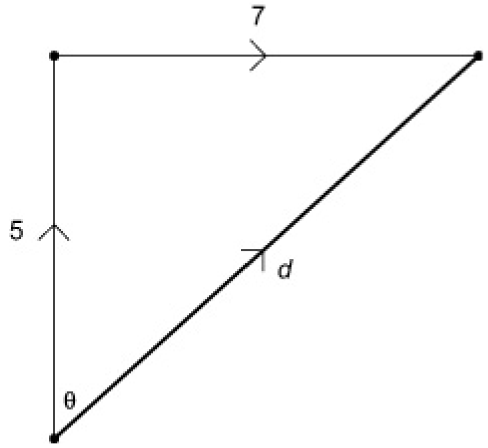
Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Vectors
Exercise A, Question 1

Question:

A bird flies 5 km due north and then 7 km due east. How far is the bird from its original position, and in what direction?

Solution:



$d = \sqrt{5^2 + 7^2} = \sqrt{25 + 49} = \sqrt{74} \approx 8.60 \text{ km}$

$\theta = \tan^{-1}\frac{7}{5} = \tan^{-1}1.4 = 54.46 \ldots {}^\circ$

The bird is 8.60 km (3 s.f.) from the starting point on a bearing of 054 ° (nearest degree).

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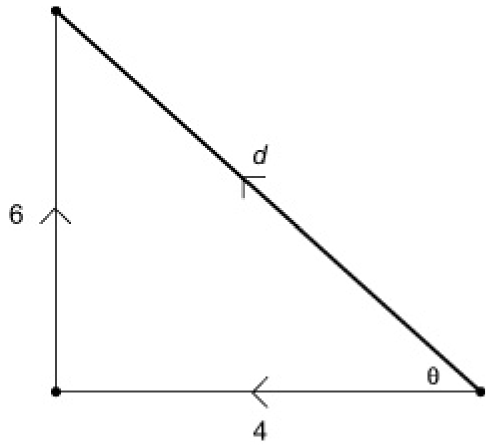
Vectors
Exercise A, Question 2

Question:

A girl cycles 4 km due west then 6 km due north. Calculate the total distance she has cycled and her displacement from her starting point.

Solution:



Distance cycled = 4 km + 6 km = 10 km

$d = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} \approx 7.2 \ldots$

$\theta = \tan^{-1} \frac{6}{4} = \tan^{-1} 1.5 = 56.3°$

bearing = 270° + 56° = 326°

The displacement is 7.2 km (3 s.f.) on a bearing of 326° (nearest degree).

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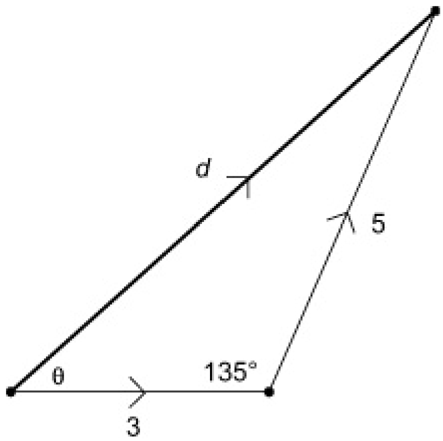
Vectors
Exercise A, Question 3

Question:

A man walks 3 km due east and then 5 km northeast. Find his distance and bearing from his original position.

Solution:



$d^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos 135° = 55.21 \ldots$

$d = 7.43$ km (3 s.f.)

$\frac{\sin \theta}{5} = \frac{\sin 135°}{d}$

$\sin \theta = \frac{5 \times \sin 135°}{d} = 0.476, \theta = 28.4°$ (3 s.f.)

$\Rightarrow$ bearing is $90° - 28° = 062°$ (nearest degree)

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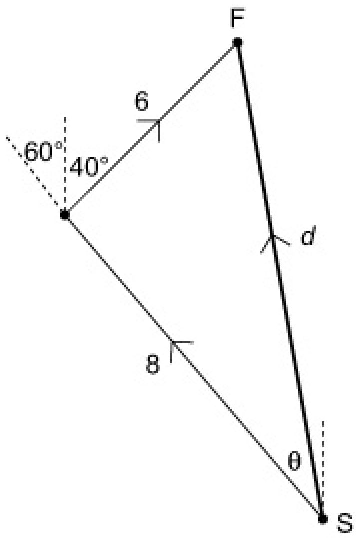
Vectors
Exercise A, Question 4

Question:

In an orienteering exercise, a team hike 8 km from the starting point, *S*, on a bearing of 300 ° then 6 km on a bearing 040 ° to the finishing point, *F*. Find the magnitude and direction of the displacement from *S* to *F*.

Solution:



$d^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 80° = 83.3 \ldots$

$d = 9.13$ km (3 s.f.)

$$\frac{\sin \theta}{6} = \frac{\sin 80°}{d}$$

$$\sin \theta = \frac{6 \times \sin 80°}{d} = 0.647 \ldots, \theta = 40.3° \text{ (3 s.f.)}$$

$\Rightarrow$ bearing is $300° + 40° = 340°$ (nearest degree)

$\Rightarrow$ the vector *SF* is 9.13 km (3 s.f.) on a bearing of 340 ° (nearest degree)

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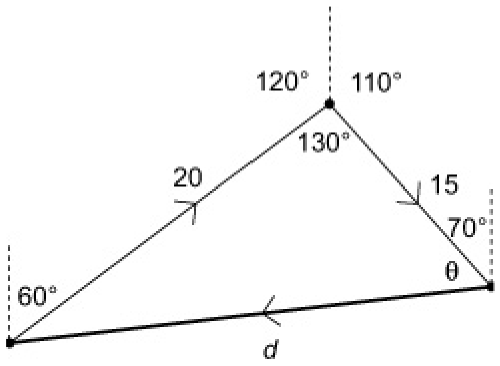
Vectors
Exercise A, Question 5

Question:

A boat travels 20 km on a bearing of 060 ° , followed by 15 km on a bearing of 110 ° . What course should it take to return to its starting point by the shortest route?

Solution:



$d^2 = 20^2 + 15^2 - 2 \times 20 \times 15 \times \cos 130°$
$= 1010.67 \ldots$

$d = 31.8$ km (3 s.f.)

$$\frac{\sin \theta}{20} = \frac{\sin 130°}{d}$$

$$\sin \theta = \frac{20 \times \sin 130°}{d} = 0.4819 \ldots , \theta = 28.8° \text{ (3 s.f.)}$$

⇒ bearing of the start $= 360° - 29° - 70° = 261°$ (nearest degree)

⇒ the return course is 31.8 km (3 s.f.) on a bearing of 261 ° (nearest degree)

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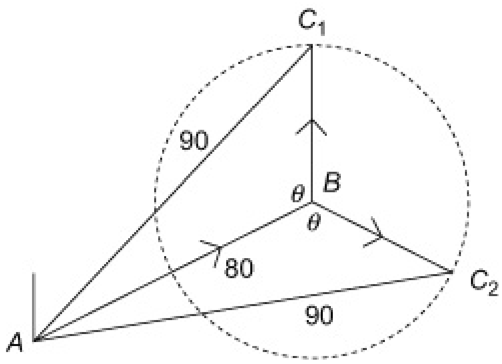
Vectors
Exercise A, Question 6

Question:

An aeroplane flies from airport A to airport B 80 km away on a bearing of 070 ° . From B the aeroplane flies to airport C, 60 km from B. Airport C is 90 km from A. Find the two possible directions for the course set by the aeroplane on the second stage of its journey.

Solution:



$\cos\theta = \frac{6^2 + 8^2 - 9^2}{2 \times 6 \times 8} = 0.19791\ldots \Rightarrow$ bearing

$\theta = 78.6°$ (3 s.f.)

of C from B is

$180° + 70° - \theta = 171.4°$ (1 d.p.) or $180° + 70° + \theta = 323.6°$ (1 d.p.)

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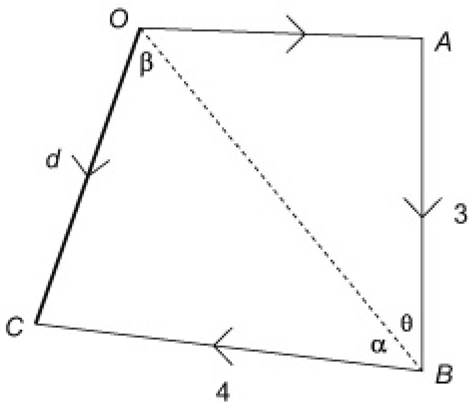
Vectors
Exercise A, Question 7

Question:

In a regatta, a yacht starts at point O, sails 2 km due east to A, 3 km due south from A to B, and then 4 km on a bearing of $280°$ from B to C. Find the displacement vector of C from O.

Solution:



Starting with the displacement OB, distance

$OB = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\theta = \tan^{-1} \frac{2}{3} = 33.7°$ (3 s.f.)

Now, looking at triangle OBC,

$\alpha = 80 - 33.7 = 46.3°$ (3 s.f.)

$d^2 = 4^2 + 13 - 2 \times 4 \times \sqrt{13} \times \cos 46.3° = 9.07 \ldots$

$d = 3.01$ km (3 s.f.)

$$\frac{\sin \beta}{4} = \frac{\sin 46.3°}{d}$$

$$\sin \beta = \frac{4 \times \sin 46.3°}{d} = 0.960 \ldots, \beta = 73.59°$$

$\Rightarrow$ bearing $= 90 + (90 - \theta) + \beta = 220°$ (nearest degree)

$\Rightarrow$ vector ie OC is 3.01 km (3 s.f.) on a bearing of $220°$ (nearest degree)

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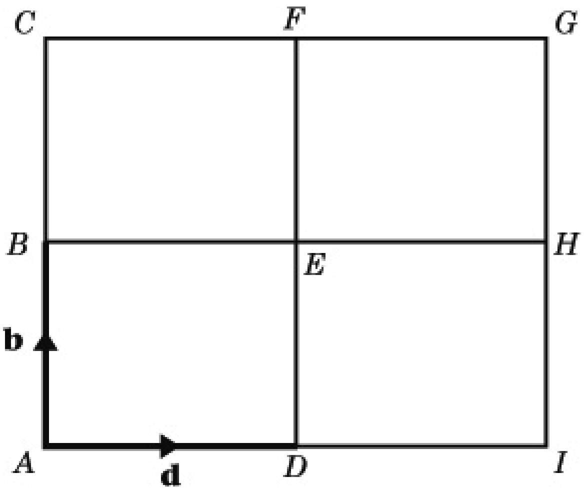
Vectors
Exercise B, Question 1

Question:



ACGI is a square, *B* is the mid-point of *AC*, *F* is the mid-point of *CG*, *H* is the mid-point of *GI*, and *D* is the mid-point of *AI*.

Vectors **b** and **d** are represented in magnitude and direction by *AB* and *AD* respectively. Find, in terms of **b** and **d**, the vectors represented in magnitude and direction by

a *AC*,	**b** *BE*,	**c** *HG*,	**d** *DF*,
e *AE*,	**f** *DH*,	**g** *HB*,	**h** *FE*,
i *AH*,	**j** *BI*,	**k** *EI*,	**l** *FB*.

Solution:

In this exercise there will usually be several correct routes to the answers because the addition law for vectors allows several options for equivalent vectors. You might reach the correct answers by a different routes to those used in these solutions.

a $AC = 2AB = 2b$

b $BE = AD$ (parallel and equal in length) $= d$

c $HG = BC$ (parallel and equal in length) $= AB$ (*B* is midpoint of *AC*) $= b$

d $DF = AC$ (parallel and equal in length) $= 2b$

e $AE = AD + DE$ (triangle law of addition)

$= AD + AB$ (*DE* and *AB* parallel and equal in length) $= d + b$

f $DH = DI + IH$ (triangle law of addition)

$= AD + AB$ ($AD = DI$ because *D* is the mid *F* point of *AI*, and *AB* is parallel and equal to *IH*)

$= d + b$

g $HB = -BH$ (same length, opposite direction)

$= -AI$ (parallel and equal in length) $= -2d$

h $FE = -EF$ (same length, opposite direction)

$= -HG$ (parallel and equal in length) $= -b$ (from part c)

i $AH = AI + IH$ (triangle law of addition) $= 2d + b$

j $BI = BA + AI = -AB + AI = -b + 2d$

k $EI = EB + BA + AI = -BE - AB + AI = -d - b + 2d = -b + d$

l $FB = FD + DA + AB = -DF - AD + AB = -2b - d + b = -b - d$

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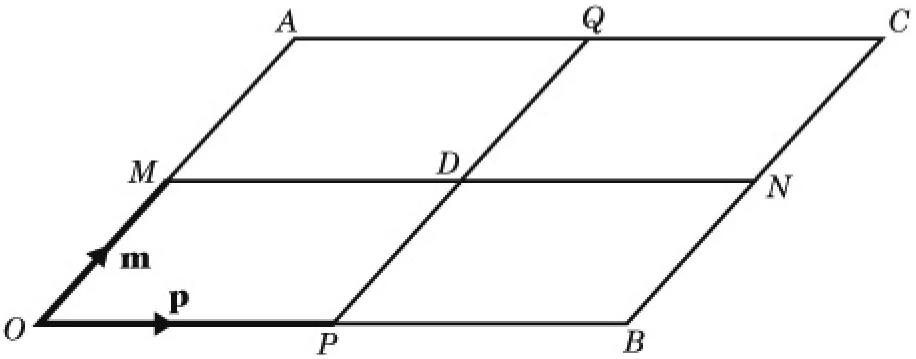
Vectors
Exercise B, Question 2

Question:



OACB is a parallelogram. *M*, *Q*, *N* and *P* are the mid-points of *OA*, *AC*, *BC* and *OB* respectively. Vectors **p** and **m** are equal to *OP* and *OM* respectively. Express in terms of **p** and **m**

a *OA*	**b** *OB*	**c** *BN*	**d** *DQ*
e *OD*	**f** *MQ*	**g** *OQ*	**h** *AD*
i *CD*	**j** *AP*	**k** *BM*	**l** *NO*.

Solution:

a *OA* = 2*OM* (*M* is the mid *F* point of *OA*) = 2*m*

b *OB* = 2*OP* (*P* is the mid *F* point of *OB*) = 2*p*

c $BN = \frac{1}{2}BC = \frac{1}{2}OA$ (opposite sides parallel and equal) = *m*

d *DQ* = *PD* (*MN* and *PQ* bisect each other)

= *OM* (line segments parallel and equal in length) = *m*

e *OD* = *OP* + *PD* (addition of vectors)

= *OP* + *OM* (*PD* and *OM* are parallel and equal in length) = *p* + *m*

f *MQ* = *MO* + *OP* + *PQ* (vector addition)

= − *OM* + *OP* + *OA* (*PQ* and *OA* are parallel and equal in length)

= − *m* + *p* + 2*m* = *p* + *m*

g *OQ* = *OP* + *PQ* = *p* + 2*m*

h *AD* = *AO* + *OD* (vector addition) = − *OA* + *OD* = − 2*m* + (*p* + *m*) = *p* − *m*

i *CD* = *CN* + *ND* (vector addition)

= *MO* + *PO* (line segments parallel and equal in length)

$= -OM + -OP = -m - p$

j $AP = AO + OP$ (vector addition) $= -OA + OP = -2m + p$

k $BM = BO + OM$ (vector addition) $= -OB + OM = -2p + m$

l $NO = NB + BO$ (vector addition)

$= MO + BO$ (MO and NB are parallel and equal in length)

$= -OM + -OB = -m - 2p$

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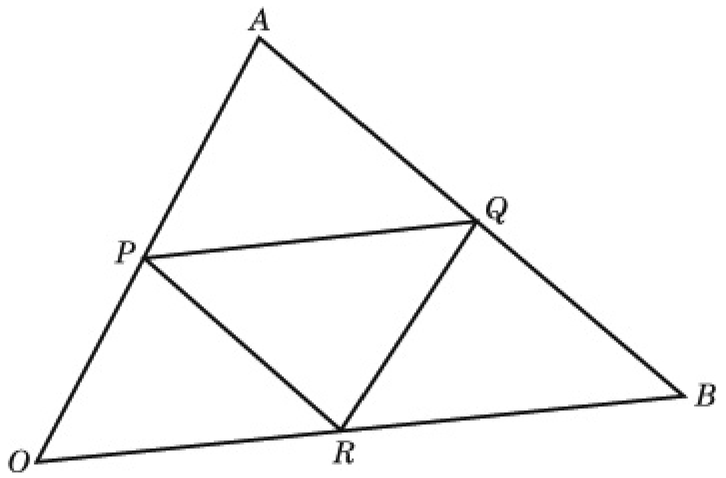
Vectors
Exercise B, Question 3

Question:



OAB is a triangle. *P*, *Q* and *R* are the mid-points of *OA*, *AB* and *OB* respectively. *OP* and *OR* are equal to **p** and **r** respectively. Find, in terms of **p** and **r**

a *OA*	**b** *OB*	**c** *AB*	**d** *AQ*
e *OQ*	**f** *PQ*	**g** *QR*	**h** *BP*.

Use parts **b** and **f** to prove that triangle *PAQ* is similar to triangle *OAB*.

Solution:

a $OA = 2OP$ (P is the mid F point of OA) $= 2p$

b $OB = 2OR$ (R is the mid F point of OB) $= 2r$

c $AB = AO + OB$ (addition of vectors) $= -OA + OB = -2p + 2r$

d $AQ = \frac{1}{2}AB$ (Q is the mid F point of AB) $= \frac{1}{2}\left(-2p + 2r\right) = -p + r$

e $OQ = OA + AQ$ (addition of vectors) $= 2p + (-p + r) = p + r$

f $PQ = PO + OQ$ (addition of vectors) $= -OP + OQ = -p + (p + r) = r$

g $QR = QO + OR$ (addition of vectors) $= -OQ + OR = -(p + r) + r = -p$

h $BP = BO + OP$ (addition of vectors) $= -OB + OP = -2r + p$

From **b** $OB = 2r$, and from **f** $PQ = r$,

$\Rightarrow$ OB and PQ are parallel

$\Rightarrow$ $\angle AOB = \angle APQ$ and $\angle ABO = \angle AQP$ (corresponding angles, parallel lines)

Angle A is common to both triangles

$\Rightarrow$ triangles PAQ and OAB are similar (three equal angles)

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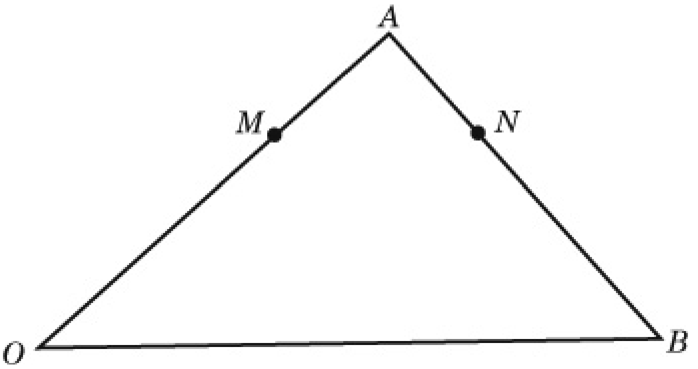
Vectors
Exercise B, Question 4

Question:



OAB is a triangle. $OA = a$ and $OB = b$. The point *M* divides *OA* in the ratio 2:1. *MN* is parallel to *OB*. Express the vector *ON* in terms of **a** and **b**.

Solution:

M divides *OA* in the ratio 2:1 $\Rightarrow OM = \frac{2}{3}a$

Using vector addition,

$ON = OA + AN = OA + \lambda AB$ (***N*** lies on ***AB***, so $AN = \lambda AB$) $= a + \lambda(-a + b)$ and $ON = OM + MN = OM + \mu OB$ (*MN* is parallel to *OB*) $= \frac{2}{3}a + \mu b$

$$\Rightarrow a + \lambda\left(-a + b\right) = \frac{2}{3}a + \mu b$$

$\Rightarrow$ (by comparing coefficients of **a** and **b**), $1 - \lambda = \frac{2}{3}$ and $\lambda = \mu$

so $\lambda = \mu = \frac{1}{3}$ and $ON = \frac{2}{3}a + \frac{1}{3}b$

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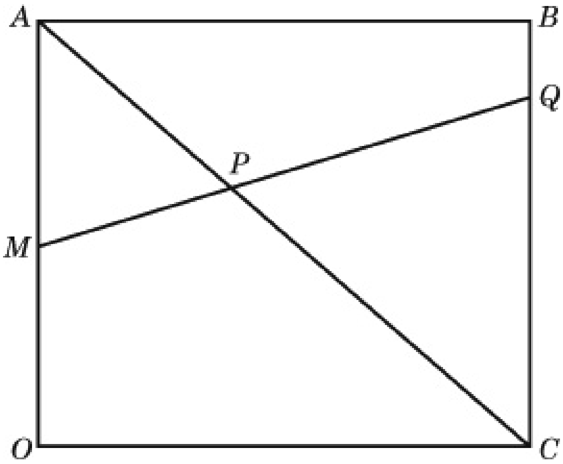
Vectors
Exercise B, Question 5

Question:



OABC is a square. *M* is the mid-point of *OA*, and *Q* divides *BC* in the ratio 1:3. *AP* and *MQ* meet at *P*. If *OA* = *a* and *OC* = *c*, express *OP* in terms of **a** and **c**.

Solution:

M is the mid *F* point of *OA*, so $OM = \frac{1}{2}OA = \frac{1}{2}a$

Using vector addition, $MQ = MA + AB + BQ$

$= MA + AB + \frac{1}{4}BC = \frac{1}{2}a + c - \frac{1}{4}a = \frac{1}{4}a + c$

and $AC = AO + OC = -a + c$

P lies on both *AC* and *MQ*, so

$OP = OM + \lambda MQ = \frac{1}{2}a + \lambda\left(\frac{1}{4}a + c\right)$

and $OP = OA + \mu AC = a + \mu(-a + c)$

$\Rightarrow \quad \frac{1}{2}a + \lambda\left(\frac{1}{4}a + c\right) = a + \mu\left(-a + c\right)$

by comparing coefficients of **a** and **c**, we get $\frac{1}{2} + \lambda\frac{1}{4} = 1 - \mu$ and $\lambda = \mu$

$\Rightarrow \frac{5}{4}\lambda = \frac{1}{2}, \lambda = \mu = \frac{2}{5}$ and $OP = \frac{3}{5}a + \frac{2}{5}b$

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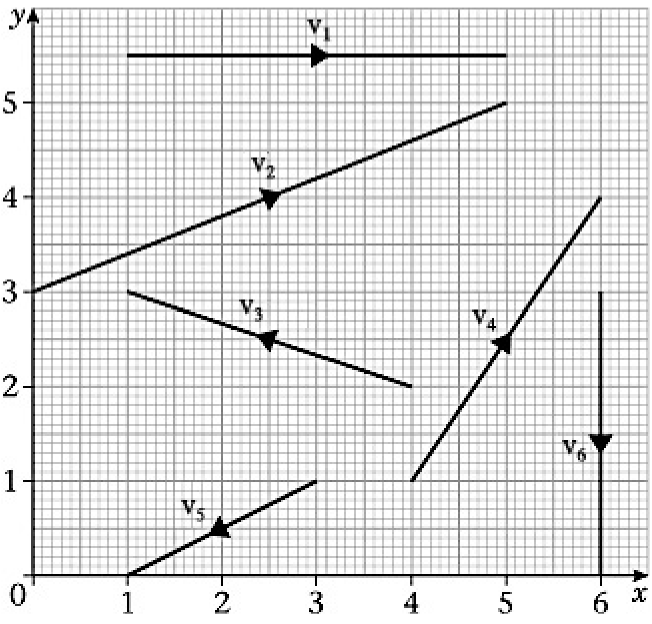
Vectors
Exercise C, Question 1

Question:

Express the vectors v_1 , v_2 , v_3 , v_4 , v_5 and v_6 using the **i**, **j** notation.



Solution:

$v_1 = 4i$, $v_2 = 5i + 2j$, $v_3 = -3i + j$, $v_4 = 2i + 3j$, $v_5 = -2i - j$, $v_6 = -3j$.

Note that some people prefer to describe vectors as 'column vectors'. In this case, the answers would be

$v_1 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, $v_4 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $v_5 = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$, $v_6 = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$.

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Edexcel AS and A Level Modular Mathematics

Vectors
Exercise D, Question 1

Question:

Given that $a = 2i + 3j$ and $b = 4i - j$, find these terms of **i** and **j**.

a $a + b$ **b** $3a + b$ **c** $2a - b$ **d** $2b + a$
e $3a - 2b$ **f** $b - 3a$ **g** $4b - a$ **h** $2a - 3b$

Solution:

a $a + b = (2i + 3j) + (4i - j) = (2 + 4)i + (3 - 1)j = 6i + 2j$

b $3a + b = 3(2i + 3j) + (4i - j) = (6i + 9j) + (4i - j) = (6 + 4)i + (9 - 1)j = 10i + 8j$

c $2a - b = 2(2i + 3j) - (4i - j) = (4i + 6j) - (4i - j) = (4 - 4)i + (6 - (-1))j = 7j$

d $2b + a = 2(4i - j) + (2i + 3j) = (8i - 2j) + (2i + 3j) = (8 + 2)i + (-2 + 3)j = 10i + j$

e

$$3a - 2b = 3(2i + 3j) - 2(4i - j) = (6i + 9j) - (8i - 2j) = (6 - 8)i + (9 + 2)j = -2i + 11j$$

f $b - 3a = (4i - j) - 3(2i + 3j) = (4i - j) - (6i + 9j) = (4 - 6)i + (-1 - 9)j = -2i - 10j$

g

$$4b - a = 4(4i - j) - (2i + 3j) = (16i - 4j) - (2i + 3j) = (16 - 2)i + (-4 + (-3))j = 14i - 7j$$

h

$$2a - 3b = 2(2i + 3j) - 3(4i - j) = (4i + 6j) - (12i - 3j) = (4 - 12)i + (6 - (-3))j = -8i + 9j$$

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Edexcel AS and A Level Modular Mathematics

Vectors
Exercise D, Question 2

Question:

Find the magnitude of each of these vectors.

a $3i + 4j$ **b** $6i - 8j$ **c** $5i + 12j$ **d** $2i + 4j$
e $3i - 5j$ **f** $4i + 7j$ **g** $-3i + 5j$ **h** $-4i - j$

Solution:

a $|3i + 4j| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

b $|6i - 8j| = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

c $|5i + 12j| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

d $|2i + 4j| = \sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} = 4.47$ (3 s.f.)

e $|3i - 5j| = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34} = 5.83$ (3 s.f.)

f $|4i + 7j| = \sqrt{4^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65} = 8.06$ (3 s.f.)

g $|-3i + 5j| = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34} = 5.83$ (3 s.f.)

h $|-4i - j| = \sqrt{4^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17} = 4.12$ (3 s.f.)

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Vectors
Exercise D, Question 3

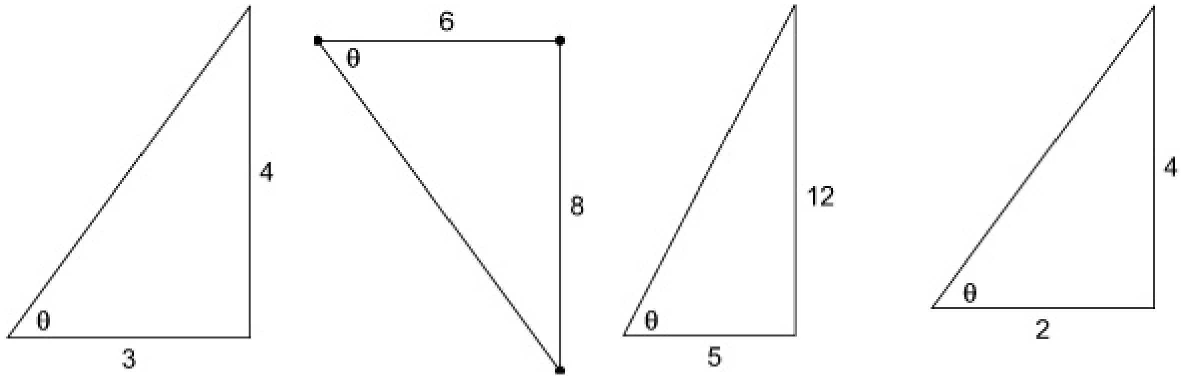
Question:

Find the angle that each of these vectors makes with the positive *x*-axis.

a $3i + 4j$ **b** $6i - 8j$ **c** $5i + 12j$ **d** $2i + 4j$

Solution:

a $3i + 4j$ **b** $6i - 8j$ **c** $5i + 12j$ **d** $2i + 4j$



$\arctan\left(\frac{4}{3}\right)$ $\arctan\left(\frac{8}{6}\right)$ $\arctan\left(\frac{12}{5}\right)$ $\arctan\left(\frac{4}{2}\right)$

$= 53.1°$ above (3 s.f.) $= 53.1°$ below (3 s.f.) $= 67.4°$ above (3 s.f.) $63.4°$ above (3 s.f.)

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Vectors
Exercise D, Question 4

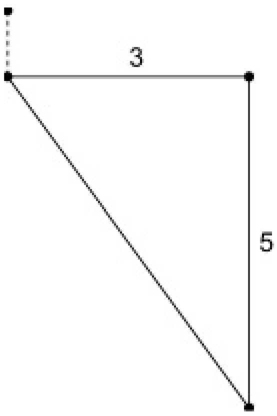
Question:

Find the angle that each of these vectors makes with the positive y-axis.

a $3i - 5j$ **b** $4i + 7j$ **c** $-3i + 5j$ **d** $-4i - j$

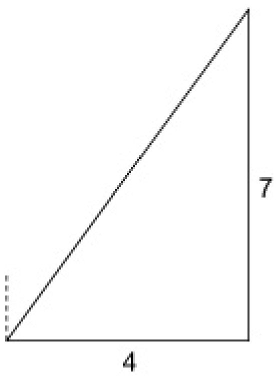
Solution:

a $3i - 5j$



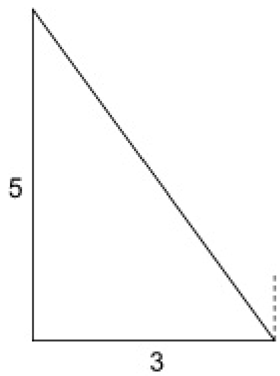
$90° + \arctan\left(\frac{5}{3}\right)$

b $4i + 7j$



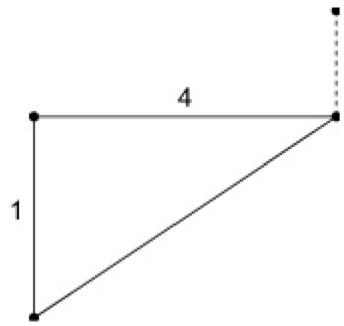
$\arctan\left(\frac{4}{7}\right)$

c $-3i + 5j$



$\arctan\left(\frac{3}{5}\right)$

d $-4i - j$



$90° + \arctan\left(\frac{1}{4}\right)$

$= 90° + 59° = 149°$ (3 s.f.) to the right $= 29.7°$ (3 s.f.) to the right $= 31.0°$ (3 s.f.) to the left $= 90° + 14° = 104°$ (3 s.f.) to the left

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Vectors
Exercise D, Question 5

Question:

Given that $a = 2i + 5j$ and $b = 3i - j$, find

a λ if $a + \lambda b$ is parallel to the vector **i**, **b** μ if $\mu a + b$ is parallel to the vector **j**,

Solution:

a $a + \lambda b = (2i + 5j) + \lambda (3i - j) = (2 + 3\lambda) i + (5 - \lambda) j$

Parallel to **i**, so $5 - \lambda = 0$, $\lambda = 5$.

b $\mu a + b = \mu (2i + 5j) + (3i - j) = (2\mu + 3) i + (5\mu - 1) j$

Parallel to **j**, so $2\mu + 3 = 0$, $\mu = \frac{-3}{2}$

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Vectors
Exercise D, Question 6

Question:

Given that $c = 3i + 4j$ and $d = i - 2j$, find

a λ if $c + \lambda d$ is parallel to $i + j$,
b μ if $\mu c + d$ is parallel to $i + 3j$,
c s if $c - sd$ is parallel to $2i + j$,
d t if $d - tc$ is parallel to $-2i + 3j$.

Solution:

a $c + \lambda d = (3i + 4j) + \lambda(i - 2j) = (3 + \lambda)i + (4 - 2\lambda)j$

Parallel to $i + j$, so $3 + \lambda = 4 - 2\lambda$

$3\lambda = 1$, $\lambda = \frac{1}{3}$

b $\mu c + d = \mu(3i + 4j) + (i - 2j) = (3\mu + 1)i + (4\mu - 2)j$

Parallel to $i + 3j$, so $4\mu - 2 = 3(3\mu + 1)$

$4\mu - 2 = 9\mu + 3$, $5\mu = -5$, $\mu = -1$

c $c - sd = (3i + 4j) - s(i - 2j) = (3 - s)i + (4 + 2s)j$

Parallel to $2i + j$, so $3 - s = 2(4 + 2s)$

$3 - s = 8 + 4s$, $-5 = 5s$, $s = -1$

d $d - tc = (i - 2j) - t(3i + 4j) = (1 - 3t)i + (-2 - 4t)j$

Parallel to $-2i + 3j$, so $-2(-2 - 4t) = 3(1 - 3t)$

$4 + 8t = 3 - 9t$, $1 = -17t$, $t = -\frac{1}{17}$

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Vectors
Exercise D, Question 7

Question:

In this question, the horizontal unit vectors **i** and **j** are directed due east and due north respectively.

Find the magnitude and bearing of these vectors.

a $2i + 3j$ **b** $4i - j$ **c** $-3i + 2j$ **d** $-2i - j$

Solution:

a $|2i + 3j| = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} = 3.61 \ (3 \text{ s.f.})$

$$\arctan\left(\frac{2}{3}\right) = 33.7°, \text{ so bearing} \approx 034°$$

b $|4i - j| = \sqrt{4^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17} = 4.12 \ (3 \text{ s.f.})$

$$\arctan\left(\frac{1}{4}\right) = 14.0°, \text{ so bearing} \approx 90° + 14° = 104°$$

c $|-3i + 2j| = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13} = 3.61 \ (3 \text{ s.f.})$

$$\arctan\left(\frac{2}{3}\right) = 33.7°, \text{ so bearing} \approx 270° + 34° = 304°$$

d $|-2i - j| = \sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5} = 2.24 \ (3 \text{ s.f.})$

$$\arctan\left(\frac{1}{2}\right) = 26.6°, \text{ so bearing} \approx 270° - 27° = 243°$$

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Vectors
Exercise E, Question 1

Question:

Find the speed of a particle moving with these velocities:

a $3i + 4j \text{ m s}^{-1}$

b $24i - 7j \text{ km h}^{-1}$

c $5i + 2j \text{ m s}^{-1}$

d $-7i + 4j \text{ cm s}^{-1}$

Solution:

a Speed = $|3i + 4j| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ m s}^{-1}$

b Speed = $|24i - 7j| = \sqrt{24^2 + (-7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25 \text{ km h}^{-1}$

c Speed = $|5i + 2j| = \sqrt{5^2 + 2^2} = \sqrt{25 + 4} = \sqrt{29} = 5.39 \text{ m s}^{-1}$ (3 s.f.)

d Speed = $|-7i + 4j| = \sqrt{(-7)^2 + 4^2} = \sqrt{49 + 16} = \sqrt{65} = 8.06 \text{ cm s}^{-1}$ (3 s.f.)

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Vectors
Exercise E, Question 2

Question:

Find the distance moved by a particle which travels for:

a 5 hours at velocity $8i + 6j \text{ km h}^{-1}$

b 10 seconds at velocity $5i - j \text{ m s}^{-1}$

c 45 minutes at velocity $6i + 2j \text{ km h}^{-1}$

d 2 minutes at velocity $-4i - 7j \text{ cm s}^{-1}$.

Solution:

a Distance = speed × time = $\sqrt{8^2 + 6^2} \times 5 = 5 \times \sqrt{64 + 36} = 5 \times \sqrt{100} = 50$ km

b Distance = speed × time = $\sqrt{5^2 + (-1)^2} \times 10 = 10 \times \sqrt{25 + 1} = 10 \times \sqrt{26} = 51.0$ m (3 s.f.)

c Distance = speed × time = $\sqrt{6^2 + 2^2} \times 0.75 = 0.75 \times \sqrt{36 + 4} = 0.75 \times \sqrt{40} = 4.74$ km (3 s.f.)

d Distance = speed × time = $\sqrt{(-4)^2 + (-7)^2} \times 120 = 120 \times \sqrt{16 + 49} = 120 \times \sqrt{65} = 967$ cm (3 s.f.)

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Vectors
Exercise E, Question 3

Question:

Find the speed and the distance travelled by a particle moving with:

a velocity $-3i + 4j \text{ m s}^{-1}$ for 15 seconds

b velocity $2i + 5j \text{ m s}^{-1}$ for 3 seconds

c velocity $5i - 2j \text{ km h}^{-1}$ for 3 hours

d velocity $12i - 5j \text{ km h}^{-1}$ for 30 minutes.

Solution:

a Speed $= \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ m s}^{-1}$,

Distance $= 5 \times 15 = 75$ m

b Speed $= \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29} = 5.39 \text{ m s}^{-1}$ (3 s.f.)

Distance $= 3 \times 5.39 = 16.2$ m (3 s.f.)

c Speed $= \sqrt{5^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29} = 5.39 \text{ km h}^{-1}$ (3 s.f.)

Distance $= 3 \times 5.39 = 16.2$ km (3 s.f.)

d Speed $= \sqrt{12^2 + (-5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ km h}^{-1}$,

Distance $= 0.5 \times 13 = 6.5$ km

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Vectors
Exercise F, Question 1

Question:

A particle P is moving with constant velocity v m s^{-1}. Initially P is at the point with position vector **r**. Find the position of P
t seconds later if:

a $r_0 = 3j$, $v = 2i$ and $t = 4$,
b $r_0 = 2i - j$, $v = -2j$ and $t = 3$,
c $r_0 = i + 4j$, $v = -3i + 2j$ and $t = 6$,
d $r_0 = -3i + 2j$, $v = 2i - 3j$ and $t = 5$.

Solution:

a Using $r = r_o + vt, r = 3j + 2i \times 4 = 8i + 3j$

b Using $r = r_o + vt, r = (2i - j) + (-2j) \times 3 = 2i - j - 6j = 2i - 7j$

c Using $r = r_o + vt, r = (i + 4j) + (-3i + 2j) \times 6 = i + 4j - 18i + 12j = -17i + 16j$

d Using $r = r_o + vt, r = (-3i + 2j) + (2i - 3j) \times 5 = -3i + 2j + 10i - 15j = 7i - 13j$

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Vectors
Exercise F, Question 2

Question:

A particle P moves with constant velocity **v**. Initially P is at the point with position vector **a**. t seconds later P is at the point with position vector **b**. Find **v** when:

a $a = 2i + 3j$, $b = 6i + 13j$, $t = 2$,

b $a = 4i + j$, $b = 9i + 16j$, $t = 5$,

c $a = 3i - 5j$, $b = 9i + 7j$, $t = 3$,

d $a = -2i + 7j$, $b = 4i - 8j$, $t = 3$,

e $a = -4i + j$, $b = -12i - 19j$, $t = 4$.

Solution:

a Using $r = r_o + vt$, $(6i + 13j) = (2i + 3j) + 2v$, $2v = (6i + 13j) - (2i + 3j) = 4i + 10j$
$v = 2i + 5j$

b Using $r = r_o + vt$, $(9i + 16j) = (4i + j) + 5v$, $5v = (9i + 16j) - (4i + j) = 5i + 15j$
$v = i + 3j$

c Using $r = r_o + vt$, $(9i + 7j) = (3i - 5j) + 3v$, $3v = (9i + 7j) - (3i - 5j) = 6i + 12j$
$v = 2i + 4j$

d Using $r = r_o + vt$, $(4i - 8j) = (-2i + 7j) + 3v$, $3v = (4i - 8j) - (-2i + 7j) = 6i - 15j$
$v = 2i - 5j$

e Using $r = r_o + vt$, $(-12i - 19j) = (-4i + j) + 4v$, $4v = (-12i - 19j) - (-4i + j) = -8i - 20j$
$v = -2i - 5j$

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Vectors
Exercise F, Question 3

Question:

A particle moving with speed $v \text{ m s}^{-1}$ in direction **d** has velocity vector **v**. Find **v** for these.

a $v = 10$, $d = 3i - 4j$
b $v = 15$, $d = -4i + 3j$
c $v = 7.5$, $d = -6i + 8j$
d $v = 5\sqrt{2}$, $d = i + j$
e $v = 2\sqrt{13}$, $d = -2i + 3j$
f $v = \sqrt{68}$, $d = 3i - 5j$
g $v = \sqrt{60}$, $d = -4i - 2j$
h $v = 15$, $d = -i + 2j$

Solution:

a $|d| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5, 10 \div 5 = 2, v = 2(3i - 4j) = 6i - 8j$

b $|d| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5, 15 \div 5 = 3, v = 3(-4i + 3j) = -12i + 9j$

c $\left| d \right| = \sqrt{(-6)^2 + 8^2} = \sqrt{100} = 10, 7.5 \div 10 = \frac{3}{4}, v = \frac{3}{4}\left(-6i + 8j\right) = -4.5i + 6j$

d $|d| = \sqrt{1^2 + 1^2} = \sqrt{2}, 5\sqrt{2} \div \sqrt{2} = 5, v = 5(i + j) = 5i + 5j$

e $|d| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}, 2\sqrt{13} \div \sqrt{13} = 2, v = 2(-2i + 3j) = -4i + 6j$

f $|d| = \sqrt{3^2 + (-5)^2} = \sqrt{34}, \sqrt{68} \div \sqrt{34} = \sqrt{2}, v = \sqrt{2}(3i - 5j) = 3\sqrt{2}i - 5\sqrt{2}j$

g $|d| = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}, \sqrt{60} \div \sqrt{20} = \sqrt{3}, v = \sqrt{3}(-4i - 2j) = -4\sqrt{3}i - 2\sqrt{3}j$

h $|d| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}, 15 \div \sqrt{5} = 3\sqrt{5}, v = 3\sqrt{5}(-i + 2j) = -3\sqrt{5}i + 6\sqrt{5}j$

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Vectors
Exercise F, Question 4

Question:

A particle P starts at the point with position vector r_0. P moves with constant velocity $v\,\text{m s}^{-1}$. After t seconds, P is at the point with position vector **r**.

a Find **r** if $r_0 = 2i$, $v = i + 3j$, and $t = 4$.

b Find **r** if $r_0 = 3i - j$, $v = -2i + j$, and $t = 5$.

c Find r_0 if $r = 4i + 3j$, $v = 2i - j$, and $t = 3$.

d Find r_0 if $r = -2i + 5j$, $v = -2i + 3j$, and $t = 6$.

e Find **v** if $r_0 = 2i + 2j$, $r = 8i - 7j$, and $t = 3$.

f Find the speed of P if $r_0 = 10i - 5j$, $r = -2i + 9j$, and $t = 4$.

g Find t if $r_0 = 4i + j$, $r = 12i - 11j$, and $v = 2i - 3j$.

h Find t if $r_0 = -2i + 3j$, $r = 6i - 3j$, and the speed of P is $4\,\text{m s}^{-1}$.

Solution:

a Using $r = r_o + vt, r = (2i) + (i + 3j) \times 4 = 2i + 4i + 12j = 6i + 12j$

b Using $r = r_o + vt, r = (3i - j) + (-2i + j) \times 5 = 3i - j - 10i + 5j = -7i + 4j$

c Using $r = r_o + vt, (4i + 3j) = r_o + (2i - j) \times 3, r_o = (4i + 3j) - (6i - 3j) = 4i + 3j - 6i + 3j = -2i + 6j$

d Using $r = r_o + vt, (-2i + 5j) = r_o + (-2i + 3j) \times 6, r_o = (-2i + 5j) - (-12i + 18j)$
$= -2i + 5j + 12i - 18j = 10i - 13j$

e Using $r = r_o + vt, (8i - 7j) = (2i + 2j) + v \times 3, 3v = (8i - 7j) - (2i + 2j) = 6i - 9j$
$v = 2i - 3j$

f Using $r = r_o + vt, (-2i + 9j) = (10i - 5j) + v \times 4, 4v = (-2i + 9j) - (10i - 5j) = -12i + 14j$ $v = -3i + 3.5j$,

$$\text{speed} = \sqrt{(-3)^2 + 3.5^2} = \sqrt{21.25} \approx 4.61\ \text{m s}^{-1}$$

g Using $r = r_o + vt, (12i - 11j) = (4i + j) + (2i - 3j) \times t, (2i - 3j) \times t = (12i - 11j) - (4i + j)$
$= 8i - 12j, t = 4$

h Using $r = r_o + vt, (6i - 3j) = (-2i + 3j) + vt, vt = (6i - 3j) - (-2i + 3j) = 8i - 6j$
$4t = |vt|\ 4t = |8i - 6j| = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10, 4t = 10, 4\ t = 2.5$

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Vectors
Exercise F, Question 5

Question:

The initial velocity of a particle P moving with uniform acceleration $a\,\text{m s}^{-2}$ is $u\,\text{m s}^{-1}$. Find the velocity and the speed of P after t seconds in these cases.

a $u = 5i$, $a = 3j$, and $t = 4$

b $u = 3i - 2j$, $a = i - j$, and $t = 3$

c $a = 2i - 3j$, $u = -2j + j$, and $t = 2$

d $t = 6$, $u = 3i - 2j$, and $a = -i$

e $a = 2i + j$, $t = 5$, and $u = -3i + 4j$

Solution:

a Using $v = u + at$, $v = (5i) + (3j) \times 4 = 5i + 12j$

speed $= \sqrt{5^2 + 12^2} = \sqrt{169} = 13\text{ m s}^{-1}$

b Using $v = u + at$, $v = (3i - 2j) + (i - j) \times 3 = 3i - 2j + 3i - 3j = 6i - 5j$

speed $= \sqrt{6^2 + (-5)^2} = \sqrt{36 + 25} = \sqrt{61} \approx 7.81\text{ m s}^{-1}$

c Using $v = u + at$, $v = (-2i + j) + (2i - 3j) \times 2 = -2i + j + 4i - 6j = 2i - 5j$

speed $= \sqrt{2^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29} \approx 5.39\text{ m s}^{-1}$

d Using $v = u + at$, $v = (3i - 2j) + (-i) \times 6 = 3i - 2j - 6i = -3i - 2j$

speed $= \sqrt{(-3)^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13} \approx 3.61\text{ m s}^{-1}$

e Using $v = u + at$, $v = (-3i + 4j) + (2i + j) \times 5 = -3i + 4j + 10i + 5j = 7i + 9j$

speed $= \sqrt{7^2 + 9^2} = \sqrt{49 + 81} = \sqrt{130} \approx 11.4\text{ m s}^{-1}$

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Vectors
Exercise F, Question 6

Question:

A constant force **F** N acts on a particle of mass 4 kg for 5 seconds. The particle was initially at rest, and after 5 seconds it has velocity $6i - 8j \text{ m s}^{-1}$. Find **F**.

Solution:

Using $v = u + at$, $\left(6i - 8j \right) = a \times 5, a = \frac{1}{5} \left(6i - 8j \right)$

Using $F = ma, F = 4 \times \frac{1}{5} \left(6i - 8j \right) = 4.8i - 6.4j$

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Vectors

Exercise F, Question 7

Question:

A force $2i - j$ N acts on a particle of mass 2 kg. If the initial velocity of the particle is $i + 3j$ m s^{-1}, find how far it moves in the first 3 seconds.

Solution:

Using $F = ma$, $\left(2i - j\right) = 2a, a = i - \frac{1}{2}j$

Using $s = ut + \frac{1}{2}at^2, s = \left(i + 3j\right) \times 3 + \frac{1}{2}\left(i - \frac{1}{2}j\right) \times 3^2 = 3i + 9j + 4\frac{1}{2}i - 2\frac{1}{4}j = 7\frac{1}{2}i + 6\frac{3}{4}j$

$$\text{distance} = \sqrt{\left(7\frac{1}{2}\right)^2 + \left(6\frac{3}{4}\right)^2} = \sqrt{56.25 + 45.5625} = \sqrt{101.8125} \approx 10.1 \text{ m}$$

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Vectors
Exercise F, Question 8

Question:

At time $t = 0$, the particle P is at the point with position vector $4\mathbf{i}$, and moving with constant velocity $i + j\ \text{m s}^{-1}$. A second particle Q is at the point with position vector $-3\mathbf{j}$ and moving with velocity $v\ \text{m s}^{-1}$. After 8 seconds, the paths of P and Q meet. Find the speed of Q.

Solution:

Using $r = r_o + vt$ for P, $r = (4i) + (i + j) \times 8 = 4i + 8i + 8j = 12i + 8j$

Using $r = r_o + vt$ for Q, $r = (-3j) + v \times 8$

Both at the same point: $12i + 8j = (-3j) + v \times 8, 8v = 12i + 8j + 3j = 12i + 11j$

$$v = \frac{1}{8}\left(12i + 11j\right) = 1.5i + 1.375j$$

$$\text{speed} = |v| = \sqrt{1.5^2 + 1.375^2} = \sqrt{2.25 + 1.890625} \approx 2.03\ \text{m s}^{-1}$$

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Vectors
Exercise F, Question 9

Question:

In questions 9 and 10 the unit vectors **i** and **j** are due east and due north respectively.

At 2 pm the coastguard spots a rowing dinghy 500 m due south of his observation point. The dinghy has constant velocity $(2i + 3j)\ \mathrm{m\,s}^{-1}$.

a Find, in terms of *t*, the position vector of the dinghy *t* seconds after 2 pm.

b Find the distance of the dinghy from the observation point at 2.05 pm.

Solution:

a Using $r = r_o + vt, r = -500j + (2i + 3j) \times t = -500j + 2ti + 3tj = 2ti + (-500 + 3t)j$

b 5 minutes = 5 × 60 seconds = 300 seconds, $r = 2 \times 300i + (-500 + 3 \times 300)j = 600i + 400j$

$$\text{distance} = \sqrt{600^2 + 400^2} = \sqrt{360\,000 + 160\,000} = \sqrt{520\,000} \approx 721 \text{ m}$$

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Vectors
Exercise F, Question 10

Question:

At noon a ferry F is 400 m due north of an observation point O moving with constant velocity $(7i + 7j)$ m s^{-1}, and a speedboat S is 500 m due east of O, moving with constant velocity $(-3i + 15j)$ m s^{-1}.

a Write down the position vectors of F and S at time t seconds after noon.

b Show that F and S will collide, and find the position vector of the point of collision.

Solution:

a

Using $r = r_o + vt$ for F, $r = 400j + (7i + 7j) \times t = 400j + 7ti + 7tj$
$= 7ti + (400 + 7t)j$

Using $r = r_o + vt$ for S, $r = 500i + (-3i + 15j) \times t = 500i - 3ti + 15tj$
$= (500 - 3t)i + 15tj$

b For F and S to collide, $7ti + (400 + 7t)j = (500 - 3t)i + 15tj$,

i components equal: $7t = 500 - 3t, 10t = 500, t = 50$

j components equal: $400 + 7t = 15t, 400 = 8t, t = 50$

Both conditions give the same value of t, so the two position vectors are equal when $t = 50$, i.e. F and S collide at $r = 7 \times 50i + (400 + 7 \times 50)j = 350i + 750j$.

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Vectors
Exercise F, Question 11

Question:

At 8 am two ships A and B are at $r_A = (i + 3j)$ km and $r_B = (5i - 2j)$ km from a fixed point P. Their velocities are $v_A = (2i - j)$ km h^{-1} and $v_B = (-i + 4j)$ km h^{-1} respectively.

a Write down the position vectors of A and B t hours later.

b Show that t hours after 8 am the position vector of B relative to A is given by $((4 - 3t)i + (-5 + 5t)j)$ km .

c Show that the two ships do not collide.

d Find the distance between A and B at 10 am.

Solution:

a Using $r = r_o + vt$ for $A, r = (i + 3j) + (2i - j) \times t = (1 + 2t)i + (3 - t)j$

Using $r = r_o + vt$ for $B, r = (5i - 2j) + (-i + 4j) \times t = (5 - t)i + (-2 + 4t)j$

b Using $AP + PB, AB = PB - PA$

$$= \{ (5 - t)i + (-2 + 4t)j \{ - \{ (1 + 2t)i + (3 - t)j \{$$
$$= (5 - t - 1 - 2t)i + (-2 + 4t - 3 + t)j = (4 - 3t)i + (-5 + 5t)j$$

c If A and B collide, the vector $\boldsymbol{AB}$ would be zero,

so $4 - 3t = 0$ and $-5 + 5t = 0$, but these two equations are not consistent ($t = 1$ and $t \neq 1$), so vector $\boldsymbol{AB}$ can never be zero, and A and B will not collide.

d At 10 am, $t = 2, AB = (4 - 3 \times 2)i + (-5 + 5 \times 2)j = -2i + 5j$,

Distance $= \sqrt{(-2)^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29} \approx 5.39$ km

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Vectors
Exercise F, Question 12

Question:

A particle A starts at the point with position vector $12i + 12j$. The initial velocity of A is $(-i + j)\ \text{m s}^{-1}$, and it has constant acceleration $(2i - 4j)\ \text{m s}^{-2}$. Another particle, B, has initial velocity $i\ \text{m s}^{-1}$ and constant acceleration $2j\ \text{m s}^{-2}$. After 3 seconds the two particles collide. Find

a the speeds of the two particles when they collide,

b the position vector of the point where the two particles collide,

c the position vector of B's starting point.

Solution:

a Using $v = u + at$ and $t = 3$,

For A:

$$\begin{aligned} v &= (-i + j) + (2i - 4j) \times 3 \\ &= (-1 + 6)i + (1 - 12)j \\ &= 5i - 11j \end{aligned}$$

$$\begin{aligned} \text{Speed} &= \sqrt{5^2 + 11^2} = \sqrt{25 + 121} \\ &= \sqrt{146} = 12.1\ \text{ms}^{-1}\ (3\text{ s.f.}) \end{aligned}$$

For B:

$$\begin{aligned} v &= i + 2j \times 3 \\ &= i + 6j \end{aligned}$$

$$\begin{aligned} \text{Speed} &= \sqrt{1^2 + 6^2} = \sqrt{1 + 36} \\ &= \sqrt{37} = 6.08\ \text{ms}^{-1}\ (3\text{ s.f.}) \end{aligned}$$

b Using $s = ut + \frac{1}{2}at^2$ for $A, s = \left(-i + j\right) \times 3 + \frac{1}{2} \times \left(2i - 4j\right) \times 9 = -3i + 3j + 9i - 18j = 6i - 15j$

So at the instant of the collision, A is at the point with position vector

$$r = (12i + 12j) + (6i - 15j) = 18i - 3j$$

c Using $s = ut + \frac{1}{2}at^2$ for B, $s = \left(i\right) \times 3 + \frac{1}{2} \times \left(2j\right) \times 9 = 3i + 9j$, so B's starting point is

$$(18i - 3j) - (3i + 9j) = 15i - 12j$$

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Vectors
Exercise G, Question 1

Question:

A particle is in equilibrium at O under the action of three forces F_1 , F_2 and F_3. Find F_3 in these cases.

a $F_1 = (2i + 7j)$ and $F_2 = (-3i + j)$ **b** $F_1 = (3i - 4j)$ and $F_2 = (2i + 3j)$

c $F_1 = (-4i - 2j)$ and $F_2 = (2i - 3j)$ **d** $F_1 = (-i - 3j)$ and $F_2 = (4i + j)$

Solution:

a $F_1 + F_2 + F_3 = 0 \Rightarrow (2i + 7j) + (-3i + j) + F_3 = 0$

$\Rightarrow F_3 = -(2i + 7j) - (-3i + j) = -2i - 7j + 3i - j = i - 8j$

b $F_1 + F_2 + F_3 = 0 \Rightarrow (3i - 4j) + (2i + 3j) + F_3 = 0$

$\Rightarrow F_3 = -(3i - 4j) - (2i + 3j) = -3i + 4j - 2i - 3j = -5i + j$

c $F_1 + F_2 + F_3 = 0 \Rightarrow (-4i - 2j) + (2i - 3j) + F_3 = 0$

$\Rightarrow F_3 = -(-4i - 2j) - (2i - 3j) = 4i + 2j - 2i + 3j = 2i + 5j$

d $F_1 + F_2 + F_3 = 0 \Rightarrow (-i - 3j) + (4i + j) + F_3 = 0$

$\Rightarrow F_3 = -(-i - 3j) - (4i + j) = i + 3j - 4i - j = -3i + 2j$

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Vectors
Exercise G, Question 2

Question:

For each part of Question 1 find the magnitude of F_3 and the angle it makes with the positive x-axis.

Solution:

a $|F_3| = \sqrt{(1)^2 + (-8)^2} = \sqrt{1 + 64} = \sqrt{65} \approx 8.06$

$$\tan\theta = \frac{8}{1} \Rightarrow \theta = 82.9° \ (3 \text{ s.f.})$$

$$\text{angle with } Ox = 82.9° \text{ below } (3 \text{ s.f.})$$

b $|F_3| = \sqrt{(-5)^2 + (1)^2} = \sqrt{25 + 1} = \sqrt{26} \approx 5.10$

$$\tan\theta = \frac{1}{5} \Rightarrow \theta = 11.3° \ (3 \text{ s.f.})$$

$$\text{angle with } Ox = 169° \text{ above } (3 \text{ s.f.})$$

c $|F_3| = \sqrt{(2)^2 + (5)^2} = \sqrt{4 + 25} = \sqrt{29} \approx 5.39$

$$\tan\theta = \frac{5}{2} \Rightarrow \theta = 68.2° \ (3 \text{ s.f.})$$

$$\text{angle with } Ox = 68.2° \text{ above } (3 \text{ s.f.})$$

d $|F_3| = \sqrt{(-3)^2 + (2)^2} = \sqrt{9 + 4} = \sqrt{13} \approx 3.61$

$$\tan\theta = \frac{2}{3} \Rightarrow \theta = 33.7° \ (3 \text{ s.f.})$$

$$\text{angle with } Ox = 146° \text{ above } (3 \text{ s.f.})$$

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Vectors
Exercise G, Question 3

Question:

Forces **P** N, **Q** N and **R** N act on a particle of m kg. Find the resultant force on the particle and the acceleration produced when

a $P = 3i + j$, $Q = 2i - 3j$, $r = i + 2j$ and $m = 2$,

b $P = 4i - 3j$, $Q = -3i + 2j$, $r = 2i - j$ and $m = 3$,

c $P = -3i + 2j$, $Q = 2i - 5j$, $r = 4i + j$ and $m = 4$,

d $P = 2i + j$, $Q = -6i - 4j$, $r = 5i - 3j$ and $m = 2$.

Solution:

a Resultant force $= P + Q + R = (3i + j) + (2i - 3j) + (i + 2j) = 6i$

$\Rightarrow$ using $F = \text{m}a$, $6i = 2 \times a$, $a = 3i \text{ m s}^{-2}$

b Resultant force $= P + Q + R = (4i - 3j) + (-3i + 2j) + (2i - j) = 3i - 2j$

$\Rightarrow$ using $F = \text{m}a$, $3i - 2j = 3 \times a$, $a = \left(i - \frac{2}{3}j \right) \text{ m s}^{-2}$

c Resultant force $= P + Q + R = (-3i + 2j) + (2i - 5j) + (4i + j) = 3i - 2j$

$\Rightarrow$ using $F = \text{m}a$, $3i - 2j = 4 \times a$, $a = \left(\frac{3}{4}i - \frac{1}{2}j \right) \text{ m s}^{-2}$

d Resultant force $= P + Q + R = (2i + j) + (-6i - 4j) + (5i - 3j) = i - 6j$

$\Rightarrow$ using $F = \text{m}a$, $i - 6j = 2 \times a$, $a = \left(\frac{1}{2}i - 3j \right) \text{ m s}^{-2}$

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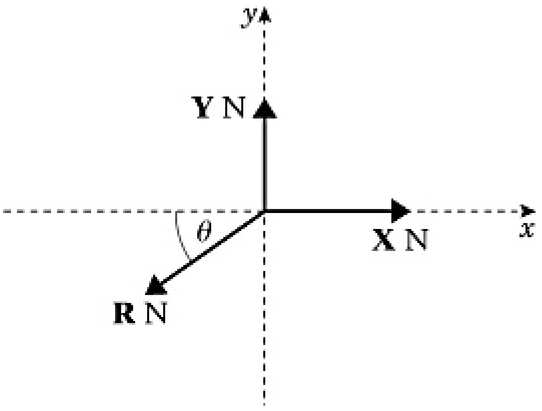
Vectors
Exercise G, Question 4

Question:

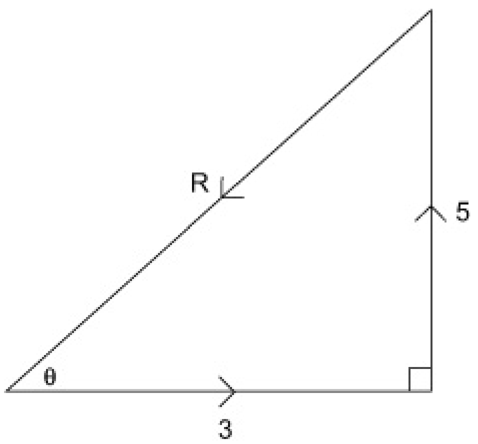


A particle is in equilibrium at O under the action of forces **X**, **Y** and **R**, as shown in the diagram. Use a triangle of forces to find the magnitude of **R** and the value of θ when:

a $|X| = 3\text{N}$, $|Y| = 5\text{N}$, **b** $|X| = 6\text{N}$, $|Y| = 2\text{N}$, **c** $|X| = 5\text{N}$, $|Y| = 4\text{N}$.

Solution:

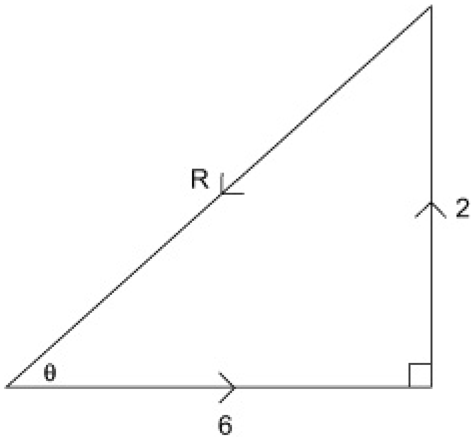
a



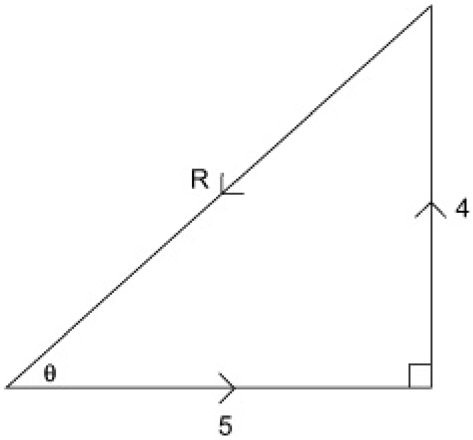
Using Pythagoras' theorem,
$|R| = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34} \approx 5.83$ N $\tan\theta = \frac{5}{3} \Rightarrow \theta = 59.0°$ (3 s.f.)

b

Using Pythagoras' theorem,
$|R| = \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40} \approx 6.32$ N $\tan\theta = \frac{2}{6} \Rightarrow \theta = 18.4°$ (3 s.f.)



c



Using Pythagoras' theorem, $|R| = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41} = 6.40$ N (3 s.f.) $\tan \theta = \frac{4}{5} \Rightarrow \theta = 38.7°$ (3 s.f.)

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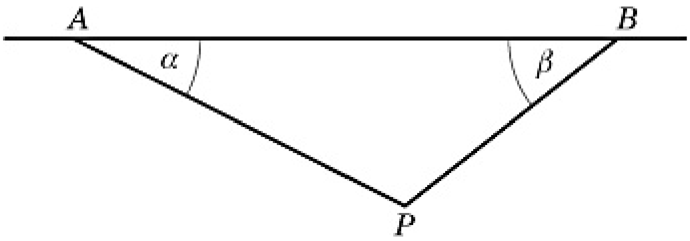
Vectors
Exercise G, Question 5

Question:



The diagram shows two strings attached to a particle *P*, of weight W N, and to two fixed points *A* and *B*. The line *AB* is horizontal and *P* is hanging in equilibrium with $\angle BAP = \alpha$ and $\angle ABP = \beta$. The magnitude of the tension in *AP* is T_AN, and the magnitude of the tension in *BP* is T_BN.

Use a triangle of vectors to find:

a T_A and T_B if $W = 5$, $\alpha = 40°$ and $\beta = 50°$,

b T_A and T_B if $W = 6$, $\alpha = 30°$ and $\beta = 45°$,

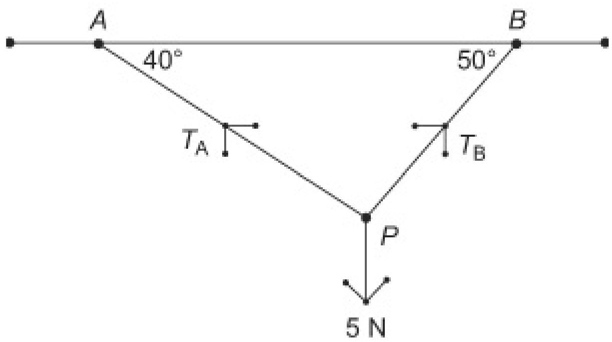
c T_A and W if $T_B = 5$, $\alpha = 40°$ and $\beta = 50°$,

d W and T_B if $T_A = 5$, $\alpha = 30°$ and $\beta = 70°$,

e T_A and α if $W = 7$, $T_B = 6$ and $\beta = 50°$.

Solution:

a

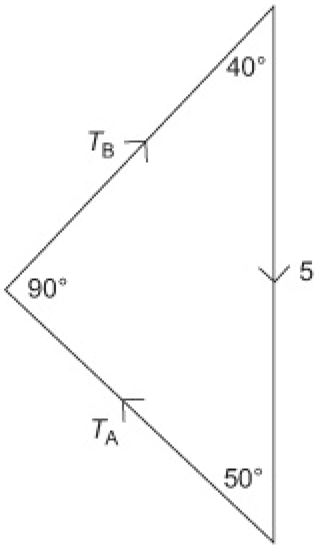


The triangle of forces is a right-angled triangle

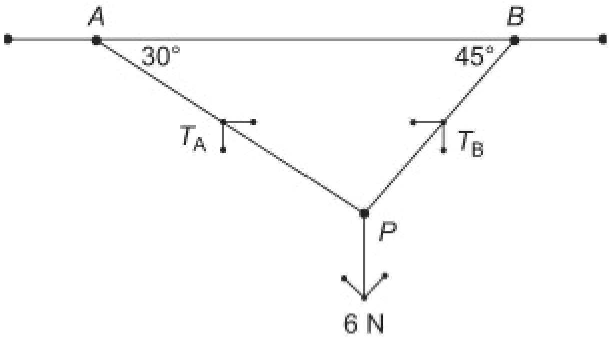
$\Rightarrow T_A = 5 \times \sin 40° = 3.21$ N (3 s.f.) and

$T_B = 5 \times \cos 40° = 3.83$ N (3 s.f.)

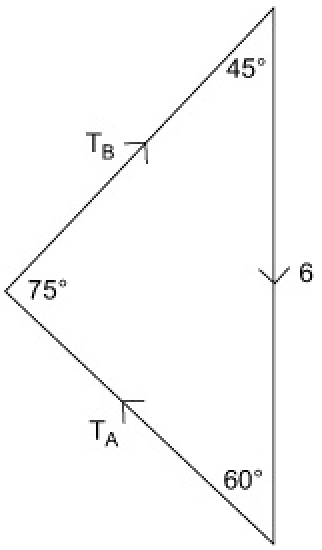
Triangle of forces



b



Triangle of forces



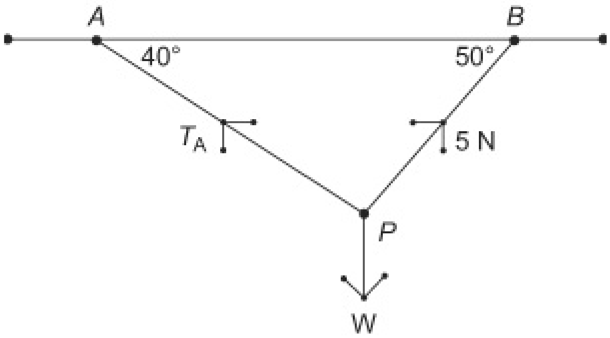
Using the Sine rule,

$$\frac{T_B}{\sin 60°} = \frac{T_A}{\sin 45°} = \frac{6}{\sin 75°}$$

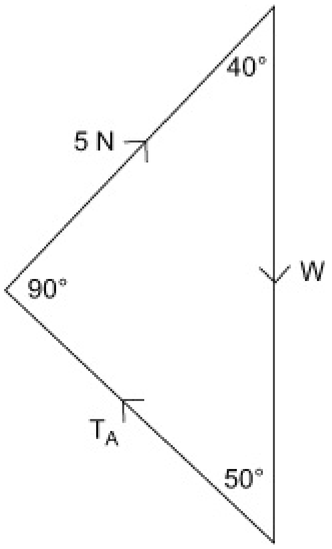
$$T_A = \frac{6 \times \sin 45°}{\sin 75°} = 4.39 \text{ N } (3 \text{ s.f.})$$

$$T_B = \frac{6 \times \sin 60°}{\sin 75°} = 5.38 \text{ N } (3 \text{ s.f.})$$

c



Triangle of forces



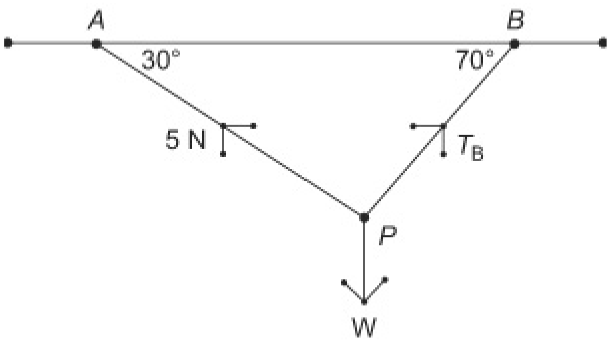
The triangle of forces is a right-angled triangle

$\Rightarrow T_A = 5 \times \tan 40° = 4.20$ N (3 s.f.)

and $5 = W \times \cos 40°$,

$W = \dfrac{5}{\cos 40°} = 6.53$ N (3 s.f.)

d



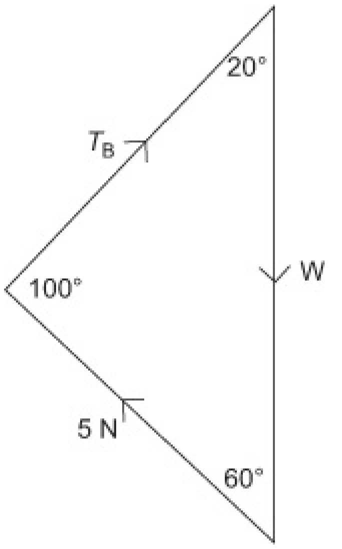
Triangle of forces

Using the Sine rule

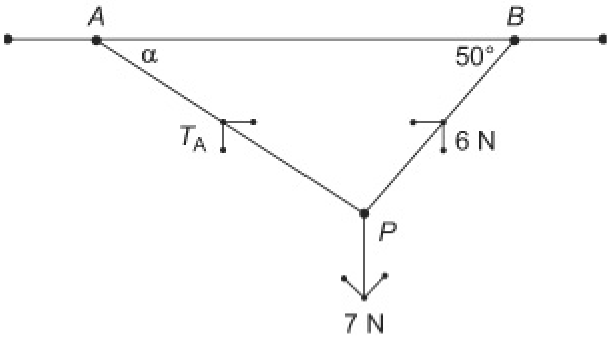
$$\frac{W}{\sin 100°} = \frac{5}{\sin 20°} = \frac{T_B}{\sin 60°}$$

$$W = \frac{5 \times \sin 100°}{\sin 20°} = 14.4 \text{ N (3 s.f.)}$$

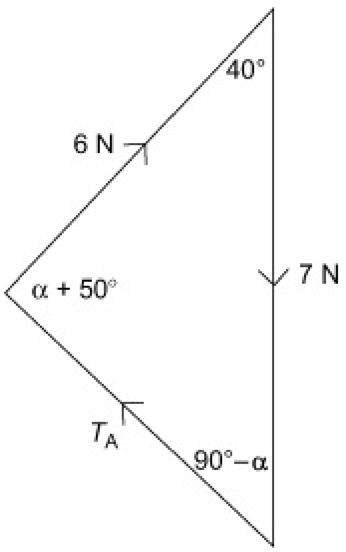
$$T_B = \frac{5 \times \sin 60°}{\sin 20°} = 12.7 \text{ N (3 s.f.)}$$



e



Triangle of forces



Using the Cosine rule:

$T_A{}^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos 40° = 20.65 \ldots$

$T_A = 4.54 \text{ N} \ (3 \text{ s.f.})$

Using the Sine rule:

$$\frac{\sin 40°}{T_A} = \frac{\sin(90° - \alpha)}{6}$$

$$\sin(90° - \alpha) = \frac{6 \times \sin 40°}{4.54} = 0.848 \ldots$$

$90° - \alpha = 58.1°$, $\alpha = 31.9°$ (3 s.f.)

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Vectors
Exercise H, Question 1

Question:

Three forces F_1 , F_2 and F_3 act on a particle. $F_1 = (-3i + 7j)$ N , $F_2 = (i - j)$ N and $F_3 = (pi + qj)$ N .

a Given that this particle is in equilibrium, determine the value of p and the value of q. The resultant of the forces F_1 and F_2 is **R**.

b Calculate, in N, the magnitude of **R**.

c Calculate, to the nearest degree, the angle between the line of action of **R** and the vector **j**.

Solution:

a $F_1 + F_2 + F_3 = 0 \Rightarrow (-3i + 7j) + (i - j) + (pi + qj) = 0$

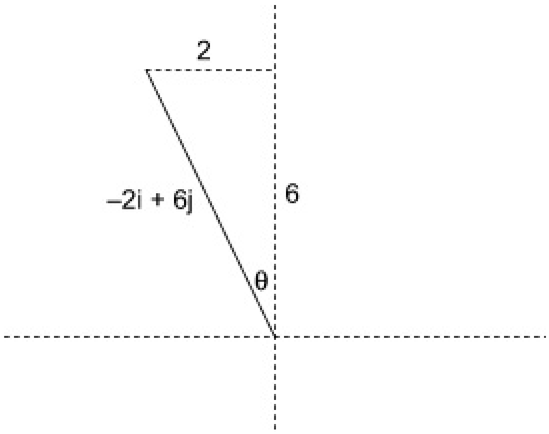
$(-3 + 1 + p) i + (7 - 1 + q) j = 0$

$p = 2, q = -6$

b $R = F_1 + F_2 = (-3i + 7j) + (i - j) = -2i + 6j$

$| R | = \sqrt{(-2)^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40} = 6.32$ N

c



$\tan \theta = \frac{2}{6}$

$\theta = 18°$

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Vectors
Exercise H, Question 2

Question:

In this question, the horizontal unit vectors **i** and **j** are directed due east and north respectively.

A coastguard station O monitors the movements of ships in a channel. At noon, the station's radar records two ships moving with constant speed. Ship A is at the point with position vector $(-3i + 10j)$ km relative to O and has velocity $(2i + 2j)$ km h^{-1}. Ship B is at the point with position vector $(6i + j)$ km and has velocity $(-i + 5j)$ km h^{-1}.

a Show that if the two ships maintain these velocities they will collide.

The coastguard radios ship A and orders it to reduce its speed to move with velocity $(i + j)$ km h^{-1}. Given that A obeys this order and maintains this new constant velocity.

b find an expression for the vector AB at time t hours after noon,

c find, to three significant figures, the distance between A and B at 1500 hours,

d find the time at which B will be due north of A.

Solution:

a At time t

$$r_A = (-3 + 2t)i + (10 + 2t)j$$
$$r_B = (6 - t)i + (1 + 5t)j$$

i components equal when $-3 + 2t = 6 - t \Rightarrow 3t = 9, t = 3$

$t = 3 : r_A = 3i + 16j$; $r_B = 3i + 16j \Rightarrow$ collide

b New $r_A = (-3 + t)i + (10 + t)j$

$\Rightarrow AB = r_B - r_A = (6 - t)i + (1 + 5t)j - \{(-3 + t)i + (10 + t)j\{$

$$= (6 - t + 3 - t)i + (1 + 5t - 10 - t)j$$
$$= (9 - 2t)i + (-9 + 4t)j$$

c $t = 3 : AB = 3i + 3j, \Rightarrow$ dist. $= \sqrt{(3^2 + 3^2)} \approx 4.24$ km

d B north of $A \Rightarrow$ no i component $\Rightarrow 9 - 2t = 0 \Rightarrow t = \frac{9}{2} \Rightarrow$ time 1630 hours

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Vectors
Exercise H, Question 3

Question:

Two ships *P* and *Q* are moving along straight lines with constant velocities. Initially *P* is at a point *O* and the position vector of *Q* relative to *O* is $(12i + 6j)$ km, where **i** and **j** are unit vectors directed due east and due north respectively. Ship *P* is moving with velocity $6i \text{ km h}^{-1}$ and ship *Q* is moving with velocity $(-3i + 6j) \text{ km h}^{-1}$. At time *t* hours the position vectors of *P* and *Q* relative to *O* are **p** km and **q** km respectively.

a Find **p** and **q** in terms of *t*.

b Calculate the distance of *Q* from *P* when $t = 4$.

c Calculate the value of *t* when *Q* is due north of *P*.

Solution:

a $p = 6ti$

$q = (12i + 6j) + (-3i + 6j)t = (12 - 3t)i + (6 + 6t)j$

b $t = 4 : p = 24i, q = 30j$

$\Rightarrow$ dist. apart $= \sqrt{24^2 + 30^2} = \sqrt{576 + 900} = \sqrt{1476} \approx 38.4$ km

c *Q* north of *P* $\Rightarrow$ *i* components match $\Rightarrow 6t = 12 - 3t \Rightarrow 9t = 12 \Rightarrow t = 1\frac{1}{3}$

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Vectors
Exercise H, Question 4

Question:

A particle P moves with constant acceleration $(-3i + j)$ m s^{-2}. At time t seconds, its velocity is v m s^{-1}. When $t = 0$, $v = 5i - 3j$.

a Find the value of t when P is moving parallel to the vector **i**.

b Find the speed of P when $t = 5$.

c Find the angle between the vector **i** and the direction of motion of P when $t = 5$.

Solution:

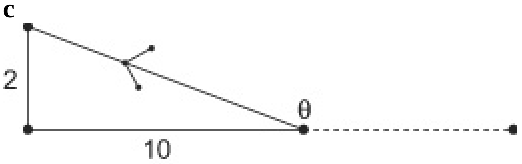
a $v = u + at$: $v = (5i - 3j) + (-3i + j) \times t = (5 - 3t) i + (-3 + t) j$

v parallel to $i \Rightarrow -3 + t = 0 \Rightarrow t = 3$ s

b $t = 5, v = -10i + 2j$

Speed $= | v | = \sqrt{104} \approx 10.2$ m s^{-1}

c



Angle $= \left(\arctan \frac{10}{2} \right) + 90^\circ = 168.7^\circ \left(1 \text{ s.f.} \right)$

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Vectors
Exercise H, Question 5

Question:

A particle P of mass 5 kg is moving under the action of a constant force **F** newtons. At $t = 0$, P has velocity $\left(5i - 3j \right)$ $\mathrm{m\,s^{-1}}$. At $t = 4\,\mathrm{s}$, the velocity of P is $(-11i + 5j)$ $\mathrm{m\,s^{-1}}$. Find

a the acceleration of P in terms of **i** and **j**,

b the magnitude of **F**.

At $t = 6\,\mathrm{s}$, P is at the point A with position vector $(28i + 6j)$ m relative to a fixed origin O. At this instant the force **F** newtons is removed and P then moves with constant velocity. Two seconds after the force has been removed, P is at the point B.

c Calculate the distance of B from O.

Solution:

a $a = \frac{v - u}{t}$: $a = \frac{1}{4}\left[\left(-11i + 5j \right) - \left(5i - 3j \right) \right] = -4i + 2j\ \mathrm{m\,s^{-2}}$

b $F = m\,a = 5 \times (-4i + 2j) = -20i + 10j$
$| F | = \sqrt{(-20)^2 + 10^2} = \sqrt{500} \approx 22.4\ \mathrm{N}$

c $t = 6, v = u + at \Rightarrow v = (5i - 3j) + (-4i + 2j) \times 6 = 5i - 3j - 24i + 12j = -19i + 9j$
At B : $r = r_0 + vt \Rightarrow r = (28i + 6j) + (-19i + 9j) \times 2 = -10i + 24j$

Distance $OB = \sqrt{(-10)^2 + 24^2} = \sqrt{100 + 576} = \sqrt{676} = 26\ \mathrm{m}$ –

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Vectors
Exercise H, Question 6

Question:

In this question the vectors **i** and **j** are horizontal unit vectors in the directions due east and due north respectively.

Two boats A and B are moving with constant velocities. Boat A moves with velocity $6i \text{ km h}^{-1}$. Boat B moves with velocity $(3i + 5j) \text{ km h}^{-1}$.

a Find the bearing on which B is moving.

At noon, A is at point O and B is 10 km due south of O. At time t hours after noon, the position vectors of A and B relative to O are **a** km and **b** km respectively.

b Find expressions for **a** and **b** in terms of t, giving your answer in the form $pi + qj$.

c Find the time when A is due east of B.

At time t hours after noon, the distance between A and B is $d$km. By finding an expression for AB,

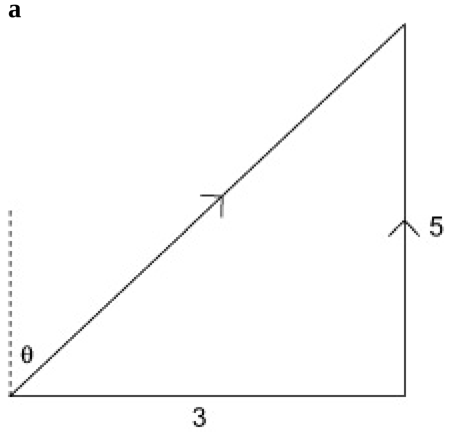
d show that $d^2 = 34t^2 - 100t + 100$.

At noon, the boats are 10 km apart.

e Find the time after noon at which the boats are again 10 km apart.

Solution:

a



$\tan\ \theta = \frac{3}{5} \Rightarrow \theta = 31°$, bearing is 031°

b

$a = 6ti$

$b = 3ti + (-10 + 5t)j$

c A due east of $B \Rightarrow j$ components match $\Rightarrow -10 + 5t = 0$

$t = 2 \Rightarrow$ 1400 hours

d

$AB = b - a = \{3ti + (-10 + 5t)j\{ -6ti = -3ti + (-10 + 5t)j$

$d^2 = |b - a|^2 = (-3t)^2 + (-10 + 5t)^2 = 9t^2 + 100 - 100t + 25t^2$

$= 34t^2 - 100t + 100$, as required.

e $d = 10 \Rightarrow d^2 = 100 \Rightarrow 34t^2 - 100t = 0$

$\Rightarrow t = 0$ or $\frac{100}{34} = 2.9411 \ldots \ldots$

$\Rightarrow$ time is 1456 hours

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Vectors
Exercise H, Question 7

Question:

A small boat S, drifting in the sea, is modelled as a particle moving in a straight line at constant speed. When first sighted at 0900, S is at a point with position vector $(-2i - 4j)$ km relative to a fixed origin O, where **i** and **j** are unit vectors due east and due north respectively. At 0940, S is at the point with position vector $(4i - 6j)$ km . At time t hours after 0900, S is at the point with position vector **s** km.

a Calculate the bearing on which S is drifting.

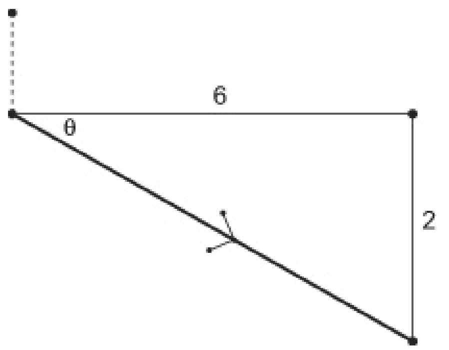
b Find an expression for **s** in terms of t.

At 1100 a motor boat M leaves O and travels with constant velocity $(pi + qj)$ km h^{-1} .

c Given that M intercepts S at 1130, calculate the value of p and the value of q.

Solution:

a $(\text{Direction of } v) = (4i - 6j) - (-2i - 4j) = 6i - 2j$



$$\tan \theta = \frac{1}{3} \Rightarrow \theta = 18.43 \ldots ^\circ$$

$$\text{Bearing} = 90^\circ + \theta = 108^\circ$$

b Expressing v in kmh^{-1}, $v = \left(6i - 2j\right) \times \frac{60}{40} = 9i - 3j$

$r = r_0 + vt \Rightarrow s = (-2i - 4j) + t(9i - 3j) = (-2 + 9t)i + (-4 - 3t)j$

c At 1130, $t = 2.5, s = (-2 + 9 \times 2.5)i + (-4 - 3 \times 2.5)j = 20.5i - 11.5j$

M has been travelling for 30 minutes $\Rightarrow m = 0.5(pi + qj)$

$s = m \Rightarrow p = 41, q = -23$

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Vectors
Exercise H, Question 8

Question:

A particle P moves in a horizontal plane. The acceleration of P is $(-2i + 3j)$ m s^{-2}. At time $t = 0$, the velocity of P is $(3i - 2j)$ m s^{-1}.

a Find, to the nearest degree, the angle between the vector **j** and the direction of motion of P when $t = 0$.

At time t seconds, the velocity of P is v m s^{-1}. Find

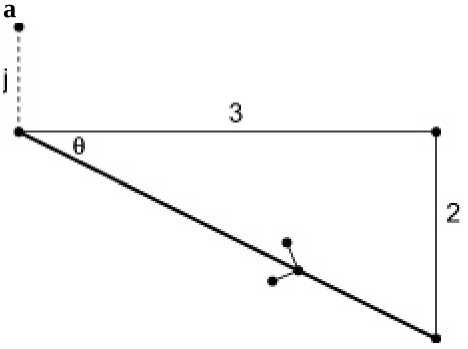
b an expression for **v** in terms of t, in the form $ai + bj$,

c the speed of P when $t = 4$,

d the time when P is moving parallel to $i + j$.

Solution:

a



$\tan\theta = \frac{2}{3} \left(\theta = 33.7^\circ \right)$

angle between v and $j = 90 + 33.7 \approx 124^\circ$

b $v = u + at$: $v = 3i - 2j + (-2i + 3j) \times t = (3 - 2t)i + (-2 + 3t)j$

c $t = 4, v = (3 - 2 \times 4)i + (-2 + 3 \times 4)j = -5i + 10j$
speed $= \sqrt{(-5)^2 + 10^2} = \sqrt{25 + 100} = \sqrt{125} \approx 11.2$ m s^{-1}

d v parallel to $i + j \Rightarrow i$ and **j** components must be equal $\Rightarrow 3 - 2t = -2 + 3t$
$\Rightarrow 5t = 5, t = 1$

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Vectors
Exercise H, Question 9

Question:

In this question, the unit vectors **i** and **j** are horizontal vectors due east and north respectively.

At time $t = 0$ a football player kicks a ball from the point A with position vector $(3i + 2j)$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(4i + 9j)$ m s^{-1}. Find

a the speed of the ball,

b the position vector of the ball after t seconds.

The point B on the field has position vector $(29i + 12j)$ m .

c Find the time when the ball is due north of B.

At time $t = 0$, another player starts running due north from B and moves with constant speed $v\ \text{m s}^{-1}$.

d Given that he intercepts the ball, find the value of v.

Solution:

a $v = 4i + 9j \Rightarrow$ speed of ball $= \sqrt{(4^2 + 9^2)} \approx 9.85\ \text{m s}^{-1}$

b $r = r_0 + vt \Rightarrow$ position vector of the ball after t seconds

$= (3i + 2j) + (4i + 9j) \times t = (3 + 4t) i + (2 + 9t) j$

c North of B when $\boldsymbol{i}$ components same, i.e. $3 + 4t = 29, 4t = 26, t = 6.5$ s

d When $t = 6.5$, position vector of the ball

$$= (3 + 4 \times 6.5) i + (2 + 9 \times 6.5) j = 29i + 60.5j$$
$$\Rightarrow j \text{ component} = 60.5$$

Distance travelled by 2nd player = $60.5 - 12 = 48.5$ m

Speed = $48.5 \div 6.5 \approx 7.46\ \text{m s}^{-1}$

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Vectors
Exercise H, Question 10

Question:

Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector $(10i + 15j)$ km relative to a fixed origin O. At the same time, Q is at the point with position vector $(-16i + 26j)$ km . Three hours later, P is at the point with position vector $(25i + 24j)$ km . The ship Q travels with velocity $12i \text{ km h}^{-1}$. At time t hours after midnight, the position vectors of P and Q are **p** km and **q** km respectively. Find

a the velocity of P in terms of **i** and **j**,

b expressions for **p** and **q** in terms of t, **i** and **j**.

At time t hours after midnight, the distance between P and Q is d km.

c By finding an expression for PQ, show that

$d^2 = 58t^2 - 430t + 797$

Weather conditions are such that an observer on P can only see the lights on Q when the distance between P and Q is 13 km or less.

d Given that when $t = 2$ the lights on Q move into sight of the observer, find the time, to the nearest minute, at which the lights on Q move out of sight of the observer.

Solution:

a $r = r_0 + vt \Rightarrow v_P = \{ (25i + 24j) - (10i + 15j) \{ / 3 = (5i + 3j) \text{ km h}^{-1}$

b

$r = r_0 + vt \Rightarrow$ after t hours,

$p = (10i + 15j) + (5i + 3j) \times t = (10 + 5t)i + (15 + 3t)j$

$q = (-16i + 26j) + 12i \times t = (-16 + 12t)i + 26j$

c

$$\begin{aligned}
PQ = q - p &= \{ (-16 + 12t)i + 26j \{ - \{ (10 + 5t)i + (15 + 3t)j \{ \\
&= (-26 + 7t)i + (11 - 3t)j \\
\Rightarrow d^2 &= (-26 + 7t)^2 + (11 - 3t)^2 \\
&= 676 - 364t + 49t^2 + 121 - 66t + 9t^2 \\
&= 58t^2 - 430t + 797, \text{ as required}
\end{aligned}$$

d $d = 13 \Rightarrow 58t^2 - 430t + 797 = 169, 58t^2 - 430t + 628 = 0$

We are given that $t = 2$ is one solution, so we know that $(t - 2)$ is a factor

$\Rightarrow (t - 2)(58t - 314) = 0$

$\Rightarrow t = \frac{314}{58} \approx 5.41$ hours, so time ≈ 0525 to the nearest minute