Probability Exercise A, Question 1

Question:

For each of the following experiments, identify the sample space and find the probability of the event specified.

Throwing a six sided die once and recording if the number face up is odd or even. Find the probability of an even number landing face up.

Solution:

O E O E O E
1 2 3 4 5 6
P(Even) =
$$\frac{3}{6} = \frac{1}{2}$$

O = odd, E = Even Score on die

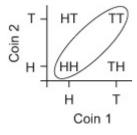
Probability Exercise A, Question 2

Question:

For the following experiment, identify the sample space and find the probability of the event specified.

Tossing two coins. Find the probability of the same outcome on each coin.

Solution:



$$P(Same outcome) = \frac{2}{4} = \frac{1}{2}$$

Probability Exercise A, Question 3

Question:

For the following experiment, identify the sample space and find the probability of the event specified.

A card is drawn from a pack of 52 playing cards. Find the probability that the card is a heart.

Solution:

 $S = \{Spades, Clubs, Diamonds, Hearts\}$

$$P(Heart) = \frac{1}{4}$$

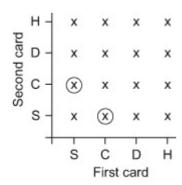
Probability Exercise A, Question 4

Question:

For the following experiment, identify the sample space and find the probability of the event specified.

A card is drawn from a pack of 52 playing cards, its suit is recorded then it is replaced and another card is drawn. Find the probability of drawing a spade and a club in any order.

Solution:



P(Spade *AND* Club) =
$$\frac{2}{16} = \frac{1}{8}$$

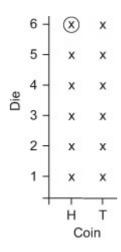
Probability Exercise A, Question 5

Question:

For the following experiment, identify the sample space and find the probability of the event specified.

Throwing a die and tossing a coin. Find the probability of a head and a 6.

Solution:



P(H and 6) =
$$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

Probability Exercise A, Question 6

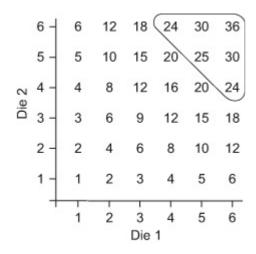
Question:

For each of the following experiments, identify the sample space and find the probability of the event specified.

Throwing two six-sided dice and recording the product of the values on the two sides that are uppermost.

Find the probability of the answer being greater than or equal to 24.

Solution:



 $P(\text{Product} \ge 24) = \frac{6}{36} = \frac{1}{6}.$

Probability Exercise B, Question 1

Question:

A card is chosen at random from a pack of 52 playing cards. C is the event 'the card chosen is a club' and K is the event 'the card chosen is a King'. Find these.

a
$$P(K)$$
 b $P(C)$ **c** $P(C \cap K)$ **d** $P(C \cup K)$ **e** $P(C')$ **f** $P(K' \cap C)$

Solution:

a.
$$P(K) = \frac{4}{52} = \frac{1}{13}$$

b. $P(C) = \frac{1}{4}$
c. $P(C \cap K) = \frac{1}{52}$
d. $P(C \cup K) = \frac{16}{52} = \frac{4}{13}$
e. $P(C') = \frac{3}{4}$
f. $P(K' \cap C) = \frac{12}{52} = \frac{3}{13}$

Probability Exercise B, Question 2

Question:

There are 25 students in a certain tutor group at Philips College. There are 16 students in the tutor group studying German, 14 studying French and six students studying both French and German.

Find the probability that a randomly chosen student in the tutor group

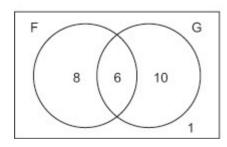
a studies French,

b studies French and German,

c studies French but not German,

d does not study French or German.

Solution:



a.
$$P(F) = \frac{14}{25}$$

b.
$$P(F \cap G) = \frac{6}{25}$$

c. P(French but not German) =
$$\frac{8}{25}$$

d. P(N of French or German) =
$$\frac{1}{25}$$

Probability Exercise B, Question 3

Question:

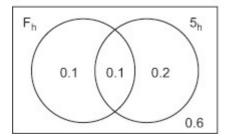
On a firing range, a rifleman has two attempts to hit a target. The probability of hitting the target with the first shot is 0.2 and the probability of hitting with the second shot is 0.3. The probability of hitting the target with both shots is 0.1.

Find the probability of

a missing the target with both shots,

b hitting with the first shot and missing with the second.

Solution:



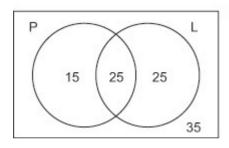
- a. P(Missing with both) = 0.6
- b. P(Hitting then Missing) = 0.1

Probability Exercise B, Question 4

Question:

Of all the households in the UK, 40% have a plasma TV and 50% have a laptop computer. There are 25% of households that have both a plasma TV and a laptop. Find the probability that a household chosen at random has either a plasma TV or a laptop computer but not both.

Solution:



P(P or L but not both) =
$$\frac{15}{100} + \frac{25}{100}$$

= 0.4

Probability Exercise B, Question 5

Question:

There are 125 diners in a restaurant who were surveyed to find out if they had ordered garlic bread, beer or cheesecake.

- 15 diners had ordered all three items
- 43 diners had ordered garlic bread
- 40 diners had ordered beer
- 44 diners had ordered cheesecake
- 20 had ordered beer and cheesecake
- 26 had ordered garlic bread and cheesecake
- 25 had ordered garlic bread and beer.

A diner is chosen at random. Find the probability that the diner ordered

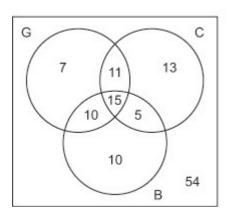
a all three items,

b beer but not cheesecake and not garlic bread,

c garlic bread and beer but not cheesecake,

d none of these items.

Solution:



a. P(all there) =
$$\frac{15}{125} = \frac{3}{25}$$

h

P(Beer but not cheesemake and not garlic bread) = $\frac{10}{125} = \frac{2}{25}$

c. P(Garlic bread and beer but not cheesemake) = $\frac{10}{125} = \frac{2}{25}$

d. P(None) = $\frac{54}{125}$

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Probability Exercise B, Question 6

Question:

A group of 275 people at a music festival were asked if they play guitar, piano or drums.

- one person plays all three instruments
- 65 people play guitar and piano
- 10 people play piano and drums
- 30 people play guitar and drums
- 15 people play piano only
- 20 people play guitar only
- 35 people play drums only

a Draw a Venn diagram to represent this information.

A festival goer is chosen at random from the group.

Find the probability that the person chosen

b plays piano

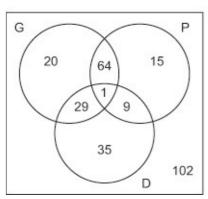
c plays at least two of guitar, piano or drums

d plays exactly one of the instruments

e plays none of the instruments.

Solution:

a.



b. P(Plays piano) =
$$\frac{89}{275}$$

d. P(Plays one) =
$$\frac{70}{275} = \frac{14}{55}$$

c. P(At least 2) =
$$\frac{103}{275}$$

e. P(Plays none) =
$$\frac{102}{275}$$

Probability Exercise C, Question 1

Question:

A and B are two events and P(A) = 0.5, P(B) = 0.2 and $P(A \cup B) = 0.1$.

Find

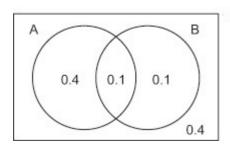
a $P(A \cup B)$,

b P(B'),

 $\mathbf{c} P(A \cap B'),$

d P($A \cup B'$).

Solution:



a. $P(A \cup B) = 0.6$

b. P(B') = 0.8

c. $P(A \cap B') = 0.4$

d. $P(A \cup B') = 0.9$

Probability

Exercise C, Question 2

Question:

A and C are two events and P(A) = 0.4, P(B) = 0.5 and $P(A \cup B) = 0.6$.

Find

a
$$P(A \cap B)$$
,

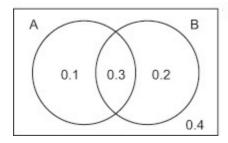
 $\mathbf{b} \; \mathbf{P}(A'),$

$$\mathbf{c} P(A \cup B'),$$

d $P(A' \cup B)$.

Solution:

a. $P(A \cap B) = 0.4 + 0.5 - 0.6 = 0.3$



b. P(A') = 0.6

c. $P(A \cup B') = 0.8$

d. $P(A' \cup B) = 0.9$

Probability

Exercise C, Question 3

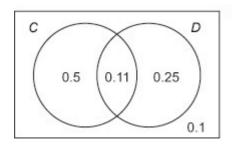
Question:

C and D are two events and P(D) = 0.4, $P(C \cap D) = 0.15$ and $P(C' \cap D') = 0.1$.

Find

a $P(C' \cap D)$, **b** $P(C \cap D')$, **c** P(C), **d** $P(C' \cap D')$.

Solution:



a.
$$P(C' \cap D) = 0.25$$

b.
$$P(C \cap D') = 0.5$$

c.
$$P(C) = 0.65$$

d.
$$P(C' \cap D') = 0.1$$

Probability Exercise C, Question 4

Question:

There are two events T and Q where $P(T) = P(Q) = 3P(T \cap Q)$ and $P(T \cup Q) = 0.75$.

Find

$$\mathbf{a} \ \mathbf{P}(T \cup Q), \qquad \mathbf{b} \ \mathbf{P}(T), \qquad \mathbf{c} \ \mathbf{P}(Q^{'}),$$

$$\mathbf{d} \ \mathbf{P}(T^{'} \cap Q^{'}), \qquad \mathbf{e} \ \mathbf{P}(T \cap Q^{'}).$$

Solution:

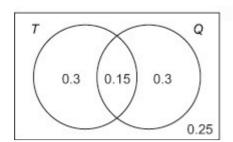
a.

$$P(T \cup Q) = P(T) + P(Q) - P(T \cap Q)$$

$$0.75 = 3P(T \cap Q) + 3P(T \cap Q) - P(T \cap Q)$$

$$5P(T \cap Q) = 0.75$$

$$P(T \cap Q) = 0.15$$



b. P(T) = 0.45

c. P(Q') = 0.55

d. $P(T' \cap Q') = 0.25$

e. $P(T \cap Q') = 0.3$

Probability Exercise C, Question 5

Question:

A survey of all the households in the town of Bury was carried out. The survey showed that 70% have a freezer and 20% have a dishwasher and 80% have either a dishwasher or a freezer or both appliances. Find the probability that a randomly chosen household in Bury has both appliances.

Solution:

$$P(F \cap D) = P(F) + P(D) - P(F \cup D)$$

= 0.7 + 0.2 - 0.8
= 0.1

Probability Exercise C, Question 6

Question:

The probability that a child in a school has blue eyes is 0.27 and the probability they have blonde hair is 0.35. The probability that the child will have blonde hair or blue eyes or both is 0.45. A child is chosen at random from the school. Find the probability that the child has

a blonde hair and blue eyes,

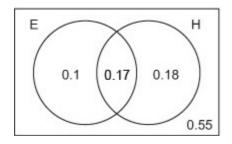
b blonde hair but not blue eyes,

c neither feature.

Solution:

a.
$$P(E \cap H) = P(E) + P(H) - P(E \cup H)$$

= 0.27 + 0.35 - 0.45
= 0.17



b. P(Blonde hair but not Blue eyes) = 0.18

c. P(Neither) = 0.55

Probability Exercise C, Question 7

Question:

A patient going in to a doctor's waiting room reads *Hiya* Magazine with probability 0.6 and *Dakor* Magazine with probability 0.4. The probability that the patient reads either one or both of the magazines is 0.7. Find the probability that the patient reads

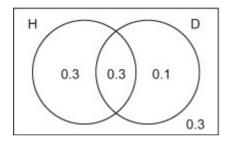
a both magazines,

b *Hiya* Magazine only.

Solution:

a.
$$P(H \cap D) = P(H) + P(D) - P(H \cup D)$$

= 0.6 + 0.4 - 0.7
= 0.3



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b. P(Hiya only) = 0.3

Probability Exercise D, Question 1

Question:

A card is drawn at random from a pack of 52 playing cards. Given that the card is a diamond, find the probability that the card is an ace.

Solution:

P(Ace | Diamond) =
$$\frac{1}{13}$$
 or P(Ace | Diamond) = $\frac{P(\text{Ace of Diamonds})}{P(\text{Diamond})} = \frac{\frac{1}{52}}{\frac{13}{52}} = \frac{1}{13}$.

Probability Exercise D, Question 2

Question:

Two coins are flipped and the results are recorded. Given that one coin lands on a head, find the probability of

a two heads, **b** a head and a tail.

Solution:

a.

P(HH |H) =
$$\frac{P(\text{Head and Head})}{P(\text{Head})}$$

= $\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$.

b.

P(Head and Tail | Head) =
$$\frac{P(\text{Head and Tail})}{P(\text{Head})}$$

= $\frac{\frac{2}{4}}{\frac{2}{4}} = \frac{2}{3}$

Probability Exercise D, Question 3

Question:

Two fair dice are thrown and the product of the numbers on the dice is recorded. Given that one die lands on 2, find the probability that the product on the dice is

a exactly 6, **b** more than 5.

Solution:

a. P(Exactly 6 | 2) =
$$\frac{P(\text{Product of 6 and 2 on the die})}{P(\text{2 on one die})}$$
$$= \frac{\frac{2}{36}}{11} = \frac{2}{11}$$

$$=\frac{\frac{2}{36}}{\frac{11}{36}}=\frac{2}{11}$$

b. P(More than
$$5 \mid 2$$
) = $\frac{\frac{8}{36}}{\frac{11}{36}} = \frac{8}{11}$

Probability Exercise D, Question 4

Question:

A and B are two events such that P(A) = 0.6, P(B) = 0.5 and $P(A \cap B) = 0.4$, find

a
$$P(A \cup B)$$
,

b
$$P(B \mid A)$$
,

$$\mathbf{c} P(A \mid B)$$

b
$$P(B \mid A)$$
, **c** $P(A \mid B)$, **d** $P(A \mid B')$.

Solution:

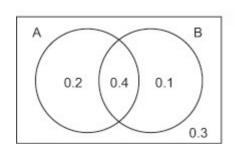
a.
$$P(A \cup B) = 0.6 + 0.5 - 0.4$$

b.
$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{0.4}{0.6} = \frac{2}{3}$$

$$= 0.7$$
c. $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.5} = 0.8$

d.
$$P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.2}{0.5}$$

$$= 0.4.$$



Probability Exercise D, Question 5

Question:

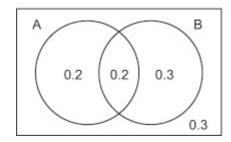
A and B are two events such that P(A) = 0.4, P(B) = 0.5 and $P(A \mid B) = 0.4$, find

a
$$P(B \mid A)$$
, **b** $P(A' \cap B')$, **c** $P(A' \cap B)$.

Solution:

$$P(A \cap B) = P(A \mid B)P(B)$$

= 0.4 × 0.5 = 0.2



a.
$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{0.2}{0.4} = \frac{1}{2}$$

b.
$$P(A' \cap B') = 0.3$$

c.
$$P(A' \cap B) = 0.3$$

Probability Exercise D, Question 6

Question:

Let A and B be events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$.

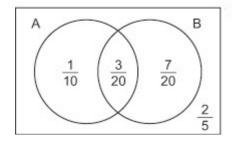
Find

a $P(A \mid B)$,

b $P(A' \cap B)$, **c** $P(A' \cap B')$.

Solution:

$$P(A \cap B) = \frac{1}{4} + \frac{1}{2} - \frac{3}{5} = \frac{3}{20}$$



a.
$$P(A \mid B) = \frac{\frac{3}{20}}{\frac{1}{2}} = \frac{3}{10}$$

b.
$$P(A' \cap B) = \frac{7}{20}$$

c.
$$P(A' \cap B') = \frac{2}{5}$$

Probability Exercise D, Question 7

Question:

C and D are two events and $P(C \mid D) = \frac{1}{3}$, $P(C \mid D') = \frac{1}{5}$ and $P(D) = \frac{1}{4}$, find

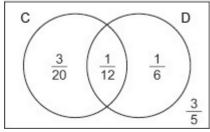
$$\mathbf{a} \ \mathbf{P}(C \cap D), \qquad \mathbf{b} \ \mathbf{P}(C \cap D'), \qquad \mathbf{c} \ \mathbf{P}(C),$$
$$\mathbf{d} \ \mathbf{P}(D \mid C), \qquad \mathbf{e} \ \mathbf{P}(D' \mid C), \qquad \mathbf{f} \ \mathbf{P}(C').$$

Solution:

a.

$$P(C \cap D) = P(C | P)P(D)$$

= $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.



e.
$$P(D' \mid C) = 1 - \frac{5}{14} = \frac{9}{14}$$

f.
$$P(C') = 1 - P(C) = \frac{7}{30} = \frac{23}{30}$$

$$\therefore P(D' \mid C') = P(C' \cap D') / P(C')$$

$$=\frac{\left(\frac{3}{5}\right)}{\left(\frac{23}{3}\right)}=\frac{18}{23}$$

b.
$$P(C \cap D') = P(C \mid D')P(D')$$

$$=\frac{1}{5}\times\frac{3}{4}=\frac{3}{20}$$

c.
$$P(C) = \frac{3}{20} + \frac{1}{12} = \frac{7}{30}$$

d.
$$P(D \mid C) = \frac{P(D \cap C)}{P(C)} = \frac{\frac{1}{12}}{\frac{7}{30}} = \frac{5}{14}$$

Probability Exercise E, Question 1

Question:

A bag contains five red and four blue tokens. A token is chosen at random, the colour recorded and the token is not replaced. A second token is chosen and the colour recorded. Find the probability that

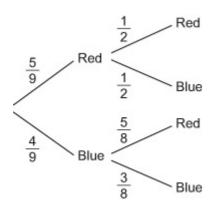
a the second token is red given the first token is blue,

b the second token is blue given the first token is red,

c both tokens chosen are blue,

d one red token and one blue token are chosen.

Solution:



a. P(Second Red | First Blue) =
$$\frac{5}{8}$$

c. P(Both Blue) =
$$\frac{4}{9} \times \frac{3}{8} = \frac{1}{6}$$

b. P(Second Blue | First Red) =
$$\frac{1}{2}$$

d. P(One Red and One Blue)

$$= P(Red Blue) + P(Blue Red)$$

$$= \frac{5}{9} \times \frac{1}{2} + \frac{4}{9} \times \frac{5}{8} = \frac{5}{9}$$

Probability Exercise E, Question 2

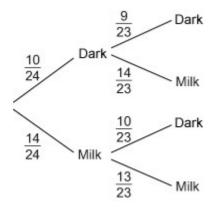
Question:

A box of 24 chocolates contains 10 dark and 14 milk chocolates. Linda chooses a chocolate at random and eats it, followed by another one. Find the probability that Linda eats

a two dark chocolates,

b one dark and one milk chocolate.

Solution:



a. P(Dark Dark) =
$$\frac{10}{24} \times \frac{9}{27} = \frac{15}{92}$$

b. P(One Dark One Milk) =
$$\frac{10}{24} \times \frac{14}{23} + \frac{14}{24} \times \frac{10}{23}$$

= $\frac{35}{69}$

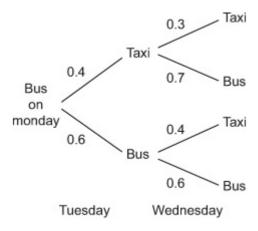
Probability Exercise E, Question 3

Question:

Jean always goes to work by bus or takes a taxi. If one day she goes to work by bus, the probability she goes to work by taxi the next day is 0.4. If one day she goes to work by taxi, the probability she goes to work by bus the next day is 0.7.

Given that Jean takes the bus to work on Monday, find the probability that she takes a taxi to work on Wednesday.

Solution:



P(Taxi on Weds) = $0.4 \times 0.3 + 0.6 \times 0.4$ = 0.36

Probability Exercise E, Question 4

Question:

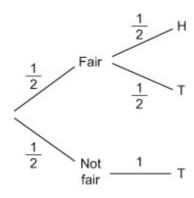
Sue has two coins. One is fair, with a head on one side and a tail on the other.

The second is a trick coin and has a tail on both sides. Sue picks up one of the coins at random and flips it.

a Find the probability that it lands heads up.

b Given that it lands tails up, find the probability that she picked up the fair coin.

Solution:



a. P(Head) =
$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

b.
$$P(Fair \mid Tail) = \frac{P(Fair \text{ and } Tail)}{P(Tail)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2}} = \frac{1}{3}$$

Probability Exercise E, Question 5

Question:

A contestant on a quiz show is asked to choose one of three doors. Behind one of the doors is the star prize of a sports car, but behind each of the other two doors there is a toy car.

The contestant chooses one of the three doors.

The host then opens one of the remaining two doors and reveals a toy car. The host then asks the contestant if they want to stick with their first choice or switch to the other unopened door.

State what you would recommend the contestant to do in order to have the greatest probability of winning the sports car. Show your working clearly.

Solution:

P(Winning if Contestant Sticks) = $\frac{1}{3}$ P(Winning if Contestant Swaps) = $\frac{2}{3}$

Contestant should Swap doors.

Probability Exercise F, Question 1

Question:

Event *A* and event *B* are mutually exclusive and P(A) = 0.2, P(B) = 0.5.

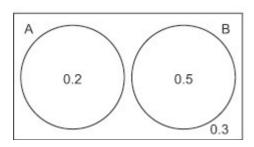
a Draw a Venn diagram to represent these two events.

b Find $P(A \cup B)$.

c Find $P(A' \cap B')$.

Solution:

a.



b.
$$P(A \cup B) = 0.7$$

c.
$$P(A' \cap B') = 0.3$$

Probability

Exercise F, Question 2

Question:

Two events *A* and *B* are independent and $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{5}$.

Find

a $P(A \cap B)$,

b $P(A \cap B')$,

 $\mathbf{c} \ \mathbf{P}(A^{'} \cap B^{'}).$

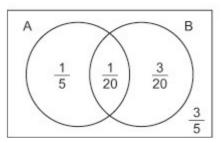
Solution:

a. $P(A \cap B) = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

b. $P(A \cap B') = \frac{1}{4} - \frac{1}{20} = \frac{1}{5}$

 $or = \frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$

c.
$$P(A' \cap B') = \frac{3}{5}$$
.



Probability

Exercise F, Question 3

Question:

Q and R are two events such that P(Q) = 0.2, P(R) = 0.4 and $P(Q' \cap R) = 0.4$.

Find

 \mathbf{a} the relationship between Q and R,

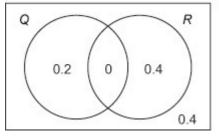
b $P(Q \cup R)$,

 $\mathbf{c} P(Q' \cap R').$

Solution:

a.

 $P(Q \cap R) = 0$ so Q and R are mutually exclusive.



b. $P(Q \cup R) = 0.4 + 0.2 = 0.6$

c. $P(Q' \cap R') = 0.4$

Probability Exercise F, Question 4

Question:

Two fair dice are rolled and the result on each die is recorded. Show that the event 'the sum of the scores on the dice is 4' and 'both dice land on the same number' are *not* mutually exclusive.

Solution:

P(Sum of 4) =
$$\frac{3}{36} = \frac{1}{12}$$

P(Same number) = $\frac{6}{36} = \frac{1}{6}$

$$P(Sum of 4) + P(Same number) = \frac{1}{6} + \frac{1}{12} = \frac{1}{4}$$

P(Sum of 4 or same number) =
$$\frac{8}{36} = \frac{2}{9}$$

 $P(Sum \text{ of } 4) + P(Same \text{ number}) \neq P(Sum \text{ of } 4 \text{ or same number})$, so the events are not mutually exclusive

Probability Exercise F, Question 5

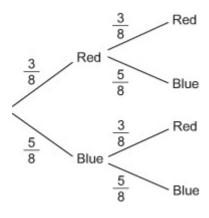
Question:

A bag contains three red beads and five blue beads. A bead is chosen at random from the bag, the colour is recorded and the bead is replaced. A second bead is chosen and the colour recorded.

a Find the probability that both beads are blue.

b Find the probability that the second bead is blue.

Solution:



a. P(Blue Blue) =
$$\frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$$

b. P(Second Blue) =
$$\frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{5}{8}$$
$$= \frac{5}{8}$$

Probability Exercise F, Question 6

Question:

A box contains 24 electrical components of which four are known to be defective. Two items are taken at random from the box. Find the probability of selecting

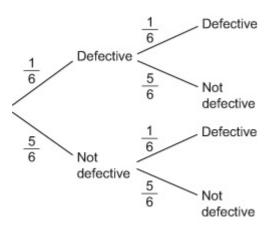
a two defective components if the first item is replaced before choosing the second item,

b two defective components if the first item is not replaced,

c one defective component and one fully functioning component if the first item is not replaced.

Solution:

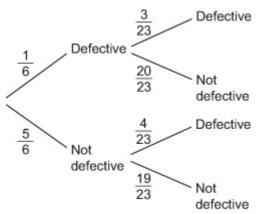
a. Replaced



P(2 defective) =
$$\frac{1}{6} \times \frac{1}{6}$$

= $\frac{1}{36}$

b. Not replaced



P(2 defective) =
$$\frac{1}{6} \times \frac{3}{23}$$

= $\frac{1}{46}$

c. P(one defective and one not defective) = $\frac{1}{6} \times \frac{20}{23} + \frac{5}{6} \times \frac{4}{23} = \frac{20}{69}$

Probability Exercise F, Question 7

Question:

A bag contains one red, two blue and three green tokens. One token is chosen at random, the colour is recorded and the token replaced. A second token is then chosen and the colour recorded.

a Draw a tree diagram showing the possible outcomes.

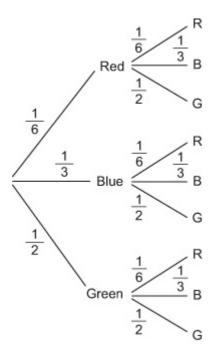
Find the probability of choosing

b two tokens of the same colour,

c two tokens that are different colours.

Solution:

a.



b. P(Same Colour)

$$= \frac{1}{6} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{7}{18}$$

c. P(Different colours)

$$= \frac{1}{6} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{11}{18} \quad or \quad 1 - \frac{7}{18} = \frac{11}{18}$$

Probability Exercise F, Question 8

Question:

Paul and Gill decide to play a board game. The probability that Paul wins the game is 0.25 and the probability that Gill wins is 0.3. They decide to play three games. Given that the results of successive games are independent, find the probability that

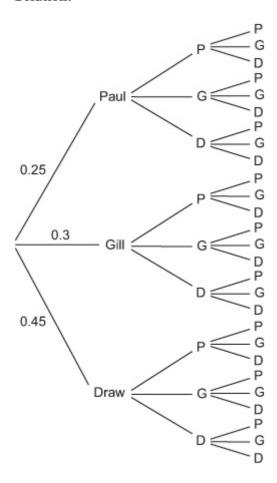
a Paul wins three games in a row,

c Gill wins two games and Paul wins one game,

b all games are drawn,

d each player wins just one game each.

Solution:



a.

P(Paul Wins 3 games) =
$$0.25 \times 0.25 \times 0.25$$

= 0.015625

b.

P(All games drown) =
$$0.45 \times 0.45 \times 0.45$$

= 0.091125

c.

P(Gill wins 2 and Paul wins 1) = $3 \times 0.3 \times 0.3 \times 0.25$ = 0.0675

d.

P(Each player wins one game) = $6 \times 0.25 \times 0.3 \times 0.45$ = 0.2025

Probability Exercise G, Question 1

Question:

The events *A* and *B* are such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$.

a Show that *A* and *B* are independent.

b Represent these probabilities in a Venn diagram.

c Find $P(A \mid B')$.

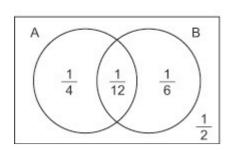
Solution:

a.
$$P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{2} = \frac{1}{12}$$

$$P(A)P(B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

Hence A and B are independent as $P(A \cap B) = P(A)P(B) = \frac{1}{12}$.

b.



c.
$$P(A \mid B') = \frac{P(A \cap B')}{P(B')}$$
$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Probability Exercise G, Question 2

Question:

A computer game has three levels and one of the objectives of every level is to collect a diamond. The probability of a randomly chosen player collecting a diamond on the first level is $\frac{4}{5}$, the second level is $\frac{2}{3}$ and the third level is $\frac{1}{2}$. The events are independent.

a Draw a tree diagram to represent collecting diamonds on the three levels of the game.

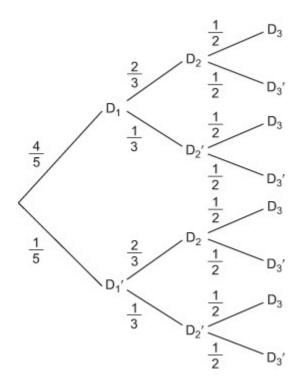
Find the probability that a randomly chosen player

b collects all three diamonds,

c collects only one diamond.

Solution:

a.



b.
$$P(D_1D_2D_3) = \frac{4}{5} \times \frac{2}{3} \times \frac{1}{2} = \frac{4}{15}$$

c. P(only 1 diamond)

$$= P(D_1D_2^{'}D_3^{'}) + P(D_1^{'}D_2D_3^{'}) + P(D_1^{'}D_2^{'}D_3)$$

$$= \frac{4}{5} \times \frac{1}{3} \times \frac{1}{2} + \frac{1}{5} \times \frac{2}{3} \times \frac{1}{2} + \frac{1}{5} \times \frac{1}{3} \times \frac{1}{2} = \frac{7}{30}$$

Probability Exercise G, Question 3

Question:

An online readers' club has 50 members. Glasses are worn by 15 members, 18 are left handed and 21 are female. There are four females who are left handed, three females who wear glasses and five members who wear glasses and are left handed. Only one member wears glasses, is left handed and female.

a Draw a Venn diagram to represent these data.

A member is selected at random. Find the probability that the member

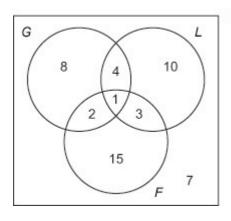
b is female, does not wear glasses and is not left handed,

c is male, does not wear glasses and is not left handed,

d wears glasses given that she is left handed and female.

Solution:

a.



b.
$$P(F \cap G' \cap L') = \frac{15}{50} = \frac{3}{10}$$

c.
$$P(F' \cap G' \cap L') = \frac{7}{50}$$

d.
$$P(G \mid L \cap F) = \frac{P(G \cap L \cap F)}{P(L \cap F)}$$

$$=\frac{1}{4}$$

Probability Exercise G, Question 4

Question:

For the events J and K,

 $P(J \cup K) = 0.5, P(J' \cap K) = 0.2, P(J \cap K') = 0.25.$

a Draw a Venn diagram to represent the events *J* and *K* and the sample space *S*.

Find

 $\mathbf{b} P(J)$,

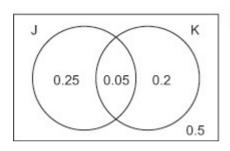
 $\mathbf{c} \ \mathrm{P}(K)$,

d $P(J \mid K)$.

e Determine whether or not *J* and *K* are independent.

Solution:

a.



b.
$$P(J) = 0.3$$

c.
$$P(K) = 0.25$$

d.
$$P(J \mid K) = \frac{P(J \cap K)}{P(K)}$$

= $\frac{0.05}{0.25} = 0.2$

Probability Exercise G, Question 5

Question:

There are 15 coloured beads in a bag; seven beads are red, three are blue and five are green. Three beads are selected at random from the bag and not replaced. Find the probability that

a the first and second beads chosen are red and the third bead is blue or green,

b one red, one blue and one green bead are chosen.

Solution:

a. P(2 red and third blue) + P(2 red and third green)

$$= \frac{7}{15} \times \frac{6}{14} \times \frac{3}{13} + \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13} = \frac{8}{65}$$

b. P(one red, one blue and one green bead) = $6 \times \frac{7}{15} \times \frac{3}{14} \times \frac{5}{13} = \frac{3}{13}$

Probability Exercise G, Question 6

Question:

A survey of a group of students revealed that 85% have a mobile phone, 60% have an MP3 player and 5% have neither phone nor MP3 player.

a Find the proportion of students who have both gadgets.

b Draw a Venn diagram to represent this information.

Given that a randomly selected student has a phone or an MP3 player,

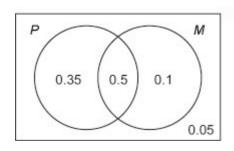
c find the probability that the student has a mobile phone.

Solution:

a. P(Phone and MP3) = 0.85 + 0.6 - 0.95

$$= 0.5$$

b.



c.
$$P(P \mid P \cup M) = \frac{0.85}{0.95} = \frac{17}{19}$$

Probability Exercise G, Question 7

Question:

In a factory, machines *A*, *B* and *C* produce electronic components. Machine *A* produces 16% of the components, machine *B* produces 50% of the components and machine *C* produces the rest. Some of the components are defective. Machine *A* produces 4%, machine *B* 3% and machine *C* 7% defective components.

a Draw a tree diagram to represent this information.

Find the probability that a randomly selected component is

b produced by machine *B* and is defective,

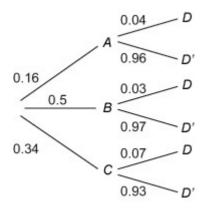
c defective.

Given that a randomly selected component is defective,

d find the probability that it was produced by machine *B*.

Solution:

a.



b. P(B and Defective) = 0.5×0.03

$$= 0.015$$

c. P(Defective) = $0.16 \times 0.04 + 0.5 \times 0.03 + 0.34 \times 0.07$

$$= 0.0452$$

d. P(B | Defective) =
$$\frac{0.015}{0.0452}$$
 = 0.332

Probability Exercise G, Question 8

Question:

A garage sells three types of fuel; U95, U98 and diesel. In a survey of 200 motorists buying fuel at the garage, 80 are female and the rest are male. Of the 90 motorists buying 'U95' fuel, 50 were female and of the 70 motorists buying diesel, 60 were male. A motorist does not buy more than one type of fuel.

Find the probability that a randomly chosen motorist

a buys U98 fuel,

b is male, given that the motorist buys U98 fuel.

Garage records indicate that 10% of the motorists buying U95 fuel, 30% of the motorists buying U98 fuel and 40% of the motorists buying diesel have their car serviced by the garage.

A motorist is chosen at random.

c Find the probability that this motorist has his or her car serviced by the garage.

d Given the motorist has his or her car serviced by the garage, find the probability that the motorist buys diesel fuel.

Solution:

a.
$$P(U98) = \frac{200 - 90 - 70}{200} = \frac{1}{5}$$

b. P(Male | U98) =
$$\frac{P(Male \text{ and buys U98})}{P(U98)} = \frac{20}{40} = \frac{1}{2}$$

c. P(Serviced) =
$$\frac{9}{20} \times \frac{1}{10} + \frac{1}{5} \times \frac{3}{10} + \frac{7}{20} \times \frac{4}{10} = \frac{49}{200}$$

d. P(Diesel | Serviced) =
$$\frac{\frac{7}{20} \times \frac{4}{10}}{\frac{49}{200}} = \frac{4}{7}$$
.

Probability Exercise G, Question 9

Question:

A study was made of a group of 150 children to determine which of three cartoons they watch on television. The following results were obtained:

- 35 watch Toontime
- 54 watch Porky
- 62 watch Skellingtons
- 9 watch Toontime and Porky
- 14 watch Porky and Skellingtons
- 12 watch Toontime and Skellingtons
- 4 watch Toontime, Porky and Skellingtons

a Draw a Venn diagram to represent these data.

Find the probability that a randomly selected child from the study watches

b none of the three cartoons,

c no more than one of the cartoons.

A child selected at random from the study only watches one of the cartoons.

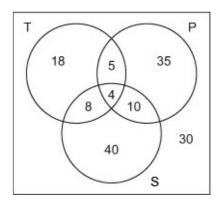
d Find the probability that it was Skellingtons.

Two different children are selected at random from the study.

e Find the probability that they both watch Skellingtons.

Solution:

a.



e. P(Both watch K) =
$$\frac{62}{150} \times \frac{61}{149} = 0.169$$

b.
$$P(None) = \frac{30}{150} = \frac{1}{5}$$

c. P(No more than one) =
$$\frac{30 + 40 + 18 + 35}{150}$$
$$= \frac{41}{50}$$

d.
$$P(S| only one) = \frac{40}{93}$$

Probability Exercise G, Question 10

Question:

The members of a wine tasting club are married couples. For any married couple in the club, the probability that the husband is retired is 0.7 and the probability that the wife is retired 0.4. Given that the wife is retired, the probability that the husband is retired is 0.8.

For a randomly chosen married couple who are members of the club, find the probability that

a both of them are retired,

b only one of them is retired,

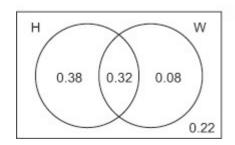
c neither of them is retired.

Two married couples are chosen at random.

d Find the probability that only one of the two husbands and only one of the two wives is retired.

Solution:

a. P(Both retired) = $0.8 \times 0.4 = 0.32$



b.
$$P(Only one) = 0.38 + 0.08$$

$$= 0.46$$

c.
$$P(Neither) = 0.22$$

d. P(only one husband and only one wife) = $(0.38 \times 0.08 + 0.32 \times 0.22) \times 2 = 0.2016$