull's eye.

**Solution:**

$X =$ number of bull's eyes in 5 arrows

$X \sim \mathrm{B}(5, \frac{1}{8})$

**a**
$$\begin{aligned} \mathrm{P}(X < 2) &= \mathrm{P}(X \leq 1) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1) \\ &= \left(\frac{7}{8}\right)^5 + 5 \times \frac{1}{8} \times \left(\frac{7}{8}\right)^4 \\ &= 0.87927 \\ &= 0.879 \end{aligned}$$

**b**
$$\begin{aligned} [\mathrm{P}(X < 2)]^2 &= 0.87927^2 \\ &= 0.773 \end{aligned}$$

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 6**

**Question:**

A completely unprepared student is given a true/false type test with 10 questions. Assuming that the student answers all the questions at random

**a** find the probability that the student gets all the answers correct.

It is decided that a pass will be awarded for 8 or more correct answers.

**b** Find the probability that the student passes the test.

**Solution:**

$X$ = number of correctly answered questions

$X \sim \mathrm{B}(10, 0.5)$

**a**

$$\begin{aligned} \mathrm{P}(X = 10) = (0.5)^{10} &= 0.00097656\ldots \\ &= 0.000977 \quad (3\text{ s.f.}) \end{aligned}$$

**b**

$$\begin{aligned} \mathrm{P}(X \geq 8) &= 1 - \mathrm{P}(X \leq 7) \\ &= 1 - 0.9453 \qquad \text{(tables)} \\ &= 0.0547 \end{aligned}$$

# Solutionbank S2
## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 7**

**Question:**

A six-sided die is biased. When the die is thrown the number 5 is twice as likely to appear as any other number. All the other faces are equally likely to appear. The die is thrown repeatedly. Find the probability that

**a** the first 5 will occur on the sixth throw,

**b** in the first eight throws there will be exactly three 5s. ***E***

**Solution:**

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\mathrm{P}(X=x)$ | $p$ | $P$ | $P$ | $P$ | $2p$ | $p$ |

$7p=1 \quad \Rightarrow \quad p=\frac{1}{7}$

**a** Not binomial

Sequence: $\bar{5}\ \bar{5}\ \bar{5}\ \bar{5}\ \bar{5}\ 5$

Probability: $\left(\frac{5}{7}\right)^5\left(\frac{2}{7}\right)$

$=0.0531$

**b** $Y=$ number of 5s in 8 throws

$Y \sim \mathrm{B}(8, \frac{2}{7})$

$$\mathrm{P}(Y=3) = \binom{8}{3}\times\left(\frac{2}{7}\right)^3\times\left(\frac{5}{7}\right)^5$$

$$= 0.24285\ldots$$

$$= 0.243 \quad (3 \text{ s.f.})$$

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 8**

**Question:**

A manufacturer produces large quantities of plastic chairs. It is known from previous records that 15% of these chairs are green. A random sample of 10 chairs is taken.

**a** Define a suitable distribution to model the number of green chairs in this sample.

**b** Find the probability of at least 5 green chairs in this sample.

**c** Find the probability of exactly 2 green chairs in this sample.

**Solution:**

$X =$ number of green chairs in sample of 10

**a** $X \sim \mathrm{B}(10, 0.15)$

**b**

$$\begin{aligned} \mathrm{P}(X \geq 5) &= 1 - \mathrm{P}(X \leq 4) \\ &= 1 - 0.9901 \qquad \text{(tables)} \\ &= 0.0099 \end{aligned}$$

**c**

$$\begin{aligned} \mathrm{P}(X = 2) &= \mathrm{P}(X \leq 2) - \mathrm{P}(X \leq 1) \\ &= 0.8202 - 0.5443 \qquad \text{(tables)} \\ &= 0.2759 \end{aligned}$$

# Solutionbank S2

## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 9**

**Question:**

A bag contains a large number of beads of which 45% are yellow. A random sample of 20 beads is taken from the bag. Use the binomial distribution to find the probability that the sample contains

**a** fewer than 12 yellow beads,

**b** exactly 12 yellow beads. ***E***

**Solution:**

$X =$ number of yellow beads in sample of 20

$X \sim \mathrm{B}(20, 0.45)$

**a**

$$\begin{aligned} \mathrm{P}(X < 12) &= \mathrm{P}(X \leq 11) \\ &= 0.8692 \end{aligned}$$

**b**

$$\begin{aligned} \mathrm{P}(X = 12) &= \mathrm{P}(X \leq 12) - \mathrm{P}(X \leq 11) \\ &= 0.9420 - 0.8692 \\ &= 0.0728 \end{aligned}$$