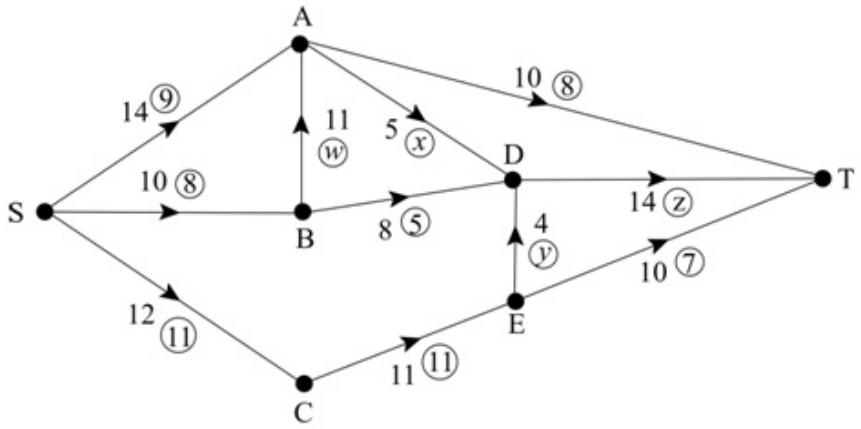
Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:



The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow pattern.

a Find the values of w, x, y and z, explaining your reasoning.
b State the value of the initial flow.
c Identify two saturated arcs.
d Write down the capacity of arc BD.
e What is the current flow along route SAT?

Solution:

a Flow into B = flow out of B $w = 3$
Flow into A = flow out of A $x = 4$
Flow into E = flow out of E $y = 4$
Flow into D = flow out of D $z = 13$
b Feasible flow = 28
c CE and ED are saturated
d BD has capacity 8
e Along SAT the current flow is 8

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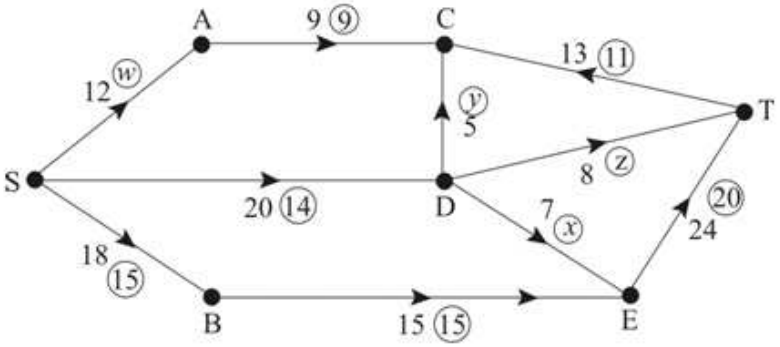
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:



The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow pattern.

a Find the values of w, x, y and z, explaining your reasoning.

b State the value of the initial flow.

c Identify two saturated arcs.

d Write down the flow along arc SD.

e What is the current flow along the route SBET?

Solution:

a Flow into A = flow out of A $w = 9$

Flow into E = flow out of E $x = 5$

Flow into C = flow out of C $y = 2$

Flow into D = flow out of D $14 = y + x + 2 \Rightarrow 14 = 2 + 5 + z \Rightarrow z = 7$

b Feasible flow = 38

c BE and AC are saturated

d Flow along SD is 14

e Flow along SBET = 15

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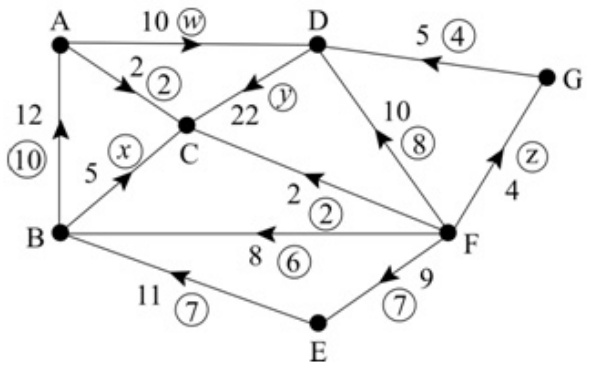
Exercise A, Question 3

Question:



The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow pattern.

a State the source vertex.
b State the sink vertex.
c Find the values of w, x, y and z, explaining your reasoning.
d State the value of the feasible flow.
e Identify three saturated arcs.
f Write down the capacity of arc FB.

Solution:

a Source vertex is F

b Sink vertex is C

c Flow into A = flow out of A $w = 8$
Flow into B = flow out of B $x = 3$
Flow into D = flow out of D $y = 20$
Flow into G = flow out of G $z = 4$

d Feasible flow = 27

e Saturated arcs are AC, FC, FG

f Capacity of FB is 8

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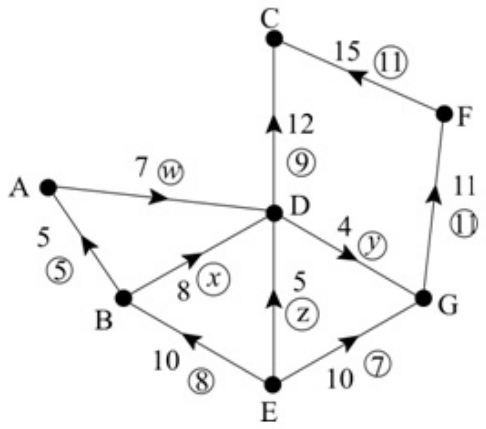
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 4

Question:



The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow pattern.

a State the source vertex.
b State the sink vertex.
c Find the values of w, x, y and z, explaining your reasoning.
d State the value of the initial flow.
e Identify four saturated arcs.
f Write down the flow along arc FC.

Solution:

a Source vertex is E
b Sink vertex is C
c Flow into A = flow out of A $w = 5$
Flow into B = flow out of B $x = 3$
Flow into G = flow out of G $y = 4$
Flow into D = flow out of D $z = 5$

d Feasible flow = 20

e Saturated arcs are BA, ED, DG, GF

f Flow along FC = 11

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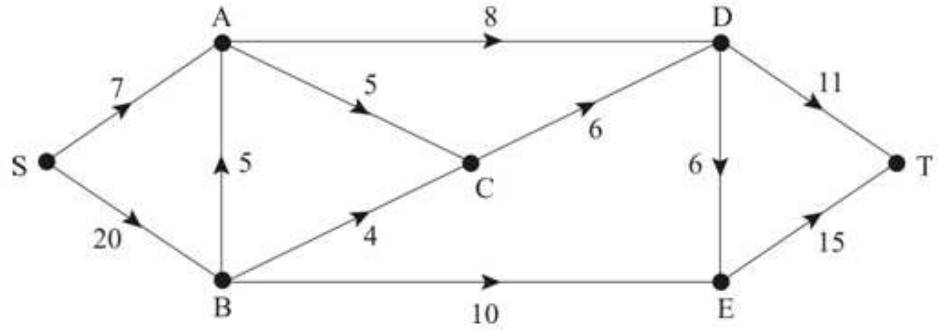
Exercise A, Question 5

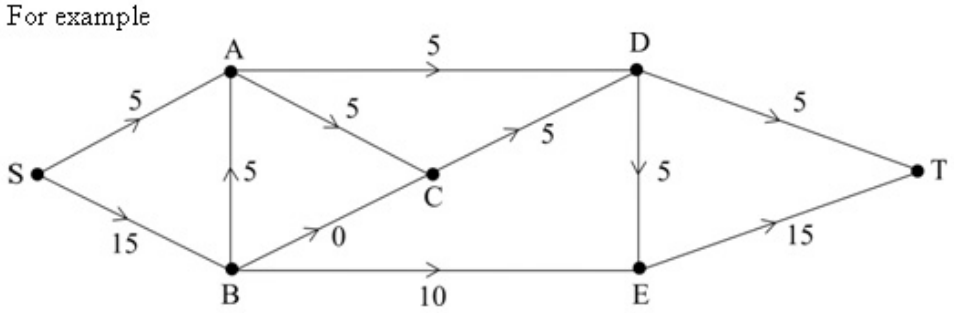
Question:



The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. Find a feasible flow of at least 20 through the network from S to T.

Solution:

For example



There are many other answers.

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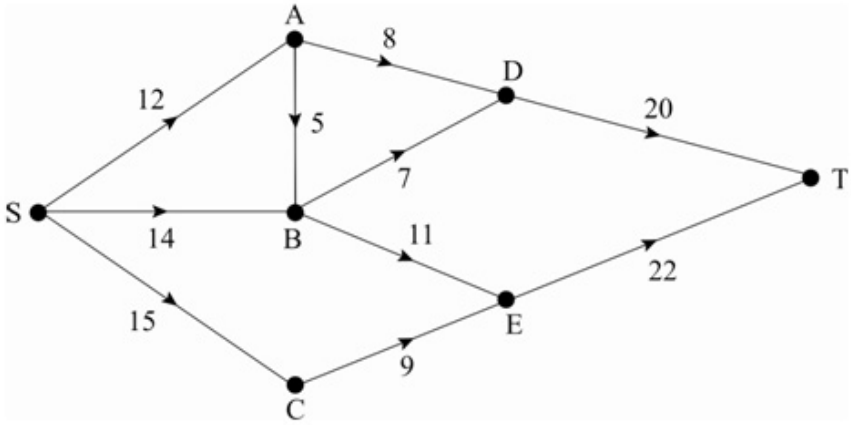
Exercise A, Question 6

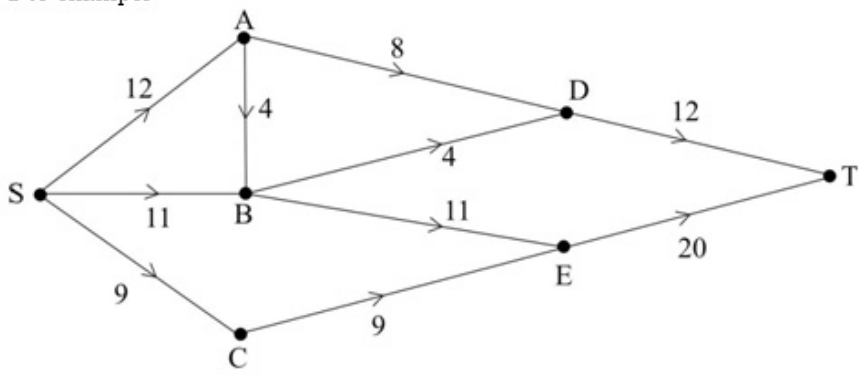
Question:



The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. Find a feasible flow of 32 through the network from S to T.

Solution:

For example



There are many other solutions.

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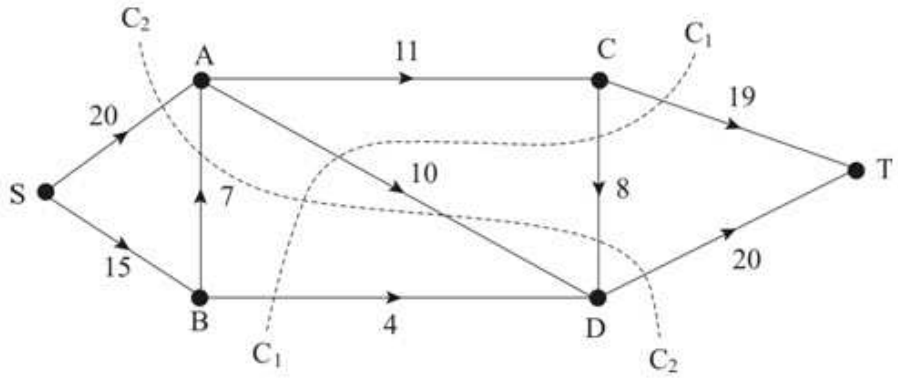
Exercise B, Question 1

Question:

The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. Where relevant, the numbers in circles represent an initial flow pattern. Evaluate the capacities of the cuts drawn.



Solution:

$\text{Cut } C_1 = 19+8+10+4=41$

$\text{Cut } C_2 = 20+7+20=47$

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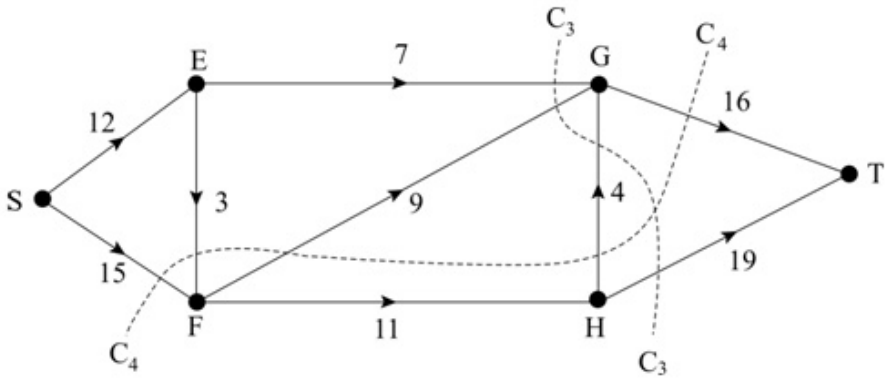
Exercise B, Question 2

Question:

The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. Where relevant, the numbers in circles represent an initial flow pattern. Evaluate the capacities of the cuts drawn.



Solution:

$$\text{Cut } C_3 = 7+9+4+19 = 39$$
$$\text{Cut } C_4 = 15+3+16 = 34$$

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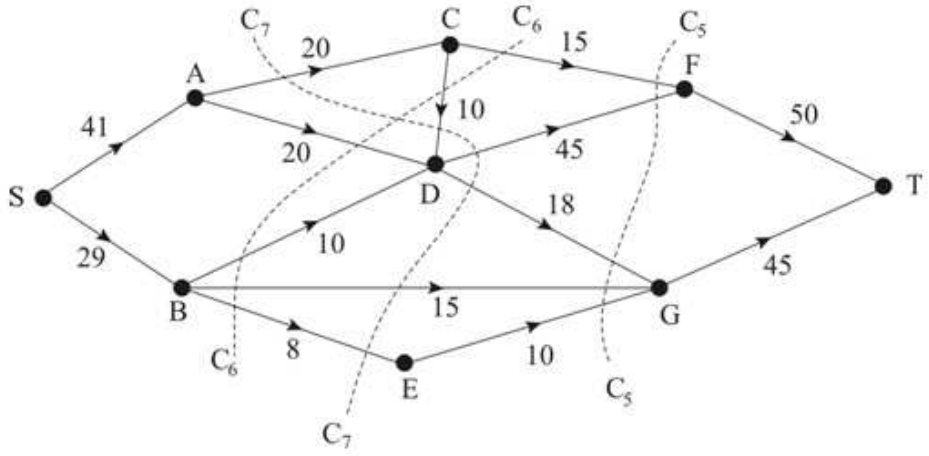
Exercise B, Question 3

Question:

The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. Where relevant, the numbers in circles represent an initial flow pattern. Evaluate the capacities of the cuts drawn.



Solution:

$$\text{Cut } C_5 = 15+45+18+15+10 = 103$$
$$\text{Cut } C_6 = 15+10+20+10+15+8 = 78$$
$$\text{Cut } C_7 = 20+45+18+15+8 = 106$$

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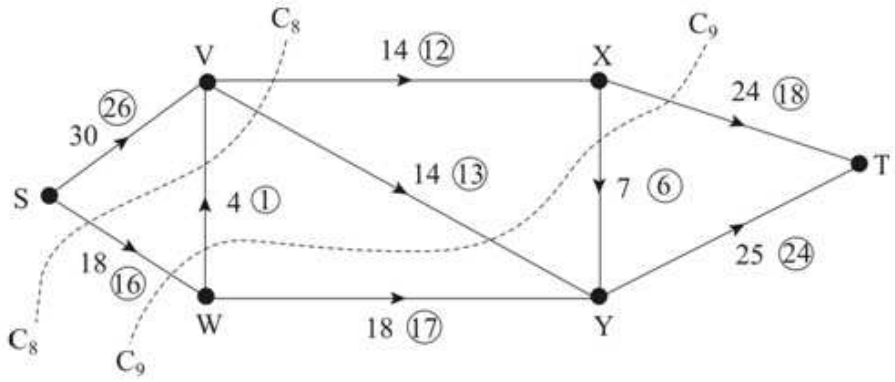
Exercise B, Question 4

Question:

The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. Where relevant, the numbers in circles represent an initial flow pattern. Evaluate the capacities of the cuts drawn.



Solution:

Cut $C_8 = 14+14+4+18=50$

Cut $C_9 = 24+14+4+18=60$

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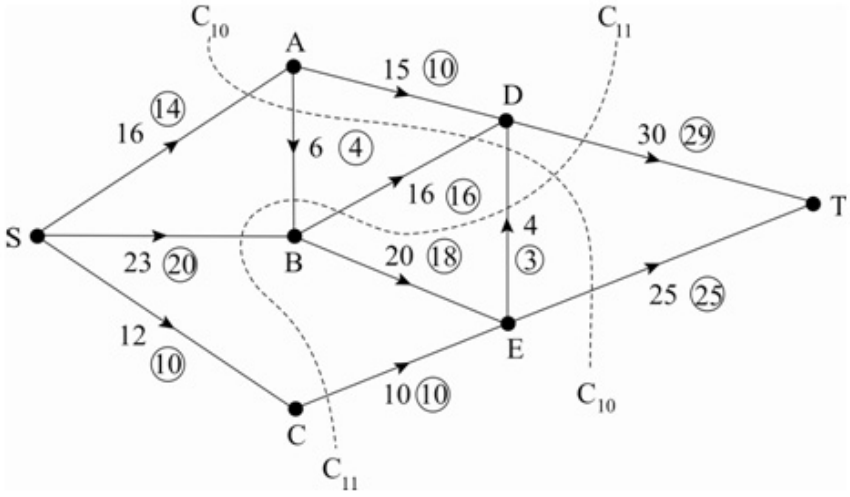
Exercise B, Question 5

Question:

The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. Where relevant, the numbers in circles represent an initial flow pattern. Evaluate the capacities of the cuts drawn.



Solution:

$$\text{Cut } C_{10} = 16+16+4+25 = 61$$
$$\text{Cut } C_{11} = 30+6+23+10 = 69$$

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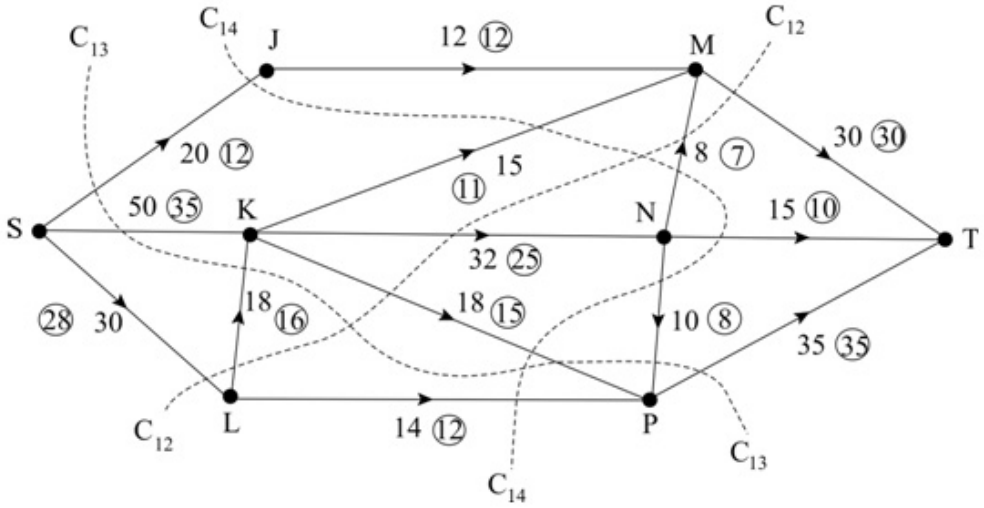
Exercise B, Question 6

Question:

The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc. Where relevant, the numbers in circles represent an initial flow pattern. Evaluate the capacities of the cuts drawn.



Solution:

$\text{Cut } C_{12} = 30+32+18+30 = 110$

$\text{Cut } C_{13} = 20+50+18+35 = 123$

$\text{Cut } C_{14} = 20+15+8+15+10+18+14 = 100$

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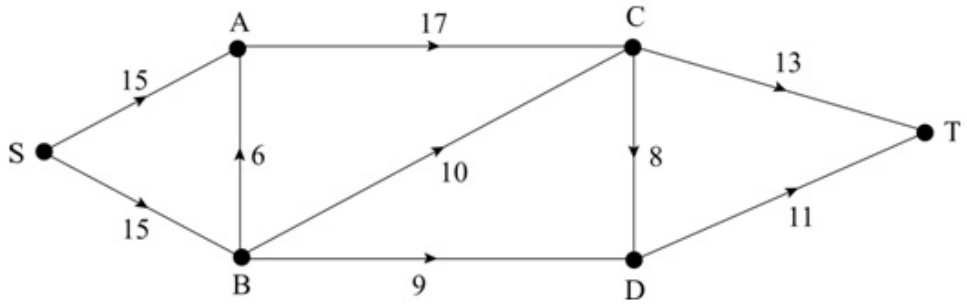
Exercise C, Question 1

Question:



The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc.

a State the maximum flows along SACT and SBCDT.

b Show these on a diagram.

Using this as your initial flow,

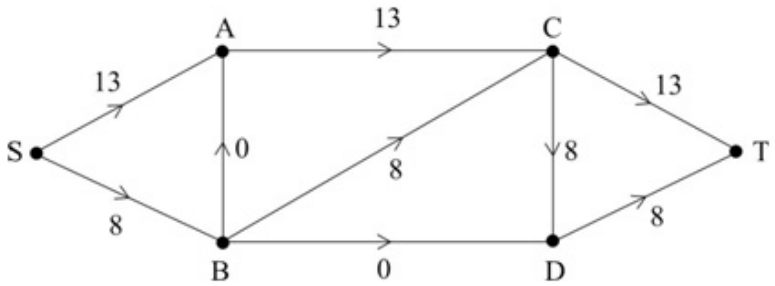
c calculate the value of the initial flow.

Solution:

a max flow along SACT = 13

max flow along SBCDT = 8

b



c Value of initial flow = 21

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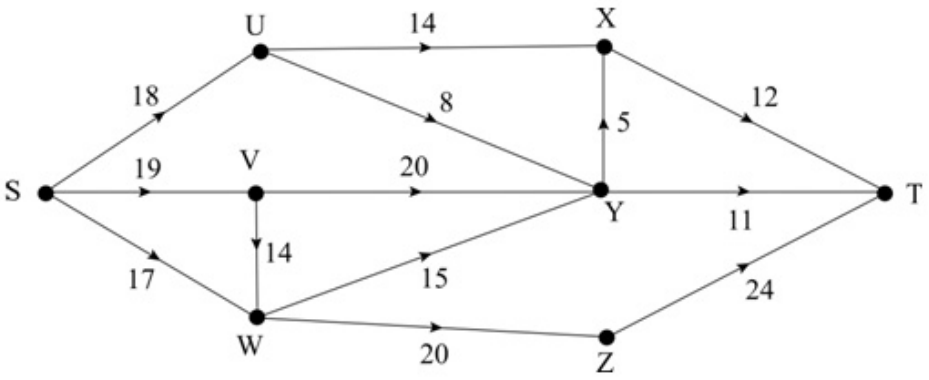
Exercise C, Question 2

Question:



The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc.

a State the maximum flows along SUXT, SWZT and SVWYT.

b Show these on a diagram.

Using this as your initial flow,

c calculate the value of the initial flow.

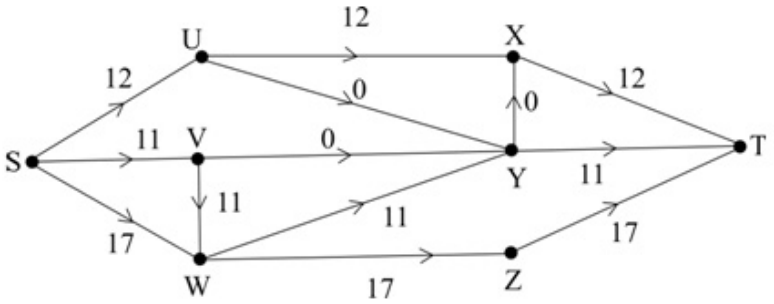
Solution:

a max flow along SUXT = 12

max flow along SWZT = 17

max flow along SVWYT = 11

b



c Value of initial flow = 40

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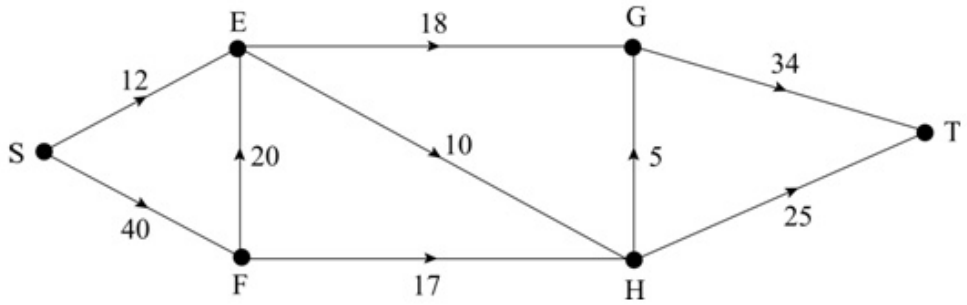
Exercise C, Question 3

Question:



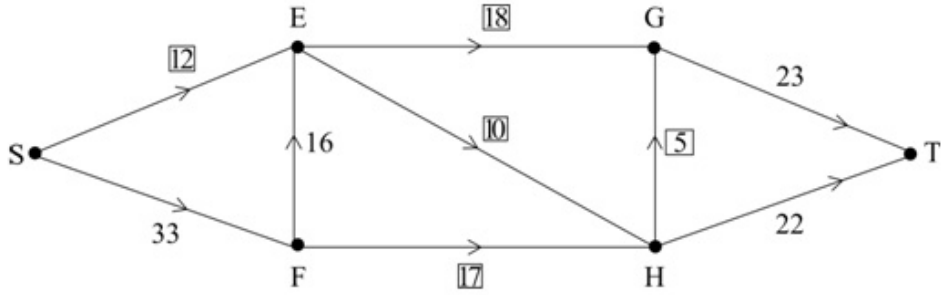
The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc.

a Given that arcs SE, EG, EH, FH and HG are saturated, draw an initial flow through the network.

b State the value of the initial flow.

Solution:

a



b Value of initial flow = 45

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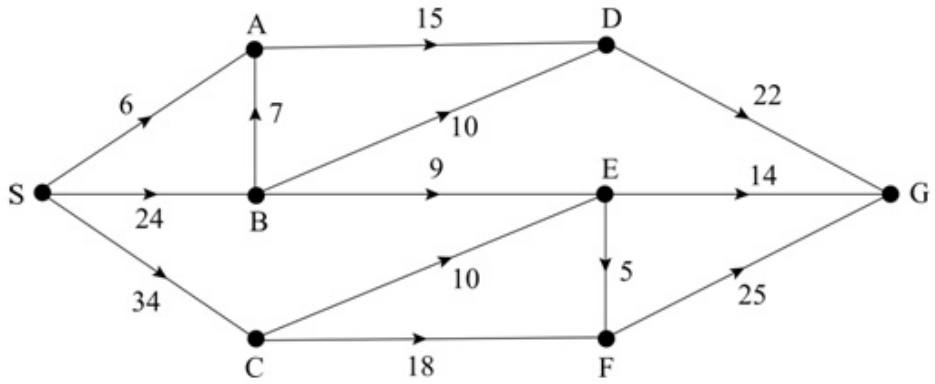
Exercise C, Question 4

Question:



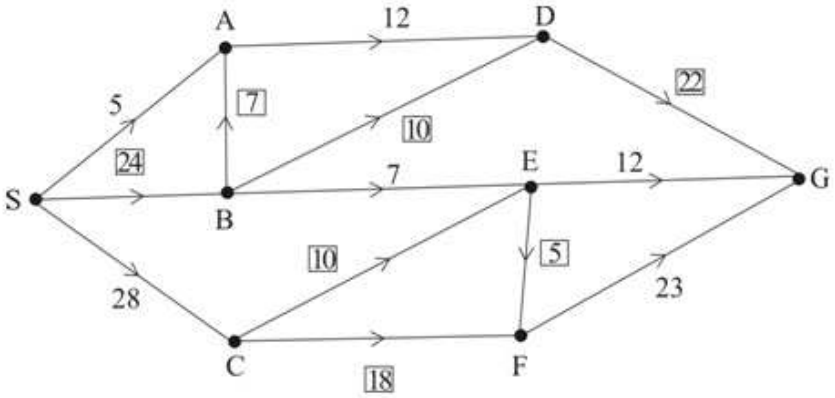
The diagram shows a capacitated directed network. The number on each arc represents the capacity of that arc.

a Given that arcs SB, BA, BD, CE, CF, EF and DG are saturated, draw an initial flow through the network.

b State the value of the initial flow.

Solution:

a



b Value of initial flow 57

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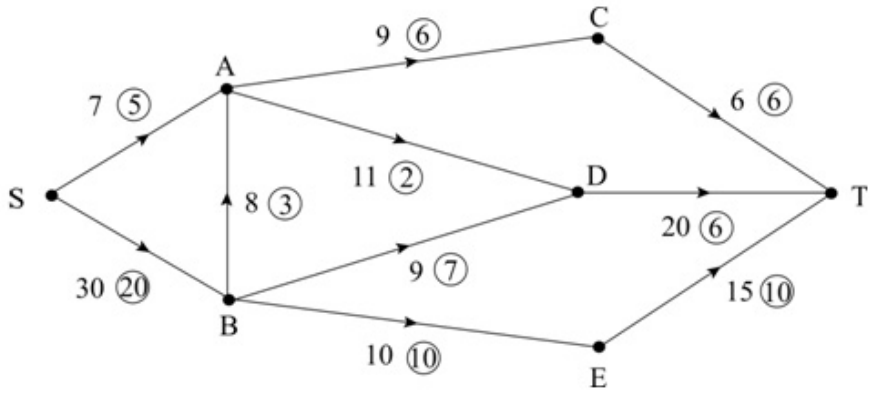
Exercise D, Question 1

Question:

The diagram shows a capacitated, directed network. The capacity of each arc is shown on each arc. The numbers in circles represent an initial flow from S to T.

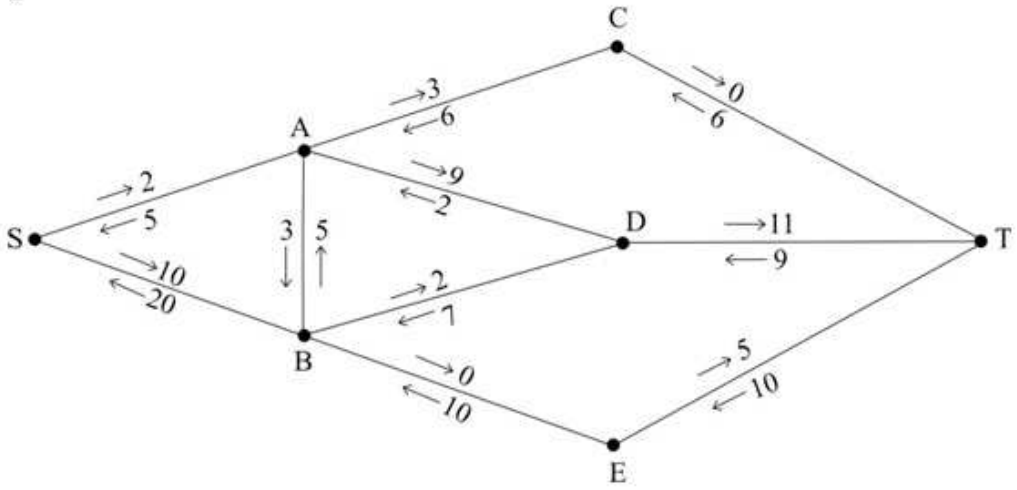
a Starting from the initial flow, use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use together with its flow.

b Draw your final flow pattern and state the value of your maximum flow.



Solution:

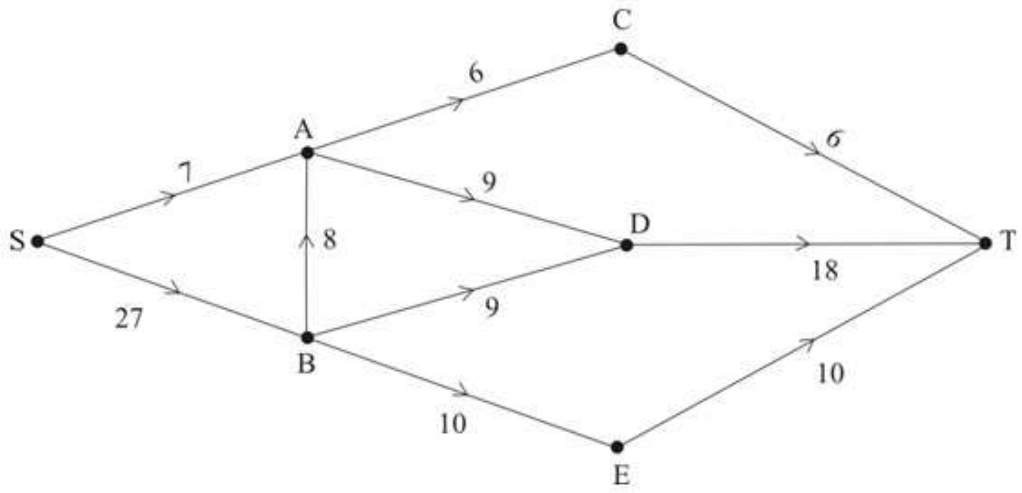
a



For example (there are many other combinations of flows possible)

SBADT – 5
SADT – 2
SBDT – 2

b



Value of maximum flow is 34

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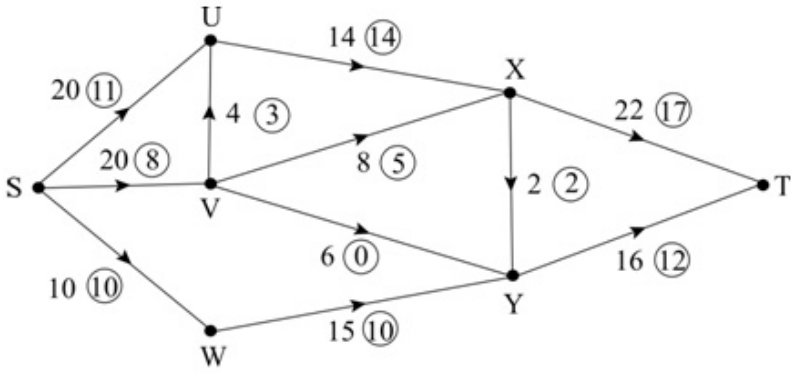
Exercise D, Question 2

Question:

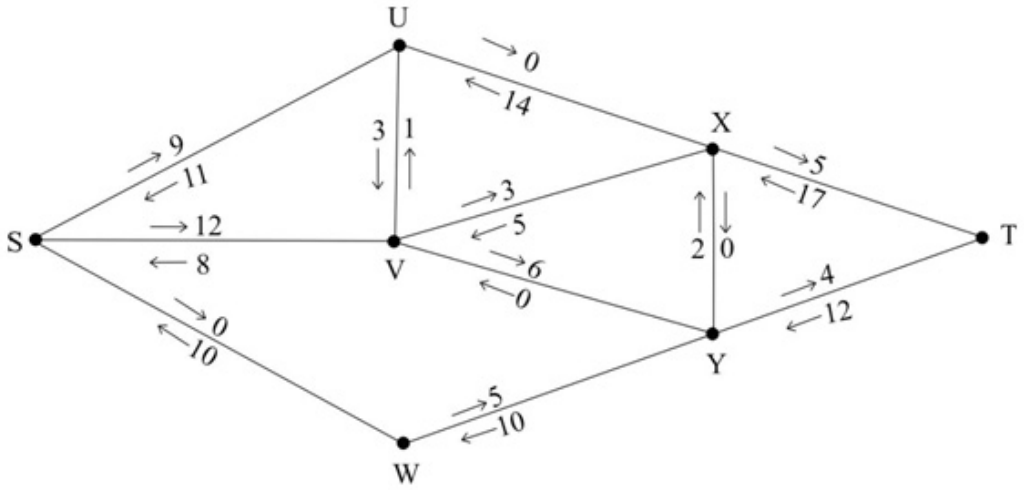
The diagram shows a capacitated, directed networks. The capacity of each arc is shown on each arc. The numbers in circles represent an initial flow from S to T.

a Starting from the initial flow, use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use together with its flow.

b Draw your final flow pattern and state the value of your maximum flow.



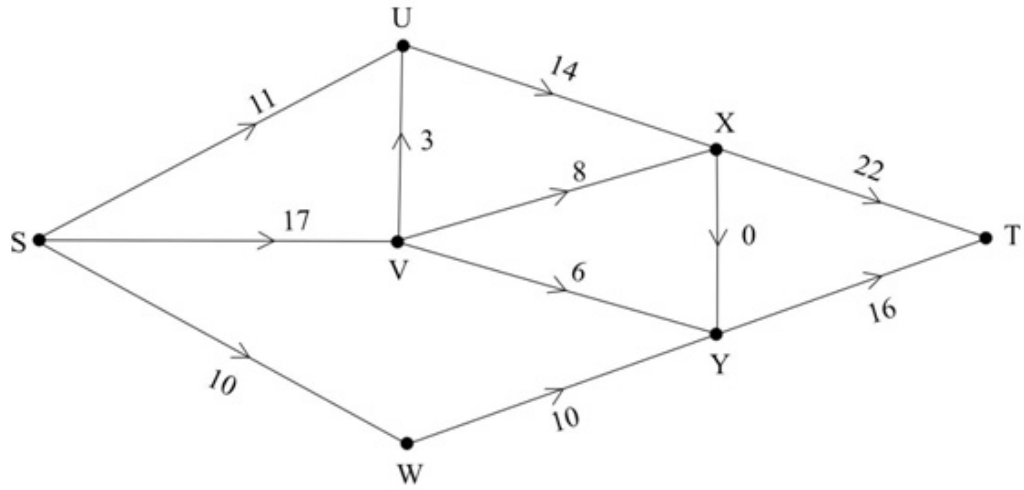
Solution:



For example(there are many other combinations of flows possible)

S V Y T – 4
S V X T – 3
S V Y X T – 2

b



Value of maximum flow is 38

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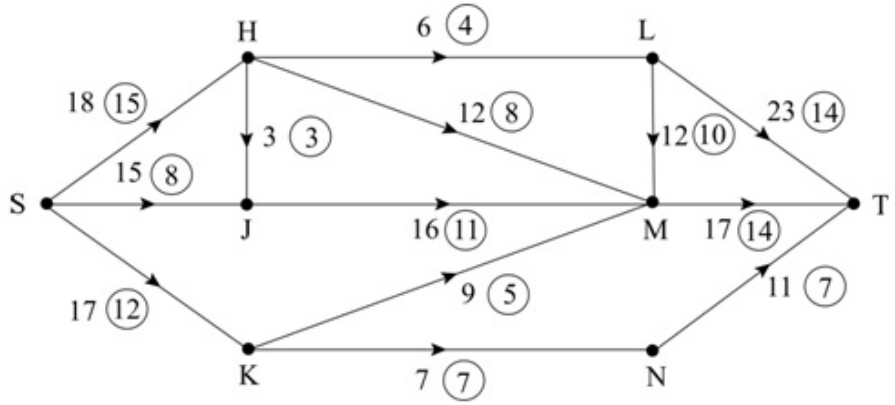
Exercise D, Question 3

Question:

The diagram shows a capacitated, directed networks. The capacity of each arc is shown on each arc. The numbers in circles represent an initial flow from S to T.

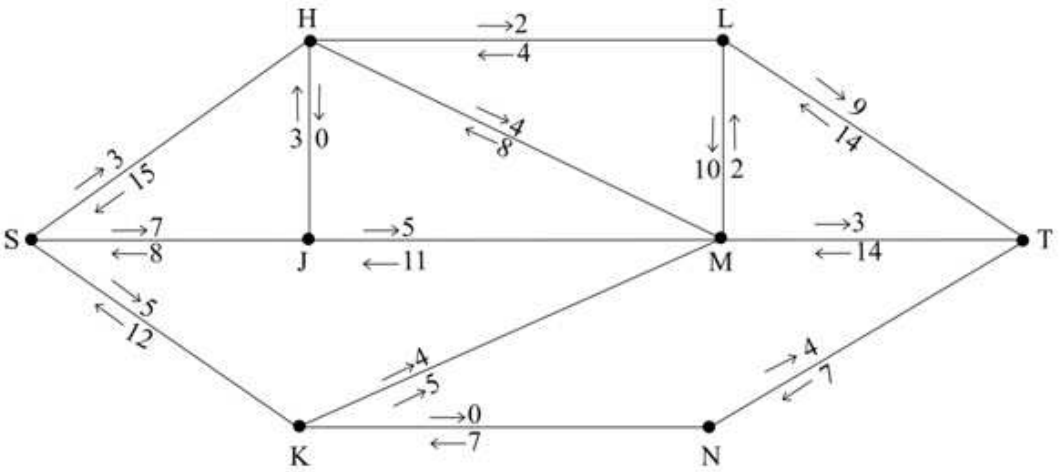
a Starting from the initial flow, use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use together with its flow.

b Draw your final flow pattern and state the value of your maximum flow



Solution:

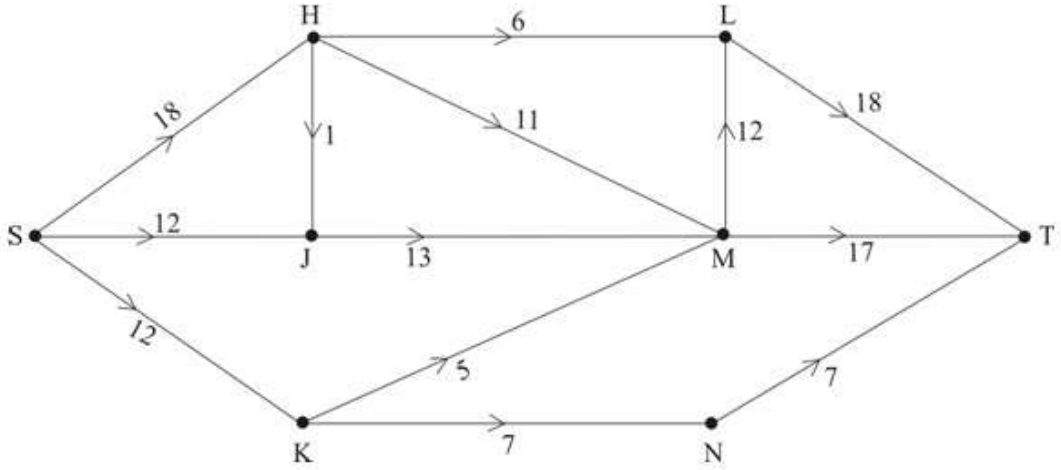
a



For example (there are many other combinations of flows)

SHMT – 3
SJMLT – 2
SJHLT – 2

b



Value of maximum flow is 42

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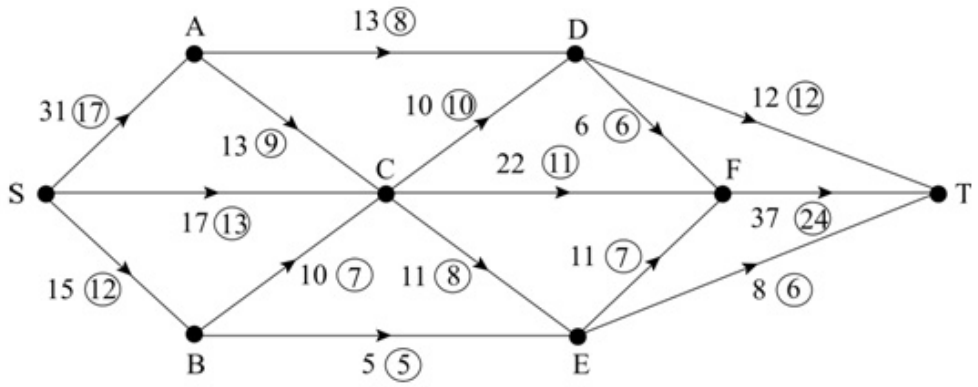
Exercise D, Question 4

Question:

The diagram shows a capacitated, directed networks. The capacity of each arc is shown on each arc. The numbers in circles represent an initial flow from S to T.

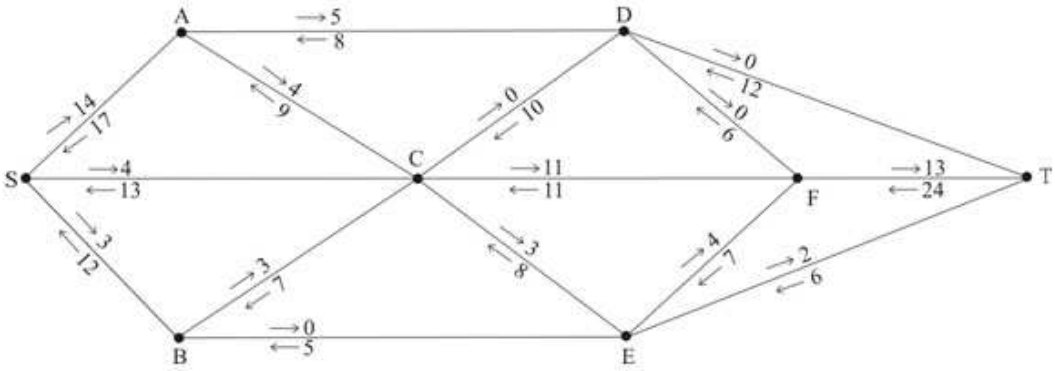
a Starting from the initial flow, use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use together with its flow.

b Draw your final flow pattern and state the value of your maximum flow



Solution:

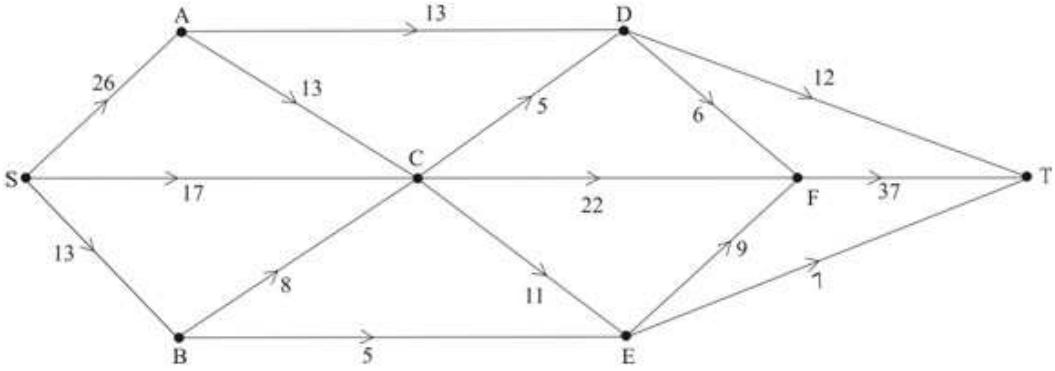
a



For example (there are many other combinations of flow)

SACFT – 4
SADCFT – 5
SCFT – 2
SCEFT – 2
SBCET – 1

b



Value of maximum flow is 56

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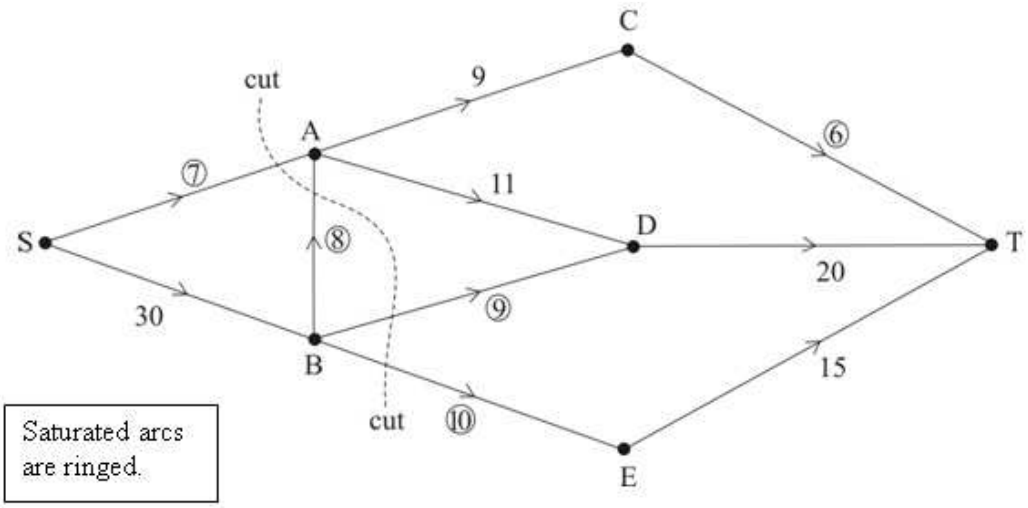
Exercise E, Question 1

Question:

Use the maximum flow-minimum cut theorem to prove that the flows you found in answer to the questions in exercise 6D are maximal.

Solution:



The diagram shows the capacity of each arc.
So by maximum flow – minimum cut theorem flow is maximal

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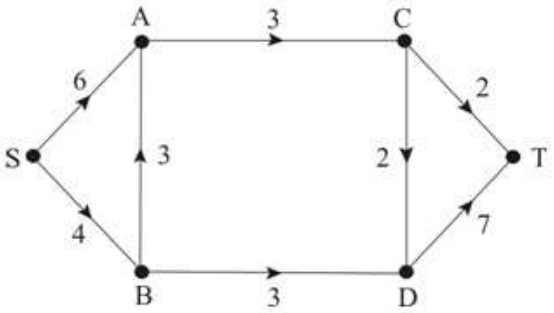
Exercise F, Question 1

Question:



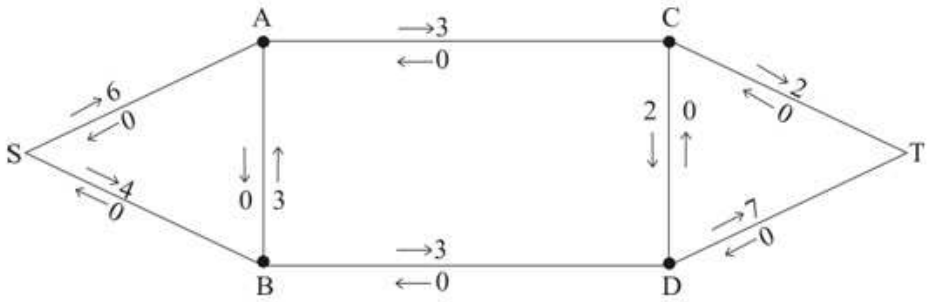
The diagram shows a capacitated, directed network. The number on each arc indicates the capacity of that arc.

a Use the labelling procedure to find the maximum flow through the network from S to T, listing each flow augmenting route you use, together with its flow.

b Verify that the flow found in part **a** is maximal.

Solution:

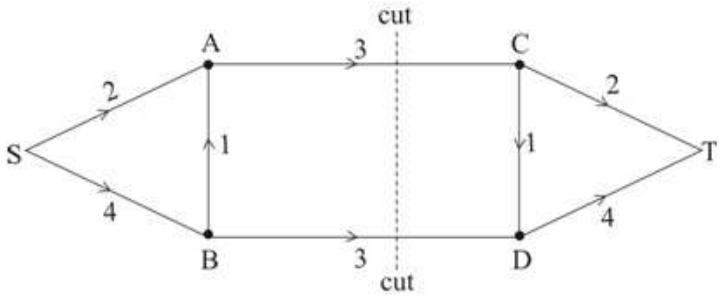
a



For example (many different flow combinations are possible)

SBDT – 3
SACT – 2
SBACDT – 1

b



Minimum cut = 6 so by maximum flow = minimum cut theorem, flow is maximum.

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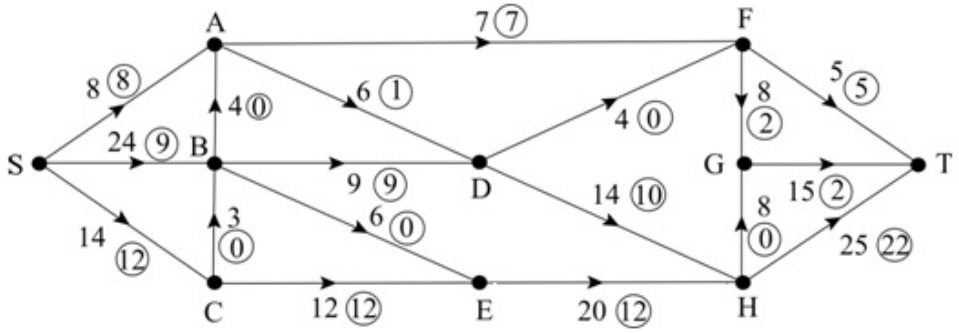
Exercise F, Question 2

Question:



The diagram shows a capacitated directed network. The number on each arc is the value of the maximum flow along that arc.

a Describe briefly a situation for which this type of network could be a suitable model.

The numbers in circles show a feasible flow of value 29 from source S to sink T. Take this as the initial flow pattern.

b Use the labelling procedure to find the maximum flow through the network from S to T. You must list each flow-augmenting route you use together with its flow.

c Indicate your maximum flow pattern and state the final flow.

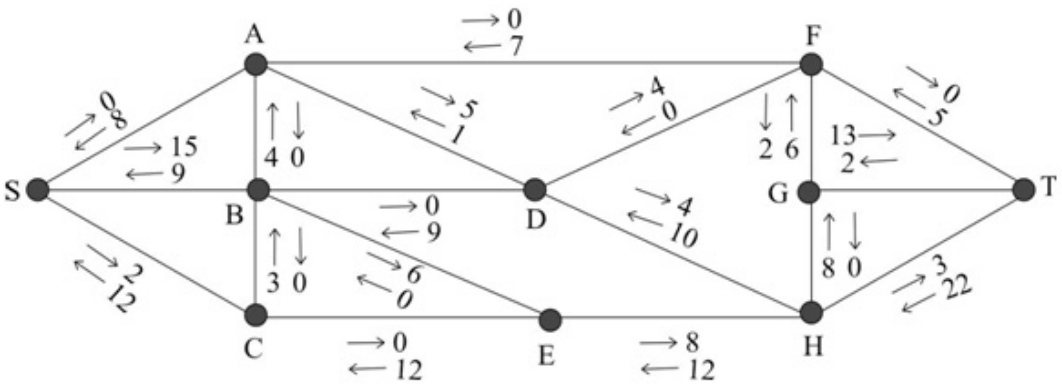
d Verify that your answer is a maximum flow by using the maximum flow–minimum cut theorem, listing the arcs through which your cut passes.

e For the maximum flow, state a property of the arcs found in **d**. **[*E*]**

Solution:

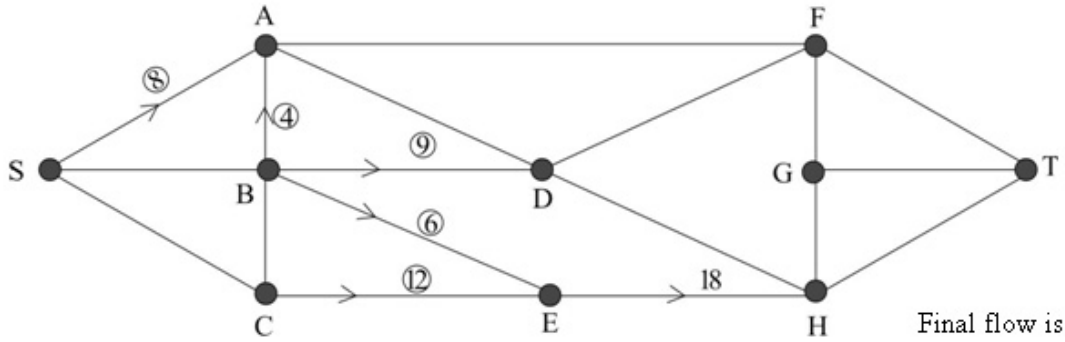
a Applied. Idea of flow through a system, idea of directed flow.
e.g. traffic moving through a one-way system of roads

b



e.g. S B E H G T – 6 and S B A D F G T – 4
or S B A D H G T – 4 and S C B E H T – 2 and S B E H G T – 4 etc.

c Many solutions are possible, but the following values must be given. Flows must be consistent.



Final flow is 39

d Cut through SA, BA, BD, BE and CE

e The arcs are saturated.

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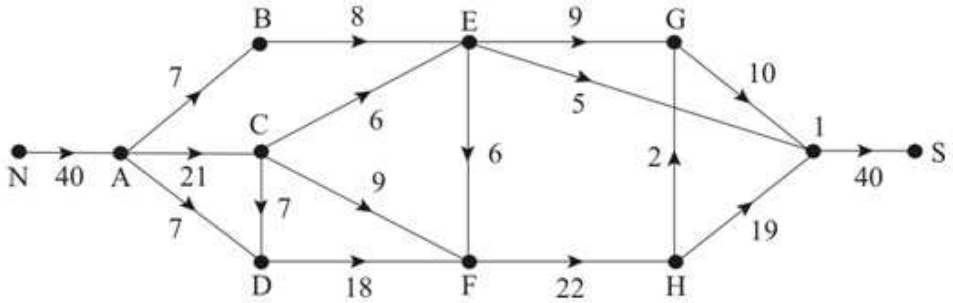
Exercise F, Question 3

Question:



The diagram shows the road routes from a bus station, N, on the north side of a town to a bus station S, on its south side. The number on each arc shows the maximum flow rate, in vehicles per minute, on that route.

a State four junctions at which there could be traffic delays, giving a reason for your answer.

Given that AB, AD, CE, CF and EI are saturated,

b show a flow of 31 from N to S that satisfies this demand.

c Taking your answer to **b** as the initial flow pattern, use the labelling procedure to find the maximum flow. You should list each flow-augmenting route you use together with its flow.

d Indicate your maximum flow pattern.

e Verify your solution using the maximum flow–minimum cut theorem, listing the arcs through which your minimum cut passes.

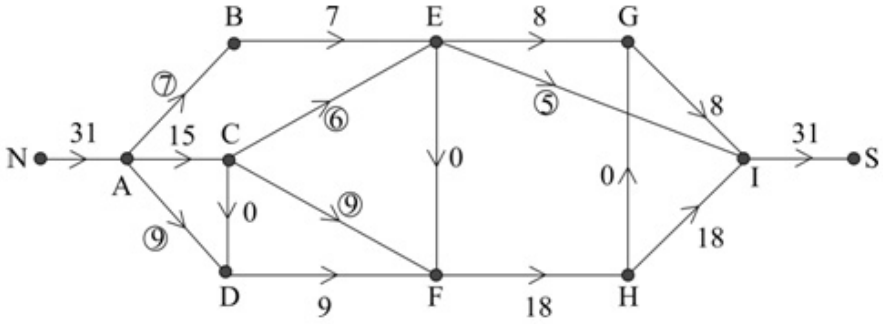
f Show that, in this case, there is a second minimum cut and list the arcs through which it passes.

[*E*]

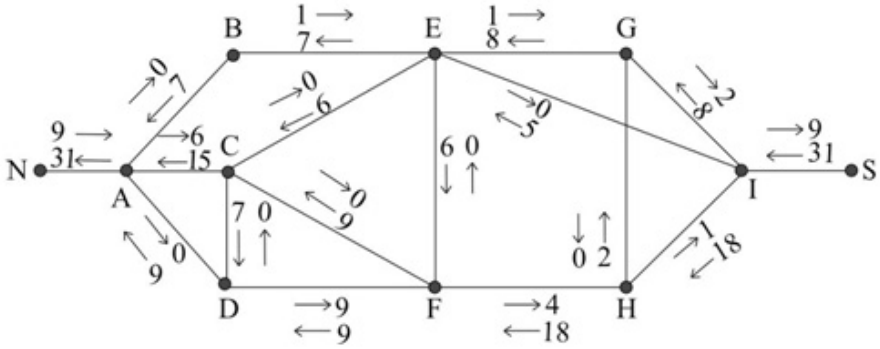
Solution:

a A F G and H, possible flow in > possible flow out.

b e.g.



c Using labelling procedure e.g.

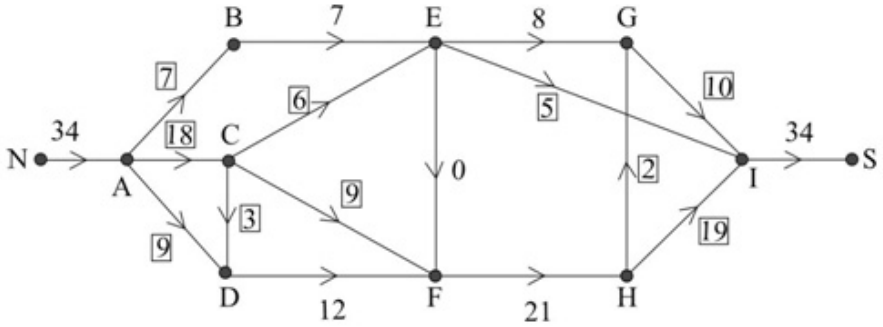


e.g.

If H I = 16	If HI = 17	If HI = 18	If H I = 19
NACDFHIS – 3	NACDFHGIS – 1	NACDFHGIS – 2	NACDFHGIS – 1
	NACDFHIS – 2	NACDFHIS – 1	NACDFEGIS – 2

Final flow 34

d e.g.



e and **f** GI EI and HI

AB CE (EF) HG and HI

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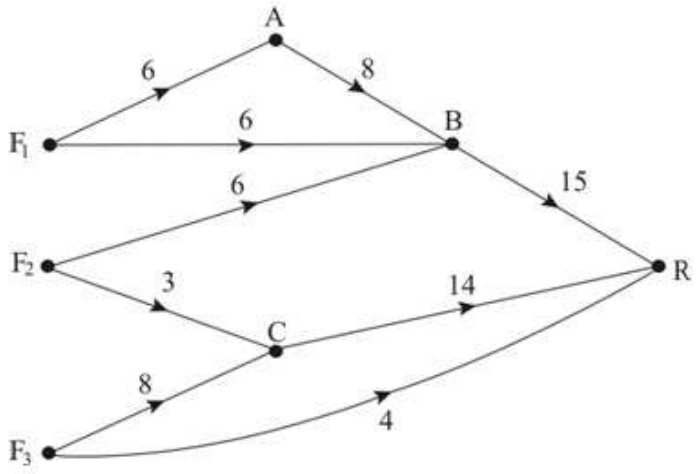
Exercise F, Question 4

Question:

A company wishes to transport its products from 3 factories F_1, F_2 and F_3 to a single retail outlet R. The capacities of the possible routes, in van loads per day, are shown.



a On the worksheet add a supersource S to obtain a capacitated network with a single source and a single sink. State the minimum capacity of each arc you have added.

b **i** State the maximum flow along SF_1ABR and SF_3CR.

ii Show these maximum flows on the worksheet, using numbers in circles.

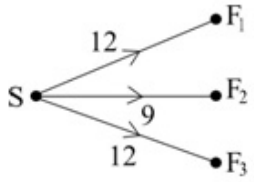
Taking your answer to part **b ii** as the initial flow pattern,

c **i** use the labelling procedure to find a maximum flow from S to R. List each flow-augmenting route you find together with its flow.

ii Prove that your final flow is maximal. ***E***

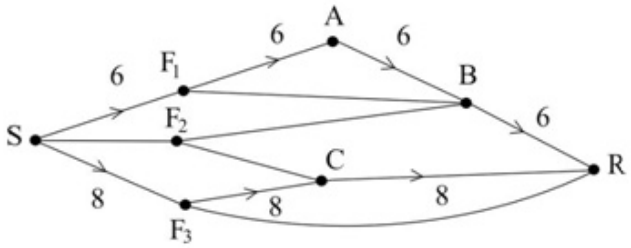
Solution:

a

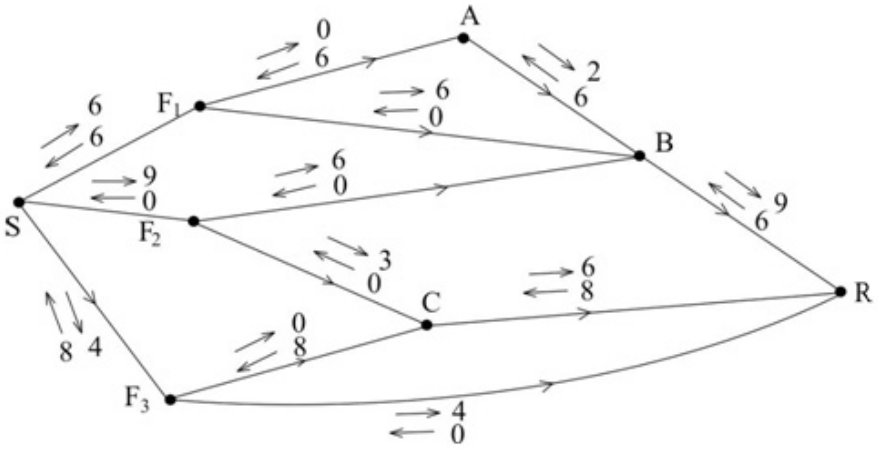


b i SF_1ABR-6

ii SF_3CR-8



c i



e.g. SF_1BR-6, SF_2BR-3, SF_2CR-3, SF_3R-4 or e.g. SF_3R-4, SF_2BR-6, SF_2CR-3, SF_1BR-3

Total flow 30

ii max flow – min cut theorem

e.g. cut BR, F_2C, F_3C, F_3R

(accept BR, F_2C, SF_3)

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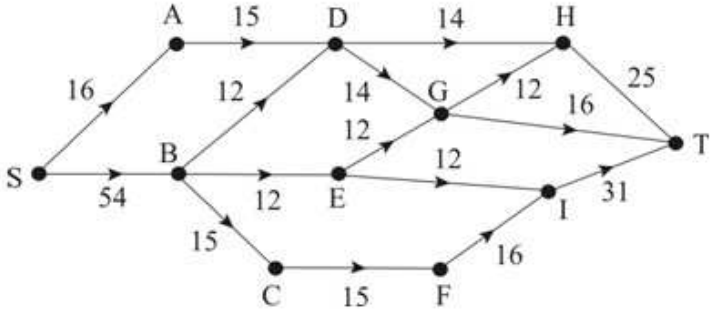
Exercise F, Question 5

Question:

The network represents a road system through a town. The number on each arc represents the maximum number of vehicles that can pass along that road every minute, i.e. the capacity of the road.



a State the maximum flow along
i S B C F I T,
ii S A D H T.

b Show these maximum flows on a diagram.

c Taking your answer to part **b** as the initial flow pattern, use the labelling procedure to find a maximum flow from S to T. List each flow-augmenting route you find, together with its flow.

d Indicate a maximum flow.

e Prove that your flow is maximal.

The council has funding to improve one of the roads to increase the flow from S to T. It can choose to increase the flow along one of BE, DH or CF.

f Making your reasoning clear, explain which one of these three roads the council should improve, given that it wishes to maximise the flow through the town.

[*E*]

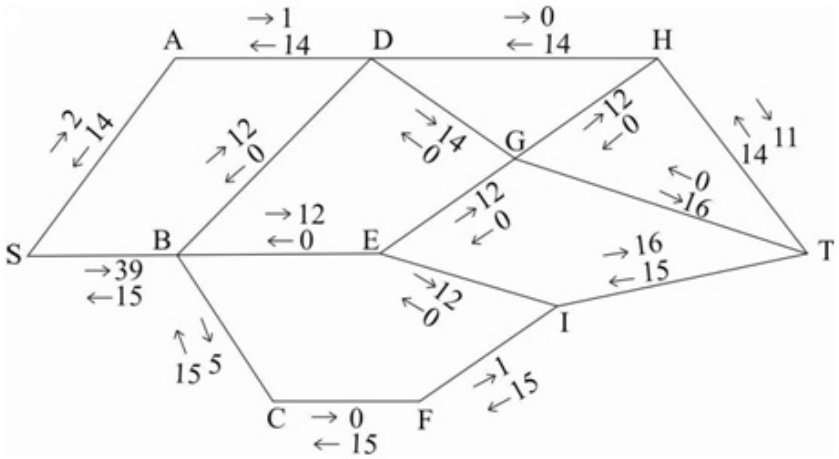
Solution:

a i Flow along S B C F I T = 15

ii Flow along S A D H T = 14

b A diagram showing the 2 flows correctly

c



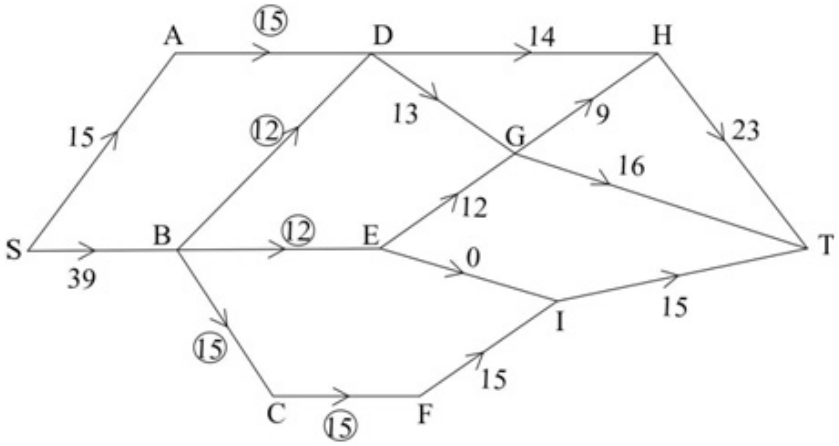
e.g. S A D G T − 1

S B D G T − 12

with S B E I T − 12 or S B E G H T − 9 and S B E G T − 3

giving a total flow of 54

d e.g.



e Max flow − min cut theorem, cut through AD, BD, BE and BC or CF

f The flow into D and into C could not increase,
so increase the flow along BE

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Exercise F, Question 6

Question:

Figure 1 shows a capacitated, directed network. The number on each arc indicates the capacity of that arc.

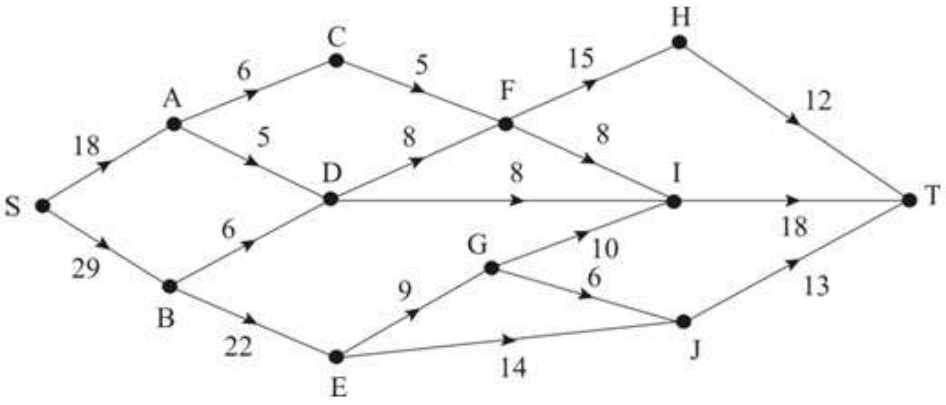


Figure 1

Figure 2 shows a feasible flow of value 29 through the same network.

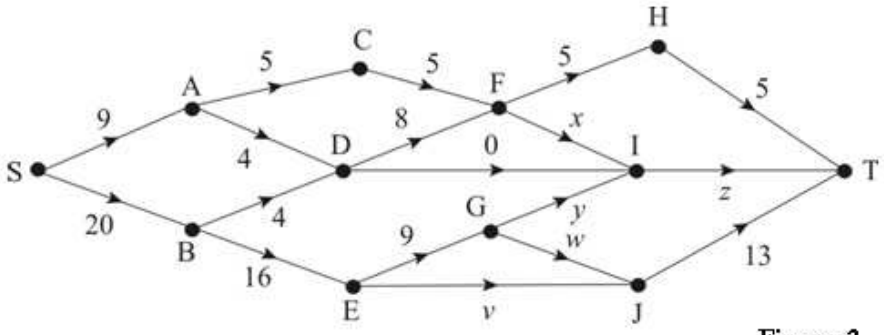


Figure 2

a Find the values of the flows v, w, x, y and z.

Start with the values in Figure 1 and your answers to part **a** as your initial flow pattern.

b Use the labelling procedure on Figure 1 to find the maximum flow through this network, listing each flow–augmenting route you use together with its flow.

c Show the maximum flow on Figure 2 and state its value.

d **i** Find the capacity of the cut which passes through the arcs HT, IT and JT.

ii Find the minimum cut, listing the arcs through which it passes.

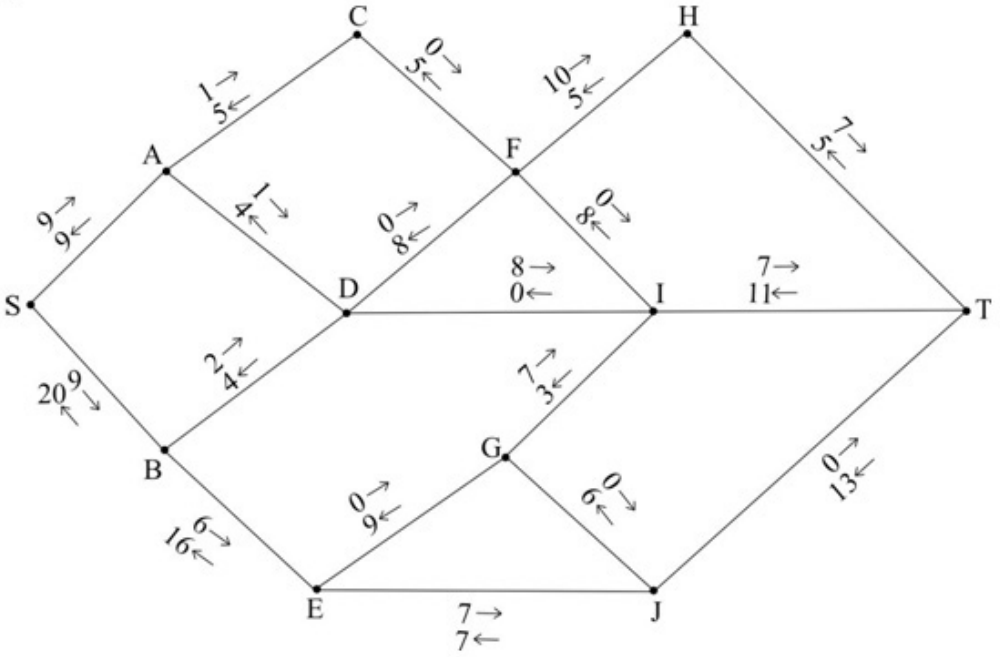
iii Explain why this proves that the flow in part **c** is a maximum.

[*E*]

Solution:

a $v=7$, $w=6$, $x=8$, $y=3$, $z=11$ (conservation of flow)

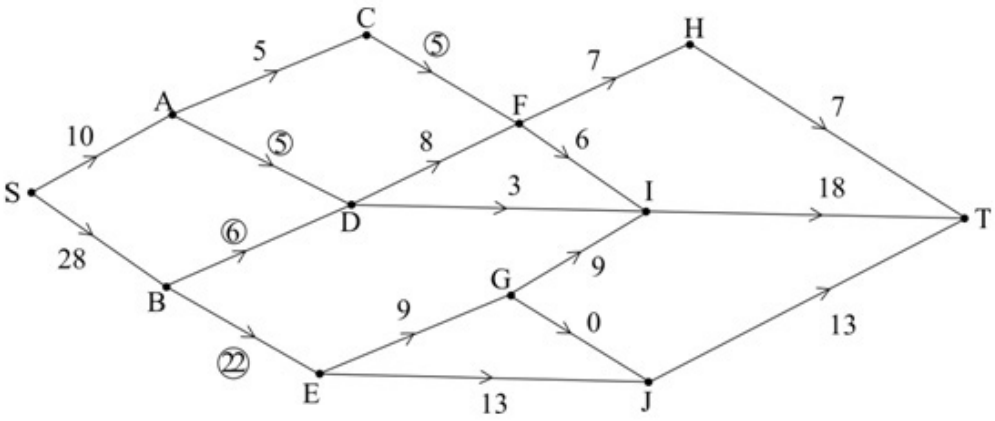
b



Increasing flow by an additional 3
e.g. S B D I T (2)
S A D I T (1)

Additional flow increase (reversing initial flow)
e.g. S B E J G I T (4)
S B E J G I F H T (2)
Flow up to maximum (38)

c E.g.



Complete, consistent flow ringed numbers (flow of 38)

d **i** $12+18+13=43$
ii CF, AD, BD, BE
iii Max flow – min cut theorem - e.g.
The minimum cut separates the source from the sink. Any additional flow must cross this cut at some point. Since all arcs in the minimum cut are saturated no additional flow can be transported along these arcs. Hence no additional flow is possible.

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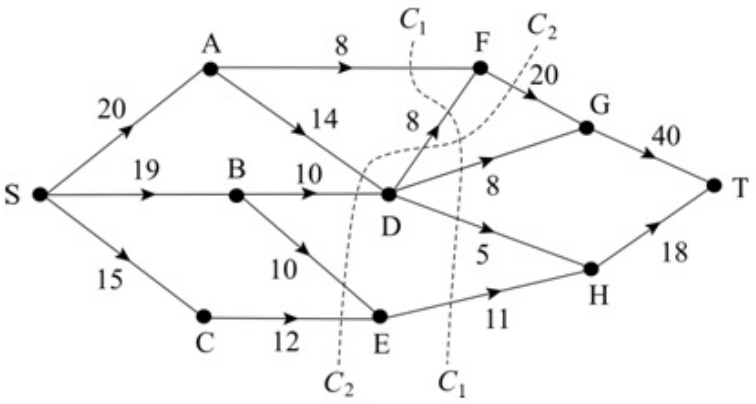
Exercise F, Question 7

Question:



The diagram shows a capacitated, directed network. The number on each arc indicates the capacity of that arc.

a Calculate the values of cuts C_1 and C_2.

Given that one of these cuts is a minimum cut,

b state the maximum flow.

c Deduce the flow along GT, making your reasoning clear.

d By considering the flow into D, deduce that there are only two possible integer values for the flow along SA.

e For each of the two values found in part **d**, draw a complete maximum flow pattern.

f Given that the flow along each arc must be an integer, determine the number of other maximum flow patterns. Give a reason for your answer.

[*E*]

Solution:

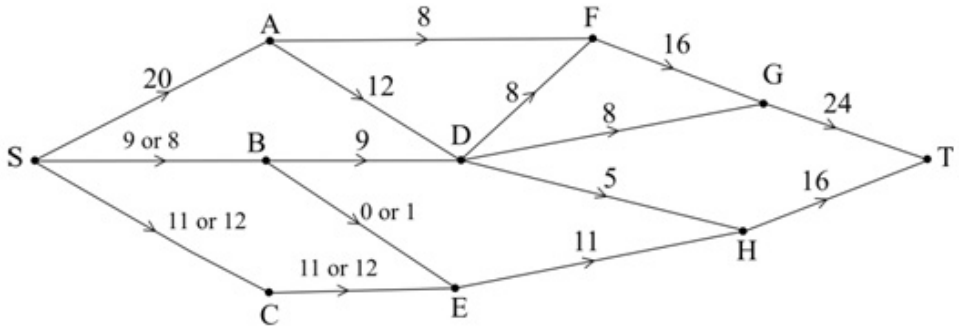
a $C_1 - 40 \quad C_2 - 56$

b max flow = min cut = 40

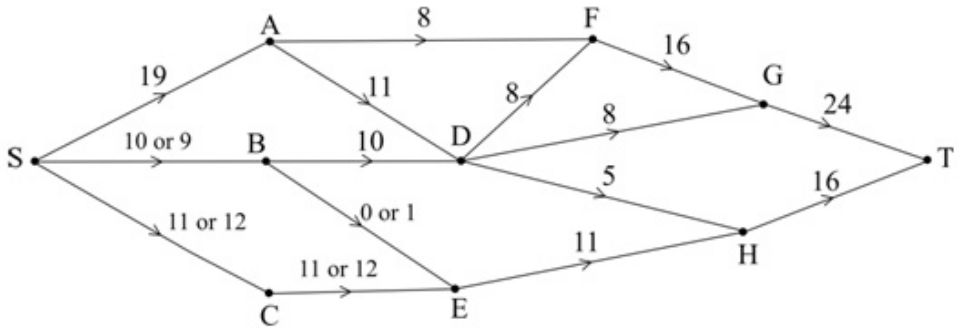
c e.g. Flow into F is 16 ∴ flow into G is 24. The flow along DG is 8 ∴ Flow along GT is 24

d e.g. Flow into A = flow out of A ∴ flow along AD ≤ 12
Flow into D = flow out of D = 21
So flow along AD + flow along BD = 21
∴ flow along AD and BD could be 12+9 or 11+10
∴ possible flows are 20 and 19

e SA = 20



SA = 19



f There are 2 more - CE could be 11 or 12 in each case, for example.

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