Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 1

Question:

A particle P is moving in a straight line. Initially P is moving through a point O with speed 4 m s^{-1}. At time t seconds after passing through O the acceleration of P is $3e^{-0.25t}$ m s^{-2} in the direction OP. Find the velocity of the particle at time t seconds.

Solution:

$$v = \int a \, dt = \int 3e^{-0.25t} \, dt$$
$$= -12e^{-0.25t} + A$$

When $t = 0, v = 4$

$$4 = -12 + A \Rightarrow A = 16$$
$$v = 16 - 12e^{-0.25t}$$

The velocity of the particle at time t seconds is $(16 - 12e^{-0.25t})$m s^{-1}.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 2

Question:

A particle P is moving along the x-axis in the direction of x increasing. At time t seconds, the velocity of P is $(t\sin t)\text{m s}^{-1}$. When $t = 0$, P is at the origin. Show that when $t = \frac{\pi}{2}$, P is 1 metre from O.

Solution:

$x = \int v\, \mathrm{d}t = \int t\sin t\, \mathrm{d}t$

Using integration by parts

$x = -t\cos t + \int \cos t\, \mathrm{d}t$

$= -t\cos t + \sin t + A$

$t = 0, x = 0$

When $0 = 0 + 0 + A \Rightarrow A = 0$

$x = -t\cos t + \sin t$

When $t = \frac{\pi}{2}$

$x = -\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$

Hence P is one metre from O, as required.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 3

Question:

At time t seconds the velocity, v m s^{-1}, of a particle moving in a straight line is given by $v = \dfrac{4}{3+2t^{0}}$ $\quad t \geq 0$.

When $t = 0$, the particle is at a point A. When $t = 3$, the particle is at the point B.

Find the distance between A and B.

Solution:

$s = \int v \, \mathrm{d}t = \int \dfrac{4}{3+2t} \, \mathrm{d}t = 2\ln(3+2t) + C$ where s is the displacement from point A.

When $t = 0, s = 0$

$$0 = 2\ln 3 + C \Rightarrow C = -2\ln 3$$

$$s = 2\ln(3+2t) - 2\ln 3 = 2\ln\left(\frac{3+2t}{3}\right)$$

When $t = 3$

$$s = 2\ln\left(\frac{3+6}{3}\right) = 2\ln 3$$

$$AB = 2\ln 3 \text{ m}$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 4

Question:

A particle P is moving along the x-axis in the positive direction. At time t seconds the acceleration of P is $4e^{\frac{1}{2}t}$ m s^{-1} in the positive direction. When $t = 0$, P is at rest. Find the distance P moves in the interval $0 \leq t \leq 2$. Give your answer to 3 significant figures.

Solution:

$v = \int a \, dt = \int 4e^{\frac{1}{2}t} \, dt = 8e^{\frac{1}{2}t} + A$

When $t = 0, v = 0$

$0 = 8 + A \Rightarrow A = -8$

$v = 8e^{\frac{1}{2}t} - 8$

The distance moved in the interval $0 \leq t \leq 2$ is given by

$s = \int v \, dt = \int_0^2 \left(8e^{\frac{1}{2}t} - 8\right) dt$

$= \left[16\,e^{\frac{1}{2}t} - 8t\right]_0^2 = (16e^1 - 16) - 16$

$= 16e - 32 \approx 11.5$

The distance moved is 11.5 m (3 s.f.).

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 5

Question:

A particle P is moving along the x-axis. At time t seconds the displacement of P from O is x m and the velocity of P is $\left(4\cos 3t \text{ m s}^{-1}\right)$, both measured in the direction Ox. When $t = 0$ the particle P is at the origin O. Find

a the magnitude of the acceleration when $t = \frac{\pi}{12}$,

b x in terms of t,

c the smallest positive value of t for which P is at O.

Solution:

a $a = \frac{dv}{dt} = -12\sin 3t$

When $t = \frac{\pi}{12}$

$$a = -12\sin\frac{\pi}{4} = -12 \times \frac{1}{\sqrt{2}} = -6\sqrt{2}$$

The magnitude of the acceleration when $t = \frac{\pi}{12}$ is $6\sqrt{2}\text{ m s}^{-2}$.

b $x = \int v \, dt = \int 4\cos 3t \, dt = \frac{4}{3}\sin 3t + A$

$t = 0, x = 0$

When $0 = \frac{4}{3} \times 0 + A \Rightarrow A = 0$

$$x = \frac{4}{3}\sin 3t$$

c When P is at O, $x = 0$

$x = \frac{4}{3}\sin 3t = 0 \Rightarrow \sin 3t = 0$

The smallest positive value of t is given by

$3t = \pi \Rightarrow t = \frac{\pi}{3}$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 6

Question:

A particle P is moving along a straight line. Initially P is at rest. At time t seconds P has velocity v m s^{-1} and acceleration a m s^{-2} where

$$a = \frac{6t}{\left(2+t^2\right)^0} \, t \geq 0.$$

Find v in terms of t.

Solution:

$$v = \int a \ \mathrm{d}t = \int \frac{6t}{(2+t^2)^2} \ \mathrm{d}t$$

Let $u = 2 + t^2$, then $\frac{\mathrm{d}u}{\mathrm{d}t} = 2t$

$$v = \int \frac{6t}{(2+t^2)^2} \ \mathrm{d}t = \int \frac{3}{(2+t^2)^2} \times 2t \ \mathrm{d}t$$

$$= \int \frac{3}{u^2} \ \mathrm{d}u = \int 3u^{-2} \ \mathrm{d}u$$

$$= \frac{3u^{-1}}{-1} + A = A - \frac{3}{u}$$

$$= A - \frac{3}{2+t^2}$$

When $t = 0, v = 0$

$$0 = A - \frac{3}{2} \Rightarrow A = \frac{3}{2}$$

$$v = \frac{3}{2} - \frac{3}{2+t^2}$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 7

Question:

A particle P is moving along the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = \begin{cases} 4, & 0 \le t \le 3 \\ 5 - \dfrac{3}{t}, & 3 < t \le 6 \end{cases}$$

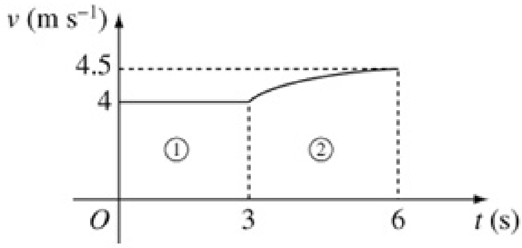
When $t = 0$, P is at the origin O.

a Sketch a velocity–time graph to illustrate the motion of P in the interval $0 \le t \le 6$.

b Find the distance of P from O when $t = 6$.

Solution:

a



b The distance moved in the first three seconds is represented by the area labelled ①.

Let this area be A_1. $A_1 = 3 \times 4 = 12$

The distance travelled in the next three seconds is represented by the area labelled ②.

Let this area be A_2.

$$A_2 = \int_3^6 \left(5 - \frac{3}{t}\right) \mathrm{d}t$$

$$= \left[5t - 3\ln t\right]_3^6 = (30 - 3\ln 6) - (15 - 3\ln 3)$$

$$= 15 - 3\ln 2$$

The distance of P from O when $t = 6$ is $(12 + 15 - 3\ln 2)$ m $= (27 - 3\ln 2)$ m .

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 8

Question:

A particle P is moving in a straight line with acceleration $\left(\sin\frac{1}{2}t\right)\text{m s}^{-2}$ at time t seconds, $t \geq 0$. The particle is initially at rest at a point O. Find

a the speed of P when $t = 2\pi$,

b the distance of P from O when $t = \frac{\pi}{2}$.

Solution:

a $v = \int a \, dt = \int \sin\frac{1}{2}t \, dt = -2\cos\frac{1}{2}t + A$

When $t = 0, v = 0$

$0 = -2 + A \Rightarrow A = 2$

$v = 2 - 2\cos\frac{1}{2}t$

When $t = 2\pi$

$v = 2 - 2\cos\pi = 2 - (2 \times -1) = 4$

The speed of P when $t = 2\pi$ is 4 m s^{-1}.

b $x = \int v \, dt = \int\left(2 - 2\cos\frac{1}{2}t\right)dt = 2t - 4\sin\frac{1}{2}t + B$

When $t = 0, x = 0$

$0 = 0 - 0 + B \Rightarrow B = 0$

$x = 2t - 4\sin\frac{1}{2}t$

When $t = \frac{\pi}{2}$

$x = 2 \times \frac{\pi}{2} - 4\sin\frac{\pi}{4} = \pi - 4 \times \frac{1}{\sqrt{2}} = \pi - 2\sqrt{2}$

The distance of P from O when $t = \frac{\pi}{2}$ is $(\pi - 2\sqrt{2})$ m.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 9

Question:

A particle P is moving along the x-axis. At time t seconds P has velocity v m s^{-1} in the direction x increasing and an acceleration of magnitude $4e^{0.2t}$ m s^{-2} in the direction x decreasing. When $t = 0$, P is moving through the origin with velocity 20 m s^{-1} in the direction x increasing. Find

a v in terms of t,
b the maximum value of x attained by P during its motion.

Solution:

a $v = \int a \, dt = \int -4e^{0.2t} \, dt = -20e^{0.2t} + A$

When $t = 0, v = 20$

$20 = -20 + A \Rightarrow A = 40$

$v = 40 - 20\,e^{0.2t}$

b $x = \int v \, dt = \int (40 - 20e^{0.2t}) \, dt = 40t - 100e^{0.2t} + B$

When $t = 0, x = 0$

$0 = 0 - 100 + B \Rightarrow B = 100$

$x = 40t - 100e^{0.2t} + 100$

The maximum value of x occurs when

$\frac{dx}{dt} = v = 40 - 20e^{0.2t} = 0$

$e^{0.2t} = 2$

$0.2t = \ln 2$

$t = 5\ln 2$

The maximum value of x is given by

$x = 40 \times 5\ln 2 - 100 \times 2 + 100 = 200\ln 2 - 100$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 10

Question:

A car is travelling along a straight road. As it passes a sign S, the driver applies the brakes. The car is modelled as a particle. At time t seconds the car is x m from S and its velocity, v m s^{-1}, is modelled by the equation $v = \frac{3200}{c + dt}$ where c and d are constants.

Given that when $t = 0$, the speed of the car is 40 m s^{-1} and its deceleration is 0.5 m s^{-2}, find

a the value of c and the value of d,

b x in terms of t.

Solution:

a $v = \frac{3200}{c + dt}$

When $t = 0, v = 40$

$$40 = \frac{3200}{c} \Rightarrow c = 80$$

$$v = \frac{3200}{80 + dt} = 3200(80 + dt)^{-1}$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -3200d(80 + dt)^{-2} = -\frac{3200d}{(80 + dt)^2}$$

When $t = 0, a = -0.5$

$$-\frac{3200d}{80^2} = -0.5 \Rightarrow d = \frac{0.5 \times 80^2}{3200} = 1$$

$$c = 80, d = 1$$

b $x = \int v \, \mathrm{d}t = \int \frac{3200}{80 + t} \mathrm{d}t = 3200 \ln(80 + t) + A$

When $t = 0, x = 0$

$$0 = 3200 \ln 80 + A \Rightarrow A = -3200 \ln 80$$

$$x = 3200 \ln(80 + t) - 3200 \ln 80 = 3200 \ln\left(\frac{80 + t}{80}\right)$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 11

Question:

A particle P is moving along a straight line. When $t = 0$, P is passing through a point A. At time t seconds after passing through A the velocity, v m s^{-1}, of P is given by $v = e^{2t} - 11e^{t} + 15t$.

Find

a the values of t for which the acceleration is zero,

b the distance of P from A when $t = \ln 3$.

Solution:

a $a = \frac{dv}{dt} = 2e^{2t} - 11e^{t} + 15 = 0$

$(2e^{t} - 5)(2e^{t} - 3) = 0$

$e^{t} = 2.5, 3$

$t = \ln 2.5, \ln 3$

b $x = \int v \, dt = \int (e^{2t} - 11e^{t} + 15t)\, dt = \frac{e^{2t}}{2} - 11e^{t} + \frac{15t^2}{2} + A$

When $t = 0, x = 0$

$0 = \frac{1}{2} - 11 + 0 + A \Rightarrow A = \frac{21}{2}$

$x = \frac{e^{2t}}{2} - 11e^{t} + \frac{15t^2}{2} + \frac{21}{2}$

When $t = \ln 3$

$x = \frac{e^{2\ln 3}}{2} - 11e^{\ln 3} + \frac{15(\ln 3)^2}{2} + \frac{21}{2}$

$= \frac{9}{2} - 33 + \frac{15(\ln 3)^2}{2} + \frac{21}{2} = \frac{15(\ln 3)^2}{2} - 18 \approx -8.95$

As distance is a positive quantity, the required distance is $\left(18 - \frac{15(\ln 3)^2}{2}\right)$ m .

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise A, Question 12

Question:

A particle P moves along a straight line. At time t seconds (where $t \geq 0$) the velocity of P is $\left[2t + \ln(t+2)\right]$ m s^{-1}. Find

a the value of t for which the acceleration has magnitude 2.2 m s^{-2}.

b the distance moved by P in the interval $1 \leq t \leq 4$.

Solution:

a $a = \frac{\mathrm{d}v}{\mathrm{d}t} = 2 + \frac{1}{t+2} = 2.2$

$\frac{1}{t+2} = 0.2 \Rightarrow t + 2 = 5 \Rightarrow t = 3$

b $x = \int v \, \mathrm{d}t = \int_1^4 (2t + \ln(t+2)) \, \mathrm{d}t$

Using integration by parts

$$\int \ln(t+2) \, \mathrm{d}t = \int 1.\ln(t+2) \, \mathrm{d}t$$

$$= t\ln(t+2) - \int \frac{t}{t+2} \, \mathrm{d}t = t\ln(t+2) - \int \left(1 - \frac{2}{t+2}\right) \mathrm{d}t$$

$$= t\ln(t+2) - t + 2\ln(t+2) = (t+2)\ln(t+2) - t$$

Hence $x = \left[t^2 + (t+2)\ln(t+2) - t\right]_1^4$

$$= (16 + 6\ln 6 - 4) - (1 + 3\ln 3 - 1)$$

$$= 12 + 6\ln 6 - 3\ln 3 = 12 + 3\ln 6^2 - 3\ln 3$$

$$= 12 + 3\ln\left(\frac{36}{3}\right) = 12 + 3\ln 12$$

The distance moved by P in the interval $1 \leq t \leq 4$ is $(12 + 3\ln 12)$ m .

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 1

Question:

A particle P moves along the x-axis. At time $t=0$, P passes through the origin O with velocity 5 m s^{-1} in the direction of x increasing. At time t seconds, the velocity of P is $v\text{ m s}^{-1}$ and $OP=x\text{ m}$. The acceleration of P is $\left(2+\frac{1}{2}x\right)\text{m s}^{-2}$, measured in the positive x direction. Find v^2 in terms of x.

Solution:

$$a=2+\frac{1}{2}x$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right)=2+\frac{1}{2}x$$
$$\frac{1}{2}v^2=\int\left(2+\frac{1}{2}x\right)\mathrm{d}x$$
$$=2x+\frac{x^2}{4}+A$$
At $x=0, v=5$
$$\frac{1}{2}\times 25=0+0+A\Rightarrow A=\frac{25}{2}$$
$$\frac{1}{2}v^2=2x+\frac{x^2}{4}+\frac{25}{2}$$
$$v^2=\frac{x^2}{2}+4x+25$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 2

Question:

A particle P moves along a straight line. When its displacement from a fixed point O on the line is x m and its velocity is v m s^{-1}, the deceleration of P is $4x$ m s^{-2}. At $x = 2$, $v = 8$. Find v in terms of x.

Solution:

$$a = -4x$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = -4x$$
$$\frac{1}{2}v^2 = \int(-4x)\,\mathrm{d}x$$
$$= -2x^2 + A$$
At $x = 2, v = 8$
$$\frac{1}{2} \times 64 = -8 + A \Rightarrow A = 40$$
$$\frac{1}{2}v^2 = -2x^2 + 40$$
$$v^2 = 80 - 4x^2$$
$$v = \pm\sqrt{(80 - 4x^2)}$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 3

Question:

A particle P is moving along the x-axis in the direction of x increasing. At $OP = x$ m$(x > 0)$, the velocity of P is v m s^{-1} and its acceleration is of magnitude $\frac{4}{x^2}$ m s^{-2} in the direction of x increasing. Given that at $x = 2, v = 6$ find the value of x for which P is instantaneously at rest.

Solution:

$$a = \frac{4}{x^2}$$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{4}{x^2}$$

$$\frac{1}{2}v^2 = \int (4x^{-2})\,dx$$

$$= \frac{4x^{-1}}{-1} + A = A - \frac{4}{x}$$

At $x = 2, v = 6$

$$\frac{1}{2} \times 36 = A - 2 \Rightarrow A = 20$$

$$\frac{1}{2}v^2 = 20 - \frac{4}{x}$$

When $v = 0$

$$0 = 20 - \frac{4}{x} \Rightarrow x = \frac{4}{20} = \frac{1}{5}$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 4

Question:

A particle P moves along a straight line. When its displacement from a fixed point O on the line is x m and its velocity is v m s^{-1}, the acceleration of P is of magnitude $25x$ m s^{-2} and is directed towards O. At $x = 0, v = 40$. In its motion P is instantaneously at rest at two points A and B. Find the distance between A and B.

Solution:

$a = -25x$

$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = -25x$

$\frac{1}{2}v^2 = \int(-25x)\,\mathrm{d}x$

$= -\frac{25}{2}x^2 + A$

At $x = 0, v = 40$

$\frac{1}{2} \times 1600 = -0 + A \Rightarrow A = 800$

$\frac{1}{2}v^2 = -\frac{25}{2}x^2 + 800$

When $v = 0$

$0 = -\frac{25}{2}x^2 + 800 \Rightarrow x^2 = 64 \Rightarrow x = \pm 8$

$AB = 16$ m

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 5

Question:

A particle P is moving along the x-axis. At $OP = x$ m, the velocity of P is v m s^{-1} and its acceleration is of magnitude kx^2 m s^{-2}, where k is a positive constant, in the direction of x decreasing. At $x = 0$, $v = 16$. The particle is instantaneously at rest at $x = 20$. Find

a the value of k,

b the velocity of P when $x = 10$.

Solution:

a $a = -kx^2$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -kx^2$$

$$\frac{1}{2}v^2 = \int(-kx^2)dx$$

$$= -\frac{kx^3}{3} + A$$

At $x = 0, v = 16$

$$\frac{1}{2} \times 256 = -0 + A \Rightarrow A = 128$$

$$\frac{1}{2}v^2 = -\frac{kx^3}{3} + 128$$

When $v = 0, x = 20$

$$0 = -\frac{8000k}{3} + 128$$

$$k = \frac{3 \times 128}{8000} = \frac{6}{125}$$

b $\frac{1}{2}v^2 = -\frac{6}{125} \times \frac{x^2}{3} + 128$

$$v^2 = 256 - \frac{4}{125}x^3$$

$$x = 10$$

When $v^2 = 256 - \frac{4}{125} \times 1000 = 224$

$$v = \pm\sqrt{224} = \pm 4\sqrt{14}$$

The velocity of P when $x = 10$ is $\pm 4\sqrt{14}$ m s^{-1} as the particle will pass through this position in both directions.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 6

Question:

A particle P is moving along the x-axis in the direction of x increasing. At $OP = x$ m, the velocity of P is v m s^{-1} and its acceleration is of magnitude $8x^3$ m s^{-2} in the direction PO. At $x = 2$, $v = 32$. Find the value of x for which $v = 8$.

Solution:

$a = -8x^3$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -8x^3$

$\frac{1}{2}v^2 = \int(-8x^3)dx$

$\quad = -2x^4 + A$

At $x = 2, v = 32$

$\frac{1}{2} \times 1024 = A - 32 \Rightarrow A = 544$

$\frac{1}{2}v^2 = 544 - 2x^4$

$v^2 = 1088 - 4x^4$

When $v = 8$

$64 = 1088 - 4x^4 \Rightarrow x^4 = 256$

$x = 256^{\frac{1}{4}} = 4$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 7

Question:

A particle P is moving along the x-axis. When the displacement of P from the origin O is x m, the velocity of P is v m s^{-1} and its acceleration is $6\sin\frac{x}{3}$ m s^{-2}. At $x=0$, $v=4$. Find

a v^2 in terms of x,

b the greatest possible speed of P.

Solution:

a $a = 6\sin\frac{x}{3}$

$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = 6\sin\frac{x}{3}$

$\frac{1}{2}v^2 = \int\left(6\sin\frac{x}{3}\right)\mathrm{d}x$

$= -18\cos\frac{x}{3} + A$

At $x=0, v=4$

$\frac{1}{2}\times 16 = -18 + A \Rightarrow A = 26$

$\frac{1}{2}v^2 = -18\cos\frac{x}{3} + 26$

$v^2 = 52 - 36\cos\frac{x}{3}$

b The greatest value of v^2 occurs when $\cos\frac{x}{3} = -1$.

The greatest value of v^2 is given by

$v^2 = 52 + 36 = 88$

$v = \pm\sqrt{88} = \pm 2\sqrt{22}$

The greatest possible speed of P is $2\sqrt{22}$ m s^{-1} (≈ 9.38 m s^{-1}).

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 8

Question:

A particle P is moving along the x-axis. At $x = 0$, the velocity of P is 2 m s^{-1} in the direction of x increasing. At $OP = x$ m, the velocity of P is $v \text{ m s}^{-1}$ and its acceleration is $(2 + 3e^{-x}) \text{ m s}^{-2}$. Find the velocity of P at $x = 3$. Give your answer to 3 significant figures.

Solution:

$a = 2 + 3e^{-x}$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2 + 3e^{-x}$

$\frac{1}{2}v^2 = \int (2 + 3e^{-x})\, dx$

$= 2x - 3e^{-x} + A$

At $x = 0, v = 2$

$\frac{1}{2} \times 4 = 0 - 3 + A \Rightarrow A = 5$

$\frac{1}{2}v^2 = 2x - 3e^{-x} + 5$

$v^2 = 4x - 6e^{-x} + 10$

At $x = 3$

$v^2 = 12 - 6e^{-3} + 10 = 21.701\ldots$

$v = \sqrt{(21.701\ldots)} = 4.658\ldots$

The velocity of P at $x = 3$ is 4.66 m s^{-1} (3 s.f.), in the direction of x increasing.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 9

Question:

A particle P moves away from the origin O along the positive x-axis. The acceleration of P is of magnitude $\frac{4}{2x+1}$ m s^{-2}, where $OP = x$ m, directed towards O. Given that the speed of P at O is 4 m s^{-1}, find

a the speed of P at $x = 10$,

b the value of x at which P is instantaneously at rest.

Give your answers to 3 significant figures.

Solution:

a $a = -\frac{4}{2x+1}$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = -\frac{4}{2x+1}$$

$$\frac{1}{2}v^2 = \int\left(-\frac{4}{2x+1}\right)\mathrm{d}x$$

$$= -2\ln(2x+1) + A$$

At $x = 0, v = 4$

$$\frac{1}{2}\times 16 = -0 + A \Rightarrow A = 8$$

$$\frac{1}{2}v^2 = -2\ln(2x+1) + 8$$

$$v^2 = 16 - 4\ln(2x+1)$$

At $x = 10$

$$v^2 = 16 - 4\ln 21 = 3.821910\ldots$$

$$v = 1.954\ldots$$

The speed of P at $x = 10$ is 1.95 m s^{-1} (3 s.f.).

b When $v = 0$

$$0 = 16 - 4\ln(2x+1) \Rightarrow \ln(2x+1) = 4$$

$$2x + 1 = e^4 \Rightarrow x = \frac{e^4 - 1}{2} = 26.799\ldots = 26.8 \text{ (3 s.f.)}.$$

Solutionbank M3

Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 10

Question:

A particle P is moving along the positive x-axis. At $OP = x$ m, the velocity of P is v m s^{-1} and its acceleration is $\left(x - \frac{4}{x^3}\right)$m s^{-2}. The particle starts from the position where $x = 1$ with velocity 3 m s^{-1} in the direction of x increasing. Find

a v in terms of x,

b the least speed of P during its motion.

Solution:

a $a = x - \frac{4}{x^3}$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x - \frac{4}{x^3}$$

$$\frac{1}{2}v^2 = \int (x - 4x^{-3})dx$$

$$= \frac{x^2}{2} - \frac{4x^{-2}}{-2} + A = \frac{x^2}{2} + \frac{2}{x^2} + A$$

At $x = 1, v = 3$

$$\frac{1}{2} \times 9 = \frac{1}{2} + 2 + A \Rightarrow A = 2$$

$$\frac{1}{2}v^2 = \frac{x^2}{2} + \frac{2}{x^2} + 2$$

$$v^2 = x^2 + 4 + \frac{4}{x^2} = \left(x + \frac{2}{x}\right)^2$$

$$v = x + \frac{2}{x}$$

b The minimum value of v occurs when $\frac{dv}{dt} = a = 0$

$x - \frac{4}{x^3} = 0 \Rightarrow x^4 = 4 \Rightarrow x = \sqrt{2}$ (as P moves on the positive x-axis, $x > 0$)

At $x = \sqrt{2}$

$$v = \sqrt{2} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$$

The least speed of P during its motion is $2\sqrt{2}$ m s^{-1}.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 11

Question:

A particle P is moving along the x-axis. Initially P is at the origin O moving with velocity 15 m s^{-1} in the direction of x increasing. When the displacement of P from O is x m, its acceleration is of magnitude $\left(10+\frac{1}{4}x\right)\text{m s}^{-2}$ directed towards O. Find the distance P moves before first coming to instantaneous rest.

Solution:

$$a = -\left(10+\frac{1}{4}x\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = -10-\frac{1}{4}x$$

$$\frac{1}{2}v^2 = \int\left(-10-\frac{1}{4}x\right)\mathrm{d}x$$

$$= -10x-\frac{x^2}{8}+A$$

At $x=0, v=15$

$$\frac{1}{2}\times 225 = -0-0+A \Rightarrow A = \frac{225}{2}$$

$$\frac{1}{2}v^2 = -10x-\frac{x^2}{8}+\frac{225}{2}$$

$$v^2 = 225-20x-\frac{x^2}{4} = -\frac{x^2+80x-900}{4} = -\frac{(x+90)(x-10)}{4}$$

$$v = 0 \Rightarrow x = 10, -90$$

As P is initially moving in the direction of x increasing, it reaches $x=10$ before $x=-90$. The distance P moves before first coming to instantaneous rest is 10 m.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise B, Question 12

Question:

A particle P is moving along the x-axis. At time t seconds, P is x m from O, has velocity v m s^{-1} and acceleration of magnitude $6x^{\frac{1}{3}}$ m s^{-2} in the direction of x increasing. When $t = 0$, $x = 8$ and $v = 12$. Find

a v in terms of x,

b x in terms of t.

Solution:

a $a = 6x^{\frac{1}{3}}$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = 6x^{\frac{1}{3}}$$

$$\frac{1}{2}v^2 = \int 6x^{\frac{1}{3}}\,\mathrm{d}x = \frac{6x^{\frac{4}{3}}}{\frac{4}{3}} + A = \frac{9}{2}x^{\frac{4}{3}} + A$$

$$v^2 = 9x^{\frac{4}{3}} + B, \text{ where } B = 2A$$

At $x = 8, v = 12$

$$144 = 9 \times 16 + B \Rightarrow B = 0$$

$$v^2 = 9x^{\frac{4}{3}}$$

$$v = 3x^{\frac{2}{3}}$$

b $v = \frac{\mathrm{d}x}{\mathrm{d}t} = 3x^{\frac{2}{3}}$

Separating the variables and integrating

$$\int x^{-\frac{2}{3}}\,\mathrm{d}x = \int 3\,\mathrm{d}t$$

$$3x^{\frac{1}{3}} = 3t + C$$

When $t = 0, x = 8$

$$3 \times 2 = 0 + C \Rightarrow C = 6$$

$$3x^{\frac{1}{3}} = 3t + 6$$

$$x^{\frac{1}{3}} = t + 2$$

$$x = (t+2)^3$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 1

Question:

A particle P moves along a straight line. When the displacement of P from a fixed point on the line is x m, its velocity is v m s^{-1} and its acceleration is of magnitude $\frac{6}{x^2}$ m s^{-2} in the direction of x increasing. At $x = 3, v = 4$.
Find v in terms of x.

Solution:

$$a = \frac{6}{x^2} = 6x^{-2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = 6x^{-2}$$

$$\frac{1}{2}v^2 = \int 6x^{-2}\ \mathrm{d}x = \frac{6x^{-1}}{-1} + A$$

$$\frac{1}{2}v^2 = A - \frac{6}{x}$$

At $x = 3, v = 4$

$$\frac{1}{2} \times 16 = A - 2 \Rightarrow A = 10$$

$$\frac{1}{2}v^2 = 10 - \frac{6}{x}$$

$$v^2 = 20 - \frac{12}{x}$$

$$v = \sqrt{\left(20 - \frac{12}{x}\right)}$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 2

Question:

A particle P is moving along the x-axis. At time t seconds, the displacement of P from the origin O is x m and the velocity of P is $4e^{0.5t}$ m s^{-1} in the direction Ox. When $t = 0$, P is at O. Find

a x in terms of t,

b the acceleration of P when $t = \ln 9$.

Solution:

a $v = \frac{dx}{dt} = 4e^{0.5t}$

$x = \int 4e^{0.5t}\, dt = 8e^{0.5t} + A$

When $t = 0, x = 0$

$0 = 8 + A \Rightarrow A = -8$

$x = 8e^{0.5t} - 8 = 8(e^{0.5t} - 1)$

b $a = \frac{dv}{dt} = 2e^{0.5t}$

When $t = \ln 9$

$a = 2e^{0.5\ln 9} = 2e^{\ln 3} = 2 \times 3 = 6$

The acceleration of P when $t = \ln 9$ is 6 m s^{-2}.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 3

Question:

A particle is moving along the x-axis. At time $t = 0$, P is passing through the origin O with velocity 8 m s^{-1} in the direction of x increasing. When P is x m from O, its acceleration is $\left(3 + \frac{1}{4}x\right) \text{m s}^{-2}$ in the direction of x decreasing.

Find the positive value of x for which P is instantaneously at rest.

Solution:

$$a = -\left(3 + \frac{1}{4}x\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = -3 - \frac{1}{4}x$$

$$\frac{1}{2}v^2 = \int\left(-3 - \frac{1}{4}x\right)\mathrm{d}x = -3x - \frac{1}{8}x^2 + A$$

At $x = 0, v = 8$

$$32 = -0 - 0 + A \Rightarrow A = 32$$

$$\frac{1}{2}v^2 = 32 - 3x - \frac{1}{8}x^2$$

When $v = 0$

$$0 = 32 - 3x - \frac{1}{8}x^2$$

$$x^2 + 24x - 256 = 0$$

$$(x+32)(x-8) = 0$$

As $x > 0$

$$x = 8$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 4

Question:

A particle P is moving on the x-axis. When P is a distance x metres from the origin O, its acceleration is of magnitude $\frac{15}{4x^2}$ m s^{-2} in the direction OP. Initially P is at the point where $x = 5$ and is moving toward O with speed 6 m s^{-1}.
Find the value of x where P first comes to rest.

Solution:

$$a = \frac{15}{4x^2} = \frac{15}{4}x^{-2}$$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{15}{4}x^{-2}$$

$$\frac{1}{2}v^2 = \int \frac{15}{4}x^{-2}\ dx = -\frac{15}{4}x^{-1} + A$$

$$\frac{1}{2}v^2 = A - \frac{15}{4x}$$

At $x = 5, v = -6$

$$18 = A - \frac{15}{20} \Rightarrow A = 18\frac{3}{4} = \frac{75}{4}$$

$$\frac{1}{2}v^2 = \frac{75}{4} - \frac{15}{4x} = \frac{15}{4}\left(5 - \frac{1}{x}\right)$$

When $v = 0$

$$5 - \frac{1}{x} = 0 \Rightarrow x = \frac{1}{5}$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 5

Question:

A particle P moves along the x-axis in the direction x increasing. At time t seconds, the velocity of P is v m s^{-1} and its acceleration is $20te^{-t^2}$ m s^{-2}. When $t = 0$ the speed of P is 8 m s^{-1}. Find

a v in terms of t,
b the limiting velocity of P.

Solution:

a $a = \frac{dv}{dt} = 20t\,e^{-t^2}$

$v = \int 20t\,e^{-t^2}\,dt = -10e^{-t^2} + A$

When $t = 0, v = 8$

$8 = -10 + A \Rightarrow A = 18$

$v = 18 - 10e^{-t^2}$

b As $t \to \infty, e^{-t^2} \to 0$ and $v \to 18$

The limiting velocity of P is 18 m s^{-1}.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 6

Question:

A particle P moves along a straight line. Initially P is at rest at a point O on the line. A time t seconds, where $t \geq 0$, the acceleration of P is $\dfrac{18}{(2t+3)^3}$ m s^{-2} directed away from O.
Find the value of t for which the speed of P is $0.48\,\text{m s}^{-1}$.

Solution:

$$a = \frac{dv}{dt} = \frac{18}{(2t+3)^3} = 18(2t+3)^{-3}$$

$$v = \int 18(2t+3)^{-3}\,dt = \frac{18}{-2\times 2}(2t+3)^{-2} + A$$

$$= A - \frac{9}{2(2t+3)^2}$$

When $t = 0, v = 0$

$$0 = A - \frac{9}{2\times 3^2} \Rightarrow A = \frac{1}{2}$$

$$v = \frac{1}{2} - \frac{9}{2(2t+3)^2}$$

When $v = 0.48$

$$0.48 = \frac{1}{2} - \frac{9}{2(2t+3)^2} \Rightarrow \frac{9}{2(2t+3)^2} = 0.02$$

$$(2t+3)^2 = \frac{9}{2\times 0.02} = 225$$

$$t \geq 0$$

As $2t+3 = \sqrt{225} = 15$

$$t = \frac{15-3}{2} = 6$$

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 7

Question:

A particle P is moving along the x-axis. At time t seconds, the velocity of P is v m s^{-1} and the acceleration of P is $(3-x)$m s^{-2} in the direction x increasing. Initially P is at the origin O and is moving with speed 4 m s^{-1} in the direction x increasing. Find

a v^2 in terms of x,
b the maximum value of v.

Solution:

a $a = 3 - x$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 3 - x$$

$$\frac{1}{2}v^2 = \int (3-x)\,dx = 3x - \frac{x^2}{2} + A$$

$$v^2 = B + 6x - x^2, \text{ where } B = 2A$$

At $x = 0, v = 4$

$$16 = B + 0 - 0 \Rightarrow B = 16$$

$$v^2 = 16 + 6x - x^2$$

b $v^2 = 16 + 6x - x^2 = 25 - 9 + 6x - x^2$

$$= 25 - (x-3)^2$$

As $(x-3)^2 \geq 0, v^2 \leq 25$

The greatest value of v is 5.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 8

Question:

A particle P is moving along the x-axis. At time $t=0$, P passes through the origin O. After t seconds the speed of P is v m s^{-1}, $OP=x$ metres and the acceleration of P is $\frac{x^2(5-x)}{2}$ m s^{-2} in the direction x increasing. At $x=10$, P is instantaneously at rest.

Find

a an expression for v^2 in terms of x,

b the speed of P when $t=0$.

Solution:

a $a=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right)=\frac{x^2(5-x)}{2}=\frac{5x^2}{2}-\frac{x^3}{2}$

$\frac{1}{2}v^2=\int\left(\frac{5x^2}{2}-\frac{x^3}{2}\right)\mathrm{d}x=\frac{5x^3}{6}-\frac{x^4}{8}+A$

$v^2=\frac{5x^3}{3}-\frac{x^4}{4}+B$, where $B=2A$

At $x=10, v=0$

$0=\frac{5000}{3}-\frac{10\,000}{4}+B\Rightarrow B=\frac{2500}{3}$

$v^2=\frac{5x^3}{3}-\frac{x^4}{4}+\frac{2500}{3}$

b When $t=0, x=0$

$v^2=\frac{2500}{3}\Rightarrow v=(\pm)\frac{50}{\sqrt{3}}=(\pm)\frac{50\sqrt{3}}{3}$

The speed of P when $t=0$ is $\frac{50\sqrt{3}}{3}$ m s^{-1}.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 9

Question:

A particle P moves away from the origin along the positive x-axis. At time t seconds, the acceleration of P is $\frac{20}{5x+2}$ m s^{-2}, where $OP = x$ m, directed away from O. Given that the speed of P is 3 m s^{-1} at $x = 0$, find, giving your answers to 3 significant figures,

a the speed of P at $x = 12$,

b the value of x when the speed of P is 5 m s^{-1}.

Solution:

a $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{20}{5x+2}$

$\frac{1}{2}v^2 = \int \frac{20}{5x+2}\,dx = 4\ln(5x+2) + A$

$v^2 = 8\ln(5x+2) + B$, where $B = 2A$

At $x = 0, v = 3$

$9 = 8\ln 2 + B \Rightarrow B = 9 - 8\ln 2$

$v^2 = 8\ln(5x+2) - 8\ln 2 + 9 = 8\ln\left(\frac{5x+2}{2}\right) + 9$

At $x = 12$

$v^2 = 8\ln 31 + 9 = 36.471\ldots$

$v = \sqrt{36.471\ldots} = 6.039\ldots$

The speed of P at $x = 12$ is 6.04 m s^{-1} (3 s.f.).

b When $v = 5$

$25 = 8\ln\left(\frac{5x+2}{2}\right) + 9$

$\ln\left(\frac{5x+2}{2}\right) = \frac{25-9}{8} = 2$

$\frac{5x+2}{2} = e^2$

$x = \frac{2e^2 - 2}{5} = 2.56$ (3 s.f.)

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 10

Question:

A car moves along a horizontal straight road. At time t seconds the acceleration of the car is $\frac{100}{(2t+5)^2}$ m s^{-2} in the direction of motion of the car. When $t = 0$, the car is at rest. Find

a an expression for v in terms of t,

b the distance moved by the car in the first 10 seconds of its motion.

Solution:

a $a = \frac{dv}{dt} = \frac{100}{(2t+5)^2} = 100(2t+5)^{-2}$

$v = \int 100(2t+5)^{-2}\, dt = \frac{100}{2\times -1}(2t+5)^{-1} + A$

$= A - \frac{50}{2t+5}$

When $t = 0, v = 0$

$0 = A - \frac{50}{5} \Rightarrow A = 10$

$v = 10 - \frac{50}{2t+5}$

b $v = \frac{dx}{dt} = 10 - \frac{50}{2t+5}$

$x = \int \left(10 - \frac{50}{2t+5}\right) dt = 10t - 25\ln(2t+5) + B$

When $t = 0, x = 0$

$0 = -25\ln 5 + B \Rightarrow B = 25\ln 5$

$x = 10t - 25\ln(2t+5) + 25\ln 5$

When $t = 10$

$x = 100 - 25\ln 25 + 25\ln 5 = 100 - 25\ln\frac{25}{5} = 100 - 25\ln 5$

The distance moved by the car in the first 10 seconds of its motion is $(100 - 25\ln 5)$ m (≈ 59.8 m).

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 11

Question:

A particle P is moving in a straight line with acceleration $\cos^2 t$ m s^{-2} at time t seconds. The particle is initially at rest at a point O.

a Find the speed of P when $t = \pi$.

b Show that the distance of P from O when $t = \frac{\pi}{4}$ is $\frac{1}{64}(\pi^2 + 8)$m.

Solution:

a $a = \frac{dv}{dt} = \cos^2 t = \frac{1}{2} + \frac{1}{2}\cos 2t$

$v = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2t \right) dt = \frac{1}{2}t + \frac{1}{4}\sin 2t + A$

When $t = 0, v = 0$

$0 = 0 + 0 + A \Rightarrow A = 0$

$v = \frac{1}{2}t + \frac{1}{4}\sin 2t$

When $t = \pi$

$v = \frac{\pi}{2} + \frac{1}{4}\sin 2\pi = \frac{\pi}{2} + 0 = \frac{\pi}{2}$

The speed of P when $t = \pi$ is $\frac{\pi}{2}$ m s^{-1}.

b The distance of P from O when $t = \frac{\pi}{4}$ is given by

$$x = \int_0^{\frac{\pi}{4}} \left(\frac{1}{2}t + \frac{1}{4}\sin 2t \right) dt = \left[\frac{1}{4}t^2 - \frac{1}{8}\cos 2t \right]_0^{\frac{\pi}{4}}$$

$$= \left(\frac{\pi^2}{64} - \cancel{\frac{1}{8}\cos\frac{\pi}{2}} \right) - \left(0 - \frac{1}{8} \right)$$

$$= \frac{\pi^2}{64} + \frac{1}{8} = \frac{1}{64}(\pi^2 + 8)$$

The distance of P from O when $t = \frac{\pi}{4}$ is $\frac{1}{64}(\pi^2 + 8)$m, as required.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 12

Question:

A particle P is moving along the x-axis. At time t seconds, the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = \begin{cases} \frac{1}{2}t^2, & 0 \le t \le 4 \\ 8e^{4-t}, & t > 4 \end{cases}$$

When $t = 0$, P is at the origin O. Find

a the acceleration of P when $t = 2.5$,

b the acceleration of P when $t = 5$,

c the distance of P from O when $t = 6$.

Solution:

a When $t = 2.5$, $v = \frac{1}{2}t^2$

$a = \frac{dv}{dt} = t$

When $t = 2.5, a = 2.5$

The acceleration of P when $t = 2.5$ is 2.5 m s^{-2} in the direction of x increasing.

b When $t = 5$, $v = 8e^{4-t}$

$a = \frac{dv}{dt} = -8e^{4-t}$

When $t = 5, a = -8e^{4-5} = -8e^{-1}$

The acceleration of P when $t = 5$ is $8e^{-1}$ m s^{-2} in the direction of x decreasing.

c The distance of P from O when $t = 6$ is given by

$$x = \int_0^4 \frac{1}{2}t^2 \, dt + \int_4^6 8e^{4-t} \, dt$$

$$= \left[\frac{t^3}{6}\right]_0^4 + \left[-8e^{4-t}\right]_4^6 = \frac{64}{6} - 8e^{-2} + 8$$

$$= \frac{56}{3} - 8e^{-2}$$

The distance of P from O when $t = 6$ is $\left(\frac{56}{3} - 8e^{-2}\right)$ m ≈ 17.6 m (3 s.f.).

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 13

Question:

A particle P is moving along the x-axis. When $t = 0$, P is passing through O with velocity 3 m s^{-1} in the direction of x increasing. When $0 \leq x \leq 4$ the acceleration is of magnitude $\left(4 + \frac{1}{2}x\right)\text{ms}^{-2}$ in the direction of x increasing. At $x = 4$, the acceleration of P changes. For $x > 4$, the magnitude of the acceleration remains $\left(4 + \frac{1}{2}x\right)\text{m s}^{-2}$ but it is now in the direction of x decreasing.

a Find the speed of P at $x = 4$.

b Find the positive value of x for which P is instantaneously at rest. Give your answer to 2 significant figures.

Solution:

a $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 4 + \frac{1}{2}x$

$\frac{1}{2}v^2 = 4x + \frac{x^2}{4} + A$

$v^2 = 8x + \frac{x^2}{2} + B$, where $B = 2A$

At $x = 0, v = 3$

$9 = 0 + 0 + B \Rightarrow B = 9$

$v^2 = 8x + \frac{x^2}{2} + 9$

At $x = 4$

$v^2 = 32 + 8 + 9 = 49 \Rightarrow v = 7$

The speed of P at $x = 4$ is 7 m s^{-1}.

b $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4 - \frac{1}{2}x$

$\frac{1}{2}v^2 = C - 4x - \frac{x^2}{4}$

$v^2 = D - 8x - \frac{x^2}{2}$, where $D = 2C$

At $x = 4, v = 7$

$49 = D - 32 - 8 \Rightarrow D = 89$

$v^2 = 89 - 8x - \frac{x^2}{2}$

When $v = 0$

$x^2 + 16x = 178 \Rightarrow x^2 + 16x + 64 = 242$

$(x+8)^2 = 242 \Rightarrow x = 11\sqrt{2} - 8$, as $x > 0$

$x = 7.6$ (2 s.f.)

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 14

Question:

A particle P is moving along the x-axis. At time t seconds, P has velocity v m s^{-1} in the direction x increasing and an acceleration of magnitude $\frac{2t+3}{t+1}$ m s^{-2} in the direction x increasing. When $t=0$, P is at rest at the origin O. Find

a v in terms of t,

b the distance of P from O when $t=2$.

Solution:

a $a=\frac{dv}{dt}=\frac{2t+3}{t+1}=2+\frac{1}{t+1}$

$v=2t+\ln(t+1)+A$

When $t=0, v=0$

$0=0+A \Rightarrow A=0$

$v=2t+\ln(t+1)$

b The distance of P from O when $t=2$ is given by

$$x=\int_0^2 (2t+\ln(t+1))\,dt$$

Using integration by parts

$$\int \ln(t+1)\,dt=\int 1\ln(t+1)\,dt=t\ln(t+1)-\int \frac{t}{t+1}\,dt$$

$$=t\ln(t+1)-\int\left(1-\frac{1}{t+1}\right)dt=t\ln(t+1)-t+\ln(t+1)$$

$$=(t+1)\ln(t+1)-t\ (+C)$$

Hence $x=\left[t^2-t+(t+1)\ln(t+1)\right]_0^2=2+3\ln 3$

The distance of P from O when $t=2$ is $(2+3\ln 3)$ m.

Solutionbank M3
Edexcel AS and A Level Modular Mathematics

Further kinematics
Exercise C, Question 15

Question:

A particle P is moving along the x-axis. At time t seconds P is x m from O, has velocity v m s^{-1} and acceleration of magnitude $(4x+6)$ m s^{-2} in the direction of x increasing. When $t=0$, P is passing through O with velocity 3 m s^{-1} in the direction of x increasing. Find

a v in terms of x,
b x in terms of t.

Solution:

a $a = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = 4x+6$

$\frac{1}{2}v^2 = 2x^2 + 6x + A$

$v^2 = 4x^2 + 12x + B$, where $B = 2A$

At $x = 0, v = 3$

$9 = 0 + 0 + B \Rightarrow B = 9$

$v^2 = 4x^2 + 12x + 9 = (2x+3)^2$

As v is increasing as x increases

$v = 2x + 3$

b $v = \frac{\mathrm{d}x}{\mathrm{d}t} = 2x + 3$

Separating the variables and integrating

$\int \frac{1}{2x+3}\,\mathrm{d}x = \int 1\,\mathrm{d}t$

$\frac{1}{2}\ln(2x+3) = t + C$

$\ln(2x+3) = 2t + 2C$

$2x + 3 = \mathrm{e}^{2t+2C} = D\mathrm{e}^{2t}$, where $D = \mathrm{e}^{2C}$

When $t = 0, x = 0$

$3 = D\mathrm{e}^0 \Rightarrow D = 3$

$2x + 3 = 3\mathrm{e}^{2t}$

$x = \frac{3}{2}(\mathrm{e}^{2t} - 1)$