Solutionbank FP1
Edexcel AS and A Level Modular Mathematics

Complex numbers
Exercise A, Question 1

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(5 + 2\text{i}) + (8 + 9\text{i})$

Solution:

$(5 + 8) + \text{i}(2 + 9) = 13 + 11\text{i}$

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Complex numbers
Exercise A, Question 2

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(4 + 10\text{i}) + (1 - 8\text{i})$

Solution:

$(4 + 1) + \text{i}(10 - 8) = 5 + 2\text{i}$

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Complex numbers
Exercise A, Question 3

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(7 + 6\text{i}) + (-3 - 5\text{i})$

Solution:

$(7 - 3) + \text{i}(6 - 5) = 4 + \text{i}$

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Complex numbers
Exercise A, Question 4

Question:

Simplify, giving your answer in the form $a + b\mathrm{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(2 - \mathrm{i}) + (11 + 2\mathrm{i})$

Solution:

$(2 + 11) + \mathrm{i}(-1 + 2) = 13 + \mathrm{i}$

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Complex numbers
Exercise A, Question 5

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(3 - 7\text{i}) + (-6 + 7\text{i})$

Solution:

$(3 - 6) + \text{i}(-7 + 7) = -3$

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Complex numbers
Exercise A, Question 6

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(20 + 12\text{i}) - (11 + 3\text{i})$

Solution:

$(20 - 11) + \text{i}(12 - 3) = 9 + 9\text{i}$

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Complex numbers
Exercise A, Question 7

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(9 + 6\text{i}) - (8 + 10\text{i})$

Solution:

$(9 - 8) + \text{i}(6 - 10) = 1 - 4\text{i}$

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Complex numbers
Exercise A, Question 8

Question:

Simplify, giving your answer in the form $a + b\mathrm{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(2 - \mathrm{i}) - (-5 + 3\mathrm{i})$

Solution:

$(2 - -5) + \mathrm{i}(-1 - 3) = 7 - 4\mathrm{i}$

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Complex numbers
Exercise A, Question 9

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(-4 - 6\text{i}) - (-8 - 8\text{i})$

Solution:

$(-4 - -8) + \text{i}(-6 - -8) = 4 + 2\text{i}$

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Complex numbers
Exercise A, Question 10

Question:

Simplify, giving your answer in the form $a + b\mathrm{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(-1 + 5\mathrm{i}) - (-1 + \mathrm{i})$

Solution:

$(-1 - -1) + \mathrm{i}(5 - 1) = 4\mathrm{i}$

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Complex numbers
Exercise A, Question 11

Question:

Simplify, giving your answer in the form $a + b\mathrm{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(3 + 4\mathrm{i}) + (4 + 5\mathrm{i}) + (5 + 6\mathrm{i})$

Solution:

$(3 + 4 + 5) + \mathrm{i}(4 + 5 + 6) = 12 + 15\mathrm{i}$

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Complex numbers
Exercise A, Question 12

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(-2 - 7\text{i}) + (1 + 3\text{i}) - (-12 + \text{i})$

Solution:

$(-2 + 1 - -12) + \text{i}(-7 + 3 - 1) = 11 - 5\text{i}$

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Complex numbers
Exercise A, Question 13

Question:

Simplify, giving your answer in the form $a + b\mathrm{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(18 + 5\mathrm{i}) - (15 - 2\mathrm{i}) - (3 + 7\mathrm{i})$

Solution:

$(18 - 15 - 3) + \mathrm{i}(5 - -2 - 7) = 0$

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Complex numbers
Exercise A, Question 14

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$2(7 + 2\text{i})$

Solution:

$14 + 4\text{i}$

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Complex numbers
Exercise A, Question 15

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$3(8 - 4\text{i})$

Solution:

$24 - 12\text{i}$

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Complex numbers
Exercise A, Question 16

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$7(1 - 3\text{i})$

Solution:

$7 - 21\text{i}$

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Complex numbers
Exercise A, Question 17

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$2(3+\text{i})+3(2+\text{i})$

Solution:

$(6+2\text{i})+(6+3\text{i})=(6+6)+\text{i}(2+3)=12+5\text{i}$

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Complex numbers
Exercise A, Question 18

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$5(4 + 3\text{i}) - 4(-1 + 2\text{i})$

Solution:

$(20 + 15\text{i}) + (4 - 8\text{i}) = (20 + 4) + \text{i}(15 - 8) = 24 + 7\text{i}$

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Complex numbers
Exercise A, Question 19

Question:

Simplify, giving your answer in the form $a + b\text{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$$\left(\frac{1}{2}+\frac{1}{3}\text{i}\right)+\left(\frac{5}{2}+\frac{5}{3}\text{i}\right)$$

Solution:

$$\left(\frac{1}{2}+\frac{5}{2}\right)+\text{i}\left(\frac{1}{3}+\frac{5}{3}\right)=3+2\text{i}$$

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Complex numbers
Exercise A, Question 20

Question:

Simplify, giving your answer in the form $a + b\mathrm{i}$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

$(3\sqrt{2} + \mathrm{i}) - (\sqrt{2} - \mathrm{i})$

Solution:

$(3\sqrt{2} - \sqrt{2}) + \mathrm{i}(1 - -1) = 2\sqrt{2} + 2\mathrm{i}$

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Complex numbers
Exercise A, Question 21

Question:

Write in the form *bi*, where $b \in \mathbb{R}$.

$\sqrt{(-9)}$

Solution:

$\sqrt{9}\sqrt{(-1)} = 3i$

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Complex numbers
Exercise A, Question 22

Question:

Write in the form *bi*, where $b \in \mathbb{R}$.

$\sqrt{(-49)}$

Solution:

$\sqrt{49}\sqrt{(-1)} = 7\text{i}$

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Complex numbers
Exercise A, Question 23

Question:

Write in the form *bi*, where $b \in \mathbb{R}$.

$\sqrt{(-121)}$

Solution:

$\sqrt{121}\sqrt{(-1)} = 11\text{i}$

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Complex numbers
Exercise A, Question 24

Question:

Write in the form bi, where $b \in \mathbb{R}$.

$\sqrt{(-10000)}$

Solution:

$\sqrt{10000}\sqrt{(-1)} = 100\text{i}$

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Complex numbers
Exercise A, Question 25

Question:

Write in the form *bi*, where $b \in \mathbb{R}$.

$\sqrt{(-225)}$

Solution:

$\sqrt{225}\sqrt{(-1)} = 15\text{i}$

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Complex numbers
Exercise A, Question 26

Question:

Write in the form *bi*, where $b \in \mathbb{R}$.

$\sqrt{(-5)}$

Solution:

$\sqrt{5}\sqrt{(-1)} = \mathrm{i}\sqrt{5}$

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Complex numbers
Exercise A, Question 27

Question:

Write in the form *bi*, where $b \in \mathbb{R}$.

$\sqrt{(-12)}$

Solution:

$\sqrt{12}\sqrt{(-1)} = \sqrt{4}\sqrt{3}\sqrt{(-1)} = 2\mathrm{i}\sqrt{3}$

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Complex numbers
Exercise A, Question 28

Question:

Write in the form bi, where $b \in \mathbb{R}$.

$\sqrt{(-45)}$

Solution:

$\sqrt{45}\sqrt{(-1)} = \sqrt{9}\sqrt{5}\sqrt{(-1)} = 3\mathrm{i}\sqrt{5}$

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Complex numbers
Exercise A, Question 29

Question:

Write in the form bi, where $b \in \mathbb{R}$.

$\sqrt{(-200)}$

Solution:

$\sqrt{200}\sqrt{(-1)} = \sqrt{100}\sqrt{2}\sqrt{(-1)} = 10\text{i}\sqrt{2}$

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Complex numbers
Exercise A, Question 30

Question:

Write in the form bi, where $b \in \mathbb{R}$.

$\sqrt{(-147)}$

Solution:

$\sqrt{147}\sqrt{(-1)} = \sqrt{49}\sqrt{3}\sqrt{(-1)} = 7\mathrm{i}\sqrt{3}$

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Complex numbers
Exercise A, Question 31

Question:

Solve these equations.

$x^2 + 2x + 5 = 0$

Solution:

$$a = 1, b = 2, c = 5$$
$$x = \frac{-2 \pm \sqrt{(4-20)}}{2} = \frac{-2 \pm 4\mathrm{i}}{2}$$
$$x = -1 \pm 2\mathrm{i}$$

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Complex numbers
Exercise A, Question 32

Question:

Solve these equations.

$x^2 - 2x + 10 = 0$

Solution:

$a = 1, b = -2, c = 10$

$x = \frac{2 \pm \sqrt{(4 - 40)}}{2} = \frac{2 \pm 6\text{i}}{2}$

$x = 1 \pm 3\text{i}$

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Complex numbers
Exercise A, Question 33

Question:

Solve these equations.

$x^2 + 4x + 29 = 0$

Solution:

$a = 1, b = 4, c = 29$

$x = \frac{-4 \pm \sqrt{(16 - 116)}}{2} = \frac{-4 \pm 10\text{i}}{2}$

$x = -2 \pm 5\text{i}$

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Complex numbers
Exercise A, Question 34

Question:

Solve these equations.

$x^2 + 10x + 26 = 0$

Solution:

$a = 1, b = 10, c = 26$

$x = \frac{-10 \pm \sqrt{(100 - 104)}}{2} = \frac{-10 \pm 2\mathrm{i}}{2}$

$x = -5 \pm \mathrm{i}$

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Complex numbers
Exercise A, Question 35

Question:

Solve these equations.

$x^2 - 6x + 18 = 0$

Solution:

$a = 1, b = -6, c = 18$

$$x = \frac{6 \pm \sqrt{(36 - 72)}}{2} = \frac{6 \pm 6\text{i}}{2}$$

$x = 3 \pm 3\text{i}$

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Complex numbers
Exercise A, Question 36

Question:

Solve these equations.

$x^2 + 4x + 7 = 0$

Solution:

$a = 1, b = 4, c = 7$

$$x = \frac{-4 \pm \sqrt{(16 - 28)}}{2} = \frac{-4 \pm \mathrm{i}\sqrt{12}}{2} = \frac{-4 \pm 2\mathrm{i}\sqrt{3}}{2}$$

$x = -2 \pm \mathrm{i}\sqrt{3}$

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Complex numbers
Exercise A, Question 37

Question:

Solve these equations.

$x^2 - 6x + 11 = 0$

Solution:

$a = 1, b = -6, c = 11$

$x = \frac{6 \pm \sqrt{(36 - 44)}}{2} = \frac{6 \pm \mathrm{i}\sqrt{8}}{2} = \frac{6 \pm 2\mathrm{i}\sqrt{2}}{2}$

$x = 3 \pm \mathrm{i}\sqrt{2}$

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Complex numbers
Exercise A, Question 38

Question:

Solve these equations.

$x^2 - 2x + 25 = 0$

Solution:

$a = 1, b = -2, c = 25$

$x = \frac{2 \pm \sqrt{(4 - 100)}}{2} = \frac{2 \pm \mathrm{i}\sqrt{96}}{2} = \frac{2 \pm 4\mathrm{i}\sqrt{6}}{2}$

$x = 1 \pm 2\mathrm{i}\sqrt{6}$

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Complex numbers
Exercise A, Question 39

Question:

Solve these equations.

$x^2 + 5x + 25 = 0$

Solution:

$a = 1, b = 5, c = 25$

$x = \frac{-5 \pm \sqrt{(25 - 100)}}{2} = \frac{-5 \pm \mathrm{i}\sqrt{75}}{2} = \frac{-5 \pm 5\mathrm{i}\sqrt{3}}{2}$

$x = \frac{-5}{2} \pm \frac{5\mathrm{i}\sqrt{3}}{2}$

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Complex numbers
Exercise A, Question 40

Question:

Solve these equations.

$x^2 + 3x + 5 = 0$

Solution:

$a = 1, b = 3, c = 5$

$x = -3 \pm \frac{\sqrt{(9-20)}}{2} = \frac{-3 \pm i\sqrt{11}}{2}$

$x = \frac{-3}{2} \pm \frac{i\sqrt{11}}{2}$

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Complex numbers
Exercise B, Question 1

Question:

Simplify these, giving your answer in the form $a + b\mathrm{i}$.

$(5 + \mathrm{i})(3 + 4\mathrm{i})$

Solution:

$5(3 + 4\mathrm{i}) + \mathrm{i}(3 + 4\mathrm{i})$
$= 15 + 20\mathrm{i} + 3\mathrm{i} + 4\mathrm{i}^2$
$= 15 + 20\mathrm{i} + 3\mathrm{i} - 4$
$= 11 + 23\mathrm{i}$

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Complex numbers
Exercise B, Question 2

Question:

Simplify these, giving your answer in the form $a + b\text{i}$.

$(6 + 3\text{i})(7 + 2\text{i})$

Solution:

$6(7 + 2\text{i}) + 3\text{i}(7 + 2\text{i})$
$= 42 + 12\text{i} + 21\text{i} + 6\text{i}^2$
$= 42 + 12\text{i} + 21\text{i} - 6$
$= 36 + 33\text{i}$

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Complex numbers
Exercise B, Question 3

Question:

Simplify these, giving your answer in the form $a + b\text{i}$.

$(5 - 2\text{i})(1 + 5\text{i})$

Solution:

$5(1 + 5\text{i}) - 2\text{i}(1 + 5\text{i})$
$= 5 + 25\text{i} - 2\text{i} - 10\text{i}^2$
$= 5 + 25\text{i} - 2\text{i} + 10$
$= 15 + 23\text{i}$

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Complex numbers
Exercise B, Question 4

Question:

Simplify these, giving your answer in the form $a + b\text{i}$.

$(13 - 3\text{i})(2 - 8\text{i})$

Solution:

$13(2 - 8\text{i}) - 3\text{i}(2 - 8\text{i})$
$= 26 - 104\text{i} - 6\text{i} + 24\text{i}^2$
$= 26 - 104\text{i} - 6\text{i} - 24$
$= 2 - 110\text{i}$

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Complex numbers
Exercise B, Question 5

Question:

Simplify these, giving your answer in the form $a + b\text{i}$.

$(-3 - \text{i})(4 + 7\text{i})$

Solution:

$-3(4 + 7\text{i}) - \text{i}(4 + 7\text{i})$
$= -12 - 21\text{i} - 4\text{i} - 7\text{i}^2$
$= -12 - 21\text{i} - 4\text{i} + 7$
$= -5 - 25\text{i}$

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Complex numbers
Exercise B, Question 6

Question:

Simplify these, giving your answer in the form $a + b\text{i}$.

$(8 + 5\text{i})^2$

Solution:

$(8 + 5\text{i})(8 + 5\text{i}) = 8(8 + 5\text{i}) + 5\text{i}(8 + 5\text{i})$
$= 64 + 40\text{i} + 40\text{i} + 25\text{i}^2$
$= 64 + 40\text{i} + 40\text{i} - 25$
$= 39 + 80\text{i}$

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Complex numbers
Exercise B, Question 7

Question:

Simplify these, giving your answer in the form $a + b\text{i}$.

$(2 - 9\text{i})^2$

Solution:

$(2 - 9\text{i})(2 - 9\text{i}) = 2(2 - 9\text{i}) - 9\text{i}(2 - 9\text{i})$
$= 4 - 18\text{i} - 18\text{i} + 81\text{i}^2$
$= 4 - 18\text{i} - 18\text{i} - 81$
$= -77 - 36\text{i}$

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Complex numbers
Exercise B, Question 8

Question:

Simplify these, giving your answer in the form $a + b\text{i}$.

$(1 + \text{i})(2 + \text{i})(3 + \text{i})$

Solution:

$(2 + \text{i})(3 + \text{i}) = 2(3 + \text{i}) + \text{i}(3 + \text{i})$
$= 6 + 2\text{i} + 3\text{i} + \text{i}^2$
$= 6 + 2\text{i} + 3\text{i} - 1$
$= 5 + 5\text{i}$
$(1 + \text{i})(5 + 5\text{i}) = 1(5 + 5\text{i}) + \text{i}(5 + 5\text{i})$
$= 5 + 5\text{i} + 5\text{i} + 5\text{i}^2$
$= 5 + 5\text{i} + 5\text{i} - 5$
$= 10\text{i}$

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Complex numbers
Exercise B, Question 9

Question:

Simplify these, giving your answer in the form $a + b\text{i}$.

$(3 - 2\text{i})(5 + \text{i})(4 - 2\text{i})$

Solution:

$(5 + \text{i})(4 - 2\text{i}) = 5(4 - 2\text{i}) + \text{i}(4 - 2\text{i})$
$= 20 - 10\text{i} + 4\text{i} - 2\text{i}^2$
$= 20 - 10\text{i} + 4\text{i} + 2$
$= 22 - 6\text{i}$
$(3 - 2\text{i})(22 - 6\text{i}) = 3(22 - 6\text{i}) - 2\text{i}(22 - 6\text{i})$
$= 66 - 18\text{i} - 44\text{i} + 12\text{i}^2$
$= 66 - 18\text{i} - 44\text{i} - 12$
$= 54 - 62\text{i}$

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Complex numbers
Exercise B, Question 10

Question:

Simplify these, giving your answer in the form $a + b\text{i}$.

$(2 + 3\text{i})^3$

Solution:

$(2 + 3\text{i})^2 = (2 + 3\text{i})(2 + 3\text{i})$
$= 2(2 + 3\text{i}) + 3\text{i}(2 + 3\text{i})$
$= 4 + 6\text{i} + 6\text{i} + 9\text{i}^2$
$= 4 + 6\text{i} + 6\text{i} - 9$
$= -5 + 12\text{i}$
$(2 + 3\text{i})^3 = (2 + 3\text{i})(-5 + 12\text{i})$
$= 2(-5 + 12\text{i}) + 3\text{i}(-5 + 12\text{i})$
$= -10 + 24\text{i} - 15\text{i} + 36\text{i}^2$
$= -10 + 24\text{i} - 15\text{i} - 36$
$= -46 + 9\text{i}$

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Complex numbers
Exercise B, Question 11

Question:

Simplify

i^6

Solution:

$i \times i \times i \times i \times i \times i$
$= i^2 \times i^2 \times i^2 = -1 \times -1 \times -1 = -1$

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Complex numbers
Exercise B, Question 12

Question:

Simplify

$(3i)^4$

Solution:

$3i \times 3i \times 3i \times 3i$
$= 81(i \times i \times i \times i) = 81(i^2 \times i^2)$
$= 81(-1 \times -1) = 81$

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Complex numbers
Exercise B, Question 13

Question:

Simplify

$i^5 + i$

Solution:

$(i \times i \times i \times i \times i) + i$
$= (i^2 \times i^2 \times i) + i = (-1 \times -1 \times i) + i$
$= i + i = 2i$

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Complex numbers
Exercise B, Question 14

Question:

Simplify

$(4\mathrm{i})^3 - 4\mathrm{i}^3$

Solution:

$(4\mathrm{i})^3 = 4\mathrm{i} \times 4\mathrm{i} \times 4\mathrm{i} = 64(\mathrm{i} \times \mathrm{i} \times \mathrm{i})$
$= 64(-1 \times \mathrm{i}) = -64\mathrm{i}$
$4\mathrm{i}^3 = 4(\mathrm{i} \times \mathrm{i} \times \mathrm{i}) = 4(-1 \times \mathrm{i}) = -4\mathrm{i}$
$(4\mathrm{i})^3 - 4\mathrm{i}^3 = -64\mathrm{i} - (-4\mathrm{i})$
$= -64\mathrm{i} + 4\mathrm{i}$
$= -60\mathrm{i}$

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Complex numbers
Exercise B, Question 15

Question:

Simplify

$(1+\mathrm{i})^8$

Solution:

$(1+\mathrm{i})^8$

$$= 1^8 + 8.1^7\mathrm{i} + 28.1^6\mathrm{i}^2 + 56.1^5\mathrm{i}^3 + 70.1^4\mathrm{i}^4 + 56.1^3\mathrm{i}^5 + 28.1^2\mathrm{i}^6 + 8.1\mathrm{i}^7 + \mathrm{i}^8$$
$$= 1 + 8\mathrm{i} + 28\mathrm{i}^2 + 56\mathrm{i}^3 + 70\mathrm{i}^4 + 56\mathrm{i}^5 + 28\mathrm{i}^6 + 8\mathrm{i}^7 + \mathrm{i}^8$$

$$\begin{aligned}
\mathrm{i}^2 &= -1 \\
\mathrm{i}^3 &= \mathrm{i}^2 \times \mathrm{i} = -\mathrm{i} \\
\mathrm{i}^4 &= \mathrm{i}^2 \times \mathrm{i}^2 = 1 \\
\mathrm{i}^5 &= \mathrm{i}^2 \times \mathrm{i}^2 \times \mathrm{i} = \mathrm{i} \\
\mathrm{i}^6 &= \mathrm{i}^2 \times \mathrm{i}^2 \times \mathrm{i}^2 = -1 \\
\mathrm{i}^7 &= \mathrm{i}^2 \times \mathrm{i}^2 \times \mathrm{i}^2 \times \mathrm{i} = -\mathrm{i} \\
\mathrm{i}^8 &= \mathrm{i}^2 \times \mathrm{i}^2 \times \mathrm{i}^2 \times \mathrm{i}^2 = 1 \\
(1+\mathrm{i})^8 &= 1 + 8\mathrm{i} - 28 - 56\mathrm{i} + 70 + 56\mathrm{i} - 28 - 8\mathrm{i} + 1 \\
&= 16
\end{aligned}$$

Note also that
$$\begin{aligned}
(1+\mathrm{i})^2 &= (1+\mathrm{i})(1+\mathrm{i}) \\
&= 1 + 2\mathrm{i} + \mathrm{i}^2 = 2\mathrm{i}
\end{aligned}$$
So $(1+\mathrm{i})^8 = (2\mathrm{i})^4 = 16\mathrm{i}^4 = 16$

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Complex numbers
Exercise C, Question 1

Question:

Write down the complex conjugate z^* for

a $z = 8 + 2\text{i}$

b $z = 6 - 5\text{i}$

c $z = \frac{2}{3} - \frac{1}{2}\text{i}$

d $z = \sqrt{5} + \text{i}\sqrt{10}$

Solution:

a $z^* = 8 - 2\text{i}$

b $z^* = 6 + 5\text{i}$

c $z^* = \frac{2}{3} + \frac{1}{2}\text{i}$

d $z^* = \sqrt{5} - \text{i}\sqrt{10}$

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Complex numbers
Exercise C, Question 2

Question:

Find $z + z^*$ and zz^* for

a $z = 6 - 3\text{i}$

b $z = 10 + 5\text{i}$

c $z = \frac{3}{4} + \frac{1}{4}\text{i}$

d $z = \sqrt{5} - 3\text{i}\sqrt{5}$

Solution:

a

$$\begin{aligned} z + z^* &= (6 - 3\text{i}) + (6 + 3\text{i}) = 12 \\ zz^* &= (6 - 3\text{i})(6 + 3\text{i}) \\ &= 6(6 + 3\text{i}) - 3\text{i}(6 + 3\text{i}) \\ &= 36 + 18\text{i} - 18\text{i} - 9\text{i}^2 = 45 \end{aligned}$$

b

$$\begin{aligned} z + z^* &= (10 + 5\text{i}) + (10 - 5\text{i}) = 20 \\ zz^* &= (10 + 5\text{i})(10 - 5\text{i}) \\ &= 10(10 - 5\text{i}) + 5\text{i}(10 - 5\text{i}) \\ &= 100 - 50\text{i} + 50\text{i} - 25\text{i}^2 = 125 \end{aligned}$$

c

$$\begin{aligned} z + z^* &= \left(\frac{3}{4} + \frac{1}{4}\text{i}\right) + \left(\frac{3}{4} - \frac{1}{4}\text{i}\right) = \frac{3}{2} \\ zz^* &= \left(\frac{3}{4} + \frac{1}{4}\text{i}\right)\left(\frac{3}{4} - \frac{1}{4}\text{i}\right) \\ &= \frac{3}{4}\left(\frac{3}{4} - \frac{1}{4}\text{i}\right) + \frac{1}{4}\text{i}\left(\frac{3}{4} - \frac{1}{4}\text{i}\right) \\ &= \frac{9}{16} - \frac{3}{16}\text{i} + \frac{3}{16}\text{i} - \frac{1}{16}\text{i}^2 \\ &= \frac{10}{16} = \frac{5}{8} \end{aligned}$$

d

$$\begin{aligned} z + z^* &= (\sqrt{5} - 3\text{i}\sqrt{5}) + (\sqrt{5} + 3\text{i}\sqrt{5}) = 2\sqrt{5} \\ zz^* &= (\sqrt{5} - 3\text{i}\sqrt{5})(\sqrt{5} + 3\text{i}\sqrt{5}) \\ &= \sqrt{5}(\sqrt{5} + 3\text{i}\sqrt{5}) - 3\text{i}\sqrt{5}(\sqrt{5} + 3\text{i}\sqrt{5}) \\ &= 5 + 15\text{i} - 15\text{i} - 45\text{i}^2 \\ &= 50 \end{aligned}$$

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Complex numbers
Exercise C, Question 3

Question:

Find these in the form $a + b\mathrm{i}$.

$(25 - 10\mathrm{i}) \div (1 - 2\mathrm{i})$

Solution:

$$\frac{25-10\mathrm{i}}{1-2\mathrm{i}} = \frac{(25-10\mathrm{i})(1+2\mathrm{i})}{(1-2\mathrm{i})(1+2\mathrm{i})}$$

$$\begin{aligned}(25-10\mathrm{i})(1+2\mathrm{i}) &= 25(1+2\mathrm{i}) - 10\mathrm{i}(1+2\mathrm{i})\\ &= 25 + 50\mathrm{i} - 10\mathrm{i} - 20\mathrm{i}^2\\ &= 45 + 40\mathrm{i}\\ (1-2\mathrm{i})(1+2\mathrm{i}) &= 1(1+2\mathrm{i}) - 2\mathrm{i}(1+2\mathrm{i})\\ &= 1 + 2\mathrm{i} - 2\mathrm{i} - 4\mathrm{i}^2\\ &= 5\\ \frac{45+40\mathrm{i}}{5} &= 9 + 8\mathrm{i}\end{aligned}$$

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Complex numbers
Exercise C, Question 4

Question:

Find these in the form $a + b\text{i}$.

$(6 + \text{i}) \div (3 + 4\text{i})$

Solution:

$$\begin{aligned}
\frac{6+\text{i}}{3+4\text{i}} &= \frac{(6+\text{i})(3-4\text{i})}{(3+4\text{i})(3-4\text{i})} \\
(6+\text{i})(3-4\text{i}) &= 6(3-4\text{i}) + \text{i}(3-4\text{i}) \\
&= 18 - 24\text{i} + 3\text{i} - 4\text{i}^2 \\
&= 22 - 21\text{i} \\
(3+4\text{i})(3-4\text{i}) &= 3(3-4\text{i}) + 4\text{i}(3-4\text{i}) \\
&= 9 - 12\text{i} + 12\text{i} - 16\text{i}^2 \\
&= 25 \\
\frac{22-21\text{i}}{25} &= \frac{22}{25} - \frac{21}{25}\text{i}
\end{aligned}$$

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Complex numbers
Exercise C, Question 5

Question:

Find these in the form $a + b\text{i}$.

$(11 + 4\text{i}) \div (3 + \text{i})$

Solution:

$$\begin{aligned}
\frac{11+4\text{i}}{3+\text{i}} &= \frac{(11+4\text{i})(3-\text{i})}{(3+\text{i})(3-\text{i})} \\
(11+4\text{i})(3-\text{i}) &= 11(3-\text{i}) + 4\text{i}(3-\text{i}) \\
&= 33 - 11\text{i} + 12\text{i} - 4\text{i}^2 \\
&= 37 + \text{i} \\
(3+\text{i})(3-\text{i}) &= 3(3-\text{i}) + \text{i}(3-\text{i}) \\
&= 9 - 3\text{i} + 3\text{i} - \text{i}^2 \\
&= 10 \\
\frac{37+\text{i}}{10} &= \frac{37}{10} + \frac{1}{10}\text{i}
\end{aligned}$$

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Complex numbers
Exercise C, Question 6

Question:

Find these in the form $a + b\mathrm{i}$.

$$\frac{1+\mathrm{i}}{2+\mathrm{i}}$$

Solution:

$$\begin{aligned}
\frac{1+\mathrm{i}}{2+\mathrm{i}} &= \frac{(1+\mathrm{i})(2-\mathrm{i})}{(2+\mathrm{i})(2-\mathrm{i})} \\
(1+\mathrm{i})(2-\mathrm{i}) &= 1(2-\mathrm{i})+\mathrm{i}(2-\mathrm{i}) \\
&= 2-\mathrm{i}+2\mathrm{i}-\mathrm{i}^2 \\
&= 3+\mathrm{i} \\
(2+\mathrm{i})(2-\mathrm{i}) &= 2(2-\mathrm{i})+\mathrm{i}(2-\mathrm{i}) \\
&= 4-2\mathrm{i}+2\mathrm{i}-\mathrm{i}^2 \\
&= 5 \\
\frac{3+\mathrm{i}}{5} &= \frac{3}{5}+\frac{1}{5}\mathrm{i}
\end{aligned}$$

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Complex numbers
Exercise C, Question 7

Question:

Find these in the form $a + b\text{i}$.

$$\frac{3-5\text{i}}{1+3\text{i}}$$

Solution:

$$\begin{aligned}
\frac{3-5\text{i}}{1+3\text{i}} &= \frac{(3-5\text{i})(1-3\text{i})}{(1+3\text{i})(1-3\text{i})} \\
(3-5\text{i})(1-3\text{i}) &= 3(1-3\text{i}) - 5\text{i}(1-3\text{i}) \\
&= 3 - 9\text{i} - 5\text{i} + 15\text{i}^2 \\
&= -12 - 14\text{i} \\
(1+3\text{i})(1-3\text{i}) &= 1(1-3\text{i}) + 3\text{i}(1-3\text{i}) \\
&= 1 - 3\text{i} + 3\text{i} - 9\text{i}^2 \\
&= 10 \\
\frac{-12-14\text{i}}{10} &= -\frac{6}{5} - \frac{7}{5}\text{i}
\end{aligned}$$

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Complex numbers
Exercise C, Question 8

Question:

Find these in the form $a + b\text{i}$.

$$\frac{3+5\text{i}}{6-8\text{i}}$$

Solution:

$$\begin{aligned}
\frac{3+5\text{i}}{6-8\text{i}} &= \frac{(3+5\text{i})(6+8\text{i})}{(6-8\text{i})(6+8\text{i})} \\
(3+5\text{i})(6+8\text{i}) &= 3(6+8\text{i})+5\text{i}(6+8\text{i}) \\
&= 18+24\text{i}+30\text{i}+40\text{i}^2 \\
&= -22+54\text{i} \\
(6-8\text{i})(6+8\text{i}) &= 6(6+8\text{i})-8\text{i}(6+8\text{i}) \\
&= 36+48\text{i}-48\text{i}-64\text{i}^2 \\
&= 100 \\
\frac{-22+54\text{i}}{100} &= \frac{-11}{50}+\frac{27}{50}\text{i}
\end{aligned}$$

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Complex numbers
Exercise C, Question 9

Question:

Find these in the form $a + b\mathrm{i}$.

$$\frac{28 - 3\mathrm{i}}{1 - \mathrm{i}}$$

Solution:

$$\begin{aligned}
\frac{28 - 3\mathrm{i}}{1 - \mathrm{i}} &= \frac{(28 - 3\mathrm{i})(1 + \mathrm{i})}{(1 - \mathrm{i})(1 + \mathrm{i})} \\
(28 - 3\mathrm{i})(1 + \mathrm{i}) &= 28(1 + \mathrm{i}) - 3\mathrm{i}(1 + \mathrm{i}) \\
&= 28 + 28\mathrm{i} - 3\mathrm{i} - 3\mathrm{i}^2 \\
&= 31 + 25\mathrm{i} \\
(1 - \mathrm{i})(1 + \mathrm{i}) &= 1(1 + \mathrm{i}) - \mathrm{i}(1 + \mathrm{i}) \\
&= 1 + \mathrm{i} - \mathrm{i} - \mathrm{i}^2 \\
&= 2 \\
\frac{31 + 25\mathrm{i}}{2} &= \frac{31}{2} + \frac{25}{2}\mathrm{i}
\end{aligned}$$

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Complex numbers
Exercise C, Question 10

Question:

Find these in the form $a + b\text{i}$.

$\frac{2+\text{i}}{1+4\text{i}}$

Solution:

$$\begin{aligned}
\frac{2+\text{i}}{1+4\text{i}} &= \frac{(2+\text{i})(1-4\text{i})}{(1+4\text{i})(1-4\text{i})} \\
(2+\text{i})(1-4\text{i}) &= 2(1-4\text{i})+\text{i}(1-4\text{i}) \\
&= 2-8\text{i}+\text{i}-4\text{i}^2 \\
&= 6-7\text{i} \\
(1+4\text{i})(1-4\text{i}) &= 1(1-4\text{i})+4\text{i}(1-4\text{i}) \\
&= 1-4\text{i}+4\text{i}-16\text{i}^2 \\
&= 17 \\
\frac{6-7\text{i}}{17} &= \frac{6}{17}-\frac{7}{17}\text{i}
\end{aligned}$$

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Complex numbers
Exercise C, Question 11

Question:

Find these in the form $a + b$i.

$$\frac{(3-4\text{i})^2}{1+\text{i}}$$

Solution:

$$\begin{aligned}
(3-4\text{i})^2 &= (3-4\text{i})(3-4\text{i}) \\
&= 3(3-4\text{i}) - 4\text{i}(3-4\text{i}) \\
&= 9 - 12\text{i} - 12\text{i} + 16\text{i}^2 \\
&= -7 - 24\text{i} \\
\frac{-7-24\text{i}}{1+\text{i}} &= \frac{(-7-24\text{i})(1-\text{i})}{(1+\text{i})(1-\text{i})} \\
(-7-24\text{i})(1-\text{i}) &= -7(1-\text{i}) - 24\text{i}(1-\text{i}) \\
&= -7 + 7\text{i} - 24\text{i} + 24\text{i}^2 \\
&= -31 - 17\text{i} \\
(1+\text{i})(1-\text{i}) &= 1(1-\text{i}) + \text{i}(1-\text{i}) \\
&= 1 - \text{i} + \text{i} - \text{i}^2 \\
&= 2 \\
\frac{-31-17\text{i}}{2} &= \frac{-31}{2} - \frac{17}{2}\text{i}
\end{aligned}$$

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Complex numbers

Exercise C, Question 12

Question:

Given that $z_1 = 1 + \mathrm{i}, z_2 = 2 + \mathrm{i}$ and $z_3 = 3 + \mathrm{i}$, find the following in the form $a + b\mathrm{i}$.

$\frac{z_1 z_2}{z_3}$

Solution:

$$
\begin{aligned}
z_1 z_2 &= (1 + \mathrm{i})(2 + \mathrm{i}) \\
&= 1(2 + \mathrm{i}) + \mathrm{i}(2 + \mathrm{i}) \\
&= 2 + \mathrm{i} + 2\mathrm{i} + \mathrm{i}^2 \\
&= 1 + 3\mathrm{i} \\
\frac{z_1 z_2}{z_3} &= \frac{1 + 3\mathrm{i}}{3 + \mathrm{i}} = \frac{(1 + 3\mathrm{i})(3 - \mathrm{i})}{(3 + \mathrm{i})(3 - \mathrm{i})} \\
(1 + 3\mathrm{i})(3 - \mathrm{i}) &= 1(3 - \mathrm{i}) + 3\mathrm{i}(3 - \mathrm{i}) \\
&= 3 - \mathrm{i} + 9\mathrm{i} - 3\mathrm{i}^2 \\
&= 6 + 8\mathrm{i} \\
(3 + \mathrm{i})(3 - \mathrm{i}) &= 3(3 - \mathrm{i}) + \mathrm{i}(3 - \mathrm{i}) \\
&= 9 - 3\mathrm{i} + 3\mathrm{i} - \mathrm{i}^2 \\
&= 10 \\
\frac{6 + 8\mathrm{i}}{10} &= \frac{3}{5} + \frac{4}{5}\mathrm{i}
\end{aligned}
$$

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Complex numbers
Exercise C, Question 13

Question:

Given that $z_1 = 1 + \text{i}, z_2 = 2 + \text{i}$ and $z_3 = 3 + \text{i}$, find the following in the form $a + b\text{i}$.

$$\frac{(z_2)^2}{z_1}$$

Solution:

$$\begin{aligned}
(z_2)^2 &= (2+\text{i})(2+\text{i}) \\
&= 2(2+\text{i}) + \text{i}(2+\text{i}) \\
&= 4 + 2\text{i} + 2\text{i} + \text{i}^2 \\
&= 3 + 4\text{i} \\
\frac{(z_2)^2}{z_1} &= \frac{3+4\text{i}}{1+\text{i}} = \frac{(3+4\text{i})(1-\text{i})}{(1+\text{i})(1-\text{i})} \\
(3+4\text{i})(1-\text{i}) &= 3(1-\text{i}) + 4\text{i}(1-\text{i}) \\
&= 3 - 3\text{i} + 4\text{i} - 4\text{i}^2 \\
&= 7 + \text{i} \\
(1+\text{i})(1-\text{i}) &= 1(1-\text{i}) + \text{i}(1-\text{i}) \\
&= 1 - \text{i} + \text{i} - \text{i}^2 \\
&= 2 \\
\frac{7+\text{i}}{2} &= \frac{7}{2} + \frac{1}{2}\text{i}
\end{aligned}$$

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Complex numbers
Exercise C, Question 14

Question:

Given that $z_1 = 1 + \mathrm{i}, z_2 = 2 + \mathrm{i}$ and $z_3 = 3 + \mathrm{i}$, find the following in the form $a + b\mathrm{i}$.

$$\frac{2z_1 + 5z_3}{z_2}$$

Solution:

$$\begin{aligned}
2z_1 + 5z_3 &= 2(1 + \mathrm{i}) + 5(3 + \mathrm{i}) \\
&= 2 + 2\mathrm{i} + 15 + 5\mathrm{i} \\
&= 17 + 7\mathrm{i} \\
\frac{2z_1 + 5z_3}{z_2} &= \frac{17 + 7\mathrm{i}}{2 + \mathrm{i}} = \frac{(17 + 7\mathrm{i})(2 - \mathrm{i})}{(2 + \mathrm{i})(2 - \mathrm{i})} \\
(17 + 7\mathrm{i})(2 - \mathrm{i}) &= 17(2 - \mathrm{i}) + 7\mathrm{i}(2 - \mathrm{i}) \\
&= 34 - 17\mathrm{i} + 14\mathrm{i} - 7\mathrm{i}^2 \\
&= 41 - 3\mathrm{i} \\
(2 + \mathrm{i})(2 - \mathrm{i}) &= 2(2 - \mathrm{i}) + \mathrm{i}(2 - \mathrm{i}) \\
&= 4 - 2\mathrm{i} + 2\mathrm{i} - \mathrm{i}^2 \\
&= 5 \\
\frac{41 - 3\mathrm{i}}{5} &= \frac{41}{5} - \frac{3}{5}\mathrm{i}
\end{aligned}$$

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Complex numbers
Exercise C, Question 15

Question:

Given that $\frac{5+2\mathrm{i}}{z} = 2 - \mathrm{i}$, find z in the form $a + b\mathrm{i}$.

Solution:

$$\frac{5+2\mathrm{i}}{z} = 2-\mathrm{i}$$
$$z = \frac{5+2\mathrm{i}}{2-\mathrm{i}} = \frac{(5+2\mathrm{i})(2+\mathrm{i})}{(2-\mathrm{i})(2+\mathrm{i})}$$
$$(5+2\mathrm{i})(2+\mathrm{i}) = 5(2+\mathrm{i}) + 2\mathrm{i}(2+\mathrm{i})$$
$$= 10 + 5\mathrm{i} + 4\mathrm{i} + 2\mathrm{i}^2$$
$$= 8 + 9\mathrm{i}$$
$$(2-\mathrm{i})(2+\mathrm{i}) = 2(2+\mathrm{i}) - \mathrm{i}(2+\mathrm{i})$$
$$= 4 + 2\mathrm{i} - 2\mathrm{i} - \mathrm{i}^2$$
$$= 5$$
$$z = \frac{8+9\mathrm{i}}{5} = \frac{8}{5} + \frac{9}{5}\mathrm{i}$$

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Complex numbers
Exercise C, Question 16

Question:

Simplify $\frac{6+8\text{i}}{1+\text{i}}+\frac{6+8\text{i}}{1-\text{i}}$, giving your answer in the form $a+b\text{i}$.

Solution:

$$\frac{6+8\text{i}}{1+\text{i}}+\frac{6+8\text{i}}{1-\text{i}}$$
$$=\frac{(6+8\text{i})(1-\text{i})+(6+8\text{i})(1+\text{i})}{(1+\text{i})(1-\text{i})}$$
$$=\frac{6(1-\text{i})+8\text{i}(1-\text{i})+6(1+\text{i})+8\text{i}(1+\text{i})}{1(1-\text{i})+\text{i}(1-\text{i})}$$
$$=\frac{6-6\text{i}+8\text{i}-8\text{i}^2+6+6\text{i}+8\text{i}+8\text{i}^2}{1-\text{i}+\text{i}-\text{i}^2}$$
$$=\frac{12+16\text{i}}{2}=6+8\text{i}$$

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Complex numbers
Exercise C, Question 17

Question:

The roots of the quadratic equation $x^2 + 2x + 26 = 0$ are α and β. Find

a α and β

b $\alpha + \beta$

c $\alpha\beta$

Solution:

$x^2 + 2x + 26 = 0$
$a = 1 \ ,b = 2 \ ,c = 26$
$$x = \frac{-2 \pm \sqrt{(4 - 104)}}{2} = \frac{-2 \pm 10\mathrm{i}}{2}$$

a $\alpha = -1 + 5\mathrm{i}, \beta = -1 - 5\mathrm{i}$ or vice versa

b $\alpha + \beta = (-1 + 5\mathrm{i}) + (-1 - 5\mathrm{i}) = -2$

c

$$\begin{aligned} \alpha\beta &= (-1 + 5\mathrm{i})(-1 - 5\mathrm{i}) \\ &= -1(-1 - 5\mathrm{i}) + 5\mathrm{i}(-1 - 5\mathrm{i}) \\ &= 1 + 5\mathrm{i} - 5\mathrm{i} - 25\mathrm{i}^2 = 26 \end{aligned}$$

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Complex numbers
Exercise C, Question 18

Question:

The roots of the quadratic equation $x^2 - 8x + 25 = 0$ are α and β. Find

a α and β

b $\alpha + \beta$

c $\alpha\beta$

Solution:

$x^2 - 8x + 25 = 0$
$a = 1 \ , b = -8 \ , c = 25$
$x = \frac{8 \pm \sqrt{(64 - 100)}}{2} = \frac{8 \pm 6\text{i}}{2}$

(a) $\alpha = 4 + 3\text{i}, \beta = 4 - 3\text{i}$ or vice versa

(b) $\alpha + \beta = (4 + 3\text{i}) + (4 - 3\text{i}) = 8$

(c) $\alpha\beta = (4 + 3\text{i})(4 - 3\text{i})$

$= 4(4 - 3\text{i}) + 3\text{i}(4 - 3\text{i})$
$= 16 - 12\text{i} + 12\text{i} - 9\text{i}^2 = 25$

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Complex numbers
Exercise C, Question 19

Question:

Find the quadratic equation that has roots $2 + 3\mathrm{i}$ and $2 - 3\mathrm{i}$.

Solution:

If roots are α and β, the equation is

$$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\begin{aligned}
\alpha + \beta &= (2 + 3\mathrm{i}) + (2 - 3\mathrm{i}) = 4 \\
\alpha\beta &= (2 + 3\mathrm{i})(2 - 3\mathrm{i}) \\
&= 2(2 - 3\mathrm{i}) + 3\mathrm{i}(2 - 3\mathrm{i}) \\
&= 4 - 6\mathrm{i} + 6\mathrm{i} - 9\mathrm{i}^2 = 13
\end{aligned}$$

Equation is $x^2 - 4x + 13 = 0$

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Complex numbers
Exercise C, Question 20

Question:

Find the quadratic equation that has roots $-5+4\mathrm{i}$ and $-5-4\mathrm{i}$.

Solution:

If roots are α and β, the equation is

$$(x-\alpha)(x-\beta) = x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$\begin{aligned}\alpha+\beta &= (-5+4\mathrm{i})+(-5-4\mathrm{i})=-10\\ \alpha\beta &= (-5+4\mathrm{i})(-5-4\mathrm{i})\\ &= -5(-5-4\mathrm{i})+4\mathrm{i}(-5-4\mathrm{i})\\ &= 25+20\mathrm{i}-20\mathrm{i}-16\mathrm{i}^2\\ &= 41\end{aligned}$$

Equation is $x^2+10x+41=0$

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Complex numbers
Exercise D, Question 1

Question:

Show these numbers on an Argand diagram.

a $7+2\mathrm{i}$

b $5-4\mathrm{i}$

c $-6-\mathrm{i}$

d $-2+5\mathrm{i}$

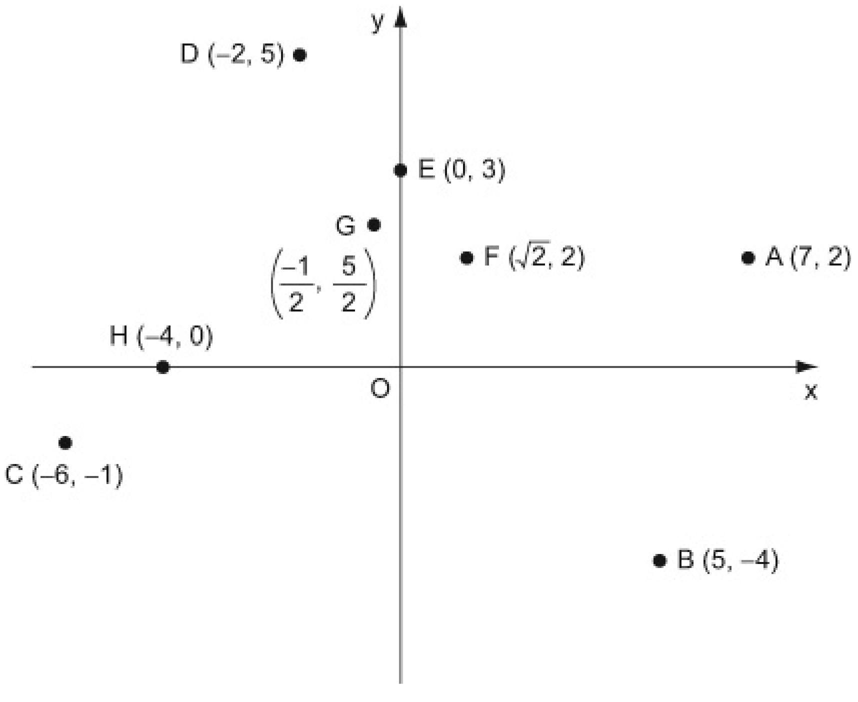
e $3\mathrm{i}$

f $\sqrt{2}+2\mathrm{i}$

g $-\frac{1}{2}+\frac{5}{2}\mathrm{i}$

h -4

Solution:



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Complex numbers
Exercise D, Question 2

Question:

Given that $z_1 = -1 - \mathrm{i}$, $z_2 = -5 + 10\mathrm{i}$ and $z_3 = 3 - 4\mathrm{i}$,

a find $z_1 z_2$, $z_1 z_3$ and $\frac{z_2}{z_3}$ in the form $a + \mathrm{i}b$.

b show $z_1, z_2, z_3, z_1 z_2, z_1 z_3$ and $\frac{z_2}{z_3}$ on an Argand diagram.

Solution:

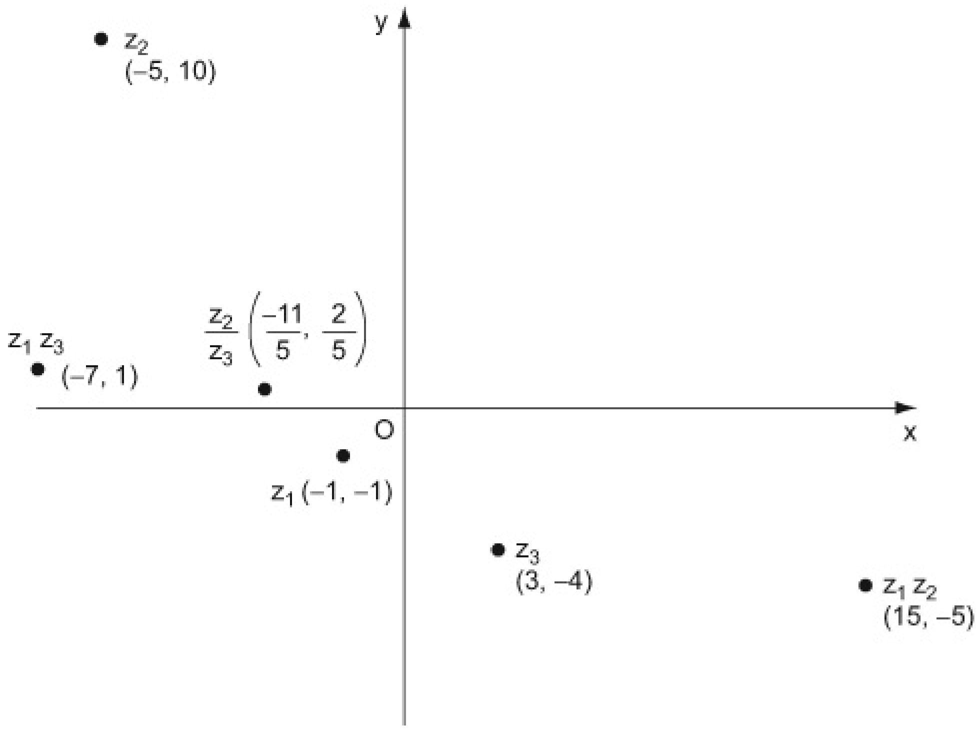
a $z_1 z_2 = (-1 - \mathrm{i})(-5 + 10\mathrm{i})$

$$= -1(-5 + 10\mathrm{i}) - \mathrm{i}(-5 + 10\mathrm{i})$$
$$= 5 - 10\mathrm{i} + 5\mathrm{i} - 10\mathrm{i}^2$$
$$= 15 - 5\mathrm{i}$$

$$z_1 z_3 = (-1 - \mathrm{i})(3 - 4\mathrm{i})$$
$$= -1(3 - 4\mathrm{i}) - \mathrm{i}(3 - 4\mathrm{i})$$
$$= -3 + 4\mathrm{i} - 3\mathrm{i} + 4\mathrm{i}^2$$
$$= -7 + \mathrm{i}$$

$$\frac{z_2}{z_3} = \frac{-5 + 10\mathrm{i}}{3 - 4\mathrm{i}} = \frac{(-5 + 10\mathrm{i})(3 + 4\mathrm{i})}{(3 - 4\mathrm{i})(3 + 4\mathrm{i})}$$
$$= \frac{-5(3 + 4\mathrm{i}) + 10\mathrm{i}(3 + 4\mathrm{i})}{3(3 + 4\mathrm{i}) - 4\mathrm{i}(3 + 4\mathrm{i})}$$
$$= \frac{-15 - 20\mathrm{i} + 30\mathrm{i} + 40\mathrm{i}^2}{9 + 12\mathrm{i} - 12\mathrm{i} - 16\mathrm{i}^2}$$
$$= \frac{-55 + 10\mathrm{i}}{25} = \frac{-11}{5} + \frac{2}{5}\mathrm{i}$$

b


y
z_2
(−5, 10)
$z_1 z_3$
(−7, 1)
$\frac{z_2}{z_3}\left(\frac{-11}{5}, \frac{2}{5}\right)$
O
x
z_1 (−1, −1)
z_3
(3, −4)
$z_1 z_2$
(15, −5)

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Complex numbers
Exercise D, Question 3

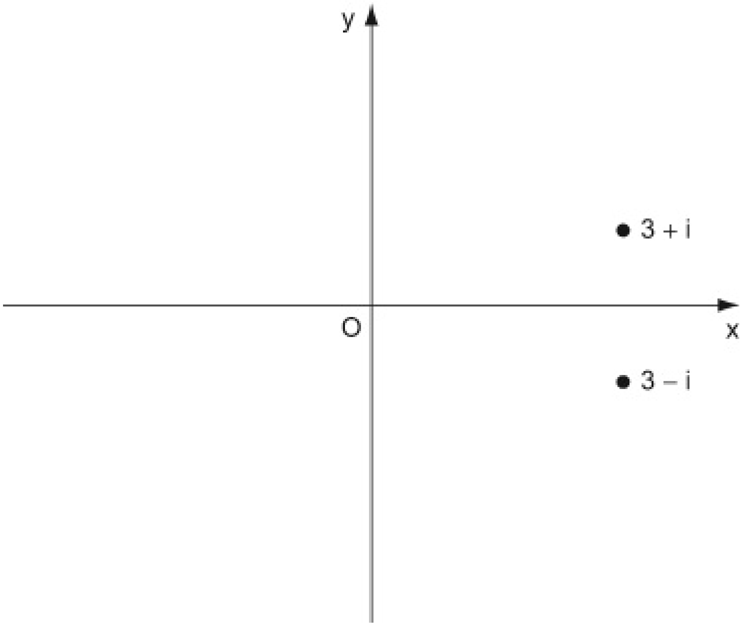
Question:

Show the roots of the equation $x^2 - 6x + 10 = 0$ on an Argand diagram.

Solution:

$x^2 - 6x + 10 = 0$
$a = 1, b = -6, c = 10$
$x = \frac{6 \pm \sqrt{(36 - 40)}}{2} = \frac{6 \pm 2i}{2}$

Roots are $3 + i$ and $3 - i$



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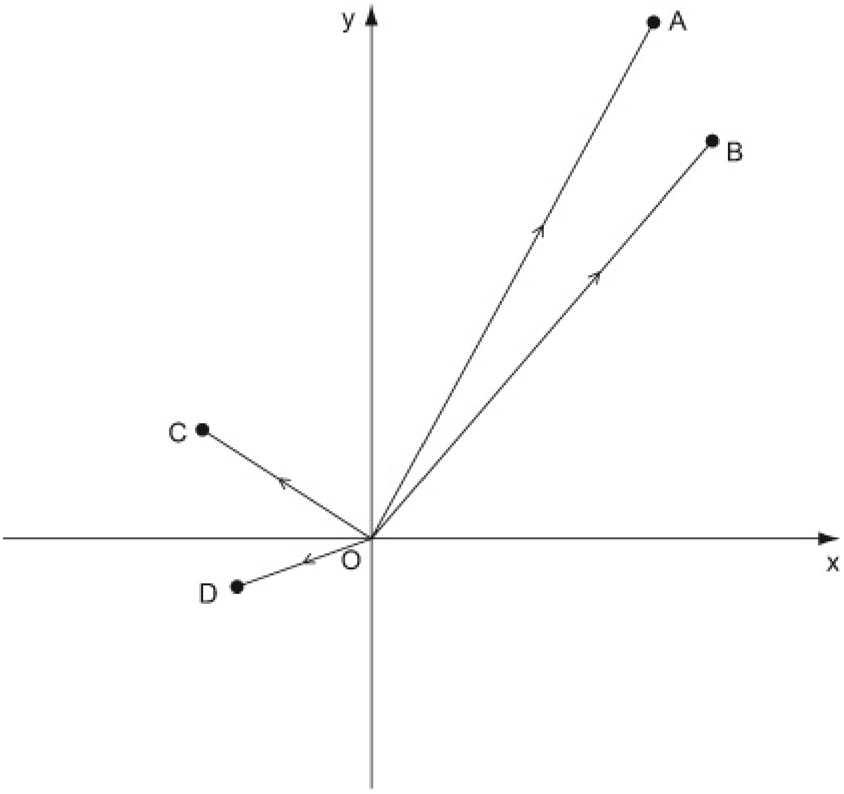
Complex numbers
Exercise D, Question 4

Question:

The complex numbers $z_1 = 5 + 12\mathrm{i}, z_2 = 6 + 10\mathrm{i}, z_3 = -4 + 2\mathrm{i}$ and $z_4 = -3 - \mathrm{i}$ are represented by the vectors $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively on an Argand diagram. Draw the diagram and calculate $|\overrightarrow{OA}|$, $|\overrightarrow{OB}|$, $|\overrightarrow{OC}|$ and $|\overrightarrow{OD}|$.

Solution:



$|\overrightarrow{OA}| = \sqrt{(5^2 + 12^2)} = \sqrt{169} = 13$

$|\overrightarrow{OB}| = \sqrt{(6^2 + 10^2)} = \sqrt{136} = \sqrt{4}\sqrt{34} = 2\sqrt{34}$

$|\overrightarrow{OC}| = \sqrt{\left((-4)^2 + 2^2\right)} = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

$|\overrightarrow{OD}| = \sqrt{\left((-3)^2 + (-1)^2\right)} = \sqrt{10}$

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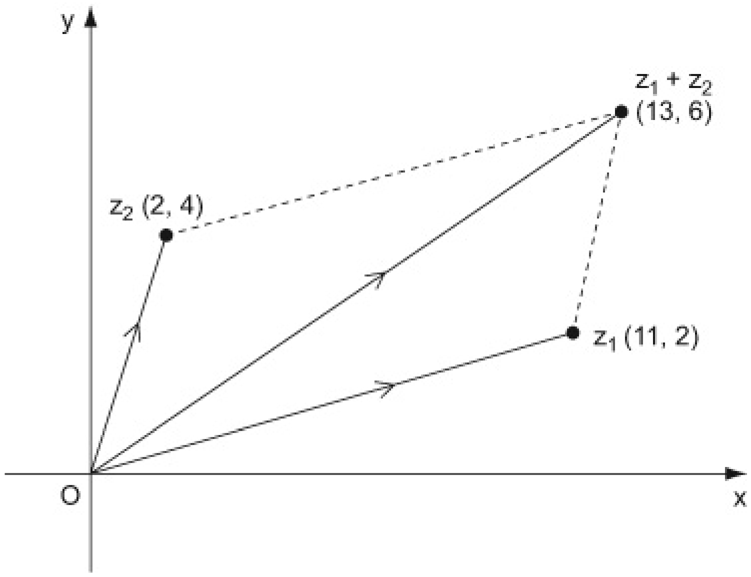
Complex numbers
Exercise D, Question 5

Question:

$z_1 = 11 + 2\text{i}$ and $z_2 = 2 + 4\text{i}$. Show z_1, z_2 and $z_1 + z_2$ on an Argand diagram.

Solution:



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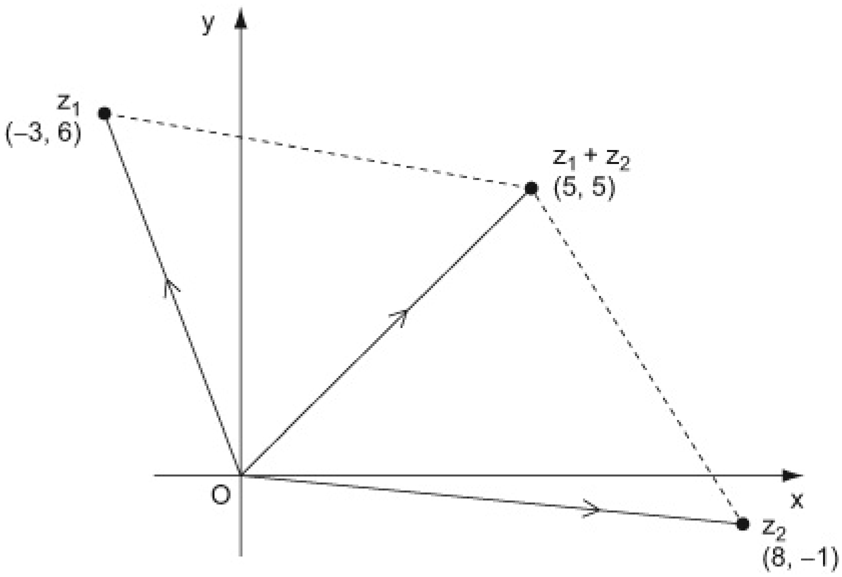
Complex numbers
Exercise D, Question 6

Question:

$z_1 = -3 + 6\text{i}$ and $z_2 = 8 - \text{i}$. Show z_1, z_2 and $z_1 + z_2$ on an Argand diagram.

Solution:



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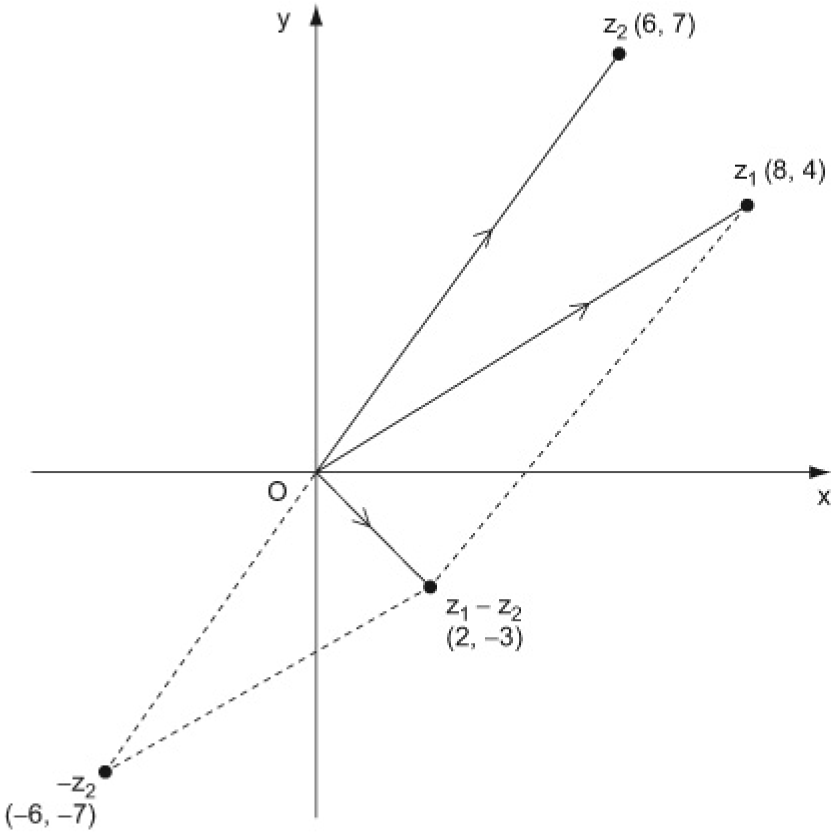
Complex numbers
Exercise D, Question 7

Question:

$z_1 = 8 + 4\text{i}$ and $z_2 = 6 + 7\text{i}$. Show z_1, z_2 and $z_1 - z_2$ on an Argand diagram.

Solution:



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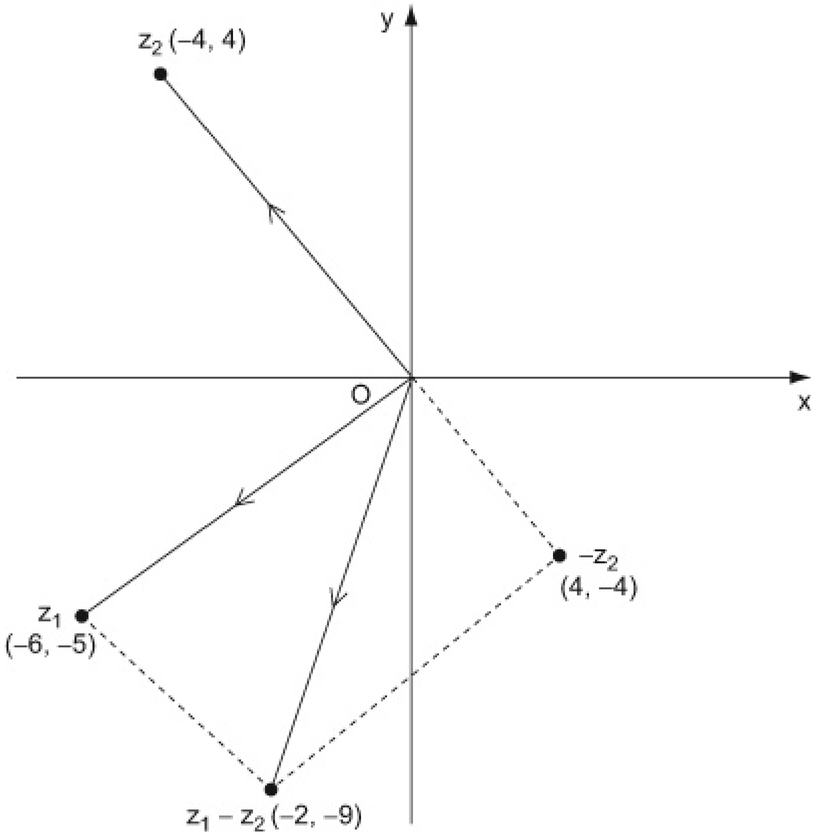
Complex numbers
Exercise D, Question 8

Question:

$z_1 = -6 - 5i$ and $z_2 = -4 + 4i$. Show z_1, z_2 and $z_1 - z_2$ on an Argand diagram.

Solution:



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Complex numbers
Exercise E, Question 1

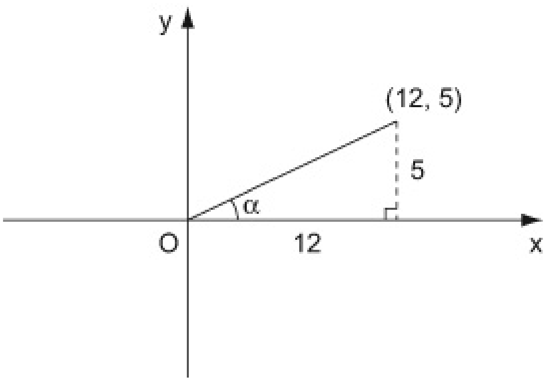
Question:

Find the modulus and argument of each of the following complex numbers, giving your answers exactly where possible, and to two decimal places otherwise.

$12 + 5\text{i}$

Solution:

$z = 12 + 5\text{i}$



$$|z| = \sqrt{(12^2 + 5^2)} = \sqrt{169} = 13$$

$$\tan\alpha = \frac{5}{12}. \quad \alpha = 0.39 \text{ rad.}$$

$$\arg z = 0.39$$

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Complex numbers
Exercise E, Question 2

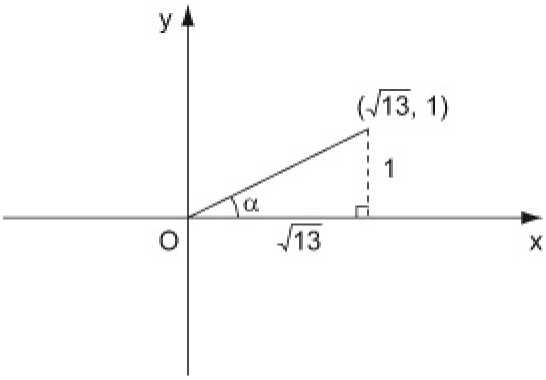
Question:

Find the modulus and argument of each of the following complex numbers, giving your answers exactly where possible, and to two decimal places otherwise.

$\sqrt{3}+\mathrm{i}$

Solution:

$z=\sqrt{3}+\mathrm{i}$



$|z| = \sqrt{\left((\sqrt{3})^2+1^2\right)} = \sqrt{4} = 2$

$\tan\alpha = \frac{1}{\sqrt{3}}. \quad \alpha = \frac{\pi}{6}.$

$\arg z = \frac{\pi}{6}.$

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Complex numbers
Exercise E, Question 3

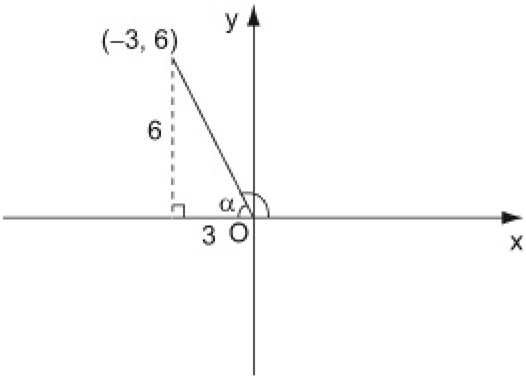
Question:

Find the modulus and argument of each of the following complex numbers, giving your answers exactly where possible, and to two decimal places otherwise.

$-3+6\text{i}$

Solution:

$z=-3+6\text{i}$



$|z| = \sqrt{\left((-3)^2 + 6^2\right)} = \sqrt{45} = 3\sqrt{5}$

$\tan\alpha = \frac{6}{3}. \quad \alpha = 1.107 \text{ rad}$

$\arg z = \pi - \alpha = 2.03$

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Complex numbers
Exercise E, Question 4

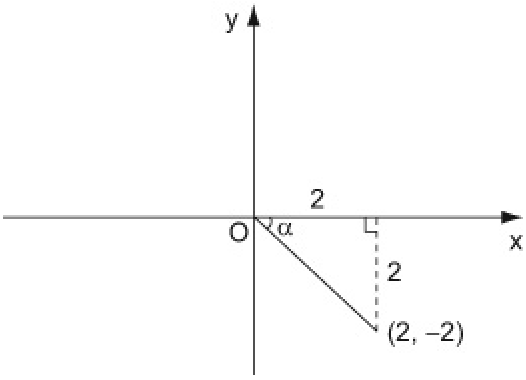
Question:

Find the modulus and argument of each of the following complex numbers, giving your answers exactly where possible, and to two decimal places otherwise.

2 − 2i

Solution:

$z = 2 - 2\text{i}$



$$|z| = \sqrt{\left(2^2 + (-2)^2\right)} = \sqrt{8} = 2\sqrt{2}$$

$$\tan\alpha = \frac{2}{2}. \quad \alpha = \frac{\pi}{4}.$$

$$\arg z = -\alpha = -\frac{\pi}{4}.$$

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Complex numbers
Exercise E, Question 5

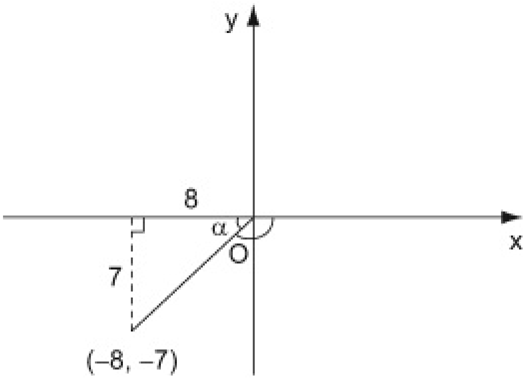
Question:

Find the modulus and argument of each of the following complex numbers, giving your answers exactly where possible, and to two decimal places otherwise.

$-8-7\text{i}$

Solution:

$z=-8-7\text{i}$



$|z|=\sqrt{\left((-8)^2+(-7)^2\right)}=\sqrt{113}$

$\tan\alpha=\frac{7}{8}.\quad \alpha=0.7188\ \text{rad}$

$\arg z=-(\pi-\alpha)=-2.42$

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Complex numbers
Exercise E, Question 6

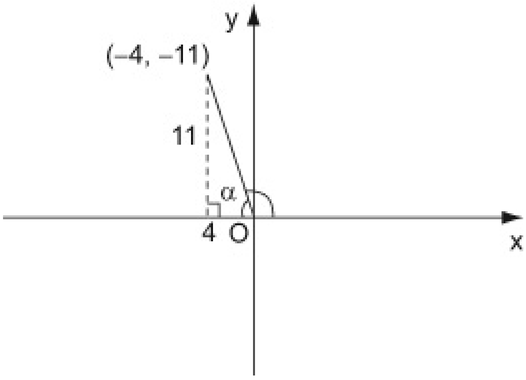
Question:

Find the modulus and argument of each of the following complex numbers, giving your answers exactly where possible, and to two decimal places otherwise.

$-4 + 11\text{i}$

Solution:

$z = -4 + 11\text{i}$



$|z| = \sqrt{\left((-4)^2 + 11^2\right)} = \sqrt{137}$

$\tan\alpha = \frac{11}{4}. \quad \alpha = 1.222 \text{ rad}$

$\arg z = \pi - \alpha = 1.92$

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Complex numbers
Exercise E, Question 7

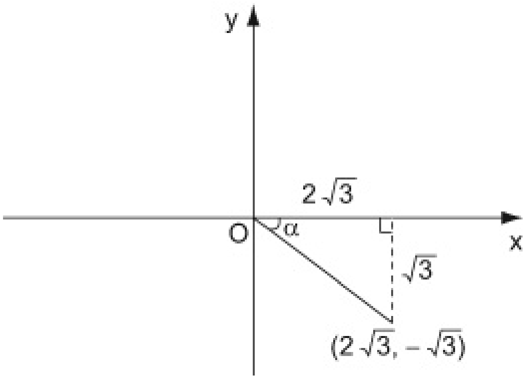
Question:

Find the modulus and argument of each of the following complex numbers, giving your answers exactly where possible, and to two decimal places otherwise.

$2\sqrt{3} - i\sqrt{3}$

Solution:

$z = 2\sqrt{3} - i\sqrt{3}$



$|z| = \sqrt{\left((2\sqrt{3})^2 + (-\sqrt{3})^2\right)} = \sqrt{15}$

$\tan\alpha = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}. \quad \alpha = 0.4636 \text{ rad.}$

$\arg z = -0.46$

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Edexcel AS and A Level Modular Mathematics

Complex numbers
Exercise E, Question 8

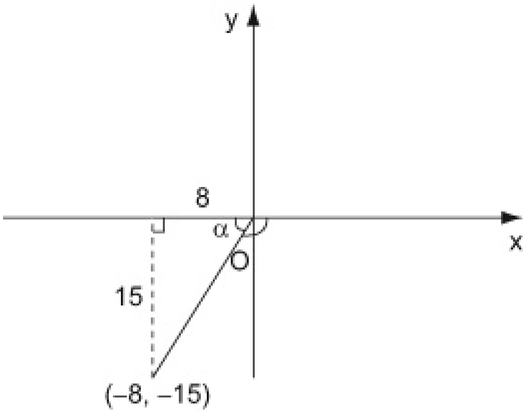
Question:

Find the modulus and argument of each of the following complex numbers, giving your answers exactly where possible, and to two decimal places otherwise.

$-8-15i$

Solution:

$z=-8-15i$



$$|z|=\sqrt{\left((-8)^2+(-15)^2\right)}=\sqrt{289}=17$$

$$\tan\alpha=\frac{15}{8}. \quad \alpha=1.0808 \text{ rad.}$$

$$\arg z=-(\pi-\alpha)=-2.06$$

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Complex numbers
Exercise F, Question 1

Question:

Express these in the form $r(\cos\ \theta + \mathrm{i}\ \sin\ \theta)$, giving exact values of r and θ where possible, or values to two decimal places otherwise.

a $2+2\mathrm{i}$

b $3\mathrm{i}$

c $-3+4\mathrm{i}$

d $1-\sqrt{3}\,\mathrm{i}$

e $-2-5\mathrm{i}$

f -20

g $7-24\mathrm{i}$

h $-5+5\mathrm{i}$

Solution:

a

$$\begin{aligned} r &= \sqrt{\left(2^2+2^2\right)} = \sqrt{8} = 2\sqrt{2} \\ \tan\alpha &= \frac{2}{2} = 1. \qquad \alpha = \frac{\pi}{4} \\ \theta &= \frac{\pi}{4} \\ 2+2\mathrm{i} &= 2\sqrt{2}\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right) \end{aligned}$$

b

$$\begin{aligned} r &= \sqrt{\left(O^2+3^2\right)} = \sqrt{9} = 3 \\ \tan\alpha &= \infty \qquad \alpha = \frac{\pi}{2} \\ \theta &= \frac{\pi}{2} \\ 3\mathrm{i} &= 3\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right) \end{aligned}$$

c

$$\begin{aligned} r &= \sqrt{\left((-3)^2+4^2\right)} = \sqrt{2}5 = 5 \\ \tan\alpha &= \frac{4}{3}. \qquad \alpha = 0.927\ \text{rad.} \\ \theta &= \pi - \alpha = 2.21 \\ -3+4\mathrm{i} &= 5(\cos 2.21 + \mathrm{i}\sin 2.21) \end{aligned}$$

d

$$r = \sqrt{\left(1^2 + (-\sqrt{3})^2\right)} = \sqrt{4} = 2$$

$$\tan\alpha = \frac{\sqrt{3}}{1}. \qquad \alpha = \frac{\pi}{3}$$

$$\theta = -\frac{\pi}{3}$$

$$1 - \sqrt{3}\,\mathrm{i} = 2\left(\cos\left(\frac{-\pi}{3}\right) + \mathrm{i}\sin\left(\frac{-\pi}{3}\right)\right).$$

e

$$r = \sqrt{\left((-2)^2 + (-5)^2\right)} = \sqrt{29}$$

$$\tan\alpha = \frac{5}{2}. \qquad \alpha = 1.190 \text{ rad}$$

$$\theta = -(\pi - \alpha) = -1.95$$

$$-2 - 5\mathrm{i} = \sqrt{29}\,(\cos(-1.95) + \mathrm{i}\sin(-1.95)).$$

f

$$r = \sqrt{\left((-20)^2 + O^2\right)} = \sqrt{400} = 20$$

$$\tan\alpha = O$$

$$\theta = \pi$$

$$-20 = 20(\cos\pi + \mathrm{i}\sin\pi)$$

g

$$r = \sqrt{\left(7^2 + (-24)^2\right)} = \sqrt{625} = 25$$

$$\tan\alpha = \frac{24}{7}. \qquad \alpha = 1.287 \text{ rad}$$

$$\theta = -1.29$$

$$7 - 24\mathrm{i} = 25(\cos(-1.29) + \mathrm{i}\sin(-1.29))$$

h

$$r = \sqrt{\left((-5)^2 + 5^2\right)} = \sqrt{50} = 5\sqrt{2}$$

$$\tan\alpha = \frac{5}{5} = 1. \qquad \alpha = \frac{\pi}{4}.$$

$$\theta = \pi - \alpha = \frac{3\pi}{4}$$

$$-5 + 5\mathrm{i} = 5\sqrt{2}\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right).$$

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Complex numbers
Exercise F, Question 2

Question:

Express these in the form $r(\cos\ \theta + \text{i}\ \sin\ \theta)$, giving exact values of r and θ where possible, or values to two decimal places otherwise.

a $\frac{3}{1+\text{i}\sqrt{3}}$

b $\frac{1}{2-\text{i}}$

c $\frac{1+\text{i}}{1-\text{i}}$

Solution:

a

$$\begin{aligned}
\frac{3}{1+\text{i}\sqrt{3}} &= \frac{3(1-\text{i}\sqrt{3})}{(1+\text{i}\sqrt{3})(1-\text{i}\sqrt{3})} \\
&= \frac{3-3\text{i}\sqrt{3}}{1(1-\text{i}\sqrt{3})+\text{i}\sqrt{3}(1-\text{i}\sqrt{3})} \\
&= \frac{3-3\text{i}\sqrt{3}}{1-\text{i}\sqrt{3}+\text{i}\sqrt{3}-3\text{i}^2} = \frac{3-3\text{i}\sqrt{3}}{4} \\
&= \frac{3}{4} - \frac{3\sqrt{3}}{4}\text{i} \\
r &= \sqrt{\left(\left(\frac{3}{4}\right)^2 + \left(-\frac{3\sqrt{3}}{4}\right)^2\right)} = \sqrt{\left(\frac{9}{16}+\frac{27}{16}\right)} \\
&= \sqrt{\left(\frac{36}{16}\right)} = \frac{3}{2} \\
\tan\alpha &= \frac{3\sqrt{3}}{4} \div \frac{3}{4} = \sqrt{3}. \qquad \alpha = \frac{\pi}{3} \\
\theta &= -\frac{\pi}{3} \\
\frac{3}{1+\text{i}\sqrt{3}} &= \frac{3}{2}\left(\cos\left(\frac{-\pi}{3}\right) + \text{i}\sin\left(\frac{-\pi}{3}\right)\right)
\end{aligned}$$

b

$$\begin{aligned}
\frac{1}{2-\text{i}} &= \frac{2+\text{i}}{(2-\text{i})(2+\text{i})} \\
&= \frac{2+\text{i}}{2(2+\text{i})-\text{i}(2+\text{i})} = \frac{2+\text{i}}{4+2\text{i}-2\text{i}-\text{i}^2} \\
&= \frac{2+\text{i}}{5} = \frac{2}{5} + \frac{1}{5}\text{i} \\
r &= \sqrt{\left(\left(\frac{2}{5}\right)^2 + \left(\frac{1}{5}\right)^2\right)} = \sqrt{\left(\frac{4}{25}+\frac{1}{25}\right)} \\
&= \sqrt{\frac{5}{25}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \\
\tan\alpha &= \frac{1}{5} \div \frac{2}{5} = \frac{1}{2}. \qquad \alpha = 0.4636\ \text{rad.} \\
\theta &= 0.46 \\
\frac{1}{2-\text{i}} &= \frac{\sqrt{5}}{5}(\cos\ 0.46 + \text{i}\sin\ 0.46)
\end{aligned}$$

c

$$\frac{1+\mathrm{i}}{1-\mathrm{i}} = \frac{(1+\mathrm{i})(1+\mathrm{i})}{(1-\mathrm{i})(1+\mathrm{i})}$$

$$= \frac{1(1+\mathrm{i})+\mathrm{i}(1+\mathrm{i})}{1(1+\mathrm{i})-\mathrm{i}(1+\mathrm{i})} = \frac{1+\mathrm{i}+\mathrm{i}+\mathrm{i}^2}{1+\mathrm{i}-\mathrm{i}-\mathrm{i}^2}$$

$$= \frac{2\mathrm{i}}{2} = \mathrm{i}$$

$$r = \sqrt{\left(0^2+1^2\right)} = 1$$

$$\tan\alpha = \infty. \qquad \alpha = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2}$$

$$\frac{1+\mathrm{i}}{1-\mathrm{i}} = 1\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)$$

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Complex numbers
Exercise F, Question 3

Question:

Write in the form $a + \mathrm{i}b$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

a $3\sqrt{2}\left(\cos\frac{\pi}{4} + \mathrm{i}\ \sin\frac{\pi}{4}\right)$

b $6\left(\cos\frac{3\pi}{4} + \mathrm{i}\ \sin\frac{3\pi}{4}\right)$

c $\sqrt{3}\left(\cos\frac{\pi}{3} + \mathrm{i}\ \sin\frac{\pi}{3}\right)$

d $7\left(\cos\left(-\frac{\pi}{2}\right) + \mathrm{i}\ \sin\left(-\frac{\pi}{2}\right)\right)$

e $4\left(\cos\left(-\frac{5\pi}{6}\right) + \mathrm{i}\ \sin\left(-\frac{5\pi}{6}\right)\right)$

Solution:

a $3\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\mathrm{i}\right) = 3 + 3\mathrm{i}$

b

$$\begin{aligned} 6\left(\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\mathrm{i}\right) &= \frac{-6}{\sqrt{2}} + \frac{6}{\sqrt{2}}\mathrm{i} \\ &= -3\sqrt{2} + 3\sqrt{2}\,\mathrm{i} \end{aligned}$$

c $\sqrt{3}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}\right) = \frac{\sqrt{3}}{2} + \frac{3}{2}\mathrm{i}$

d $7(0 + (-1)\mathrm{i}) = -7\mathrm{i}$

e $4\left(\frac{-\sqrt{3}}{2} + \left(\frac{-1}{2}\right)\mathrm{i}\right) = -2\sqrt{3} - 2\mathrm{i}$

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Complex numbers
Exercise F, Question 4

Question:

In each case, find $|z_1|, |z_2|$ and z_1z_2, and verify that $|z_1z_2| = |z_1||z_2|$.

a $z_1 = 3 + 4\mathrm{i}$ $\quad z_2 = 4 - 3\mathrm{i}$

b $z_1 = -1 + 2\mathrm{i}$ $\quad z_2 = 4 + 2\mathrm{i}$

c $z_1 = 5 + 12\mathrm{i}$ $\quad z_2 = 7 + 24\mathrm{i}$

d $z_1 = \sqrt{3} + \mathrm{i}\sqrt{2}$ $\quad z_2 = -\sqrt{2} + \mathrm{i}\sqrt{3}$

Solution:

a

$$
\begin{aligned}
|z_1| &= \sqrt{\left(3^2 + 4^2\right)} = \sqrt{25} = 5 \\
|z_2| &= \sqrt{\left(4^2 + (-3)^2\right)} = \sqrt{25} = 5 \\
z_1z_2 &= (3 + 4\mathrm{i})(4 - 3\mathrm{i}) \\
&= 3(4 - 3\mathrm{i}) + 4\mathrm{i}(4 - 3\mathrm{i}) \\
&= 12 - 9\mathrm{i} + 16\mathrm{i} - 12\mathrm{i}^2 \\
&= 24 + 7\mathrm{i} \\
|z_1z_2| &= \sqrt{\left(24^2 + 7^2\right)} = \sqrt{625} = 25 \\
|z_1||z_2| &= 5 \times 5 = 25 = |z_1z_2|
\end{aligned}
$$

b

$$
\begin{aligned}
|z_1| &= \sqrt{\left((-1)^2 + 2^2\right)} = \sqrt{5} \\
|z_2| &= \sqrt{\left(4^2 + 2^2\right)} = \sqrt{20} = 2\sqrt{5} \\
z_1z_2 &= (-1 + 2\mathrm{i})(4 + 2\mathrm{i}) \\
&= -1(4 + 2\mathrm{i}) + 2\mathrm{i}(4 + 2\mathrm{i}) \\
&= -4 - 2\mathrm{i} + 8\mathrm{i} + 4\mathrm{i}^2 \\
&= -8 + 6\mathrm{i} \\
|z_1z_2| &= \sqrt{\left((-8)^2 + 6^2\right)} = \sqrt{100} = 10 \\
|z_1||z_2| &= \sqrt{5} \times 2\sqrt{5} = 10 = |z_1z_2|
\end{aligned}
$$

c

$$
\begin{aligned}
|z_1| &= \sqrt{\left(5^2+12^2\right)} = \sqrt{169} = 13 \\
|z_2| &= \sqrt{\left(7^2+24^2\right)} = \sqrt{625} = 25 \\
z_1z_2 &= (5+12\text{i})(7+24\text{i}) \\
&= 5(7+24\text{i})+12\text{i}(7+24\text{i}) \\
&= 35+120\text{i}+84\text{i}+288\text{i}^2 \\
&= -253+204\text{i} \\
|z_1z_2| &= \sqrt{\left((-253)^2+204^2\right)} = \sqrt{105625} = 325 \\
|z_1||z_2| &= 13\times 25 = 325 = |z_1z_2|
\end{aligned}
$$

d

$$
\begin{aligned}
|z_1| &= \sqrt{\left((\sqrt{3})^2+(\sqrt{2})^2\right)} = \sqrt{5} \\
|z_2| &= \sqrt{\left((-\sqrt{2})^2+(\sqrt{3})^2\right)} = \sqrt{5} \\
z_1z_2 &= (\sqrt{3}+\text{i}\sqrt{2})(-\sqrt{2}+\text{i}\sqrt{3}) \\
&= \sqrt{3}(-\sqrt{2}+\text{i}\sqrt{3})+\text{i}\sqrt{2}(-\sqrt{2}+\text{i}\sqrt{3}) \\
&= -\sqrt{6}+3\text{i}-2\text{i}+\text{i}^2\sqrt{6} \\
&= -2\sqrt{6}+\text{i} \\
|z_1z_2| &= \sqrt{\left((-2\sqrt{6})^2+1^2\right)} = \sqrt{(24+1)} = 5 \\
|z_1||z_2| &= \sqrt{5}\times\sqrt{5} = 5 = |z_1z_2|.
\end{aligned}
$$

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Complex numbers
Exercise G, Question 1

Question:

$a+2b+2a\mathrm{i}=4+6\mathrm{i}$, where a and b are real.

Find the value of a and the value of b.

Solution:

Real parts: $a+2b=4$

Imaginary parts: $2a=6$

$a=3$

$$\begin{aligned} 3+2b &= 4 \\ 2b &= 1 \\ b &= \frac{1}{2} \end{aligned}$$

$a=3$ and $b=\frac{1}{2}$

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Complex numbers
Exercise G, Question 2

Question:

$(a-b)+(a+b)\text{i}=9+5\text{i}$, where a and b are real.

Find the value of a and the value of b.

Solution:

Real parts : $a-b=9$
Imaginary parts : $a+b=5$
Adding : $2a=14$
$a=7$

$7-b=9$
$b=-2$

$a=7$ and $b=-2$.

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Edexcel AS and A Level Modular Mathematics

Complex numbers
Exercise G, Question 3

Question:

$(a+b)(2+\mathrm{i}) = b+1+(10+2a)\mathrm{i}$, where a and b are real.

Find the value of a and the value of b.

Solution:

Real parts : $2(a+b) = b+1$

$$2a+2b = b+1$$
$$2a+b = 1 \qquad \text{(i)}$$

Imaginary parts : $a+b = 10+2a$

$$-a+b = 10 \qquad \text{(ii)}$$

(i) − (ii) : $3a = -9$

$$a = -3$$

Substitute into (i) : $-6+b = 1$

$$b = 7$$

$a = -3$ and $b = 7$

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Complex numbers
Exercise G, Question 4

Question:

$(a+\text{i})^3 = 18 + 26\text{i}$, where a is real.

Find the value of a.

Solution:

$$\begin{aligned}(a+\text{i})^3 &= a^3 + 3a^2\text{i} + 3a\text{i}^2 + \text{i}^3 \\ &= (a^3 - 3a) + \text{i}(3a^2 - 1)\end{aligned}$$

Imaginary part : $3a^2 - 1 = 26$

$3a^2 = 27$

$a^2 = 9$

$a = 3$ or -3

Real part : $a = 3$ gives $27 - 9 = 18$. Correct.

$a = -3$ gives $-27 + 9 = -18$. Wrong.

So $a = 3$.

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Complex numbers
Exercise G, Question 5

Question:

$ab\mathrm{i} = 3a - b + 12\mathrm{i}$, where a and b are real.

Find the value of a and the value of b.

Solution:

Real parts: $O = 3a - b$ (i)

Imaginary parts : $ab = 12$ (ii)

From (ii), $b = \frac{12}{a}$

Substitute into (i) :

$$\begin{aligned} O &= 3a - \frac{12}{a} \\ 3a^2 - 12 &= 0 \\ a^2 &= 4 \\ a &= 2 \text{ or } -2 \end{aligned}$$

If $a = 2$,$b = \frac{12}{2} = 6$

If $a = -2$,$b = \frac{12}{-2} = -6$

Either $a = 2$ and $b = 6$
or $a = -2$ and $b = -6$.

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Complex numbers
Exercise G, Question 6

Question:

Find the real numbers x and y, given that

$$\frac{1}{x+\mathrm{i}y} = 3 - 2\mathrm{i}$$

Solution:

$(3-2\mathrm{i})(x+\mathrm{i}y) = 1$

$3(x+\mathrm{i}y) - 2\mathrm{i}(x+\mathrm{i}y) = 1$
$3x + 3y\mathrm{i} - 2x\mathrm{i} - 2\mathrm{i}^2 y = 1$
$(3x+2y) + \mathrm{i}(3y-2x) = 1$

Real parts: $3x + 2y = 1$ (i)

Imaginary parts : $3y - 2x = 0$ (ii)

$2 \times \text{(i)} + 3 \times \text{(ii)}$:

$$\begin{aligned} 6x + 4y + 9y - 6x &= 2 \\ 13y &= 2 \\ y &= \frac{2}{13} \end{aligned}$$

Substitute into (i) : $3x + \frac{4}{13} = 1$

$$\begin{aligned} 3x &= \frac{9}{13}. \\ x &= \frac{3}{13} \end{aligned}$$

$x = \frac{3}{13}$ and $y = \frac{2}{13}$

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Complex numbers
Exercise G, Question 7

Question:

Find the real numbers x and y, given that

$(x+\mathrm{i}y)(1+\mathrm{i})=2+\mathrm{i}$

Solution:

$$\begin{aligned}(x+\mathrm{i}y)(1+\mathrm{i}) &= x(1+\mathrm{i})+\mathrm{i}y(1+\mathrm{i})\\ &= x+x\mathrm{i}+\mathrm{i}y+\mathrm{i}^2y\\ &= (x-y)+\mathrm{i}(x+y)\end{aligned}$$

Real parts : $x-y=2$
Imaginary parts : $x+y=1$

Adding : $2x=3$

$x=\frac{3}{2}$

$\frac{3}{2}+y=1$, $y=-\frac{1}{2}$

$x=\frac{3}{2}$ and $y=-\frac{1}{2}$

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Complex numbers
Exercise G, Question 8

Question:

Solve for real x and y

$(x+\mathrm{i}y)(5-2\mathrm{i})=-3+7\mathrm{i}$

Hence find the modulus and argument of $x+\mathrm{i}y$.

Solution:

$$\begin{aligned}(x+\mathrm{i}y)(5-2\mathrm{i}) &= x(5-2\mathrm{i})+\mathrm{i}y(5-2\mathrm{i})\\ &= 5x-2x\mathrm{i}+5y\mathrm{i}-2y\mathrm{i}^2\\ &= (5x+2y)+\mathrm{i}(-2x+5y)\end{aligned}$$

Real parts: $5x+2y=-3$ (i)

Imaginary parts : $-2x+5y=7$ (ii)

$$\begin{aligned}\text{(i)}\times 2: \quad 10x+4y &= -6\\ \text{(ii)}\times 5: \quad -10x+25y &= 35\\ \text{Adding}: \quad 29y &= 29\\ y &= 1\end{aligned}$$

$$\begin{aligned}\text{Substitute into (i)}: 5x+2 &= -3\\ 5x &= -5\\ x &= -1\end{aligned}$$

$x=-1$ and $y=1$

$|-1+\mathrm{i}|=\sqrt{((-1)^2+1^2)}=\sqrt{2}$

$$\begin{aligned}\arg(-1+\mathrm{i}) &= \pi-\arctan 1\\ &= \pi-\frac{\pi}{4}=\frac{3\pi}{4}\end{aligned}$$

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Complex numbers
Exercise G, Question 9

Question:

Find the square roots of $7+24\mathrm{i}$.

Solution:

$$\begin{aligned}(a+\mathrm{i}b)^2 &= 7+24\mathrm{i}\\ a(a+\mathrm{i}b)+\mathrm{i}b(a+\mathrm{i}b) &= 7+24\mathrm{i}\\ a^2+ab\mathrm{i}+ab\mathrm{i}+b^2\mathrm{i}^2 &= 7+24\mathrm{i}\\ (a^2-b^2)+2ab\mathrm{i} &= 7+24\mathrm{i}\end{aligned}$$

Real parts: $a^2-b^2=7$ (i)

Imaginary parts: $2ab=24$ (ii)

From (ii), $b=\frac{24}{2a}=\frac{12}{a}$

Substituting into (i) : $a^2-\frac{144}{a^2}=7$

$$\begin{aligned}a^4-144&=7a^2\\ a^4-7a^2-144&=0\\ (a^2-16)(a^2+9)&=0\\ a^2=16 \text{ or } a^2&=-9\end{aligned}$$

Since a is real, $a=4$ or $a=-4$

When $a=4, b=\frac{12}{a}=\frac{12}{4}=3$

When $a=-4, b=\frac{12}{-4}=-3$

Square roots are $4+3\mathrm{i}$ and $-(4+3\mathrm{i})$, i .e . $\pm(4+3\mathrm{i})$

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Complex numbers
Exercise G, Question 10

Question:

Find the square roots of $11 + 60\mathrm{i}$.

Solution:

$$
\begin{aligned}
(a + \mathrm{i}b)^2 &= 11 + 60\mathrm{i} \\
a(a + \mathrm{i}b) + \mathrm{i}b(a + \mathrm{i}b) &= 11 + 60\mathrm{i} \\
a^2 + ab\mathrm{i} + ab\mathrm{i} + b^2\mathrm{i}^2 &= 11 + 60\mathrm{i} \\
(a^2 - b^2) + 2ab\mathrm{i} &= 11 + 60\mathrm{i}
\end{aligned}
$$

Real parts: $a^2 - b^2 = 11$ (i)

Imaginary parts: $2ab = 60$ (ii)

From (ii): $b = \frac{60}{2a} = \frac{30}{a}$

Substituting into (i): $a^2 - \frac{900}{a^2} = 11$

$$
\begin{aligned}
a^4 - 900 &= 11a^2 \\
a^4 - 11a^2 - 900 &= 0 \\
(a^2 - 36)(a^2 + 25) &= 0 \\
a^2 = 36 \text{ or } a^2 &= -25
\end{aligned}
$$

Since a is real, $a = 6$ or $a = -6$.

When $a = 6, b = \frac{30}{a} = \frac{30}{6} = 5$

When $a = -6, b = \frac{30}{-6} = -5$.

Square roots are $6 + 5\mathrm{i}$ and $-(6 + 5\mathrm{i})$,

i. e. $\pm(6 + 5\mathrm{i})$

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Complex numbers
Exercise G, Question 11

Question:

Find the square roots of $5 - 12\mathrm{i}$.

Solution:

$$(a + \mathrm{i}b)^2 = 5 - 12\mathrm{i}$$
$$a(a + \mathrm{i}b) + \mathrm{i}b(a + \mathrm{i}b) = 5 - 12\mathrm{i}$$
$$a^2 + ab\mathrm{i} + ab\mathrm{i} + b^2\mathrm{i}^2 = 5 - 12\mathrm{i}$$
$$(a^2 - b^2) + 2ab\mathrm{i} = 5 - 12\mathrm{i}$$

Real parts: $a^2 - b^2 = 5$ (i)

Imaginary parts: $2ab = -12$ (ii)

From (ii): $b = \frac{-12}{2a} = \frac{-6}{a}$

Substituting into (i): $a^2 - \frac{36}{a^2} = 5$

$$a^4 - 36 = 5a^2$$
$$a^4 - 5a^2 - 36 = 0$$
$$(a^2 - 9)(a^2 + 4) = 0$$
$$a^2 = 9 \text{ or } a^2 = -4.$$

Since a is real, $a = 3$ or $a = -3$

When $a = 3, b = \frac{-6}{a} = \frac{-6}{3} = -2$

When $a = -3, b = \frac{-6}{-3} = 2$

Square roots are $3 - 2\mathrm{i}$ and $-(3 - 2\mathrm{i})$,

i. e. $\pm(3 - 2\mathrm{i})$

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Complex numbers
Exercise G, Question 12

Question:

Find the square roots of 2i.

Solution:

$$(a+\mathrm{i}b)^2 = 2\mathrm{i}$$
$$a(a+\mathrm{i}b)+\mathrm{i}b(a+\mathrm{i}b) = 2\mathrm{i}$$
$$a^2+ab\mathrm{i}+ab\mathrm{i}+b^2\mathrm{i}^2 = 2\mathrm{i}$$
$$(a^2-b^2)+2ab\mathrm{i} = 2\mathrm{i}$$

Real parts: $a^2-b^2=0$ (i)

Imaginary parts: $2ab=2$ (ii)

From (ii): $b=\frac{2}{2a}=\frac{1}{a}$

Substituting into (i) : $a^2-\frac{1}{a^2}=0$

$$a^4-1=0$$
$$a^4=1$$

Real solutions are $a=1$ or $a=-1$.

When $a=1, b=\frac{1}{a}=\frac{1}{1}=1$

When $a=-1, b=\frac{1}{-1}=-1$.

Square roots are $1+\mathrm{i}$ and $-(1+\mathrm{i})$,

i. e. $\pm(1+\mathrm{i})$

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Edexcel AS and A Level Modular Mathematics

Complex numbers
Exercise H, Question 1

Question:

Given that $1 + 2\mathrm{i}$ is one of the roots of a quadratic equation, find the equation.

Solution:

The other root is $1 - 2\mathrm{i}$.

If the roots are α and β, the equation is

$$(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

$$\begin{aligned} \alpha+\beta &= (1+2\mathrm{i}) + (1-2\mathrm{i}) = 2 \\ \alpha\beta &= (1+2\mathrm{i})(1-2\mathrm{i}) \\ &= 1(1-2\mathrm{i}) + 2\mathrm{i}(1-2\mathrm{i}) \\ &= 1 - 2\mathrm{i} + 2\mathrm{i} - 4\mathrm{i}^2 = 5 \end{aligned}$$

Equation is $x^2 - 2x + 5 = 0$

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Complex numbers
Exercise H, Question 2

Question:

Given the $3 - 5\mathrm{i}$ is one of the roots of a quadratic equation, find the equation.

Solution:

The other root is $3 + 5\mathrm{i}$.

If the roots are α and β, the equation is

$$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

$$\begin{aligned} \alpha + \beta &= (3 - 5\mathrm{i}) + (3 + 5\mathrm{i}) = 6 \\ \alpha\beta &= (3 - 5\mathrm{i})(3 + 5\mathrm{i}) \\ &= 3(3 + 5\mathrm{i}) - 5\mathrm{i}(3 + 5\mathrm{i}) \\ &= 9 + 15\mathrm{i} - 15\mathrm{i} - 25\mathrm{i}^2 = 34 \end{aligned}$$

Equation is $x^2 - 6x + 34 = 0$

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Complex numbers
Exercise H, Question 3

Question:

Given that $a + 4\text{i}$, where a is real, is one of the roots of a quadratic equation, find the equation.

Solution:

The other root is $a - 4\text{i}$.

If the roots are α and β, the equation is

$$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

$$\begin{aligned} \alpha + \beta &= (a + 4\text{i}) + (a - 4\text{i}) = 2a \\ \alpha\beta &= (a + 4\text{i})(a - 4\text{i}) \\ &= a(a - 4\text{i}) + 4\text{i}(a - 4\text{i}) \\ &= a^2 - 4a\text{i} + 4a\text{i} - 16\text{i}^2 = a^2 + 16 \end{aligned}$$

Equation is $x^2 - 2ax + a^2 + 16 = 0$

Solutionbank FP1
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Complex numbers
Exercise H, Question 4

Question:

Show that $x = -1$ is a root of the equation $x^3 + 9x^2 + 33x + 25 = 0$.

Hence solve the equation completely.

Solution:

When $x = -1$,

$x^3 + 9x^2 + 33x + 25 = -1 + 9 - 33 + 25 = 0$

So $x = -1$ is a root.

So $(x + 1)$ is a factor

$x^3 + 9x^2 + 33x + 25 = (x + 1)(x^2 + 8x + 25) = 0$

$a = 1, \; b = 8, \; c = 25.$

$x = \frac{-8 \pm \sqrt{(64 - 100)}}{2} = \frac{-8 \pm 6\mathrm{i}}{2} = -4 \pm 3\mathrm{i}$

Roots are -1, $-4 + 3\mathrm{i}$ and $-4 - 3\mathrm{i}$

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Complex numbers
Exercise H, Question 5

Question:

Show that $x = 3$ is a root of the equation $2x^3 - 4x^2 - 5x - 3 = 0$.

Hence solve the equation completely.

Solution:

When $x = 3$,

$2x^3 - 4x^2 - 5x - 3 = 54 - 36 - 15 - 3 = 0$.

So $x = 3$ is a root.

So $(x - 3)$ is a factor.

$2x^3 - 4x^2 - 5x - 3 = (x - 3)(2x^2 + 2x + 1) = 0$

$a = 2,\ b = 2,\ c = 1.$

$$x = \frac{-2 \pm \sqrt{(4-8)}}{4} = \frac{-2 \pm 2i}{4} = \frac{-1}{2} \pm \frac{1}{2}i$$

Roots are 3, $\frac{-1}{2} + \frac{1}{2}i$ and $\frac{-1}{2} - \frac{1}{2}i$

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Complex numbers
Exercise H, Question 6

Question:

Show that $x = -\frac{1}{2}$ is a root of the equation $2x^3 + 3x^2 + 3x + 1 = 0$.

Hence solve the equation completely.

Solution:

When $x = \frac{-1}{2}$,

$$\begin{aligned} 2x^3 + 3x^2 + 3x + 1 &= 2\left(\frac{-1}{8}\right) + 3\left(\frac{1}{4}\right) + 3\left(\frac{-1}{2}\right) + 1 \\ &= \frac{-1}{4} + \frac{3}{4} - \frac{3}{2} + 1 = 0 \end{aligned}$$

So $x = -\frac{1}{2}$ is a root.

So $(2x + 1)$ is a factor.

$2x^3 + 3x^2 + 3x + 1 = (2x + 1)(x^2 + x + 1) = 0$

$a = 1, \ b = 1, \ c = 1$

$$x = \frac{-1 \pm \sqrt{(1-4)}}{2} = \frac{-1 \pm \mathrm{i}\sqrt{3}}{2} = \frac{-1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}$$

Roots are $\frac{-1}{2}$, $\frac{-1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$ and $\frac{-1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}$.

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Complex numbers
Exercise H, Question 7

Question:

Given that $-4+\mathrm{i}$ is one of the roots of the equation $x^3+4x^2-15x-68=0$, solve the equation completely.

Solution:

Another root is $-4-\mathrm{i}$

The equation with roots α and β is

$(x-\alpha)(x-\beta)=x^2-(\alpha+\beta)x+\alpha\beta=0.$

$$\begin{aligned}\alpha+\beta &= (-4+\mathrm{i})+(-4-\mathrm{i})=-8\\ \alpha\beta &= (-4+\mathrm{i})(-4-\mathrm{i})\\ &= -4(-4-\mathrm{i})+\mathrm{i}(-4-\mathrm{i})\\ &= 16+4\mathrm{i}-4\mathrm{i}-\mathrm{i}^2=17\end{aligned}$$

Quadratiz equation is $x^2+8x+17=0$.

So $(x^2+8x+17)$ is a factor of $(x^3+4x^2-15x-68)$.

$(x^3+4x^2-15x-68)=(x^2+8x+17)(x-4)$

Roots are 4, $-4+\mathrm{i}$ and $-4-\mathrm{i}$.

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Complex numbers
Exercise H, Question 8

Question:

Given that $x^4 - 12x^3 + 31x^2 + 108x - 360 = (x^2 - 9)(x^2 + bx + c)$, find the values of b and c, and hence find all the solutions of the equation $x^4 - 12x^3 + 31x^2 + 108x - 360 = 0$.

Solution:

$$x^4 - 12x^3 + 31x^2 + 108x - 360 = (x^2 - 9)(x^2 + bx + c)$$

x^3 terms : $-12 = b$
$b = -12$

Constant term : $-360 = -9c$
$c = 40$

$$(x^2 - 9)(x^2 - 12x + 40) = 0$$

$x^2 - 9 = 0$: $x^2 = 9$
$x = 3$ or $x = -3$

$$x^2 - 12x + 40 = 0$$

$a = 1,\ b = -12,\ c = 40$

$$x = \frac{12 \pm \sqrt{(144 - 160)}}{2} = \frac{12 \pm 4\mathrm{i}}{2} = 6 \pm 2\mathrm{i}$$

Roots are 3 , -3, $6 + 2\mathrm{i}$ and $6 - 2\mathrm{i}$

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Complex numbers
Exercise H, Question 9

Question:

Given that $2+3\text{i}$ is one of the roots of the equation $x^4+2x^3-x^2+38x+130=0$, solve the equation completely.

Solution:

Another root is $2-3\text{i}$

The equation with roots α and β is

$$(x-\alpha)(x-\beta)=x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$\begin{aligned}\alpha+\beta &= (2+3\text{i})+(2-3\text{i})=4\\ \alpha\beta &= (2+3\text{i})(2-3\text{i})\\ &= 2(2-3\text{i})+3\text{i}(2-3\text{i})\\ &= 4-6\text{i}+6\text{i}-9\text{i}^2=13\end{aligned}$$

Quadratic equation is $x^2-4x+13=0$.

So $(x^2-4x+13)$ is a factor of $(x^4+2x^3-x^2+38x+130)$.

$$(x^4+2x^3-x^2+38x+130)=(x^2-4x+13)(x^2+6x+10)$$

$$x^2+6x+10=0$$

$a=1,\ b=6,\ c=10$

$$x=\frac{-6\pm\sqrt{(36-40)}}{2}=\frac{-6\pm 2\text{i}}{2}=-3\pm\text{i}$$

Roots are $2+3\text{i},\ 2-3\text{i},\ -3+\text{i}$ and $-3-\text{i}$.

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Complex numbers
Exercise H, Question 10

Question:

Find the four roots of the equation $x^4 - 16 = 0$.

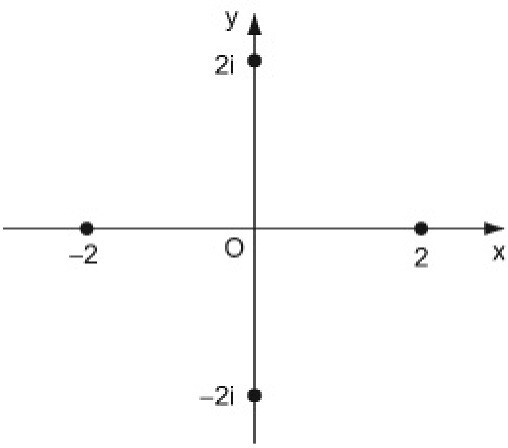
Show these roots on an Argand diagram.

Solution:

$x^4 - 16 = 0$
$(x^2 - 4)(x^2 + 4) = 0$

$x^2 = 4$ or $x^2 = -4$

$x = 2$, -2, $2i$ or $-2i$



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Complex numbers
Exercise H, Question 11

Question:

Three of the roots of the equation $ax^5 + bx^4 + cx^3 + dx^2 + ex + \mathrm{f} = 0$ are -2, $2\mathrm{i}$ and $1 + \mathrm{i}$. Find the values of a, b, c, d, e and f.

Solution:

The other two roots are $-2\mathrm{i}$ and $1 - \mathrm{i}$

The equation with roots α and β is

$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta = 0.$

Using $2i$ and $-2\mathrm{i}$,

$$\begin{aligned} \alpha + \beta &= 2\mathrm{i} - 2\mathrm{i} = 0 \\ \alpha\beta &= (2\mathrm{i})(-2\mathrm{i}) = -4\mathrm{i}^2 = 4 \end{aligned}$$

Quadratic equation is $x^2 + 4 = 0$

Using $1 + \mathrm{i}$ and $1 - \mathrm{i}$,

$$\begin{aligned} \alpha + \beta &= (1 + \mathrm{i}) + (1 - \mathrm{i}) = 2 \\ \alpha\beta &= (1 + \mathrm{i})(1 - \mathrm{i}) \\ &= 1(1 - \mathrm{i}) + \mathrm{i}(1 - \mathrm{i}) \\ &= 1 - \mathrm{i} + \mathrm{i} - \mathrm{i}^2 = 2. \end{aligned}$$

Quadratic equation is $x^2 - 2x + 2 = 0$

The required equation is

$$(x + 2)(x^2 + 4)(x^2 - 2x + 2) = 0$$
$$(x^3 + 2x^2 + 4x + 8)(x^2 - 2x + 2) = 0$$

$$x^3(x^2 - 2x + 2) + 2x^2(x^2 - 2x + 2) + 4x(x^2 - 2x + 2) + 8(x^2 - 2x + 2) = 0$$
$$x^5 - 2x^4 + 2x^3 + 2x^4 - 4x^3 + 4x^2 + 4x^3 - 8x^2 + 8x + 8x^2 - 16x + 16 = 0$$

$$x^5 + 2x^3 + 4x^2 - 8x + 16 = 0$$

$a = 1,\ b = 0,\ c = 2,\ d = 4,\ e = -8,\ f = 16.$

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Complex numbers
Exercise I, Question 1

Question:

a Find the roots of the equation $z^2 + 2z + 17 = 0$ giving your answers in the form $a + \mathrm{i}b$, where a and b are integers.

b Show these roots on an Argand diagram.

Solution:

a

$$z^2 + 2z + 17 = 0$$
$$z^2 + 2z = -17$$
$$z^2 + 2z + 1 = -17 + 1 = -16$$

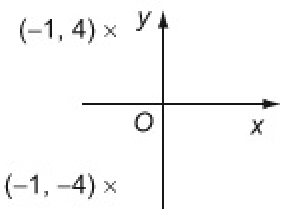
You may use any accurate method of solving a quadratic equation. Completing the square works well when the coefficient of z^2 is one and the coefficient of z is even.

$$(z + 1)^2 = -16$$
$$z + 1 = \pm 4\mathrm{i}$$

$\sqrt{(-16)} = 4\sqrt{(-1)} = 4\mathrm{i}$

$z = -1 - 4\mathrm{i}, -1 + 4\mathrm{i}$

b



In the Argand diagram, you must place points representing conjugate complex numbers symmetrically about the real x-axis.

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Complex numbers
Exercise I, Question 2

Question:

$z_1 = -\mathrm{i}, z_2 = 1 + \mathrm{i}\sqrt{3}$

a Find the modulus of

i $z_1 z_2$

ii $\frac{z_1}{z_2}$.

b Find the argument of

i $z_1 z_2$

ii $\frac{z_1}{z_2}$.

Give your answers in radians as exact multiples of π.

Solution:

a i

$$\begin{aligned} z_1 z_2 &= -\mathrm{i}(1 + \mathrm{i}\sqrt{3}) \\ &= -\mathrm{i} + \sqrt{3} \\ &= \sqrt{3} - \mathrm{i} \end{aligned}$$

$-\mathrm{i} \times \mathrm{i}\sqrt{3} = -(-1)\sqrt{3} = \sqrt{3}$

$$\begin{aligned} |z_1 z_2|^2 &= (\sqrt{3})^2 + (-1)^2 = 3 + 1 = 4 \\ |z_1 z_2| &= 2 \end{aligned}$$

You find the modulus of complex numbers using the result that, if $z = a + \mathrm{i}b$, then $|z|^2 = a^2 + b^2$. This result is essentially the same as Pythagoras' Theorem and so is easy to remember.

ii

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{-\mathrm{i}}{1 + \mathrm{i}\sqrt{3}} \times \frac{1 - \mathrm{i}\sqrt{3}}{1 - \mathrm{i}\sqrt{3}} \\ &= \frac{-\mathrm{i} - \sqrt{3}}{1^2 + (\sqrt{3})^2} = -\frac{\sqrt{3}}{4} - \frac{1}{4}\mathrm{i} \end{aligned}$$

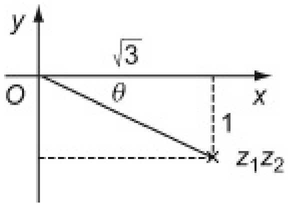
To simplify a quotient, you multiply the numerator and denominator by the conjugate complex of the denominator. The conjugate complex of this denominator, $1 + \mathrm{i}\sqrt{3}$, is $1 - \mathrm{i}\sqrt{3}$.

$$\left|\frac{z_1}{z_2}\right|^2 = \left(-\frac{\sqrt{3}}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{3}{16} + \frac{1}{16} = \frac{1}{4}$$

$$\left|\frac{z_1}{z_2}\right| = \frac{1}{2}$$

b i

$z_1 z_2 = \sqrt{3} - \mathrm{i}$



You draw a sketch of the Argand diagram to check which quadrant your complex number is in.

$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$

You usually work out an angle in a right angled triangle using a tangent.

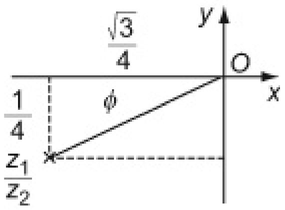
$z_1 z_2$ is in the fourth quadrant.

$\arg(z_1 z_2) = -\frac{\pi}{6}$

You then adjust you angle to the correct quadrant. The argument is measured from the positive x-axis. This is clockwise and, hence, negative.

ii

$\frac{z_1}{z_2} = -\frac{\sqrt{3}}{4} - \frac{1}{4}\mathrm{i}$

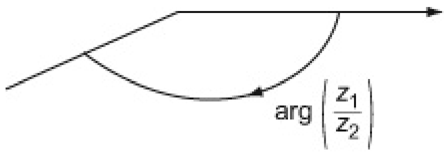


$\tan\phi = \frac{\frac{1}{4}}{\frac{\sqrt{3}}{4}} = \frac{1}{\sqrt{3}} \Rightarrow \phi = \frac{\pi}{6}$

This complex number is in the third quadrant. Again the argument is negative.

$\frac{z_1}{z_2}$ is in the third quadrant.

$\arg\left(\frac{z_1}{z_2}\right) = -\left(\pi - \frac{\pi}{6}\right) - \frac{5\pi}{6}$



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Edexcel AS and A Level Modular Mathematics

Complex numbers
Exercise I, Question 3

Question:

$z = \frac{1}{2+\mathrm{i}}$.

a Express in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$,

i z^2

ii $z - \frac{1}{z}$.

b Find $|z^2|$.

c Find $\arg\left(z - \frac{1}{z}\right)$, giving your answer in degrees to one decimal place.

Solution:

a i

$$\begin{aligned} z &= \frac{1}{2+\mathrm{i}} \times \frac{2-\mathrm{i}}{2-\mathrm{i}} = \frac{2-\mathrm{i}}{5} \\ &= \frac{2}{5} - \frac{1}{5}\mathrm{i} \end{aligned}$$

It is useful to be able to write down the product of a complex number and its conjugate without doing a lot of working. $(a + \mathrm{i}b)(a - \mathrm{i}b) = a^2 + b^2$ This is sometimes called the formula for the sum of two squares. It has a similar pattern to the formula for the difference of two squares. $(a + b)(a - b) = a^2 - b^2$

$$\begin{aligned} z^2 &= \left(\frac{2}{5} - \frac{1}{5}\mathrm{i}\right)^2 \\ &= \frac{4}{25} - \frac{4}{25}\mathrm{i} + \left(\frac{1}{5}\mathrm{i}\right)^2 \\ &= \frac{4}{25} - \frac{4}{25}\mathrm{i} - \frac{1}{25} \\ &= \frac{3}{25} - \frac{4}{25}\mathrm{i} \end{aligned}$$

You square using the formula $(a - b)^2 = a^2 - 2ab + b^2$

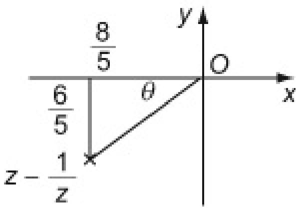
ii

$$\begin{aligned} z - \frac{1}{z} &= \frac{2}{5} - \frac{1}{5}\mathrm{i} - (2+\mathrm{i}) \\ &= \frac{2}{5} - \frac{1}{5}\mathrm{i} - 2 - \mathrm{i} \\ &= -\frac{8}{5} - \frac{6}{5}\mathrm{i} \end{aligned}$$

b

$$|z^2|^2 = \left(\frac{3}{25}\right)^2 + \left(-\frac{4}{25}\right)^2$$
$$= \frac{9}{625} + \frac{16}{625} = \frac{25}{625} = \frac{1}{25}$$
$$|z^2| = \frac{1}{5}$$

c



You should draw a sketch to help you decide which quadrant the complex number is in.

$$\tan\theta = \frac{\frac{6}{5}}{\frac{8}{5}} = \frac{3}{4} \Rightarrow \theta \approx 36.87^\circ$$

$z - \frac{1}{z}$ is in the third quadrant

$$\arg\left(z - \frac{1}{z}\right) = -(180^\circ - \theta)$$
$$= -143.1^\circ, \text{to 1 d.p.}$$

Arguments are measured from the positive x-axis. Angles measured clockwise are negative.

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Complex numbers
Exercise I, Question 4

Question:

The real and imaginary parts of the complex number $z = x + iy$ satisfy the equation $(2 - i)x - (1 + 3i)y - 7 = 0$.

a Find the value of x and the value of y.

b Find the values of

i $|z|$

ii arg z.

Solution:

a

$2x - xi - y - 3yi - 7 = 0$

You find two simultaneous equations by equating the real and imaginary parts of the equation.

$(2x - y - 7) + (-x - 3y)i = 0 + 0i$

You think of 0 as 0 + 0i, a number which has both its real and imaginary parts zero.

Equating real and imaginary parts

Real $2x - y - 7 = 0$

Imaginary $-x - 3y = 0$

$2x - y = 7$ **(1)**

$x + 3y = 0$ **(2)**

$2 \times$ **(2)** $2x + 6y = 0$ **(3)**

(3) − (1) $7y = -7 \Rightarrow y = -1$

The simultaneous equations are solved in exactly the same way as you learnt for GCSE.

Substitute into **(2)**

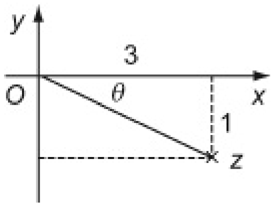
$$\begin{aligned} x - 3 &= 0 \Rightarrow x = 3 \\ x &= 3, y = -1 \end{aligned}$$

b i

$$\begin{aligned} z &= 3 - i \\ |z| &= 3^2 + (-1)^2 = 10 \\ |z| &= \sqrt{10} \end{aligned}$$

ii

As the question has not specified that you should work in radians or degrees, you could work in either and $-18.4°$ would also be an



$\tan\theta = \frac{1}{3} \Rightarrow \theta \approx 0.322$, in radians

z is in the fourth quadrant.

$\arg\ z = -0.322$, in radians to 3 d.p.

acceptable answer.

The question did not specify any accuracy. 3 significant figures is a sensible accuracy but you could give more.

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Complex numbers
Exercise I, Question 5

Question:

Given that $2 + \mathrm{i}$ is a root of the equation $z^3 - 11z + 20 = 0$, find the other roots of the equation.

Solution:

One other root is $2 - \mathrm{i}$.

If $a + \mathrm{i}b$ is a root, then $a - \mathrm{i}b$ must also be a root. The complex roots of polynomials with real coefficients occur as complex conjugate pairs.

The cubic equation must be identical to

$$(z - 2 - \mathrm{i})(z - 2 + \mathrm{i})(z - \gamma) = 0$$

If α, β and γ are the roots of a cubic equation, then the equation must have the form $(x - \alpha)(x - \beta)(x - \gamma) = 0$.

$$((z - 2) - \mathrm{i})((z - 2) + \mathrm{i}) = (z - 2)^2 - \mathrm{i}^2$$

$$= z^2 - 4z + 4 + 1 = z^2 - 4z + 5$$

You know the first two roots, α and β, so the only remaining problem is finding the third root γ.

Hence

$$\left(z^2 - 4z + 5\right)(z - \gamma) = z^3 - 11z + 20$$

You need not multiply the brackets on the left hand side of this equation out fully. If the brackets were multiplied out, the only term without a z would be when $+5$ is multiplied by $-\gamma$ and the product of these, -5γ, equals the term without z on the right hand side, $+20$.

Equating constant coefficients

$$-5\gamma = 20 \Rightarrow \gamma = -4$$

The other roots are $2 - \mathrm{i}$ and -4.

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Complex numbers
Exercise I, Question 6

Question:

Given that $1+3\text{i}$ is a root of the equation $z^3+6z+20=0$,

a find the other two roots of the equation,

b show, on a single Argand diagram, the three points representing the roots of the equation,

c prove that these three points are the vertices of a right-angled triangle.

Solution:

a One other root is $1-3\text{i}$

If $a+\text{i}b$ is a root, then $a-\text{i}b$ must also be a root. The complex roots of polynomials with real coefficients occur as complex conjugate pairs.

The cubic equation must be identical to
$(z-1-3\text{i})(z-1+3\text{i})(z-\gamma)=0$

If α, β and γ are the roots of a cubic equation, then the equation must have the form $(x-\alpha)(x-\beta)(x-\gamma)=0$.

$$((z-1)-3\text{i})((z-1)+3\text{i})=(z-1)^2-(3\text{i})^2$$
$$=z^2-2z+1+9=z^2-2z+10$$

You know the first two roots, α and β, so the only remaining problem is finding the third γ.

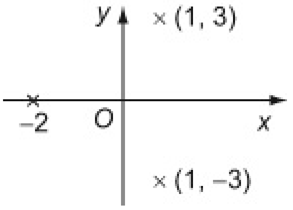
Hence
$\left(z^2-2z+10\right)(z-\gamma)=z^3+6z+20$

You need not multiply the brackets on the left hand side of this equation out fully. If the brackets were multiplied out, the only term without a z would be when $+10$ is multiplied by $-\gamma$ and the product of these, -10γ, equals the term without z on the right hand side, $+20$.

Equating constant coefficients
$-10\gamma=20 \Rightarrow \gamma=-2$

The other roots are $1-3\text{i}$ and -2.

b



c

The gradient of the line joining $(-2,0)$ to $(1,3)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{1-(-2)} = \frac{3}{3} = 1$$

You prove the result in part (c) using the methods of Coordinate Geometry that you learnt for the C1 module. These can be found in Edexcel Modular Mathematics for AS and A-level Core Mathematics 1, Chapter 5.

The gradient of the line joining $(-2,0)$ to $(1,-3)$ is given by

$$m' = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{1-(-2)} = \frac{-3}{3} = -1$$

Hence $mm' = -1$, which is the condition for perpendicular lines.

Two sides of the triangle are at right angles to each other and the triangle is right-angled.

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Edexcel AS and A Level Modular Mathematics

Complex numbers
Exercise I, Question 7

Question:

$z_1 = 4 + 2i, z_2 = -3 + i$

a Display points representing z_1 and z_2 on the same Argand diagram.

b Find the exact value of $|z_1 - z_2|$.

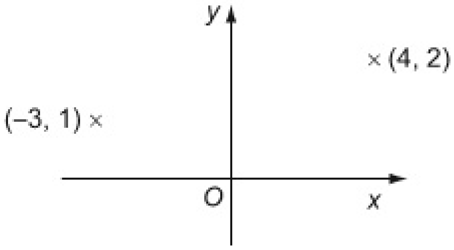
Given that $w = \frac{z_1}{z_2}$,

c express w in the form $a + ib$, where $a, b \, \varepsilon \, \mathbb{R}$,

d find arg w, giving your answer in radians.

Solution:

a



b

$$\begin{aligned} z_1 - z_2 &= 4 + 2i - (-3 + i) \\ &= 4 + 2i + 3 - i = 7 + i \end{aligned}$$

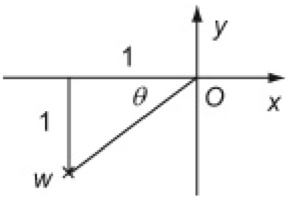
$z_1 - z_2$ could be represented by the vector joining the point (−3,1) to the point (4, 2). $|z_1 - z_2|$ is then the distance between these two points.

$$\begin{aligned} |z_1 - z_2|^2 &= 7^2 + 1^2 = 50 \\ |z_1 - z_2| &= \sqrt{50} = 5\sqrt{2} \end{aligned}$$

The question specifies an exact answer, so decimals would not be acceptable.

c

$$\begin{aligned} w &= \frac{4 + 2i}{-3 + i} \times \frac{-3 - i}{-3 - i} = \frac{-12 - 4i - 6i + 2}{(-3)^2 + 1^2} \\ &= \frac{-10 - 10i}{10} = -1 - i \end{aligned}$$



$\tan\ \theta = \dfrac{\frac{1}{4}}{\frac{1}{4}} = 1 \Rightarrow \theta = \dfrac{\pi}{4}$

w is in the third quadrant.

$\arg\ w = -\left(\pi - \dfrac{\pi}{4}\right) = -\dfrac{3\pi}{4}$

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Edexcel AS and A Level Modular Mathematics

Complex numbers
Exercise I, Question 8

Question:

Given that $3-2\mathrm{i}$ is a solution of the equation

$x^4-6x^3+19x^2-36x+78=0$,

a solve the equation completely,

b show on a single Argand diagram the four points that represent the roots of the equation.

Solution:

a

Let $\mathrm{f}(x)=x^4-6x^3+19x^2-36x+78$

When you have to refer to a long expression, like this quartic equation, several times in a solution, it saves time to call the expression, say, $\mathrm{f}(x)$. It is much quicker to write $\mathrm{f}(x)$ than $x^4-6x^3+19x^2-36x+78$!

As $3-2\mathrm{i}$ is a root of $\mathrm{f}(x)$, $3+2\mathrm{i}$ is also a root of $\mathrm{f}(x)$.

$$
\begin{aligned}
(x-3+2\mathrm{i})(x-3+2\mathrm{i}) &= (x-3)^2+4 \\
&= x^2-6x+9+4 \\
&= x^2-6x+13
\end{aligned}
$$

If $a-\mathrm{i}\,b$ is a root, then $a+\mathrm{i}\,b$ must also be a root. The complex roots of polynomials with real coefficients occur as complex conjugate pairs.

$$
\begin{array}{r|l}
 & \quad\quad\quad\ \ x^2+6 \\
\cline{2-2}
x^2-6x+13 & x^4-6x^3+19x^2-36x+78 \\
 & \underline{x^4-6x^3+13x^2} \\
 & \quad\quad\quad\quad 6x^2-36x+78 \\
 & \quad\quad\quad\quad \underline{6x^2-36x+78}
\end{array}
$$

If α and β are roots of $\mathrm{f}(x)$, then $\mathrm{f}(x)$ must have the form $(x-\alpha)(x-\beta)\left(x^2+ax+b\right)$ and the remaining two roots can be found by solving $x^2+ax+b=0$. The method used here is finding a and b by long division. In this case $a=0$ and $b=6$.

Hence

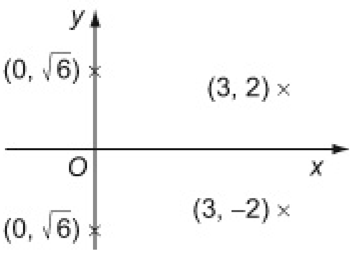
$$\mathrm{f}(x)=\left(x^2-6x+13\right)\left(x^2+6\right)=0$$

$$x^2+6=0 \Rightarrow x=\pm\mathrm{i}\sqrt{6}$$

The solutions of $\mathrm{f}(x)=0$ are

$$3-2\mathrm{i}, 3+2\mathrm{i}, \mathrm{i}\sqrt{6}, -\mathrm{i}\sqrt{6}$$

b


y
(0, √6)
(3, 2)
O
x
(3, –2)
(0, √6)

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Complex numbers
Exercise I, Question 9

Question:

$z = \frac{a+3\text{i}}{2+a\text{i}}, \quad a \varepsilon \mathbb{R}.$

a Given that $a = 4$, find $|z|$.

b Show that there is only one value of a for which $\arg z = \frac{\pi}{4}$, and find this value.

Solution:

a

$$\begin{aligned} z &= \frac{a+3\text{i}}{2+a\text{i}} = \frac{a+3\text{i}}{2+a\text{i}} \times \frac{2-a\text{i}}{2-a\text{i}} \\ &= \frac{2a-a^2\text{i}+6\text{i}+3a}{4+a^2} \\ &= \frac{5a}{4+a^2} + \frac{6-a^2}{4+a^2}\text{i} \ldots\ldots * \end{aligned}$$

You could substitute $a = 4$ into the expression for z at the beginning of part (a) and this would actually make this part easier. However you can use the expression marked * once in this part and three times in part (b) as well. It often pays to read quickly right through a question before starting.

Substitute $a = 4$

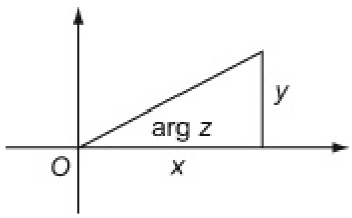
$$z = \frac{20}{20} + \frac{-10}{20}\text{i} = 1 - \frac{1}{2}\text{i}$$

$$|z|^2 = 1^2 + \left(-\frac{1}{2}\right)^2 = \frac{5}{4}$$

$$|z| = \frac{\sqrt{5}}{2}$$

b

$$\tan(\arg z) = \frac{\frac{5a}{4+a^2}}{\frac{6-a^2}{4+a^2}} = \frac{5a}{6-a^2}$$



If $z = x + \text{i}\,y$, then $\tan(\arg z) = \frac{y}{x}$.

Also from the data in the question

$$\tan(\arg z) = \tan\frac{\pi}{4} = 1$$

Hence

$$\frac{5a}{6-a^2} = 1 \Rightarrow 5a = 6 - a^2 \Rightarrow a^2 + 5a - 6 = 0$$

$$(a-1)(a+6) = 0 \Rightarrow a = 1\,, -6$$

If $a = -6$, substituting into the result $*$ in part (a)

$$z = \frac{30}{40} - \frac{30}{40}\text{i} = \frac{3}{4} - \frac{3}{4}\text{i}$$

This is in the third quadrant and has a negative

At this point you have two answers. The question asks you to show that there is only one value of a. You must test both and choose the one that satisfies the condition $\arg z = \frac{\pi}{4}$. The other value occurs because argument $\left(-\frac{3\pi}{4}\right)$, so $a = -6$ is rejected.

$\tan\frac{\pi}{4}$ and $\tan\left(-\frac{3\pi}{4}\right)$ are both 1.

If $a = 1$, substituting into the result $*$ in part (a)

$z = \frac{5}{5} + \frac{5}{5}\mathrm{i} = 1 + \mathrm{i}$

This is in the first quadrant and does have an argument $\frac{\pi}{4}$.

$a = 1$ is the only possible value of a.