

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise A, Question 1

Question:

Use tables of the normal distribution to find the following.

a $P(Z < 2.12)$

b $P(Z < 1.36)$

c $P(Z > 0.84)$

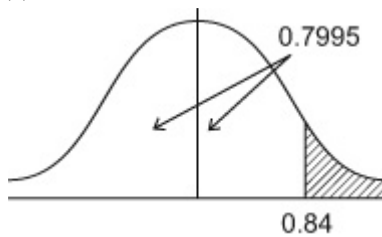
d $P(Z < -0.38)$

Solution:

(a) $P(z < 2.12) = \underline{0.9830}$

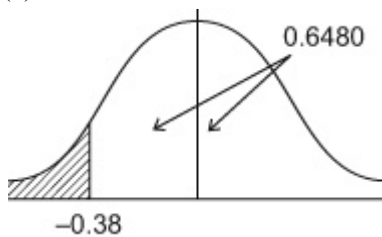
(b) $P(z < 1.36) = \underline{0.9131}$

(c)



$$\begin{aligned} P(Z > 0.84) &= 1 - 0.7995 \\ &= \underline{0.2005} \end{aligned}$$

(d)



$$\begin{aligned} P(Z < -0.38) &= 1 - 0.6480 \\ &= \underline{0.352} \end{aligned}$$

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Exercise A, Question 2

Question:

Use tables of the normal distribution to find the following.

a $P(Z > 1.25)$

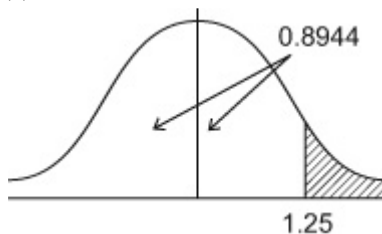
b $P(Z > -1.68)$

c $P(Z < -1.52)$

d $P(Z < 3.15)$

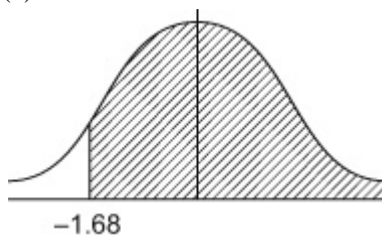
Solution:

(a)



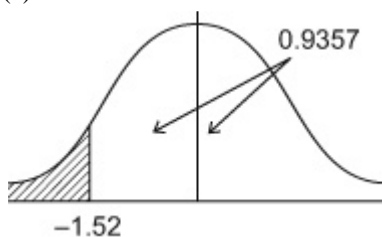
$$P(Z > 1.25) = 1 - 0.8944 \\ = \underline{\underline{0.1056}}$$

(b)



$$P(Z > -1.68) = \underline{\underline{0.9535}}$$

(c)



$$P(Z < -1.52) = 1 - 0.9357 \\ = \underline{\underline{0.0643}}$$

(d) $P(Z < 3.15) = \underline{\underline{0.9992}}$

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Exercise A, Question 3

Question:

Use tables of the normal distribution to find the following.

a $P(Z > -2.24)$

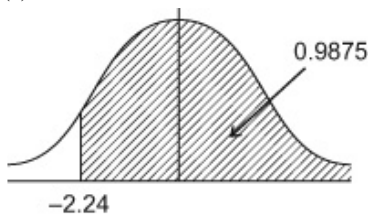
b $P(0 < Z < 1.42)$

c $P(-2.30 < Z < 0)$

d $P(Z < -1.63)$

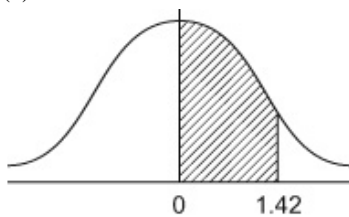
Solution:

(a)



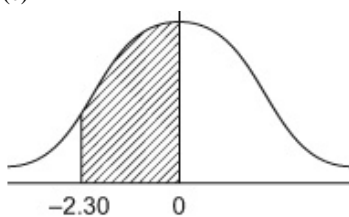
$$P(Z > -2.24) = \underline{\underline{0.9875}}$$

(b)



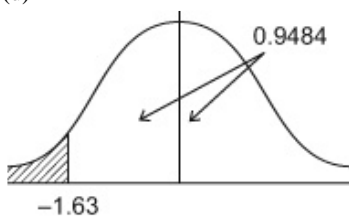
$$\begin{aligned} P(0 < Z < 1.42) &= 0.9222 - 0.5 \\ &= \underline{\underline{0.4222}} \end{aligned}$$

(c)



$$\begin{aligned} P(-2.30 < Z < 0) &= 0.9893 - 0.5 \\ &= \underline{\underline{0.4893}} \end{aligned}$$

(d)



$$\begin{aligned} P(Z < -1.63) &= 1 - 0.9484 \\ &= \underline{\underline{0.0516}} \end{aligned}$$

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Exercise A, Question 4

Question:

Use tables of the normal distribution to find the following.

a $P(1.25 < Z < 2.16)$

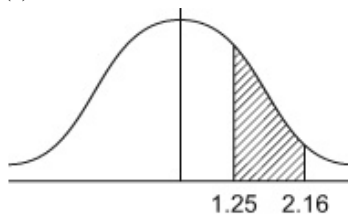
b $P(-1.67 < Z < 2.38)$

c $P(-2.16 < Z < -0.85)$

d $P(-1.57 < Z < 1.57)$

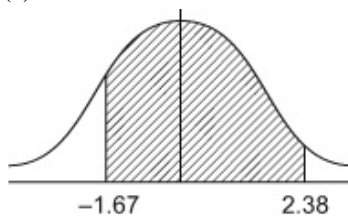
Solution:

(a)



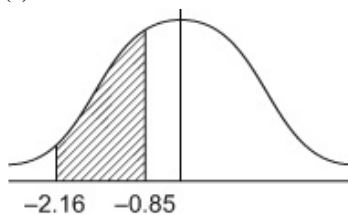
$$\begin{aligned} P(1.25 < Z < 2.16) \\ &= P(Z < 2.16) - P(Z < 1.25) \\ &= 0.9846 - 0.8944 \\ &= \underline{0.0902} \end{aligned}$$

(b)



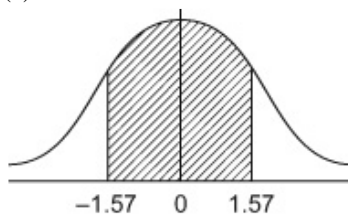
$$\begin{aligned} P(-1.67 < Z < 2.38) \\ &= P(Z < 2.38) - [1 - P(Z < 1.67)] \\ &= 0.9913 - [1 - 0.9525] \\ &= 0.9913 - 0.0475 \\ &= \underline{0.9438} \end{aligned}$$

(c)



$$\begin{aligned} P(-2.16 < Z < -0.85) \\ &= P(Z < 2.16) - P(Z < 0.85) \\ &= 0.9846 - 0.8023 \\ &= \underline{0.1823} \end{aligned}$$

(d)



$$\begin{aligned} P(-1.57 < Z < 1.57) \\ &= 2 \times P(0 < Z < 1.57) \\ &= 2 \times [0.9418 - 0.5] \\ &= 2 \times 0.4418 \\ &= \underline{0.8836} \end{aligned}$$

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Exercise B, Question 1

Question:

Find the value of a in the following.

a $P(Z < a) = 0.9082$

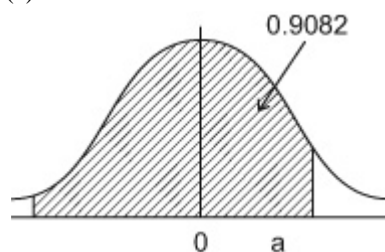
b $P(Z > a) = 0.0314$

c $P(Z < a) = 0.3372$

d $P(Z > a) = 0.6879$

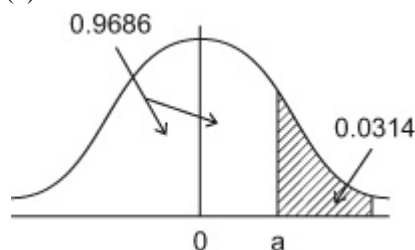
Solution:

(a)



$$\underline{a = 1.33}$$

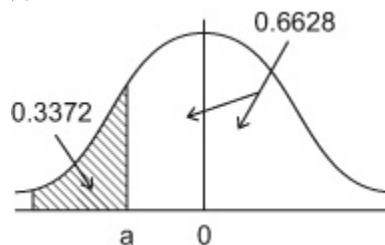
(b)



$$1 - 0.0314 = 0.9686$$

$$\therefore \underline{a = 1.86}$$

(c)



$$1 - 0.3372 = 0.6628$$

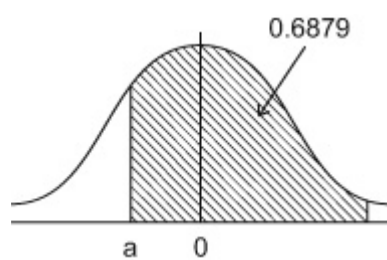
$$\text{N.B. } 0.3372 < 0.5 \therefore a < 0$$

$$\therefore \underline{a = -0.42}$$

(d)

$$[\text{N.B. } a < 0 \text{ again}]$$

$$\underline{\underline{a = -0.49}}$$



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Exercise B, Question 2

Question:

Find the value of a in the following.

a $P(Z < a) = 0.9938$

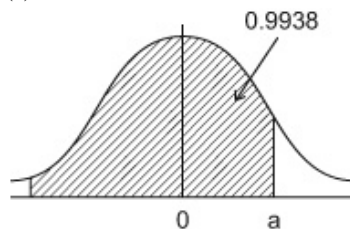
b $P(Z > a) = 0.4129$

c $P(Z > a) = 0.7611$

d $P(Z > a) = 0.2000$

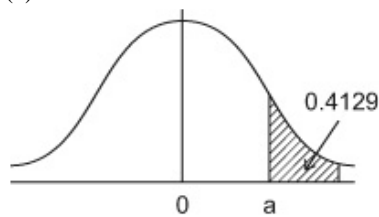
Solution:

(a)



$$\underline{\underline{a = 2.50}}$$

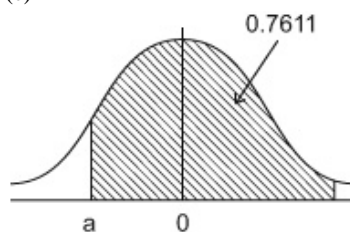
(b)



$$1 - 0.4129 = 0.5871$$

$$\therefore \underline{\underline{a = 0.22}}$$

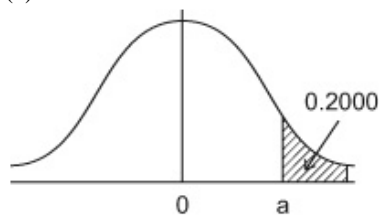
(c)



$$[\text{N.B. } a < 0 \because P(Z > a) > 0.5]$$

$$\underline{\underline{a = -0.71}}$$

(d)



Using table of percentage points

$$\underline{\underline{a = 0.8416}}$$

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Exercise B, Question 3

Question:

Find the value of a in the following.

a $P(Z > a) = 0.1500$

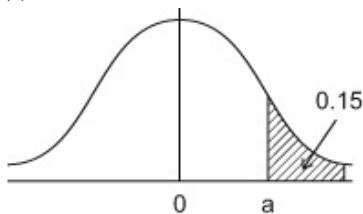
b $P(Z > a) = 0.9500$

c $P(Z > a) = 0.1112$

d $P(Z < a) = 0.9990$

Solution:

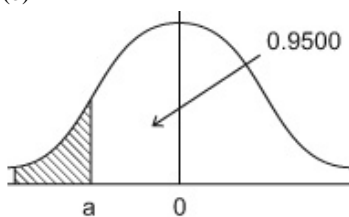
(a)



Use table of percentage points with $p = 0.15$

$$\underline{\underline{a = 1.0364}}$$

(b)

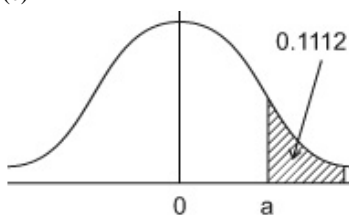


$$1 - 0.9500 = 0.05 = p$$

Using table of percentage points

$$\underline{\underline{a = -1.6449}}$$

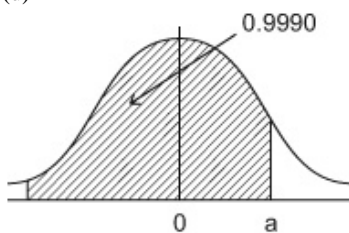
(c)



$$1 - 0.1112 = 0.8888$$

$$\underline{\underline{a = 1.22}}$$

(d)



$$1 - 0.9990 = 0.0010$$

Use table of percentage points with

$$p = 0.0010$$

$$\underline{\underline{a = 3.0902}}$$

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Exercise B, Question 4

Question:

Find the value of a in the following.

a $P(0 < Z < a) = 0.3554$

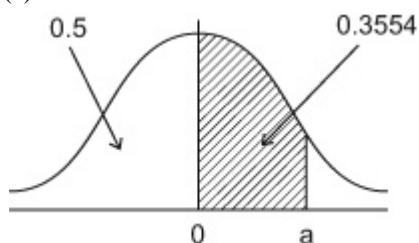
b $P(0 < Z < a) = 0.4946$

c $P(-a < Z < a) = 0.5820$

d $P(-a < Z < a) = 0.8230$

Solution:

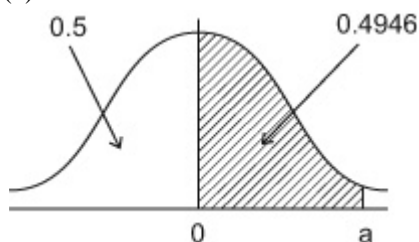
(a)



$$P(Z < a) = 0.8554$$

$$\therefore \underline{a = 1.06}$$

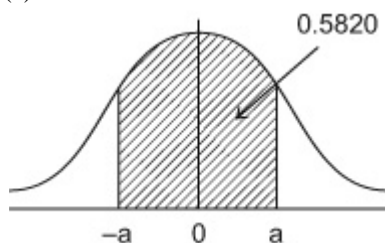
(b)



$$P(Z < a) = 0.9946$$

$$\therefore \underline{a = 2.55}$$

(c)



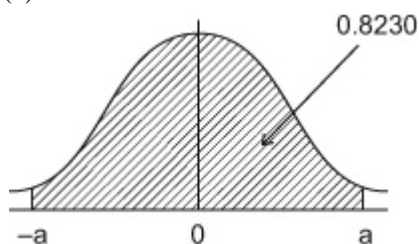
$$P(0 < Z < a) = \frac{1}{2} \times 0.5820$$

$$= 0.2910$$

$$\therefore P(Z < a) = 0.7910$$

$$\therefore \underline{a = 0.81}$$

(d)



$$P(0 < Z < a) = \frac{1}{2} \times 0.8230$$

$$= 0.4115$$

$$\therefore P(Z < a) = 0.9115$$

$$\therefore \underline{a = 1.35}$$

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Exercise B, Question 5

Question:

Find the value of a in the following.

a $p(0 < Z < a) = 0.10$

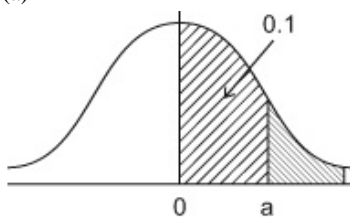
b $p(0 < Z < a) = 0.35$

c $p(-a < Z < a) = 0.80$

d $p(-a < Z < a) = 0.40$

Solution:

(a)

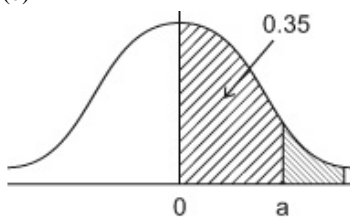


$$p = 0.5 - 0.1 = 0.4000$$

Use table of percentage points

$$\underline{\underline{a = 0.2533}}$$

(b)

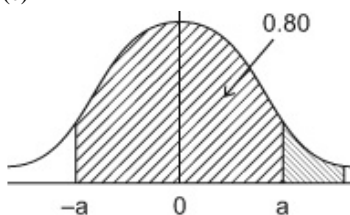


$$p = 0.5 - 0.35 = 0.1500$$

Use table of percentage points

$$\underline{\underline{a = 1.0364}}$$

(c)

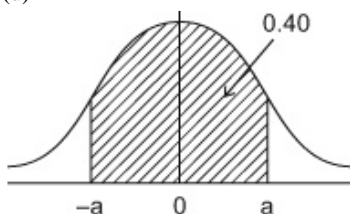


$$p = 0.5 - \frac{1}{2} \times 0.8 = 0.1000$$

Use table of percentage points

$$\underline{\underline{a = 1.2816}}$$

(d)



$$P(0 < Z < a) = \frac{1}{2} \times 0.4 = 0.20$$

$$p = 0.5 - 0.2 = 0.3000$$

Use table of percentage points

$$\underline{\underline{a = 0.5244}}$$

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Exercise C, Question 1

Question:

The random variable $X \sim N(30, 2^2)$.

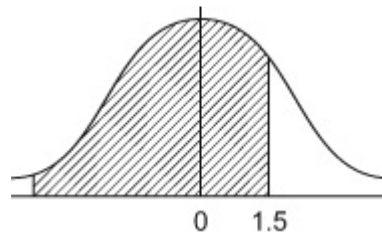
Find **a** $P(X < 33)$,

b $P(X > 26)$.

Solution:

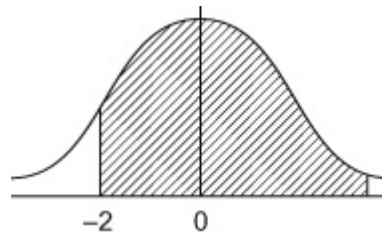
(a)

$$\begin{aligned} P(X < 33) &= P\left(Z < \frac{33-30}{2}\right) \\ &= P(Z < 1.5) \\ &= \underline{\underline{0.9332}} \end{aligned}$$



(b)

$$\begin{aligned} P(X > 26) &= P\left(Z > \frac{26-30}{2}\right) \\ &= P(Z > -2) \\ &= \underline{\underline{0.9772}} \end{aligned}$$



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Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise C, Question 2

Question:

The random variable $X \sim N(40, 9)$.

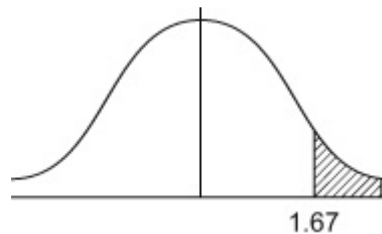
Find **a** $P(X > 45)$,

b $P(X < 38)$.

Solution:

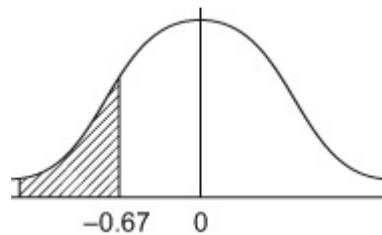
(a)

$$\begin{aligned} P(X > 45) &= P\left(Z > \frac{45 - 40}{\sqrt{9}}\right) \\ &= P(Z > 1.67) \\ &= 1 - 0.9525 \\ &= \underline{\underline{0.0475}} \text{ (allow AWRT } \underline{\underline{0.048}}) \end{aligned}$$



(b)

$$\begin{aligned} P(X < 38) &= P\left(Z < \frac{38 - 40}{3}\right) \\ &= P(Z < -0.67) \\ &= 1 - 0.7486 \\ &= \underline{\underline{0.2514}} \text{ (allow AWRT } 0.251 \text{ or } 0.252) \end{aligned}$$



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Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise C, Question 3

Question:

The random variable $Y \sim N(25, 25)$.

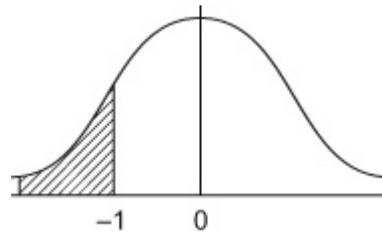
Find **a** $P(Y < 20)$,

b $P(18 < Y < 26)$.

Solution:

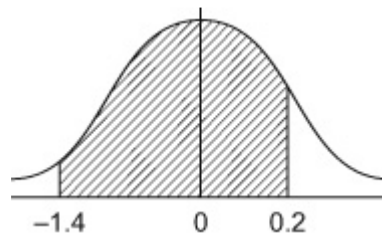
(a)

$$\begin{aligned} P(Y < 20) &= P\left(Z < \frac{20-25}{\sqrt{25}}\right) \\ &= P(Z < -1) \\ &= 1 - 0.8413 \\ &= \underline{\underline{0.1587}} \end{aligned}$$



(b)

$$\begin{aligned} P(18 < Y < 26) &= P\left(\frac{18-25}{5} < Z < \frac{26-25}{5}\right) \\ &= P(-1.4 < Z < 0.2) \\ &= (0.5793 - 0.5) + (0.9192 - 0.5) \\ &= \underline{\underline{0.4985}} \end{aligned}$$



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Exercise C, Question 4

Question:

The random variable $X \sim N(18, 10)$.

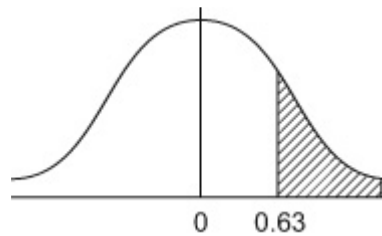
Find **a** $P(X > 20)$,

b $P(X < 15)$.

Solution:

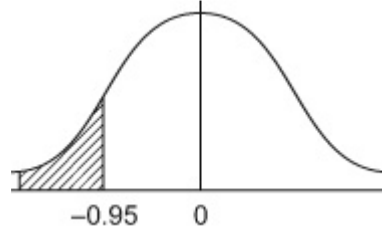
(a)

$$\begin{aligned}
 P(X > 20) &= P\left(Z > \frac{20-18}{\sqrt{10}}\right) \\
 &= P(Z > 0.6324 \dots) \text{ Use } 0.63 \\
 &= 1 - 0.7357 \\
 &= \underline{0.2643} \text{ (Calculator } 0.26354 \dots) \\
 &\text{allow AWRT } \underline{0.264} \text{ or } \underline{0.263}
 \end{aligned}$$



(b)

$$\begin{aligned}
 P(X < 15) &= P\left(Z < \frac{15-18}{\sqrt{10}}\right) \\
 &= P(Z < -0.9486 \dots) \text{ [Use } -0.95] \\
 &= 1 - 0.8289 \\
 &= \underline{0.1711} \text{ (Calculator: } 0.17139 \dots) \\
 &\text{allow AWRT } \underline{0.171}
 \end{aligned}$$



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Exercise C, Question 5

Question:

The random variable $X \sim N(20, 8)$.

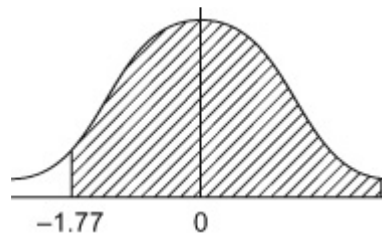
Find **a** $P(X > 15)$,

b the value of a such that $P(X < a) = 0.8051$.

Solution:

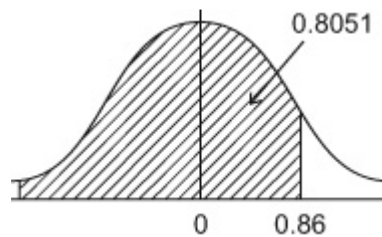
(a)

$$\begin{aligned} P(X > 15) &= P\left(Z > \frac{15-20}{\sqrt{8}}\right) \\ &= P(Z > -1.767 \dots) \text{ Use } -1.77 \\ &= 0.9616 \text{ (Calculator: } 0.96145 \dots) \\ &\text{allow AWRT } \underline{0.961 \text{ or } 0.962} \end{aligned}$$



(b)

$$\begin{aligned} P(X < a) &= 0.8051 \\ P\left(Z < \frac{a-20}{\sqrt{8}}\right) &= 0.8051 \\ \therefore \frac{a-20}{\sqrt{8}} &= 0.86 \\ \therefore a &= \underline{\underline{22.43}} \text{ (allow AWRT } 22.4) \end{aligned}$$



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Exercise C, Question 6

Question:

The random variable $Y \sim N(30, 5^2)$.

Find the value of a such that $P(Y > a) = 0.30$.

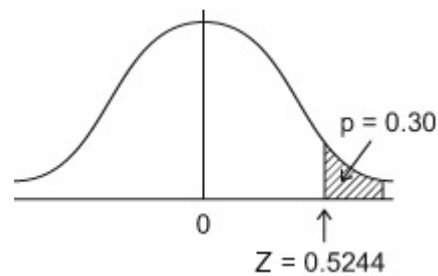
Solution:

$$P(Y > a) = 0.30$$

$$\frac{a - 30}{5} = 0.5244$$

$$\therefore a = 5 \times 0.5244 + 30$$

$$a = 32.622 \text{ or } \underline{\underline{32.6}} \text{ (3sf)}$$



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Exercise C, Question 7

Question:

The random variable $X \sim N(15, 3^2)$.

Find the value of a such that $P(X > a) = 0.15$.

Solution:

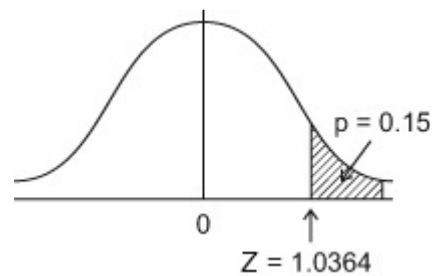
$$p(X > a) = 0.15$$

$$\frac{a - 15}{3} = 1.0364$$

$$\therefore a = 3 \times 1.0364 + 15$$

$$a = 18.1092$$

$$a = \underline{\underline{18.1}} \text{ (3sf)}$$



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Exercise C, Question 8

Question:

The random variable $X \sim N(20, 12)$.

Find the value of a and the value of b such that

a $P(X < a) = 0.40$,

b $P(X > b) = 0.6915$.

c Write down $P(b < X < a)$.

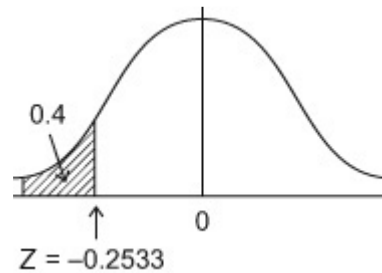
Solution:

(a)

$$P(X < a) = 0.40 \quad \text{Use } P = 0.4000$$

$$\frac{a-20}{\sqrt{12}} = -0.2533$$

$$a = 19.122 \dots \quad \therefore a = \underline{\underline{19.1}} \quad (3\text{sf})$$

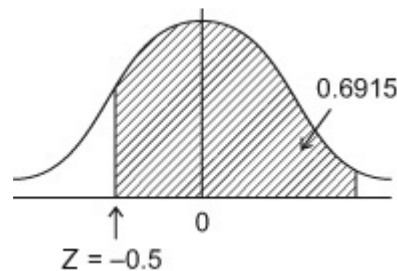


(b)

$$P(X > b) = 0.6915$$

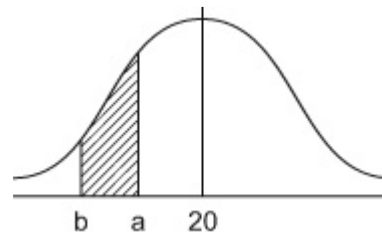
$$\frac{b-20}{\sqrt{12}} = -0.5$$

$$\therefore b = 18.267 \dots \quad \therefore b = \underline{\underline{18.3}} \quad (3\text{sf})$$



(c)

$$\begin{aligned} P(b < X < a) \\ &= 0.40 - [1 - 0.6915] \\ &= \underline{\underline{0.0915}} \end{aligned}$$



Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise C, Question 9

Question:

The random variable $Y \sim N(100, 15^2)$.

Find the value of a and the value of b such that

a $P(Y > a) = 0.975$,

b $P(Y < b) = 0.10$.

c Write down $P(a < Y < b)$.

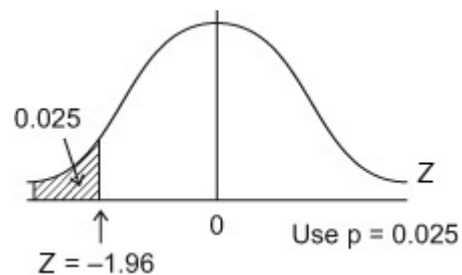
Solution:

(a)

$$P(Y > a) = 0.975$$

$$\therefore \frac{a - 100}{15} = -1.96$$

$$\therefore a = \underline{\underline{70.6}}$$

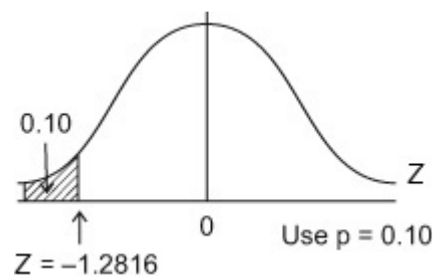


(b)

$$P(Y < b) = 0.10$$

$$\therefore \frac{b - 100}{15} = -1.2816$$

$$\therefore b = 80.776 \text{ or } \underline{\underline{80.8}} \text{ (3sf)}$$

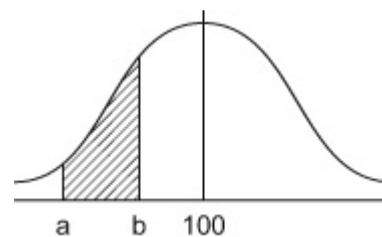


(c)

$$P(a < Y < b)$$

$$= 0.10 - 0.025$$

$$= \underline{\underline{0.075}}$$



Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise C, Question 10

Question:

The random variable $X \sim N(80, 16)$.

Find the value of a and the value of b such that

a $P(X > a) = 0.40$,

b $P(X < b) = 0.5636$.

c Write down $P(b < X < a)$.

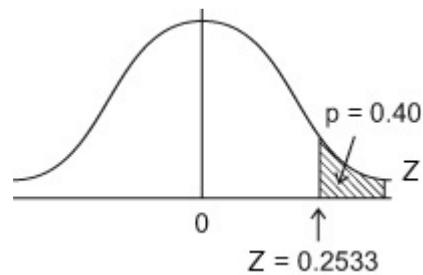
Solution:

(a)

$$P(X > a) = 0.40$$

$$\therefore \frac{a - 80}{\sqrt{16}} = 0.2533$$

$$\therefore a = \underline{\underline{81.0}} \text{ (3sf)}$$

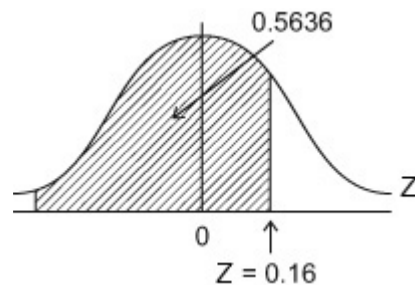


(b)

$$P(X < b) = 0.5636$$

$$\therefore \frac{b - 80}{4} = 0.16$$

$$\therefore b = \underline{\underline{80.64}}$$



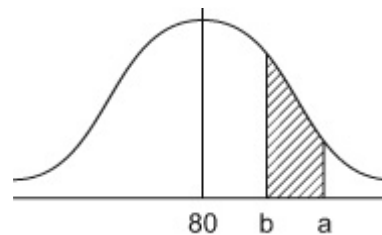
(c)

$$P(b < X < a)$$

$$= [1 - 0.4] - 0.5636$$

$$= 0.6 - 0.5636$$

$$= \underline{\underline{0.0364}}$$



Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

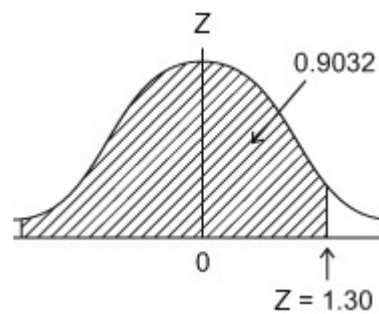
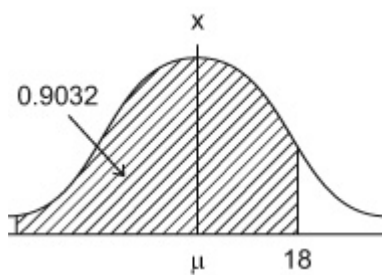
Exercise D, Question 1

Question:

The random variable $X \sim N(\mu, 5^2)$ and $P(X < 18) = 0.9032$.

Find the value of μ .

Solution:



$$Z = \frac{X - \mu}{\sigma} \quad \Rightarrow \quad 1.30 = \frac{18 - \mu}{5}$$

$$\therefore \quad \mu = 18 - 5 \times 1.30$$

$$\mu = \underline{\underline{11.5}}$$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

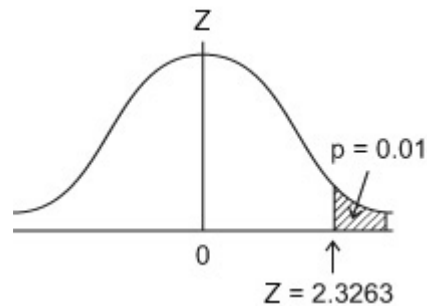
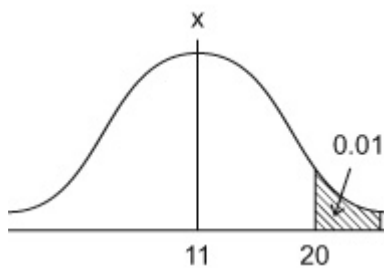
Exercise D, Question 2

Question:

The random variable $X \sim N(11, \sigma^2)$ and $P(X > 20) = 0.01$.

Find the value of σ .

Solution:



$$\begin{aligned}
 Z &= \frac{X - \mu}{\sigma} \Rightarrow 2.3263 = \frac{20 - 11}{\sigma} \\
 \therefore \sigma &= \frac{9}{2.3263} \\
 &= 3.8688 \dots \\
 \therefore \sigma &= \underline{3.87} \text{ (3sf)}
 \end{aligned}$$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

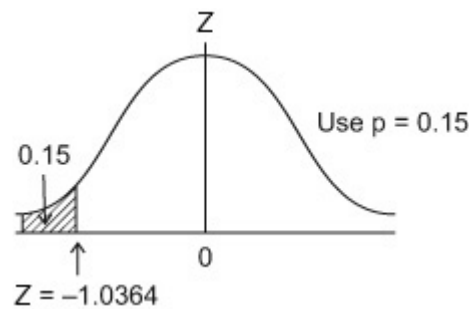
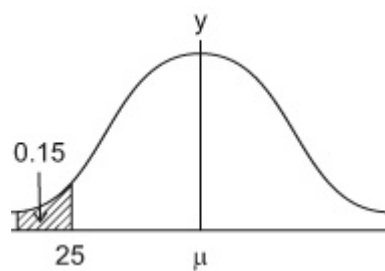
Exercise D, Question 3

Question:

The random variable $Y \sim N(\mu, 40)$ and $P(Y < 25) = 0.15$.

Find the value of μ .

Solution:



$$Z = \frac{X - \mu}{\sigma} \quad \Rightarrow \quad -1.0364 = \frac{25 - \mu}{\sqrt{40}}$$

$$\mu = 31.554 \dots$$

$$= \underline{\underline{31.6}} \quad (3\text{sf})$$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

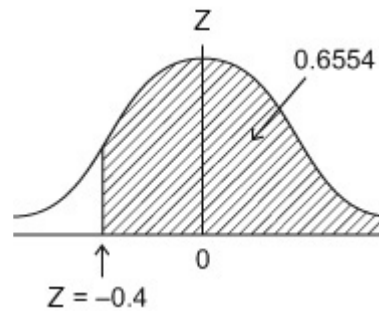
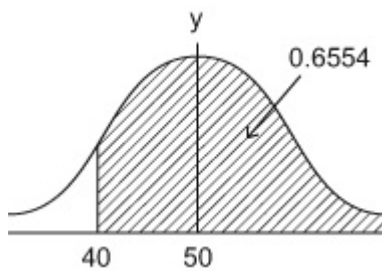
Exercise D, Question 4

Question:

The random variable $Y \sim N(50, \sigma^2)$ and $P(Y > 40) = 0.6554$.

Find the value of σ .

Solution:



$$Z = \frac{X - \mu}{\sigma} \quad \Rightarrow \quad -0.4 = \frac{40 - 50}{\sigma}$$

$$\therefore \quad \sigma = \frac{10}{0.4}$$

$$\sigma = \underline{\underline{25}}$$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

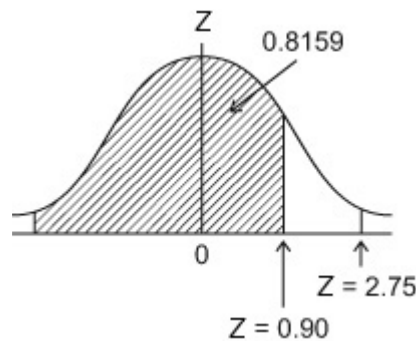
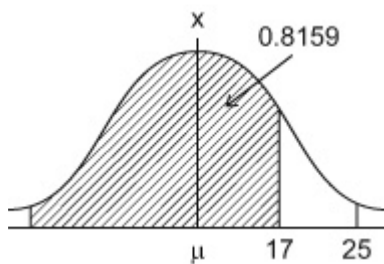
Exercise D, Question 5

Question:

The random variable $X \sim N(\mu, \sigma^2)$.

Given that $P(X < 17) = 0.8159$ and $P(X < 25) = 0.9970$, find the value of μ and the value of σ .

Solution:



$$Z = \frac{X - \mu}{\sigma} \Rightarrow 0.90 \quad \sigma = 17 - \mu$$

$$\underline{2.75\sigma = 25 - \mu}$$

Subtract $1.85 \sigma = 8$

$$\therefore \sigma = \frac{8}{1.85} = 4.3243$$

$$\mu = 17 - 0.90 \times \sigma$$

$$= 13.1081$$

$$\therefore \underline{\mu = 13.1} \quad \sigma = \underline{4.32} \quad (3\text{sf})$$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

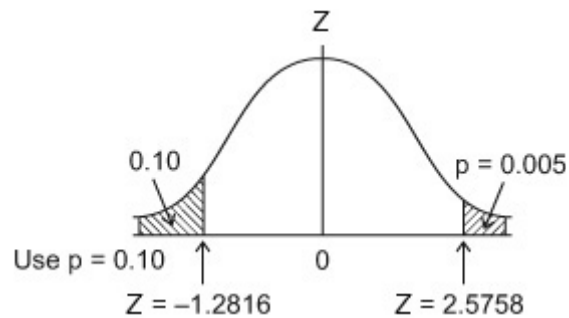
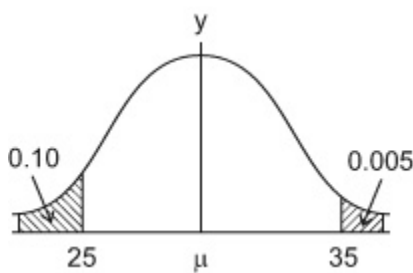
Exercise D, Question 6

Question:

The random variable $Y \sim N(\mu, \sigma^2)$.

Given that $P(Y < 25) = 0.10$ and $P(Y > 35) = 0.005$, find the value of μ and the value of σ .

Solution:



$$Z = \frac{X - \mu}{\sigma} \Rightarrow 2.5758\sigma = 35 - \mu$$

$$\underline{-1.2816\sigma = 25 - \mu.}$$

Subtract $3.8574\sigma = 10$

$$\therefore \sigma = 2.59241 \dots$$

$$\therefore \mu = 35 - 2.5758 \times 2.59241 \dots$$

$$= 28.322 \dots$$

$$\therefore \mu = \underline{28.3}, \quad \sigma = \underline{2.59} \quad (3sf)$$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

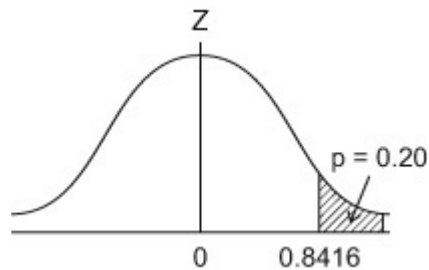
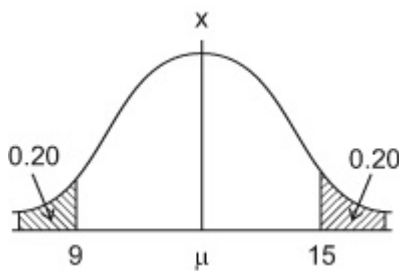
Exercise D, Question 7

Question:

The random variable $X \sim N(\mu, \sigma^2)$.

Given that $P(X > 15) = 0.20$ and $P(X < 9) = 0.20$, find the value of μ and the value of σ .

Solution:



By symmetry $\mu = \frac{1}{2}(9 + 15) = \underline{\underline{12}}$

$$Z = \frac{X - \mu}{\sigma} \Rightarrow 0.8416 = \frac{15 - \mu}{\sigma} = \frac{3}{\sigma}$$

$$\therefore \sigma = \frac{3}{0.8416}$$

$$\sigma = \underline{\underline{3.56}}$$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise D, Question 8

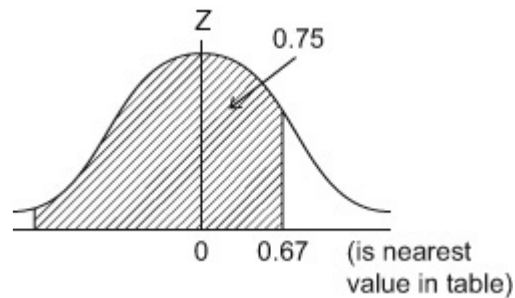
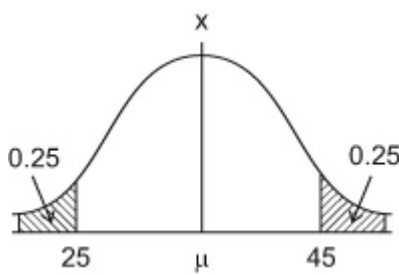
Question:

The random variable $X \sim N(\mu, \sigma^2)$.

The lower quartile of X is 25 and the upper quartile of X is 45.

Find the value of μ and the value of σ .

Solution:



By Symmetry $\mu = \frac{1}{2}(25 + 45) = \underline{35}$

$$Z = \frac{X - \mu}{\sigma} \Rightarrow 0.67 = \frac{45 - 35}{\sigma}$$

$$\therefore \sigma = \frac{10}{0.67}$$

$$\sigma = 14.92 \quad (\text{or } 14.826 \dots \text{ on Calculator})$$

\therefore accept $\sigma = \underline{14.9 \text{ or } 14.8}$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

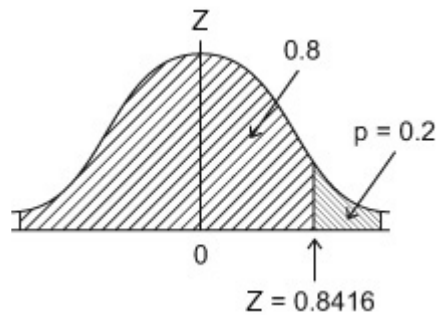
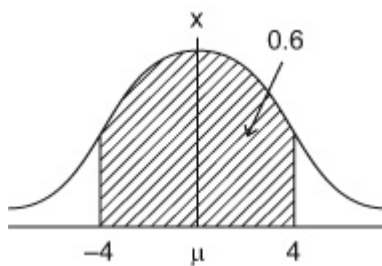
Exercise D, Question 9

Question:

The random variable $X \sim N(0, \sigma^2)$.

Given that $P(-4 < X < 4) = 0.6$, find the value of σ .

Solution:



By Symmetry $\mu = \underline{0}$

$$Z = \frac{X - \mu}{\sigma} \Rightarrow 0.8416 = \frac{4}{\sigma}$$

$$\therefore \sigma = \frac{4}{0.8416}$$

$$\sigma = \underline{\underline{4.75}} \quad 3\text{sf}$$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

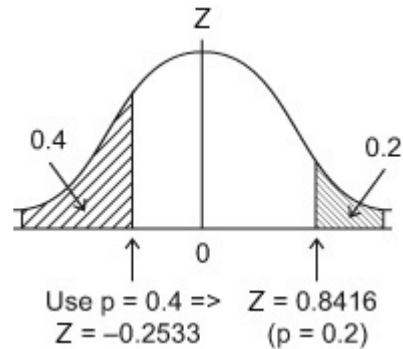
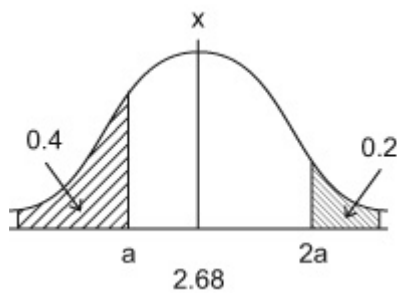
Exercise D, Question 10

Question:

The random variable $X \sim N(2.68, \sigma^2)$.

Given that $P(X > 2a) = 0.2$ and $P(X < a) = 0.4$, find the value of σ and the value of a .

Solution:



$$Z = \frac{X - \mu}{\sigma} \Rightarrow 0.8416\sigma = 2a - 2.68$$

$$\underline{-0.2533\sigma = a - 2.68 \times 2}$$

$$\underline{-0.5066\sigma = 2a - 5.36}$$

$$\underline{0.8416\sigma = 2a - 2.68}$$

Subtract :

$$1.3482\sigma = 2.68$$

$$\sigma = 1.9878 \dots$$

$$\sigma = \underline{1.99} \quad 3\text{sf}$$

$$a = 2.68 - 0.2533\sigma$$

$$\therefore a = 2.176 \dots$$

$$a = \underline{2.18} \quad 3\text{sf}$$

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise E, Question 1

Question:

The heights of a large group of men are normally distributed with a mean of 178 cm and a standard deviation of 4 cm.

A man is selected at random from this group.

a Find the probability that he is taller than 185 cm.

A manufacturer of door frames wants to ensure that fewer than 0.005 men have to stoop to pass through the frame.

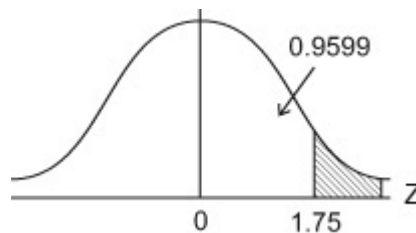
b On the basis of this group, find the minimum height of a door frame.

Solution:

$$M \sim N(178, 4^2)$$

(a)

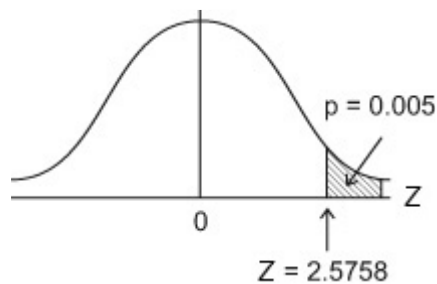
$$\begin{aligned} P(M > 185) &= P\left(Z > \frac{185 - 178}{4}\right) \\ &= P(Z > 1.75) \\ &= 1 - 0.9599 \\ &= \underline{\underline{0.0401}} \end{aligned}$$



(b)

$$P(M > h) = 0.005$$

$$\begin{aligned} Z = \frac{X - \mu}{\sigma} &\Rightarrow 2.5758 = \frac{h - 178}{4} \\ \therefore h &= 188.3032 \\ \therefore h &= \underline{\underline{188\text{cm}}} \end{aligned}$$



Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise E, Question 2

Question:

The weights of steel sheets produced by a factory are known to be normally distributed with mean 32.5 kg and standard deviation 2.2 kg.

a Find the percentage of sheets that weigh less than 30 kg.

Bob requires sheets that weigh between 31.6 kg and 34.8 kg.

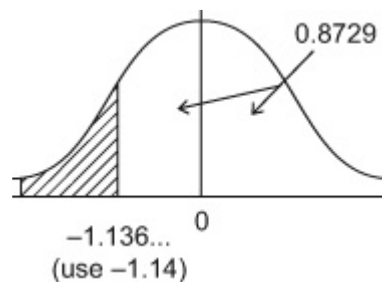
b Find the percentage of sheets produced that satisfy Bob's requirements.

Solution:

$$W \sim N(32.5, 2.2^2)$$

(a)

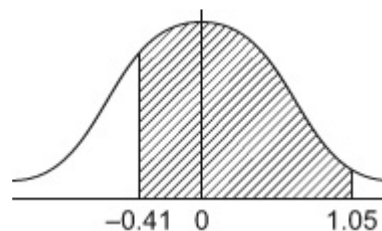
$$\begin{aligned} P(W < 30) &= P\left(Z < \frac{30 - 32.5}{2.2}\right) \\ &= P(Z < -1.14) \\ &= 1 - 0.8729 \\ &= 0.1271 \quad \underline{\underline{12.7\% \text{ or } 12.8\%}} \end{aligned}$$



(Calculator gives 0.1279.. so allow AWRT (0.127 - 0.128))

(b)

$$\begin{aligned} P(31.6 < W < 34.8) \\ &= P\left(\frac{31.6 - 32.5}{2.2} < Z < \frac{34.8 - 32.5}{2.2}\right) \\ &= P(-0.41 < Z < 1.05) \\ &= 0.8531 - (1 - 0.6591) \\ &= 0.5122 \end{aligned}$$



(Calculator gives 0.510856 ... So allow AWRT 0.511 or 0.512)

So 51.1% or 51.2% of sheets satisfy Bob's requirements

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise E, Question 3

Question:

The time a mobile phone battery lasts before needing to be recharged is assumed to be normally distributed with a mean of 48 hours and a standard deviation of 8 hours.

a Find the probability that a battery will last for more than 60 hours.

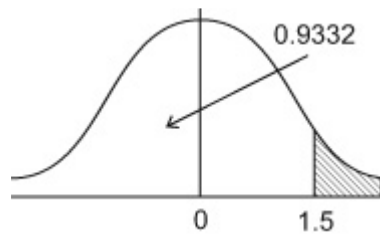
b Find the probability that the battery lasts less than 35 hours.

Solution:

$$T \sim N(48, 8^2)$$

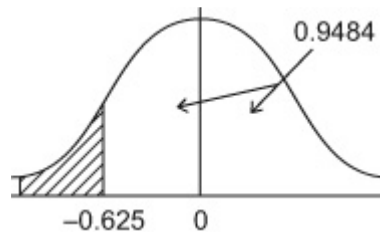
(a)

$$\begin{aligned} P(T > 60) &= P\left(Z > \frac{60 - 48}{8}\right) \\ &= P(Z > 1.5) \\ &= 1 - 0.9332 \\ &= \underline{\underline{0.0668}} \end{aligned}$$



(b)

$$\begin{aligned} P(T < 35) &= P\left(Z < \frac{35 - 48}{8}\right) \\ &= P(Z < -1.63) \\ &= 1 - 0.9484 \\ &= 0.0516 \end{aligned}$$



(Calculator gives 0.05208... so allow AWRT 0.052)

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise E, Question 4

Question:

The random variable $X \sim N(24, \sigma^2)$.

Given that $P(X > 30) = 0.05$, find

a the value of σ ,

b $P(X < 20)$,

c the value of d so that $P(X > d) = 0.01$.

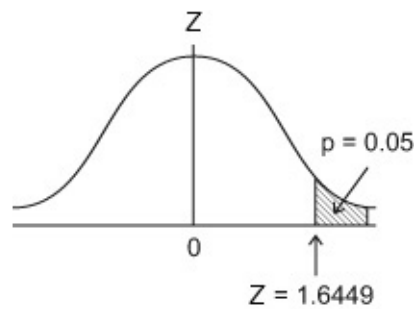
Solution:

$$X \sim N(24, \sigma^2)$$

(a)

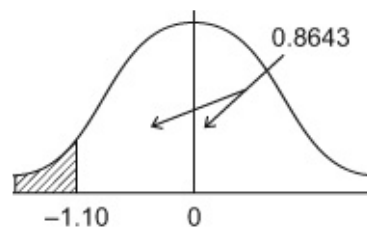
$$P(X > 30) = 0.05$$

$$\begin{aligned} Z = \frac{X - \mu}{\sigma} &\Rightarrow 1.6449 = \frac{30 - 24}{\sigma} \\ \therefore \sigma &= \frac{6}{1.6449} = 3.6476 \dots \\ \therefore \sigma &= \underline{3.65(3 \text{ sf})} \end{aligned}$$



(b)

$$\begin{aligned} P(X < 20) &= P\left(Z < \frac{20 - 24}{\sigma}\right) \\ &= P(Z < -1.10) \\ &= 1 - 0.8643 \\ &= \underline{0.1357} \end{aligned}$$

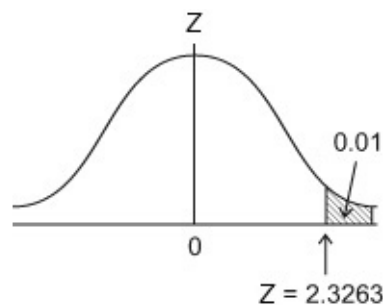


(Calculator gives 0.1364... so allow AWRT 0.136)

(c)

$$P(X > d) = 0.01$$

$$\begin{aligned} Z = \frac{X - \mu}{\sigma} &\Rightarrow 2.3263 = \frac{d - 24}{\sigma} \\ \therefore d &= 32.485 \dots \\ d &= \underline{32.5} \quad (3\text{sf}) \end{aligned}$$



Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

Exercise E, Question 5

Question:

A machine dispenses liquid into plastic cups in such a way that the volume of liquid dispensed is normally distributed with a mean of 120 ml. The cups have a capacity of 140 ml and the probability that the machine dispenses too much liquid so that the cup overflows is 0.01.

a Find the standard deviation of the volume of liquid dispensed.

b Find the probability that the machine dispenses less than 110 ml.

Ten percent of customers complain that the machine has not dispensed enough liquid.

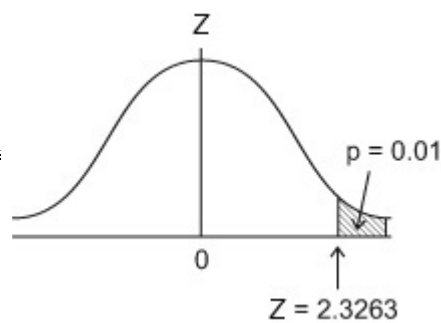
c Find the largest volume of liquid that will lead to a complaint.

Solution:

$$L \sim N(120, \sigma^2)$$

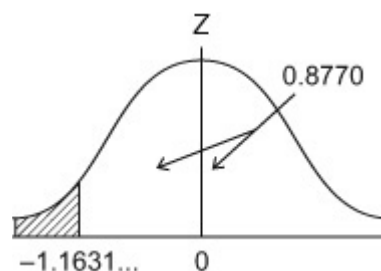
(a)

$$\begin{aligned} P(L > 140) &= 0.01 \\ Z = \frac{X - \mu}{\sigma} &\Rightarrow 2.3263 = \frac{140 - 120}{\sigma} \\ \therefore \sigma &= 8.5973 \dots = \underline{\underline{8.60 \text{ ml}}} \end{aligned}$$



(b)

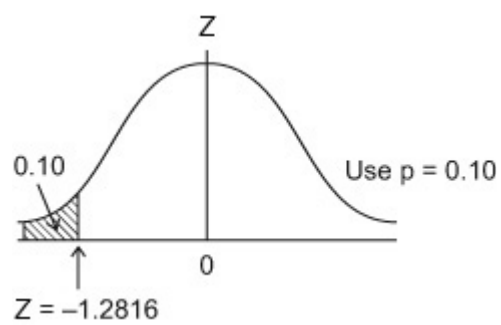
$$\begin{aligned} P(L < 110) &= P\left(Z < \frac{110 - 120}{\sigma}\right) \\ &= P(Z < -1.16) \\ &= 1 - 0.8770 \\ &= 0.123 \end{aligned}$$



(Calculator gives 0.12238... so allow AWRT 0.122 or 0.123)

(c)

$$\begin{aligned} P(L < c) &= 0.10 \\ Z = \frac{X - \mu}{\sigma} &\Rightarrow -1.2816 = \frac{c - 120}{\sigma} \\ \therefore c &= 108.98 \dots \\ &= \underline{\underline{109 \text{ ml}}} \quad (3\text{sf}) \end{aligned}$$



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Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Normal distribution

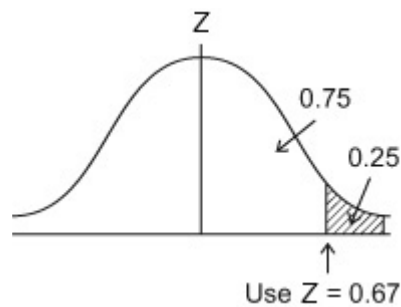
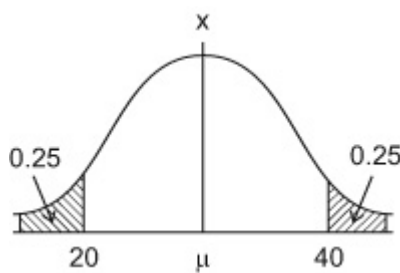
Exercise E, Question 6

Question:

The random variable $X \sim N(\mu, \sigma^2)$. The lower quartile of X is 20 and the upper quartile is 40. Find μ and σ .

Solution:

$$X \sim N(\mu, \sigma^2)$$



By symmetry $\mu = \frac{1}{2}(20 + 40) = \underline{30}$

$$Z = \frac{X - \mu}{\sigma} \Rightarrow \frac{40 - 30}{\sigma} = 0.67$$

$$\therefore \sigma = 14.925 \dots$$

(Calculator gives 14.82... so allow AWRT 14.8 or 14.9)

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

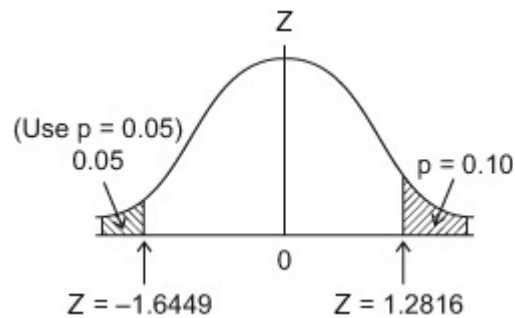
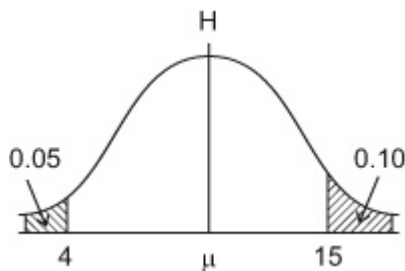
Normal distribution

Exercise E, Question 7

Question:

The heights of seedlings are normally distributed. Given that 10% of the seedlings are taller than 15 cm and 5% are shorter than 4 cm, find the mean and standard deviation of the heights.

Solution:



$$\begin{array}{rcl}
 Z\sigma = X - \mu & \Rightarrow & -1.6449 \sigma = 4 - \mu \\
 & & 1.2816 \sigma = 15 - \mu \\
 \text{Subtract} & & 2.9265 \sigma = 11 \\
 \therefore & & \sigma = 3.7587 \dots = \underline{3.76} \text{ cm (3sf)} \\
 \mu = 15 - 1.2816 \sigma & & \\
 \therefore & & \mu = 10.2 \text{ cm (3sf)}
 \end{array}$$

Solutionbank S1

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Normal distribution

Exercise E, Question 8

Question:

A psychologist gives a student two different tests. The first test has a mean of 80 and a standard deviation of 10 and the student scored 85.

a Find the probability of scoring 85 or more on the first test.

The second test has a mean of 100 and a standard deviation of 15. The student scored 105 on the second test.

b Find the probability of a score of 105 or more on the second test.

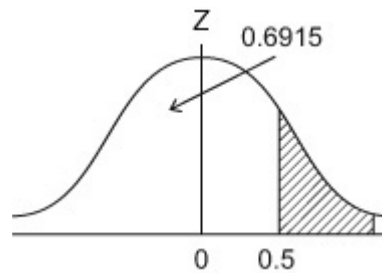
c State, giving a reason, which of the student's two test scores was better.

Solution:

$T \sim N(80, 10^2)$

(a)

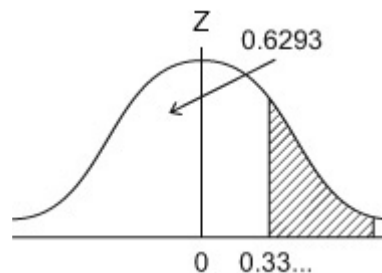
$$\begin{aligned} P(T > 85) &= P\left(Z > \frac{85 - 80}{10}\right) \\ &= P(Z > 0.5) \\ &= 1 - 0.6915 \\ &= \underline{\underline{0.3085}} \end{aligned}$$



(b)

$S \sim N(100, 15^2)$

$$\begin{aligned} P(S > 105) &= P\left(Z > \frac{105 - 100}{15}\right) \\ &= P(Z > 0.33) \\ &= 1 - 0.6293 \\ &= 0.3707 \end{aligned}$$



(Calculator gives 0.36944... so allow 0.369, 0.370 or 0.371)

(c) 1st score is best since a lower proportion of scores will beat it. (or Z value of 1st test is higher so this is the better result)

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Exercise E, Question 9

Question:

Jam is sold in jars and the mean weight of the contents is 108 grams. Only 3% of jars have contents weighing less than 100 grams. Assuming that the weight of jam in a jar is normally distributed find

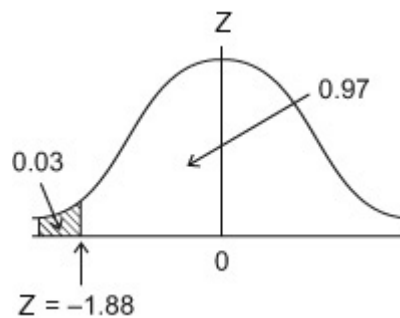
- a** the standard deviation of the weight of jam in a jar,
- b** the proportion of jars where the contents weigh more than 115 grams.

Solution:

$$J \sim N(108, \sigma^2)$$

(a)

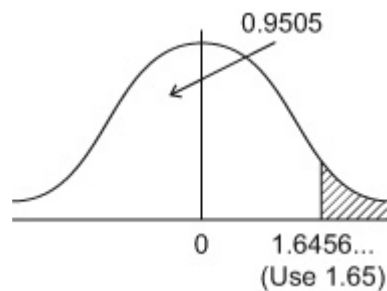
$$\begin{aligned} P(J < 100) &= 0.03 \\ Z = \frac{X - \mu}{\sigma} &\Rightarrow -1.88 = \frac{100 - 108}{\sigma} \\ \therefore \sigma &= 4.255 \dots \end{aligned}$$



(Calculator gives 4.2535... so allow AWRT 4.25–4.26)

(b)

$$\begin{aligned} P(J > 115) &= P\left(Z > \frac{115 - 108}{\sigma}\right) \\ &= P(Z > 1.65) \\ &= 1 - 0.9505 \\ &= 0.0495 \end{aligned}$$



(Calculator gives: 0.0499... so allow AWRT 0.050)

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Exercise E, Question 10

Question:

The waiting time at a doctor's surgery is assumed to be normally distributed with standard deviation of 3.8 minutes. Given that the probability of waiting more than 15 minutes is 0.0446, find

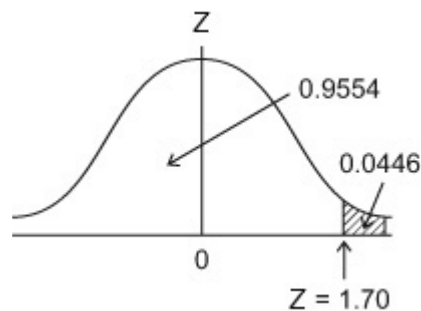
a the mean waiting time,

b the probability of waiting fewer than 5 minutes.

Solution:

$$T \sim N(\mu, 3.8^2)$$

$$P(T > 15) = 0.0446$$

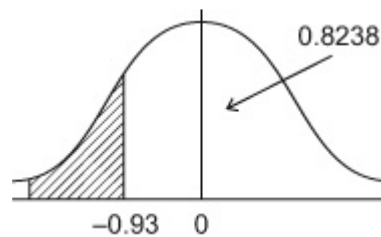


$$(a) \quad Z = \frac{X - \mu}{\sigma} \Rightarrow 1.70 = \frac{15 - \mu}{3.8}$$

$$\begin{aligned} \therefore \mu &= 15 - 3.8 \times 1.70 \\ \mu &= \underline{8.54} \text{ (3sf) minutes} \end{aligned}$$

(b)

$$\begin{aligned} P(T < 5) &= P\left(Z < \frac{5 - 8.54}{3.8}\right) \\ &= P(Z < -0.93 \dots) \\ &= 1 - 0.8238 \\ &= \underline{0.1762} \end{aligned}$$



(Calculator gives 0.17577... so allow AWRT 0.176)

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Normal distribution

Exercise E, Question 11

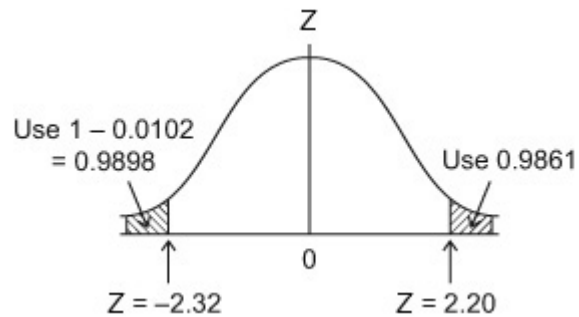
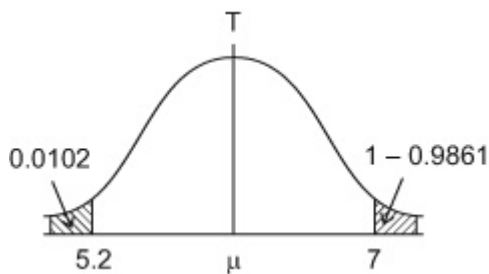
Question:

The thickness of some plastic shelving produced by a factory is normally distributed. As part of the production process the shelving is tested with two gauges. The first gauge is 7 mm thick and 98.61% of the shelving passes through this gauge. The second gauge is 5.2 mm thick and only 1.02% of the shelves pass through this gauge.

Find the mean and standard deviation of the thickness of the shelving.

Solution:

$$T \sim N(\mu, \sigma^2)$$



$$Z\sigma = X - \mu \Rightarrow 2.20 \sigma = 7 - \mu$$

$$\underline{-2.32 \sigma = 5.2 - \mu}$$

Subtract $4.52 \sigma = 1.8$
 $\sigma = 0.3982 \dots$

$$\mu = 7 - 2.20 \sigma \Rightarrow \mu = 6.1238 \dots$$

$$\therefore \underline{\mu = 6.12 \text{ mm}, \sigma = 0.398 \text{ mm}}$$

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Exercise E, Question 12

Question:

The random variable $X \sim N(14, 9)$. Find

a $P(X \geq 11)$,

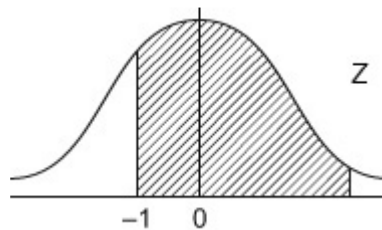
b $P(9 < X < 11)$.

Solution:

$$X \sim N(14, 3^2)$$

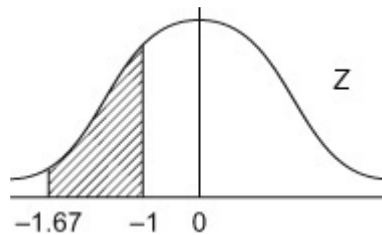
(a)

$$\begin{aligned} P(X \geq 11) &= P\left(Z \geq \frac{11-14}{3}\right) \\ &= P(Z \geq -1) \\ &= \underline{\underline{0.8413}} \end{aligned}$$



(b)

$$\begin{aligned} P(9 < X < 11) &= P\left(\frac{9-14}{3} < Z < \frac{11-14}{3}\right) \\ &= P(-1.67 < Z < -1) \\ &= 0.9525 - 0.8413 \\ &= 0.1112 \end{aligned}$$



(Calculator gives: 0.11086 ... so allow AWRT 0.111)

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Normal distribution

Exercise E, Question 13

Question:

The random variable $X \sim N(20, 5^2)$. Find

a $P(X \leq 16)$,

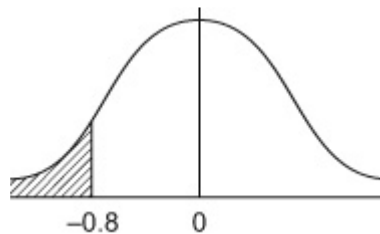
b the value of d such that $P(X < d) = 0.95$.

Solution:

$$X \sim N(20, 5^2)$$

(a)

$$\begin{aligned} P(X \leq 16) &= P\left(Z < \frac{16-20}{5}\right) \\ &= P(Z < -0.8) \\ &= 1 - 0.7881 \\ &= \underline{\underline{0.2119}} \end{aligned}$$



(b)

$$P(X < d) = 0.95$$

$$\begin{aligned} Z = \frac{X - \mu}{\sigma} &\Rightarrow 1.6449 = \frac{d - 20}{5} \\ \therefore d &= 20 + 5 \times 1.6449 \\ d &= 28.2 \end{aligned}$$

