Exercise A, Question 1

Question:

Solve the following inequality

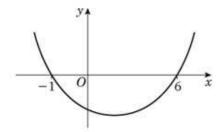
$$x^2 < 5x + 6$$

Solution:

$$x^2 - 5x - 6 < 0$$
$$(x - 6)(x + 1) < 0$$

critical values x = -1 or 6

sketch



solution is -1 < x < 6

Exercise A, Question 2

Question:

Solve the following inequality

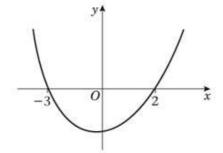
$$x(x+1) \ge 6$$

Solution:

$$x^2 + x \ge 6$$
$$(x+3)(x-2) \ge 0$$

critical values x = 2 or -3

sketch



solution is $x \ge 2$ or $x \le -3$

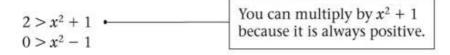
Exercise A, Question 3

Question:

Solve the following inequality

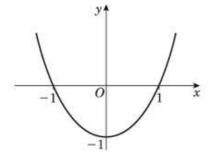
$$\frac{2}{x^2+1} > 1$$

Solution:



critical values $x = \pm 1$

sketch



solution is -1 < x < 1

Exercise A, Question 4

Question:

Solve the following inequality

$$\frac{2}{x^2-1} > 1$$

Solution:

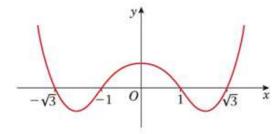
$$\frac{2}{(x^2-1)} \times (x^2-1)^2 > (x^2-1)^2$$

$$0 > (x^2-1)[x^2-1-2]$$

$$0 > (x-1)(x+1)(x-\sqrt{3})(x+\sqrt{3})$$

critical values $x = \pm 1, \pm \sqrt{3}$

sketch



solution is $-\sqrt{3} < x < -1$ or $1 < x < \sqrt{3}$

Exercise A, Question 5

Question:

Solve the following inequality

$$\frac{x}{x-1} \le 2x \quad x \ne 1$$

Solution:

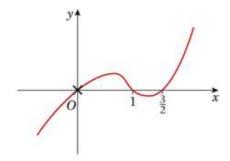
$$\frac{x}{(x-1)} \times (x-1)^{2} \le 2x(x-1)^{2}$$

$$0 \le x(x-1)[2x-2-1]$$

$$0 \le x(x-1)(2x-3)$$

critical values $x = 0, 1, \frac{3}{2}$

sketch



solution is $x > \frac{3}{2}$ or 0 < x < 1

Exercise A, Question 6

Question:

Solve the following inequality

$$\frac{3}{x+1} < \frac{2}{x}$$

Solution:

$$\frac{3}{(x+1)} \times (x+1)^2 x^2 < \frac{2}{x} \times (x+1)^2 x^2$$

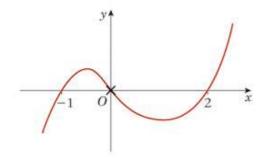
$$x(x+1)[3x - 2(x+1)] < 0$$

$$x(x+1)(x-2) < 0$$

critical values x = 0, -1, 2

$$x = 0, -1, 2$$

sketch



solution is x < -1 or 0 < x < 2

Exercise A, Question 7

Question:

Solve the following inequality

$$\frac{3}{(x+1)(x-1)} < 1$$

Solution:

$$\frac{3}{(x+1)(x-1)} \times (x+1)^{2}(x-1)^{2} < (x+1)^{2}(x-1)^{2}$$

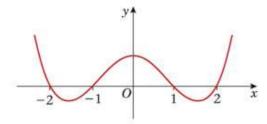
$$0 < (x+1)(x-1)[x^{2}-1-3]$$

$$0 < (x+1)(x-1)(x-2)(x+2)$$

critical values

$$x = \pm 1, \pm 2$$

sketch



solution is x < -2 or -1 < x < 1 or x > 2

Exercise A, Question 8

Question:

Solve the following inequality

$$\frac{2}{x^2} \ge \frac{3}{(x+1)(x-2)}$$

Solution:

$$\frac{2}{x^2} \times (x+1)^2 (x-2)^2 \ge \frac{3(x+1)^2 (x-2)^2}{(x+1)(x-2)}$$

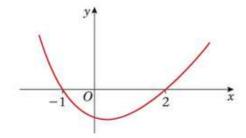
$$(x+1)(x-2)[2x^2 - 2x - 4 - 3x^2] \ge 0 \qquad \text{You can multiply across } x^2 \text{ since it is positive.}$$

$$(x+1)(x-2)(-4-2x-x^2) \ge 0$$
or
$$0 \ge (x+1)(x-2)(x^2+2x+4)$$

 $x^2 + 2x + 4$ has no real roots

 \therefore critical values x = 2 or -1

sketch



solution is
$$-1 < x < 2$$
 $x \ne 0$
or $-1 < x < 0$ or $0 < x < 2$

NB x = 2 and x = -1, x = 0 are invalid in the original expression.

Exercise A, Question 9

Question:

Solve the following inequality

$$\frac{2}{x-4} < 3$$

Solution:

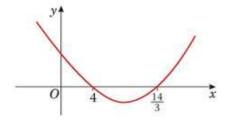
$$\frac{2}{x-4} \times (x-4)^{2} < 3(x-4)^{2}$$

$$0 < (x-4)[3x-12-2]$$

$$0 < (x-4)(3x-14)$$

critical values $x = 4, \frac{14}{3}$

sketch



solution is x < 4 or $x > \frac{14}{3}$

Exercise A, Question 10

Question:

Solve the following inequality

$$\frac{3}{x+2} > \frac{1}{x-5}$$

Solution:

$$\frac{3}{(x+2)} \times (x+2)^{2}(x-5)^{2} > \frac{1}{(x-5)} \times (x+2)^{2}(x-5)^{2}$$

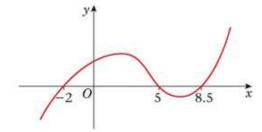
$$(x+2)(x-5)[3x-15-(x+2)] > 0$$

$$(x+2)(x-5)(2x-17) > 0$$

critical values

$$x = -2, 5, 8.5$$

sketch



solution is -2 < x < 5 or x > 8.5

Exercise A, Question 11

Question:

Solve the following inequality

$$\frac{3x^2 + 5}{x + 5} > 1$$

Solution:

$$\frac{3x^2 + 5}{(x + 5)^2} \times (x + 5)^2 > (x + 5)^2$$

$$(x + 5)[3x^2 + 5 - (x + 5)] > 0$$

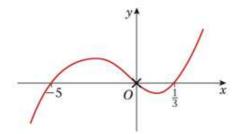
$$(x + 5)(3x^2 - x) > 0$$

$$(x + 5)x(3x - 1) > 0$$

critical values

$$x=0,\tfrac{1}{3},\,-5$$

sketch



solution is -5 < x < 0 or $x > \frac{1}{3}$

Exercise A, Question 12

Question:

Solve the following inequality

$$\frac{3x}{x-2} > x$$

Solution:

$$\frac{3x}{x-2} \times (x-2)^2 > x(x-2)^2$$

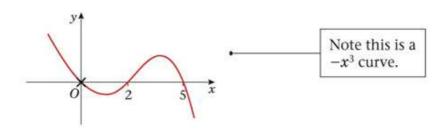
$$x(x-2)[3-(x-2)] > 0$$

$$x(x-2)(5-x) > 0$$

critical values x = 0, 2, 5

$$x = 0, 2, 5$$

sketch



solution is x < 0 or 2 < x < 5

Exercise A, Question 13

Question:

Solve the following inequality

$$\frac{1+x}{1-x} > \frac{2-x}{2+x}$$

Solution:

$$\frac{1+x}{1-x} \times (1-x)^{2}(2+x)^{2} > \frac{2-x}{2+x} \times (1-x)^{2}(2+x)^{2}$$

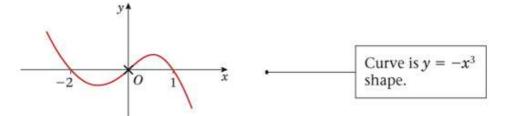
$$(1-x)(2+x)[(1+x)(2+x) - (2-x)(1-x)] > 0$$

$$(1-x)(2+x)(x^{2}+3x+2-(x^{2}-3x+2)) > 0$$

$$(1-x)(2+x)6x > 0$$

critical values x = 1, -2, 0

sketch



solution is x < -2 or 0 < x < 1

Exercise A, Question 14

Question:

Solve the following inequality

$$\frac{x^2 + 7x + 10}{x + 1} > 2x + 7$$

Solution:

$$\frac{x^2 + 7x + 10}{x + 1} \times (x + 1)^{2} > (2x + 7) \times (x + 1)^{2}$$

$$(x + 1)[x^2 + 7x + 10 - (2x + 7)(x + 1)] > 0$$

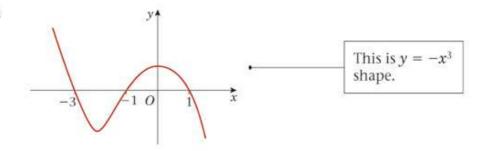
$$(x + 1)[x^2 + 7x + 10 - 2x^2 - 9^x - 7] > 0$$

$$(x + 1)(3 - 2x - x^2) > 0$$

$$(x + 1)(1 - x)(x + 3) > 0$$

critical values x = -1, 1, -3

sketch



solution is x < -3 or -1 < x < 1

Exercise A, Question 15

Question:

Solve the following inequalities

a
$$\frac{x+1}{x^2} > 6$$

b
$$\frac{x^2}{x+1} > \frac{1}{6}$$

Solution:

$$\frac{x+1}{x^2} > 6$$

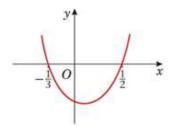
$$0 > 6x^2 - x - 1 \leftarrow$$

$$0 > (3x+1)(2x-1)$$
You can multiply by x^2 since it is > 0 .

critical values $x = -\frac{1}{3}, \frac{1}{2}$

$$x = -\frac{1}{3}, \frac{1}{2}$$

sketch



solution is
$$-\frac{1}{3} < x < \frac{1}{2}$$
 But $x \neq 0$
or $-\frac{1}{3} < x < 0$ or $0 < x < \frac{1}{2}$

$$\frac{x^2}{x+1} \times (x+1)^2 > \frac{1}{6}(x+1)^2$$

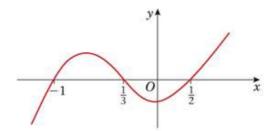
$$(x+1)[6x^2 - (x+1)] > 0$$

$$(x + 1)(3x + 1)(2x - 1) > 0$$

critical values

$$x = -1, \frac{1}{2}, -\frac{1}{3}$$

sketch



solution is $-1 < x < -\frac{1}{3}$ or $x > \frac{1}{2}$

Exercise B, Question 1

Question:

Solve the following inequality

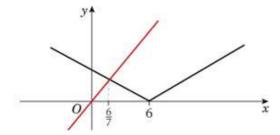
$$|x - 6| > 6x$$

Solution:

$$|x - 6| > 6x$$

$$x - 6 = 6x$$
 or $-(x - 6) = 6x$
 $\Rightarrow -6 = 5x$ $\Rightarrow 6 = 7x$
 $-1.2 = x$ $\Rightarrow \frac{6}{7} = x$

sketch



only $x = \frac{6}{7}$ is valid

solution is $x < \frac{6}{7}$

Exercise B, Question 2

Question:

Solve the following inequality

$$|t-3| > t^2$$

Solution:

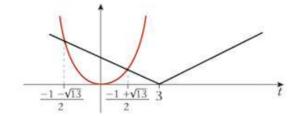
$$|t - 3| > t^2$$

$$t-3 = t^{2} \qquad \text{or} \qquad -(t-3) = t^{2}$$

$$\Rightarrow \qquad 0 = t^{2} - t + 3 \qquad \Rightarrow \qquad 0 = t^{2} + t - 3$$

$$t = \text{no solution} \qquad \qquad t = \frac{-1 \pm \sqrt{1 + 12}}{2}$$

sketch



$$|t - 3|$$
 is above t^2 for $\frac{-1 - \sqrt{13}}{2} < t < \frac{-1 + \sqrt{13}}{2}$

Exercise B, Question 3

Question:

Solve the following inequality

$$|(x-2)(x+6)| < 9$$

Solution:

$$|(x-2)(x+6)| < 9$$

$$x^{2} + 4x - 12 = 9$$

$$\Rightarrow x^{2} + 4x - 21 = 0$$

$$(x-3)(x+7) = 0$$

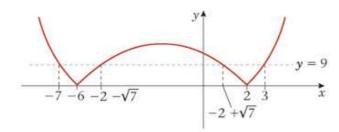
$$x = 3 \text{ or } -7$$
or
$$-(x^{2} + 4x - 12) = 9$$

$$0 = x^{2} + 4x - 3$$

$$x = \frac{-4 \pm \sqrt{16 + 12}}{2}$$

$$x = -2 \pm \sqrt{7}$$

sketch



Line y = 9 is above curve for $-7 < x < -2 - \sqrt{7}$ or $-2 + \sqrt{7} < x < 3$

Exercise B, Question 4

Question:

Solve the following inequality

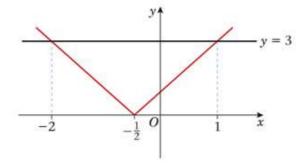
$$|2x+1| \ge 3$$

Solution:

$$|2x+1| \ge 3$$

$$2x + 1 = 3$$
 or $-(2x + 1) = 3$
 $\Rightarrow 2x = 2$ $-4 = 2x$
 $x = 1$ $-2 = x$

sketch



solution is y = 3 is below the **V** when

$$x \le -2 \text{ or } x \ge 1$$

Exercise B, Question 5

Question:

Solve the following inequality

$$|2x| + x > 3$$

Solution:

$$|2x| + x > 3$$

Rearrange: |2x| > 3 - x

$$2x = 3 - x \qquad \text{or} \qquad$$

$$-2x = 3 - x$$

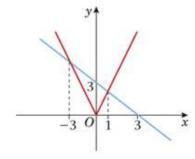
$$\Rightarrow$$
 3x = 3

$$-x = 3$$

$$\Rightarrow x = 1$$

$$x = -3$$

sketch



$$y = 3 - x$$
 is below \vee for

$$x < -3$$
 or $x > 1$

Exercise B, Question 6

Question:

Solve the following inequality

$$\frac{x+3}{|x|+1} < 2$$

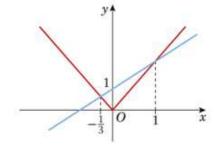
Solution:

 $\frac{x+3}{|x|+1} < 2$ Rearrange: x+3 < 2|x|+2 ...
i.e. x+1 < 2|x|

Because |x| + 1 is positive you can multiply across.

x + 1 = 2x or x + 1 = -2x $\Rightarrow 1 = x$ $\Rightarrow 3x = -1$ $x = -\frac{1}{3}$

sketch



Line y = x + 1 is below **V** when $x < -\frac{1}{3}$ or x > 1

Exercise B, Question 7

Question:

Solve the following inequality

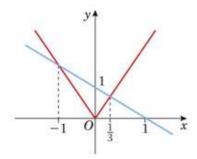
$$\frac{3-x}{|x|+1} > 2$$

Solution:

Rearrange: 3 - x > 2|x| + 2 1 - x > 2|x| 1 - x > 2|x|You can multiply by |x| + 1 since it is > 0.

 $1 - x = 2x \qquad \text{or} \qquad 1 - x = -2x$ $\Rightarrow \qquad 1 = 3x \qquad \qquad x = -1$ $\frac{1}{3} = x$

sketch



The line y = 1 - x is above the **V** for $-1 < x < \frac{1}{3}$

Exercise B, Question 8

Question:

Solve the following inequality

$$\left|\frac{x}{x+2}\right| < 1-x$$

Solution:

$$\left|\frac{x}{x+2}\right| < 1 - x$$

$$\frac{x}{x+2} = 1 - x \qquad \text{or} \qquad -\frac{x}{x+2} = 1 - x$$

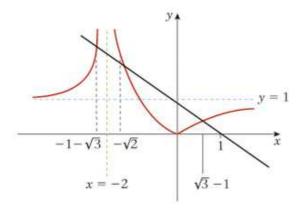
$$\Rightarrow \qquad x = (1-x)(x+2) \qquad -x = (1-x)(x+2)$$

$$x^2 + 2x - 2 = 0 \qquad x^2 - 2 = 0$$

$$x = \frac{-2 \pm \sqrt{12}}{2} \qquad x = \pm \sqrt{2}$$

$$x = -1 \pm \sqrt{3}$$

sketch



NB $x = +\sqrt{2}$ is invalid.

The line y = 1 - x is above the curve for $x < -1 - \sqrt{3}$

or
$$-\sqrt{2} < x < -1 + \sqrt{3}$$

Exercise B, Question 9

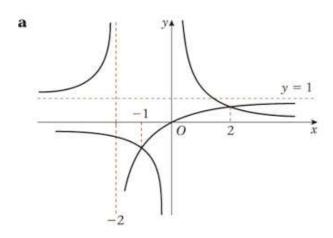
Question:

Solve the following inequalities

a On the same axes sketch the graphs of $y = \frac{1}{x}$ and $y = \frac{x}{x+2}$.

b Solve
$$\frac{1}{x} > \frac{x}{x+2}$$
.

Solution:



b
$$\frac{1}{x} = \frac{x}{x+2}$$
 \Rightarrow $x+2 = x^2$
i.e. $0 = x^2 - x - 2$
 $0 = (x-2)(x+1)$
 $x = 2 \text{ or } -1$

$$\frac{1}{x}$$
 is above $\frac{x}{x+2}$ for $-2 < x < -1$ or $0 < x < 2$

Exercise B, Question 10

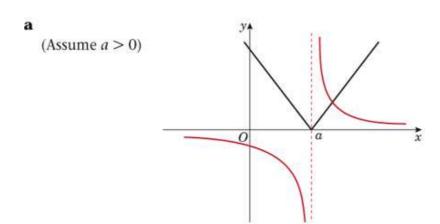
Question:

Solve the following inequalities

a On the same axes sketch the graphs of $y = \frac{1}{x - a}$ and y = 4|x - a|.

b Solve, giving your answers in terms of the constant a, $\frac{1}{x-a} < 4|x-a|$.

Solution:



Only this case needs to be considered because the right hand branch of V has the intersection.

$$\frac{1}{4} = (x - a)^2$$

$$\pm \frac{1}{2} = x - a$$

$$x = a \pm \frac{1}{2}$$
From sketch $x = a + \frac{1}{2}$.

V is above when x < a or $x > a + \frac{1}{2}$

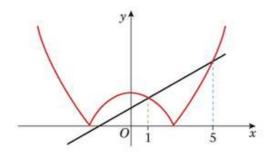
Exercise C, Question 1

Question:

Solve the inequality $|x^2 - 7| < 3(x + 1)$

Solution:

sketch:



$$|x^2 - 7| < 3(x + 1)$$

$$x^{2} - 7 = 3x + 3$$
 or $-(x^{2} - 7) = 3x + 3$
 $\Rightarrow x^{2} - 3x - 10 = 0$ $\Rightarrow 0 = x^{2} + 3x - 4$
 $(x - 5)(x + 2) = 0$ $0 = (x + 4)(x - 1)$
 $x = -2 \text{ or } 5$ $x = -4 \text{ or } 1$

From the sketch, only x = 1 and x = 5 are valid.

Line is above the curve for 1 < x < 5

Exercise C, Question 2

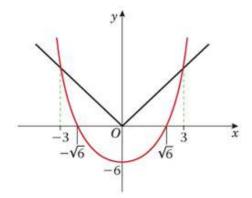
Question:

Solve the inequality $\frac{x^2}{|x|+6} < 1$

Solution:

Rearrange: $x^{2} < |x| + 6$ or $x^{2} - 6 < |x|$ Multiply by |x| + 6 since it is positive.

sketch:



$$x^2 - 6 = x$$
 or $x^2 - 6 = -x$
 $\Rightarrow x^2 - x - 6 = 0$ $x^2 + x - 6 = 0$
 $(x - 3)(x + 2) = 0$ $(x + 3)(x - 2) = 0$
 $x = -2 \text{ or } 3$ $x = 2 \text{ or } -3$

From the sketch the intersections are $> \sqrt{6}$ $\therefore x = \pm 3$

Curve is below V for -3 < x < 3

Exercise C, Question 3

Question:

Find the set of values of x for which |x-1| > 6x - 1

Solution:

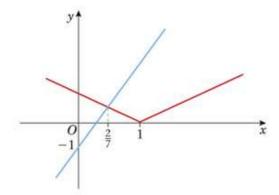
$$|x-1| > 6x-1$$

$$x - 1 = 6x - 1 \qquad \text{or} \qquad -(x - 1) = 6x - 1$$

$$\Rightarrow \qquad 0 = 5x \qquad \qquad 2 = 7x$$

$$\Rightarrow \qquad x = 0 \qquad \qquad \frac{2}{7} = x$$

sketch:



x = 0 is not valid so only critical value is $x = \frac{2}{7}$

V is above the line for $x < \frac{2}{7}$

Exercise C, Question 4

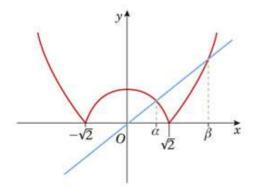
Question:

Find the complete set of values of x for which $|x^2 - 2| > 2x$

Solution:

$$|x^2 - 2| > 2x$$

sketch:



$$x^{2} - 2 = 2x$$

$$\Rightarrow x^{2} - 2x - 2 = 0$$

$$x = \frac{2 \pm \sqrt{12}}{2}$$

$$x = \frac{1}{2}$$

$$x = 1 \pm \sqrt{3}$$

 $-(x^2-2)=2x$

$$0 = x^2 + 2x - 2$$

$$x = \frac{-2 \pm \sqrt{12}}{2}$$

$$x = -1 \pm \sqrt{3}$$

 β is a solution of this equation α is a solution of this equation so must be $1 + \sqrt{3}$

so must be $\sqrt{3} - 1$

The line is below the curve for $x > 1 + \sqrt{3}$ or $x < \sqrt{3} - 1$

Exercise C, Question 5

Question:

Find the set of values of x for which $\frac{x+1}{2x-3} < \frac{1}{x-3}$

Solution:

$$\frac{x+1}{2x-3} \times (2x-3)^{2}(x-3)^{2} < \frac{1}{x-3} \times (2x-3)^{2}(x-3)^{2}$$

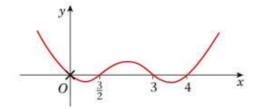
$$(2x-3)(x-3)[(x+1)(x-3) - (2x-3)] < 0$$

$$(2x-3)(x-3)(x^2-2x-x-2x+x)<0$$

$$(2x - 3)(x - 3)x(x - 4) < 0$$

critical values $x = \frac{3}{2}$, 3, 4, 0

sketch



$$0 < x < \frac{3}{2}$$
 or $3 < x < 4$

Exercise C, Question 6

Question:

Solve
$$\frac{(x+3)(x+9)}{x-1} > 3x-5$$

Solution:

$$\frac{(x+3)(x+9)}{x-1} \times (x-1)^{2} > (3x-5) \times (x-1)^{2}$$

$$(x-1)[x^{2}+12x+27-(3x^{2}-8x+5)] > 0$$

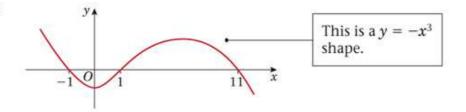
$$(x-1)(22+20x-2x^{2}) > 0$$

$$(x-1)(11+10x-x^{2}) > 0 \qquad \qquad \text{Divide by 2.}$$

$$(x-1)(11-x)(1+x) > 0$$

critical values x = 1, -1, 11

sketch:



$$x < -1$$
 or $1 < x < 11$

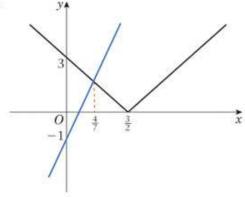
Exercise C, Question 7

Question:

a Sketch, on the same axes, the graph with equation y = |2x - 3|, and the line with equation

b Solve the inequality |2x - 3| < 5x - 1

Solution:



b |2x-3| < 5x-1

$$2x - 3 = 5x - 1$$
 or $-(2x - 3) = 5x - 1$

$$-(2x - 3) = 5x - 1$$

$$\Rightarrow$$
 $-2 = 3x$

$$4 = /x$$

$$-\frac{2}{3}=x$$

$$\frac{4}{7} = \lambda$$

From sketch this is not valid.

Line is above **V** for $x > \frac{4}{7}$

Solutionbank FP2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 8

Question:

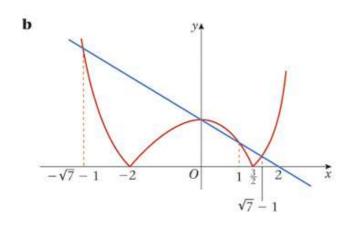
a Use algebra to find the exact solution of $|2x^2 + x - 6| = 6 - 3x$

b On the same diagram, sketch the curve with equation $y = |2x^2 + x - 6|$ and the line with equation y = 6 - 3x

c Find the set of values of x for which $|2x^2 + x - 6| > 6 - 3x$

Solution:

a
$$2x^2 + x - 6 = 6 - 3x$$
 or $-(2x^2 + x - 6) = 6 - 3x$
 $2x^2 + 4x - 12 = 0$ $0 = 2x^2 - 2x$
 $2(x^2 + 2x - 6) = 0$ $0 = 2x(x - 1)$
 $x = \frac{-2 \pm \sqrt{28}}{2}$ $x = 0 \text{ or } 1$
 $= -1 \pm \sqrt{7}$



$$2x^{2} + x - 6 = 0$$

$$(2x - 3)(x + 2) = 0$$

$$x = -2 \text{ or } \frac{3}{2}$$

c The line is below the curve for $x > \sqrt{7} - 1$ or 0 < x < 1 or $x < -\sqrt{7} - 1$

Solutionbank FP2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 9

Question:

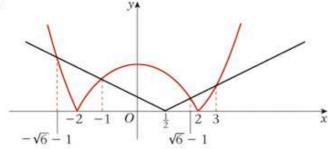
a On the same diagram, sketch the graphs of $y = |x^2 - 4|$ and y = |2x - 1|, showing the coordinates of the points where the graphs meet the *x*-axis.

b Solve $|x^2 - 4| = |2x - 1|$, giving your answers in surd form where appropriate.

c Hence, or otherwise, find the set of values of x for which $|x^2 - 4| > |2x - 1|$

Solution:

a



b
$$x^2 - 4 = 2x - 1$$

$$x^2 - 4 = -(2x - 1)$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow$$

$$x^2 + 2x - 5 = 0$$

$$(x-3)(x+1)=0$$

$$x = \frac{-2 \pm \sqrt{24}}{2}$$

$$x = -1$$
 or 3

$$x = -1 \pm \sqrt{6}$$

c V is below the curve for

$$|x^2 - 4| > |2x - 1|$$

when
$$x > 3$$
 or $-1 < x < \sqrt{6} - 1$ or $x < -\sqrt{6} - 1$