

Solutionbank S1

Edexcel AS and A Level Modular Mathematics

Representation and summary of data – measures of dispersion

Exercise A, Question 1

Question:

15 students do a mathematics test. Their marks are shown opposite.

7	4	9	7	6
10	12	11	3	8
5	9	8	7	3

a Find the value of the median.

b Find Q_1 and Q_3 .

c Work out the interquartile range.

Solution:

a **3,3,4,5,6,7,7,7,8,8,9,9,10,11,12**

Median: $\frac{15}{2} = 7.5$ therefore 8th value is needed. Median = **7**

b $Q_1 : \frac{15}{4} = 3.75$ therefore the 4th value is needed $Q_1 = \mathbf{5}$

$Q_3 : 3 \times \frac{15}{4} = 11.25$ therefore the 12th value is needed $Q_3 = \mathbf{9}$

c IQR = $9 - 5 = \mathbf{4}$

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Exercise A, Question 2

Question:

A group of workers were asked to write down their weekly wage. The wages were:

£550 £400 £260 £320 £500 £450 £460 £480 £510 £490 £505

a Work out the range for these wages.

b Find Q_1 and Q_3 .

c Work out the interquartile range.

Solution:

a Range = £550 – £260 = **£290**

b

260 320 400 450 460 480 490 500 505 510 550

Q_1 : $\frac{11}{4} = 2.75$ therefore the 3rd value is needed $Q_1 = \mathbf{400}$

Q_3 : $3 \times \frac{11}{4} = 8.25$ therefore the 9th value is needed $Q_3 = \mathbf{505}$

c IQR = 505 – 400 = **105**

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Exercise A, Question 3

Question:

A superstore records the number of hours overtime worked by their employees in one particular week. The results are shown in the table.

a Fill in the cumulative frequency column and work out how many employees the superstore had in that week.

b Find Q_1 and Q_3 .

c Work out the interquartile range.

Number of hours	Frequency	Cumulative frequency
0	25	
1	10	
2	20	
3	10	
4	25	
5	10	

Solution:

a

$$CF = 25 \ 35 \ 55 \ 65 \ 90 \ 100$$

Superstore had 100 employees

b $Q_1 : \frac{100}{4} = 25$ therefore the average of the 25th and 26th values is $Q_1 = 0.5$

$Q_3 : 3 \times \frac{100}{4} = 75$ therefore the average of the 75th and 76th values is $Q_3 = 4$

c $IQR = 4 - 0.5 = 3.5$

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Exercise A, Question 4

Question:

A moth trap was set every night for five weeks. The number of moths caught in the trap was recorded. The results are shown in the table.

Number of moths	Frequency
7	2
8	5
9	9
10	14
11	5

Find the interquartile range.

Solution:

$$CF = 2 \ 7 \ 16 \ 30 \ 35$$

$$Q_1 : \frac{35}{4} = 8.75 \text{ therefore the } 9^{\text{th}} \text{ value is needed } Q_1 = 9$$

$$Q_3 : 3 \times \frac{35}{4} = 26.25 \text{ therefore the } 27^{\text{th}} \text{ value is needed } Q_3 = 10$$

$$IQR = 10 - 9 = 1$$

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Exercise A, Question 5

Question:

The weights of 31 Jersey cows were recorded to the nearest kilogram. The weights are shown in the table.

Weight of cattle (kg)	Frequency	Cumulative frequency
300–349	3	
350–399	6	
400–449	10	
450–499	7	
500–549	5	

a Complete the cumulative frequency column in the table.

b Find the lower quartile, Q_1 .

c Find the upper quartile, Q_3 .

d Find the interquartile range.

Solution:

a

$$\mathbf{CF = 3 \ 9 \ 19 \ 26 \ 31}$$

b Data is continuous so:

$$Q_1 : \frac{31}{4} = 7.75^{\text{th}} \text{ value so } Q_1 \text{ is in class } 350 - 399$$

$$\begin{aligned} \frac{Q_1 - 349.5}{399.5 - 349.5} &= \frac{7.75 - 3}{9 - 3} \\ \frac{Q_1 - 349.5}{50} &= \frac{4.75}{6} \\ Q_1 &= 39.58 + 349.5 \\ Q_1 &= \mathbf{389.1} \end{aligned}$$

$$\mathbf{c} \quad Q_3 : 3 \times \frac{31}{4} = 23.25^{\text{th}} \text{ value so } Q_3 \text{ is in class } 450 - 499$$

$$\begin{aligned} \frac{Q_3 - 449.5}{499.5 - 449.5} &= \frac{23.25 - 19}{26 - 19} \\ \frac{Q_3 - 449.5}{50} &= \frac{4.25}{7} \\ Q_3 &= 30.36 + 449.5 \\ &= \mathbf{479.9} \end{aligned}$$

$$\mathbf{d} \quad \text{IQR} = 479.86 - 389.08 = \mathbf{90.8}$$

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Exercise A, Question 6

Question:

The number of visitors to a hospital in a week was recorded. The results are shown in the table.

Number of visitors	Frequency
500–1000	10
1000–1500	25
1500–2000	15
2000–2500	5
2500–3000	5

Giving your answers to the nearest whole number find:

a the lower quartile Q_1 , **b** the upper quartile Q_3 , **c** the interquartile range.

Solution:

Data is continuous so

a

$$CF = 10 \ 35 \ 50 \ 55 \ 60$$

$$Q_1 : \frac{60}{4} = 15^{\text{th}} \text{ value so } Q_1 \text{ is in class } 1000 - 1500$$

$$\begin{aligned} \frac{Q_1 - 1000}{1500 - 1000} &= \frac{15 - 10}{35 - 10} \\ Q_1 - 1000 &= 100 \\ Q_1 &= \mathbf{1100} \end{aligned}$$

$$\mathbf{b} \ Q_3 : 3 \times \frac{60}{4} = 45^{\text{th}} \text{ value so } Q_3 \text{ is in class } 1500 - 2000$$

$$\begin{aligned} \frac{Q_3 - 1500}{2000 - 1500} &= \frac{45 - 35}{50 - 35} \\ Q_3 - 1500 &= 333.33 \\ Q_3 &= \mathbf{1833} \end{aligned}$$

$$\mathbf{c} \ IQR = 1833 - 1100 = \mathbf{733}$$

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Exercise A, Question 7

Question:

The lengths of a number of slow worms were measured, to the nearest mm. The results are shown in the table.

a Work out how many slow worms were measured.

b Find the interquartile range for the lengths of the slow worms.

Lengths of slow worms (mm)	Frequency
125–139	4
140–154	4
155–169	2
170–184	7
185–199	20
200–214	24
215–229	10

Solution:

Data is continuous

a

$$CF = 4 \ 8 \ 10 \ 17 \ 37 \ 61 \ 71$$

71 slow worms

b $Q_1 : \frac{71}{4} = 17.75^{\text{th}}$ value so Q_1 is in class 185 – 199

$$\frac{Q_1 - 184.5}{199.5 - 184.5} = \frac{17.75 - 17}{37 - 17}$$

$$Q_1 - 184.5 = 0.5625$$

$$Q_1 = 185.0625$$

$Q_3 : 3 \times \frac{71}{4} = 53.25^{\text{th}}$ value so Q_3 is in class 200–214

$$\frac{Q_3 - 199.5}{214.5 - 199.5} = \frac{53.25 - 37}{61 - 37}$$

$$Q_3 - 199.5 = 243.75/24$$

$$Q_3 = 209.656$$

$$IQR = 209.656 - 185.0625 = \mathbf{24.6 \text{ to } 3SF}$$

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Exercise B, Question 1

Question:

A gardener counted the peas in a number of pea pods. The results are shown in the table.

Number of peas	Frequency	Cumulative frequency
6	8	
7	12	
8	36	
9	18	
10	15	
11	10	

a Complete the cumulative frequency column.

b Calculate the 80th percentile.

c Calculate the 40th percentile

d Calculate the 65th percentile.

Solution:

a CF **8, 20, 56, 74, 89, 99**

b $80^{\text{th}} : \frac{80}{100} \times 99 = 79.2$ therefore 80^{th} term needed $P_{80} = \mathbf{10}$

c $40^{\text{th}} : \frac{40}{100} \times 99 = 39.6$ therefore 40^{th} term needed $P_{40} = \mathbf{8}$

d $65^{\text{th}} : \frac{65}{100} \times 99 = 64.35$ therefore 65^{th} term needed $P_{65} = \mathbf{9}$

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Exercise B, Question 2

Question:

A shopkeeper goes to a clothes fair. He records the costs of jeans. The costs are shown in the table.

Cost of jeans (£'s)	Frequency	Cumulative frequency
10–15	11	
16–20	35	
21–25	34	
26–30	16	
31–35	10	
36–40	5	

- a Complete the cumulative frequency table.
- b Calculate P_{20} .
- c Calculate P_{80} .
- d Calculate the 20% to 80% interpercentile range.

Solution:

a CF 11, 46, 80, 96, 106, 111

$$20^{\text{th}} : \frac{20}{100} \times 111 = 22.2$$

$$\text{b} \quad \frac{P_{20} - 15.5}{20.5 - 15.5} = \frac{22.2 - 11}{46 - 11}$$

$$P_{20} = \mathbf{17.1}$$

$$80^{\text{th}} : \frac{80}{100} \times 111 = 88.8$$

$$\text{c} \quad \frac{P_{80} - 25.5}{30.5 - 25.5} = \frac{88.8 - 80}{96 - 80}$$

$$P_{80} = \mathbf{28.25}$$

d 20% to 80% interpercentile = $28.25 - 17.1 = \mathbf{11.15}$

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Exercise B, Question 3

Question:

The table shows the monthly income for a number of workers in a factory.

Calculate the 34% to 66% interpercentile range.

Monthly income (£'s)	Frequency
900–1000	3
1000–1100	24
1100–1200	28
1200–1300	15

Solution:

$$34^{\text{th}} : \frac{34}{100} \times 70 = 23.8$$

$$\frac{P_{34} - 1000}{1100 - 1000} = \frac{23.8 - 3}{27 - 3}$$

$$P_{34} = 1086.7$$

$$66^{\text{th}} : \frac{66}{100} \times 70 = 46.2$$

$$\frac{P_{66} - 1100}{1200 - 1100} = \frac{46.2 - 27}{55 - 27}$$

$$P_{66} = 1168.57$$

$$34\% \text{ to } 66\% \text{ interpercentile} = 1168.57 - 1086.7 = \mathbf{81.9}$$

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Exercise B, Question 4

Question:

A train travelled from Lancaster to Preston. The times, to the nearest minute, it took for the journey were recorded over a certain period. The times are shown in the table.

Time for journey (minutes)	15–16	17–18	19–20	21–22
Frequency	5	10	35	10

Calculate the 5% to 95% interpercentile range.

Solution:

$$5^{\text{th}} : \frac{5}{100} \times 60 = 3$$

$$\frac{P_5 - 14.5}{16.5 - 14.5} = \frac{3 - 0}{5 - 0}$$

$$P_5 = 15.7$$

$$95^{\text{th}} : \frac{95}{100} \times 60 = 57$$

$$\frac{P_{95} - 20.5}{22.5 - 20.5} = \frac{57 - 50}{60 - 50}$$

$$P_{95} = 21.9$$

$$5\% \text{ to } 95\% \text{ interpercentile} = 21.9 - 15.7 = \mathbf{6.2}$$

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Exercise B, Question 5

Question:

A roadside assistance firm kept a record over a week of the amount of time, in minutes, people were kept waiting for assistance in a particular part of the country. The time taken was from the time the phone call was received to the arrival of the breakdown mechanic. The times are shown below.

Time Waiting (minutes)	20–30	30–40	40–50	50–60	60–70
Frequency	6	10	18	13	2

a Work out the number of people who called for assistance.

b Calculate the 30th percentile.

c Calculate the 65th percentile.

Solution:

a CF 6, 16, 34, 47, 49

$$30^{\text{th}} : \frac{30}{100} \times 49 = 14.7$$

$$\mathbf{b} \quad \frac{P_{30} - 30}{40 - 30} = \frac{14.7 - 6}{16 - 6}$$

$$P_{30} = \mathbf{38.7}$$

$$65^{\text{th}} : \frac{65}{100} \times 49 = 31.85$$

$$\mathbf{c} \quad \frac{P_{80} - 40}{50 - 40} = \frac{31.85 - 16}{34 - 16}$$

$$P_{65} = \mathbf{48.8}$$

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Representation and summary of data – measures of dispersion
Exercise C, Question 1

Question:

Given that for a variable x :

$$\Sigma x = 24 \quad \Sigma x^2 = 78 \quad n = 8$$

Find:

a The mean. **b** The variance σ^2 . **c** The standard deviation σ .

Solution:

a Mean = $\frac{24}{8} = 3$

b Variance = $\frac{78}{8} - 3^2 = 0.75$

c Standard deviation = $\sqrt{0.75} = 0.866$

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Exercise C, Question 2

Question:

Ten collie dogs are weighed (w kg). The following summary data for the weights are shown below:

$$\Sigma w = 241 \quad \Sigma w^2 = 5905$$

Use this summary data to find the standard deviation of the collies' weights.

Solution:

$$\text{Standard deviation} = \sqrt{\frac{5905}{10} - \left(\frac{241}{10}\right)^2} = \mathbf{3.11}$$

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Representation and summary of data – measures of dispersion Exercise C, Question 3

Question:

Eight students' heights (h cm) are measured. They are as follows:

165 170 190 180 175 185 176 184

a Work out the mean height of the students.

b Given $\Sigma h^2 = 254\,307$ work out the variance. Show all your working.

c Work out the standard deviation.

Solution:

a $\Sigma h = 165 + 170 + 190 + 180 + 175 + 185 + 176 + 184 = 1425$

$$\text{Mean} = \frac{1425}{8} = \mathbf{178.125 = 178}$$

b $\text{Variance} = \frac{254307}{8} - 178.125^2 = \mathbf{59.9}$

c $\text{Standard deviation} = \sqrt{59.9} = \mathbf{7.74}$

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Exercise C, Question 4

Question:

For a set of 10 numbers:

$$\Sigma x = 50 \quad \Sigma x^2 = 310$$

For a set of 15 numbers:

$$\Sigma x = 86 \quad \Sigma x^2 = 568$$

Find the mean and the standard deviation of the combined set of 25 numbers.

Solution:

$$\Sigma x = 50 + 86 = 136$$

$$\Sigma x^2 = 310 + 568 = 878$$

$$\text{Mean} = \frac{136}{25} = \mathbf{5.44}$$

$$\text{Standard deviation} = \sqrt{\frac{878}{25} - \left(\frac{136}{25}\right)^2} = \mathbf{2.35}$$

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Exercise C, Question 5

Question:

The number of members (m) in six scout groups was recorded. The summary statistics for these data are:

$$\Sigma m = 150 \quad \Sigma m^2 = 3846$$

- a** Work out the mean number of members in a scout group.
- b** Work out the standard deviation of the number of members in the scout groups.

Solution:

a Mean = $\frac{150}{6} = 25$

b Standard deviation = $\sqrt{\frac{3846}{6} - \left(\frac{150}{6}\right)^2} = 4$

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Exercise C, Question 6

Question:

There are two routes for a worker to get to his office. Both the routes involve hold ups due to traffic lights. He records the time it takes over a series of six journeys for each route. The results are shown in the table.

Route 1	15	15	11	17	14	12
Route 2	11	14	17	15	16	11

a Work out the mean time taken for each route.

b Calculate the variance and the standard deviation of each of the two routes.

c Using your answers to **a** and **b** suggest which route you would recommend. State your reason clearly.

Solution:

$$\mathbf{a} \text{ Route 1: Mean} = \frac{84}{6} = \mathbf{14}$$

$$\text{Route 2: Mean} = \frac{84}{6} = \mathbf{14}$$

$$\mathbf{b} \text{ Route 1: Standard deviation} = \sqrt{\frac{1200}{6} - \left(\frac{84}{6}\right)^2} = \mathbf{2}$$

$$\text{Route 2: Standard deviation} = \sqrt{\frac{1208}{6} - \left(\frac{84}{6}\right)^2} = \mathbf{2.31}$$

c Route 1 as although the means are the same the standard deviation is less meaning the times are more consistent and the journey is less likely to take longer than expected.

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Representation and summary of data – measures of dispersion
Exercise D, Question 1

Question:

For a certain set of data:

$$\Sigma fx = 1975 \quad \Sigma fx^2 = 52\,325 \quad n = 100$$

Work out the variance for these data.

Solution:

$$\text{Variance} = \frac{52325}{100} - \left(\frac{1975}{100}\right)^2 = 133.2 = 133$$

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Representation and summary of data – measures of dispersion
Exercise D, Question 2

Question:

For a certain set of data

$$\Sigma fx = 264 \quad \Sigma fx^2 = 6456 \quad n = 12$$

Work out the standard deviation for these data.

Solution:

$$\text{Standard deviation} = \sqrt{\frac{6456}{12} - \left(\frac{264}{12}\right)^2} = 7.35$$

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Exercise D, Question 3

Question:

Nahab asks the students in his year group how much pocket money they get per week. The results, rounded to the nearest pound, are shown in the table.

Number of £'s (x)	Number of students f	fx	fx^2
8	14		
9	8		
10	28		
11	15		
12	20		
Totals			

a Complete the table.

b Using the formula $\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$ work out the variance for these data.

c Work out the standard deviation for these data.

Solution:

a fx : 112, 72, 280, 165, 240, total 869

fx^2 : 896, 648, 2800, 1815, 2880, total 9039

total f : 85

b Variance = $\frac{9039}{85} - \left(\frac{869}{85}\right)^2 = 1.82$

c Standard deviation = $\sqrt{1.82} = 1.35$

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Exercise D, Question 4

Question:

In a student group, a record was kept of the number of days absence each student had over one particular term. The results are shown in the table.

Number days absent (x)	Number of students f	fx	fx^2
0	12		
1	20		
2	10		
3	7		
4	5		

- a** Complete the table.
- b** Calculate the variance for these data.
- c** Work out the standard deviation for these data.

Solution:

a fx : 0, 20, 20, 21, 20

fx^2 : 0, 20, 40, 63, 80

b Variance = $\frac{203}{54} - \left(\frac{81}{54}\right)^2 = 1.51$

c Standard deviation = $\sqrt{1.51} = 1.23$

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Exercise D, Question 5

Question:

A certain type of machine contained a part that tended to wear out after different amounts of time. The time it took for 50 of the parts to wear out was recorded. The results are shown in the table.

Lifetime in hours	Number of parts	Mid-point x	fx	fx^2
$5 < h \leq 10$	5			
$10 < h \leq 15$	14			
$15 < h \leq 20$	23			
$20 < h \leq 25$	6			
$25 < h \leq 30$	2			

a Complete the table.

b Calculate an estimate for the variance and the standard deviation for these data.

Solution:

a midpoint: 7.5, 12.5, 17.5, 22.5, 27.5

fx : 37.5, 175, 402.5, 135, 55

fx^2 : 281.25, 2187.5, 7043.75, 3037.5, 1512.5

$$\text{b Variance} = \frac{14062.5}{50} - \left(\frac{805}{50}\right)^2 = 22.0$$

$$\text{Standard deviation} = \sqrt{22.04} = 4.69$$

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Exercise D, Question 6

Question:

The heights (x cm) of a group of 100 women were recorded.

The summary data is as follows:

$$\Sigma fx = 17\,100 \quad \Sigma fx^2 = 2\,926\,225$$

Work out the variance and the standard deviation.

Solution:

$$\text{Variance} = \frac{2926225}{100} - \left(\frac{17100}{100}\right)^2 = \mathbf{21.25}$$

$$\text{Standard deviation} = \sqrt{21.25} = \mathbf{4.61}$$

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Exercise E, Question 1

Question:

a Work out the standard deviation of the following data.

x 11 13 15 20 25

b Use the following coding to find the standard deviation of the data. Show all the working.

i $x - 10$ **ii** $\frac{x}{10}$ **iii** $\frac{x-3}{2}$

Solution:

a Standard deviation = $\sqrt{\frac{1540}{5} - \left(\frac{84}{5}\right)^2} = 5.08$

b i coded data 1, 3, 5, 10, 15

Standard deviation of coded data = $\sqrt{\frac{360}{5} - \left(\frac{34}{5}\right)^2} = 5.08$

Standard deviation = **5.08**

ii coded data 1.1, 1.3, 1.5, 2.0, 2.5

Standard deviation of coded data = $\sqrt{\frac{15.4}{5} - \left(\frac{8.4}{5}\right)^2} = 0.508$

Standard deviation = $0.508 \times 10 = 5.08$

iii coded data 4, 5, 6, 8.5, 11

Standard deviation of coded data = $\sqrt{\frac{270.25}{5} - \left(\frac{34.5}{5}\right)^2} = 2.5377$

Standard deviation = $2.5377 \times 2 = 5.08$

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Exercise E, Question 2

Question:

Use the following codings to find the standard deviation of the following weights, w .

$$w \quad 210 \quad 260 \quad 310 \quad 360 \quad 410$$

i $\frac{w}{10}$

ii $w - 200$

iii $\frac{w - 10}{200}$

Solution:

i coded data 21, 26, 31, 36, 41

$$\text{Standard deviation of coded data} = \sqrt{\frac{5055}{5} - \left(\frac{155}{5}\right)^2} = 7.07$$

$$\text{Standard deviation} = 7.07 \times 10 = \mathbf{70.7}$$

ii coded data 10, 60, 110, 160, 210

$$\text{Standard deviation of coded data} = \sqrt{\frac{85500}{5} - \left(\frac{550}{5}\right)^2} = 70.7$$

$$\text{Standard deviation} = \mathbf{70.7}$$

iii coded data 1, 1.25, 1.5, 1.75, 2

$$\text{Standard deviation of coded data} = \sqrt{\frac{11.875}{5} - \left(\frac{7.5}{5}\right)^2} = 0.354$$

$$\text{Standard deviation} = 0.354 \times 200 = \mathbf{70.7}$$

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Exercise E, Question 3

Question:

- a** The coding $y = \frac{x-50}{28}$ gives a standard deviation of 0.01 for y . Work out the standard deviation for x .
- b** The coding $y = \frac{h}{15}$ produced a standard deviation for y of 0.045. What is the standard deviation of h ?
- c** The coding $y = x - 14$ produced a standard deviation for y of 2.37. What is the standard deviation of x ?
- d** The coding $y = \frac{s-5}{10}$ produced a standard deviation for y of 0.65. What is the standard deviation of s ?

Solution:

- a** Standard deviation = $0.01 \times 28 = \mathbf{0.28}$
- b** Standard deviation = $0.045 \times 15 = \mathbf{0.675}$
- c** Standard deviation = $\mathbf{2.37}$
- d** Standard deviation = $0.65 \times 10 = \mathbf{6.5}$

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Exercise E, Question 4

Question:

The coding $y = x - 40$ gives a standard deviation for y of 2.34. Write down the standard deviation of x .

Solution:

Standard deviation = **2.34**

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Exercise E, Question 5

Question:

The lifetime, x , in hours, of 70 light bulbs is shown in the table.

Lifetime in hours	Number of light bulbs
20–22	3
22–24	12
24–26	40
26–28	10
28–30	5
total	70

Using the coding $y = \frac{x-1}{20}$ estimate the standard deviation of the actual lifetime in hours of a light bulb.

Solution:

Lifetime in hours	Number of lightbulbs	Midpoint	$y = \frac{x-1}{20}$	fy	fy^2
20 – 22	3	21	1	3	3
22 – 24	12	23	1.1	13.2	14.52
24 – 26	40	25	1.2	48	57.6
26 – 28	10	27	1.3	13	16.9
28 – 30	5	29	1.4	7	9.8
total	70			84.2	101.82

$$\text{Standard deviation of coded data} = \sqrt{\frac{101.82}{70} - \left(\frac{84.2}{70}\right)^2} = 0.0878$$

$$\text{Standard deviation} = 0.0878 \times 20 = \mathbf{1.76}$$

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Representation and summary of data – measures of dispersion

Exercise E, Question 6

Question:

The weekly income, i , of 100 women workers was recorded.

The data were coded using $y = \frac{i-90}{100}$ and the following summations were obtained.

$$\Sigma y = 131, \Sigma y^2 = 176.84$$

Work out an estimate for the standard deviation of the actual women workers' weekly income.

Solution:

$$\text{Standard deviation of coded data} = \sqrt{\frac{176.84}{100} - \left(\frac{131}{100}\right)^2} = 0.229$$

$$\text{Standard deviation} = 0.229 \times 100 = \mathbf{22.9}$$

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Representation and summary of data – measures of dispersion

Exercise E, Question 7

Question:

A meteorologist collected data on the annual rainfall, x mm, at six randomly selected places. The data were coded using $s = 0.01x - 10$ and the following summations were obtained.

$$\Sigma s = 16.1, \Sigma s^2 = 147.03$$

Work out an estimate for the standard deviation of the actual annual rainfall.

Solution:

$$\text{Standard deviation of coded data} = \sqrt{\frac{147.03}{6} - \left(\frac{16.1}{6}\right)^2} = 4.16$$

$$\text{Standard deviation} = 4.16 \div 0.01 = \mathbf{416}$$

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Representation and summary of data – measures of dispersion

Exercise F, Question 1

Question:

For the set of numbers:

2 5 3 8 9 10 3 2 15 10 5 7 6 7 5

- a** work out the median,
- b** work out the lower quartile,
- c** work out the upper quartile,
- d** work out the interquartile range.

Solution:

2, 2, 3, 3, 5, 5, 5, 6, 7, 7, 8, 9, 10, 10, 15

a Median: $\frac{15}{2} = 7.5$ therefore 8th value needed = **6**

b Q_1 : $\frac{15}{4} = 3.75$ therefore 4th value needed = **3**

c Q_3 : $3 \times \frac{15}{4} = 11.25$ therefore 12th value needed = **9**

d IQR = $9 - 3 = 6$

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Representation and summary of data – measures of dispersion

Exercise F, Question 2

Question:

A frequency distribution is shown below

Class interval	1–20	21–40	41–60	61–80	81–100
Frequency (<i>f</i>)	5	10	15	12	8

Work out the interquartile range.

Solution:

$$\text{CF} = 5 \ 15 \ 30 \ 42 \ 50$$

Data is continuous so:

$$Q_1 : \frac{50}{4} = 12.5^{\text{th}} \text{ value so } Q_1 \text{ is in class } 21 - 40$$

$$\frac{Q_1 - 20.5}{40.5 - 20.5} = \frac{12.5 - 5}{15 - 5}$$

$$\frac{Q_1 - 20.5}{20} = \frac{7.5}{10}$$

$$Q_1 = 35.5$$

$$Q_3 : 3 \times \frac{50}{4} = 37.5^{\text{th}} \text{ value so } Q_3 \text{ is in class } 61 - 80$$

$$\frac{Q_3 - 60.5}{80.5 - 60.5} = \frac{37.5 - 30}{42 - 30}$$

$$Q_3 = 73$$

$$\text{IQR} = 73 - 35.5 = \mathbf{37.5}$$

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Representation and summary of data – measures of dispersion

Exercise F, Question 3

Question:

A frequency distribution is shown below.

Class interval	1–10	11–20	21–30	31–40	41–50
Frequency (<i>f</i>)	10	20	30	24	16

- a** Work out the value of the 30th percentile.
- b** Work out the value of the 70th percentile.
- c** Calculate the 30% to 70% interpercentile range.

Solution:

$$\mathbf{a} \quad 30^{\text{th}} : \frac{30}{100} \times 100 = 30$$

$$P_{30} = \mathbf{20.5}$$

$$\mathbf{b} \quad 70^{\text{th}} : \frac{70}{100} \times 100 = 70$$

$$\frac{P_{70} - 30.5}{40.5 - 30.5} = \frac{70 - 60}{84 - 60}$$

$$P_{66} = \mathbf{34.7}$$

$$\mathbf{c} \quad 30\% \text{ to } 70\% \text{ interpercentile} = 34.7 - 20.5 = \mathbf{14.2}$$

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Representation and summary of data – measures of dispersion

Exercise F, Question 4

Question:

The heights (h) of 10 mountains were recorded to the nearest 10 m and are shown in the table below.

h	1010	1030	1050	1000	1020	1030	1030	1040	1020	1000
-----	------	------	------	------	------	------	------	------	------	------

Use the coding $y = \frac{h-1000}{10}$ to find the standard deviation of the heights.

Solution:

coded data 10, 10.2, 10.4, 9.9 10.1, 10.2, 10.2, 10.3, 10.1, 9.9

$$\text{Standard deviation of coded data} = \sqrt{\frac{1026.41}{10} - \left(\frac{101.3}{10}\right)^2} = 0.155$$

$$\text{Standard deviation} = 0.155 \times 100 = \mathbf{15.5}$$

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Representation and summary of data – measures of dispersion

Exercise F, Question 5

Question:

The times it took a random sample of runners to complete a race are summarised in the table.

Time taken (t minutes)	Frequency
20–29	5
30–39	10
40–49	36
50–59	20
60–69	9

a Work out the lower quartile.

b Work out the upper quartile.

c Work out the interquartile range.

The mid-point of each class was represented by x and its corresponding frequency by f giving:

$$\Sigma fx = 3740 \quad \Sigma fx^2 = 183\,040$$

d Estimate the variance and standard deviation for these data.

Solution:

$$\mathbf{CF = 5 \ 15 \ 51 \ 71 \ 80}$$

a Data is continuous so: $Q_1 : \frac{80}{4} = 20^{\text{th}}$ value so Q_1 is in class 40 – 49

$$\frac{Q_1 - 39.5}{49.5 - 39.5} = \frac{20 - 15}{51 - 15}$$

$$Q_1 = 40.9$$

b $Q_3 : 3 \times \frac{80}{4} = 60^{\text{th}}$ value so Q_3 is in class 50 – 59

$$\frac{Q_3 - 49.5}{59.5 - 49.5} = \frac{60 - 51}{71 - 51}$$

$$Q_3 = 54$$

$$\mathbf{c \ IQR = 54 - 40.9 = 13.1}$$

$$\mathbf{d \ \text{Variance} = \frac{183040}{80} - \left(\frac{3740}{80}\right)^2 = 102.4375 = 102}$$

$$\text{Standard deviation} = \sqrt{102.4375} = \mathbf{10.1}$$

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Representation and summary of data – measures of dispersion Exercise F, Question 6

Question:

20 birds were caught for ringing.

Their wing spans (x) were measured to the nearest centimetre.

The following summary data was worked out:

$$\Sigma x = 316 \quad \Sigma x^2 = 5078$$

a Work out the mean and the standard deviation of the wing spans of the 20 birds. One more bird was caught. It had a wing span of 13 centimetres

b Without doing any further working say how you think this extra wing span will affect the mean wing span.

Solution:

a Mean = $\frac{316}{20} = 15.8$

$$\text{Standard deviation} = \sqrt{\frac{5078}{20} - \left(\frac{316}{20}\right)^2} = 2.06$$

b It will decrease the mean wing span since $13 < 15.8$

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Representation and summary of data – measures of dispersion

Exercise F, Question 7

Question:

The heights of 50 clover flowers are summarised in the table.

Heights in mm (h)	Frequency
90–95	5
95–100	10
100–105	26
105–110	8
110–115	1

- a** Find Q_1 .
- b** Find Q_2 .
- c** Find the interquartile range.
- d** Use $\Sigma fx = 5075$ and $\Sigma fx^2 = 516\,112.5$ to find the standard deviation.

Solution:

CF = 5 15 41 49 50

a Data is continuous so:

$$Q_1 : \frac{50}{4} = 12.5^{\text{th}} \text{ value so } Q_1 \text{ is in class } 95 - 100$$

$$\frac{Q_1 - 95}{100 - 95} = \frac{12.5 - 5}{15 - 5}$$

$$Q_1 = \mathbf{98.75}$$

$$\mathbf{b} \quad Q_3 : 3 \times \frac{50}{4} = 37.5^{\text{th}} \text{ value so } Q_3 \text{ is in class } 100 - 105$$

$$\frac{Q_3 - 100}{105 - 100} = \frac{37.5 - 15}{41 - 15}$$

$$Q_3 = \mathbf{104.33}$$

$$\mathbf{c} \quad \text{IQR} = 104.33 - 98.75 = \mathbf{5.58}$$

$$\mathbf{d} \quad \text{Standard deviation} = \sqrt{\frac{516112.5}{50} - \left(\frac{5075}{50}\right)^2} = \mathbf{4.47}$$