

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

The curve **C**, with equation $y = x^2 \ln x$, $x > 0$, has a stationary point P. Find, in terms of e , the coordinates of P. (7)

Solution:

$$y = x^2 \ln x, x > 0$$

Differentiate as a product:

$$\frac{dy}{dx} = x^2 \times \frac{1}{x} + 2x \ln x = x + 2x \ln x = x(1 + 2 \ln x)$$

$$\frac{dy}{dx} = 0 \Rightarrow 1 + 2 \ln x = 0 \text{ as } x > 0$$

$$\Rightarrow 2 \ln x = -1$$

$$\Rightarrow \ln x = -\frac{1}{2}$$

$$\Rightarrow x = e^{-\frac{1}{2}}$$

Substituting $x = e^{-\frac{1}{2}}$, in $y = x^2 \ln x$

$$\Rightarrow y = \left(e^{-\frac{1}{2}}\right)^2 \ln e^{-\frac{1}{2}} = -\frac{1}{2}e^{-1}$$

So coordinates are $\left(e^{-\frac{1}{2}}, -\frac{1}{2}e^{-1}\right)$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

$$f(x) = e^{2x-1}, x \geq 0$$

The curve **C** with equation $y = f(x)$ meets the y -axis at **P**.

The tangent to **C** at **P** crosses the x -axis at **Q**.

(a) Find, to 3 decimal places, the area of triangle **POQ**, where **O** is the origin. (5)

The line $y = 2$ intersects **C** at the point **R**.

(b) Find the exact value of the x -coordinate of **R**. (3)

Solution:

(a) **C** meets y -axis where $x = 0$

$$\Rightarrow y = e^{-1}$$

Find gradient of curve at **P**.

$$\frac{dy}{dx} = 2e^{2x-1}$$

$$\text{At } x = 0, \frac{dy}{dx} = 2e^{-1}$$

Equation of tangent is $y - e^{-1} = 2e^{-1}x$

This meets x -axis at **Q**, where $y = 0$

$$\Rightarrow Q \equiv \left(-\frac{1}{2}, 0 \right)$$

$$\text{Area of } \triangle POQ = \frac{1}{2} \times \frac{1}{2} \times e^{-1} = \frac{1}{4}e^{-1} = 0.092$$

(b) At **R**, $y = 2 \Rightarrow 2 = e^{2x-1}$

$$\Rightarrow 2x - 1 = \ln 2$$

$$\Rightarrow 2x = 1 + \ln 2$$

$$\Rightarrow x = \frac{1}{2} (1 + \ln 2)$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 3

Question:

$$f(x) = \frac{3x}{x+1} - \frac{x+7}{x^2-1}, x > 1$$

(a) Show that $f(x) = 3 - \frac{4}{x-1}, x > 1$. (5)

(b) Find $f^{-1}(x)$. (4)

(c) Write down the domain of $f^{-1}(x)$. (1)

Solution:

(a) $\frac{3x}{x+1} - \frac{x+7}{(x+1)(x-1)}, x > 1$

$$\equiv \frac{3x(x-1) - (x+7)}{(x+1)(x-1)}$$

$$\equiv \frac{3x^2 - 4x - 7}{(x+1)(x-1)}$$

$$\equiv \frac{(3x-7)(x+1)}{(x+1)(x-1)}$$

$$\equiv \frac{3x-7}{x-1}$$

$$\equiv \frac{3(x-1) - 4}{x-1}$$

$$\equiv 3 - \frac{4}{x-1}$$

(b) Let $y = 3 - \frac{4}{x-1}$

$$\Rightarrow \frac{4}{x-1} = 3 - y$$

$$\Rightarrow \frac{x-1}{4} = \frac{1}{3-y}$$

$$\Rightarrow x-1 = \frac{4}{3-y}$$

$$\Rightarrow x = 1 + \frac{4}{3-y} \text{ or } \frac{7-y}{3-y}$$

$$\text{So } f^{-1}(x) = 1 + \frac{4}{3-x} \text{ or } \frac{7-x}{3-x}$$

(c) Domain of $f^{-1}(x)$ is the range of $f(x)$.

$$x > 1 \Rightarrow \frac{4}{x-1} > 0 \Rightarrow f(x) = 3 - \frac{4}{x-1} < 3$$

So the domain of $f^{-1}(x)$ is $x < 3$

© Pearson Education Ltd 2008

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 4

Question:

(a) Sketch, on the same set of axes, for $x > 0$, the graphs of

$$y = -1 + \ln 3x \text{ and } y = \frac{1}{x} \quad (2)$$

The curves intersect at the point P whose x -coordinate is p .
Show that

(b) p satisfies the equation

$$p \ln 3p - p - 1 = 0 \quad (1)$$

(c) $1 < p < 2$ (2)

The iterative formula

$$x_{n+1} = \frac{1}{3}e \left(1 + \frac{1}{x_n} \right), x_0 = 2$$

is used to find an approximation for p .

(d) Write down the values of x_1, x_2, x_3 and x_4 giving your answers to 4 significant figures. (3)

(e) Prove that $p = 1.66$ correct to 3 significant figures. (2)

Solution:

(a)

(b) At P, $-1 + \ln 3p = \frac{1}{p}$

$$\Rightarrow -p + p \ln 3p = 1$$

$$\Rightarrow p \ln 3p - p - 1 = 0$$

(c) Let $f(p) \equiv p \ln 3p - p - 1$

$$f(1) = \ln 3 - 2 = -0.901\dots$$

$$f(2) = 2 \ln 6 - 3 = +0.5835\dots$$

Sign change implies root between 1 and 2, so $1 < p < 2$.

(d) $x_{n+1} = \frac{1}{3}e \left(1 + \frac{1}{x_n} \right), x_0 = 2$

$$x_1 = \frac{1}{3}e^{\frac{3}{2}} = 1.494 \text{ (4 s.f.)}$$

$$x_2 = 1.770 \text{ (4 s.f.)}$$

$$x_3 = 1.594 \text{ (4 s.f.)}$$

$$x_4 = 1.697 \text{ (4 s.f.)}$$

$$\text{(e) } f(1.665) = +0.013$$

$$f(1.655) = -0.003$$

\Rightarrow root between 1.655 and 1.665

$$\text{So } p = 1.66 \text{ (3 s.f.)}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 5

Question:

The curve C_1 has equation

$$y = \cos 2x - 2 \sin^2 x$$

The curve C_2 has equation

$$y = \sin 2x$$

(a) Show that the x -coordinates of the points of intersection of C_1 and C_2 satisfy the equation

$$2 \cos 2x - \sin 2x = 1 \quad (3)$$

(b) Express $2 \cos 2x - \sin 2x$ in the form $R \cos (2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving the exact value of R and giving α in radians to 3 decimal places. (4)

(c) Find the x -coordinates of the points of intersection of C_1 and C_2 in the interval $0 \leq x < \pi$, giving your answers in radians to 2 decimal places. (5)

Solution:

(a) Where C_1 and C_2 meet

$$\cos 2x - 2 \sin^2 x = \sin 2x$$

$$\text{Using } \cos 2x \equiv 1 - 2 \sin^2 x \quad \Rightarrow \quad -2 \sin^2 x \equiv \cos 2x - 1$$

$$\text{So } \cos 2x + (\cos 2x - 1) = \sin 2x$$

$$\Rightarrow 2 \cos 2x - \sin 2x = 1$$

(b) Let $2 \cos 2x - \sin 2x \equiv R \cos (2x + \alpha)$

$$\equiv R \cos 2x \cos \alpha - R \sin 2x \sin \alpha$$

$$\text{Compare: } R \cos \alpha = 2, R \sin \alpha = 1$$

$$\text{Divide: } \tan \alpha = \frac{1}{2} \quad \Rightarrow \quad \alpha = 0.464 \text{ (3 d.p.)}$$

$$\text{Square and add: } R^2 (\cos^2 \alpha + \sin^2 \alpha) = 2^2 + 1^2 = 5$$

$$\Rightarrow R = \sqrt{5}$$

$$\text{So } 2 \cos 2x - \sin 2x \equiv \sqrt{5} \cos (2x + 0.464)$$

(c) $2 \cos 2x - \sin 2x = 1$

$$\Rightarrow \sqrt{5} \cos (2x + 0.464) = 1$$

$$\Rightarrow \cos \left(2x + 0.464 \right) = \frac{1}{\sqrt{5}}$$

$$\Rightarrow 2x + 0.464 = 1.107, 5.176 \quad 0.464 \leq 2x + 0.464 < 6.747$$

$$\Rightarrow 2x = 0.643, 4.712$$

$$\Rightarrow x = 0.32, 2.36 \text{ (2 d.p.)}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 6

Question:

(a) Given that $y = \ln \sec x$, $-\frac{\pi}{2} < x \leq 0$, use the substitution $u = \sec x$, or otherwise, to show that $\frac{dy}{dx} = \tan x$. (3)

The curve **C** has equation $y = \tan x + \ln \sec x$, $-\frac{\pi}{2} < x \leq 0$.

At the point **P** on **C**, whose x -coordinate is p , the gradient is 3.

(b) Show that $\tan p = -2$. (6)

(c) Find the exact value of $\sec p$, showing your working clearly. (2)

(d) Find the y -coordinate of **P**, in the form $a + k \ln b$, where a , k and b are rational numbers. (2)

Solution:

(a) $y = \ln \sec x$

Let $u = \sec x \Rightarrow \frac{du}{dx} = \sec x \tan x$

so $y = \ln u \Rightarrow \frac{dy}{du} = \frac{1}{u}$

Using $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = \frac{1}{u} \times \sec x \tan x = \frac{1}{\sec x} \sec x \tan x = \tan x$

(b) $\frac{dy}{dx} = \sec^2 x + \tan x$

So $\sec^2 p + \tan p = 3$

$$\Rightarrow 1 + \tan^2 p + \tan p = 3$$

$$\Rightarrow \tan^2 p + \tan p - 2 = 0$$

$$\Rightarrow (\tan p - 1)(\tan p + 2) = 0$$

As $-\frac{\pi}{2} < x \leq 0$ (4th quadrant), $\tan p$ is negative

So $\tan p = -2$

$$(c) \sec^2 p = 1 + \tan^2 p = 5 \Rightarrow \sec p = + \sqrt{5} \quad (4\text{th quadrant})$$

$$(d) y = \ln \sqrt{5} + (-2) = -2 + \frac{1}{2} \ln 5$$

© Pearson Education Ltd 2008

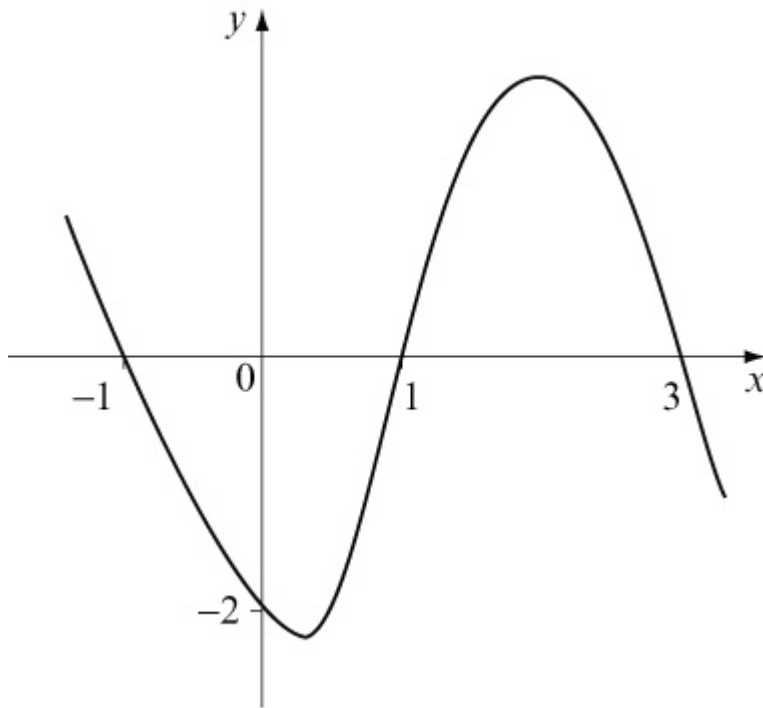
Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 7

Question:

The diagram shows a sketch of part of the curve with equation $y = f(x)$. The curve has no further turning points.



On separate diagrams show a sketch of the curve with equation

(a) $y = 2f(-x)$ (3)

(b) $y = |f(2x)|$ (3)

In each case show the coordinates of points in which the curve meets the coordinate axes.

The function g is given by

$$g : x \rightarrow |x + 1| - k, x \in \mathbb{R}, k > 1$$

(c) Sketch the graph of g , showing, in terms of k , the y -coordinate of the point of intersection of the graph with the y -axis. (3)

Find, in terms of k ,

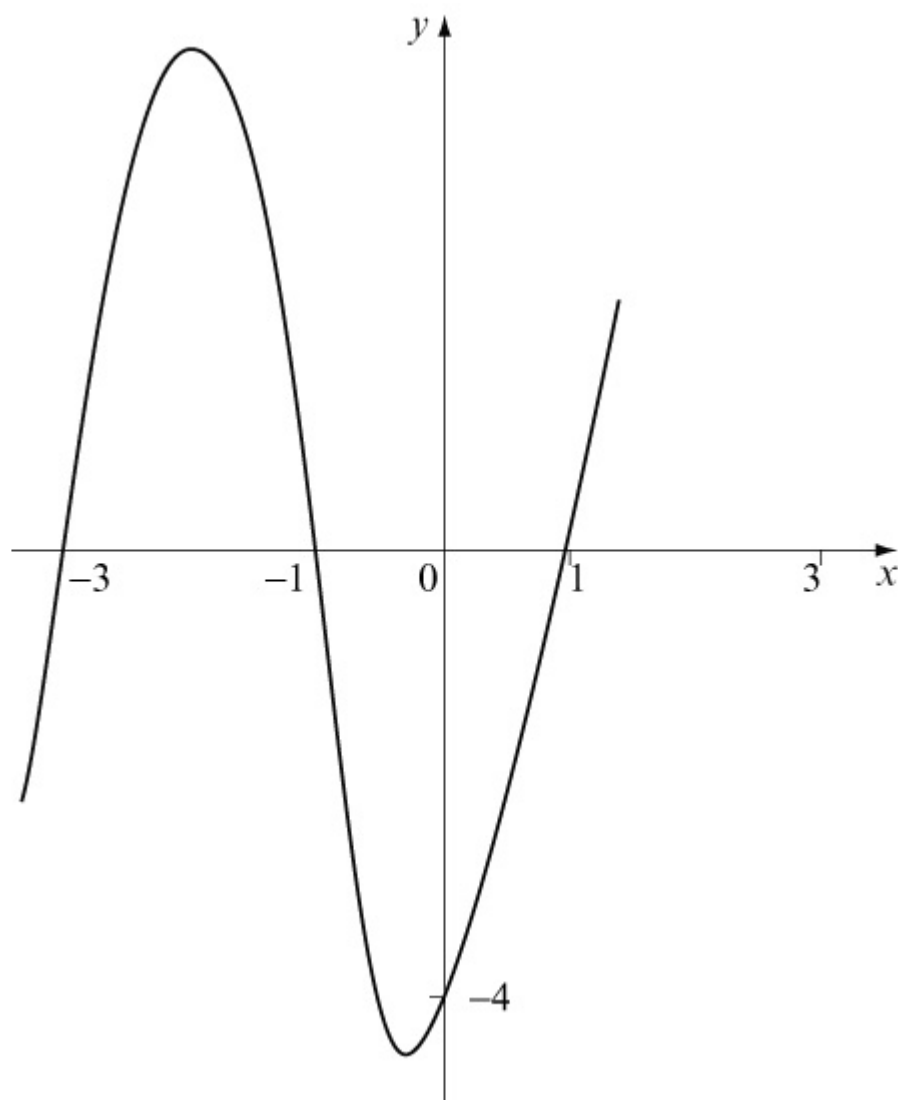
(d) the range of $g(x)$ (1)

(e) $gf(0)$ (2)

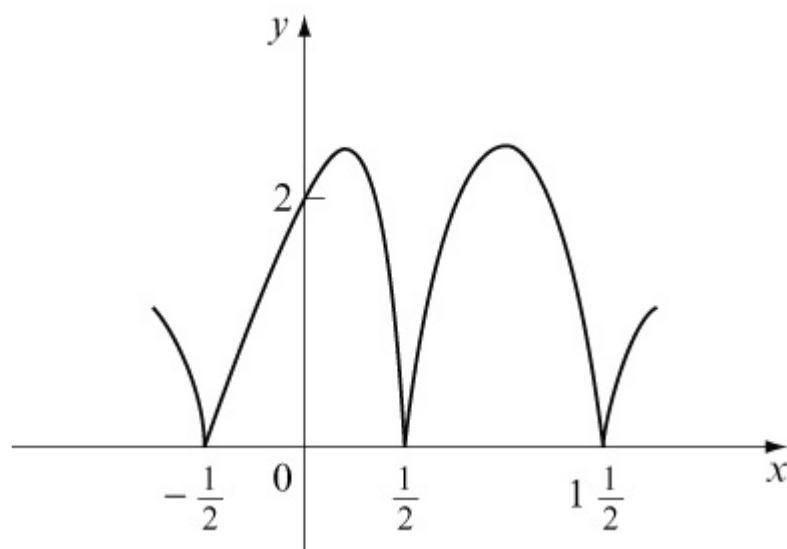
(f) the solution of $g(x) = x(3)$

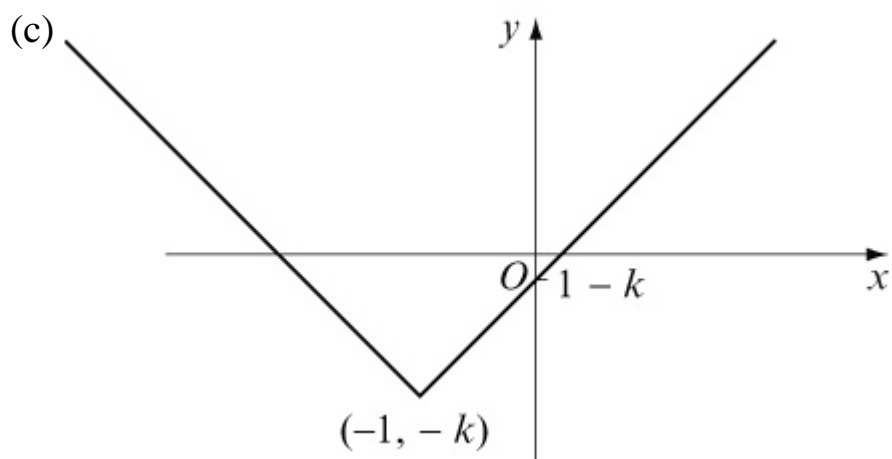
Solution:

(a)



(b)





(d) $g(x) \geq -k$

(e) $gf(0) = g(-2) = |-1| - k = 1 - k$

(f) $y = x$ meets $y = |x + 1| - k$

where $x = -(x + 1) - k$

$$\Rightarrow 2x = -(1 + k)$$

$$\Rightarrow x = -\frac{1+k}{2}$$