Solutionbank FP1
Edexcel AS and A Level Modular Mathematics

Numerical solutions of equations
Exercise A, Question 1

Question:

Use interval bisection to find the positive square root of $x^2 - 7 = 0$, correct to one decimal place.

Solution:

$x^2 - 7 = 0$

So roots lies between 2 and 3 as $f(2) = -3$ and $f(2) = +$ Using table method.

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\frac{(a+b)}{2}$
2	−3	3	+2	2.5	−0.75
2.5	−0.75	3	+2	2.75	0.5625
2.5	−0.75	2.75	0.5625	2.625	−0.109375
2.625	−0.109375	2.75	0.5625	2.6875	0.2226562
2.625	−0.109375	2.6875	0.2226562	2.65625	0.055664
2.625	−0.109375	2.65625	0.055664	2.640625	−0.0270996

Hence $x^2 - 7 = 0$ when $x = 2.6$ to 1decimal place

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Numerical solutions of equations
Exercise A, Question 2

Question:

a Show that one root of the equation $x^3 - 7x + 2 = 0$ lies in the interval [2, 3].

b Use interval bisection to find the root correct to two decimal places.

Solution:

a $f(2) = 8 - 14 + 2 = -4$ $\quad f(x) = x^3 - 7x + 2$

$f(3) = 27 - 21 + 2 = +8$

Hence change of sign, implies roots between 2 and 3.

b Using table method.

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	−4	3	+8	2.5	0.125
2	−4	2.5	0.125	2.25	−2.359375
2.25	−2.359375	2.5	0.125	2.375	−1.2285156
2.375	−1.2285156	2.5	0.125	2.4375	−0.5803222
2.4375	−0.5803222	2.5	0.125	2.46875	−0.2348938
2.46875	−0.2348938	2.5	0.125	2.484375	−0.0567665
2.484375	−0.0567665	2.5	0.125	2.4921875	0.0336604
2.484375	−0.0567665	2.4921875	0.0336604	2.4882813	−0.0116673

Hence $x = 2.49$ to 2 decimal places.

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Numerical solutions of equations
Exercise A, Question 3

Question:

a Show that the largest positive root of the equation $0 = x^3 + 2x^2 - 8x - 3$ lies in the interval [2, 3].

b Use interval bisection to find this root correct to one decimal place.

Solution:

a $f(2) = 8 + 8 - 16 - 3 = -3$ $\quad f(x) = x^3 + 2x^2 - 8x - 3$

$f(3) = 27 + 18 - 24 - 3 = 18$

Change of sign implies root in interval [2,3]

b

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	−3	3	18	2.5	5.125
2	−3	2.5	5.125	2.25	0.51562
2	−3	2.25	0.515625	2.125	−1.37304
2.125	−1.3730469	2.25	0.515625	2.1875	−0.46215

Hence solution = 2.2 to 1decimal place

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Numerical solutions of equations
Exercise A, Question 4

Question:

a Show that the equation $f(x) = 1 - 2\sin x$ has one root which lies in the interval [0.5, 0.8].

b Use interval bisection four times to find this root. Give your answer correct to one decimal place.

Solution:

a $f(0.5) = +0.0411489$

$f(0.8) = -0.4347121$

Change of sign implies root between 0.5 and 0.8

b

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$\frac{f(a+b)}{2}$
0.5	0.0411489	0.8	−0.4347121	0.65	−0.2103728
0.5	0.0411489	0.65	−0.2103728	0.575	−0.0876695
0.5	0.0411489	0.575	−0.0876696	0.5375	−0.0239802

0.5 to 1 decimal place.

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Numerical solutions of equations
Exercise A, Question 5

Question:

a Show that the equation $0 = \frac{x}{2} - \frac{1}{x}$, $x > 0$, has a root in the interval [1, 2].

b Obtain the root, using interval bisection two times. Give your answer to two significant figures.

Solution:

a f(1) = −0.5 $p = \frac{1}{2} + x - \frac{1}{x}$

f(2) = +0.5

Change of sign implies root between interval [1,2]

b

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
1	−0.5	2	+0.5	1.5	0.0833
1	−0.5	1.5	0.083	1.25	−0.175
1.25	−0.175	1.5	0.083	1.375	−0.0397727
1.375	−0.0397727	1.5	0.083	1.4375	0.0230978

Hence $x = 1.4$ to 2 significant figures

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Numerical solutions of equations
Exercise A, Question 6

Question:

$f(x) = 6x - 3^x$

The equation $f(x) = 0$ has a root between $x = 2$ and $x = 3$. Starting with the interval [2, 3] use interval bisection three times to give an approximation to this root.

Solution:

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	3	3	−9	2.5	−0.588457
2	3	2.5	−0.5884572	2.25	1.65533
2.25	1.6553339	2.5	−0.5884572	2.375	0.66176
2.375	0.6617671	2.5	−0.5884572	2.4375	0.0708
2.4375	0.0709769	2.5	−0.5844572	2.46875	−0.2498
2.4375	0.0709769	2.46875	−0.2498625	2.453125	−0.08726
2.4375	0.0709769	2.453125	−0.0872613	2.4453125	−0.0076

2.4 correct to 1 decimal place.

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Numerical solutions of equations
Exercise B, Question 1

Question:

a Show that a root of the equation $x^3 - 3x - 5 = 0$ lies in the interval [2, 3].

b Find this root using linear interpolation correct to one decimal place.

Solution:

a $f(2) = 8 - 6 - 5 = -3 \quad f(x) = x^3 - 3x - 5$

$f(3) = 27 - 9 - 5 = +13$

Change of size therefore root in interval [2, 3]

b Using linear interpolation and similar triangle taking x_1 as the first root.

$$\frac{3 - x_1}{x_1 - 2} = \frac{3}{13} \quad x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

so

$$\begin{aligned} 13(3 - x_1) &= 3(x_1 - 2) \\ 39 - 13x_1 &= 3x_1 - 6 \\ 16x_1 &= 45 \\ x_1 &= 2.8125 \quad f(x_1) = 8.8098 \end{aligned}$$

Using interval (2, 2.8125)

$$\begin{aligned} \frac{2.8125 - x_2}{x_2 - 2} &= \frac{3}{8.8098} \\ x_2 &= 2.606 \quad f(x_2) = 4.880 \end{aligned}$$

Using interval (2, 2.606)

$$\begin{aligned} \frac{2.606 - x_3}{x_3 - 2} &= \frac{3}{4.880} \\ x_2 &= 2.375 \quad f(x_2) = 1.276 \end{aligned}$$

Using interval (2, 2.375)

$$\begin{aligned} \frac{2.375 - x_4}{x_4 - 2} &= \frac{3}{1.276} \\ x_2 &= 2.112 \quad f(x_4) = -1.915 \end{aligned}$$

Using interval (2.112, 2.375)

$$\begin{aligned} \frac{2.375 - x_5}{x_5 - 2.112} &= \frac{1.915}{1.276} \\ &= 2.218 \quad f(x_5) = -0.736 \end{aligned}$$

Using interval (2.218, 2.375)

$$\frac{2.375 - x_6}{x_6 - 2.218} = \frac{0.736}{1.276}$$
$$= 2.318 \quad f(x_6) = 0.494$$

Using interval (2.218, 2.318)

$$\frac{2.318 - x_7}{x_7 - 2.218} = \frac{0.736}{0.494}$$
$$= 2.25 \quad f(x_7) = -0.229$$

2.3 to 1 decimal place.

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Numerical solutions of equations
Exercise B, Question 2

Question:

a Show that a root of the equation $5x^3 - 8x^2 + 1 = 0$ has a root between $x = 1$ and $x = 2$.

b Find this root using linear interpolation correct to one decimal place.

Solution:

a $f(1) = 5 - 8 + 1 = -2 \quad f(x) = 5x^3 - 8x^2 + 1$

$f(2) = 40 - 32 + 1 = +9$

Therefore root in interval [1, 2] as sign change.

b Using linear interpolation.

$$\frac{2 - x_1}{x_1 - 1} = \frac{2}{9}$$
$$x_1 = 1.818 \quad f(x_1) = 4.612.$$

Using interval (1, 1.818)

$$\frac{1.818 - x_2}{x_2 - 1} = \frac{2}{4.612}$$
$$x_2 = 1.570 \quad f(x_2) = 0.647$$

Using interval (1, 1.570)

$$\frac{1.570 - x_3}{x_3 - 1} = \frac{2}{0.647}$$
$$x_3 = 1.139 \quad f(x_3) = -1.984$$

Using interval (1.139, 1.570)

$$\frac{1.570 - x_4}{x_4 - 1.139} = \frac{1.984}{0.647}$$
$$x_4 = 1.447 \quad f(x_4) = -0.590$$

Use interval (1.447, 1.570)

$$\frac{1.570 - x_5}{x_5 - 1.447} = \frac{0.590}{0.647}$$
$$= 1.511 \quad f(x_5) = -0.0005.$$

Ans 1.5 correct to 1 decimal place.

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Numerical solutions of equations
Exercise B, Question 3

Question:

a Show that a root of the equation $\frac{3}{x}+3=x$ lies in the interval [3, 4].

b Use linear interpolation to find this root correct to one decimal place.

Solution:

a $f(3)=1 \quad f(x)=\frac{3}{x}+3-x$

$f(4)=-0.25$

Hence root as sign change in interval [3, 4]

b Using linear interpolation

$$\frac{4-x_1}{x_1-3}=\frac{0.25}{1}$$
$$x_1=3.8 \quad f(x_1)=-0.011$$

Using interval [3, 3.8]

$$\frac{3.8-x_2}{x_2-3}=\frac{0.0111}{1}$$
$$x_2=3.791 \quad f(x_2)=-0.0004579$$

Ans = 3.8 to 1decimal place

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Numerical solutions of equations
Exercise B, Question 4

Question:

a Show that a root of the equation $2x\cos x - 1 = 0$ lies in the interval [1, 1.5].

b Find this root using linear interpolation correct to two decimal places.

Solution:

a f(1) = 0.0806

f(1.5) = −0.788

Hence root between (1, 1.5) as sign change

b Using linear interpolation

$$\frac{1.5 - x_1}{x_1 - 1} = \frac{0.788}{1}$$
$$x_1 = 1.280 \quad f(1.280) = -0.265$$

Use interval [1, 1.28]

$$\frac{1.28 - x_2}{x_2 - 1} = \frac{0.265}{1}$$
$$x_2 = 1.221 \quad f(1.221) = -0.164$$

Use interval [1, 1.221]

$$\frac{1.221 - x_2}{x_3 - 1} = \frac{0.164}{1}$$
$$x_3 = 1.190 \quad f(1.190) = -0.115$$

Use interval [1, 1.190]

$$\frac{1.190 - x_4}{x_4 - 1} = \frac{0.115}{1}$$
$$x_4 = 1.170 \quad f(1.170) = 0.088$$

Use interval [1, 1.170]

$$\frac{1.170 - x_5}{x_5 - 1} = \frac{0.088}{1}$$
$$x_5 = 1.156 \quad f(1.156) = -0.068$$

Root 1.10 to 2 decimal places.

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Numerical solutions of equations
Exercise B, Question 5

Question:

a Show that the largest possible root of the equation $x^3 - 2x^2 - 3 = 0$ lies in the interval [2, 3].

b Find this root correct to one decimal place using interval interpolation.

Solution:

a $f(2) = 8 - 8 - 3 = -3 \quad f(x) = x^3 - 2x^2 - 3$

$f(3) = 27 - 18 - 3 = 6$

Hence root lies in interval [2, 3] and $\forall\, x \in\ x \geq 3 f(x) < 0$.

b Using linear interpolation

$$\frac{3 - x_1}{x_1 - 2} = \frac{6}{3}$$
$$x_1 = 2.333 \quad f(x_1) = -1.185$$

$$\frac{3 - x_2}{x_2 - 2.333} = \frac{6}{1.185}$$
$$x_2 = 2.443 \quad f(x_2) = -0.356$$

$$\frac{3 - x_3}{x_3 - 2.443} = \frac{6}{0.356}$$
$$x_3 = 2.474 \quad f(x_3) = -0.095$$

$$\frac{3 - x_4}{x_4 - 2.474} = \frac{6}{0.095}$$
$$x_4 = 2.482$$

Hence root = 2.5 to 1 d.p

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Numerical solutions of equations
Exercise B, Question 6

Question:

$f(x) = 2^x - 3x - 1$

The equation $f(x) = 0$ has a root in the interval [3, 4].

Using this interval find an approximation to x.

Solution:

Let root be α

$f(3) = -2$
$f(4) = 3$

$\frac{4-\alpha}{\alpha-3} = \frac{3}{2}$
$\alpha = 3.4$ is the approximation.

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Numerical solutions of equations
Exercise C, Question 1

Question:

Show that the equation $x^3 - 2x - 1 = 0$ has a root between 1 and 2. Find the root correct to two decimal places using the Newton–Raphson process.

Solution:

$f(1) = -2 \qquad f(x) = x^3 - 2x - 1$
$f(2) = 3 \quad f(2) = 3$ is correct

Hence root in interval [1,2] as sign change

$$\begin{aligned} f(x) &= x^3 - 2x - 1 \\ f'(x) &= 3x^2 - 2 \end{aligned}$$

Let $x_0 = 2$.

$$\begin{aligned} \text{Then } x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ x_1 &= 2 - \frac{3}{10} \\ x_1 &= 1.7 \\ x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ x_2 &= 1.88 - \frac{1.885}{8.6032} \\ &= 1.661 \\ x_3 &= x_2 - \frac{f(x_2)}{f'(x_2)} \\ x_3 &= 1.661 - \frac{0.2597}{6.2767} \\ &= 1.6120 \\ x_4 &= 1.620 - \frac{f(1.620)}{f'(1.620)} \\ x_4 &= 1.62 - \frac{0.0115}{5.8732} \\ &= 1.618 \\ \text{Solution } &= 1.62 \text{ to 2 decimal places} \end{aligned}$$

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Numerical solutions of equations
Exercise C, Question 2

Question:

Use the Newton–Raphson process to find the positive root of the equation $x^3 + 2x^2 - 6x - 3 = 0$ correct to two decimal places.

Solution:

$$\begin{aligned} f(0) &= -3 \qquad f(x) = x^3 + 2x^2 - 6x - 3 \\ f(1) &= 1 + 2 - 6 - 3 = -6 \\ f(2) &= 8 + 8 - 12 - 3 = 1 \end{aligned}$$

Hence root in interval [1,2]

Using Newton Raphson

$$\begin{aligned} f(x) &= x^3 + 2x^2 - 6x - 3 \\ f'(x) &= 3x^2 + 4x - 6 \\ x_0 &= 2 \end{aligned}$$

$$\begin{aligned} \text{Then } x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 2 - \frac{1}{14} \\ &= 1.92857 \\ x_2 &= 1.92857 - \frac{0.0404494}{12.872427} \\ &= 1.92857 - 0.00314 \\ &= 1.9254 \end{aligned}$$

Root = 1.93 to 2 decimal places.

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Numerical solutions of equations
Exercise C, Question 3

Question:

Find the smallest positive root of the equation $x^4 + x^2 - 80 = 0$ correct to two decimal places. Use the Newton–Raphson process.

Solution:

$$\begin{aligned} f(x) &= x^4 + x^2 - 80 \\ f'(x) &= 4x^3 + 2x \end{aligned}$$

$$\begin{aligned} \text{Let } x_0 &= 3 \qquad\qquad f(3) = 10 \\ \text{So } x_1 &= 3 - \frac{f(x_0)}{f'(x_0)} \\ x_1 &= 3 - \frac{10}{114} \\ &= 2.912 \\ \text{Then } x_2 &= 2.912 - \frac{0.1768}{104.388} \\ &= 2.908 \end{aligned}$$

Hence root = 2.91 to 2 decimal places.

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Numerical solutions of equations
Exercise C, Question 4

Question:

Apply the Newton–Raphson process to find the negative root of the equation $x^3 - 5x + 2 = 0$ correct to two decimal places.

Solution:

$$\begin{aligned} f(x) &= x^3 - 5x + 2 \\ f'(x) &= 3x^2 - 5 \end{aligned}$$

$$\begin{aligned} f(0) &= 2 \\ f(-1) &= -1 + 5 + 2 = 6 \\ f(-2) &= -8 + 10 + 2 = 4 \\ f(-3) &= -27 + 15 + 2 = -10 \end{aligned}$$

Hence root between interval [−2,−3]

Let $x_0 = -2$

$$\begin{aligned} \text{Then } x_1 &= -2 - \frac{f(x_0)}{f'(x_0)} \\ &= -2 - \frac{4}{7} \\ &= -2.5714 \\ x_2 &= -2.571 - \frac{f(x_1)}{f'(x_1)} \\ &= -2.571 - \frac{2.1394}{14.83} \\ &= -2.4267 \\ x_3 &= -2.4267 - \frac{0.1570}{12.6662} \\ &= -2.4267 - 0.01234 \\ &= -2.439 \\ x_4 &= -2.439 - \frac{0.00163}{12.846} \\ &= -2.4391 \end{aligned}$$

Root = −2.44 correct to 2 decimal places.

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Numerical solutions of equations
Exercise C, Question 5

Question:

Show that the equation $2x^3 - 4x^2 - 1 = 0$ has a root in the interval [2, 3]. Taking 3 as a first approximation to this root, use the Newton–Raphson process to find this root correct to two decimal places.

Solution:

$$\begin{aligned} f(x) &= 2x^3 - 4x^2 - 1. \\ f(2) &= 16 - 16 - 1 = -1 \\ f(3) &= 54 - 36 - 1 = 17 \end{aligned}$$

Sign change implies root in interval [2,3]

$f'(x) = 6x^2 - 8x$

Let $x_0 = 3$

$$\begin{aligned} \text{Then} \quad x_1 &= 3 - \frac{f(x_0)}{f'(x_0)} \\ &= 3 - \frac{17}{30} \\ &= 2.43 \\ x_2 &= 2.43 - \frac{f(2.43)}{f'(2.43)} \\ &= 2.43 - \frac{4.078}{16.05} \\ &= 2.43 - 0.254 \\ &= 2.179 \\ x_3 &= 2.179 - \frac{f(2.179)}{f'(2.179)} \\ &= 2.179 - \frac{0.6998}{11.056} \\ &= 2.179 - 0.063296 \\ &= 2.116 \\ x_4 &= 2.116 - \frac{f(2.116)}{f'(2.116)} \\ &= 2.116 - \frac{0.0388}{9.937} = 2.112 \\ x_5 &= 2.112 - \frac{f(2.112)}{f'(2.112)} \\ &= 2.112 - \frac{-0.00084}{9.8672} \\ &= 2.112 \end{aligned}$$

Ans = 2.11 correct to 2 decimal place.

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Numerical solutions of equations
Exercise C, Question 6

Question:

$f(x) = x^3 - 3x^2 + 5x - 4$

Taking 1.4 as a first approximation to a root, x, of this equation, use Newton–Raphson process once to obtain a second approximation to x. Give your answer to three decimal places.

Solution:

$$\begin{aligned} f(x) &= x^3 - 3x^2 + 5x - 4 \\ f'(x) &= 3x^2 - 6x + 5 \end{aligned}$$

Let $x_0 = 1.4$

Using Newton Raphson

$$\begin{aligned} x_1 &= 1.4 - \frac{f(1.4)}{f'(1.4)} \\ &= 1.4 - \frac{-0.136}{2.48} \\ &= 1.4 + 0.0548 \\ &= 1.455 \text{ to } 3 \text{ decimal places} \end{aligned}$$

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Numerical solutions of equations
Exercise C, Question 7

Question:

Use the Newton–Raphson process twice to find the root of the equation $2x^3 + 5x = 70$ which is near to $x = 3$. Give your answer to three decimal places.

Solution:

$$\begin{aligned} f(x) &= 2x^3 + 5x - 70 \\ f'(x) &= 6x^2 + 5 \end{aligned}$$

Let $x_0 = 3$

Using Newton Raphson

$$\begin{aligned} x_1 &= 3 - \frac{f(3)}{f'(3)} \\ &= 3 - \frac{-1}{59} \\ &= 3.02 \\ x_2 &= 3.02 - \frac{f(3.02)}{f'(3.02)} \\ &= 3.02 - \frac{0.1872}{59.72} \\ &-3.017 \text{ to } 3 \text{ decimal places.} \end{aligned}$$

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Numerical solutions of equations
Exercise D, Question 1

Question:

Given that $f(x) = x^3 - 2x + 2$ has a root in the interval $[-1, -2]$, use interval bisection on the interval $[-1, -2]$ to obtain the root correct to one decimal place.

Solution:

$$\begin{aligned} f(x) &= x^3 - 2x + 2 \\ f(-1) &= -1 + 2 + 2 = +3 \\ f(-2) &= -8 + 4 + 2 = -2 \end{aligned}$$

Hence root in interval [−1, −2] as sign change

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$\frac{f(a+b)}{2}$
−1	+3	−2	−2	−1.5	+1.625
−1.5	1.625	−2	−2	−1.75	0.141
−1.75	0.141	−2	−2	−1.875	−0.842
−1.75	0.141	−1.875	−0.841	−1.8125	−0.329
−1.75	0.141	−1.8125	−0.329	−1.78125	

Hence solution is −1.8 to 1 decimal place.

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Numerical solutions of equations
Exercise D, Question 2

Question:

Show that the equation $x^3 - 12x - 7.2 = 0$ has one positive and two negative roots. Obtain the positive root correct to three significant figures using the Newton–Raphson process.

Solution:

$f(x) = x^3 - 12x - 7.2 = 0$

$f(0) = -7.2$	$f(-1) = 3.8$
$f(1) = -18.2$	$f(-2) = 8.8$
$f(2) = -23.2$	$f(-3) = 1.8$
$f(3) = -16.2$	$f(-4) = -23.2$
$f(4) = 8.8$	

positive root between [3, 4]

negative roots between [0, −1], [−3, −4] Let $x_0 = 4$

Using $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

where $f(x) = x^3 - 12x - 7.2$

$f'(x) = 3x^2 - 12$

So $x_1 = 4 - \frac{8.8}{36}$

$$\begin{aligned} x_1 &= 3.756 \text{ to 3d.p.} \\ x_2 &= 3.756 - \frac{0.716}{30.322} \\ x_2 &= 3.732 \\ x_3 &= 3.732 - \frac{0.011}{30.323} \\ x_3 &= 3.7316 \end{aligned}$$

Hence root = 3.73 to 3 significant figures

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Numerical solutions of equations
Exercise D, Question 3

Question:

Find, correct to one decimal place, the real root of $x^3 + 2x - 1 = 0$ by using the Newton–Raphson process.

Solution:

$$\begin{aligned} f(x) &= x^3 + 2x - 1 \\ f(0) &= -1 \\ f(1) &= 2 \end{aligned}$$

Hence root interval [0, 1]

Using $f(x) = x^3 + 2x - 1$

$$\begin{aligned} f'(x) &= 3x^2 + 2 \quad \text{and} \quad x_0 = 1 \\ x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ x_1 &= 1 - \frac{2}{5} \\ x_1 &= 0.6 \\ x_2 &= 0.6 - \frac{0.416}{3.08} \\ x_2 &= 0.465 \\ x_3 &= 0.465 - \frac{0.031}{2.647} \\ x_3 &= 0.453 \end{aligned}$$

Hence root is 0.5 to 1decimal place.

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Numerical solutions of equations
Exercise D, Question 4

Question:

Use the Newton–Raphson process to find the real root of the equation $x^3 + 2x^2 + 4x - 6 = 0$, taking $x = 0.9$ as the first approximation and carrying out one iteration.

Solution:

$$\begin{aligned}
\mathrm{f}(x) &= x^3 + 2x^2 + 4x - 6 \\
\mathrm{f}'(x) &= 3x^2 + 4x + 4 \\
\mathrm{f}(0.9) &= -0.051 \\
\mathrm{f}'(0.9) &= 10.03 \\
x_1 &= x_0 - \frac{\mathrm{f}(x_0)}{\mathrm{f}'(x_1)} \\
&= 0.9 - \frac{-0.051}{10.03} \\
&= 0.905 \text{ to 3 decimal places}
\end{aligned}$$

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Numerical solutions of equations
Exercise D, Question 5

Question:

Use linear interpolation to find the positive root of the equation $x^3 - 5x + 3 = 0$ correct to one decimal place.

Solution:

$$\begin{aligned} f(x) &= x^3 - 5x + 3 \\ f(1) &= -1 \\ f(2) &= +1. \end{aligned}$$

Hence positive root in interval [1, 2] Using linear interpolation and x, as the 1st approximation

$$\begin{aligned} \frac{2 - x_1}{x_1 - 1} &= \frac{1}{1} \\ 2 - x_1 &= x_1 - 1 \\ 2x_1 &= 3 \\ x_1 &= 1.5 \qquad f(x_1) = 1.125 \end{aligned}$$

Then

$$\begin{aligned} \frac{2 - x_2}{x_2 - 1.5} &= \frac{1}{1.125} \\ x_2 &= 1.882 \qquad f(x_2) = 0.260 \\ \frac{1.882 - x_2}{x_2 - 1.5} &= \frac{0.260}{1.125} \\ x_2 &= 1.810 \qquad f(x_3) = -0.117 \\ \frac{1.882 - x_4}{x_2 - 1.810} &= \frac{0.260}{0.117} \\ &= 1.832 \end{aligned}$$

root = 1.8 to 1 decimal place

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Numerical solutions of equations
Exercise D, Question 6

Question:

$f(x) = x^3 + x^2 - 6.$

a Show that the real root of $f(x) = 0$ lies in the interval [1, 2].

b Use the linear interpolation on the interval [1, 2] to find the first approximation to x.

c Use the Newton–Raphson process on $f(x)$ once, starting with your answer to **b**, to find another approximation to x, giving your answer correct to two decimal places.

Solution:

a

$$\begin{aligned} f(x) &= x^3 + x^2 - 6 \\ f(1) &= -4 \\ f(2) &= 6 \end{aligned}$$

Hence root in interval [1, 2]

b

$$\begin{aligned} \frac{2 - x_1}{x_1 - 1} &= \frac{6}{4} \\ x_1 &= 1.4 \end{aligned}$$

c

$$\begin{aligned} x_0 &= 1.4 \\ f(x) &= x^3 + x^2 - 6 \\ f'(x) &= 3x^2 + 2x \\ x_1 &= x_0 - \frac{f(x_0)}{f'(x_1)} \\ &= 1.4 - \frac{-1.296}{8.68} \\ &= 1.55 \text{ to 2 decimal places} \end{aligned}$$

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Numerical solutions of equations
Exercise D, Question 7

Question:

The equation $\cos x = \frac{1}{4}x$ has a root in the interval [1.0, 1.4]. Use linear interpolation once in the interval [1.0, 1.4] to find an estimate of the root, giving your answer correct to two decimal places.

Solution:

$\cos x = \frac{1}{4}x \Rightarrow \mathrm{f}(x) = \frac{1}{4}x - \cos x$

$\mathrm{f}(1) = -0.29$

$\mathrm{f}(1.4) = 0.180$

$\frac{1.4 - x_1}{x_1 - 1} = \frac{-0.290}{-0.180}$

$x_1 = 1.153$

$x_1 = 1.15$ to 2 decimal places

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Numerical solutions of equations
Exercise D, Question 8

Question:

$f(x) = x^3 - 3x - 6$

Use the Newton–Raphson process to find the positive root of this equation correct to two decimal places.

Solution:

$$\begin{aligned} f(x) &= x^3 - 3x - 6 \\ f'(x) &= 3x^2 - 3 \end{aligned}$$

$$\begin{aligned} f(0) &= -5 \quad f(1) = -7 \\ f(2) &= -3 \quad f(3) = +13 \end{aligned}$$

Hence root in interval [2, 3]

Let $x_0 = 2$

Then

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_1)} \\ &= 2 - \frac{-3}{9} \\ x_1 &= 2.333 \\ x_2 &= -\frac{4.301}{16.500} \\ x_2 &= 2.297 \\ x_3 &= 2.297 - \frac{0.228}{12.828} \\ x_3 &= 2.279 \\ x_4 &= 2.279 - \frac{-0.000236}{12.582} \\ &= 2.279 + 0.000019 \\ x_4 &= 2.2790 \end{aligned}$$

Ans = 2.28 to 2 decimal places