Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Discrete random variables
Exercise A, Question 1

Question:

Write down whether or not each of the following is a discrete random variable.

Give a reason for your answer.

a The average lifetime of a battery.

b The number of days in a week.

c The number of moves it takes to complete a game of chess.

Solution:

i) This **is not** a discrete random variable. Time is a continuous variable.

ii) This **is not** a discrete random variable. It is always 7, so does not vary

iii) This **is** a discrete random variable. It is always a whole number but it does vary.

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Discrete random variables
Exercise A, Question 2

Question:

A fair die is thrown four times and the number of times it falls with a 6 on the top, Y, is noted. Write down all the possible values of y.

Solution:

y: 0 1 2 3 4

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Edexcel AS and A Level Modular Mathematics

Discrete random variables
Exercise A, Question 3

Question:

A bag contains two discs with the number 2 on them and two discs with the number 3 on them. A disc is drawn at random from the bag and the number noted. The disc is returned to the bag. A second disc is then drawn from the bag and the number noted.

a Write down the sample space.

The discrete random variable X is the sum of the two numbers.

b Write down the probability distribution for X.

c Write down the probability function for X.

Solution:

a S = (2,2), (2,3), (3,2), (3,3)

b

x	4	5	6
$\mathrm{P}(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

c $\mathrm{P}(X = x) = \begin{cases} \frac{1}{4} & x = 4,6, \\ \frac{1}{2} & x = 5. \end{cases}$

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Discrete random variables
Exercise A, Question 4

Question:

A discrete random variable X has the following probability distribution:

x	1	2	3	4
$\mathbf{P}(X = x)$	$\frac{1}{3}$	$\frac{1}{3}$	k	$\frac{1}{4}$

Find the value of k.

Solution:

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{4} + k = 1$$

$$k = 1 - \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{4}\right)$$

$$= 1 - \frac{11}{12} = \mathbf{\frac{1}{12}}$$

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Discrete random variables
Exercise A, Question 5

Question:

The random variable X has a probability function

$P(X = x) = kx \qquad x = 1, 2, 3, 4.$

Show that $k = \frac{1}{10}$.

Solution:

x	1	2	3	4
$P(X = x)$	k	$2k$	$3k$	$4k$

$k + 2k + 3k + 4k = 1$

$10k = 1$

$\boldsymbol{k = \frac{1}{10}}$

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Discrete random variables
Exercise A, Question 6

Question:

The random variable X has the probability function

$$P(X = x) = \frac{x-1}{10} \qquad x = 1, 2, 3, 4, 5.$$

Construct a table giving the probability distribution of X.

Solution:

x	1	2	3	4	5
$P(X = x)$	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

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Discrete random variables
Exercise A, Question 7

Question:

The random variable X has a probability function

$$P(X=x)=\begin{cases} kx & x=1,3 \\ k(x-1) & x=2,4 \end{cases}$$

where k is a constant.

a Find the value of k.

b Construct a table giving the probability distribution of X.

Solution:

a

x	1	2	3	4
$P(X=x)$	k	k	$3k$	$3k$

Using the fact that the probabilities add up to 1:

$$k+k+3k+3k=1$$

$$8k=1$$

$$\boldsymbol{k=\frac{1}{8}}$$

b

x	**1**	**2**	**3**	**4**
$P(X=x)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$

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Discrete random variables
Exercise A, Question 8

Question:

The discrete random variable X has a probability function

$$P(X = x) = \begin{cases} 0.1 & x = -2, -1 \\ \beta & x = 0, 1 \\ 0.2 & x = 2 \end{cases}$$

a Find the value of β.

b Construct a table giving the probability distribution of X.

Solution:

x	− 2	−1	0	1	2
$P(X = x)$	0.1	0.1	β	β	0.2

The probabilities add up to 1.

$0.1 + 0.1 + \beta + \beta + 0.2 = 1$

$2\beta = 1 - 0.4$

$\beta = \mathbf{0.3}$

b

x	−2	−1	0	1	2
$P(X = x)$	0.1	0.1	0.3	0.3	0.2

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Discrete random variables
Exercise A, Question 9

Question:

A discrete random variable has the probability distribution shown in the table below.

x	0	1	2
$\mathbf{P}(X = x)$	$\frac{1}{4} - a$	a	$\frac{1}{2} + a$

Find the value of a.

Solution:

$$\frac{1}{4} - a + a + \frac{1}{2} + a = 1$$

$$a = 1 - \frac{3}{4} = \frac{1}{4}$$

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Discrete random variables
Exercise B, Question 1

Question:

A discrete random variable X has probability distribution

x	0	1	2	3	4	5
$\mathbf{P}(X = x)$	0.1	0.1	0.3	0.3	0.1	0.1

a Find the probability that $X < 3$.

b Find the probability that $X > 3$.

c Find the probability that $1 < X < 4$.

Solution:

a $P(X < 3) = P(0) + P(1) + P(2) = 0.1 + 0.1 + 0.3 = \mathbf{0.5}$

b $P(X > 3) = P(4) + P(5) = 0.1 + 0.1 = \mathbf{0.2}$

c $P(1 < X < 4) = P(2) + P(3) = 0.3 + 0.3 = \mathbf{0.6}$

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Discrete random variables
Exercise B, Question 2

Question:

A discrete random variable X has probability distribution

x	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$

Find

a $\mathrm{P}(1 < X \le 3)$, **b** $\mathrm{P}(X < 2)$.

Solution:

a $\mathrm{P}(1 < X \le 3) = \mathrm{P}(2) + \mathrm{P}(3) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

b $\mathrm{P}(X < 2) = \mathrm{P}(0) + \mathrm{P}(1) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

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Discrete random variables
Exercise B, Question 3

Question:

A discrete random variable X has a probability distribution

x	1	2	3	4	5	6
$\mathrm{P}(X = x)$	0.1	0.1	0.15	0.25	0.3	0.1

a Draw up a table to show the cumulative distribution function F(x).
b Write down F(5). **c** Write down F(2.2).

Solution:

a

x	**1**	**2**	**3**	**4**	**5**	**6**
$\mathrm{F}(x_0)$	**0.1**	**0.2**	**0.35**	**0.60**	**0.9**	**1**

b F(5) = **0.9**

c F(2.2) = **0.2**

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Discrete random variables
Exercise B, Question 4

Question:

A discrete random variable has a cumulative distribution function F(x) given in the table.

x	0	1	2	3	4	5	6
F(x)	0	0.1	0.2	0.45	0.5	0.9	1

a Draw up a table to show the probability distribution X.
b Write down $P(X < 5)$. **c** Find $P(2 \leq X < 5)$.

Solution:

a

x	**1**	**2**	**3**	**4**	**5**	**6**
P($X = x$)	**0.1**	**0.1**	**0.25**	**0.05**	**0.4**	**0.1**

b $P(X < 5) = 0.5$

c $P(2 \leq X < 5) = 0.1 + 0.25 + 0.05 = 0.4$

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Discrete random variables
Exercise B, Question 5

Question:

5 The random variable X has a probability function

$$P(X = x) = \begin{cases} kx & x = 1,3,5 \\ k(x-1) & x = 2,4,6 \end{cases}$$

where k is a constant.

a Find the value of k.

b Draw a table giving the probability distribution of X.

c Find $P(2 \le X < 5)$.

d Find F(4).

e Find F(1.6).

Solution:

a

	1	2	3	4	5	6
P(*X* = *x*)	k	k	$3k$	$3k$	$5k$	$5k$

$k + k + 3k + 3k + 5k + 5k = 1$

$$\mathbf{k = \frac{1}{18}}$$

b

x	1	2	3	4	5	6
P(*X* = *x*)	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{5}{18}$

c $P(2 \le X < 5) = P(2) + P(3) + P(4) = \frac{1}{18} + \frac{1}{6} + \frac{1}{6} = \mathbf{\frac{7}{18}}$

d Remember F means the cumulative function

$F(4) = 1 - (P(6) + P(5)) = 1 - \left(\frac{5}{18} + \frac{5}{18}\right) = \frac{8}{18}$ or $\mathbf{\frac{4}{9}}$

(This could also be done by adding P(1) P(2) P(3) and P(4).)

e 1.6 lies below 2 but above 1. Because this is a **discrete** random variable F(1.6) is the same as F(1) which is $\mathbf{\frac{1}{18}}$

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Discrete random variables
Exercise B, Question 6

Question:

The discrete random variable X has the probability function

$$P(X = x) = \begin{cases} 0.1 & x = -2, -1 \\ \alpha & x = 0, 1 \\ 0.3 & x = 2 \end{cases}$$

a Find the value of α

b Draw a table giving the probability distribution of X.

c Write down the value of F(0.3).

Solution:

a

x	−2	−1	0	1	2
P(X = x)	**0.1**	**0.1**	α	α	**0.3**

$$\begin{aligned} 0.1 + 0.1 + \alpha + \alpha + 0.2 &= 1 \\ 2\alpha &= 0.6 \\ \alpha &= 0.3 \end{aligned}$$

b

x	−2	−1	0	1	2
P(X = x)	**0.1**	**0.1**	**0.25**	**0.25**	**0.3**

c F(0.3) = F (0) = 0.1 + 0.1 + 0.25 = 0.45

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Discrete random variables
Exercise B, Question 7

Question:

The discrete random variable X has a cumulative distribution function F(x) defined by

$$F(x) = \begin{cases} 0 & x = 0 \\ \frac{1+x}{6} & x = 1,2,3,4,5 \\ 1 & x > 5 \end{cases}$$

a Find $P(X \le 4)$.

b Show that $P(X = 4)$ is $\frac{1}{6}$.

c Find the probability distribution for X.

Solution:

a

x	1	2	3	4	5
F(x)	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1

$P(X \le 4) = \frac{5}{6}$

b $P(X = 4) = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

c

x	1	2	3	4	5
$P(X = x)$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

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Discrete random variables
Exercise B, Question 8

Question:

The discrete random variable X has a cumulative distribution function F(x) defined by $F(x) = \begin{cases} 0 & x = 0 \\ \frac{(x+k)^2}{16} & x = 1, 2 \text{ and } 3 \\ 1 & x > 3 \end{cases}$

a Find the value of k.

b Find the probability distribution for X.

Solution:

a $\frac{(x+k)^2}{16} = 1$ when $x = 3$

$\frac{(3+k)^2}{16} = 1$

$(3 + k)^2 = 16$

$3 + k = \pm 4$

$\boldsymbol{k = 1}$ (negative probabilities do not exist)

b

x	1	2	3
F(x)	$\frac{4}{16}$	$\frac{9}{16}$	1

So Probability distribution is

x	1	2	3
P($X = x$)	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{7}{16}$

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Discrete random variables
Exercise C, Question 1

Question:

Find $E(X)$ and $E(X^2)$ for the following distributions of x.

a

x	2	4	6	8
$P(X = x)$	0.3	0.3	0.2	0.2

b

x	1	2	3	4
$P(X = x)$	0.1	0.4	0.1	0.4

Solution:

a $E(X) = (2 \times 0.3) + (4 \times 0.3) + (6 \times 0.2) + (8 \times 0.2)$

$= 0.6 + 1.2 + 1.2 + 1.6 = \mathbf{4.6}$

$E(X^2) = (4 \times 0.3) + (16 \times 0.3) + (36 \times 0.2) + (64 \times 0.2)$

$= 1.2 + 4.8 + 7.2 + 12.8 = \mathbf{26}$

b $E(X) = (1 \times 0.1) + (2 \times 0.4) + (3 \times 0.1) + (4 \times 0.4)$

$= 0.1 + 0.8 + 0.3 + 1.6 = \mathbf{2.8}$

$E(X^2) = (1 \times 0.1) + (4 \times 0.4) + (9 \times 0.1) + (16 \times 0.4)$

$= 0.1 + 1.6 + 0.9 + 6.4 = \mathbf{9}$

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Discrete random variables
Exercise C, Question 2

Question:

A biased die has the probability distribution

x	1	2	3	4	5	6
$\mathbf{P}(X = x)$	0.1	0.1	0.1	0.2	0.4	0.1

Find E(X) and E(X^2).

Solution:

$$\mathrm{E}(X) = (1 \times 0.1) + (2 \times 0.1) + (3 \times 0.1) + (4 \times 0.2) + (5 \times 0.4) + (6 \times 0.1)$$

$$= 0.1 + 0.2 + 0.3 + 0.8 + 2.0 + 0.6 = \mathbf{4}$$

$$\mathrm{E}(X^2) = (1 \times 0.1) + (4 \times 0.1) + (9 \times 0.1) + (16 \times 0.2) + (25 \times 0.4) + (36 \times 0.1)$$

$$= 0.1 + 0.4 + 0.9 + 3.2 + 10 + 3.6 = \mathbf{18.2}$$

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Discrete random variables
Exercise C, Question 3

Question:

The random variable X has a probability function

$$P(X = x) = \begin{cases} \frac{1}{x} & x = 2,3,6 \\ 0 & \text{all other values} \end{cases}$$

a Construct a table giving the probability distribution of X.

b Work out E(X) and E(X^2).

c State with a reason whether or not $(E(X))^2 = E(X^2)$.

Solution:

a

x	2	3	6
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

b

$$E(X) = \left(2 \times \frac{1}{2}\right) + \left(3 \times \frac{1}{3}\right) + \left(6 \times \frac{1}{6}\right)$$

$$= 1 + 1 + 1 = \mathbf{3}$$

$$E(X^2) = \left(4 \times \frac{1}{2}\right) + \left(9 \times \frac{1}{3}\right) + \left(36 \times \frac{1}{6}\right)$$

$$= 2 + 3 + 6 = \mathbf{11}$$

c

$(E(X))^2 = 3 \times 3 = 9$

$E(X^2) = 11$

Therefore $(E(X))^2$ does not equal $E(X^2)$

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Discrete random variables
Exercise C, Question 4

Question:

Two coins are tossed 50 times. The number of heads is noted each time.

a Construct a probability distribution table for the number of heads when the two coins are tossed once, assuming that the two coins are unbiased.

b Work out how many times you would expect to get 0, 1 or 2 heads.

The following table shows the actual results.

Number of heads (h)	0	1	2
Frequency (f)	7	22	21

c State whether or not the actual frequencies obtained support the statement that the coins are unbiased. Give a reason for your answer.

Solution:

a

Number of heads (h)	0	1	2
$P(H = h)$	0.25	0.5	0.25

b

$0.25 \times 50 = 12.5$

$0.5 \times 50 = 25$

We would expect to get **1 head 25 times** and **0 or 2 heads 12.5 times each**.

c The coins would appear to be biased. There were far too many times when 2 heads appeared and not enough when 0 heads appeared.

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Discrete random variables
Exercise C, Question 5

Question:

The random variable X has the following probability distribution.

x	1	2	3	4	5
P(X = x)	0.1	a	b	0.2	0.1

Given E(X) = 2.9 find the value of a and the value of b.

Solution:

x	1	2	3	4	5
P(X = x)	0.1	a	b	0.2	0.1

The probabilities add up to 1 so

$0.1 + a + b + 0.2 + 0.1 = 1$

$a + b = 0.6$ **(1)**

and

$2.9 = (1 \times 0.1) + (2 \times a) + (3 \times b) + (4 \times 0.2) + (5 \times 0.1)$

$2.9 = 0.1 + 2\,a + 3\,b + 0.8 + 0.5$

$2\,a + 3\,b = 1.5$ **(2)**

multiply **(1)** by **(2)**

$2a + 2b = 1.2$ **(3)**

(2) minus **(3)**

$b =$ **0.3**

so from 3 $2a + 0.6 = 1.2$

$a =$ **0.3**

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Discrete random variables
Exercise C, Question 6

Question:

A fair spinner with equal sections numbered 1 to 5 is thrown 500 times. Work out how many times it can be expected to land on 3.

Solution:

x	1	2	3	4	5
$P(X = x)$	0.2	0.2	0.2	0.2	0.2

$0.2 \times 500 = 100$

We can expect it to land on 3 100 times.

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Discrete random variables
Exercise D, Question 1

Question:

For the following probability distribution

x	-1	0	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

a write down E(X).

b find Var(X).

Solution:

a By symmetry $E(X) = 1$

b $\text{Var } X = E(X^2) - (E(X))^2$

$E(X^2) = \frac{1}{5} + 0 + \frac{1}{5} + \frac{4}{5} + \frac{9}{5} = 3$

$(E(X))^2 = 1^2 = 1$

$\text{Var } X = 3 - 1 = \mathbf{2}$

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Discrete random variables
Exercise D, Question 2

Question:

Find the expectation and variance of each of the following distributions of X.

a

x	1	2	3
P(X = x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

b

x	−1	0	1
P(x = x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

c

x	−2	−1	1	2
P(X = x)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

Solution:

a Mean $= \text{E}(X) = \left(1\times\frac{1}{3}\right)+\left(2\times\frac{1}{2}\right)+\left(3\times\frac{1}{6}\right)=\frac{1}{3}+1+\frac{1}{2}=\mathbf{1\frac{5}{6}}$

$\text{E}(X^2)=\frac{1}{3}+2+\frac{9}{6}=3\frac{5}{6}$

$\text{Var }X=3\frac{5}{6}-\left(1\frac{5}{6}\right)^2=\mathbf{\frac{138}{36}}-\mathbf{\frac{121}{36}}=\mathbf{\frac{17}{36}}$

b Mean $= \text{E}(X) = \left(-1\times\frac{1}{4}\right)+\left(0\times\frac{1}{2}\right)+\left(1\times\frac{1}{4}\right)=0$ (or by symmetry = 0)

$\text{E}(X^2)=\left(1\times\frac{1}{4}\right)+\left(0\times\frac{1}{2}\right)+\left(1\times\frac{1}{4}\right)=\frac{1}{2}$

$\text{Var }X=\frac{1}{2}-0=\mathbf{\frac{1}{2}}$

c Mean $= \text{E}(X) = \left(-2\times\frac{1}{3}\right)+\left(-1\times\frac{1}{3}\right)+\left(1\times\frac{1}{6}\right)+\left(2\times\frac{1}{6}\right)=-\mathbf{\frac{1}{2}}$

$\text{E}(X^2)=\left(4\times\frac{1}{3}\right)+\left(1\times\frac{1}{3}\right)+\left(1\times\frac{1}{6}\right)+\left(4\times\frac{1}{6}\right)=\mathbf{2\frac{1}{2}}$

$\text{Var }X=2\frac{1}{2}-\left(-\frac{1}{2}\right)^2=\mathbf{2\frac{1}{4}}$

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Discrete random variables
Exercise D, Question 3

Question:

Given that Y is the score when a single unbiased die is rolled, find E(Y) and Var(Y).

Solution:

y	1	2	3	4	5	6
$P(Y = y)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(Y) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = 3\frac{1}{2}$$

$$E(X^2) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} = 15\frac{1}{6}$$

$$\text{Var } X = 15\frac{1}{6} - \left(3\frac{1}{2}\right)^2 = \mathbf{2\frac{11}{12}}$$

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Discrete random variables
Exercise D, Question 4

Question:

Two fair cubical dice are rolled and S is the sum of their scores.

a Find the distribution of S.

b Find E(S).

c Find Var(S).

Solution:

a

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

s	2	3	4	5	6	7	8	9	10	11	12
$P(S = s)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

b $E(S) = \frac{2+6+12+20+30+42+40+36+30+22+12}{36} = 7$ (or by symmetry $= 7$)

c $E(S^2) = \frac{4+18+48+100+180+294+320+324+300+242+144}{36}$

$= 54\frac{5}{6}$

$\text{Var}\, S = 54\frac{5}{6} - 7^2 = \mathbf{5\frac{5}{6}}$

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Discrete random variables
Exercise D, Question 5

Question:

Two fair tetrahedral (four-sided) dice are rolled and D is the difference between their scores.

a Find the distribution of D and show that $P(D = 3) = \frac{1}{8}$.

b Find E(D).

c Find Var(D).

Solution:

a

Diff.	1	2	3	4
1	0	1	2	3
2	1	0	1	2
3	2	1	0	1
4	3	2	1	0

d	0	1	2	3
$P(D = d)$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

From Distribution Table it can be seen that $P(D = 3) = \mathbf{\frac{1}{8}}$

b $E(D) = 0 + \frac{3}{8} + \frac{1}{2} + \frac{3}{8} = \mathbf{1\frac{1}{4}}$

c $E(D^2) = 0 + \frac{3}{8} + 1 + 1\frac{1}{8} = \mathbf{2\frac{1}{2}}$

$\text{Var}\,D = 2\frac{1}{2} - \left(1\frac{1}{4}\right)^2 = \mathbf{\frac{15}{16}}$

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Edexcel AS and A Level Modular Mathematics

Discrete random variables
Exercise D, Question 6

Question:

A fair coin is tossed repeatedly until a head appears or three tosses have been made. The random variable T represents the number of tosses of the coin.

a Show that the distribution of T is

t	1	2	3
P(T = t)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

b Find the expectation and variance of T.

Solution:

a $\mathrm{P(H)} = \frac{1}{2}$

$\mathrm{P(TH)} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\mathrm{P(TTH)} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$\mathrm{P(TTT)} = \frac{1}{8}$

$\mathrm{P}(T = 1) = \frac{1}{2}$

$\mathrm{P}(T = 2) = \frac{1}{4}$

$\mathrm{P}(T = 3) = \frac{1}{4}$

b $\mathrm{E}(T) = 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{4} = \mathbf{1\frac{3}{4}}$

$\mathrm{Var}(T) = 1 \times \frac{1}{2} + 4 \times \frac{1}{4} + 9 \times \frac{1}{4} - \left(1\frac{3}{4}\right)^2 = \mathbf{\frac{11}{16}}$

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Edexcel AS and A Level Modular Mathematics

Discrete random variables
Exercise D, Question 7

Question:

The random variable X has the following distribution:

x	1	2	3
P(X = x)	a	b	a

where a and b are constants.

a Write down E(X).

b Given that Var(X) = 0.75, find the values of a and b.

Solution:

a E(X) = 2 by symmetry

b $\sum p(x) = 2a + b = 1 \qquad (1)$

$$\begin{aligned}\text{Var}(X) &= \text{E}(X^2) - (E(X))^2\\ &= 10a + 4b - 2^2\\ &= 10a + 4b - 4 = \frac{3}{4} \qquad (2)\end{aligned}$$

$$\begin{aligned}10a + 4b &= 4\frac{3}{4} \qquad \text{from (2)}\\ \underline{8a + 4b = 4} & \qquad \text{from (1)} \times 4\\ 2a &= \frac{3}{4}\end{aligned}$$

$$\begin{aligned}a &= \frac{3}{8}\\ b &= 1 - 2a\\ &= 1 - \frac{3}{4}\\ &= \mathbf{\frac{1}{4}}\end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Discrete random variables
Exercise E, Question 1

Question:

$E(X) = 4$, $Var(X) = 10$

Find

a $E(2X)$,

b $Var(2X)$.

Solution:

Remember mean is $E(X)$ and variance is $Var\ X$.

a $E(2X) = 2\ E(X) = 2 \times 4 =$ **8**

b $Var(2X) = 2^2\ Var\ X = 4 \times 10 =$ **40**

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Discrete random variables
Exercise E, Question 2

Question:

$E(X) = 2$, $\text{Var}(X) = 6$

Find

a $E(3X)$,

b $E(3X + 1)$,

c $E(X - 1)$,

d $E(4 - 2X)$,

e $\text{Var}(3X)$,

f $\text{Var}(3X + 1)$,

g $\text{Var}(X - 1)$.

Solution:

a $E(3X) = 3\,E(X) = 3 \times 2 = \mathbf{6}$

b $E(3X + 1) = 3\,E(X) + 1 = (3 \times 2) + 1 = \mathbf{7}$

c $E(X - 1) = E(X) - 1 = 2 - 1 = \mathbf{1}$

d $E(4 - 2X) = 4 - 2E(X) = 4 - 2 \times 2 = \mathbf{0}$

e $\text{Var}(3X) = 3^2\,\text{Var}\,X = 9 \times 6 = \mathbf{54}$

f $\text{Var}(3X + 1) = 3^2\,\text{Var}\,X = \mathbf{54}$

g $\text{Var}(X - 1) = \text{Var}\,X = \mathbf{6}$

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Edexcel AS and A Level Modular Mathematics

Discrete random variables
Exercise E, Question 3

Question:

The random variable X has an expectation of 3 and a variance of 9.

Find

a $E(2X + 1)$,

b $E(2 + X)$,

c $\text{Var}(2X + 1)$,

d $\text{Var}(2 + X)$.

Solution:

a $E(2X + 1) = 2\,E(X) + 1 = (2 \times 3) + 1 = \mathbf{7}$

b $E(2 + X) = E(X + 2) = E(X) + 2 = 3 + 2 = \mathbf{5}$

c $\text{Var}(2X + 1) = 2^2\,\text{Var}(X) = 4 \times 9 = \mathbf{36}$

d $\text{Var}(2 + X) = \text{Var}(X + 2) = \text{Var}(X) = \mathbf{9}$

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Edexcel AS and A Level Modular Mathematics

Discrete random variables

Exercise E, Question 4

Question:

The random variable X has a mean μ and standard deviation σ.

Find, in terms of μ and σ

a $E(4X)$,

b $E(2X + 2)$,

c $E(2X - 2)$,

d $Var(2X + 2)$,

e $Var(2X - 2)$.

Solution:

a $E(4X) = 4\ E(X) = 4\mu$

b $E(2X + 2) = 2\ E(X) + 2 = \mathbf{2\ \mu + 2}$

c $E(2X - 2) = 2\ E(X) - 2 = \mathbf{2\ \mu - 2}$

d $Var(2X + 2) = 2^2\ Var(X) = \mathbf{4\sigma^2}$ (Remember Standard deviation is σ so variance is σ^2)

e $Var(2X - 2) = 2^2\ Var(X) = \mathbf{4\sigma^2}$

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Discrete random variables
Exercise E, Question 5

Question:

The random variable Y has mean 2 and variance 9.

Find:

a $E(3Y + 1)$,

b $E(2 - 3Y)$,

c $Var(3Y + 1)$,

d $Var(2 - 3Y)$,

e $E(Y^2)$,

f $E[(Y - 1)(Y + 1)]$.

Solution:

a $E(3Y + 1) = 3E(Y) + 1 = 3 \times 2 + 1 = \mathbf{7}$

b $E(2 - 3Y) = 2 - 3E(Y) = 2 - 3 \times 2 = -\mathbf{4}$

c $Var(3Y + 1) = 3^2 Var(Y) = 9 \times 9 = \mathbf{81}$

d $Var(2 - 3Y) = (-3)^2\ Var(Y) = 9 \times 9 = \mathbf{81}$

e $E(Y^2) = Var(Y) + [E(Y)]^2 = 9 + 2^2 = \mathbf{13}$

f $E[(Y - 1)(Y + 1)] = E(Y^2 - 1) = E(Y^2) - 1 = 13 - 1 = \mathbf{12}$

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Discrete random variables
Exercise E, Question 6

Question:

The random variable T has a mean of 20 and a standard deviation of 5.

It is required to scale T by using the transformation $S = 3T + 4$.

Find $E(S)$ and $Var(S)$.

Solution:

$E(S) = E(3T + 4) = 3\ E(S) + 4 = (3 \times 20) + 4 = \mathbf{64}$

$Var(T) = 5 \times 5 = 25$

$Var\ (S) = Var(3T + 4) = 3^2 Var\ T = 9 \times 25 = \mathbf{225}$

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Discrete random variables
Exercise E, Question 7

Question:

A fair spinner is made from the disc in the diagram and the random variable X represents the number it lands on after being spun.

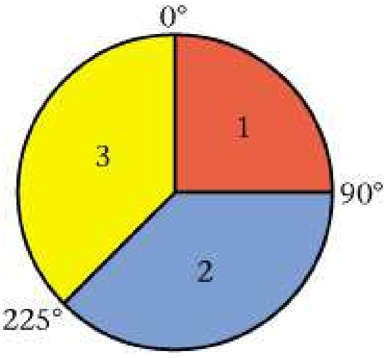
a Write down the distribution of X.

b Work out E(X).

c Find Var(X).

d Find E(2X + 1).

e Find Var(3X − 1).



Solution:

a

x	1	2	3
P($X = x$)	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$

b $\mathrm{E}(X) = \left(1 \times \frac{1}{4}\right) + \left(2 \times \frac{3}{8}\right) + \left(3 \times \frac{3}{8}\right) = \mathbf{2\frac{1}{8}}$

c $\mathrm{E}(X^2) = \left(1 \times \frac{1}{4}\right) + \left(4 \times \frac{3}{8}\right) + \left(9 \times \frac{3}{8}\right) = \mathbf{5\frac{1}{8}}$

$\mathrm{Var}(X) = 5\frac{1}{8} - \left(2\frac{1}{8}\right)^2 = \mathbf{\frac{39}{64}}$

d $\mathrm{E}(2X + 1) = 2\mathrm{E}(X) + 1 = \left(2 \times 2\frac{1}{8}\right) + 1 = \mathbf{5\frac{1}{4}}$

e $\mathrm{Var}(3X - 1) = 3^2\,\mathrm{Var}\,X = 9 \times \mathbf{\frac{39}{64}} = \mathbf{5\frac{31}{64}}$

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Discrete random variables
Exercise E, Question 8

Question:

The discrete variable X has the probability distribution

x	−1	0	1	2
P(*X* = *x*)	0.2	0.5	0.2	0.1

a Find E(X),

b Find Var (X),

c Find $E(\frac{1}{3}X+1)$,

d Find $\mathrm{Var}(\frac{1}{3}X+1)$.

Solution:

a E(X) = − 0.2 + 0 + 0.2 + 0.2 = **0.2**

b E(X^2) = 0.2 + 0 + 0.2 + 0.4 = **0.8**

Var(X) = 0.8 − 0.2^2 = 0.8 − 0.04 = **0.76**

c $E\left(\frac{1}{3}X+1\right)=\frac{1}{3}E(X)+1=\left(\frac{1}{3}\times 0.2\right)+1=\left(\frac{1}{3}\times\frac{1}{5}\right)+1=\mathbf{1\frac{1}{15}}(1.0\dot{6})$

d $\mathrm{Var}\left(\frac{1}{3}X+1\right)=\left(\frac{1}{3}\right)^2\mathrm{Var}X=\frac{1}{9}\times 0.76=\frac{1}{9}\times\frac{19}{25}=\mathbf{\frac{19}{225}}(0.08\dot{4})$

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Discrete random variables
Exercise F, Question 1

Question:

X is a discrete uniform distribution over the numbers 1, 2, 3, 4 and 5. Work out the expectation and variance of X.

Solution:

Expectation $= \frac{n+1}{2} = \frac{5+1}{2} = \mathbf{3}$

Variance $= \frac{(n+1)(n-1)}{12} = \frac{(5+1)(5-1)}{12} = \mathbf{2}$

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Discrete random variables
Exercise F, Question 2

Question:

Seven similar balls are placed in a bag. The balls have the numbers 1 to 7 on them. A ball is drawn out of the bag. The variable X represents the number on the ball.

a Find E(X).

b Work out Var(X).

Solution:

a $n = 7$

$\mathrm{E}(X) = \frac{n+1}{2} = 4$

b $\mathrm{Var}(X) = \frac{(n+1)(n-1)}{12} = \mathbf{4}$

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Discrete random variables
Exercise F, Question 3

Question:

A fair die is thrown once and the random variable X represents the value on the upper face.

a Find the expectation and variance of X.

b Calculate the probability that X is within one standard deviation of the expectation.

Solution:

a Expectation $= \frac{n+1}{2} = \frac{6+1}{2} = \mathbf{3\frac{1}{2}}$

$\mathrm{Var}(X) = \frac{(n+1)(n-1)}{12} = \frac{7 \times 5}{12} = \mathbf{2\frac{11}{12}} = \mathbf{2.91\dot{6}}$

b

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\sigma = \sqrt{2.91\dot{6}} = 1.7078$

Therefore we want between 3.5 − 1.7078 = 1.7922 and 3.5 + 1.7078 = 5.2078

$P(1.7922 \le X \le 5.2078) = p(2) + p(3) + p(4) + p(5) = \mathbf{\frac{2}{3}}$

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Discrete random variables
Exercise F, Question 4

Question:

A card is selected at random from a pack of cards containing the even numbers 2, 4, 6, …, 20. The variable X represents the number on the card.

a Find $P(X > 15)$.

b Find the expectation and variance of X.

Solution:

a This is a uniform distribution.

x	2	4	6	8	10	12	14	16	18	20
$P(X = x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$P(X > 15) = P(16) + P(18) + P(20) = \mathbf{\frac{3}{10}}$

b Let R be a uniform distribution over the numbers 1,2, … 10.

Then $X = 2R$

$E(R) = \frac{n+1}{2} = 5.5$

$\text{Var}(R) = \frac{(n+1)(n-1)}{12} = \frac{99}{12} = 8\frac{1}{4} = 8.25$

$E(X) = 2E(R) = 2 \times 5.5 = \mathbf{11}$

$\text{Var}(X) = \text{Var}(2R) = 2^2\text{Var}(R) = 4 \times 8.25 = \mathbf{33}$

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Discrete random variables
Exercise F, Question 5

Question:

A card is selected at random from a pack of cards containing the odd numbers 1, 3, 5, …, 19. The variable X represents the number on the card.

a Find $P(X > 15)$.

b Find the expectation and variance of X.

Solution:

y	1	3	5	7	9	11	13	15	17	19
$P(Y = y)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

a $P(X > 15) = P(17) + P(19) = \mathbf{\frac{1}{5}}$

b $Y = X - 1$ (X as in previous question)

$E(Y) = E(X - 1) = E(X) - 1 = 11 - 1 = \mathbf{10}$

$\text{Var}(Y) = \text{Var}(X - 1) = \text{Var}(X) = \mathbf{33}$

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Discrete random variables
Exercise F, Question 6

Question:

A straight line is drawn on a piece of paper. The line is divided into four equal lengths and the segments are marked 1, 2, 3 and 4. In a party game a person is blindfolded and asked to mark a point on the line and the number of the segment is recorded. A discrete uniform distribution over the set (1, 2, 3, 4) is suggested as model for this distribution. Comment on this suggestion.

Solution:

A discrete uniform distribution is not likely to be a good model for this distribution. The game depends on the skill of the player. The points are quite likely to cluster around the middle.

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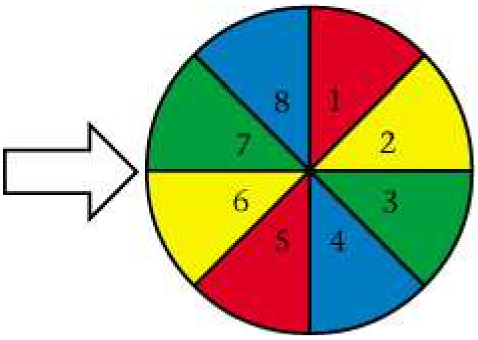
Discrete random variables
Exercise F, Question 7

Question:

In a fairground game the spinner shown is used.

It cost 5p to have a go on the spinner.

The spinner is spun and the number of pence shown is returned to the contestant.



If X is the number which comes up on the next spin,

a name a suitable model for X,

b find E(X),

c find Var(X),

d explain why you should not expect to make money at this game if you have a large number of goes.

Solution:

a A discrete uniform distribution

b $n = 8$

$\mathrm{E}(X) = \frac{n+1}{2} = \mathbf{4.5}$

c $\mathrm{Var}(X) = \frac{(n+1)(n-1)}{12} = \frac{63}{12} = \mathbf{5\frac{1}{4}}$ or **5.25**

d The expected winnings are less than the 5p stake.

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Edexcel AS and A Level Modular Mathematics

Discrete random variables
Exercise G, Question 1

Question:

The random variable X has probability function

$P(X = x) = \frac{x}{21} x = 1, 2, 3, 4, 5, 6.$

a Construct a table giving the probability distribution of X.

Find

b $P(2 < X \le 5)$,

c $E(X)$,

d $Var(X)$,

e $Var(3 - 2X)$.

Solution:

a

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$

b $P(3) + P(4) + P(5) = \frac{3}{21} + \frac{4}{21} + \frac{5}{21} = \mathbf{\frac{12}{21}}$

c $E(X) = \left(1 \times \frac{1}{21}\right) + \left(2 \times \frac{2}{21}\right) + \left(3 \times \frac{3}{21}\right) + \left(4 \times \frac{4}{21}\right) + \left(5 \times \frac{5}{21}\right) + \left(6 \times \frac{6}{21}\right)$

$= \frac{91}{21} = \mathbf{4\frac{1}{3}} (4.\dot{3})$

d $E(X^2) = \left(1 \times \frac{1}{21}\right) + \left(4 \times \frac{2}{21}\right) + \left(9 \times \frac{3}{21}\right) + \left(16 \times \frac{4}{21}\right) + \left(25 \times \frac{5}{21}\right) + \left(36 \times \frac{6}{21}\right)$

$= \frac{441}{21} = \mathbf{21}$

$Var(X) = 21 - \left(4\frac{1}{3}\right)^2 = 21 - 18\frac{7}{9} = \mathbf{2\frac{2}{9}}$

e $Var(3 - 2X) = Var(-2X + 3) = (-2)^2 Var(X) = 4 \times 2\frac{2}{9} = \mathbf{8\frac{8}{9}}$

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Discrete random variables
Exercise G, Question 2

Question:

The discrete random variable X has the probability distribution shown.

x	−2	−1	0	1	2	3
P(X = x)	0.1	0.2	0.3	r	0.1	0.1

Find

a r,

b $P(-1 \le X < 2)$,

c F(0.6),

d $E(2X + 3)$,

e $Var(2X + 3)$.

Solution:

a $0.1 + 0.2 + 0.3 + r + 0.1 + 0.1 = 1$

$r = 1 - 0.8 =$ **0.2**

b $P(-1) + P(0) + P(1) = 0.2 + 0.3 + 0.2 =$ **0.7**

c

x	−2	−1	0	1	2	3
F(X)	0.1	0.3	0.6	0.8	0.9	1

$F(0.6) = F(0) =$ **0.6**

d $E(X) = (-0.2) + (-0.2) + 0 + 0.2 + 0.2 + 0.3 = 0.3$

$E(2X + 3) = 2E(X) + 3 = (2 \times 0.3) + 3 =$ **3.6**

e $E(X^2) = 0.4 + 0.2 + 0 + 0.2 + 0.4 + 0.9 = 2.1$

$Var(X) = 2.1 - 0.3^2 = 2.01$

$Var(2X + 3) = 2^2 \, Var\, X = 4 \times 2.01 =$ **8.04**

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Discrete random variables
Exercise G, Question 3

Question:

A discrete random variable X has the probability distribution shown in the table below.

x	0	1	2
$\mathbf{P}(X = x)$	$\frac{1}{5}$	b	$\frac{1}{5} + b$

a Find the value of b.

b Show that $\mathrm{E}(X) = 1.3$.

c Find the exact value of $\mathrm{Var}(X)$.

d Find the exact value of $\mathrm{P}(X \le 1.5)$.

Solution:

a

x	0	1	2
$\mathrm{P}(X = x)$	$\frac{1}{5}$	b	$b + \frac{1}{5}$

$$\frac{1}{5} + b + b + \frac{1}{5} = 1$$

$$2b = 1 - \frac{2}{5} = \frac{3}{5}$$

$$b = \frac{3}{10}$$

b $\mathrm{E}(X) = \left(0 \times \frac{1}{5}\right) + \left(1 \times \frac{3}{10}\right) + \left(2 \times \frac{5}{10}\right) = 0 + \frac{3}{10} + 1 = 1.3$

c $\mathrm{E}(X^2) = \left(0 \times \frac{1}{5}\right) + \left(1 \times \frac{3}{10}\right) + \left(4 \times \frac{5}{10}\right) = 0 + \frac{3}{10} + 2 = 2.3$

$\mathrm{Var}(X) = 2.3 - 1.3^2 = 0.61$

d $\mathrm{P}(0) + \mathrm{P}(1) = \frac{1}{5} + \frac{3}{10} = 0.5$

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Discrete random variables
Exercise G, Question 4

Question:

The discrete random variable X has a probability function

$$P(X = x) = \begin{cases} k(1-x) & x = 0,1 \\ k(x-1) & x = 2,3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

a Show that $k = \frac{1}{4}$.

b Find E(X) and show that E(X^2) = 5.5.

c Find Var(2X − 2).

Solution:

a $k(1 - 0) + k(1 - 1) + k(2 - 1) + k(3 - 1) = 1$

$k + k + 2k = 1$

$4k = 1$

$k = \mathbf{\frac{1}{4}}$

b

x	0	1	2	3
$P(X = x)$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$

$$E(X) = 0 + 0 + \frac{1}{2} + \frac{3}{2} = 2$$

$$E(X^2) = \left(0 \times \frac{1}{4}\right) + (1 \times 0) + \left(4 \times \frac{1}{4}\right) + \left(9 \times \frac{1}{2}\right) = 1 + 4.5 = \mathbf{5.5}$$

c Var (X) = 5.5 − 4 = 1.5

Var (2X − 2) = 4 × 1.5 = **6**

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Edexcel AS and A Level Modular Mathematics

Discrete random variables
Exercise G, Question 5

Question:

A discrete random variable X has the probability distribution,

x	0	1	2	3
$\mathbf{P}(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$

Find

a $P(1 < X \le 2)$,

b $F(1.5)$,

c $E(X)$,

d $E(3X - 1)$,

e $\mathrm{Var}(X)$.

Solution:

a $P(1 < X \le 2) = P(2) = \mathbf{\frac{1}{8}}$

b $F(1.5) = F(1) = \frac{1}{4} + \frac{1}{2} = \mathbf{\frac{3}{4}}$

c $E(X) = 0 + \frac{1}{2} + \frac{1}{4} + \frac{3}{8} = \frac{9}{8} = \mathbf{1\frac{1}{8}}$

d $E(3X - 1) = 3\,E(X) - 1 = 3\frac{3}{8} - 1 = \mathbf{2\frac{3}{8}}$

e $E(X^2) = 0 + \frac{1}{2} + \frac{1}{2} + \frac{9}{8} = \mathbf{2\frac{1}{8}}$

$\mathrm{Var}(X) = 2\frac{1}{8} - \left(1\frac{1}{8}\right)^2 = \mathbf{\frac{55}{64}}$

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Discrete random variables
Exercise G, Question 6

Question:

A discrete random variable is such that each of its values is assumed to be equally likely.

a Write the name of the distribution.

b Give an example of such a distribution.

A discrete random variable X as defined above can take values 0, 1, 2, 3, and 4.

Find

c E(X),

d Var (X).

Solution:

a A discrete uniform distribution

b Any distribution where all the probabilities are the same. An example is throwing a fair die.

c

x	0	1	2	3	4
P($X = x$)	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\mathrm{E}(X) = 0 + \frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} = \mathbf{2}$ (or $\frac{5+1}{2} - 1 = 2$) (OR use symmetry)

d $\mathrm{E}(X^2) = 0 + \frac{1}{5} + \frac{4}{5} + \frac{9}{5} + \frac{16}{5} = \mathbf{6}$ $\left(\text{or Var}(X) = \frac{(5+1)(5-1)}{12} = 2\right)$

Var (X) = 6 − 4 = **2**

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Discrete random variables
Exercise G, Question 7

Question:

The random variable X has a probability distribution

x	1	2	3	4	5
P(X = x)	0.1	p	q	0.3	0.1

a Given that E(X) = 3.1, write down two equations involving p and q.

Find

b the value of p and the value of q,

c Var(X),

d Var($2X - 3$).

Solution:

a $0.1 + p + q + 0.3 + 0.1 = 1$

$p + q = 0.5$ **1**

$0.1 + 2p + 3q + 1.2 + 0.5 = 3.1$

$\mathbf{2p + 3q = 1.3}$ **2**

b $2p + 3q = 1.3$ (equation 2)

$2p + 2q = 1$ (equation 1 times 2) **3**

$\mathbf{q = 0.3}$ (equation 2–3)

$p + 0.3 = 0.5$

$\mathbf{p = 0.2}$

c

x	1	2	3	4	5
P(X = x)	0.1	0.2	0.3	0.3	0.1

E(X) = 0.1 + 0.4 + 0.9 + 1.2 + 0.5 = 3.1

E(X^2) = 0.1 + 0.8 + 2.7 + 4.8 + 2.5 = 10.9

Var (X) = $10.9 - 3.1^2$ = **1.29**

d Var $(2X + 3) = 4$ Var $(X) = 4 \times 1.29 =$ **5.16**

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Discrete random variables
Exercise G, Question 8

Question:

The random variable X has probability function

$$\mathrm{P}(X = x) = \begin{cases} kx & x = 1,2 \\ k(x-2) & x = 3,4,5 \end{cases}$$

where k is a constant.

a Find the value of k.

b Find the exact value of $\mathrm{E}(X)$.

c Show that, to three significant figures, $\mathrm{Var}(X) = 2.02$.

d Find, to one decimal place, $\mathrm{Var}(3 - 2X)$.

Solution:

a

x	1	2	3	4	5
$\mathrm{P}(X = x)$	k	$2k$	k	$2k$	$3k$

$k + 2k + k + 2k + 3k = 1$

$9k = 1$

k$= \frac{1}{9}$

b $\mathrm{E}(X) = \frac{1}{9} + \frac{4}{9} + \frac{3}{9} + \frac{8}{9} + \frac{15}{9} = \frac{31}{9} = \mathbf{3\frac{4}{9}}$

c $\mathrm{E}\left(X^2\right) = \frac{1}{9} + \frac{8}{9} + 1 + \frac{32}{9} + \frac{75}{9} = \frac{125}{9} = 13\frac{8}{9}$

$\mathrm{Var}\ (X) = 13\frac{8}{9} - \left(3\frac{4}{9}\right)^2 = 13.888 - 11.864 = \mathbf{2.02}$ to 3 sig figs

d $\mathrm{Var}\ (3 - 2X) = (-2)^2\ \mathrm{Var}\ (X) = 4 \times 2.02 = \mathbf{8.1}$ to 1dp

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Discrete random variables
Exercise G, Question 9

Question:

The random variable X has the discrete uniform distribution

$\mathrm{P}(X = x) = \frac{1}{6}$ $x = 1,2,3,4,5,6.$

a Write down E(X) and show that $\mathrm{Var}(X) = \frac{35}{12}$.

b Find E($2X - 1$).

c Find Var($3 - 2X$).

Solution:

a $\mathrm{E}(X) = 3.5 = 3\frac{1}{2}$

$\mathrm{E}\left(X^2\right) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} = \frac{91}{6} = 15\frac{1}{6}$

$\mathrm{Var}\ (X) = 15\frac{1}{6} - \left(3\frac{1}{2}\right)^2 = \frac{91}{6} - \frac{49}{4} = \mathbf{\frac{35}{12}}$

b $\mathrm{E}(2X - 1) = 2\ \mathrm{E}(X) - 1 = 7 - 1 = \mathbf{6}$

c $\mathrm{Var}\ (3 - 2X) = 4\ \mathrm{Var}\ (X) = 4 \times \frac{35}{12} = \frac{35}{3} = \mathbf{11\frac{2}{3}}$ or **11.67** to 2 dp

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Discrete random variables
Exercise G, Question 10

Question:

The random variable X has probability function

$\mathrm{p}(x) = \frac{(3x-1)}{26}$ $x = 1,2,3,4.$

a Construct a table giving the probability distribution of X.

Find

b $\mathrm{P}(2 < X \leq 4)$,

c the exact value of $\mathrm{E}(X)$.

d Show that $\mathrm{Var}(X) = 0.92$ to two significant figures.

e Find $\mathrm{Var}(1 - 3X)$.

Solution:

a

x	1	2	3	4
$\mathrm{P}(X = x)$	$\frac{2}{26}$	$\frac{5}{26}$	$\frac{8}{26}$	$\frac{11}{26}$

b $\mathrm{P}(2 < X \leq 4) = \mathrm{P}(3) + \mathrm{P}(4) = \mathbf{\frac{19}{26}}$

c $\mathrm{E}(X) = \frac{2}{26} + \frac{10}{26} + \frac{24}{26} + \frac{44}{26} = \frac{80}{26} = \mathbf{3\frac{1}{13}}$

d $\mathrm{E}\left(X^2\right) = \frac{2}{26} + \frac{20}{26} + \frac{72}{26} + \frac{176}{26} = \frac{270}{26} = 10\frac{10}{26} = \mathbf{10\frac{5}{13}}$

$\mathrm{Var}\ (X) = 10\frac{5}{13} - \left(3\frac{1}{13}\right)^2 = 10.385\ldots - 9.467\ldots = \mathbf{0.92}$

e $\mathrm{Var}(1 - 3X) = (-3)^2\ \mathrm{Var}\ (X) = 9 \times 0.92 = \mathbf{8.28}$