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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

In this exercise **i** and **j** are perpendicular unit vectors.

A particle of mass 0.25 kg is moving with velocity $(12\mathbf{i}+4\mathbf{j})$ m s^{-1} when it receives an impulse $(8\mathbf{i}-7\mathbf{j})$ Ns. Find the new velocity of the particle.

Solution:

$$8\mathbf{i}-7\mathbf{j}=0.25\mathbf{v}-0.25(12\mathbf{i}+4\mathbf{j})$$
$$\therefore 8\mathbf{i}-7\mathbf{j}=0.25\mathbf{v}-3\mathbf{i}-\mathbf{j}$$
$$\therefore 0.25\mathbf{v}=11\mathbf{i}-6\mathbf{j}$$
$$\therefore \mathbf{v}=44\mathbf{i}-24\mathbf{j}$$

Use impulse $= m\mathbf{v}-m\mathbf{u}$, then make **v** the subject of the formula.

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Exercise A, Question 2

Question:

A particle of mass 0.5 kg is moving with velocity $(2\mathbf{i}-2\mathbf{j})$ m s^{-1} when it receives an impulse $(3\mathbf{i}+5\mathbf{j})$ Ns. Find the new velocity of the particle.

Solution:

$$3\mathbf{i}+5\mathbf{j} = 0.5\mathbf{v} - 0.5(2\mathbf{i}-2\mathbf{j})$$
$$= 0.5\mathbf{v} - \mathbf{i} + \mathbf{j}$$
$$\therefore 0.5\mathbf{v} = 4\mathbf{i}+4\mathbf{j}$$
$$\therefore \mathbf{v} = 8\mathbf{i}+8\mathbf{j}$$

Use impulse $= m\mathbf{v} - m\mathbf{u}$ (change in momentum).

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Exercise A, Question 3

Question:

A particle of mass 2 kg moves with velocity $(3\mathbf{i}+2\mathbf{j})$ m s^{-1} immediately after it has received an impulse $(4\mathbf{i}+8\mathbf{j})$ Ns. Find the original velocity of the particle.

Solution:

$$\begin{aligned} 4\mathbf{i}+8\mathbf{j} &= 2\times(3\mathbf{i}+2\mathbf{j})-2\mathbf{u} \\ &= 6\mathbf{i}+4\mathbf{j}-2\mathbf{u} \\ \therefore 2\mathbf{u} &= 6\mathbf{i}+4\mathbf{j}-4\mathbf{i}-8\mathbf{j} \\ &= 2\mathbf{i}-4\mathbf{j} \\ \therefore \mathbf{u} &= \mathbf{i}-2\mathbf{j} \end{aligned}$$

Use impulse = change in momentum.

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Exercise A, Question 4

Question:

A particle of mass 1.5 kg moves with velocity $(5\mathbf{i} - 8\mathbf{j})$ m s^{-1} immediately after it has received an impulse $(3\mathbf{i} - 6\mathbf{j})$ Ns. Find the original velocity of the particle.

Solution:

$$3\mathbf{i} - 6\mathbf{j} = 1.5(5\mathbf{i} - 8\mathbf{j}) - 1.5\mathbf{u}$$
$$\therefore 1.5\mathbf{u} = 7.5\mathbf{i} - 12\mathbf{j} - 3\mathbf{i} + 6\mathbf{j}$$
$$= 4.5\mathbf{i} - 6\mathbf{j}$$
$$\therefore \mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$$

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Exercise A, Question 5

Question:

A body of mass 3 kg is initially moving with a constant velocity of $(\mathbf{i}+\mathbf{j})$ m s^{-1} when it is acted on by a force of $(6\mathbf{i}-8\mathbf{j})$ N for 3 seconds. Find the impulse exerted on the body and find its velocity when the force ceases to act.

Solution:

$$\text{impulse} = \text{force} \times \text{time}$$
$$\therefore \text{impulse} = (6\mathbf{i}-8\mathbf{j}) \times 3$$
$$= 18\mathbf{i} - 24\mathbf{j}$$

Use impulse = force × time

But impulse = change in momentum

$$\therefore 18\mathbf{i} - 24\mathbf{j} = 3(\mathbf{v} - (\mathbf{i}+\mathbf{j}))$$
$$\therefore 18\mathbf{i} - 24\mathbf{j} + 3\mathbf{i} + 3\mathbf{j} = 3\mathbf{v}$$
$$\therefore 3\mathbf{v} = 21\mathbf{i} - 21\mathbf{j}$$
$$\therefore \mathbf{v} = 7\mathbf{i} - 7\mathbf{j}$$

Then use
impulse = change in momentum
= $m\mathbf{v} - m\mathbf{u}$

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Exercise A, Question 6

Question:

A body of mass 0.5 kg is initially moving with a constant velocity of $(5\mathbf{i}+12\mathbf{j})$ m s^{-1} when it is acted on by a force of $(2\mathbf{i}-\mathbf{j})$ N for 5 seconds. Find the impulse exerted on the body and find its velocity when the force ceases to act.

Solution:

$$\text{impulse} = \text{force} \times \text{time}$$
$$= (2\mathbf{i}-\mathbf{j}) \times 5$$
$$= 10\mathbf{i}-5\mathbf{j}$$

Use impulse = force × time.

But impulse = change in momentum

$$\therefore 10\mathbf{i}-5\mathbf{j} = 0.5(\mathbf{v}-(5\mathbf{i}+12\mathbf{j}))$$
$$\therefore 10\mathbf{i}-5\mathbf{j}+2.5\mathbf{i}+6\mathbf{j} = 0.5\mathbf{v}$$
$$\therefore 0.5\mathbf{v} = 12.5\mathbf{i}-\mathbf{j}$$
$$\therefore \mathbf{v} = 25\mathbf{i}-2\mathbf{j}$$

Then impulse = change in momentum.

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Exercise A, Question 7

Question:

A particle of mass 2 kg is moving with velocity $(5\mathbf{i}+3\mathbf{j})$ m s^{-1} when it hits a wall. It rebounds with velocity $(-\mathbf{i}-3\mathbf{j})$ m s^{-1}. Find the impulse exerted by the wall on the particle.

Solution:

$$\begin{aligned}\text{impulse} &= \text{change in momentum}\\ &= 2(-\mathbf{i}-3\mathbf{j})-2(5\mathbf{i}+3\mathbf{j})\\ &= -12\mathbf{i}-12\mathbf{j}\end{aligned}$$

Use impulse $= m\mathbf{v} - m\mathbf{u}$.

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Exercise A, Question 8

Question:

A particle of mass 0.5 kg is moving with velocity $(11\mathbf{i} - 2\mathbf{j})$ m s^{-1} when it hits a wall. It rebounds with velocity $(-\mathbf{i} + 7\mathbf{j})$ m s^{-1}. Find the impulse exerted by the wall on the particle.

Solution:

$$\begin{aligned}\text{impulse} &= \text{change in momentum}\\ &= 0.5\times(-\mathbf{i}+7\mathbf{j}) - 0.5\times(11\mathbf{i}-2\mathbf{j})\\ &= -6\mathbf{i} + 4\tfrac{1}{2}\mathbf{j}\end{aligned}$$

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Exercise A, Question 9

Question:

A particle P of mass 3 kg receives an impulse Q Ns. Immediately before the impulse the velocity of P is $5\mathbf{i}$ m s^{-1} and immediately afterwards it is $(13\mathbf{i} - 6\mathbf{j})$ m s^{-1}. Find the magnitude of Q and the angle between Q and $\mathbf{i}$.

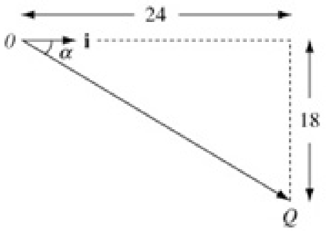
Solution:

$$\begin{aligned} Q &= m\mathbf{v} - m\mathbf{u} \\ &= 3(13\mathbf{i} - 6\mathbf{j}) - 3(5\mathbf{i}) \\ &= 24\mathbf{i} - 18\mathbf{j} \end{aligned}$$

Use impulse = change in momentum

$$\begin{aligned} \therefore |Q| &= \sqrt{(24)^2 + (-18)^2} \\ &= 30 \end{aligned}$$

Find the magnitude of Q by using Pythagoras' Theorem, and find the angle between Q and **i** by using trigonometry.



Let α be the acute angle between **i** and Q.

Then

$$\tan \alpha = \frac{18}{24}$$

$$\therefore \alpha = 36.9° \text{ (3 s.f.)}$$

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Exercise A, Question 10

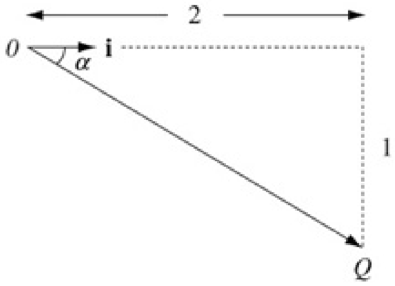
Question:

A particle P of mass 0.5 kg receives an impulse Q Ns. Immediately before the impulse the velocity of P is $(-\mathbf{i} - 2\mathbf{j})$ m s^{-1} and immediately afterwards it is $(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find the magnitude of Q and the angle between Q and $\mathbf{i}$.

Solution:

Use impulse = change in momentum.

$$\begin{aligned} \mathrm{Q} &= 0.5(3\mathbf{i} - 4\mathbf{j}) - 0.5(-\mathbf{i} - 2\mathbf{j}) \\ &= 2\mathbf{i} - \mathbf{j} \\ \therefore \ |\mathrm{Q}| &= \sqrt{2^2 + (-1)^2} \\ &= \sqrt{5} = 2.24 \ (3 \text{ s.f.}) \end{aligned}$$



Let α be the acute angle between Q and $\mathbf{i}$.

Use

$$\tan \alpha = \frac{1}{2}$$
$$\therefore \alpha = 26.6° \ (3 \text{ s.f.})$$

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Exercise A, Question 11

Question:

A cricket ball of mass 0.5 kg is hit by a bat. Immediately before being hit the velocity of the ball is $(20\mathbf{i}-4\mathbf{j})$ m s^{-1} and immediately afterwards it is $(-16\mathbf{i}+8\mathbf{j})$ m s^{-1}. Find the magnitude of the impulse exerted on the ball by the bat.

Solution:

$$\begin{aligned}\text{Impulse} &= \text{change in momentum}\\ &= m\mathbf{v}-m\mathbf{u}\\ &= 0.5\times(-16\mathbf{i}+8\mathbf{j})-0.5\times(20\mathbf{i}-4\mathbf{j})\\ &= -8\mathbf{i}+4\mathbf{j}-10\mathbf{i}+2\mathbf{j}\\ &= -18\mathbf{i}+6\mathbf{j}\end{aligned}$$

$$\begin{aligned}\therefore \text{Magnitude of the impulse} &= \sqrt{(-18)^2+6^2}=6\sqrt{10}\\ &= 19.0 \text{ Ns (3 s.f.)}\end{aligned}$$

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Exercise A, Question 12

Question:

A ball of mass 0.2 kg is hit by a bat. Immediately before being hit by the bat the velocity of the ball is $-15\mathbf{i}$ m s^{-1} and the bat exerts an impulse of $(2\mathbf{i}+6\mathbf{j})$ Ns on the ball. Find the velocity of the ball after the impact.

Solution:

Use impulse = change in momentum

$$\begin{aligned}2\mathbf{i}+6\mathbf{j} &= 0.2\mathbf{v}-0.2(-15\mathbf{i})\\ &= 0.2\mathbf{v}+3\mathbf{i}\\ \therefore 0.2\mathbf{v} &= 2\mathbf{i}+6\mathbf{j}-3\mathbf{i}\\ &= -\mathbf{i}+6\mathbf{j}\\ \therefore \mathbf{v} &= -5\mathbf{i}+30\mathbf{j}\end{aligned}$$

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Exercise A, Question 13

Question:

A particle of mass 0.25 kg has velocity $\mathbf{v}$ m s^{-1} at time t s where $\mathbf{v} = (t^2 - 3)\mathbf{i} + 4t\mathbf{j}$.

When $t = 3$, the particle receives an impulse of $2\mathbf{i} + 2\mathbf{j}$ Ns. Find the velocity of the particle immediately after the impulse.

Solution:

$\mathbf{v} = (t^2 - 3)\mathbf{i} + 4t\mathbf{j}$

When $t = 3$ let $\mathbf{v} = \mathbf{u}$

Substitute $t = 3$ into expression for velocity, to find the velocity before the impact.

$\mathbf{u} = 6\mathbf{i} + 12\mathbf{j}$

Use impulse = change in momentum

Then $2\mathbf{i} + 2\mathbf{j} = 0.25\mathbf{v} - 0.25(6\mathbf{i} + 2\mathbf{j})$

$\therefore 0.25\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + 0.25(6\mathbf{i} + 2\mathbf{j})$

$= 3.5\mathbf{i} + 2.5\mathbf{j}$

$\therefore \mathbf{v} = 14\mathbf{i} + 10\mathbf{j}$

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Exercise A, Question 14

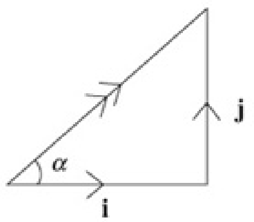
Question:

A ball of mass 2 kg is initially moving with a velocity of $(\mathbf{i}+\mathbf{j})$ m s^{-1}. It receives an impulse of $2\mathbf{j}$ Ns. Find the velocity immediately after the impulse and the angle through which the ball is deflected as a result. Give your answer to the nearest degree.

Solution:

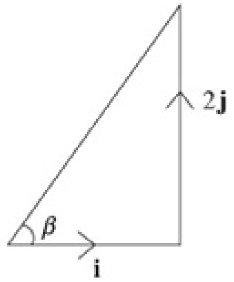
Use impulse = change in momentum

$$\therefore 2\mathbf{j} = 2\mathbf{v} - 2(\mathbf{i}+\mathbf{j})$$
$$\therefore 2\mathbf{v} = 2\mathbf{j} + 2(\mathbf{i}+\mathbf{j})$$
$$= 2\mathbf{i} + 4\mathbf{j}$$
$$\therefore \mathbf{v} = \mathbf{i} + 2\mathbf{j}$$



Before impact the velocity was $\mathbf{i}+\mathbf{j}$ and so the direction of the ball was at an angle α with **i**, where $\tan\alpha = \frac{1}{1}$, i.e. $\alpha = 45°$.

Find the angle between the direction of the velocity and the direction **i**, both before and after the impulse.



After impact the velocity is $\mathbf{i}+2\mathbf{j}$ and so the direction of the ball is at an angle β with **i**, where $\tan\beta = \frac{2}{1}$, i.e. $\beta = 63.4°$.

Then calculate the angle of deflection.

$\therefore$ The ball is deflected through an angle of 18.4°.

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Exercise A, Question 15

Question:

A particle of mass 0.5 kg moving with velocity $3\mathbf{i}$ m s^{-1} collides with a particle of mass 0.25 kg moving with velocity $12\mathbf{i}$ m s^{-1}. The two particles coalesce and move as one particle of mass 0.75 kg. Find the velocity of the combined particle.

Solution:

Let the new velocity be $x\mathbf{i}$.

Let the new velocity be $x\mathbf{i}$ and use conservation of momentum. Equate $\mathbf{i}$ component to find x.

Using conservation of momentum

$$0.5 \times 3\mathbf{i} + 0.25 \times 12\mathbf{i} = 0.75x\mathbf{i}$$
$$\therefore 1.5\mathbf{i} + 3\mathbf{i} = 0.75x\mathbf{i}$$
$$\therefore 0.75x\mathbf{i} = 4.5\mathbf{i}$$
$$\therefore x = \frac{4.5}{0.75}$$
$$= 6$$

So the velocity of the combined particle is $6\mathbf{i}$ m s^{-1}.

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Exercise A, Question 16

Question:

A particle of mass 5 kg moving with velocity $(\mathbf{i}-\mathbf{j})$ m s^{-1} collides with a particle of mass 2 kg moving with velocity $(-\mathbf{i}+\mathbf{j})$ m s^{-1}. The two particles coalesce and move as one particle of mass 7 kg. Find the magnitude of the velocity $\mathbf{v}$ m s^{-1} of the combined particle.

Solution:

Use conservation of momentum to find $\mathbf{v}$, then use Pythagoras' Theorem and trigonometry to find $|\mathbf{v}|$ and the angle between $\mathbf{v}$ and $\mathbf{i}$.

Let the new velocity be $x\mathbf{i}+y\mathbf{j}$.

Use conservation of momentum:

$5(\mathbf{i}-\mathbf{j})+2(-\mathbf{i}+\mathbf{j})=7(x\mathbf{i}+y\mathbf{j})$

Then $5\mathbf{i}-5\mathbf{j}-2\mathbf{i}+2\mathbf{j}=7x\mathbf{i}+7y\mathbf{j}$

$\therefore 3\mathbf{i}-3\mathbf{j}=7x\mathbf{i}+7y\mathbf{j}$

Equate coefficients of $\mathbf{i}$ and $\mathbf{j}$ to give

$7x=3$ and $7y=-3$

$\therefore x=\frac{3}{7}$ and $y=-\frac{3}{7}$

$\therefore$ velocity is $\frac{3}{7}\mathbf{i}-\frac{3}{7}\mathbf{j}$

The magnitude of the velocity is $\sqrt{\left(\frac{3}{7}\right)^2+\left(-\frac{3}{7}\right)^2}=\frac{3}{7}\sqrt{2}=0.606$ m s^{-1}(3 s.f.)

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Exercise B, Question 1

Question:

In each part of this question the two diagrams show the speeds of two particles A and B just before and just after a collision. The particles move on a smooth horizontal plane. Find the coefficient of restitution e in each case.

	Before collision		After collision	
a	6 ms^{-1} → A	At rest B	At rest A	4 ms^{-1} → B
b	4 ms^{-1} → A	2 ms^{-1} → B	2 ms^{-1} → A	3 ms^{-1} → B
c	9 ms^{-1} → A	6 ms^{-1} ← B	3 ms^{-1} ← A	2 ms^{-1} → B

Solution:

a $e = \frac{4-0}{6-0} = \frac{2}{3}$

b $e = \frac{3-2}{4-2} = \frac{1}{2}$

c

$$e = \frac{2-(-3)}{9-(-6)}$$
$$= \frac{5}{15}$$
$$= \frac{1}{3}$$

Use
$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{speed of separation}}{\text{speed of approach}}.$$

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Exercise B, Question 2

Question:

In each part of this question the two diagrams show the speeds of two particles A and B just before and just after a collision. The particles move on a smooth horizontal plane. The masses of A and B and the coefficients of restitution e are also given. Find the speeds v_1 and v_2 in each case.

		Before collision		After collision	
a	$e = \frac{1}{2}$	6 m s^{-1} → A (0.25kg)	At rest B (0.5kg)	$v_1\text{ m s}^{-1}$ → A (0.25kg)	$v_2\text{ m s}^{-1}$ → B (0.5kg)
b	$e = 0.25$	4 m s^{-1} → A (2kg)	2 m s^{-1} → B (3kg)	$v_1\text{ m s}^{-1}$ → A (2kg)	$v_2\text{ m s}^{-1}$ → B (3kg)
c	$e = \frac{1}{7}$	8 m s^{-1} → A (3kg)	6 m s^{-1} ← B (1kg)	$v_1\text{ m s}^{-1}$ → A (3kg)	$v_2\text{ m s}^{-1}$ → B (1kg)
d	$e = \frac{2}{3}$	6 m s^{-1} → A (400g)	6 m s^{-1} ← B (400g)	$v_1\text{ m s}^{-1}$ → A (400g)	$v_2\text{ m s}^{-1}$ → B (400g)
e	$e = \frac{1}{5}$	3 m s^{-1} → A (5kg)	12 m s^{-1} ← B (4kg)	$v_1\text{ m s}^{-1}$ → A (5kg)	$v_2\text{ m s}^{-1}$ → B (4kg)

Solution:

Use conservation of linear momentum and Newton's Law of Restitution.

a Using conservation of linear momentum:

$$0.25\times6+0.5\times0=0.25v_1+0.5v_2$$

Multiply equation by 4.

$$6=v_1+2v_2 \qquad (1)$$

Using Newton's Law of Restitution:

$$e=\frac{\text{speed of separation}}{\text{speed of approach}}$$

i.e. $\frac{1}{2}=\frac{v_2-v_1}{6-0}$

$$\therefore 3=v_2-v_1 \qquad (2)$$

Add equations (1) and (2)

Then

$$9=3v_2$$
$$\therefore v_2=3$$

Substitute into equation (1)

$$\therefore 6=v_1+2\times3$$
$$\therefore v_1=0$$

$\therefore$ A is at rest and B moves at 3 m s^{-1} after the collision.

b Using conservation of linear momentum:

$$2\times4+3\times2=2v_1+3v_2$$
$$\therefore 14=2v_1+3v_2 \qquad (1)$$

Using Newton's Law of Restitution:

$$e=\frac{\text{speed of separation}}{\text{speed of approach}}$$

$$\therefore 0.25=\frac{v_2-v_1}{4-2}$$
$$\therefore 0.5=v_2-v_1 \qquad (2)$$

Multiply equation (2) by 2 and add to equation (1).

$$\therefore 15=5v_2$$
$$\therefore v_2=3$$

Substitute into equation (1) $v_1=2\frac{1}{2}$

$\therefore$ A and B move with speeds $2\frac{1}{2}\text{ m s}^{-1}$ and 3 m s^{-1} respectively after the collision.

c Using conservation of linear momentum:

$3\times 8+1\times(-6)=3v_1+1v_2$

$\therefore 18=3v_1+v_2 \quad (1)$

Remember that the speed 6 m s^{-1} appears in the equations as -6 because it is directed to the left in the diagram.

Using Newton's Law of Restitution:

$e=\frac{1}{7}=\frac{v_2-v_1}{8-(-6)}$

$\therefore \frac{1}{7}=\frac{v_2-v_1}{14}$

$\therefore 2=v_2-v_1 \quad (2)$

Subtract to give equation (1) − equation (2)

$\therefore 16=4v_1$

$\therefore v_1 = 4$

Substitute in equation (1) to give $v_2=6$.

[This answer may be checked in equation (2).]

$\therefore$ Speed of A is 4 m s^{-1} and speed of B is 6 m s^{-1} after the collision.

d Using conservation of linear momentum:

$0.4\times 6-0.4\times 6=0.4v_1+0.4v_2$

$\therefore 0=v_1+v_2 \quad (1)$

Using Newton's Law of Restitution:

$e=\frac{2}{3}=\frac{v_2-v_1}{6-(-6)}$

i.e. $\frac{2}{3}=\frac{v_2-v_1}{12}$

$\therefore v_2-v_1=8 \quad (2)$

Add equations (1) and (2)

$\therefore 2v_2=8$

i.e. $v_2=4$

Substitute into equation (1)

$\therefore v_1=-4.$

The speeds of A and B are 4 m s^{-1} after the collision, and both change direction after the collision.

e Using conservation of linear momentum:

> Remember that a particle moving in the opposite direction (i.e. to the left) has a *negative* velocity in the equations.

$$5 \times 3 - 4 \times 12 = 5v_1 + 4v_2$$
$$\therefore -33 = 5v_1 + 4v_2 \quad (1)$$

Using Newton's Law of Restitution:

$$e = \frac{1}{5} = \frac{v_2 - v_1}{3 - (-12)}$$
$$\therefore \frac{1}{5} = \frac{v_2 - v_1}{15}$$
$$\therefore 3 = v_2 - v_1 \quad (2)$$

Multiply equation (2) by 5 and add to equation (1)

$$\therefore -18 = 9v_2$$
$$\therefore v_2 = -2$$

Substitute into equation (1)

$$\therefore -33 = 5v_1 - 8$$
$$\therefore -25 = 5v_1$$
[Check your answers in equation (2).]
$$\therefore v_1 = -5$$

The speeds of A and B are 5 m s^{-1} and 2 m s^{-1} after the collision and both change direction.

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Exercise B, Question 3

Question:

A small smooth sphere A of mass 1 kg is travelling along a straight line on a smooth horizontal plane with speed 4 m s^{-1} when it collides with a second smooth sphere B of the same radius, with mass 2 kg and travelling in the same direction as A with speed 2.5 m s^{-1}.

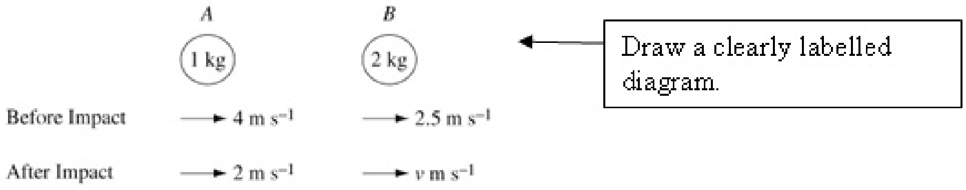
After the collision, A continues in the same direction with speed 2 m s^{-1}.

a Find the speed of B after the collision.

b Find the coefficient of restitution for the spheres.

Solution:

a



Let the speed of B, after the collision, be $v \text{ m s}^{-1}$.

Use conservation of linear momentum:

$$1\times 4+2\times 2.5=1\times 2+2v$$
$$\therefore 9=2+2v$$
$$\therefore 2v=7$$
$$v=3.5$$

Speed of B after the collision is 3.5 m s^{-1}.

b Use Newton's Law of Restitution:

$$e=\frac{v-2}{4-2.5}$$
$$=\frac{3.5-2}{4-2.5}$$
$$=\frac{1.5}{1.5}$$
$$=1$$

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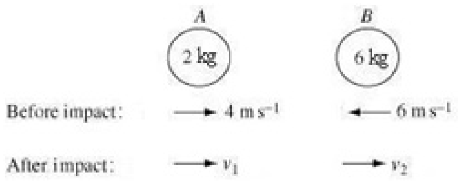
Exercise B, Question 4

Question:

Two spheres A and B are of equal radius and have masses 2 kg and 6 kg respectively. A and B move towards each other along the same straight line on a smooth horizontal surface with velocities 4 m s^{-1} and 6 m s^{-1} respectively. If the coefficient of friction is $\frac{1}{5}$, find the velocities of the spheres after the collision and the magnitude of the impulse given to each sphere.

Solution:



Using conservation of linear momentum:

> Use conservation of linear momentum and Newton's Law of Restitution.

$2\times4-6\times6=2v_1+6v_2$

$\therefore -28=2v_1+6v_2$

$\div$ by 2

Before impact:
$\therefore -14=v_1+3v_2 \quad (1)$

Using Newton's Law of Restitution:

$e=\frac{1}{5}=\frac{v_2-v_1}{4-(-6)}$

$\therefore \frac{1}{5}=\frac{v_2-v_1}{10}$

i.e. $2=v_2-v_1 \quad (2)$

Add equations (1) and (2)

$-12=4v_2$

$\therefore v_2=-3$

Substitute into equation (2)

$2=-3-v_1$

$\therefore v_1=-5$ [Check in equation (1).]

The speed of A and B are 5 m s^{-1} and 3 m s^{-1} both now in the direction that B was moving before the impact.

> Use impulse = change of momentum for each particle separately.

The impulse given to sphere A = change in momentum of sphere A

$=2\times(-5)-2\times4$

$=-18\text{ Ns}$

The impulse given to sphere B = change in momentum of sphere B

$=6\times(-3)-6\times(-6)$

$=18\text{ Ns}$

$\therefore$ A and B experience equal and opposite impulses − to the left on A and to the right on B.

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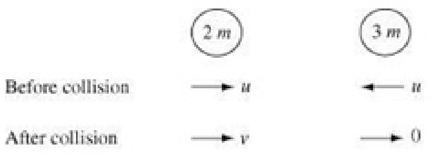
Exercise B, Question 5

Question:

Two particles of mass $2m$ and $3m$ respectively are moving towards each other with speed u. If the $3m$ mass is brought to rest by the collision, find the speed of the $2m$ mass after the collision and the coefficient of restitution between the particles.

Solution:



Let the speed of the $2m$ mass be $v \text{ m s}^{-1}$ after the collision.

Using conservation of linear momentum:

$$2mu - 3mu = 2mv + 3m \times 0$$
$$\therefore -mu = 2mv$$
$$\therefore v = -\frac{u}{2}$$

Using Newton's Law of Restitution:

$$e = \frac{0 - v}{u - (-u)} = \frac{\frac{u}{2}}{2u} = \frac{1}{4}$$

$\therefore$ The $2m$ mass moves with speed $\frac{u}{2} \text{ m s}^{-1}$ after the collision, having changed direction.

The coefficient of restitution is $\frac{1}{4}$.

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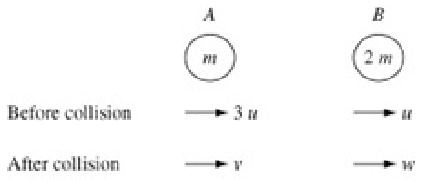
Exercise B, Question 6

Question:

Two particles A and B are travelling along the same straight line in the same direction on as smooth horizontal surface with speeds $3u$ and u respectively. Particle A catches up and collides with particle B. If the mass of B is twice that of A and the coefficient of restitution is e find, in terms of e and u, expressions for the speeds of A and B after the collision.

Solution:



Let the speeds of A and B, after the collision, be v and w respectively.
Using conservation of linear momentum:

$m \times 3u + 2m \times u = mv + 2mw$
$\div$ through by m
$\therefore v + 2w = 5u \quad (1)$

Using Newton's Law of Restitution:

$$\frac{w - v}{3u - u} = e$$

$\therefore w - v = 2ue \quad (2)$

Add equations (1) and (2)

$3w = u(5 + 2e)$

$$\therefore w = \frac{u}{3}(5 + 2e)$$

Subtract twice equation (2) from equation (1)

$\therefore 3v = 5u - 4ue$

$$\therefore v = \frac{u}{3}(5 - 4e)$$

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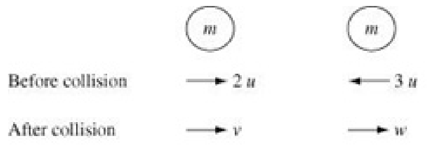
Exercise B, Question 7

Question:

Two identical particles of mass m are projected towards each other along the same straight line on a smooth horizontal surface with speeds of $2u$ and $3u$. After the collision the directions of motion of both particles are reversed. Show that this implies that the coefficient of restitution e satisfies the inequality $e > \frac{1}{5}$.

Solution:



Using conservation of linear momentum:

$$m \times 2u + m(-3u) = mv + mw$$

$$\therefore v + w = -u \quad (1)$$

Using Newton's Law of Restitution:

$$\frac{w - v}{2u + 3u} = e$$

$$\therefore w - v = 5eu \quad (2)$$

Add equations (1) and (2)

$$2w = 5eu - u$$

$$\therefore w = \frac{u}{2}(5e - 1)$$

Subtract equation (2) from equation (1)

$$2v = -u - 5eu.$$

$$\therefore v = \frac{u}{2}(-1 - 5e).$$

As directions of both particles are reversed, $v < 0$ and $w > 0$.

As $v = -\frac{u}{2}(1 + 5e)$, then $v < 0$ for all values of e.

$w = \frac{u}{2}(5e - 1)$, then $w > 0$ implies $5e - 1 > 0$

i.e. $e > \frac{1}{5}$

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Exercise B, Question 8

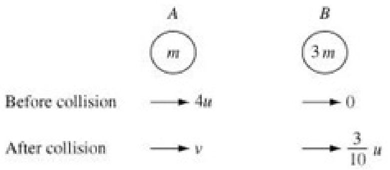
Question:

Two particles A and B of mass m and km respectively are placed on a smooth horizontal plane. Particle A is made to move on the plane with speed u so as to collide directly with B which is at rest. After the collision B moves with speed $\frac{3}{10}u$.

a Find, in terms of u and the constant k, the speed of A after the collision.

b By using Newton's Law of Restitution show that $\frac{7}{3} \le k \le \frac{17}{3}$.

Solution:



a Let the speed of A after the collision be v.

Using conservation of linear momentum:

$$mu + km \times 0 = mv + km \times \frac{3}{10}u$$

$$\therefore mv = mu - \frac{3}{10}kmu$$

$\div$ through by m

$$\therefore v = u - \frac{3}{10}ku \quad \text{or} \quad \frac{u}{10}(10 - 3k)$$

b Using Newton's Law of Restitution:

$$\frac{\frac{3}{10}u - v}{u - 0} = e$$

$$\therefore \frac{3}{10}u - \left(u - \frac{3}{10}ku\right) = eu$$

$$\therefore \frac{3}{10}ku - \frac{7}{10}u = eu$$

Multiply both sides by 10 and $\div$ by u

$$\therefore 3k = 7 + 10e$$

$$\therefore k = \frac{7 + 10e}{3}$$

As $0 \le e \le 1$, then $\frac{7}{3} \le k \le \frac{17}{3}$.

To show the inequality, you will need to use $0 \le e \le 1$.

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Exercise B, Question 9

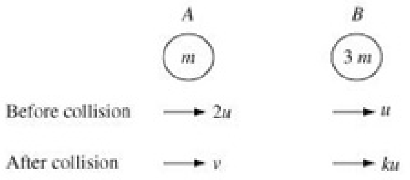
Question:

Two particles A and B of mass m and $3m$ respectively are placed on a smooth horizontal plane. Particle A is made to move on the plane with speed $2u$ so as to collide directly with B which is moving in the same direction with speed u. After the collision B moves with speed ku, where k is a positive constant.

a Find, in terms of u and the constant k, the speed of A after the collision.

b By using Newton's Law of Restitution show that $\frac{5}{4} \le k \le \frac{3}{2}$.

Solution:



a Let the velocity of A after the collision be v.

Use conservation of linear momentum:

$$m2u + 3mu = mv + 3mku$$
$$\therefore v + 3ku = 5u$$
$$\text{i.e. } v = u(5 - 3k)$$

b Using Newton's Law of Restitution:

$$\frac{ku - v}{2u - u} = e$$
$$\therefore ku - v = eu$$
$$\therefore ku - u(5 - 3k) = eu$$
$$\therefore 4ku - 5u = eu$$
$$\therefore k = \frac{e + 5}{4}$$

But $0 \le e \le 1$ $\therefore \frac{5}{4} \le k \le \frac{3}{2}$

You will need to use the condition $0 \le e \le 1$.

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Exercise B, Question 10

Question:

A particle P of mass m is moving with speed $4u$ on a smooth horizontal plane. The particle collides directly with a particle Q of mass $3m$ moving with speed $2u$ in the same direction as P. The coefficient of restitution between P and Q is e.

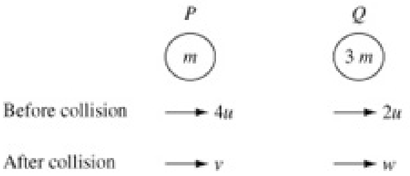
a Show that the speed of Q after the collision is $\frac{u}{2}(5+e)$.

b Find the speed of P after the collision, giving your answer in terms of e.

c Show that the direction of motion of P is unchanged by the collision, provided that $e < \frac{3}{5}$.

d Given that the magnitude of the impulse of P on Q is $2mu$, find the value of e.

Solution:



a Let the speeds of P and Q after the collision be v and w respectively.

Using conservation of linear momentum:

$$m \times 4u + 3m \times 2u = mv + 3mw$$
$$\therefore v + 3w = 10u \quad (1)$$

Using Newton's Law of Restitution:

$$\frac{w - v}{4u - 2u} = e$$
$$\therefore w - v = 2ue \quad (2)$$

Add equations (1) and (2).

$$4w = 10u + 2ue$$
$$\therefore w = \frac{2u}{4}(5 + e) = \frac{u}{2}(5 + e)$$

b Substitute into equation (1).

$$\therefore \quad v + \frac{3u}{2}(5 + e) = 10u$$
$$\therefore \quad v = 10u - \frac{15u}{2} - \frac{3ue}{2}$$
$$\therefore \quad v = \frac{u}{2}(5 - 3e)$$

[Check that w and v satisfy equation (2).]

c The direction of motion of P is unchanged provided that $\frac{u}{2}(5 - 3e) > 0$

i.e. $e < \frac{3}{5}$.

Use $v > 0$, where v is velocity of P after the collision.

d Change of momentum of Q is

Use impulse = change in momentum of Q.

$$3m(w - 2u) = 3m\left(\frac{5u}{2} + \frac{eu}{2} - 2u\right)$$
$$= \frac{3mu}{2}(1 + e)$$

But impulse of P on Q is $2mu$

$$\therefore 2mu = \frac{3mu}{2}(1 + e)$$
$$\therefore 1 + e = \frac{4}{3}$$
$$\therefore e = \frac{1}{3}$$

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Exercise C, Question 1

Question:

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speeds of the sphere before and after collision. In each case find the value of the coefficient of restitution e.

a	Before impact		After impact	
	10 m s^{-1} →	Wall	4 m s^{-1} ←	Wall

b	Before impact		After impact	
	6 m s^{-1} →	Wall	3 m s^{-1} ←	Wall

Solution:

a $e = \frac{\text{speed of rebound}}{\text{speed of approach}}$

$= \frac{4}{10}$

$\therefore e = \frac{2}{5}$

Use $e = \frac{\text{speed of rebound}}{\text{speed of approach}}$.

b $e = \frac{\text{speed of rebound}}{\text{speed of approach}}$

$= \frac{3}{6}$

$\therefore e = \frac{1}{2}$

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Exercise C, Question 2

Question:

A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speed of the sphere before and after the collision. The value of e is given in each case.

Find the speed of the sphere after the collision in each case.

a $e = \frac{1}{2}$	Before impact		After impact	
	7 ms^{-1} →	Wall	$v\text{ ms}^{-1}$ ←	Wall
b $e = \frac{1}{4}$	Before impact		After impact	
	12 ms^{-1} →	Wall	$V\text{ ms}^{-1}$ ←	Wall

Solution:

a Use

Use $e = \frac{\text{speed of rebound}}{\text{speed of approach}}$ and make speed of rebound the subject of the formula.

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$

$$\therefore \frac{1}{2} = \frac{v}{7}$$

$$\therefore v = 7 \times \frac{1}{2}$$

$$= 3.5$$

Speed of sphere after collision is 3.5 m s^{-1}.

b Use

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$

$$\therefore \frac{1}{4} = \frac{V}{12}$$

$$\therefore V = \frac{1}{4} \times 12$$

$$= 3$$

$\therefore$ speed after collision is 3 m s^{-1}.

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Exercise C, Question 3

Question:

A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speed of the sphere before and after the collision. The value of e is given in each case.

Find the speed of the sphere before the collision in each case.

a $e=\frac{1}{2}$	Before impact		After impact	
	u ms^{-1} → ○	Wall	4 ms^{-1} ← ○	Wall
b $e=\frac{3}{4}$	Before impact		After impact	
	u ms^{-1} → ○	Wall	6 ms^{-1} ← ○	Wall

Solution:

a Use

$$e=\frac{\text{speed of rebound}}{\text{speed of approach}}$$
$$\therefore \frac{1}{2}=\frac{4}{u}$$
$$\therefore u\times\frac{1}{2}=4$$
$$\therefore u=4\times 2$$
$$=8$$

Speed before the collision is 8 m s^{-1}.

b Use

$$e=\frac{\text{speed of rebound}}{\text{speed of approach}}$$
$$\therefore \frac{1}{4}=\frac{V}{12}$$
$$\therefore V=\frac{1}{4}\times 12$$
$$=3$$

$\therefore$ speed after collision is 3 m s^{-1}.

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Exercise C, Question 4

Question:

A small smooth sphere of mass 0.3 kg is moving on a smooth horizontal table with a speed of 10 m s^{-1} when it collides normally with a fixed smooth wall. It rebounds with a speed of 7.5 m s^{-1}. Find the coefficient of restitution between the sphere and the wall.

Solution:

Use

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$

$$\therefore e = \frac{7.5}{10}$$

i.e. $e = 0.75$

The coefficient of restitution is 0.75 or $\frac{3}{4}$.

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Exercise C, Question 5

Question:

A particle falls 2.5 m from rest on to a smooth horizontal plane. It then rebounds to a height of 1.5 m. Find the coefficient of restitution between the particle and the plane. Give your answer to 2 s.f.

Solution:

The particle falls under gravity:

$s = 2.5, a = g, u = 0, v = ?$

Find velocity when particle hits plane by using constant acceleration formula.

Use $v^2 = u^2 + 2as$ as motion is under constant acceleration.

$\therefore v^2 = 2 \times g \times 2.5$

$= 5g$

$\therefore v = \sqrt{5g} = \sqrt{5 \times 9.8} = \sqrt{49} = 7$

Particle strikes the plane with velocity 7 m s^{-1}.

After it rebounds it moves under gravity to a height of 1.5 m.

After the impact the motion is again under gravity.

$u = ?\ s = 1.5\ a = -g\ v = 0$

Use

$v^2 = u^2 + 2as$

$\therefore 0 = u^2 - 2g \times 1.5$

$\therefore u^2 = 3g$

$= 3 \times 9.8$

$= 29.4$

$\therefore u = \sqrt{29.4}$

$= 5.422$

The velocity after impact is 5.422 m s^{-1}.

Using

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$

$$e = \frac{5.422}{7} = 0.78 \text{ (2 s.f.)}$$

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Exercise C, Question 6

Question:

A particle falls 3 m from rest onto a smooth horizontal plane. It then rebounds to a height h m. The coefficient of restitution between the particle and the plane is 0.25. Find the value of h.

Solution:

The particle falls under gravity

$s = 3, u = 0, a = g, v = ?$

Use

$$\therefore v^2 = u^2 + 2as$$
$$v^2 = 2 \times g \times 3$$
$$= 6g$$
$$v = \sqrt{6g}$$

Use $v^2 = u^2 + 2as$ for the motion before impact and for the motion after impact.

It hits the ground and rebounds. The velocity after the impact is ev.
i.e. new velocity is $0.25\sqrt{6g}$.
It rebounds and moves under gravity.
$u = 0.25\sqrt{6g}, a = -g, s = h, v = 0$
Use

$$v^2 = u^2 + 2as$$
$$\therefore 0 = \left(0.25\sqrt{6g}\right)^2 - 2gh$$
$$\therefore 2gh = \frac{1}{16} \times 6g$$
$$\therefore h = \frac{6g}{16} \div 2g$$
$$\therefore h = \frac{3}{16}$$

So the particle rebounds to a height of $\frac{3}{16}$ m $= 18.75$ cm.

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Exercise C, Question 7

Question:

A small smooth sphere falls from rest onto a smooth horizontal plane. It takes 2 seconds to reach the plane then another 2 seconds to reach the plane a second time. Find the coefficient of restitution between the particle and the plane.

Solution:

The sphere falls under gravity.

$u = 0, t = 2, a = g, v = ?$

Use

$$v = u + at$$

$$\therefore v = 2g$$

Use $v = u + at$ to find the sphere's velocity when it hits the plane.

This is its velocity when it hits the plane.

The sphere then bounces and its new velocity is $2ge$, where e is the coefficient of restitution.

The sphere then moves under gravity for 2 seconds.

$u = 2ge, a = -g, t = 2, s = 0$

Use

$$s = ut + \frac{1}{2}at^2$$

$$\therefore 0 = 2ge \times 2 - \frac{1}{2}g \times 4$$

$$= 4ge - 2g.$$

$$\therefore 4ge = 2g$$

$$e = \frac{2g}{4g} = \frac{1}{2}$$

For the motion after impact, use $s = ut + \frac{1}{2}at^2$ with $s = 0$ and $a = -g$.

The coefficient of restitution is $\frac{1}{2}$.

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Exercise C, Question 8

Question:

A small smooth sphere falls from rest onto a smooth horizontal plane. It takes 3 seconds to reach the plane. The coefficient of restitution between the particle and the plane is 0.49.

Find the time it takes for the sphere to reach the plane a second time.

Solution:

The sphere falls under gravity.

$u = 0, t = 3, v = ?, a = g$

Use

$$v = u + at$$

$$\therefore v = 3g$$

After impact the new velocity is $0.49 \times 3g$.

The sphere then moves again under gravity.

$u = 0.49 \times 3g, t = ?, a = -g, s = 0$

Use

$$s = ut + \frac{1}{2}at^2$$

$$\therefore 0 = 1.47gt - \frac{1}{2}gt^2$$

$$\therefore t = \frac{2 \times 1.47g}{g}$$

$$= 2.94 \text{ s}$$

So the time is 2.9 s (2 s.f.)

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Exercise D, Question 1

Question:

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

Three small smooth spheres A, B and C of equal radius move along the same straight line on a horizontal plane. Sphere A collides with sphere B and then sphere B collides with sphere C.

The diagrams show the velocities before the first collision, after the first collision between A and B and then after the collision between B and C.

a Find the values of u, v, x and y if $e = \frac{1}{2}$ for both collisions.

Before collision			After A and B have collided			After B and C have collided		
5 m s^{-1} →	1 m s^{-1} →	4 m s^{-1} →	$u \text{ m s}^{-1}$ →	$v \text{ m s}^{-1}$ →	4 m s^{-1} →	$u \text{ m s}^{-1}$ →	$x \text{ m s}^{-1}$ →	$y \text{ m s}^{-1}$ →
A (2kg)	B (1kg)	C (2kg)	A (2kg)	B (1kg)	C (2kg)	A (2kg)	B (1kg)	C (2kg)

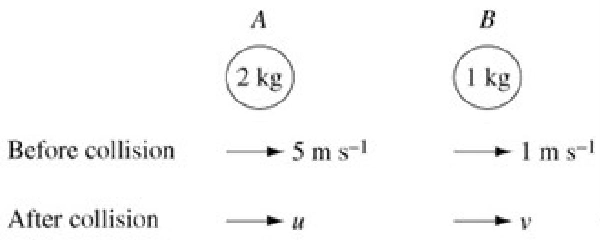
b Find the values of u, v, x and y if $e = \frac{1}{6}$ for the collision between A and B and $e = \frac{1}{2}$ for the collision between B and C.

Before collision			After A and B have collided			After B and C have collided		
10 m s^{-1} →	2 m s^{-1} ←	3 m s^{-1} →	$u \text{ m s}^{-1}$ →	$v \text{ m s}^{-1}$ →	3 m s^{-1} →	$u \text{ m s}^{-1}$ →	$x \text{ m s}^{-1}$ →	$y \text{ m s}^{-1}$ →
A (1.5kg)	B (2kg)	C (1kg)	A (1.5kg)	B (2kg)	C (1kg)	A (1.5kg)	B (2kg)	C (1kg)

Solution:

a

First collision (between A and B)



First consider the collision between A and B and use conservation of linear momentum and Newton's Law of Restitution to find u and v. Then repeat the process for the collision between B and C.

Use conservation of linear momentum:

$$2\times5+1\times1=2u+1v$$
$$\therefore 11=2u+v \quad (1)$$

Use Newton's Law of Restitution:

$$e=\frac{1}{2}=\frac{v-u}{5-1}$$
$$\therefore v-u=\tfrac{1}{2}\times4$$
$$v-u=2 \quad (2)$$

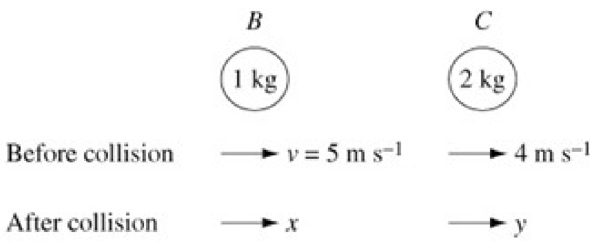
Subtract equation (2) from equation (1).

$$9=3u$$
$$\therefore u=3$$

Substitute into equation (1)

$$11=6+v$$
$$\therefore v = 5$$ [Check in equation (2).]

Second collision (between B and C)



Use conservation of linear momentum:

$$1\times5+2\times4=1\times x+2\times y$$
$$\therefore 13 = x+2y \quad (3)$$

Use Newton's Law of Restitution:

$$e=\frac{1}{2}=\frac{y-x}{5-4}$$
$$\therefore \frac{1}{2}=y-x \quad (4)$$

Add equations (3) and (4)

$13\frac{1}{2} = 3y$

$\therefore y = 4\frac{1}{2}$

Substitute back into equation (4)

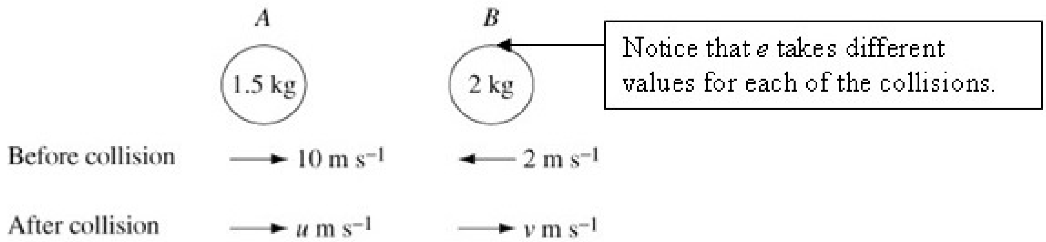
$\frac{1}{2} = 4\frac{1}{2} - x$

$\therefore x = 4$ [Check in equation (3).]

$\therefore u = 3, v = 5, x = 4, y = 4\frac{1}{2}$

b

First collision



Use conservation of linear momentum:

$1.5 \times 10 - 2 \times 2 = 1.5u + 2v$

$\therefore 11 = 1.5u + 2v \quad (1)$

Use Newton's Law of Restitution:

$e = \frac{1}{6} = \frac{v - u}{10 - (-2)}$

$\therefore \frac{1}{6} \times 12 = v - u$

i.e. $2 = v - u \quad (2)$

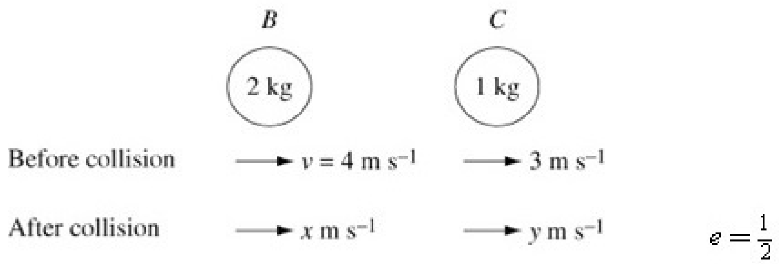
Add (1) to $1.5 \times (2)$.

$14 = 3.5v$

$\therefore v = 4$

Substitute into equation (1) to give $u = 2$.
[Check your answers in equation (2).]

Second collision



Use conservation of linear momentum:

$$2\times4+1\times3=2x+y$$
$$\therefore 11=2x+y \quad (1)$$

Use Newton's Law of Restitution:

$$e=\frac{1}{2}=\frac{y-x}{4-3}$$
$$\therefore \frac{1}{2}=y-x \quad (2)$$

Subtract equation (2) from equation (1)

$$10\frac{1}{2}=3x$$
$$\therefore x=3\frac{1}{2}$$

Substitute into equation (2)

$$\therefore y=4$$
$$u=2, v=4, x=3\frac{1}{2}, y=4$$

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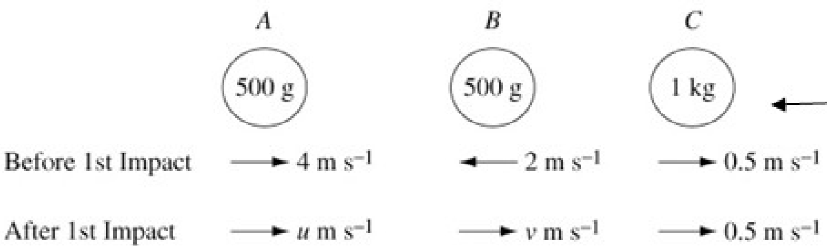
Exercise D, Question 2

Question:

Three small smooth spheres A, B and C of equal radius have masses 500 g, 500 g and 1 kg respectively. The spheres move along the same straight line on a horizontal plane with A following B which is following C. Initially the velocities of A, B and C are $4\mathbf{i}$ m s^{-1}, $-2\mathbf{i}$ m s^{-1} and $0.5\mathbf{i}$ m s^{-1} respectively, where $\mathbf{i}$ is a unit vector in the direction ABC. Sphere A collides with sphere B and then sphere B collides with sphere C. The coefficient of restitution between A and B is $\frac{2}{3}$ and between B and C is $\frac{1}{2}$. Find the velocities of the three spheres after all of the collisions have taken place.

Solution:



Draw clearly labelled diagrams for each collision separately. Make it clear which speed refers to which particle.

Impact between *A* and *B*

Using conservation of linear momentum:

$$0.5\times4-0.5\times2=0.5u+0.5v$$
$$1=0.5u+0.5v \quad (1)$$

Using Newton's Law of Restitution:

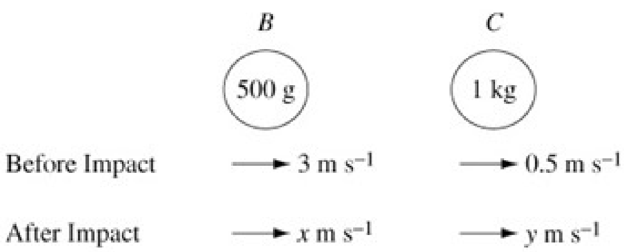
$$e=\frac{2}{3}=\frac{v-u}{4-(-2)}$$
$$\therefore \frac{2}{3}\times6=v-u$$
$$\text{i.e. } 4=v-u \qquad (2)$$

Multiply equation (1) by 2 and add to equation (2)

$$\therefore 6=2v$$
$$\therefore v=3$$

Substitute into equation (2) $\therefore 4=3-u$ and $u=-1$
[Check u and v in equation (1).]

Impact between *B* and *C*



Using conservation of linear momentum:

$$0.5\times3+1\times0.5=0.5x+1y$$
$$\therefore 2=0.5x+y \quad (1)'$$

Using Newton's Law of Restitution:

$$e = \frac{1}{2} = \frac{y-x}{3-0.5}$$

$$\therefore \frac{1}{2} \times 2.5 = y - x$$

$$\therefore 1.25 = y - x \quad (2)'$$

Subtract equation $(2)'$ from equation $(1)'$

$$0.75 = 1.5x$$

$$x = \frac{0.75}{1.5}$$

$$= 0.5$$

Substitute back into equation $(2)'$

$$\therefore y = 0.5 + 1.25 = 1.75$$

So $u = -1, v = 3, x = 0.5, y = 1.75$

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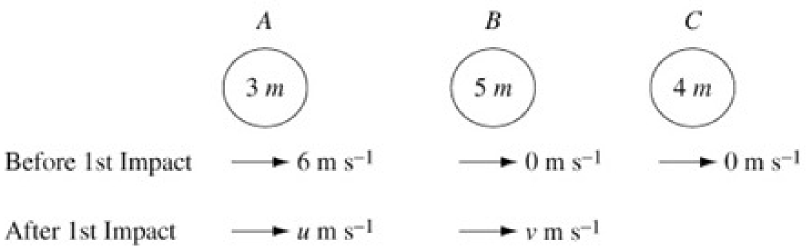
Exercise D, Question 3

Question:

Three perfectly elastic particles A, B and C of masses $3m$, $5m$ and $4m$ respectively lie at rest on a straight line on a smooth horizontal table with B between A and C. Particle A is projected directly towards B with speed 6 m s^{-1} and after A has collided with B, B then collides with C. Find the speed of each particle after the second impact.

Solution:



Use conservation of linear momentum:

$$3m\times 6+5m\times 0=3mu+5mv$$
$$\therefore 18=3u+5v \quad (1)$$

Use Newton's Law of Restitution:

$$e=1=\frac{v-u}{6}$$
$$\therefore 6=v-u \quad (2)$$

'Perfectly elastic' means $e=1$.

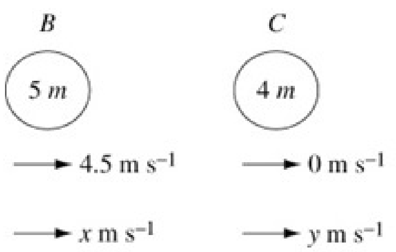
Add equation (1) to 3 times equation (2).

$$\therefore 36=8v$$
$$\therefore v=\frac{36}{8}=4.5$$

Substitute into equation (2):

$$\therefore 6=4.5-u$$
$$\therefore u=-1.5$$

Second impact:



Use conservation of linear momentum:

$$5m\times 4.5+4m\times 0=5mx+4my$$
$$\therefore 22.5=5x+4y \quad (1)'$$

Use Newton's Law of Restitution:

$$e = 1 = \frac{y - x}{4.5 - 0}$$
$$\therefore 4.5 = y - x \quad (2)'$$

Add equation (1)' to 5 times equation (2)'

$$\therefore 45 = 9y$$
$$\therefore y = 5$$

Substitute into equation (2)'

Then

$$4.5 = 5 - x$$
$$\therefore x = 0.5$$

$$\therefore u = -1.5, v = 4.5, x = 0.5, y = 5$$

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Exercise D, Question 4

Question:

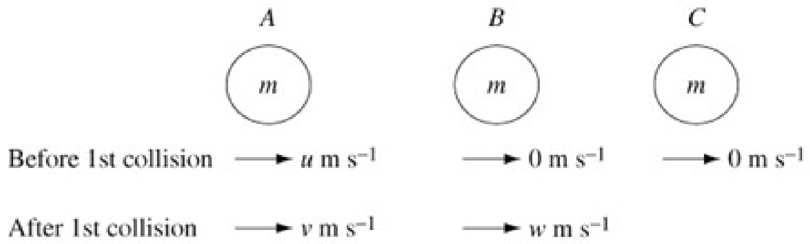
Three identical smooth spheres A, B and C, each of mass m, lie at rest on a straight line on a smooth horizontal table. Sphere A is projected with speed u to strike sphere B directly. Sphere B then strikes sphere C directly. The coefficient of restitution between any two sphere is e, $e \neq 1$.

a Find the speeds in terms of u and e of the spheres after these two collision.

b Show that A will catch up with B and there will be a further collision.

Solution:

a



Using conservation of linear momentum:

$$mu = mv + mw$$
$$\therefore u = v + w \quad (1)$$
$$e = \frac{w - v}{u}$$
$$\therefore eu = w - v \quad (2)$$

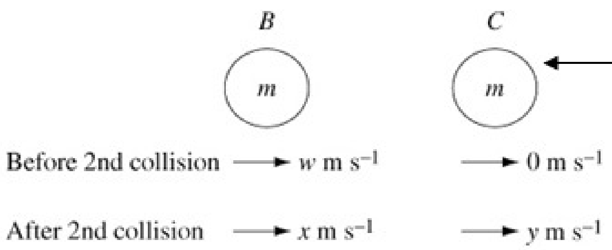
Add $(1) + (2)$

$$\therefore u(1+e) = 2w \rightarrow w = \frac{1}{2}u(1+e)$$

Subtract $(1) - (2)$

$$\therefore u(1-e) = 2v \rightarrow v = \frac{1}{2}u(1-e)$$

Consider second collision



The second collision here is similar to the first. The only difference is that the moving sphere has a different velocity. Replacing 'u' by this velocity will give expressions for velocities of B and C after the second impact.

By similar reasoning to above $y = \frac{1}{2}w(1+e)$ and $x = \frac{1}{2}w(1-e)$

And as $w = \frac{1}{2}u(1+e)$

$y = \frac{1}{4}u(1+e)^2$ and $x = \frac{1}{4}u(1+e)(1-e)$

$\therefore$ The speeds of A, B and C after the two collisions are

$\frac{1}{2}u(1-e), \frac{1}{4}u(1+e)(1-e)$ and $\frac{1}{4}u(1+e)^2$ respectively.

b A will catch up with B provided that

$\frac{1}{2}u(1-e) > \frac{1}{4}u(1+e)(1-e)$

i.e. provided that $2 > 1+e$

Since $e < 1$ this condition holds and A will catch up with B resulting in a further collision.

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Exercise D, Question 5

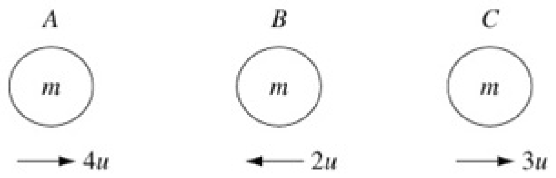
Question:

Three identical spheres A, B and C of equal mass m, and equal radius move along the same straight line on a horizontal plane. B is between A and C. A and B are moving towards each other with velocities $4u$ and $2u$ respectively while C moves away from B with velocity $3u$.

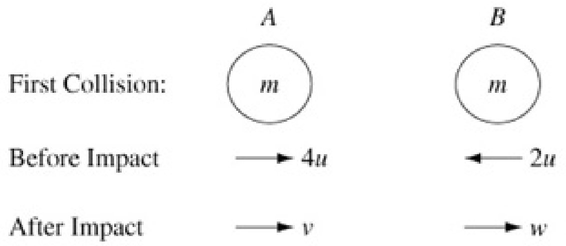
a If the coefficient of restitution between any two of the spheres is e, show that B will only collide with C if $e > \frac{2}{3}$.

b Find the direction of motion of A after collision, if $e > \frac{2}{3}$.

Solution:



First collision:



a Let velocities of A and B be v and w after impact.

Using conservation of linear momentum:

$$m\times 4u - m\times 2u = mv + mw$$

$$\therefore 2u = v + w \qquad (1)$$

Using Newton's Law of Restitution:

$$e = \frac{w-v}{4u-(-2u)}$$

$$\therefore 6ue = w - v \qquad (2)$$

Add equations (1) and (2)

$$2w = 2u + 6ue$$

$$\therefore w = u(1+3e)$$

B will then collide with C if

$$w > 3u$$

$$u(1+3e) > 3u$$

$$\text{i.e. } 3e > 2 \rightarrow e > \frac{2}{3}$$

B collides with C if the velocity of B after the collision is greater than $3u$.

b Subtract (2) from (1)

$$2u - 6ue = 2v$$

$$\therefore v = 2u(1-3e)$$

If $e > \frac{2}{3}$ then $v < 0$ $\therefore A$ moves to the left.

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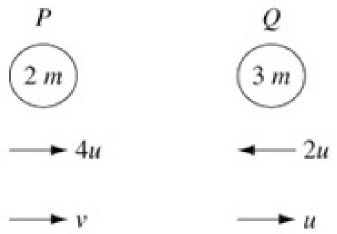
Exercise D, Question 6

Question:

Two particles P of mass $2m$ and Q of mass $3m$ are moving towards each other with speeds $4u$ and $2u$ respectively. The direction of motion of Q is reversed by the impact and its speed after impact is u. This particle then hits a smooth vertical wall perpendicular to its direction of motion. The coefficient of restitution between Q and the wall is $\frac{2}{3}$. In the subsequent motion, there is a further collision between Q and P. Find the speeds of P and Q after this collision.

Solution:



The **first collision** is between P and Q. Let the velocity of P after the collision be v.

Using conservation of momentum:

$$2m\times 4u-3m\times 2u = 2mv+3mu$$
$$\therefore 2mu = 2mv+3mu$$
$$\therefore 2v = -u$$
$$\therefore v = -\frac{1}{2}u$$

Using Newton's Law of Restitution:

From the information given find the coefficient of restitution between particles P and Q.

$$e=\frac{u-v}{4u-(-2u)}$$
$$=\frac{1\frac{1}{2}u}{6u}$$
$$=\frac{1}{4}$$

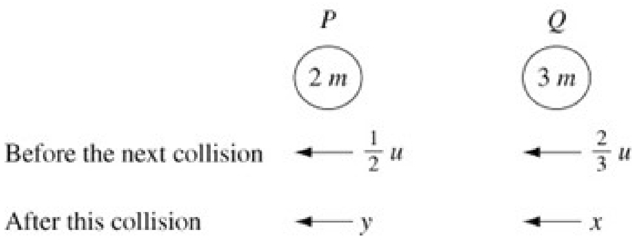
$\therefore$ coefficient of restitution between P and Q is $\frac{1}{4}$.

The **second collision** is between Q and the wall.

Q rebounds from the wall with velocity $e'u=\frac{2}{3}u$.

(as e', the coefficient of restitution between Q and the wall, is $\frac{2}{3}$).

The **next collision** is between P and Q.



Taking direction to left as positive: let velocities of Q and P after this collision be x and y respectively.

Using conservation of linear momentum $\leftarrow$:

$$2m\times\frac{1}{2}u+3m\times\frac{2}{3}u = 2my+3mx$$
$$\therefore mu+2mu = 2my+3mx$$
$$\text{i.e. } 3u = 2y+3x \quad (1)$$

Using Newton's Law of Restitution:

$$e = \frac{1}{4} = \frac{y - x}{\frac{2}{3}u - \frac{1}{2}u}$$

$$\therefore \frac{1}{4} \times \frac{1}{6}u = y - x$$

$$\text{i.e. } \frac{u}{24} = y - x \quad (2)$$

Add (1) to 3 times (2)

Then

$$3\frac{3}{24}u = 5y$$

$$\text{i.e. } 5y = \frac{75u}{24}$$

$$\therefore y = \frac{15u}{24} = \frac{5u}{8}$$

Substitute into equation (2) $x = \frac{5u}{8} - \frac{u}{24} = \frac{7u}{12}$

$\therefore P$ has speed $\frac{5u}{8}$ and Q has speed $\frac{7u}{12}$ both to the left.

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Exercise D, Question 7

Question:

Two small smooth spheres P and Q of equal radius have masses m and $3m$ respectively.

Sphere P is moving with speed $12u$ on a smooth horizontal table when it collides directly with Q which is at rest on the table. The coefficient of restitution between P and Q is $\frac{2}{3}$.

a Find the speeds of P and Q immediately after the collision.

After the collision Q hits a smooth vertical wall perpendicular to the direction of its motion.

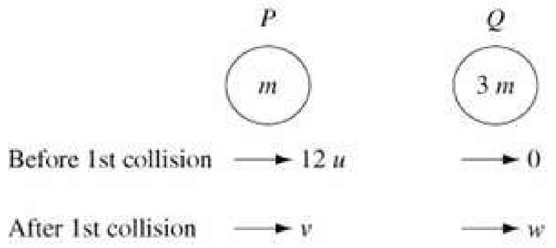
The coefficient of restitution between Q and the wall is $\frac{4}{5}$.

Q then collides with P a second time.

b Find the speeds of P and Q after the second collision between P and Q.

Solution:

a



Use conservation of linear momentum:

$m \times 12u = mv + 3mw$

$\therefore 12u = v + 3w \quad (1)$

Using Newton's Law of Restitution

$$e = \frac{2}{3} = \frac{w - v}{12u - 0}$$

$\therefore \frac{2}{3} \times 12u = w - v$

$\therefore 8u = w - v \quad (2)$

Add $(1) + (2)$

$\therefore 20u = 4w \rightarrow w = 5u$

Substitute into equation (2):

$\therefore 8u = 5u - v$

i.e. $v = -3u$

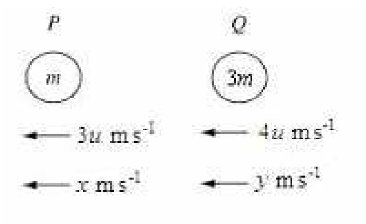
The speeds of P and Q after the first collision are $3u$ m s^{-1} to the left and $5u$ m s^{-1} to the right, respectively.

b

Q then hits a wall and rebounds with speed $4u$ m s^{-1}$(= \frac{4}{5} \times 5u)$.

Let the speeds of P and Q be x m s^{-1} and y m s^{-1} after the second collision between them.

Second collision



Use conservation of linear momentum $\leftarrow$:

$m \times 3u + 3m \times 4u = mx + 3my$

$15mu = mx + 3my$

i.e. $15u = x + 3y \quad (1)'$

Use Newton's Law of Restitution:

$$e = \frac{2}{3} = \frac{x - y}{4u - 3u}$$

$$\therefore \frac{2}{3}u = x - y \qquad (2)'$$

Subtract $(1)' - (2)'$

$$\therefore 14\frac{1}{3}u = 4y$$

$$\therefore y = \frac{43}{3}u \div 4 = \frac{43u}{12}$$

Substitute into $(2)'$ to give

$$x = \frac{2}{3}u + \frac{43u}{12}$$

$$= \frac{51u}{12} = \frac{17u}{4}$$

$\therefore$ After the second collision the speeds of P and Q are $\frac{17u}{4}$ m s^{-1} and $\frac{43u}{12}$ m s^{-1} respectively. Both particles are moving away from the wall.

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Exercise D, Question 8

Question:

A small smooth table tennis ball, which may be modelled as a particle, falls from rest at a height 40 cm onto a smooth horizontal plane. The coefficient of restitution between the ball and the plane is 0.7.

a Find the height to which the ball rebounds after the first bounce.

b Find the height to which the ball rebounds after the second bounce.

c Find the total distance travelled by the ball before it comes to rest.

Solution:

a First stage of motion under gravity:

$u = 0$, $a = g = 9.8$, $s = 0.4$, $v = ?$

Use $v^2 = u^2 + 2as$. Then

$$v^2 = 2 \times 9.8 \times 0.4$$

$$\therefore v = 2.8$$

The ball hits the plane with velocity 2.8 m s^{-1} and rebounds with velocity $0.7 \times 2.8 \text{ m s}^{-1}$ i.e. 1.96 m s^{-1}.

The ball then moves up under gravity to a height h.

This time

$u = 1.96$, $a = -9.8$, $s = h$, $v = 0$.

Use

$$v^2 = u^2 + 2as$$

$$\therefore 0 = 1.96^2 - 2 \times 9.8 \times h$$

$$\therefore h = \frac{1.96 \times 1.96}{2 \times 9.8}$$

$$= 0.196$$

i.e. it rebounds to a height of 19.6 cm $[= 40 \times 0.7 \times 0.7]$.

b After the second bounce the ball rebounds to a height $19.6 \times 0.7 \times 0.7 = 9.604$ cm.

c The total distance travelled is

$$0.4 + 2 \times 0.4 \times 0.7^2 + 2 \times 0.4 \times 0.7^4 + \ldots$$

$$= 0.4 + 2 \times 0.4 \times 0.7^2 (1 + 0.7^2 + 0.7^4 + \ldots)$$

You will need to use the sum of an infinite G. P. to find the answer.

The sum in the bracket is an infinite G. P. with sum $= \frac{1}{1 - 0.7^2}$

$$\therefore \text{Total distance} = 0.4 + 0.392(1.96\ldots)$$

$$= 1.169 \text{ m or } 1.17 \text{ m (3 s.f.)}$$

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Exercise D, Question 9

Question:

A small smooth ball, which may be modelled as a particle, falls from rest at a height H onto a smooth horizontal plane. The coefficient of restitution between the ball and the plane is e.

a Find in terms of H and e the height to which the ball rebounds after the first bounce.

b Find in terms of H and e the height to which the ball rebounds after the second bounce.

c Find an expression for the total distance travelled by the ball before it comes to rest.

Solution:

a First part of motion is under gravity:

$u = 0 \; s = H \; a = g \; v = ?$

Using $v^2 = u^2 + 2as \rightarrow v^2 = 2gH$

$\therefore$ The ball hits the plane with velocity $\sqrt{2gH}$.

It rebounds with velocity $e\sqrt{2gH}$.

Then it moves under gravity to height H'.

Using $u = e\sqrt{2gH}$, $s = H'$, $a = -g$, $v = 0$ with

$$v^2 = u^2 + 2as$$
$$0 = e^2(2gH) - 2gH'$$

$\therefore H' = e^2 H$.

The ball rebounds to a height $e^2 H$.

b After the second bounce it rebounds to a height $e^4 H$.

You can deduce the answer to **b** without repeating all the work done in **a**.

c Total distance travelled is:

$$H + 2e^2H + 2e^4H + 2e^6H + \ldots$$
$$= H + 2e^2H(1 + e^2 + e^4 + \ldots)$$

You need to use the sum to infinity of a Geometric Progression.

The expression in the bracket is a Geometric Progression. Using $S_\infty = \frac{a}{1-r}$, the

expression $= \frac{1}{1-e^2}$

$\therefore$ Total distance travelled is $H + 2e^2H \times \frac{1}{1-e^2} = \frac{H(1+e^2)}{(1-e^2)}$

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Exercise E, Question 1

Question:

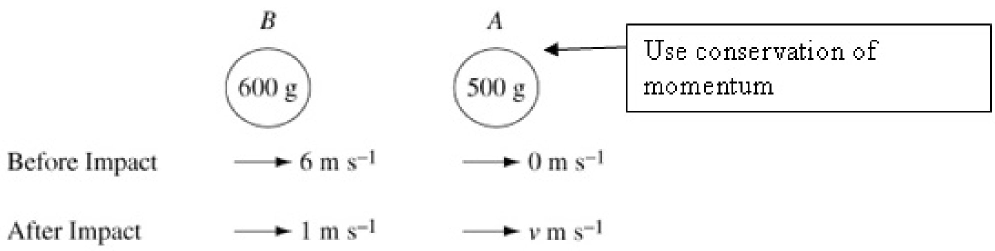
A particle A of mass 500 g lies at rest on a smooth horizontal table. A second particle B of mass 600 g is projected along the table with velocity 6 m s^{-1} and collides directly with A.

If the collision reduces the speed of B to 1 m s^{-1}, without changing its direction, find

a the speed of A after the collision,

b the loss of kinetic energy due to the collision.

Solution:



a Using conservation of momentum $\rightarrow$:

$$0.6\times 6+0.5\times 0=0.6\times 1+0.5\times v$$
$$\therefore 3.6-0.6=0.5v$$
$$\therefore v=6$$

The speed of A after the collision is 6 m s^{-1}.

b Total kinetic energy before collision

$$=\frac{1}{2}\times 0.6\times 6^2$$
$$=10.8 \text{ J}$$

Use loss of K.E. = K.E. before impact − K.E. after impact

Total kinetic energy after collision

$$=\frac{1}{2}\times 0.6\times 1^2+\frac{1}{2}\times 0.5\times v^2$$
$$=0.3+9$$
$$=9.3 \text{ J}$$

$$\text{The loss of K.E}=(10.8-9.3)\text{ J}$$
$$=1.5 \text{ J}$$

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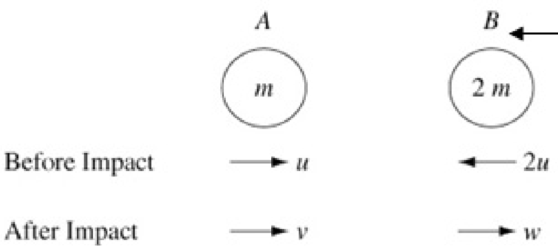
Exercise E, Question 2

Question:

Two particles A and B of mass m and $2m$ respectively move toward each other with speeds u and $2u$. If the coefficient of restitution between the spheres is $\frac{2}{3}$, find the speeds of A and of B after the collision. Find also, in terms of m and u, the loss of kinetic energy due to the collision.

Solution:



Use conservation of linear momentum and Newton's Law of Restitution to find the velocities after the impact.

Let the speeds of A and B after the collision be v and w respectively.

Use conservation of linear momentum $\rightarrow$:

$$mu - 2m \times 2u = mv + 2mw$$
$$\therefore -3u = v + 2w \qquad (1)$$

Use Newton's Law of Restitution:

$$\frac{w - v}{u - (-2u)} = \frac{2}{3}$$
$$\therefore w - v = \frac{2}{3} \times 3u$$
$$\therefore 2u = w - v \qquad (2)$$

Add equations (1) and (2)

$$\therefore -u = 3w$$
$$\text{i.e. } w = -\frac{u}{3}$$

Substitute into equation (2)

$$\therefore v = w - 2u = -\frac{7u}{3}.$$

The speed of A after impact is $\frac{7u}{3}$ in the direction away from B.

The speed of B after impact is $\frac{u}{3}$ towards A.

The kinetic energy before impact

$$= \frac{1}{2}mu^2 + \frac{1}{2} \times 2m(-2u)^2$$
$$= \frac{9}{2}mu^2$$

The kinetic energy after impact

$$= \frac{1}{2}mv^2 + \frac{1}{2} \times 2mw^2$$
$$= \frac{1}{2}\left(\frac{49}{9}u^2\right) + \frac{1}{2} \times 2m\left(\frac{u^2}{9}\right)$$
$$= \frac{51}{18}mu^2$$

$$\therefore \text{The loss of kinetic energy} = \frac{9}{2}mu^2 - \frac{51}{18}mu^2$$
$$= \frac{5}{3}mu^2$$

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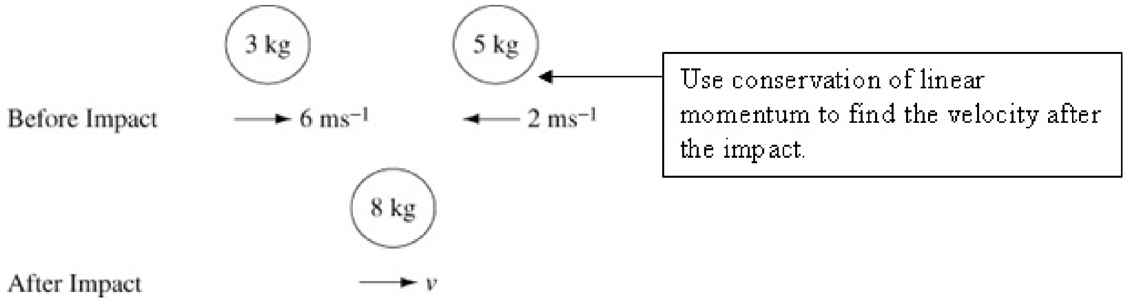
Exercise E, Question 3

Question:

A particle of mass 3 kg moving with velocity 6 m s^{-1} collides directly with a particle of mass 5 kg moving in the opposite direction with velocity 2 m s^{-1}. The particles coalesce and move with velocity v after the collision. Find the loss of kinetic energy due to the impact.

Solution:



Let the velocity of the combined particle be v.

Using conservation of momentum:

$$3\times 6+5\times(-2)=8\times v$$
$$\therefore 8=8v$$
$$\therefore v=1$$

i.e. the combined particle moves with velocity 1 m s^{-1}.

$$\text{The total K.E. before impact} =\tfrac{1}{2}\times 3\times 6^2+\tfrac{1}{2}\times 5\times(-2)^2$$
$$=54+10$$
$$=64\text{ J}$$

$$\text{The total K.E. after impact} =\tfrac{1}{2}\times 8\times v^2$$
$$=4\times 1^2$$
$$=4\text{ J}$$

$\therefore$ Loss of kinetic energy due to the impact is $64\text{ J}-4\text{ J}=60\text{ J}$.

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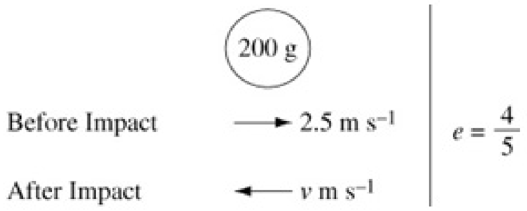
Exercise E, Question 4

Question:

A billiard ball of mass 200 g strikes a smooth cushion at right angles. Its velocity before the impact is 2.5 m s^{-1} and the coefficient of restitution is $\frac{4}{5}$. Find the loss in kinetic energy of the billiard ball due to the impact.

Solution:



After impact with the cushion the velocity of the billiard ball is $v \text{ m s}^{-1}$, where

$$\frac{v}{2.5} = \frac{4}{5}$$
$$\therefore v = 2$$

$\therefore$ The loss in kinetic energy is:

$$\frac{1}{2} \times 0.2 \times 2.5^2 - \frac{1}{2} \times 0.2 \times 2^2$$
$$= 0.625 - 0.4$$
$$= 0.225 \text{ J}$$

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Exercise E, Question 5

Question:

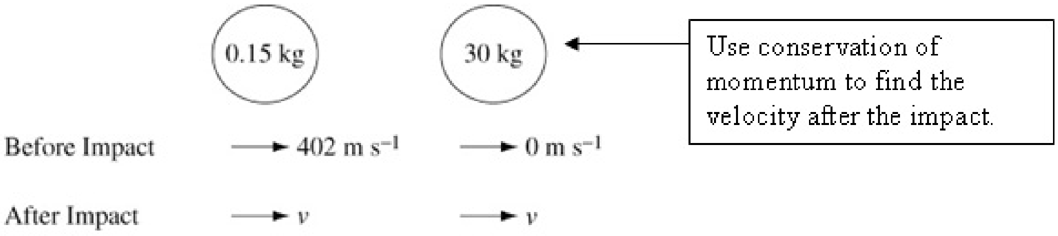
A bullet of mass 0.15 kg moving horizontally at 402 m s^{-1} embeds itself in a sandbag of mass 30 kg, which is suspended freely. Assuming that the sandbag is stationary before the impact, find

a the common velocity of the bullet and the sandbag,

b the loss of kinetic energy due to the impact.

Solution:

a



The common velocity after the impact is v.

Using conservation of linear momentum:

$$0.15 \times 402 + 0 = 30.15v$$
$$\therefore v = \frac{0.15 \times 402}{30.15}$$
$$= 2$$

$\therefore$ Common velocity is 2 m s^{-1}.

b Kinetic energy before the impact is $\frac{1}{2} \times 0.15 \times 402^2 = 12\,120.3 \text{ J}$

Kinetic energy after the impact is $\frac{1}{2} \times 30.15 \times 2^2 = 60.3 \text{ J}$

$\therefore$ Loss of kinetic energy $= 12\,060 \text{ J} = 12.06 \text{ kJ}$.

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Exercise E, Question 6

Question:

A particle of mass 0.4 kg is moving with velocity $(\mathbf{i}-4\mathbf{j})\text{ m s}^{-1}$ when it receives an impulse $(3\mathbf{i}+2\mathbf{j})$ Ns. Find the new velocity of the particle and the change in kinetic energy of the particle as a result of the impulse.

Solution:

Let the velocity after the impulse be $\mathbf{v}$.

Using impulse = change in momentum

Use impulse = change in momentum.

$$\begin{aligned} 3\mathbf{i}+2\mathbf{j} &= 0.4\mathbf{v}-0.4(\mathbf{i}-4\mathbf{j}) \\ \therefore 0.4\mathbf{v} &= 3\mathbf{i}+2\mathbf{j}+0.4\mathbf{i}-1.6\mathbf{j} \\ &= 3.4\mathbf{i}+0.4\mathbf{j} \\ \therefore \mathbf{v} &= 8.5\mathbf{i}+\mathbf{j} \end{aligned}$$

$\therefore$ The new velocity of the particle is $(8.5\mathbf{i}+\mathbf{j})\text{ m s}^{-1}$.

$$\begin{aligned} \text{The K.E. before the impulse} &= \tfrac{1}{2}\times 0.4\times(1^2+(-4)^2) \\ &= 3.4\text{ J} \\ \text{The K.E. after the impulse} &= \tfrac{1}{2}\times 0.4\times(8.5^2+1^2) \\ &= 14.65\text{ J} \end{aligned}$$

$\therefore$ The change in K.E. is an increase of 11.25 J.

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Exercise E, Question 7

Question:

A squash ball of mass 0.025 kg is moving with velocity $(22\mathbf{i}+37\mathbf{j})$ m s^{-1} when it hits a wall.

It rebounds with velocity $(10\mathbf{i}-11\mathbf{j})$ m s^{-1}. Find the change in kinetic energy of the squash ball.

Solution:

Kinetic energy before it hits the wall is
$\frac{1}{2}\times 0.25\times(22^2+37^2)=231.625$ J

A mass m kg moving with velocity $(v_1\mathbf{i}+v_2\mathbf{j})$ m s^{-1} has K.E $=\frac{1}{2}m(v_1^2+v_2^2)$ J.

Kinetic energy after rebound is $\frac{1}{2}\times 0.25\times(10^2+(-11)^2)=27.625$ J

$\therefore$ The loss in K.E. = 204 J

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Exercise E, Question 8

Question:

A particle of mass 0.2 kg is moving with velocity $(5\mathbf{i}+25\mathbf{j})$ m s^{-1} when it collides with a particle of mass 0.1 kg moving with velocity $(2\mathbf{i}+10\mathbf{j})$ m s^{-1}. The two particles coalesce and form one particle of mass 0.3 kg. Find the velocity of the combined particle and find the loss in kinetic energy as a result of the collision.

Solution:

Let the velocity of the combined particle be $(v_1\mathbf{i}+v_2\mathbf{j})$ m s^{-1}.

Using conservation of momentum:

Use conservation of momentum to find the velocity of the combined particle.

$$0.2(5\mathbf{i}+25\mathbf{j})+0.1(2\mathbf{i}+10\mathbf{j})=0.3(v_1\mathbf{i}+v_2\mathbf{j})$$

Equate $\mathbf{i}$ components:

$$1\mathbf{i}+0.2\mathbf{i}=0.3v_1\mathbf{i}$$
$$\therefore 1.2=0.3v_1$$
$$\Rightarrow v_1=4$$

Equate $\mathbf{j}$ components:

$$5\mathbf{j}+\mathbf{j}=0.3v_2\mathbf{j}$$
$$\therefore 6=0.3v_2$$
$$\Rightarrow v_2=20$$

$\therefore$ The required velocity is $(4\mathbf{i}+20\mathbf{j})$ m s^{-1}.

$$\text{The initial K.E.}=\tfrac{1}{2}\times0.2(5^2+25^2)+\tfrac{1}{2}\times0.1(2^2+10^2)$$
$$=65+5.2$$
$$=70.2\text{ J}$$
$$\text{The K.E. after impact}=\tfrac{1}{2}\times0.3\times(4^2+20^2)$$
$$=62.4\text{ J}$$
$$\therefore\text{The loss in K.E.}=70.2\text{ J}-62.4\text{ J}$$
$$=7.8\text{ J}$$

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Exercise E, Question 9

Question:

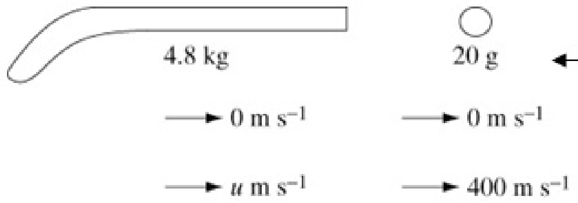
A bullet is fired horizontally from a rifle. The rifle has mass 4.8 kg and the bullet has mass 20 g. The initial speed of the bullet is 400 m s^{-1}. Find

a the initial speed with which the rifle recoils,

b the total kinetic energy generated as a result of firing the bullet.

Solution:

a



Convert the mass of the bullet into kg for your calculations.

Use conservation of momentum:

$$0 = 4.8u + 0.02 \times 400$$

$$\therefore u = \frac{-0.02 \times 400}{4.8}$$

$$= -\frac{5}{3}$$

$\therefore$ The rifle recoils (moves back) with a speed of $\frac{5}{3} \text{ m s}^{-1}$.

b The total K.E. before firing $= 0$

Total K.E. after firing $= \frac{1}{2} \times 4.8 \times \left(\frac{5}{3}\right)^2 + \frac{1}{2} \times (0.02) \times 400^2$

$= 6.\dot{6} + 1600$

$= 1606.\dot{6}$ J

$\therefore$ K.E. generated is $1606.\dot{6}$ J.

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Exercise E, Question 10

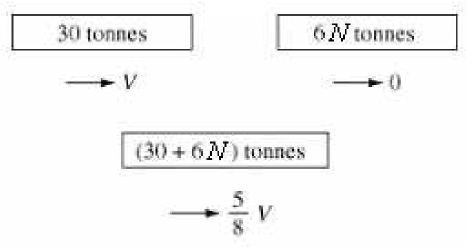
Question:

A train of mass 30 tonnes moving with a small velocity V impacts upon a number of stationary carriages each weighing 6 tonnes. The complete train and carriages now move forward with a velocity of $\frac{5}{8}V$. Find

a the number of stationary carriages,

b the fraction of the original kinetic energy lost in the impact.

Solution:



a Let the number of carriages be N.

Using conservation of momentum:

$$30V + 6N \times 0 = (30 + 6N)\frac{5}{8}V$$

Let the mass of the train and carriages be $(30 + 6N)$ tonnes.

$$\therefore 30 = \frac{150}{8} + \frac{30N}{8}$$

$$\therefore \frac{90}{8} = \frac{30N}{8}$$

$$\therefore N = 3$$

b

$$\text{Energy before impact} = \frac{1}{2} \times 30\,000V^2 = 15\,000V^2$$

$$\text{Energy after impact} = \frac{1}{2}(48\,000) \times \left(\frac{5V}{8}\right)^2$$

$$= 9375V^2$$

$$\therefore \text{Energy lost} = 5625V^2$$

$$\therefore \text{Fraction of original K.E. lost} = \frac{5625V^2}{15\,000V^2}$$

$$= \frac{3}{8}$$

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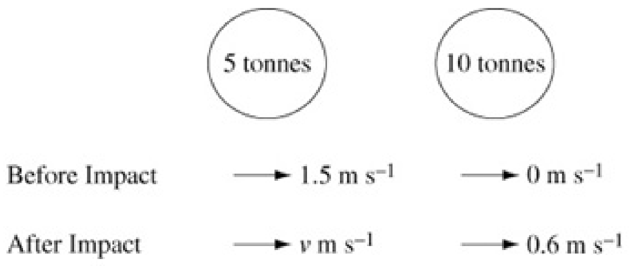
Exercise E, Question 11

Question:

A truck of mass 5 tonnes is moving at 1.5 m s^{-1} when it hits a second truck of mass 10 tonnes which is at rest. After the impact the second truck moves at 0.6 m s^{-1}. Find the speed of the first truck after the impact and the total loss of kinetic energy due to the impact.

Solution:



Let the speed of the first truck be $v\text{ m s}^{-1}$, after the impact.

Use conservation of momentum:

$$5\times1.5+0=5v+10\times0.6$$
$$\therefore 5v=7.5-6$$
$$=1.5$$
$$\therefore v=0.3$$

$\therefore$ The first truck moves with a speed of 0.3 m s^{-1}.

The total K.E. before impact $=\frac{1}{2}\times5000\times1.5^2=5625\text{ J}$

The total K.E. after impact $=\frac{1}{2}\times5000\times0.3^2+\frac{1}{2}\times10\,000\times0.6^2$
$=2025\text{ J}$

Convert the masses of the trucks into kg to calculate the K.E. in Joules.

$\therefore$ The loss of K.E. due to the impact $=3600\text{ J}$

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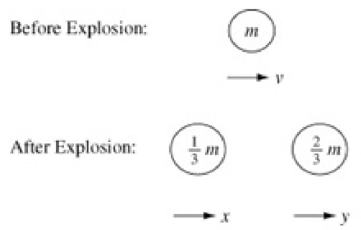
Exercise E, Question 12

Question:

A particle of mass m moves in a straight line with velocity v when it explodes into two parts, one of mass $\frac{1}{3}m$ and the other of mass $\frac{2}{3}m$ both moving in the same direction as before.
If the explosion increases the energy of the system by $\frac{1}{4}mu^2$, where u is a positive constant, find the velocities of the particles immediately after the explosion. Give your answers in terms of u and v.

Solution:



Let the velocities of the $\frac{1}{3}m$ mass and the $\frac{2}{3}m$ mass be x and y respectively.
Use conservation of momentum:

> Use conservation of momentum, and energy equation to give simultaneous equations–one linear and one quadratic.

$$mv = \frac{1}{3}mx + \frac{2}{3}my$$

$\therefore x + 2y = 3v \quad (1)$

The energy of the system is increased by $\frac{1}{4}mu^2$.

$$\therefore \frac{1}{2}mv^2 + \frac{1}{4}mu^2 = \frac{1}{2}\left(\frac{1}{3}m\right)x^2 + \frac{1}{2}\left(\frac{2}{3}m\right)y^2$$

$$\therefore x^2 + 2y^2 = 3v^2 + \frac{3}{2}u^2 \quad (2)$$

From equation (1), $x = 3v - 2y$
Substitute into equation (2)

$$\therefore (3v - 2y)^2 + 2y^2 = 3v^2 + \frac{3}{2}u^2$$

$$\therefore 9v^2 - 12vy + 6y^2 = 3v^2 + \frac{3}{2}u^2$$

$$\therefore 6y^2 - 12vy + 6v^2 = \frac{3}{2}u^2$$

$$\therefore y^2 - 2vy + v^2 = \frac{1}{4}u^2$$

$$\therefore (y - v)^2 = \left(\pm\frac{1}{2}u\right)^2$$

$$\therefore y = v \pm \frac{1}{2}u$$

Substitute back into (1) $\therefore x = v \mp u$

But $x < y, \therefore x = v - u$ and $y = v + \frac{1}{2}u$

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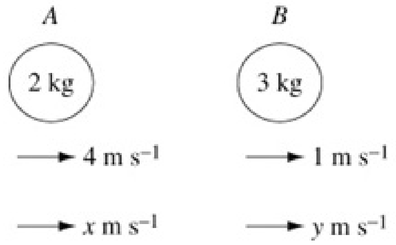
Exercise E, Question 13

Question:

A small smooth sphere A of mass 2 kg moves at 4 m s^{-1} on a smooth horizontal table. It collides directly with a second equal-sized smooth sphere B of mass 3 kg, which is moving away from A in the same direction at a speed of 1 m s^{-1}. If the loss of kinetic energy due to the collision is 3 J find the speeds and the directions of the two spheres after the collision.

Solution:



Use conservation of momentum:

$2 \times 4 + 3 \times 1 = 2x + 3y$

$\therefore 2x + 3y = 11 \quad (1)$

Use conservation of momentum and a change in energy equation to give simultaneous equations – one linear and one quadratic.

The energy before the collision $= \frac{1}{2} \times 2 \times 4^2 + \frac{1}{2} \times 3 \times 1^2$

$= 17.5\text{ J}$

The energy after collision $= \frac{1}{2} \times 2x^2 + \frac{1}{2} \times 3y^2$

$= x^2 + 1.5y^2$

As the loss of K.E. $= 3\text{ J}$

$\therefore \quad x^2 + 1.5y^2 = 14.5 \quad (2)$

From (1) $x = \dfrac{11 - 3y}{2}$ – substitute into equation (2).

$\therefore \dfrac{121 - 66y + 9y^2}{4} + 1.5y^2 = 14.5$

Multiply by 4

$\therefore 15y^2 - 66y + 121 = 58$

$\therefore 5y^2 - 22y + 21 = 0$

$\therefore (5y - 7)(y - 3) = 0$

$\therefore y = 3$ or $\frac{7}{5}$

Substitute value of y into equation (1)

$\therefore x = 1$ or $x = 3.4$.

But $x < y \quad \therefore x = 1$ and $y = 3$

$\therefore$ The 2 kg mass moves with speed 1 m s^{-1} and the 3 kg mass moves with speed 3 m s^{-1}.

They both move in the same direction as before the impact.

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Exercise E, Question 14

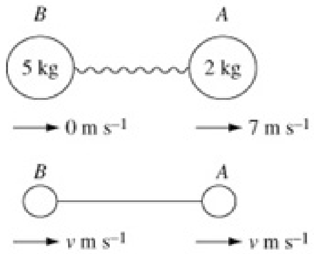
Question:

Two particles, A and B, of masses 2 kg and 5 kg respectively, are connected by a light inextensible string. The particles are side by side on a smooth floor and A is projected with speed 7 m s^{-1} directly away from B. When the string becomes taut particle B is jerked into motion and A and B then move with a common speed in the direction of the original velocity of A. Find

a the common speed of the particles after the string becomes taut,

b the loss of total kinetic energy due to the jerk.

Solution:



a Using conservation of momentum →:

Use conservation of momentum to find the common speed after the string becomes taut.

$2 \times 7 = 2v + 5v$

$= 7v$

$\therefore v = 2$

b K.E. before the jerk $= \frac{1}{2} \times 2 \times 7^2 = 49$ J

K.E. after the jerk $= \frac{1}{2} \times 5v^2 + \frac{1}{2} \times 2v^2$

$= \frac{7}{2} \times 2^2$

$= 14$ J

$\therefore$ Loss of K.E. due to the jerk $= 14$ J

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Exercise E, Question 15

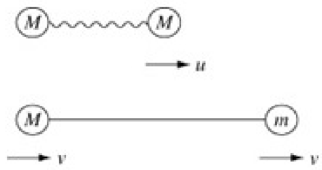
Question:

Two particles, A and B, of masses m and M respectively, are connected by a light inextensible string. The particles are side by side on a smooth floor and A is projected with speed u directly away from B. When the string becomes taut particle B is jerked into motion and A and B then move with a common speed in the direction of the original velocity of A.

Find the common speed of the particles after the string becomes taut, and show that the loss of total kinetic energy due to the jerk is $\frac{mMu^2}{2(m+M)}$.

Solution:

String slack:



After string becomes taut:

Let the speed after the string becomes taut be v.

Use conservation of momentum $\rightarrow$:

$\therefore mu = Mv + mv$

$\therefore mu = v(M+m)$

$\therefore v = \frac{mu}{M+m}$ ← This is the common speed required.

Energy before the jerk $= \frac{1}{2}mu^2$

Energy after the jerk $= \frac{1}{2}Mv^2 + \frac{1}{2}mv^2$

$$= \frac{1}{2}(M+m)\left[\frac{mu}{M+m}\right]^2$$

$$\therefore \text{Loss of energy} = \frac{1}{2}mu^2 - \frac{1}{2}\frac{m^2u^2}{(M+m)}$$

$$= \frac{1}{2}mu^2\left[1 - \frac{m}{M+m}\right]$$

$$= \frac{1}{2}mu^2\left[\frac{M}{M+m}\right]$$

$$= \frac{mMu^2}{2(m+M)} \text{ as required}$$

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Exercise E, Question 16

Question:

Two particles of masses 3 kg and 5 kg lie on a smooth table and are connected by a slack inextensible string. The first particle is projected along the table with a velocity of 20 m s^{-1} directly away from the second particle.

a Find the velocity of each particle after the string has become taut.

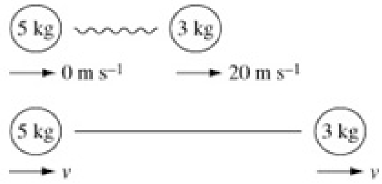
b Find the difference between the kinetic energies of the system when the string is slack and when it is taut.

The second particle is attached to a third particle of unknown mass by another slack string, and the velocity of the whole system after both strings have become taut is 6 m s^{-1}.

c Find the mass of the third particle.

Solution:

a String slack:



After string becomes taut:

Let the speed after the string becomes taut be v.

Using conservation of momentum $\rightarrow$:

Consider conservation of momentum before and after first string becomes taut.

$3\times 20 = 5v + 3v$

$\therefore 8v = 60$

$\therefore v = \frac{60}{8}$

$= 7.5$

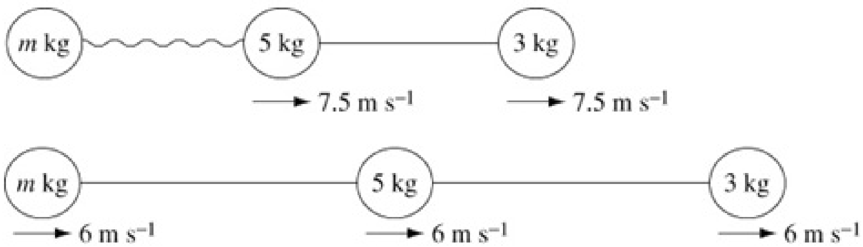
i.e. the common velocity is 7.5 m s^{-1}.

b Difference in K.E. $= \frac{1}{2}\times 3\times 20^2 - \left[\frac{1}{2}\times 5\times 7.5^2 + \frac{1}{2}\times 3\times 7.5^2\right]$

$= 600 - [225]$

$= 375\text{ J}$

c Second string slack.



After second string becomes taut

Let mass of third particle be m kg.

Use conservation of momentum:

Use conservation of momentum again before and after second string becomes taut.

$5\times 7.5 + 3\times 7.5 = 6m + 5\times 6 + 3\times 6$

$\therefore 60 = 6m + 48$

$\therefore 6m = 12$

$\therefore m = 2$

The mass of the third particle is 2 kg.

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Exercise E, Question 17

Question:

Three small spheres of mass 20 g, 40 g and 60 g respectively lie in order in a straight line on a large smooth table. The distance between adjacent spheres is 10 cm. Two slack strings, each 70 cm in length, connect the first sphere with the second, and the second sphere with the third. The 60 g sphere is projected with a speed of 5 m s^{-1}, directly away from the other two.

a Find the time which elapses before the 20 g sphere begins to move and the speed with which it starts.

b Find the loss in kinetic energy resulting from the two jerks.

Solution:

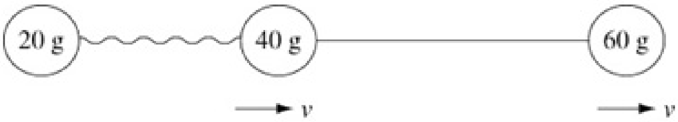
a

Stage 1: 60g particle moves 60 cm at 5 m s^{-1}

The particles are 10 cm apart. The strings are each 70 cm long. So the particle moves 60 cm at constant speed before the string becomes taut.

This takes time $t_1 = 0.6 \div 5 = 0.12\text{ s}$

Stage 2: 40 g mass is jerked into motion.



Let speed after jerk be v.

Use conservation of momentum.

$$\therefore 60 \times 5 = 40 \times v + 60 \times v$$
$$300 = 100v$$
$$\therefore v = 3$$

Stage 3: 60 g and 40 g particle move 60 cm at 3 m s^{-1}.

This takes time $t_2 = 0.6 \div 3 = 0.2\text{ s}$

Stage 4: 20 g mass is jerked into motion.

Let speed after jerk be w.

Use conservation of momentum.

$$\therefore 60 \times 3 + 40 \times 3 = 20w + 40w + 60w$$
$$300 = 120w$$
$$\therefore w = 2.5$$

i.e. after a time $t_1 + t_2 = 0.32\text{ s}$ the 20 g mass moves with a velocity 2.5 m s^{-1}.

b

$$\text{Final K.E.} = \tfrac{1}{2} \times 0.02w^2 + \tfrac{1}{2} \times 0.04w^2 + \tfrac{1}{2} \times 0.06w^2$$
$$= 0.06w^2$$
$$= 0.375\text{ J}$$
$$\text{Initial K.E.} = \tfrac{1}{2} \times 0.06 \times 5^2$$
$$= 0.75\text{ J}$$
$$\therefore \text{Loss in K.E.} = 0.75\text{ J} - 0.375\text{ J}$$
$$= 0.375\text{ J}$$

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Exercise F, Question 1

Question:

A cricket ball of mass 0.5 kg is struck by a bat. Immediately before being struck the velocity of the ball is $-25\mathbf{i}$ m s^{-1}. Immediately after being struck the velocity of the ball is $(23\mathbf{i}+20\mathbf{j})$ m s^{-1}. Find the magnitude of the impulse exerted on the ball by the bat and the angle between the impulse and the direction of $\mathbf{i}$.

Solution:

$$\begin{aligned}\text{Impulse} &= \text{change in momentum}\\ &= 0.5(23\mathbf{i}+20\mathbf{j})-0.5(-25\mathbf{i})\\ &= (24\mathbf{i}+10\mathbf{j})\text{ Ns}\end{aligned}$$

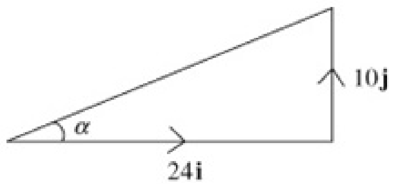
Use impulse = change in momentum.

∴ Magnitude of the impulse

$$= \sqrt{24^2+10^2}\text{ Ns}$$
$$= 26\text{ Ns}$$

Angle between the impulse and the direction $\mathbf{i}$ is α where

$$\tan\alpha = \frac{10}{24}$$
$$\therefore \alpha = 23^\circ \text{ (nearest degree)}$$



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Exercise F, Question 2

Question:

A ball of mass 0.2 kg is hit by a bat which gives it an impulse of $(2.4\mathbf{i} + 3.6\mathbf{j})$ Ns.

The velocity of the ball immediately after being hit is $(12\mathbf{i} + 5\mathbf{j})$ m s^{-1}. Find the velocity of the ball immediately before it is hit.

Solution:

Let velocity before being hit be $\mathbf{u}$ m s^{-1}.

Use impulse = change in momentum.

$$\text{impulse} = \text{change in momentum}$$
$$\therefore 2.4\mathbf{i} + 3.6\mathbf{j} = 0.2(12\mathbf{i} + 5\mathbf{j}) - 0.2\mathbf{u}$$
$$\therefore 0.2\mathbf{u} = 2.4\mathbf{i} + \mathbf{j} - 2.4\mathbf{i} - 3.6\mathbf{j}$$
$$= -2.6\mathbf{j}$$
$$\therefore \quad \mathbf{u} = -13\mathbf{j}$$

The velocity of the ball immediately before it is hit is $-13\mathbf{j}$ m s^{-1}.

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Exercise F, Question 3

Question:

A particle P of mass 0.3 kg is moving so that its position vector $\mathbf{r}$ metres at time t seconds is given by

$$\mathbf{r} = (t^3 + t^2 + 4t)\mathbf{i} + (11t)\mathbf{j}$$

a Calculate the speed of P when $t = 4$.

When $t = 4$, the particle is given an impulse $(2.4\mathbf{i} + 3.6\mathbf{j})$ Ns.

b Find the velocity of P immediately after the impulse.

Solution:

a $\mathbf{r} = (t^3 + t^2 + 4t)\mathbf{i} + 11t\mathbf{j}$

$\therefore \mathbf{v} = \dot{\mathbf{r}} = (3t^2 + 2t + 4)\mathbf{i} + 11\mathbf{j}$

Differentiate displacement vector to give the velocity vector.

When $t = 4, \mathbf{v} = (60)\mathbf{i} + 11\mathbf{j}$

the magnitude of $\mathbf{v} = \sqrt{60^2 + 11^2}$
$= 61$

$\therefore$ The speed when $t = 4$ is 61 m s^{-1}.

b Let the velocity immediately after the impulse be $\mathbf{V} \text{ m s}^{-1}$.

Then as impulse = change in momentum

$$\begin{aligned}
\therefore 2.4\mathbf{i} + 3.6\mathbf{j} &= 0.3\mathbf{V} - 0.3(60\mathbf{i} + 11\mathbf{j}) \\
\therefore 0.3\mathbf{V} &= 2.4\mathbf{i} + 3.6\mathbf{j} + 18\mathbf{i} + 3.3\mathbf{j} \\
&= 20.4\mathbf{i} + 6.9\mathbf{j} \\
\therefore \mathbf{V} &= 68\mathbf{i} + 23\mathbf{j}
\end{aligned}$$

Use impulse = change in momentum.

$\therefore$ velocity after the impulse is $(68\mathbf{i} + 23\mathbf{j}) \text{ m s}^{-1}$.

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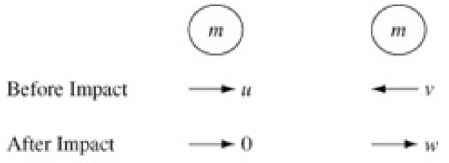
Exercise F, Question 4

Question:

Two identical spheres, moving in opposite directions, collide directly. As a result of the impact one of the spheres is brought to rest. The coefficient of restitution between the spheres is $\frac{1}{3}$. Show that the ratio of the speeds of the spheres before the impact is 2 : 1.

Solution:



$e = \frac{1}{3}$

Let the spheres each have mass m and let their speeds before the impact be $u \text{ m s}^{-1}$ and $v \text{ m s}^{-1}$ (towards each other).

Let the speeds after impact be 0 and $w \text{ m s}^{-1}$.

Using conservation of momentum $\rightarrow$:

Let the speeds before impact be u and v and after impact be 0 and w, then use conservation of momentum and the restitution equation.

$mu - mv = mw$

i.e. $u - v = w$ (1)

Using Newton's Law of Restitution:

$$e = \frac{1}{3} = \frac{w - 0}{u - (-v)}$$

$\therefore u + v = 3w$ (2)

Add (1) + (2)

$\therefore 2u = 4w \rightarrow u = 2w$

Substitute into (1)

$\therefore v = w$

$\therefore$ The ratio of the speeds before impact is $u : v = 2w : w = 2 : 1$ as required.

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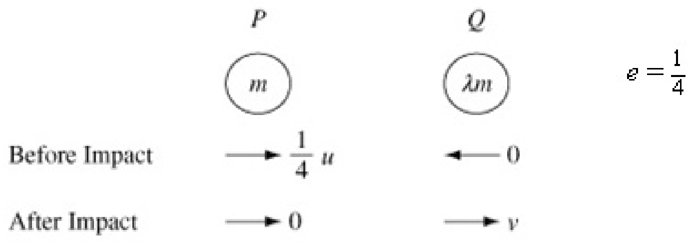
Exercise F, Question 5

Question:

A particle P of mass m is moving in a straight line with speed $\frac{1}{4}u$ at the instant when it collides directly with a particle Q of mass λm, which is at rest. The coefficient of restitution between P and Q is $\frac{1}{4}$. Given that P comes to rest immediately after hitting Q find the value of λ.

Solution:



Let the velocity of Q after impact be v.

Use conservation of linear momentum:

$$m\times\frac{1}{4}u+0=0+\lambda mv$$

$$\therefore \lambda v=\frac{1}{4}u \quad (1)$$

Use Newton's Law of Restitution:

$$e=\frac{1}{4}=\frac{v-0}{\frac{1}{4}u}$$

$$\therefore v=\frac{1}{16}u \quad (2)$$

Solving equations (1) and (2) $\lambda=4$

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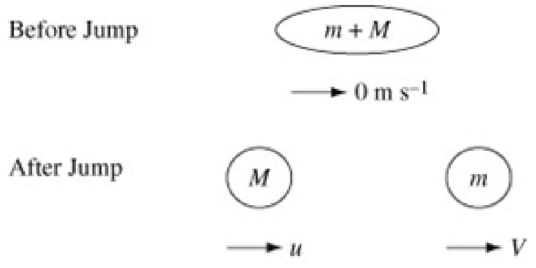
Exercise F, Question 6

Question:

A boy of mass m dives off a boat of mass M which was previously at rest. Immediately after diving off, the boy has a horizontal speed of V. Calculate the speed with which the boat begins to move. Prove that the total kinetic energy of the boy and the boat is $\frac{m(m+M)V^2}{2M}$.

Solution:



Let the boat move with velocity u.

Using conservation of linear momentum:

Use conservation of linear momentum to find the velocity of the boat.

$$Mu + mV = 0$$

$$\therefore u = -\frac{mV}{M}$$

(Note that the boat moves in the opposite direction to the boy.)

Total K.E. of boy and boat

$$= \frac{1}{2}mV^2 + \frac{1}{2}M\left(-\frac{mV}{M}\right)^2$$

$$= \frac{mMV^2 + m^2V^2}{2M}$$

$$= \frac{m(m+M)V^2}{2M}, \text{ as required}$$

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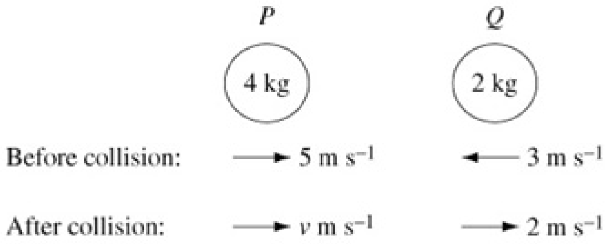
Exercise F, Question 7

Question:

Two spheres P and Q of equal radius and masses 4 kg and 2 kg respectively are travelling towards each other along a straight line on a smooth horizontal surface. Initially, P has a speed of 5 m s^{-1} and Q has a speed of 3 m s^{-1}. After the collision the direction of Q is reversed and it is travelling at a speed of 2 m s^{-1}. Find the speed of P after the collision and the loss of kinetic energy due to the collision.

Solution:



Use conservation of linear momentum:

$$4 \times 5 - 2 \times 3 = 4v + 2 \times 2$$
$$\therefore 4v = 10$$
$$\therefore v = 2.5 \text{ m s}^{-1}$$

Use conservation of linear momentum to find the speed of P after the collision.

$$\text{The total K.E. before collision} = \frac{1}{2} \times 4 \times 5^2 + \frac{1}{2} \times 2 \times (-3)^2$$
$$= 50 + 9$$
$$= 59 \text{ J}$$

$$\text{The total K.E. after collision} = \frac{1}{2} \times 4 \times 2.5^2 + \frac{1}{2} \times 2 \times 2^2$$
$$= 12.5 + 4$$
$$= 16.5 \text{ J}$$

$\therefore$ Loss of K.E. due to the collision is 42.5 J.

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Exercise F, Question 8

Question:

A body P of mass 4 kg is moving with velocity $(2\mathbf{i}+16\mathbf{j})$ m s^{-1} when it collides with a body Q of mass 3 kg moving with velocity $(-\mathbf{i}-8\mathbf{j})$ m s^{-1}. Immediately after the collision the velocity of P is $(-4\mathbf{i}-32\mathbf{j})$ m s^{-1}. Find the velocity of Q immediately after the collision.

Solution:

Let the velocity of Q after the collision be $\mathbf{v}$ m s^{-1}.

Use conservation of momentum: ← Use conservation of momentum.

$$4(2\mathbf{i}+16\mathbf{j})+3(-\mathbf{i}-8\mathbf{j}) = 4(-4\mathbf{i}-32\mathbf{j})+3\mathbf{v}$$
$$\therefore 5\mathbf{i}+40\mathbf{j} = -16\mathbf{i}-128\mathbf{j}+3\mathbf{v}$$
$$\therefore 3\mathbf{v} = 21\mathbf{i}+168\mathbf{j}$$
$$\therefore \mathbf{v} = 7\mathbf{i}+56\mathbf{j}$$

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Exercise F, Question 9

Question:

A particle P of mass $3m$ is moving in a straight line with speed u at the instant when it collides directly with a particle Q of mass m which is at rest. The coefficient of restitution between P and Q is e.

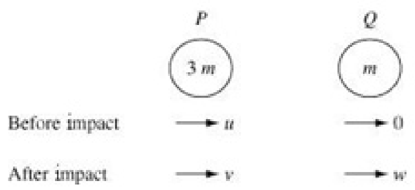
a Show that after the collision P is moving with speed $\frac{u(3-e)}{4}$.

b Show that the loss of kinetic energy due to the collision is $\frac{3mu^2(1-e^2)}{8}$.

c Find in terms of m, u and e the impulse exerted on Q by P in the collision.

Solution:

a



Let the velocities of P and Q be v and w respectively, after the impact.

Use conservation of momentum:

$$3mu + 0 = 3mv + mw$$
$$\therefore 3v + w = 3u \quad (1)$$

Use Newton's Law of Restitution:

$$e = \frac{w - v}{u - 0}$$
$$\therefore w - v = eu \quad (2)$$

Subtract $(1) - (2)$

$$\therefore 4v = 3u - eu$$
$$\therefore v = \frac{u(3-e)}{4}$$

Substitute into equation (2) to give

$$w = eu + \frac{u(3-e)}{4}$$
$$w = \frac{3u}{4}(1+e).$$

b Loss of K.E.

$$= \frac{1}{2} \times 3mu^2 - \frac{1}{2} \times 3mv^2 - \frac{1}{2}mw^2$$

Find the velocity of Q as well as the velocity of P so that you can calculate the total kinetic energy.

$$= \frac{m}{2}\left[3u^2 - 3\frac{u^2(3-e)^2}{16} - 9\frac{u^2(1+e)^2}{16}\right]$$
$$= \frac{3mu^2}{32}\left[16 - (9 - 6e + e^2) - (3 + 6e + 3e^2)\right]$$
$$= \frac{3mu^2}{32}\left[4 - 4e^2\right]$$
$$= \frac{3mu^2}{8}(1 - e^2)$$

c Impulse exerted on Q is change of momentum of Q

$$= \frac{3mu(1+e)}{4} \text{ Ns}.$$

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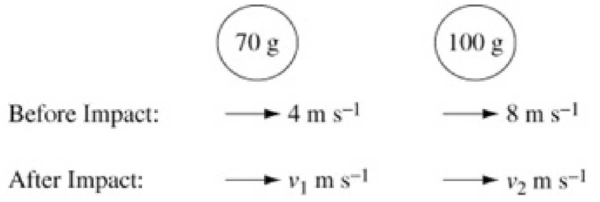
Exercise F, Question 10

Question:

Two spheres of mass 70 g and 100 g are moving along a straight line towards each other with velocities 4 m s^{-1} and 8 m s^{-1} respectively. Their coefficient of restitution is $\frac{5}{12}$. Find their velocities after impact and the amount of kinetic energy lost in the collision.

Solution:



Let the velocities after impact be $v_1 \text{ m s}^{-1}$ and $v_2 \text{ m s}^{-1}$.

Using conservation of momentum:

$$0.07\times 4 + 0.1\times(-8) = 0.07v_1 + 0.1v_2$$
$$\therefore 7v_1 + 10v_2 = -52 \quad (1)$$

Use conservation of momentum and Newton's Law of Restitution to find the velocities after the impact.

Using Newton's Law of Restitution:

$$e = \frac{5}{12} = \frac{v_2 - v_1}{4-(-8)}$$
$$\therefore v_2 - v_1 = 5 \quad (2)$$

Solving equation (1) and (2)

$$v_1 = -6 \text{ and } v_2 = -1$$

So the velocities after impact are 6 m s^{-1} and 1 m s^{-1} in the direction of the 100 g mass prior to the impact.

Loss of K.E.

$$= \frac{1}{2}\times 0.07\times 4^2 + \frac{1}{2}\times 0.1\times(-8)^2 - \left[\frac{1}{2}\times 0.07\times(-6)^2 + \frac{1}{2}\times 0.1\times(-1)^2\right]$$
$$= [0.56 + 3.2] - [1.26 + 0.05]$$
$$= 2.45 \text{ J}$$

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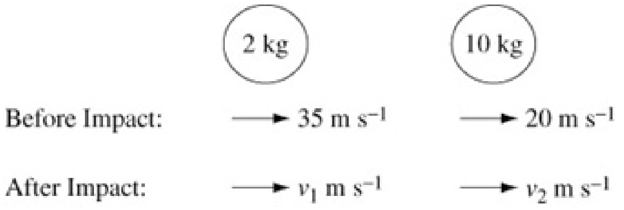
Exercise F, Question 11

Question:

A mass of 2 kg moving at 35 m s^{-1} catches up and collides with a mass of 10 kg moving in the same direction at 20 m s^{-1}. Five seconds after the impact the 10 kg mass encounters a fixed barrier which reduces it to rest. Assuming the coefficient of restitution between the masses is $\frac{3}{5}$, find the further time that will elapse before the 2 kg mass strikes the 10 kg mass again.

You may assume that the masses are moving on a smooth surface and have constant velocity between collisions.

Solution:



Using conservation of momentum $\rightarrow$:

$$2\times35+10\times20=2v_1+10v_2$$
$$\therefore 2v_1+10v_2=270$$
$$\text{or } v_1+5v_2=135 \quad (1)$$

Using Newton's Law of Restitution:

$$e=\frac{3}{5}=\frac{v_2-v_1}{35-20}$$
$$\therefore v_2-v_1=9 \quad (2)$$

Add equations $(1)+(2)$

$$6v_2=144$$
$$\therefore v_2=24$$

Substitute into (1) $\therefore v_1=15$

Five seconds after the impact the 10 kg mass has moved a distance $24\times5=120\text{ m}$.

It takes the 2 kg mass a time of $\frac{120}{15}$ to travel 120 m, i.e. 8 seconds.

$\therefore$ The *further* time required $=8\text{ s}-5\text{ s}=3\text{ s}$

After the impact, assume that the particles move at constant speed and use speed $\times$ time $=$ distance.

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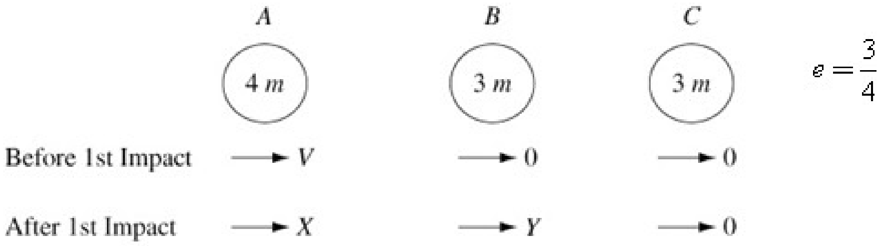
Exercise F, Question 12

Question:

Three balls A, B and C of masses $4m$, $3m$ and $3m$, respectively and of equal radius lie at rest on a smooth horizontal table with their centres in a straight line. Their coefficient of restitution is $\frac{3}{4}$. Show that if A is projected towards B with speed V there are three impacts and the final velocities are $\frac{5}{32}V, \frac{1}{4}V$ and $\frac{7}{8}V$ respectively.

Solution:



Use conservation of momentum for 1st impact:

First consider impact of A with B, then of B with C, then of A with B again.

$$4mV = 4mX + 3mY$$

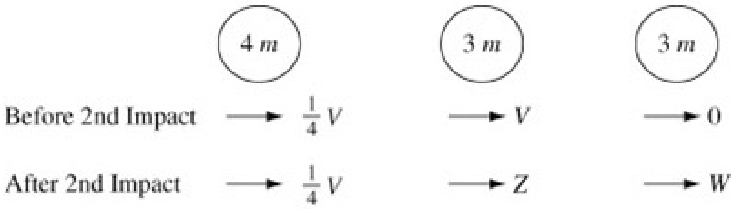
$\therefore 4X + 3Y = 4V \quad (1)$

Use Newton's Law of Restitution for 1st impact:

$$\frac{3}{4} = \frac{Y - X}{V}$$

$\therefore Y - X = \frac{3}{4}V \quad (2)$

Solve (1) and (2) to give $Y = V$, $X = \frac{1}{4}V$.



Use conservation of momentum for impact between B and C:

$$3mV = 3mZ + 3mW$$

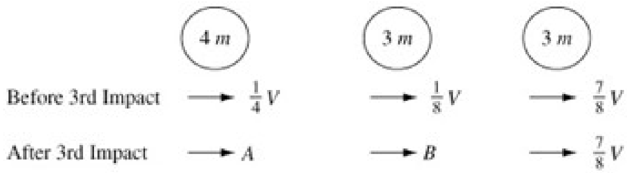
$\therefore Z + W = V \quad (3)$

Use Newton's Law of Restitution for this impact:

$$\frac{3}{4} = \frac{W - Z}{V}$$

$\therefore W - Z = \frac{3}{4}V \quad (4)$

Solve (3) and (4) to give $W = \frac{7}{8}V$, $Z = \frac{1}{8}V$.



Use conservation of momentum for 3rd impact between A and B.

$$4m \times \frac{1}{4}V + 3m \times \frac{1}{8}V = 4mA + 3mB$$

$$\therefore 4A + 3B = \frac{11}{8}V \quad (5)$$

Use Newton's Law of Restitution for this impact:

$$\frac{3}{4} = \frac{B - A}{\frac{1}{4}V - \frac{1}{8}V}$$

$$\therefore B - A = \frac{3}{32}V \quad (6)$$

Equation $(5) + 4 \times$ Equation $(6) \Rightarrow 7B = \frac{14}{8}V \Rightarrow B = \frac{1}{4}V$

Substitute into (6) give $A = \frac{1}{4}V - \frac{3}{32}V = \frac{5}{32}V$

$\therefore$ After 3 collisions the velocities are $\frac{5}{32}V, \frac{1}{4}V$ and $\frac{7}{8}V$ for the particles A, B and C respectively.

As $\frac{5}{32}V < \frac{1}{4}V < \frac{7}{8}V$ there are no further collisions.

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Exercise F, Question 13

Question:

A bullet of mass 60 g is fired horizontally at a fixed vertical metal barrier. The bullet hits the barrier when it is travelling at 600 m s^{-1} and then rebounds.

a Find the kinetic energy lost at the impact if $e = 0.4$.

b Give one possible form of energy into which the lost kinetic energy has been transformed.

Solution:

a Velocity of bullet after hitting the barrier

$$= 600 \times 0.4$$
$$= 240 \text{ m s}^{-1}.$$

Kinetic energy lost

$$= \frac{1}{2} \times 0.06 \times 600^2 - \frac{1}{2} \times 0.06 \times 240^2$$
$$= 9072 \text{ N}$$

b *Either* heat *or* sound.

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Exercise F, Question 14

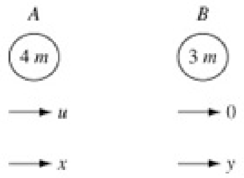
Question:

A particle A of mass $4m$ moving with speed u on a horizontal plane strikes directly a particle B of mass $3m$ which is at rest on the plane. The coefficient of restitution between A and B is e.

a Find, in terms of e and u, the speeds of A and B immediately after the collision.

b Given that the magnitude of the impulse exerted by A on B is $2mu$ show that $e = \frac{1}{6}$.

Solution:



a Let the speeds after the collision be x and y.

Use conservation of momentum:

$$4mu + 0 = 4mx + 3my$$
$$\therefore 4x + 3y = 4u \quad (1)$$

Use Newton's Law of Restitution:

$$e = \frac{y - x}{u}$$
$$\therefore y - x = eu \quad (2)$$

Add $(1) + 4 \times (2)$

$$\Rightarrow \quad 7y = 4u + 4eu$$
$$\therefore y = \frac{4}{7}u(1+e)$$

Substitute into (2)

$$\therefore x = \frac{4}{7}u(1+e) - eu$$
$$= \frac{4u}{7} - \frac{3ue}{7}$$
$$x = \frac{u}{7}(4 - 3e)$$

b Impulse = change in momentum of B

$$\therefore 2mu = 3m \times \frac{4}{7}u(1+e)$$
$$\therefore 1 + e = \frac{14}{12}$$
$$\therefore e = \frac{1}{6}$$

Use impulse = change in momentum of particle B.

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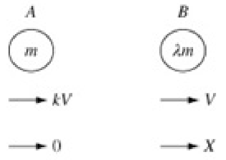
Exercise F, Question 15

Question:

A ball of mass m moving with speed kV on a smooth table hits another ball of mass λm moving with speed V travelling in the same direction on the table. The impact reduces the first ball to rest. Show that the coefficient of restitution is $\frac{\lambda+k}{\lambda(k-1)}$ and that λ must be greater than $\frac{k}{k-2}$ and k must be greater than 2.

Solution:



Let the speed of the second ball, after impact, be X.

Use conservation of momentum:

$$mkV + \lambda mV = \lambda mX$$

$$\therefore X = \frac{(\lambda + k)V}{\lambda} \quad *$$

Use Newton's Law of Restitution:

Let e = coefficient of restitution

Then $e = \dfrac{X}{kV - V}$

Substituting the value for X given in $*$:

$$e = \frac{(\lambda + k)V}{\lambda(k-1)V}$$

$$\therefore e = \frac{\lambda + k}{\lambda(k-1)}$$

As $e < 1$ $\therefore \dfrac{\lambda + k}{\lambda(k-1)} < 1$

$\therefore \lambda + k < \lambda k - \lambda$ (as $\lambda > 0$ and $k > 1$)

$\therefore 2\lambda + k < \lambda k$

$\lambda(k-2) > k$

Use the condition $e < 1$ to prove the inequalities.

As $k > 0$ and $\lambda > 0$, $\therefore k > 2$

$$\therefore \lambda > \frac{k}{k-2}$$

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Exercise F, Question 16

Question:

A ball is dropped from zero velocity and after falling for 1 s under gravity meets another equal ball which is moving upwards at 7 m s^{-1}.

a Taking the value of g as 9.8 m s^{-2}, calculate the velocity of each ball after the impact, given that the coefficient of restitution is $\frac{1}{4}$.

b Find the percentage loss in kinetic energy due to the impact, giving your answer to 2 significant figures.

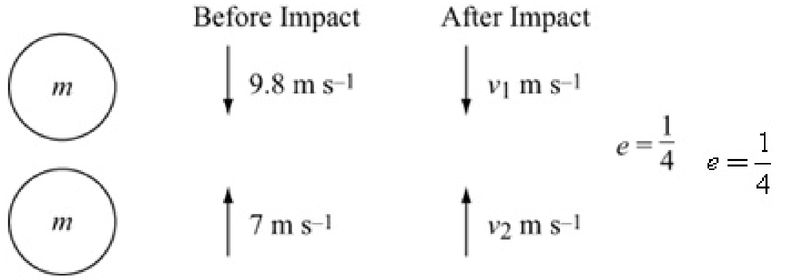
Solution:

a Find the velocity of the first ball before impact:

$u = 0, t = 1, a = 9.8, v = ?$

Use $v = u + at$
Then $v = 9.8$

Use constant acceleration formulae to find the velocity of the first ball prior to the impact.



Let mass of each ball be m.

Use conservation of momentum:

$9.8m - 7m = mv_1 + mv_2$

$\therefore v_1 + v_2 = 2.8 \quad (1)$

Use Newton's Law of Restitution:

$$e = \frac{1}{4} = \frac{v_2 - v_1}{9.8 - (-7)}$$

$\therefore \frac{1}{4} \times 16.8 = v_2 - v_1$

i.e. $v_2 - v_1 = 4.2 \quad (2)$

Add $(1) + (2)$

$\therefore 2v_2 = 7$

i.e. $v_2 = 3.5$

Substitute in (1)

$\therefore v_1 = -0.7$

Both balls change directions, the first moves up with speed 0.7 m s^{-1} and the second moves down with speed 3.5 m s^{-1}.

b

$$\text{K.E. before impact} = \frac{1}{2}m \times 9.8^2 + \frac{1}{2}m \times 7^2$$
$$= 72.52m \text{ J}$$
$$\text{K.E. after impact} = \frac{1}{2}m \times 0.7^2 + \frac{1}{2}m \times 3.5^2$$
$$= 6.37m \text{ J}$$
$$\therefore \%\text{loss in K.E.} = \frac{72.52 - 6.37}{72.52} = 91.2\%$$
$$= 91\% \text{ (2 s.f.)}$$

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Exercise F, Question 17

Question:

A particle falls from a height 8 m onto a fixed horizontal plane. The coefficient of restitution between the particle and the plane is $\frac{1}{4}$.

a Find the height to which the particle rises after impact.

b Find the time the particle takes from leaving the plane after impact to reach the plane again.

c What is the velocity of the particle after the second rebound?

You may leave your answers in terms of g.

Solution:

a Stage one: Particle falls under gravity ↓:

$u = 0, s = 8, a = g, v = ?$

Use constant acceleration formulae for the motion under gravity.

Use

$v^2 = u^2 + 2as$

$\therefore v^2 = 16g$

Stage two: First impact:

The particle rebounds with velocity $\frac{1}{4}\sqrt{16g}$.

Stage three: Particle moves under gravity ↑:

$u = \frac{1}{4}\sqrt{16g}, v = 0, s = ?, t = ?, a = -g$

Use

$v^2 = u^2 + 2as$

$\therefore 0 = \frac{1}{16} \times 16g - 2gs$

$\therefore s = 0.5\text{ m}$

i.e. the height to which particle rises is 0.5 m.

b Also use

$v = u + at$

$\therefore 0 = \frac{1}{4}\sqrt{16g} - gt$

$\therefore t = \frac{\sqrt{g}}{g} = 0.319$

$\therefore$ The time taken to reach the plane again

$= 2 \times 0.319$

$= 0.639\text{ s or } \frac{2}{\sqrt{g}}\text{ s}$

c The particle returns to the plane with velocity $\frac{1}{4}\sqrt{16g}$.

Stage four: Second impact

The velocity after the second rebound from the plane is $\frac{1}{4} \times \frac{1}{4}\sqrt{16g} = \frac{1}{4}\sqrt{g}\text{ m s}^{-1}$.

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Exercise F, Question 18

Question:

A particle falls from a height h onto a fixed horizontal plane. If e is the coefficient of restitution between the particle and the plane, show that the total time taken before the particle finishes bouncing is $\frac{1+e}{1-e} \times \sqrt{\frac{2h}{g}}$.

Solution:

Stage one: Particle falls under gravity ↓:

$u = 0, s = h, a = g, t = ?, v = ?$

Use

$$v^2 = u^2 + 2as$$

$$\therefore v^2 = 2gh$$

∴ Particle hits plane with velocity $\sqrt{2gh}$.

Use

$$s = ut + \frac{1}{2}at^2$$

$$\therefore h = \frac{1}{2}gt^2$$

$$\therefore t_1 = \sqrt{\frac{2h}{g}}$$ ← This is the time to the first bounce.

Stage two: Particle rebounds from plane.

The particle rebounds with velocity $e\sqrt{2gh}$.

Stage three: Particle moves under gravity until it hits the plane again ↑:

$u = e\sqrt{2gh}, s = 0, a = -g, t = ?, v = ?$

Use

$$s = ut + \frac{1}{2}at^2$$

$$\therefore 0 = e\sqrt{2gh}\,t - \frac{1}{2}gt^2$$

$$\therefore t = \frac{2e\sqrt{2gh}}{g}$$

$$\therefore t_2 = 2e\sqrt{\frac{2h}{g}}$$ ← This is time from first to second bounce.

and $v = -e\sqrt{2gh}$ ← This is velocity before second impact.

Stage four: Particle rebounds (again) from plane. It rebounds with velocity $e^2\sqrt{2gh}$.

By similar working you find that $t_3 = 2e^2\sqrt{\frac{2h}{g}}$ and $t_4 = 2e^3\sqrt{\frac{2h}{g}}, \ldots$ [Times between successive impacts form a G.P.]

$\therefore$ Total time taken by the particle is:

$$\sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + 2e^3\sqrt{\frac{2h}{g}} + \ldots$$
$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h}{g}}\left(e + e^2 + e^3 + \ldots\right)$$

The times between successive impacts form a geometric progression.

The expression in the bracket is a G.P. with $a = e$ and $r = e$. It is an infinite G.P. and $S_\infty = \frac{a}{1-r} = \frac{e}{1-e}$.

Use the formula $\frac{a}{1-r}$ to find S_∞ for a G.P.

$$\therefore \text{Total time} = \sqrt{\frac{2h}{g}}\left[1 + \frac{2e}{1-e}\right]$$
$$= \frac{1+e}{1-e} \times \sqrt{\frac{2h}{g}}$$

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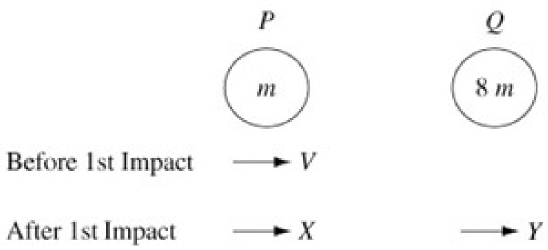
Exercise F, Question 19

Question:

A sphere P of mass m lies on a smooth table between a sphere Q of mass $8m$ and a fixed vertical plane. Sphere P is projected towards sphere Q. The coefficient of restitution between the two spheres is $\frac{7}{8}$. Given that sphere P is reduced to rest by the second impact with sphere Q find the coefficient of restitution between sphere P and the fixed vertical plane.

Solution:



Let initial velocity of P be V and use conservation of momentum and Newton's Law of Restitution to find velocities of P and Q after impact.

Let initial speed of P be V and let speeds of P and Q be X and Y after the 1st impact.

Use conservation of momentum:

$$\therefore mV = mX + 8mY$$

i.e. $X + 8Y = V$ (1)

Use Newton's Law of Restitution:

$$\frac{7}{8} = \frac{Y - X}{V}$$

$\therefore Y - X = \frac{7}{8}V$ (2)

Add equations (1) and (2)

$$\therefore 9Y = \frac{15}{8}V$$

i.e. $Y = \frac{5}{24}V$

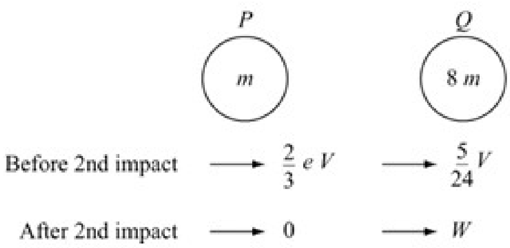
Substitute into (2)

$$\therefore X = \frac{5}{24}V - \frac{7}{8}V$$
$$= -\frac{2}{3}V$$

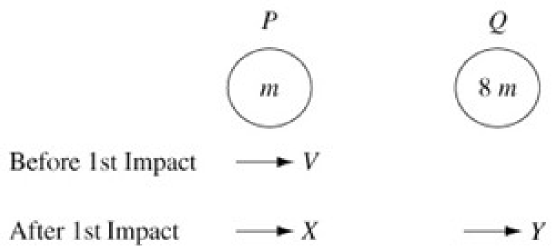
P then hits the vertical plane with speed $\frac{2}{3}V$ and rebounds with speed $\frac{2}{3}eV$.

Then consider impact of P with the vertical plane to find new velocity of P.

2nd impact between P and Q:



After the 2nd impact between P and Q let the velocity of Q be W and the velocity of P be 0.



Let initial velocity of P be V and use conservation of momentum and Newton's Law of Restitution to find velocities of P and Q after impact.

Use conservation of momentum:

$$m\times\frac{2}{3}eV+8m\times\frac{5}{24}V=8mW$$

$$\therefore 2eV+5V=24W \quad (1)$$

Use Newton's Law of Restitution:

$$\frac{7}{8}=\frac{W-0}{\frac{2}{8}eV-\frac{5}{24}V}$$

$$\therefore \frac{7}{8}\left(\frac{2}{8}eV-\frac{5}{24}V\right)=W$$

$$\text{i.e. } 14eV-\frac{35}{8}V=24W \quad (2)$$

Subtract $(1)-(2)$

$$\therefore -12eV+5V+\frac{35}{8}V=0$$

$$\therefore 12eV=\frac{75V}{8}$$

$$\therefore e=\frac{75}{96}$$

$$=\frac{25}{32}$$

Finally consider 2nd impact of P and Q.

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