Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 1

Question:

A particle is moving in a straight line with constant acceleration 3 m s^{-2}. At time $t = 0$, the speed of the particle is 2 m s^{-1}. Find the speed of the particle at time $t = 6$ s.

Solution:

$$a = 3,\ u = 2,\ t = 6,\ v = ?$$
$$v = u + at$$
$$= 2 + 3 \times 6 = 2 + 18 = 20$$

The speed of the particle at time $t = 6$ s is 20 m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 2

Question:

A particle is moving in a straight line with constant acceleration. The particle passes a point with speed 1.2 m s $^{-1}$. Four seconds later the particle has speed 7.6 m s $^{-1}$. Find the acceleration of the particle.

Solution:

$$u = 1.2\ ,\ t = 4\ ,\ v = 7.6\ ,\ a = \ ?$$
$$v = u + at$$
$$7.6 = 1.2 + a \times 4$$
$$a = \frac{7.6 - 1.2}{4} = 1.6$$

The acceleration of the particle is 1.6 m s $^{-2}$.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 3

Question:

A car is approaching traffic lights. The car is travelling with speed 10 m s^{-1}. The driver applies the brakes to the car and the car comes to rest with constant deceleration in 16 s. Modelling the car as a particle, find the deceleration of the car.

Solution:

$$u = 10\ ,\ v = 0\ ,\ t = 16\ ,\ a = \ ?$$
$$v = u + at$$
$$0 = 10 + a \times 16$$
$$a = -\frac{10}{16} = -0.625$$

The deceleration of the car is 0.625 m s^{-2}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise A, Question 4

Question:

A particle moves in a straight line from a point *A* to point *B* with constant acceleration. The particle passes *A* with speed 2.4 m s^{-1}. The particle passes *B* with speed 8 m s^{-1}, five seconds after it passed *A*. Find the distance between *A* and *B*.

Solution:

$u = 2.4 , v = 8 , t = 5 , s = ?$

$$s = \left(\frac{u + v}{2} \right) t$$

$$= \frac{2.4 + 8}{2} \times 5 = 5.2 \times 5 = 26$$

The distance between *A* and *B* is 26 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise A, Question 5

Question:

A car accelerates uniformly while travelling on a straight road. The car passes two signposts 360 m apart. The car takes 15 s to travel from one signpost to the other. When passing the second signpost, it has speed 28 m s^{-1}. Find the speed of the car at the first signpost.

Solution:

$s = 360$, $t = 15$, $v = 28$, $u =$?

$$s = \left(\frac{u + v}{2} \right) t$$

$$360 = \frac{u + 28}{2} \times 15$$

$$u + 28 = \frac{360 \times 2}{15} = 48$$

$$u = 48 - 28 = 20$$

The speed of the car at the first sign post is 20 m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 6

Question:

A particle is moving along a straight line with constant deceleration. The points X and Y are on the line and $XY = 120$ m. At time $t = 0$, the particle passes X and is moving towards Y with speed 18 m s $^{-1}$. At time $t = 10$ s, the particle is at Y. Find the velocity of the particle at time $t = 10$ s.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 7

Question:

A cyclist is moving along a straight road from *A* to *B* with constant acceleration 0.5 m s $^{-2}$. Her speed at *A* is 3 m s $^{-1}$ and it takes her 12 seconds to cycle from *A* to *B*. Find **a** her speed at *B*, **b** the distance from *A* to *B*.

Solution:

a

$a = 0.5$, $u = 3$, $t = 12$, $v = ?$

$v = u + at$

$= 3 + 0.5 \times 12 = 3 + 6 = 9$

The speed of the cyclist at *B* is 9 m s $^{-1}$.

b

$u = 3$, $v = 9$, $t = 12$, $s = ?$

$s = \left(\frac{u + v}{2} \right) t$

$= \left(\frac{3 + 9}{2} \right) \times 12 = 6 \times 12 = 72$

The distance from *A* to *B* is 72 m.

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Kinematics of a particle moving in a straight line
Exercise A, Question 8

Question:

A particle is moving along a straight line with constant acceleration from a point A to a point B, where $AB = 24$ m. The particle takes 6 s to move from A to B and the speed of the particle at B is 5 m s^{-1}. Find **a** the speed of the particle at A, **b** the acceleration of the particle.

Solution:

a

$s = 24$, $t = 6$, $v = 5$, $u = ?$

$$s = \left(\frac{u + v}{2} \right) t$$

$$24 = \left(\frac{u + 5}{2} \right) \times 6$$

$$u + 5 = \frac{24 \times 2}{6} = 8$$

$$u = 8 - 5 = 3$$

The speed of the particle at A is 3 m s^{-1}.

b

$u = 3$, $v = 5$, $t = 6$, $a = ?$

$$v = u + at$$

$$5 = 3 + 6a$$

$$a = \frac{5 - 3}{6} = \frac{1}{3}$$

The acceleration of the particle is $\frac{1}{3}$ m s^{-2}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise A, Question 9

Question:

A particle moves in a straight line from a point A to a point B with constant deceleration 1.2 m s^{-2}. The particle takes 6 s to move from A to B. The speed of the particle at B is 2 m s^{-1} and the direction of motion of the particle has not changed. Find **a** the speed of the particle at A, **b** the distance from A to B.

Solution:

a

$$a = -1.2,\ t = 6,\ v = 2,\ u = ?$$
$$v = u + at$$
$$2 = u - 1.2 \times 6 = u - 7.2$$
$$u = 2 + 7.2 = 9.2$$

The speed of the particle at A is 9.2 m s^{-1}.

b

$$u = 9.2,\ v = 2,\ t = 6,\ s = ?$$
$$s = \left(\frac{u+v}{2}\right) t$$
$$= \left(\frac{9.2+2}{2}\right) \times 6 = 5.6 \times 6 = 33.6$$

The distance from A to B is 33.6 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 10

Question:

A train, travelling on a straight track, is slowing down with constant deceleration 0.6 m s^{-2}. The train passes one signal with speed 72 km h^{-1} and a second signal 25 s later. Find **a** the speed, in km h^{-1}, of the train as it passes the second signal, **b** the distance between the signals.

Solution:

a

$$72 \text{ km h}^{-1} = 72 \times 1000 \text{ m h}^{-1} = \frac{72 \times 1000}{3600} \text{ m s}^{-1} = 20 \text{ m s}^{-1}$$

$$u = 20 , a = -0.6 , t = 25 , v = ?$$
$$v = u + at$$
$$= 20 - 0.6 \times 25 = 20 - 15 = 5 (\text{m s}^{-1})$$

$$5 \text{ m s}^{-1} = 5 \times 3600 \text{ m h}^{-1} = \frac{5 \times 3600}{1000} \text{ km h}^{-1} = 18 \text{ km h}^{-1}$$

The speed of the train as it passes the second signal is 18 km h^{-1}.

b

$$u = 20 , v = 5 , t = 25 , s = ?$$
$$s = \left(\frac{u + v}{2} \right) t$$
$$= \left(\frac{20 + 5}{2} \right) \times 25 = 12.5 \times 25 = 312.5$$

The distance between the signals is 312.5 m.

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Kinematics of a particle moving in a straight line
Exercise A, Question 11

Question:

A particle moves in a straight line from a point *A* to a point *B* with a constant deceleration of 4 m s $^{-2}$. At *A* the particle has speed 32 m s $^{-1}$ and the particle comes to rest at *B*. Find **a** the time taken for the particle to travel from *A* to *B*, **b** the distance between *A* and *B*.

Solution:

a

$a = -4 , u = 32 , v = 0 , t = ?$

$v = u + at$

$0 = 32 - 4t$

$t = \frac{32}{4} = 8$

The time taken for the particle to move from *A* to *B* is 8 *s*.

b

$u = 32 , v = 0 , t = 8 , s = ?$

$s = \left(\frac{u + v}{2} \right) t$

$= \left(\frac{32 + 0}{2} \right) \times 8 = 16 \times 8 = 128$

The distance between *A* and *B* is 128 m.

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Kinematics of a particle moving in a straight line
Exercise A, Question 12

Question:

A skier travelling in a straight line up a hill experiences a constant deceleration. At the bottom of the hill, the skier has a speed of 16 m s^{-1} and, after moving up the hill for 40 s, he comes to rest. Find **a** the deceleration of the skier, **b** the distance from the bottom of the hill to the point where the skier comes to rest.

Solution:

a

$u = 16,\ t = 40,\ v = 0,\ a = ?$

$v = u + at$

$0 = 16 + 40a$

$a = -\frac{16}{40} = -0.4$

The deceleration of the skier is 0.4 m s^{-2}.

b

$u = 16,\ t = 40,\ v = 0,\ s = ?$

$s = \left(\frac{u+v}{2}\right)t$

$= \left(\frac{16+0}{2}\right) \times 40 = 8 \times 40 = 320$

The distance from the bottom of the hill to the point where the skier comes to rest is 320 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 13

Question:

A particle is moving in a straight line with constant acceleration. The points *A*, *B* and *C* lie on this line. The particle moves from *A* through *B* to *C*. The speed of the particle at *A* is 2 m s $^{-1}$ and the speed of the particle at *B* is 7 m s $^{-1}$. The particle takes 20 s to move from *A* to *B*.

a Find the acceleration of the particle.

The speed of the particle is *C* is 11 m s $^{-1}$. Find

b the time taken for the particle to move from *B* to *C*,

c the distance between *A* and *C*.

Solution:

a

$u = 2 , v = 7 , t = 20 , a = ?$

$v = u + at$

$7 = 2 + 20a$

$a = \frac{7-2}{20} = 0.25$

The acceleration of the particle is 0.25 m s $^{-2}$.

b
From *B* to *C*

$u = 7 , v = 11 , a = 0.25 , t = ?$

$v = u + at$

$11 = 7 + 0.25t$

$t = \frac{11-7}{0.25} = 16$

The time taken for the particle to move from *B* to *C* is 16 s.

c The time taken to move from *A* to *C* is $(20 + 16)$ s = 36 s

From *A* to *C*

$u = 2 , v = 11 , t = 36 , s = ?$

$s = \left(\frac{u+v}{2} \right) t$

$= \left(\frac{2+11}{2} \right) \times 36 = 6.5 \times 36 = 234$

The distance between *A* and *C* is 234 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 14

Question:

A particle moves in a straight line from *A* to *B* with constant acceleration 1.5 m s^{-2}. It then moves, along the same straight line, from *B* to *C* with a different acceleration. The speed of the particle at *A* is 1 m s^{-1} and the speed of the particle at *C* is 43 m s^{-1}. The particle takes 12 s to move from *A* to *B* and 10 s to move from *B* to *C*. Find

a the speed of the particle at *B*,

b the acceleration of the particle as it moves from *B* to *C*,

c the distance from *A* to *C*.

Solution:

a
From *A* to *B*

$$a = 1.5,\ u = 1,\ t = 12,\ v = ?$$
$$v = u + at$$
$$= 1 + 1.5 \times 12 = 1 + 18 = 19$$

The speed of the particle at *B* is 19 m s^{-1}.

b
From *B* to *C*

$$u = 19,\ v = 43,\ t = 10,\ a = ?$$
$$v = u + at$$
$$43 = 19 + 10a$$
$$a = \frac{43 - 19}{10} = 2.4$$

The acceleration from *B* to *C* is 2.4 m s^{-2}.

c
The distance from *A* to *B*

$$u = 1,\ v = 19,\ t = 12,\ s = ?$$
$$s = \left(\frac{u + v}{2}\right) t$$
$$= \left(\frac{1 + 19}{2}\right) \times 12 = 10 \times 12 = 120$$

The distance from *B* to *C*

$$u = 19,\ v = 43,\ t = 10,\ s = ?$$
$$s = \left(\frac{u + v}{2}\right) t$$
$$= \left(\frac{19 + 43}{2}\right) \times 10 = 31 \times 10 = 310$$

The distance from *A* to *C* is $(120 + 310)$ m = 430 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 15

Question:

A cyclist travels with constant acceleration x m s^{-2}, in a straight line, from rest to 5 m s^{-1} in 20 s. She then decelerates from 5 m s^{-1} to rest with constant deceleration $\frac{1}{2}x$ m s^{-2}. Find **a** the value of x, **b** the total distance she travelled.

Solution:

a

$u = 0, v = 5, t = 20, a = x$

$v = u + at$

$5 = 0 + 20x$

$x = \frac{5}{20} = 0.25$

b
While accelerating

$u = 0, v = 5, t = 20, s = ?$

$s = \left(\frac{u + v}{2}\right) t$

$= \left(\frac{0 + 5}{2}\right) \times 20 = 2.5 \times 20 = 50$

While decelerating

The value of t is found first.

$u = 5, v = 0, a = -\frac{1}{2}x = -0.125, t = ?$

$v = u + at$

$0 = 5 - 0.125t$

$t = \frac{5}{0.125} = 40$

To find the value of s while decelerating.

$u = 5, v = 0, t = 40, s = ?$

$s = \left(\frac{u + v}{2}\right) t$

$= \left(\frac{5 + 0}{2}\right) \times 40 = 2.5 \times 40 = 100$

The total distance travelled is the distance travelled while accelerating added to the distance travelled while decelerating = $(50 + 100)$ m = 150 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise A, Question 16

Question:

A particle is moving with constant acceleration in a straight line. It passes through three points, *A*, *B* and *C* with speeds 20 m s^{-1}, 30 m s^{-1} and 45 m s^{-1} respectively. The time taken to move from *A* to *B* is t_1 seconds and the time taken to move from *B* to *C* is t_2 seconds.

a Show that $\frac{t_1}{t_2} = \frac{2}{3}$.

Given also that the total time taken for the particle to move from *A* to *C* is 50 s,

b find the distance between *A* and *B*.

Solution:

a
From *A* to *B*

$$v = u + at$$
$$30 = 20 + at_1$$
$$at_1 = 10 \qquad (1)$$

From *B* to *C*

$$v = u + at$$
$$45 = 30 + at_2$$
$$at_2 = 15 \qquad (2)$$

Dividing equation (1) by equation (2)

$$\frac{\cancel{a}t_1}{\cancel{a}t_2} = \frac{\cancel{10}^2}{\cancel{15}_3}$$
$$\frac{t_1}{t_2} = \frac{2}{3}\text{, as required}$$

b
From the result in part **a**

$$t_2 = \frac{3}{2}t_1$$
$$t_1 + t_2 = t_1 + \frac{3}{2}t_1 = \frac{5}{2}t_1 = 50\text{, given.}$$
$$t_1 = \frac{2}{5} \times 50 = 20$$

From *A* to *B*

$u = 20$, $v = 30$, $t = 20$, $s = ?$

$$s = \left(\frac{u+v}{2} \right) t$$

$$= \left(\frac{20+30}{2} \right) \times 20 = 25 \times 20 = 500$$

The distance from *A* to *B* is 500 m.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 1

Question:

A particle is moving in a straight line with constant acceleration 2.5 m s^{-2}. It passes a point *A* with speed 3 m s^{-1} and later passes through a point *B*, where *AB* = 8 m. Find the speed of the particle as it passes through *B*.

Solution:

$$a = 2.5, u = 3, s = 8, v = ?$$
$$v^2 = u^2 + 2as$$
$$= 3^2 + 2 \times 2.5 \times 8 = 9 + 40 = 49$$
$$v = \sqrt{49} = 7$$

The speed of the particle as it passes through *B* is 7 m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 2

Question:

A car is accelerating at a constant rate along a straight horizontal road. Travelling at 8 m s $^{-1}$, it passes a pillar box and 6 s later it passes a sign. The distance between the pillar box and the sign is 60 m. Find the acceleration of the car.

Solution:

$u = 8, t = 6, s = 60, a = ?$

$s = ut + \frac{1}{2}at^2$

$60 = 8 \times 6 + \frac{1}{2} \times a \times 6^2 = 48 + 18a$

$a = \frac{60 - 48}{18} = \frac{12}{18} = \frac{2}{3}$

The acceleration of the car is $\frac{2}{3}$ m s $^{-2}$.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 3

Question:

A cyclist travelling at 12 m s $^{-1}$ applies her brakes and comes to rest after travelling 36 m in a straight line. Assuming that the brakes cause the cyclist to decelerate uniformly, find the deceleration.

Solution:

$$u = 12, v = 0, s = 36, a = ?$$
$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 + 2 \times a \times 36 = 144 + 72a$$
$$a = -\frac{144}{72} = -2$$

The deceleration is 2 m s $^{-2}$.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise B, Question 4

Question:

A particle moves along a straight line from P to Q with constant acceleration 1.5 m s^{-2}. The particle takes 4 s to pass from
P to Q and PQ = 22 m. Find the speed of the particle at Q.

Solution:

$$a = 1.5, t = 4, s = 22, v = ?$$

$$s = vt - \frac{1}{2}at^2$$

$$22 = 4v - \frac{1}{2} \times 1.5 \times 4^2 = 4v - 12$$

$$v = \frac{22 + 12}{4} = \frac{34}{4} = 8.5$$

The speed of the particle at Q is 8.5 m s^{-1}.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 5

Question:

A particle is moving along a straight line *OA* with constant acceleration 2 m s $^{-2}$. At *O* the particle is moving towards *A* with speed 5.5 m s $^{-1}$. The distance *OA* is 20 m. Find the time the particle takes to move from *O* to *A*.

Solution:

$a = 2, u = 5.5, s = 20, t = ?$

$s = ut + \frac{1}{2}at^2$

$20 = 5.5t + \frac{1}{2} \times 2 \times t^2$

$t^2 + 5.5t - 20 = 0$

$2t^2 + 11t - 40 = (2t - 5)(t + 8) = 0$

$t = 2.5, -8$

Reject the negative answer.

The time the particle takes to move from *O* to *A* is 2.5 s.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 6

Question:

A train is moving along a straight horizontal track with constant acceleration. The train passes a signal at 54 km h $^{-1}$ and a second signal at 72 km h $^{-1}$. The distance between the two signals is 500 m. Find, in m s $^{-2}$, the acceleration of the train.

Solution:

$$54 \text{ km h}^{-1} = \frac{54 \times 1000}{3600} \text{ m s}^{-1} = 15 \text{ m s}^{-1}$$

$$72 \text{ km h}^{-1} = \frac{72 \times 1000}{3600} \text{ m s}^{-1} = 20 \text{ m s}^{-1}$$

$u = 15, v = 20, s = 500, a = ?$

$v^2 = u^2 + 2as$

$20^2 = 15^2 + 2 \times a \times 500$

$400 = 225 + 1000a$

$a = \frac{400 - 225}{1000} = 0.175$

The acceleration of the train is 0.175 m s $^{-2}$.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 7

Question:

A particle moves along a straight line, with constant acceleration, from a point A to a point B where $AB = 48$ m. At A the particle has speed 4 m s^{-1} and at B it has speed 16 m s^{-1}. Find **a** the acceleration of the particle, **b** the time the particle takes to move from A to B.

Solution:

a

$$s = 48, u = 4, v = 16, a = ?$$
$$v^2 = u^2 + 2as$$
$$16^2 = 4^2 + 2 \times a \times 48$$
$$256 = 16 + 96a$$
$$a = \frac{256 - 16}{96} = 2.5$$

The acceleration of the particle is 2.5 m s^{-2}.

b

$$u = 4, v = 16, a = 2.5, t = ?$$
$$v = u + at$$
$$16 = 4 + 2.5t$$
$$t = \frac{16 - 4}{2.5} = 4.8$$

The time taken to move from A to B is 4.8 s.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 8

Question:

A particle moves along a straight line with constant acceleration 3 m s $^{-2}$. The particle moves 38 m in 4 s. Find **a** the initial speed of the particle, **b** the final speed of the particle.

Solution:

a

$$a = 3, s = 38, t = 4, u = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$38 = 4u + \frac{1}{2} \times 3 \times 4^2 = 4u + 24$$

$$u = \frac{38 - 24}{4} = 3.5$$

The initial speed of the particle is 3.5 m s $^{-1}$.

b

$$v = u + at$$
$$= 3.5 + 3 \times 4 = 15.5$$

The final speed of the particle is 15.5 m s $^{-1}$.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 9

Question:

The driver of a car is travelling at 18 m s^{-1} along a straight road when she sees an obstruction ahead. She applies the brakes and the brakes cause the car to slow down to rest with a constant deceleration of 3 m s^{-2}. Find **a** the distance travelled as the car decelerates, **b** the time it takes for the car to decelerate from 18 m s^{-1} to rest.

Solution:

a

$$u = 18, v = 0, a = -3, s = ?$$
$$v^2 = u^2 + 2as$$
$$0^2 = 18^2 - 2 \times 3 \times s = 324 - 6s$$
$$s = \frac{324}{6} = 54$$

The distance travelled as the car decelerates is 54 m.

b

$$u = 18, v = 0, a = -3, t = ?$$
$$v = u + at$$
$$0 = 18 - 3t$$
$$t = \frac{18}{3} = 6$$

The time taken for the car to decelerate is 6 s.

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 10

Question:

A stone is sliding across a frozen lake in a straight line. The initial speed of the stone is 12 m s $^{-1}$. The friction between the stone and the ice causes the stone to slow down at a constant rate of 0.8 m s $^{-2}$. Find **a** the distance moved by the stone before coming to rest, **b** the speed of the stone at the instant when it has travelled half of this distance.

Solution:

a

$$u = 12, v = 0, a = -0.8, s = ?$$
$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 - 2 \times 0.8 \times s = 144 - 1.6s$$
$$s = \frac{144}{1.6} = 90$$

The distance moved by the stone is 90 m.

b

$$u = 12, a = -0.8, s = 45, v = ?$$

Half the distance in **a** is 45 m.

$$v^2 = u^2 + 2as$$
$$= 12^2 - 2 \times 0.8 \times 45 = 144 - 72 = 72$$
$$v = \sqrt{72} \approx 8.49$$

The speed of the stone is 8.49 m s $^{-1}$ (3 s.f.).

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 11

Question:

A particle is moving along a straight line *OA* with constant acceleration 2.5 m s $^{-2}$. At time $t = 0$, the particle passes through *O* with speed 8 m s $^{-1}$ and is moving in the direction *OA*. The distance *OA* is 40 m. Find **a** the time taken for the particle to move from *O* to *A*, **b** the speed of the particle at *A*. Give your answers to one decimal place.

Solution:

a

$$a = 2.5, u = 8, s = 40, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$40 = 8t + 1.25t^2$$

$1.25t^2 + 8t - 40 = 0$

$$t = \frac{-8 + \sqrt{(8^2 + 4 \times 1.25 \times 40)}}{2.5} = \frac{-8 + \sqrt{264}}{2.5}$$

$$= 3.299 \ldots$$

The negative solution is discounted.

The time taken for the particle to move from *O* to *A* 3.3 s (1 d.p.).

b

$$a = 2.5, u = 8, s = 40, v = ?$$

$$v^2 = u^2 + 2as$$

$$= 8^2 + 2 \times 2.5 \times 40 = 264$$

$$v = \sqrt{264} = 16.24 \ldots$$

The speed of the particle at *A* is 16.2 m s $^{-1}$ (1 d.p.).

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Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise B, Question 12

Question:

A particle travels with uniform deceleration 2 m s^{-2} in a horizontal line. The points A and B lie on the line and $AB = 32$ m. At time $t = 0$, the particle passes through A with velocity 12 m s^{-1} in the direction AB. Find **a** the values of t when the particle is at B, **b** the velocity of the particle for each of these values of t.

Solution:

a

$$a = -2, s = 32, u = 12, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$32 = 12t - t^2$$

$$t^2 - 12t + 32 = (t - 4)(t - 8) = 0$$

$$t = 4, 8$$

b

$$\text{When } t = 4, v = u + at$$
$$= 12 - 2 \times 4 = 4$$

The velocity is 4 m s^{-1} in the direction AB.

$$\text{When } t = 8, v = u + at$$
$$= 12 - 2 \times 8 = -4$$

The velocity is 4 m s^{-1} in the direction BA.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 13

Question:

A particle is moving along the x-axis with constant deceleration 5 m s^{-2}. At time $t = 0$, the particle passes through the origin O with velocity 12 m s^{-1} in the positive direction. At time t seconds the particle passes through the point A with x-coordinate 8. Find **a** the values of t, **b** the velocity of the particle as it passes through the point with x-coordinate -8.

Solution:

a

$$a = -5, u = 12, s = 8, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$8 = 12t - 2.5t^2$$

$$2.5t^2 - 12t + 8 = 0$$

$$5t^2 - 24t + 16 = (5t - 4)(t - 4) = 0$$

$$t = 0.8, 4$$

b

$$a = -5, u = 12, s = -8, v = ?$$

$$v^2 = u^2 + 2as$$

$$= 12^2 + 2 \times (-5) \times (-8) = 144 + 80 = 224$$

$$v = \sqrt{224} = 14.966 \ldots$$

The speed at $x = -8$ is 15.0 m s^{-1} (3 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 14

Question:

A particle P is moving on the x-axis with constant deceleration 4 m s^{-2}. At time $t = 0$, P passes through the origin O with velocity 14 m s^{-1} in the positive direction. The point A lies on the axis and $OA = 22.5$ m. Find **a** the difference between the times when P passes through A, **b** the total distance travelled by P during the interval between these times.

Solution:

a

$$a = -4, u = 14, s = 22.5, t = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$22.5 = 14t - 2t^2$$
$$2t^2 - 14t + 22.5 = 0$$
$$4t^2 - 28t + 45 = (2t - 5)(2t - 9) = 0$$
$$t = 2.5, 4.5$$

The difference between the times is $(4.5 - 2.5)$ s = 2 s.

b

Find the time when P reverses direction.

$$a = -4, u = 14, v = 0, t = ?$$
$$v = u + at$$
$$0 = 14 - 4t \Rightarrow t = \frac{14}{4} = 3.5$$

Find the displacement when $t = 3.5$.

$$s = ut + \frac{1}{2}at^2$$
$$= 14 \times 3.5 - 2 \times 3.5^2 = 24.5$$

Between the times when $t = 2.5$ and $t = 4.5$ the particle moves $2(24.5 - 22.5)$ m = 4 m.

Solutionbank M1

Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise B, Question 15

Question:

A car is travelling along a straight horizontal road with constant acceleration. The car passes over three consecutive points A, B and C where $AB = 100$ m and $BC = 300$ m. The speed of the car at B is 14 m s^{-1} and the speed of the car at C is 20 m s^{-1}. Find **a** the acceleration of the car, **b** the time taken for the car to travel from A to C.

Solution:

a From B to C

$$u = 14, v = 20, s = 300, a = ?$$
$$v^2 = u^2 + 2as$$
$$20^2 = 14^2 + 2 \times a \times 300$$
$$a = \frac{20^2 - 14^2}{600} = 0.34$$

The acceleration of the car is 0.34 m s^{-2}.

b From A to C

$$v = 20, s = 400, a = 0.34, u = ?$$
$$v^2 = u^2 + 2as$$
$$20^2 = u^2 + 2 \times 0.34 \times 400 = u^2 + 272$$
$$u^2 = 400 - 272 = 128$$
$$u = \sqrt{128} = 8\sqrt{2} \quad \text{Assuming the car is not in reverse at } A$$
$$v = u + at$$
$$20 = 8\sqrt{2} + 0.34t$$
$$t = \frac{20 - 8\sqrt{2}}{0.34} \approx 25.5$$

The time taken for the car to travel from A to C is 25.5 s (3 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise B, Question 16

Question:

Two particles *P* and *Q* are moving along the same straight horizontal line with constant accelerations 2 m s^{-2} and 3.6 m s^{-2} respectively. At time $t = 0$, *P* passes through a point *A* with speed 4 m s^{-1}. One second later *Q* passes through *A* with speed 3 m s^{-1}, moving in the same direction as *P*.

a Write down expressions for the displacements of *P* and *Q* from *A*, in terms of *t*, where *t* seconds is the time after *P* has passed through *A*.

b Find the value of *t* where the particles meet.

c Find the distance of *A* from the point where the particles meet.

Solution:

a For *P*:

$s = ut + \frac{1}{2}at^2 \Rightarrow s = 4t + t^2$

The displacement of *P* is $(4t + t^2)$ m.

For *Q*:

Q has been moving for $(t - 1)$ seconds since passing through *A*.

$s = ut + \frac{1}{2}at^2 \Rightarrow s = 3(t-1) + 1.8(t-1)^2$

The displacement of *Q* is $[3(t-1) + 1.8(t-1)^2]$ m.

b

$$4t + t^2 = 3(t-1) + 1.8(t-1)^2$$
$$4t + t^2 = 3t - 3 + 1.8t^2 - 3.6t + 1.8$$
$$0.8t^2 - 4.6t - 1.2 = 0$$

Divide throughout by 0.2.

$$4t^2 - 23t - 6 = 0$$
$$(t-6)(4t+1) = 0$$

$t = 6$, the negative solution is rejected (*t* is the time after passing through *A*).

c

Substitute $t = 6$ into the expression for the displacement of *P*.

$s = 4t + t^2 = 4 \times 6 + 6^2 = 60$

The distance of *A* from the point where the particles meet is 60 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 1

Question:

A ball is projected vertically upwards from a point O with speed 14 m s^{-1}. Find the greatest height above O reached by the ball.

Solution:

Take upwards as the positive direction.

$u = 14$, $v = 0$, $a = -9.8$, $s = ?$

$v^2 = u^2 + 2as$

$0^2 = 14^2 - 2 \times 9.8 \times s$

$s = \frac{14^2}{2 \times 9.8} = 10$

The greatest height above O reached by the ball is 10 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 2

Question:

A well is 50 m deep. A stone is released from rest at the top of the well. Find how long the stone takes to reach the bottom of the well.

Solution:

Take downwards as the positive direction.

$s = 50$, $u = 0$, $a = 9.8$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$50 = 0 + 4.9t^2$

$t^2 = \frac{50}{4.9} = 10.204 \ldots \Rightarrow t = 3.194 \ldots \approx 3.2$

The stone takes 3.2 s (2 s.f.) to reach the bottom of the well.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 3

Question:

A book falls from the top shelf of a bookcase. It takes 0.6 s to reach the floor. Find how far it is from the top shelf to the floor.

Solution:

Take downwards as the positive direction.

$u = 0$, $t = 0.6$, $a = 9.8$, $s =$?

$s = ut + \frac{1}{2}at^2$

$= 0 + 4.9 \times 0.6^2 = 1.764 \approx 1.8$

The top shelf is 1.8 m (2 s.f.) from the floor.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 4

Question:

A particle is projected vertically upwards with speed 20 m s^{-1} from a point on the ground. Find the time of flight of the particle.

Solution:

Take upwards as the positive direction.

$u = 20$, $a = -9.8$, $s = 0$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 20t - 4.9t^2 = t(20 - 4.9t)$, $t \neq 0$

$t = \frac{20}{4.9} = 4.081 \ldots \approx 4.1$

The time of flight of the particle is 4.1 s (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 5

Question:

A ball is thrown vertically downward from the top of a tower with speed 18 m s^{-1}. It reaches the ground in 1.6 s. Find the height of the tower.

Solution:

Take downwards as the positive direction.

$u = 18$, $a = 9.8$, $t = 1.6$, $s =$?

$s = ut + \frac{1}{2}at^2$

$= 18 \times 1.6 + 4.9 \times 1.6^2 = 41.344 \approx 41$

The height of the tower is 41 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 6

Question:

A pebble is catapulted vertically upwards with speed 24 m s^{-1}. Find **a** the greatest height above the point of projection reached by the pebble, **b** the time taken to reach this height.

Solution:

a

Take upwards as the positive direction.

$$u = 24\ ,\ a = -9.8\ ,\ v = 0\ ,\ s = \ ?$$
$$v^2 = u^2 + 2as$$
$$0^2 = 24^2 - 2 \times 9.8 \times s$$
$$s = \frac{24^2}{2 \times 9.8} = 29.387 \ldots \approx 29$$

The greatest height above the point of projection reached by the pebble is 29 m (2 s.f.).

b

$$u = 24\ ,\ a = -9.8\ ,\ v = 0\ ,\ t = \ ?$$
$$v = u + at$$
$$0 = 24 - 9.8t$$
$$t = \frac{24}{9.8} = 2.448 \ldots \approx 2.4$$

The time taken to reach the greatest height is 2.4 s (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 7

Question:

A ball is projected upwards from a point which is 4 m above the ground with speed 18 m s^{-1}. Find **a** the speed of the ball when it is 15 m above its point of projection, **b** the speed with which the ball hits the ground.

Solution:

a

Take upwards as the positive direction.

$$u = 18, \ a = -9.8, \ s = 15, \ v = ?$$
$$v^2 = u^2 + 2as$$
$$= 18^2 - 2 \times 9.8 \times 15 = 30$$
$$v = \sqrt{30} = \pm 5.477 \ldots \approx \pm 5.5$$

The speed of the ball when it is 15 m above its point of projection is 5.5 m s^{-1} (2 s.f.).

b

$$u = 18, \ a = -9.8, \ s = -4, \ v = ?$$
$$v^2 = u^2 + 2as$$
$$= 18^2 + 2 \times (-9.8) \times (-4)$$
$$= 324 + 78.4 = 402.4$$
$$v = -\sqrt{402.2} = -20.059 \ldots \approx -20$$

The speed with which the ball hits the ground is 20 m s^{-1} (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 8

Question:

A particle P is projected vertically downwards from a point 80 m above the ground with speed 4 m s^{-1}. Find **a** the speed with which P hits the ground, **b** the time P takes to reach the ground.

Solution:

a

Take downwards as the positive direction.

$$s = 80 , u = 4 , a = 9.8 , v = ?$$
$$v^2 = u^2 + 2as$$
$$= 4^2 + 2 \times 9.8 \times 80 = 1584$$
$$v = \sqrt{1584} = 39.799 \ldots \approx 40$$

The speed with which P hits the ground is 40 m s^{-1} (2 s.f.).

b

$$u = 4 , a = 9.8 , v = \sqrt{1584} , t = ?$$
$$v = u + at$$
$$\sqrt{1584} = 4 + 9.8t$$
$$t = \frac{\sqrt{1584} - 4}{9.8} = 3.653 \ldots \approx 3.7$$

The time P takes to reach the ground is 3.7 s (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 9

Question:

A particle P is projected vertically upwards from a point X. Five seconds later P is moving downwards with speed 10 m s^{-1}. Find **a** the speed of projection of P, **b** the greatest height above X attained by P during its motion.

Solution:

a

Take upwards as the positive direction.

$$v = -10,\ a = -9.8,\ t = 5,\ u = ?$$
$$v = u + at$$
$$-10 = u - 9.8 \times 5$$
$$u = 9.8 \times 5 - 10 = 39$$

The speed of projection of P is 39 m s^{-1}.

b

$$u = 39,\ v = 0,\ a = -9.8,\ s = ?$$
$$v^2 = u^2 + 2as$$
$$0^2 = 39^2 - 2 \times 9.8 \times s$$
$$s = \frac{39^2}{2 \times 9.8} = 77.602\ldots \approx 78$$

The greatest height above X attained by P during its motion is 78 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 10

Question:

A ball is thrown vertically upwards with speed 21 m s^{-1}. It hits the ground 4.5 s later. Find the height above the ground from which the ball was thrown.

Solution:

Take upwards as the positive direction.

$u = 21$, $t = 4.5$, $a = -9.8$, $s = ?$

$s = ut + \frac{1}{2}at^2$

$= 21 \times 4.5 - 4.9 \times 4.5^2 = -4.725 \approx -4.7$

The height above the ground from which the ball was thrown is 4.7 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 11

Question:

A stone is thrown vertically upward from a point which is 3 m above the ground, with speed 16 m s $^{-1}$. Find **a** the time of flight of the stone, **b** the total distance travelled by the stone.

Solution:

a

Take upwards as the positive direction.

$$s = -3 \text{ , } u = 16 \text{ , } a = -9.8 \text{ , } t = \text{ ?}$$

$$s = ut + \frac{1}{2}at^2$$

$$-3 = 16t - 4.9t^2$$

$4.9t^2 - 16t - 3 = 0$

Using $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$$t = \frac{16 \pm \sqrt{(16^2 + 4 \times 4.9 \times 3)}}{2 \times 4.9}$$ Only the positive solution need be considered.

$$= 3.443 \ldots \approx 3.4$$

The time of flight of the stone is 3.4 s (2 s.f.).

b

Find the greatest height, say *h*, reached by the stone.

$$u = 16 \text{ , } v = 0 \text{ , } a = -9.8 \text{ , } s = h$$

$$v^2 = u^2 + 2as$$

$$0^2 = 16^2 - 2 \times 9.8 \times h$$

$$h = \frac{16^2}{2 \times 4.9} = 13.061 \ldots \approx 13$$

The total distance travelled by the stone is $(2 \times 13 + 3)$ m = 29 m (2 s.f.).

Solutionbank M1

Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise C, Question 12

Question:

A particle is projected vertically upwards with speed 24.5 m s^{-1}. Find the total time for which it is 21 m or more above its point of projection.

Solution:

Take upwards as the positive direction.

$u = 24.5$, $a = -9.8$, $s = 21$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$21 = 24.5t - 4.9t^2$$

$4.9t^2 - 24.5t + 21 = 0$

Using $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$$t = \frac{24.5 \pm \sqrt{(24.5^2 - 4 \times 4.9 \times 21)}}{2 \times 9.8}$$

$$= 1.0984\ldots, 3.9015\ldots$$

The difference between these times is $(3.9015\ldots - 1.0984\ldots)$ s $= 2.803\ldots$ s

The total time for which the particle is 21 m or more above its point of projection is 2.8 s (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 13

Question:

A particle is projected vertically upwards from a point *O* with speed u m s^{-1}. Two seconds later it is still moving upwards and its speed is $\frac{1}{3}u$ m s^{-1}. Find **a** the value of u, **b** the time from the instant that the particle leaves *O* to the instant that it returns to *O*.

Solution:

a

Take upwards as the positive direction.

$$u = u\ ,\ v = \frac{1}{3}u\ ,\ a = -9.8\ ,\ t = 2$$

$$v = u + at$$

$$\frac{1}{3}u = u - 9.8 \times 2$$

$$\frac{2}{3}u = 19.6 \Rightarrow u = \frac{3}{2} \times 19.6 = 29.4$$

$$u = 29 \quad (2 \text{ s.f.})$$

b

$$u = 29.4\ ,\ s = 0\ ,\ a = -9.8\ ,\ t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 29.4t - 4.9t^2 = t(29.4 - 4.9t) \quad ,\ t \neq 0$$

$$t = \frac{29.4}{4.9} = 6$$

The time from the instant that the particle leaves *O* to the instant that it returns to *O* is 6 s.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 14

Question:

A ball *A* is thrown vertically downwards with speed 5 m s^{-1} from the top of a tower block 46 m above the ground. At the same time as *A* is thrown downwards, another ball *B* is thrown vertically upwards from the ground with speed 18 m s^{-1}. The balls collide. Find the distance of the point where *A* and *B* collide from the point where *A* was thrown.

Solution:

For *A*, take downwards as the positive direction.

$$s_A = ut + \frac{1}{2}at^2 = 5t + 4.9t^2 \ldots \quad *$$

For *B*, take upwards as the positive direction.

$$s_B = ut + \frac{1}{2}at^2 = 18t - 4.9t^2$$

$$s_A + s_B = 46$$

$$5t + 4.9t^2 + 18t - 4.9t^2 = 46$$

$$23t = 46 \Rightarrow t = 2$$

Substitute into $*$

$$s_A = 5 \times 2 + 4.9 \times 2^2 = 29.6$$

The distance of the point where *A* and *B* collide from the point where *A* was thrown is 30 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 15

Question:

A ball is released from rest at a point which is 10 m above a wooden floor. Each time the ball strikes the floor, it rebounds with three-quarters of the speed with which it strikes the floor. Find the greatest height above the floor reached by the ball **a** the first time it rebounds from the floor, **b** the second time it rebounds from the floor.

Solution:

a

Find the speed, u_1 say, immediately before the ball strikes the floor.

$$u = 0 \, , a = 9.8 \, , s = 10 \, , v = u_1$$
$$v^2 = u^2 + 2as$$
$$u_1^{\,2} = 0^2 + 2 \times 9.8 \times 10 = 196$$
$$u_1 = \sqrt{196} = 14$$

The speed of the first rebound, u_2 say, is given by

$$u_2 = \frac{3}{4}u_1 = \frac{3}{4} \times 14 = 10.5$$

Find the maximum height, h_1 say, reached after the first rebound.

$$u = 10.5 \, , v = 0 \, , a = -9.8 \, , s = h_1$$
$$v^2 = u^2 + 2as$$
$$0^2 = 10.5^2 - 2 \times 9.8 \times h_1 \Rightarrow h_1 = \frac{10.5^2}{2 \times 9.8} = 5.625$$

The greatest height above the floor reached by the ball the first time it rebounds from the floor is 5.6 m (2 s.f.).

b

Immediately before the ball strikes the floor for the second time, its speed is again $u_2 = 10.5$.

The speed of the second rebound, u_3 say, is given by

$$u_3 = \frac{3}{4}u_2 = \frac{3}{4} \times 10.5 = 7.875$$

Find the maximum height, h_2 say, reached after the second rebound.

$u = 7.875$, $v = 0$, $a = -9.8$, $s = h_2$

$v^2 = u^2 + 2as$

$0^2 = 7.875^2 - 2 \times 9.8 \times h_2 \Rightarrow h_2 = \frac{7.875^2}{2 \times 9.8} = 3.164 \ldots$

The greatest height above the floor reached by the ball the second time it rebounds from the floor is 3.2 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise C, Question 16

Question:

A particle *P* is projected vertically upwards from a point *O* with speed 12 m s $^{-1}$. One second after *P* has been projected from *O*, another particle *Q* is projected vertically upwards from *O* with speed 20 m s $^{-1}$. Find **a** the time between the instant that *P* is projected from *O* and the instant when *P* and *Q* collide, **b** the distance of the point where *P* and *Q* collide from *O*.

Solution:

a

Take upwards as the positive direction.

For *P*,

$s = ut + \frac{1}{2}at^2$

$s_P = 12t - 4.9t^2 \ldots \ldots *$

For *Q*,

$s = ut + \frac{1}{2}at^2$ Q has been moving for one less second than P.

$s_Q = 20(t-1) - 4.9(t-1)^2$

At the point of collision $s_P = s_Q$.

$12t - 4.9t^2 = 20(t-1) - 4.9(t-1)^2$

$12t - 4.9t^2 = 20t - 20 - 4.9t^2 + 9.8t - 4.9$

$24.9 = 17.8t \Rightarrow t = \frac{24.9}{17.8} = 1.3988\ldots \approx 1.4$

The time between the instant that *P* is projected from *O* and the instant when *P* and *Q* collide is 1.4 s (2 s.f.).

b

Substitute for t in $*$

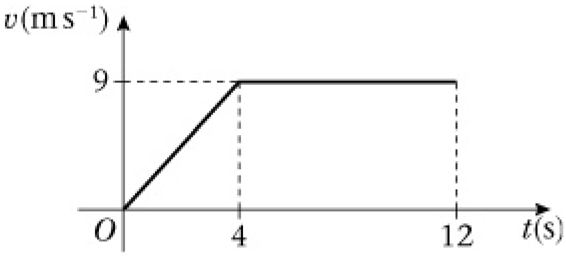
$s_P = 12t - 4.9t^2 \approx 12 \times 1.4 - 4.9 \times 1.4^2 \approx 7.2$

The distance of the point where *P* and *Q* collide from *O* is 7.2 m (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 1

Question:



The diagram shows the speed-time graph of the motion of an athlete running along a straight track. For the first 4 s, he accelerates uniformly from rest to a speed of 9 m s^{-1}. This speed is then maintained for a further 8 s. Find

a the rate at which the athlete accelerates,

b the total distance travelled by the athlete in 12 s.

Solution:

a $a = \frac{9}{4} = 2.25$

The athlete accelerates at a rate of 2.25 m s^{-2}.

b

$$s = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2}(8+12) \times 9 = 90$$

The total distance travelled by the athlete is 90 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 2

Question:

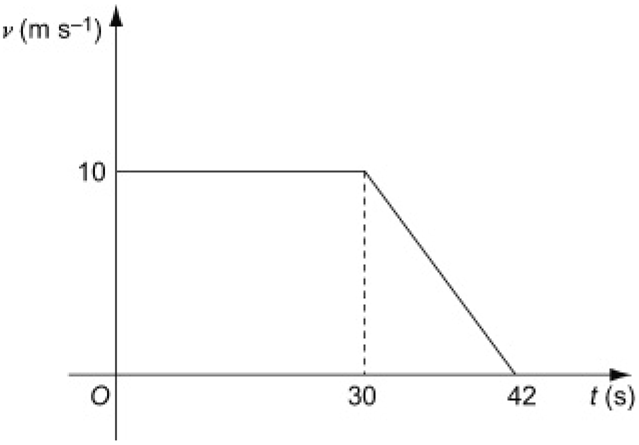
A car is moving along a straight road. When $t = 0$ s, the car passes a point A with speed 10 m s^{-1} and this speed is maintained until $t = 30$ s. The driver then applies the brakes and the car decelerates uniformly, coming to rest at the point B when $t = 42$ s.

a Sketch a speed-time graph to illustrate the motion of the car.

b Find the distance from A to B.

Solution:

a



b

$$s = \frac{1}{2}(a + b)h$$

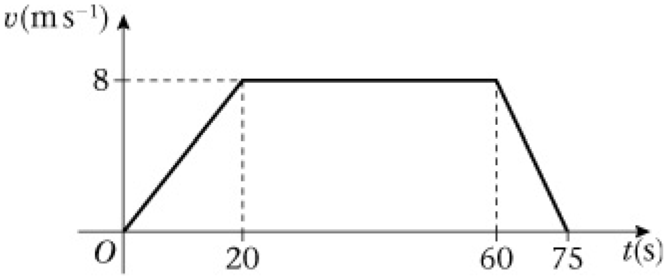
$$= \frac{1}{2}(30 + 42) \times 10 = 360$$

The distance from A to B is 360 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 3

Question:



The diagram shows the speed-time graph of the motion of a cyclist riding along a straight road. She accelerates uniformly from rest to 8 m s^{-1} in 20 s. She then travels at a constant speed of 8 m s^{-1} for 40 s. She then decelerates uniformly to rest in 15 s. Find

a the acceleration of the cyclist in the first 20 s of motion,

b the deceleration of the cyclist in the last 15 s of motion,

c the total distance travelled in 75 s.

Solution:

a $a = \frac{8}{20} = 0.4$

The acceleration of the cyclist is 0.4 m s^{-2}.

b $a = \frac{-8}{15} \ (= -0.53)$

The deceleration of the cyclist is $\frac{8}{15}$ m s^{-2}.

c

$$s = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2}(40+75) \times 8 = 460$$

The total distance travelled is 460 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 4

Question:

A car accelerates at a constant rate, starting from rest at a point *A* and reaching a speed of 45 km h^{-1} in 20 s. This speed is then maintained and the car passes a point *B* 3 minutes after leaving *A*.

a Sketch a speed-time graph to illustrate the motion of the car.

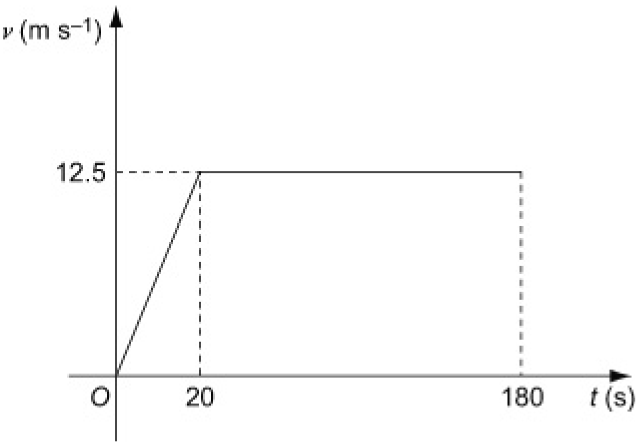
b Find the distance from *A* to *B*.

Solution:

a

$$45 \text{ km h}^{-1} = \frac{45 \times 1000}{3600} \text{ m s}^{-1} = 12.5 \text{ m s}^{-1}$$

3 min = 180 s



b

$$s = \frac{1}{2}(a+b)h$$

$$= \frac{1}{2}(160+180) \times 12.5 = 2125$$

The distance from *A* to *B* is 2125 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 5

Question:

A motorcyclist starts from rest at a point *S* on a straight race track. He moves with constant acceleration for 15 s, reaching a speed of 30 m s $^{-1}$. He then travels at a constant speed of 30 m s $^{-1}$ for *T* seconds. Finally he decelerates at a constant rate coming to rest at a point *F*, 25 s after he begins to decelerate.

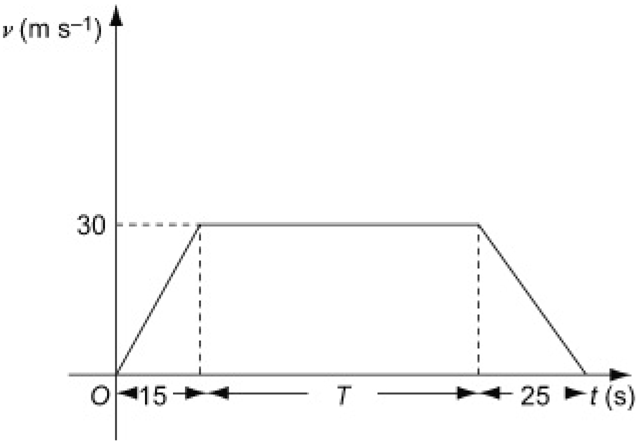
a Sketch a speed-time graph to illustrate the motion.

Given that the distance between *S* and *F* is 2.4 km,

b calculate the time the motorcyclist takes to travel from *S* to *F*.

Solution:

a



b

$$s = \frac{1}{2}(a+b)h$$

$$2400 = \frac{1}{2}(T + (15 + T + 25)) \times 30 = 15(2T + 40)$$

$$2T + 40 = \frac{2400}{15} = 160$$

$$T = \frac{160 - 40}{2} = 60$$

The taken to travel from *S* to *F* is $(15 + T + 25)$ s = 100 s.

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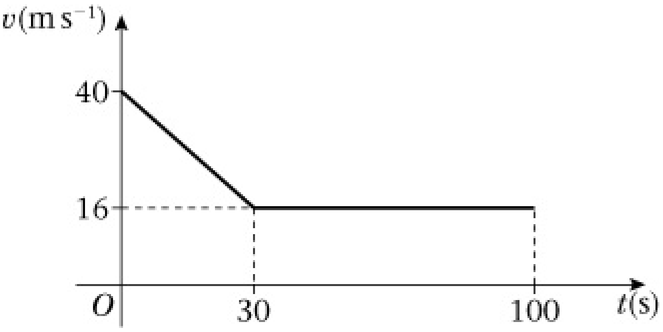
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 6

Question:



A train is travelling along a straight track. To obey a speed restriction, the brakes of the train are applied for 30 s reducing the speed of the train from 40 m s^{-1} to 16 m s^{-1}. The train then continues at a constant speed of 16 m s^{-1} for a further 70s. The diagram shows a speed-time graph illustrating the motion of the train for the total period of 100 s. Find

a the retardation of the train in the first 30 s,

b the total distance travelled by the train in 100 s.

Solution:

a $a = \frac{16 - 40}{30} = -\frac{24}{30} = -0.8$

The retardation (deceleration) is 0.8 m s^{-2}.

b The area under the graph is made up of a triangle, with sides 30 × 24, and a rectangle, with sides 100 × 16.

$s = \frac{1}{2} \times 30 \times 24 + 100 \times 16 = 360 + 1600 = 1960$

The total distance travelled by the train is 1960 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 7

Question:

A train starts from a station *X* and moves with constant acceleration 0.6 m s^{-2} for 20 s. The speed it has reached after 20 s is then maintained for *T* seconds. The train then decelerates from this speed to rest in a further 40 s, stopping at a station *Y*.

a Sketch a speed-time graph to illustrate the motion of the train.

Given that the distance between the stations is 4.2 km, find

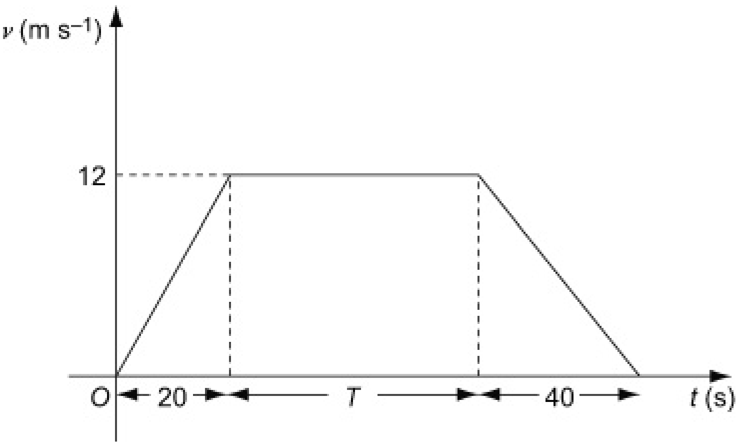
b the value of *T*,

c the distance travelled by the train while it is moving with constant speed.

Solution:

a The speed after 20 s is given by

$$v = u + at$$
$$= 0 + 0.6 \times 20 = 12$$



b

$$s = \frac{1}{2}(a + b)h$$

$$4200 = \frac{1}{2}(T + (20 + T + 40)) \times 12$$

$$4200 = 6(2T + 60) = 12T + 360$$

$$T = \frac{4200 - 360}{12} = 320$$

c at constant speed, distance = speed × time

= 12 × 320 = 3840

The distance travelled at a constant speed is 3840 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 8

Question:

A particle moves along a straight line. The particle accelerates from rest to a speed of 10 m s^{-1} in 15 s. The particle then moves at a constant speed of 10 m s^{-1} for a period of time. The particle then decelerates uniformly to rest. The period of time for which the particle is travelling at a constant speed is 4 times the period of time for which it is decelerating.

a Sketch a speed-time graph to illustrate the motion of the particle.

Given that the total distance travelled by the particle is 480 m,

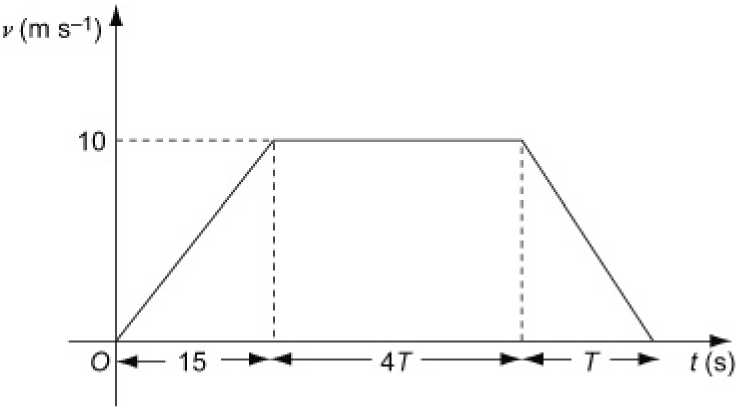
b find the total time for which the particle is moving,

c sketch an acceleration-time graph illustrating the motion of the particle.

Solution:

a Let the time for which the particle decelerates be T seconds.

Then the time for which the particle moves at a constant speed is 4T seconds.



b

$$s = \frac{1}{2}(a+b)h$$

$$480 = \frac{1}{2}(4T + (15 + 4T + T)) \times 10 = 5(9T + 15)$$

$$45T + 75 = 480$$

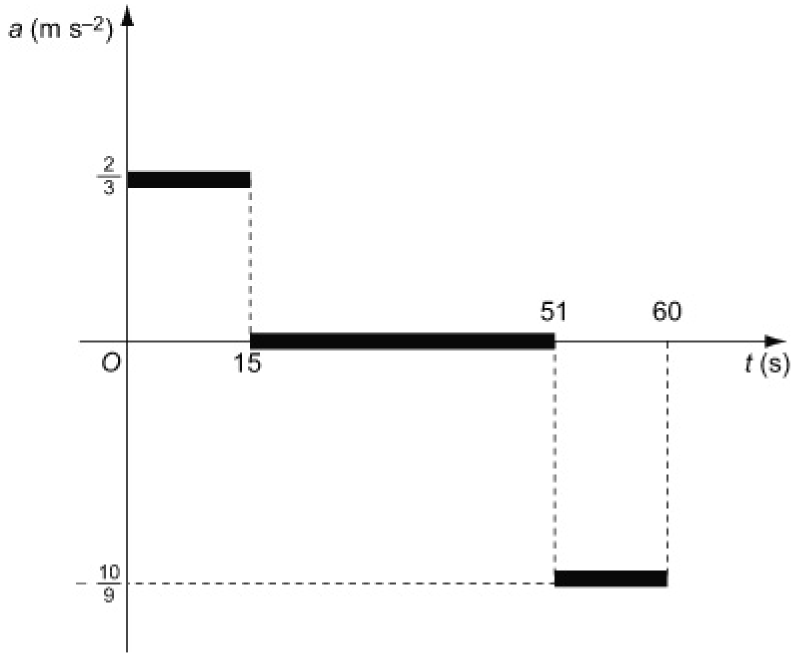
$$T = \frac{480 - 75}{45} = 9$$

The total time for which the particle is moving is

$(5T + 15)$ s $= (5 \times 9 + 15)$ s $= 60$ s

c While accelerating, $a = \frac{10}{15} = \frac{2}{3}$

While decelerating, $a = -\frac{10}{9}$



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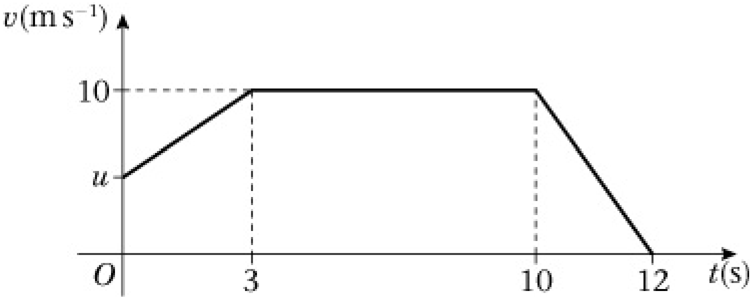
Kinematics of a particle moving in a straight line
Exercise D, Question 9

Question:



A particle moves 100 m in a straight line. The diagram is a sketch of a speed-time graph of the motion of the particle. The particle starts with speed u m s $^{-1}$ and accelerates to a speed 10 m s $^{-1}$ in 3 s. The speed of 10 m s $^{-1}$ is maintained for 7 s and then the particle decelerates to rest in a further 2 s. Find

a the value of u,

b the acceleration of the particle in the first 3 s of motion.

Solution:

a Area = trapezium + rectangle + triangle

$$100 = \frac{1}{2}(u + 10) \times 3 + 7 \times 10 + \frac{1}{2} \times 2 \times 10$$

$$100 = \frac{3}{2}(u + 10) + 70 + 10$$

$$\frac{3}{2}(u + 10) = 100 - 70 - 10 = 20$$

$$u + 10 = 20 \times \frac{2}{3} = \frac{40}{3}$$

$$u = \frac{40}{3} - 10 = \frac{10}{3}$$

b $a = \dfrac{10 - \frac{10}{3}}{3} = \dfrac{20}{9}$

The acceleration of the particle is $\frac{20}{9}$ m s $^{-2}$.

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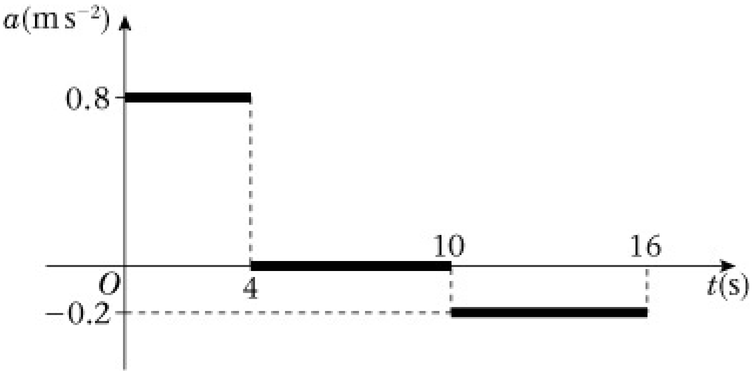
Kinematics of a particle moving in a straight line
Exercise D, Question 10

Question:

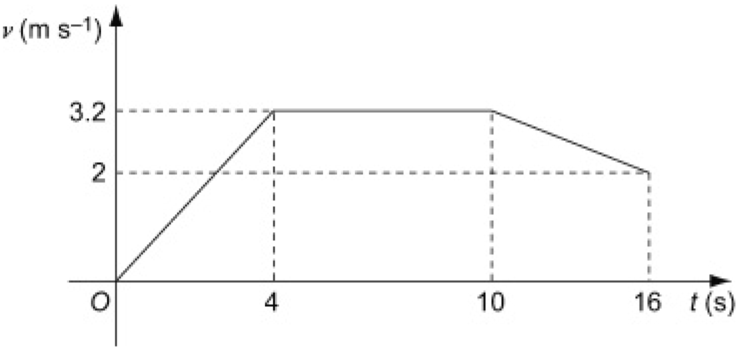


The diagram is an acceleration-time graph to show the motion of a particle. At time $t = 0$ s, the particle is at rest. Sketch a speed-time graph for the motion of the particle.

Solution:

From $t = 0$ to $t = 4$ $\quad v = u + at$
$= 0 + 0.8 \times 4 = 3.2$

From $t = 10$ to $t = 16$ $\quad v = u + at$
$= 3.2 - 0.2 \times 6 = 2$



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Kinematics of a particle moving in a straight line
Exercise D, Question 11

Question:

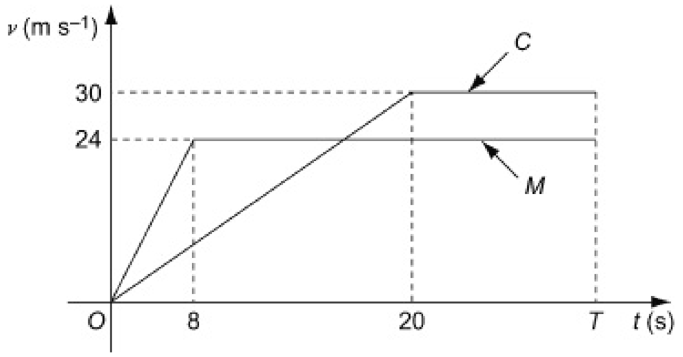
A motorcyclist *M* leaves a road junction at time $t = 0$ s. She accelerates at a rate of 3 m s^{-2} for 8 s and then maintains the speed she has reached. A car *C* leaves the same road junction as *M* at time $t = 0$ s. The car accelerates from rest to 30 m s^{-1} in 20 s and then maintains the speed of 30 m s^{-1}. *C* passes *M* as they both pass a pedestrian.

a On the same diagram, sketch speed-time graphs to illustrate the motion of *M* and *C*.

b Find the distance of the pedestrian from the road junction.

Solution:

a For *M*, $v = u + at = 0 + 3 \times 8 = 24$



b Let *C* overtake *M* at time *T* seconds.

The distance travelled by *M* is given by

$$s = \frac{1}{2}(a+b)h = \frac{1}{2}(T-8+T) \times 24 = 12(2T-8)$$

The distance travelled by *C* is given by

$$s = \frac{1}{2}(a+b)h = \frac{1}{2}(T-20+T) \times 30 = 15(2T-20)$$

At the point of overtaking the distances are equal

$$12(2T-8) = 15(2T-20)$$
$$24T - 96 = 30T - 300$$
$$6T = 204$$
$$T = \frac{204}{6} = 34$$

$$s = 12(2T-8) = 12(2 \times 34 - 8) = 720$$

The distance of the pedestrian from the road junction is 720 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise D, Question 12

Question:

A particle is moving on an axis Ox. From time $t = 0$ s to time $t = 32$ s, the particle is travelling with constant speed 15 m s^{-1}. The particle then decelerates from 15 m s^{-1} to rest in T seconds.

a Sketch a speed-time graph to illustrate the motion of the particle.

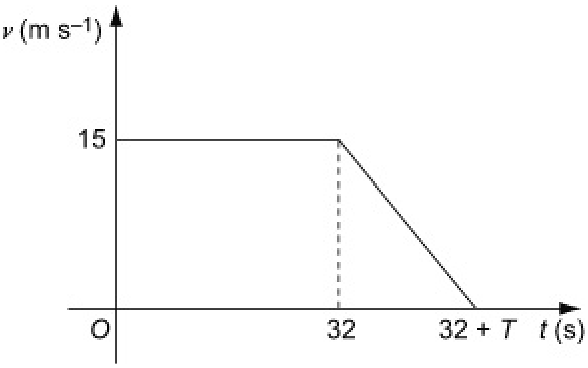
The total distance travelled by the particle is 570 m.

b Find the value of T.

c Sketch a distance-time graph illustrating the motion of the particle.

Solution:

a



b

$$s = \frac{1}{2}(a + b)h$$

$$570 = \frac{1}{2}(32 + 32 + T) \times 15$$

$$\frac{15}{2}(T + 64) = 570$$

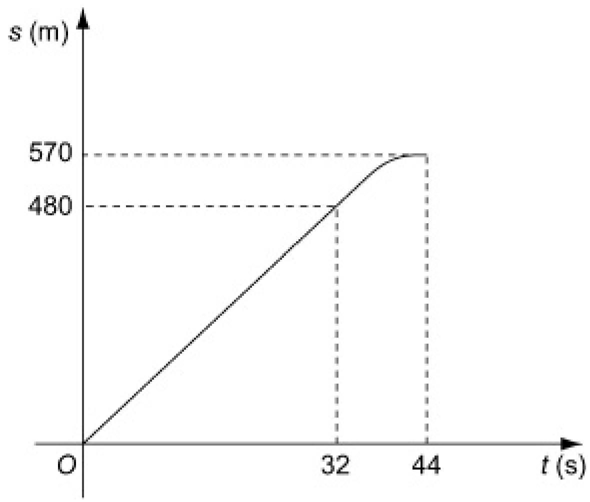
$$T + 64 = \frac{570 \times 2}{15} = 76$$

$$T = 76 - 64 = 12$$

c At $t = 32$, $s = 32 \times 15 = 480$

At $t = 44$, $s = 480 +$ area of the triangle

$$= 480 + \frac{1}{2} \times 12 \times 15 = 570$$


s (m)
570
480
O
32
44
t (s)

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 1

Question:

A car travelling along a straight road at 14 m s^{-1} is approaching traffic lights. The driver applies the brakes and the car comes to rest with constant deceleration. The distance from the point where the brakes are applied to the point where the car comes to rest is 49 m. Find the deceleration of the car.

Solution:

$$u = 14, v = 0, s = 49, a = ?$$
$$v^2 = u^2 + 2as$$
$$0^2 = 14^2 + 2 \times a \times 49$$
$$a = -\frac{14^2}{2 \times 49} = -2$$

The deceleration of the car is 2 m s^{-2}.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 2

Question:

A ball is thrown vertically downward from the top of a tower with speed 6 m s^{-1}. The ball strikes the ground with speed 25 m s^{-1}. Find the time the ball takes to move from the top of the tower to the ground.

Solution:

Take downwards as the positive direction.

$$u = 6, v = 25, a = 9.8, t = \ ?$$

$$v = u + at$$

$$25 = 6 + 9.8t$$

$$t = \frac{25 - 6}{9.8} = 1.938 \ldots \approx 1.9$$

The ball takes 1.9 s (2 s.f.) to move from the top of the tower to the ground.

Solutionbank M1
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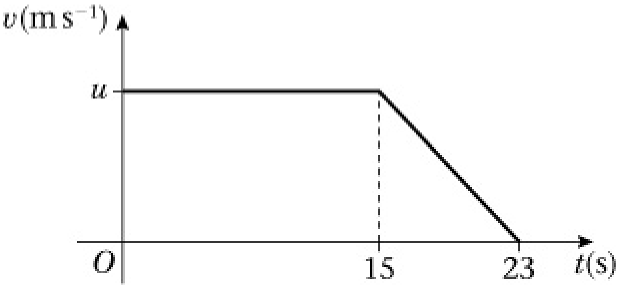
Kinematics of a particle moving in a straight line
Exercise E, Question 3

Question:



The diagram is a speed-time graph representing the motion of a cyclist along a straight road. At time t = 0 s, the cyclist is moving with speed u m s $^{-1}$. The speed is maintained until time t = 15 s, when she slows down with constant deceleration, coming to rest when t = 23 s. The total distance she travels in 23 s is 152 m. Find the value of u.

Solution:

$$s = \frac{1}{2}(a+b)h$$

$$152 = \frac{1}{2}(15+23)u = 19u$$

$$u = \frac{152}{19} = 8$$

Solutionbank M1

Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line

Exercise E, Question 4

Question:

A stone is projected vertically upwards with speed 21 m s^{-1}. Find

a the greatest height above the point of projection reached by the stone,

b the time between the instant that the stone is projected and the instant that it reaches its greatest height.

Solution:

a

Take upwards as the positive direction.

$$u = 21, v = 0, a = -9.8, s = ?$$
$$v^2 = u^2 + 2as$$
$$0^2 = 21^2 - 2 \times 9.8 \times s$$
$$s = \frac{21^2}{2 \times 9.8} = 22.5$$

The greatest height above the point of projection reached is 23 m (2 s.f.).

b

$$u = 21, v = 0, a = -9.8, t = ?$$
$$v = u + at$$
$$0 = 21 - 9.8t$$
$$t = \frac{21}{9.8} = 2.14 \ldots \approx 2.1$$

The time between the instant that the stone is projected and the instant that it reaches its greatest height is 2.1 s (2 s.f.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 5

Question:

A train is travelling with constant acceleration along a straight track. At time $t = 0$ s, the train passes a point O travelling with speed 18 m s^{-1}. At time $t = 12$ s, the train passes a point P travelling with speed 24 m s^{-1}. At time $t = 20$ s, the train passes a point Q. Find

a the speed of the train at Q,

b the distance from P to Q.

Solution:

a From O to P

$$u = 18, v = 24, t = 12, a = ?$$
$$v = u + at$$
$$24 = 18 + 12a$$
$$a = \frac{24 - 18}{12} = \frac{1}{2}$$

From O to Q

$$u = 18, t = 20, a = \frac{1}{2}, v = ?$$
$$v = u + at$$
$$= 18 + \frac{1}{2} \times 20 = 28$$

The speed of the train at Q is 28 m s^{-1}.

b From P to Q

$$u = 24, v = 28, t = 8, s = ?$$
$$s = \left(\frac{u + v}{2}\right) t$$
$$= \left(\frac{24 + 28}{2}\right) \times 8 = 208$$

The distance from P to Q is 208 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 6

Question:

A car travelling on a straight road slows down with constant deceleration. The car passes a road sign with speed 40 km h^{-1} and a post box with speed of 24 km h^{-1}. The distance between the road sign and the post box is 240 m. Find, in m s^{-2}, the deceleration of the car.

Solution:

$$40 \text{ km h}^{-1} = \frac{40 \times 1000}{3600} \text{ m s}^{-1} = \frac{100}{9} \text{ m s}^{-1}$$

$$24 \text{ km h}^{-1} = \frac{24 \times 1000}{3600} \text{ m s}^{-1} = \frac{20}{3} \text{ m s}^{-1}$$

$$u = \frac{100}{9}, v = \frac{20}{3}, s = 240, a = \ ?$$

$$v^2 = u^2 + 2as$$

$$\left(\frac{20}{3}\right)^2 = \left(\frac{100}{9}\right)^2 + 2 \times a \times 240$$

$$a = \left(\frac{\left(\frac{20}{3}\right)^2 - \left(\frac{100}{9}\right)^2}{2 \times 240}\right) = -0.1646 \ldots$$

The deceleration of the car is 0.165 m s^{-2} (3 d.p.).

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 7

Question:

A skier is travelling downhill along a straight path with constant acceleration. At time $t = 0$ s, she passes a point A with speed 6 m s^{-1}. She continues with the same acceleration until she reaches a point B with speed 15 m s^{-1}. At B, the path flattens out and she travels from B to a point C at the constant speed of 15 m s^{-1}. It takes 20 s for the skier to travel from B to C and the distance from A to C is 615 m.

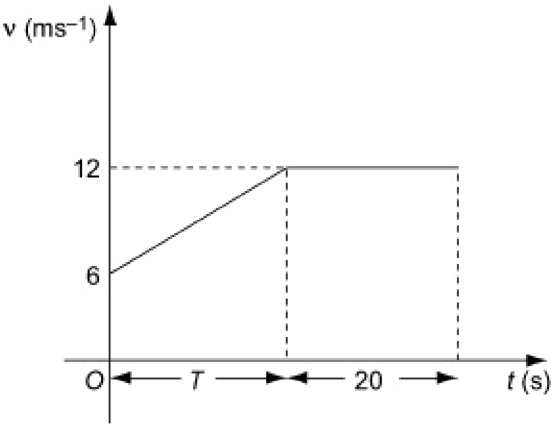
a Sketch a speed-time graph to illustrate the motion of the skier.

b Find the distance from A to B.

c Find the time the skier took to travel from A to B.

Solution:

a



b The distance from B to C is given by distance = speed × time = 15 × 20 = 300 (m)

The distance from A to B is (615 − 300) m = 315 m.

c

Let T seconds be the time for which the skier is accelerating.

$$s = \frac{1}{2}(a+b)h$$

$$315 = \frac{1}{2}(6+15)T = \frac{21}{2}T$$

$$T = \frac{315 \times 2}{21} = 30$$

The time the skier took to travel from A to B is 30 s.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 8

Question:

A child drops a ball from a point at the top of a cliff which is 82 m above the sea. The ball is initially at rest. Find

a the time taken for the ball to reach the sea,

b the speed with which the ball hits the sea.

c State one physical factor which has been ignored in making your calculation.

Solution:

a

Take downwards as the positive direction.

$$u = 0, s = 82, a = 9.8, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$82 = 0 + 4.9t^2$$

$$t^2 = \frac{82}{4.9} = 16.73\ldots$$

$$t = \sqrt{16.73\ldots} \approx 4.1$$

The time taken for the ball to reach the sea is 4.1 s (2 s.f.).

b

$$u = 0, s = 82, a = 9.8, v = ?$$

$$v^2 = u^2 + 2as$$

$$= 0^2 + 2 \times 9.8 \times 82 = 1607.2$$

$$v = \sqrt{1607.2} \approx 40.09$$

The speed with which the ball hits the sea is 40 m s^{-1} (2 s.f.).

c Air resistance

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 9

Question:

A particle moves along a straight line, from a point *X* to a point *Y*, with constant acceleration. The distance from *X* to *Y* is 104 m. The particle takes 8 s to move from *X* to *Y* and the speed of the particle at *Y* is 18 m s^{-1}. Find

a the speed of the particle at *X*,

b the acceleration of the particle.

The particle continues to move with the same acceleration until it reaches a point *Z*. At *Z* the speed of the particle is three times the speed of the particle at *X*.

c Find the distance *XZ*.

Solution:

a

$$s = 104, t = 8, v = 18, u = ?$$

$$s = \left(\frac{u+v}{2}\right) t$$

$$104 = \left(\frac{u+18}{2}\right) \times 8 = (u+18)\,4 = 4u + 72$$

$$u = \frac{104 - 72}{4} = 8$$

The speed of the particle at *X* is 8 m s^{-1}.

b

$$s = 104, t = 8, v = 18, a = ?$$

$$s = vt - \frac{1}{2}at^2$$

$$104 = 18 \times 8 - \frac{1}{2}a \times 8^2 = 144 - 32a$$

$$a = \frac{144 - 104}{32} = 1.25$$

The acceleration of the particle is 1.25 m s^{-2}.

c From *X* to *Z*

$u = 8, v = 24, a = 1.25, s = \ ?$

$v^2 = u^2 + 2as$

$24^2 = 8^2 + 2 \times 1.25 \times s$

$s = \frac{24^2 - 8^2}{2.5} = 204.8$

$XZ = 204.8$ m

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 10

Question:

A pebble is projected vertically upwards with speed 21 m s^{-1} from a point 32 m above the ground. Find

a the speed with which the pebble strikes the ground,

b the total time for which the pebble is more than 40 m above the ground.

Solution:

a

Take upwards as the positive direction.

$$u = 21, s = -32, a = -9.8, v = ?$$
$$v^2 = u^2 + 2as$$
$$= 21^2 + 2 \times (-9.8) \times (-32) = 441 + 627.2 = 1068.2$$
$$v = \sqrt{1068.2} = 32.68 \ldots$$

the speed with which the pebble strikes the ground is 33 m s^{-1} (2 s.f.).

b

40 m above the ground is 8 m above the point of projection.

$$u = 21, s = 8, a = -9.8, t = ?$$
$$s = ut + \frac{1}{2}at^2$$
$$8 = 21t - 4.9t^2$$

$$4.9t^2 - 21t + 8 = 0$$
$$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} = \frac{21 \pm \sqrt{(21^2 - 4 \times 4.9 \times 8)}}{9.8} = \frac{21 \pm \sqrt{284.2}}{9.8} = 3.86, 0.42$$

the pebble is above 40 m between these times; 3.86 − 0.42 = 3.44

the pebble is more than 40 m above the ground for 3.4 s (2 s.f.)

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Kinematics of a particle moving in a straight line
Exercise E, Question 11

Question:

A particle P is moving along the x-axis with constant deceleration 2.5 m s^{-2}. At time $t = 0$ s, P passes through the origin with velocity 20 m s^{-1} in the direction of x increasing. At time $t = 12$ s, P is at the point A. Find

a the distance OA,

b the total distance P travels in 12 s.

Solution:

a

$a = -2.5, u = 20, t = 12, s = ?$

$s = ut + \frac{1}{2}at^2$

$= 20 \times 12 - \frac{1}{2} \times 2.5 \times 12^2 = 240 - 180 = 60$

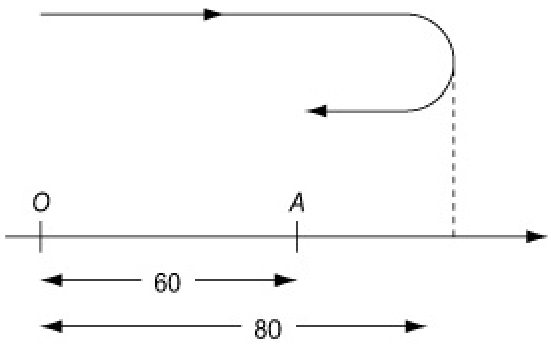
$OA = 60$ m

b The particle will turn round when $v = 0$.

$a = -2.5, u = 20, v = 0, s = ?$

$v^2 = u^2 + 2as$

$0^2 = 20^2 - 5s \Rightarrow s = 80$



The total distance P travels is $(80 + 20)$ m = 100 m.

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Kinematics of a particle moving in a straight line
Exercise E, Question 12

Question:

A train starts from rest at a station *P* and moves with constant acceleration for 45 s reaching a speed of 25 m s^{-1}. The train then maintains this speed for 4 minutes. The train then uniformly decelerates, coming to rest at a station *Q*.

a Sketch a speed-time graph illustrating the motion of the train from *P* to *Q*.

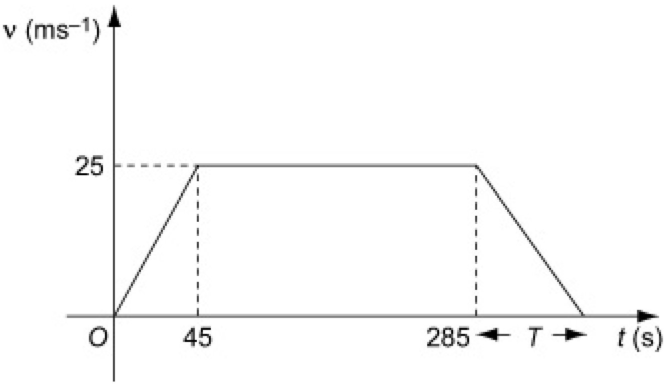
The distance between the stations is 7 km.

b Find the deceleration of the train.

c Sketch an acceleration-time graph illustrating the motion of the train from *P* to *Q*.

Solution:

a



b

$$s = \frac{1}{2}(a+b)h$$

$$7000 = \frac{1}{2}(240 + 285 + T) \times 25$$

$$525 + T = \frac{7000 \times 2}{25} = 560 \Rightarrow T = 35$$

The deceleration in the last 35 s is given by the gradient of the line.

$$a = \frac{-25}{35} = -\frac{5}{7}$$

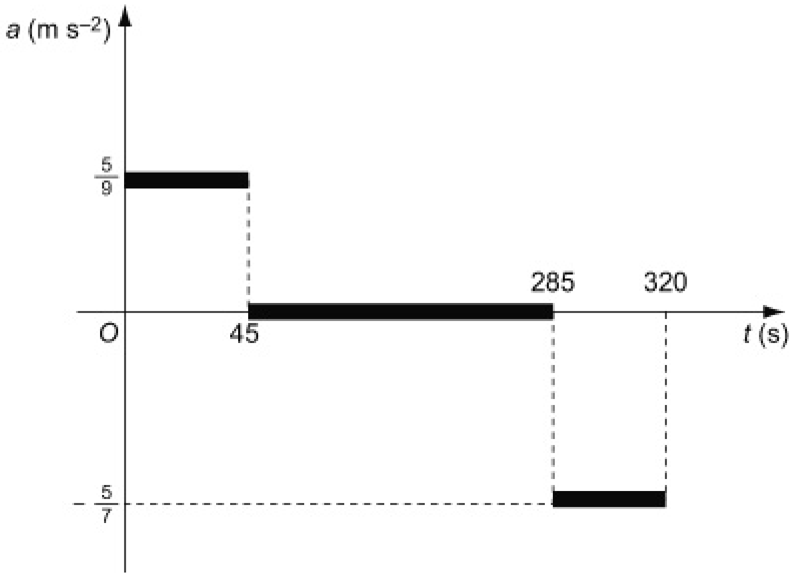
the deceleration of the train is $\frac{5}{7}$ m s^{-2}

c

The acceleration of the train in the first 40

s is given by the gradient of the line.

$a = \frac{25}{45} = \frac{5}{9}$



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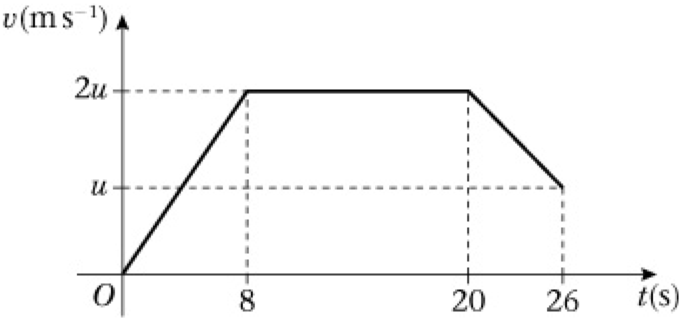
Kinematics of a particle moving in a straight line
Exercise E, Question 13

Question:



A particle moves 451 m in a straight line. The diagram shows a speed-time graph illustrating the motion of the particle. The particle starts at rest and accelerates at a constant rate for 8 s reaching a speed of $2u$ m s^{-1}. This speed is then maintained until $t = 20$ s. The particle then decelerates to a speed of u m s^{-1} at time $t = 26$ s. Find

a the value of u,

b the distance moved by the particle while its speed is less than u m s^{-1}.

Solution:

a

distance = area of triangle + area of rectangle + area of trapezium

$$451 = \frac{1}{2} \times 8 \times 2u + 12 \times 2u + \frac{1}{2} \times (u + 2u) \times 6$$
$$= 8u + 24u + 9u = 41u$$
$$u = \frac{451}{41} = 11$$

b

The particle is moving with speed less than u m s^{-1} for the first 4 s.

$$s = \frac{1}{2} \times 4 \times 11 = 22$$

The distance moved with speed less than 11 m s^{-1} is 22 m.

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Kinematics of a particle moving in a straight line
Exercise E, Question 14

Question:

A particle is moving in a straight line. The particle starts with speed 5 m s^{-1} and accelerates at a constant rate of 2 m s^{-1} for
8 s. It then decelerates at a constant rate coming to rest in a further 12 s.

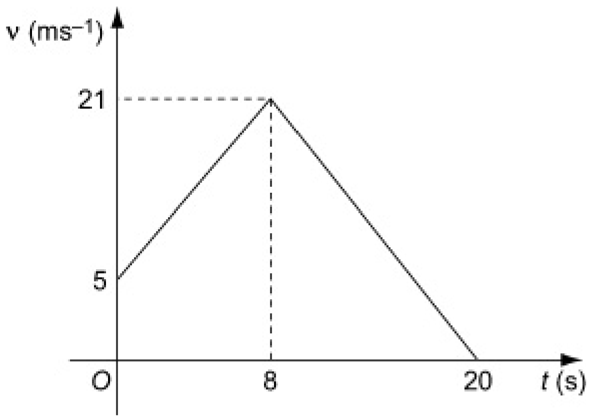
a Sketch a speed-time graph illustrating the motion of the particle.

b Find the total distance moved by the particle during its 20 s of motion.

c Sketch a distance-time graph illustrating the motion of the particle.

Solution:

a



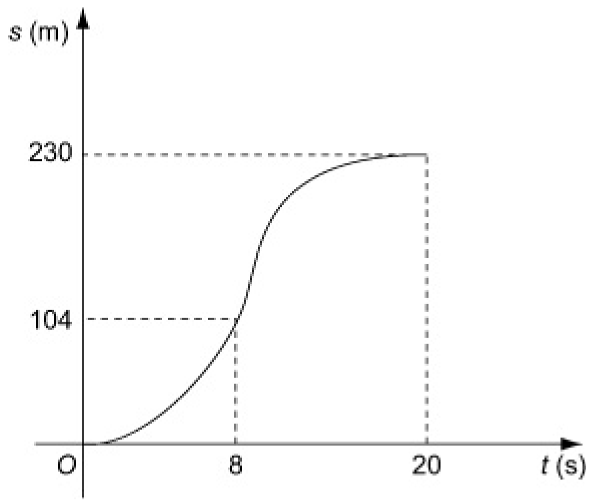
$v = u + at$
$v = 5 + 2 \times 8$
$= 21 \text{ m s}^{-1}$

b
distance = area of trapezium + area of triangle

$$s = \frac{1}{2}(5 + 21) \times 8 + \frac{1}{2} \times 12 \times 21 = 104 + 126 = 230$$

The distance moved in 20 s is 230 m.

c After 8 s, $s = 104$


s (m)
230
104
O
8
20
t (s)

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Kinematics of a particle moving in a straight line
Exercise E, Question 15

Question:

A boy projects a ball vertically upwards with speed 10 m s $^{-1}$ from a point *X*, which is 50 m above the ground. *T* seconds after the first ball is projected upwards, the boy drops a second ball from *X*. Initially the second ball is at rest. The balls collide
25 m above the ground. Find the value of *T*.

Solution:

Find the time taken by the first ball to reach 25 m below its point of projection, take upwards as the positive direction.

$u = 10, s = -25, a = -9.8, t = ?$

$s = ut + \frac{1}{2}at^2$

$-25 = 10t - 4.9t^2$

$4.9t^2 - 10t - 25 = 0$

$t = \frac{10 + \sqrt{(10^2 + 4 \times 4.9 \times 25)}}{9.8} = 3.4989 \ldots$

The negative solution is not possible.

Find the time taken by the second ball to reach 25 m below its point of projection. Take downwards as the positive direction.

$u = 0, s = 25, a = 9.8, t = t'$

$s = ut + \frac{1}{2}at^2$

$25 = 4.9t'^2 \Rightarrow t' \approx 2.2587 \ldots$

$T = t - t' = 1.240 \ldots = 1.2 \text{ s } (2 \text{ s.f.})$

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Kinematics of a particle moving in a straight line
Exercise E, Question 16

Question:

A car is moving along a straight road with uniform acceleration. The car passes a check-point A with speed 12 m s^{-1} and another check-point C with speed 32 m s^{-1}. The distance between A and C is 1100 m.

a Find the time taken by the car to move from A to C.

Given that B is the mid-point of AC,

b find the speed with which the car passes B.

Solution:

a

$$u = 12, v = 32, s = 1100, t = ?$$
$$s = \left(\frac{u+v}{2}\right)t$$

$$1100 = \frac{1}{2}(12 + 32)t = 22t$$

$$t = \frac{1100}{22} = 50$$

The time taken by the car to move from A to C is 50 s.

b

Find a first.

From A to C

$$u = 12, v = 32, s = 1100, a = ?$$
$$v^2 = u^2 + 2as$$
$$32^2 = 12^2 + 2a \times 1100$$
$$a = \frac{32^2 - 12^2}{2200} = 0.4$$

From A to B

$$u = 12, s = 550, a = 0.4, v = ?$$
$$v^2 = u^2 + 2as$$
$$= 12^2 + 2 \times 0.4 \times 550 = 584 \Rightarrow v = 24.166\ldots$$

The car passes C with speed 24.2 m s^{-1} (3 s.f.).

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Kinematics of a particle moving in a straight line
Exercise E, Question 17

Question:

A particle is projected vertically upwards with a speed of 30 m s^{-1} from a point *A*. The point *B* is *h* metres above *A*. The particle moves freely under gravity and is above *B* for a time 2.4 s. Calculate the value of *h*.

Solution:

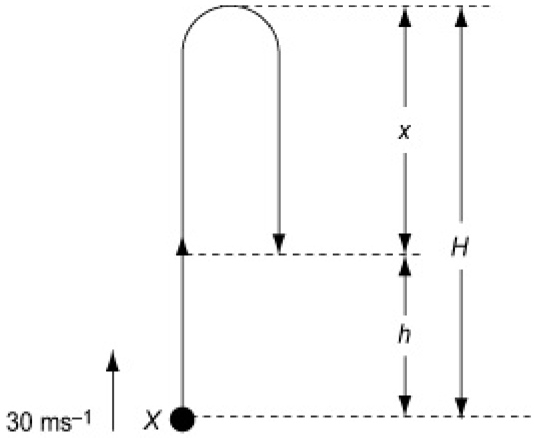
To find greatest height reached *H*, take upwards as the positive direction.

$u = 30, v = 0, a = -9.8, s = H$

$v^2 = u^2 + 2as$

$0^2 = 30^2 - 2 \times 9.8 \times H$

$H = \frac{30^2}{2 \times 9.8} \approx 45.918 \ldots$



By symmetry, the particle takes $\frac{2.4}{2} = 1.2$ s to fall the distance, say *x*, from the highest point to a point *h* m above the point of projection.

Take downwards as the positive direction.

$u = 0, t = 1.2, a = 9.8, s = x$

$s = ut + \frac{1}{2}at^2$

$x = 0 + 4.9 \times 1.2^2 = 7.056$

From diagram,

$h = H - x = 45.918 - 7.056 = 38.862$

$h = 39$ (2 s.f.)

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Kinematics of a particle moving in a straight line
Exercise E, Question 18

Question:

Two cars *A* and *B* are moving in the same direction along a straight horizontal road. At time $t = 0$, they are side by side, passing a point *O* on the road. Car *A* travels at a constant speed of 30 m s^{-1}. Car *B* passes *O* with a speed of 20 m s^{-1}, and has constant acceleration of 4 m s^{-2}. Find

a the speed of *B* when it has travelled 78 m from *O*,

b the distance from *O* of *A* when *B* is 78 m from *O*,

c the time when *B* overtakes *A*.

Solution:

a

$$u = 20, a = 4, s = 78, v = ?$$
$$v^2 = u^2 + 2as$$
$$= 20^2 + 2 \times 4 \times 78 = 1024$$
$$v = \sqrt{1024} = 32$$

The speed of *B* when it has travelled 78 m is 32 m s^{-1}.

b

Find time for *B* to reach the point 78 m from *O*.

$$v = 32, u = 20, a = 4, t = ?$$
$$v = u + at$$
$$32 = 20 + 4t \Rightarrow t = \frac{32 - 20}{4} = 3$$

For *A*, distance = speed × time

$s = 30 \times 3 = 90$

The distance from *O* of *A* when *B* is 78 m from *O* is 90 m.

c At time *t* seconds, for *A*, $s = 30t$

$$\text{for } B, s = ut + \frac{1}{2}at^2 = 20t + 2t^2$$

On overtaking the distances are the same.

$$20t + t^2 = 30t$$
$$t^2 - 10t = t(t - 10) = 0$$

For overtaking $t > 0$, $t = 10$

B overtakes *A* 10 s after passing *O*.

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Kinematics of a particle moving in a straight line
Exercise E, Question 19

Question:

A car is being driven on a straight stretch of motorway at a constant speed of 34 m s $^{-1}$, when it passes a speed restriction sign *S* warning of road works ahead and requiring speeds to be reduced to 22 m s $^{-1}$. The driver continues at her speed for 2 s after passing *S*. She then reduces her speed to 22 m s $^{-1}$ with constant deceleration of 3 m s $^{-2}$, and continues at the lower speed.

a Draw a speed-time graph to illustrate the motion of the car after it passes *S*.

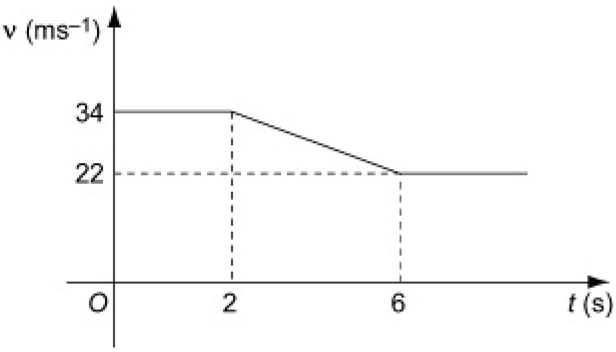
b Find the shortest distance before the road works that *S* should be placed on the road to ensure that a car driven in this way has had its speed reduced to 22 m s $^{-1}$ by the time it reaches the start of the road works.

Solution:

a

To find time decelerating.

$$u = 34, v = 22, a = -3, t = ?$$
$$v = u + at$$
$$22 = 34 - 3t \Rightarrow t = \frac{34 - 22}{3} = 4$$



b

distance = rectangle + trapezium

$$s = 34 \times 2 + \frac{1}{2}(22 + 34) \times 4$$
$$= 68 + 112 = 180$$

Distance required is 180 m.

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Kinematics of a particle moving in a straight line
Exercise E, Question 20

Question:

A train starts from rest at station *A* and accelerates uniformly at $3x$ m s $^{-2}$ until it reaches a speed of 30 m s $^{-1}$. For the next
T seconds the train maintains this constant speed. The train then retards uniformly at x m s $^{-2}$ until it comes to rest at a station *B*. The distance between the stations is 6 km and the time taken from *A* to *B* is 5 minutes.

a Sketch a speed-time graph to illustrate this journey.

b Show that $\frac{40}{x} + T = 300$.

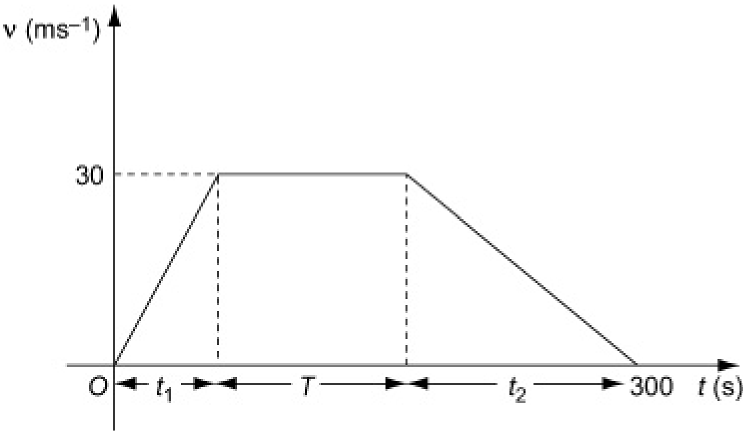
c Find the value of *T* and the value of *x*.

d Calculate the distance the train travels at constant speed.

e Calculate the time taken from leaving *A* until reaching the point half-way between the stations.

Solution:

a



b

Acceleration is the gradient of a line.

$3x = \frac{30}{t_1} \Rightarrow t_1 = \frac{30}{3x} = \frac{10}{x}$

$-x = \frac{-30}{t_2} \Rightarrow t_2 = \frac{30}{x}$

$t_1 + T + t_2 = 300$

$\frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300$, as required

c $s = \frac{1}{2}(a+b)h$

$6000 = \frac{1}{2}(T + 300) \times 30 = 15T + 4500$

$T = \frac{6000 - 4500}{15} = 100$

Substitute into the result in part **b**.

$\frac{40}{x} + 100 = 300 \quad \Rightarrow \quad \frac{40}{x} = 200$

$x = \frac{40}{200} = 0.2$

d at constant speed, distance = speed × time = 30 × 100 = 3000 (m)

The distance travelled at a constant speed in 3 km.

e $t_1 = \frac{10}{x} = \frac{10}{0.2} = 50$

while accelerating, distance travelled is $\left(\frac{1}{2} \times 50 \times 30\right)$ m = 750 m

To reach half way, the train must travel 2250 m at a constant speed.

at constant speed, time = $\frac{\text{distance}}{\text{speed}} = \frac{2250}{30}$ s = 75 s

Time for train to reach half-way is (50 + 75) s = 125 s.