Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Show that each of these equations $f(x) = 0$ has a root in the given interval(s):

(a) $x^3 - x + 5 = 0 \qquad -2 < x < -1.$

(b) $3 + x^2 - x^3 = 0 \qquad 1 < x < 2.$

(c) $x^2 - \sqrt{x} - 10 = 0 \qquad 3 < x < 4.$

(d) $x^3 - \frac{1}{x} - 2 = 0 \qquad -0.5 < x < -0.2$ and $1 < x < 2.$

(e) $x^5 - 5x^3 - 10 = 0 \qquad -2 < x < -1.8,\ -1.8 < x < -1$ and $2 < x < 3.$

(f) $\sin x - \ln x = 0 \qquad 2.2 < x < 2.3$

(g) $e^x - \ln x - 5 = 0 \qquad 1.65 < x < 1.75.$

(h) $\sqrt[3]{x} - \cos x = 0 \qquad 0.5 < x < 0.6.$

Solution:

(a) Let $f(x) = x^3 - x + 5$

$f(-2) = (-2)^3 - (-2) + 5 = -8 + 2 + 5 = -1$

$f(-1) = (-1)^3 - (-1) + 5 = -1 + 1 + 5 = 5$

$f(-2) < 0$ and $f(-1) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = -2$ and $x = -1$.

(b) Let $f(x) = 3 + x^2 - x^3$

$f(1) = 3 + (1)^2 - (1)^3 = 3 + 1 - 1 = 3$

$f(2) = 3 + (2)^2 - (2)^3 = 3 + 4 - 8 = -1$

$f(1) > 0$ and $f(2) < 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 1$ and $x = 2$.

(c) Let $f(x) = x^2 - \sqrt{x} - 10$

$f(3) = 3^2 - \sqrt{3} - 10 = -2.73$

$f(4) = 4^2 - \sqrt{4} - 10 = 4$

$f(3) < 0$ and $f(4) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 3$ and $x = 4$.

(d) Let $f(x) = x^3 - \frac{1}{x} - 2$

[1] $f(-0.5) = (-0.5)^3 - \frac{1}{-0.5} - 2 = -0.125$

$f(-0.2) = (-0.2)^3 - \frac{1}{-0.2} - 2 = 2.992$

$f(-0.5) < 0$ and $f(-0.2) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = -0.5$ and $x = -0.2$.

[2] $f(1) = (1)^3 - \frac{1}{1} - 2 = -2$

$f(2) = (2)^3 - \frac{1}{2} - 2 = 5\frac{1}{2}$

$f(1) < 0$ and $f(2) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 1$ and $x = 2$.

(e) Let $f(x) = x^5 - 5x^3 - 10$

[1] $f(-2) = (-2)^5 - 5(-2)^3 - 10 = -2$

$f(-1.8) = (-1.8)^5 - 5(-1.8)^3 - 10 = 0.26432$

$f(-2) < 0$ and $f(-1.8) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = -2$ and $x = -1.8$.

[2] $f(-1.8) = 0.26432$

$f(-1) = (-1)^5 - 5(-1)^3 - 10 = -6$

$f(-1.8) > 0$ and $f(-1) < 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = -1.8$ and $x = -1$.

[3] $f(2) = (2)^5 - 5(2)^3 - 10 = -18$

$f(3) = (3)^5 - 5(3)^3 - 10 = 98$

$f(2) < 0$ and $f(3) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 2$ and $x = 3$.

(f) Let $f(x) = \sin x - \ln x$

$f(2.2) = \sin 2.2 - \ln 2.2 = 0.0200$

$f(2.3) = -0.0872$

$f(2.2) > 0$ and $f(2.3) < 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 2.2$ and $x = 2.3$.

(g) Let $f(x) = e^x - \ln x - 5$

$f(1.65) = e^{1.65} - \ln 1.65 - 5 = -0.294$

$f(1.75) = e^{1.75} - \ln 1.75 - 5 = 0.195$

$f(1.65) < 0$ and $f(1.75) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 1.65$ and $x = 1.75$.

(h) Let $f(x) = \sqrt[3]{x} - \cos x$

$f(0.5) = \sqrt[3]{0.5} - \cos 0.5 = -0.0839$

$f(0.6) = \sqrt[3]{0.6} - \cos 0.6 = 0.0181$

$f(0.5) < 0$ and $f(0.6) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 0.5$ and $x = 0.6$.

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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

Given that $f(x) = x^3 - 5x^2 + 2$, show that the equation $f(x) = 0$ has a root near to $x = 5$.

Solution:

Let $f(x) = x^3 - 5x^2 + 2$

$f(4.9) = (4.9)^3 - 5(4.9)^2 + 2 = -0.401$

$f(5.0) = 2$

$f(4.9) < 0$ and $f(5) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 4.9$ and $x = 5$.

Solutionbank

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Exercise A, Question 3

Question:

Given that $f(x) \equiv 3 - 5x + x^3$, show that the equation $f(x) = 0$ has a root $x = a$, where a lies in the interval $1 < a < 2$.

Solution:

Let $f(x) = 3 - 5x + x^3$

$f(1) = 3 - 5(1) + (1)^3 = -1$

$f(2) = 3 - 5(2) + (2)^3 = 1$

$f(1) < 0$ and $f(2) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 1$ and $x = 2$.

So if the root is $x = a$, then $1 < a < 2$.

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Exercise A, Question 4

Question:

Given that $f(x) \equiv e^x \sin x - 1$, show that the equation $f(x) = 0$ has a root $x = r$, where r lies in the interval $0.5 < r < 0.6$.

Solution:

$f(x) = e^x \sin x - 1$
$f(0.5) = e^{0.5} \sin 0.5 - 1 = -0.210$
$f(0.6) = e^{0.6} \sin 0.6 - 1 = 0.0288$
$f(0.5) < 0$ and $f(0.6) > 0$ so there is a change of sign.
$\Rightarrow$ There is a root between $x = 0.5$ and $x = 0.6$.
So if the root is $x = r$, then $0.5 < r < 0.6$.

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Exercise A, Question 5

Question:

It is given that $f(x) \equiv x^3 - 7x + 5$.

(a) Copy and complete the table below.

x	-3	-2	-1	0	1	2	3
$f(x)$							

(b) Given that the negative root of the equation $x^3 - 7x + 5 = 0$ lies between α and $\alpha + 1$, where α is an integer, write down the value of α.

Solution:

(a)

x	-3	-2	-1	0	1	2	3
$f(x)$	-1	11	11	5	-1	-1	11

(b) $f(-3) < 0$ and $f(-2) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = -3$ and $x = -2$.

So $\alpha = -3$. (**Note.** $\alpha + 1 = -2$).

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Exercise A, Question 6

Question:

Given that $f(x) \equiv x - (\sin x + \cos x)^{\frac{1}{2}}$, $0 \leq x \leq \frac{3}{4}\pi$, show that the equation $f(x) = 0$ has a root lying between $\frac{\pi}{3}$ and $\frac{\pi}{2}$.

Solution:

$f(x) = x - (\sin x + \cos x)^{\frac{1}{2}}$

$f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - \left(\sin\frac{\pi}{3} + \cos\frac{\pi}{3}\right)^{\frac{1}{2}} = -0.122$

$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} - \left(\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right)^{\frac{1}{2}} = 0.571$

$f\left(\frac{\pi}{3}\right) < 0$ and $f\left(\frac{\pi}{2}\right) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = \frac{\pi}{3}$ and $x = \frac{\pi}{2}$.

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Exercise A, Question 7

Question:

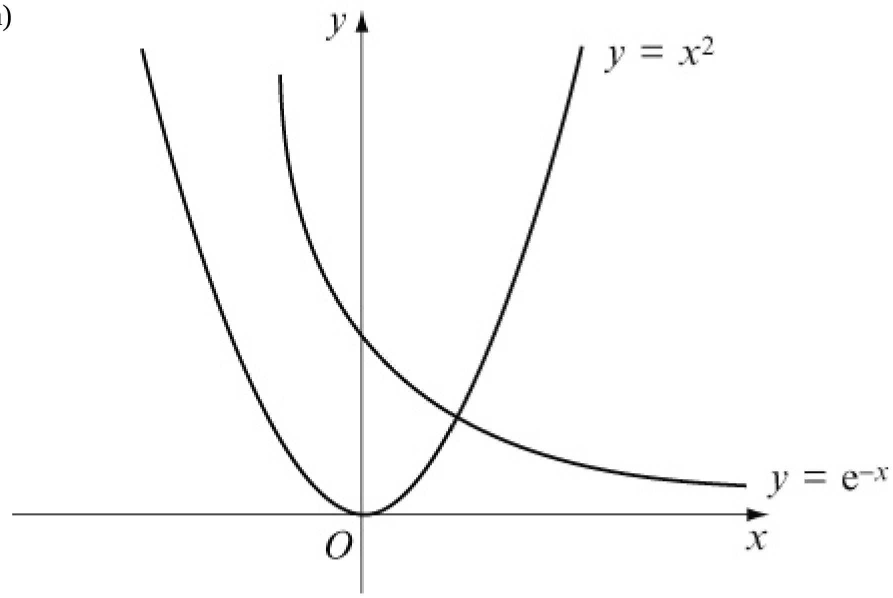
(a) Using the same axes, sketch the graphs of $y = e^{-x}$ and $y = x^2$.

(b) Explain why the equation $e^{-x} = x^2$ has only one root.

(c) Show that the equation $e^{-x} = x^2$ has a root between $x = 0.70$ and $x = 0.71$.

Solution:

(a)



(b) The curves meet where $e^{-x} = x^2$

The curves meet at one point, so there is one value of x that satisfies the equation $e^{-x} = x^2$.

So $e^{-x} = x^2$ has one root.

(c) Let $f(x) = e^{-x} - x^2$

$f(0.70) = e^{-0.70} - 0.70^2 = 0.00659$

$f(0.71) = e^{-0.71} - 0.71^2 = -0.0125$

$f(0.70) > 0$ and $f(0.71) < 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 0.70$ and $x = 0.71$.

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Exercise A, Question 8

Question:

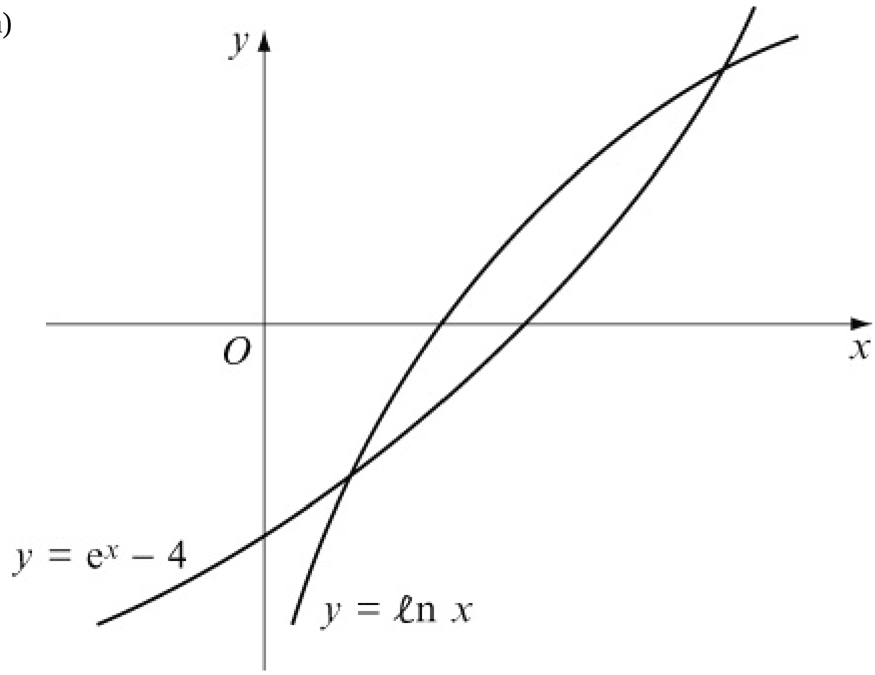
(a) On the same axes, sketch the graphs of $y = \ln x$ and $y = e^x - 4$.

(b) Write down the number of roots of the equation $\ln x = e^x - 4$.

(c) Show that the equation $\ln x = e^x - 4$ has a root in the interval (1.4, 1.5).

Solution:

(a)



(b) The curves meet at two points, so there are two values of x that satisfy the equation $\ln x = e^x - 4$.
So $\ln x = e^x - 4$ has two roots.

(c) Let $f(x) = \ln x - e^x + 4$
$f(1.4) = \ln 1.4 - e^{1.4} + 4 = 0.281$
$f(1.5) = \ln 1.5 - e^{1.5} + 4 = -0.0762$
$f(1.4) > 0$ and $f(1.5) < 0$ so there is a change of sign.
$\Rightarrow$ There is a root between $x = 1.4$ and $x = 1.5$.

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Exercise A, Question 9

Question:

(a) On the same axes, sketch the graphs of $y = \sqrt{x}$ and $y = \frac{2}{x}$.

(b) Using your sketch, write down the number of roots of the equation $\sqrt{x} = \frac{2}{x}$.

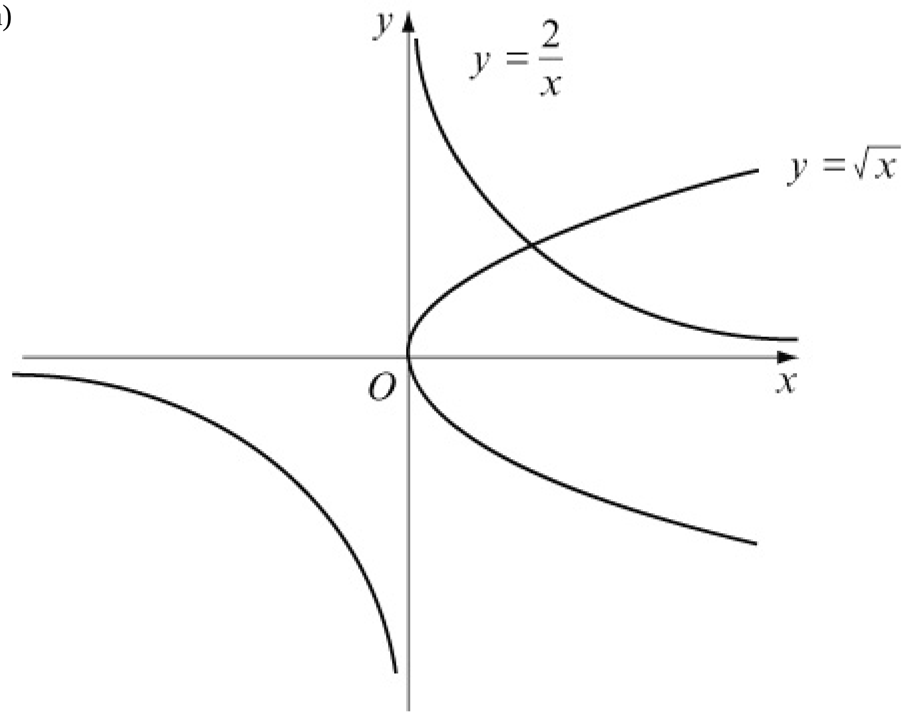
(c) Given that $f(x) \equiv \sqrt{x} - \frac{2}{x}$, show that $f(x) = 0$ has a root r, where r lies between $x = 1$ and $x = 2$.

(d) Show that the equation $\sqrt{x} = \frac{2}{x}$ may be written in the form $x^p = q$, where p and q are integers to be found.

(e) Hence write down the exact value of the root of the equation $\sqrt{x} - \frac{2}{x} = 0$.

Solution:

(a)



(b) The curves meet at one point, so there is one value of x that satisfies the equation $\sqrt{x} = \frac{2}{x}$.

So $\sqrt{x} = \frac{2}{x}$ has **one** root.

(c) $f(x) = \sqrt{x} - \frac{2}{x}$

$f(1) = \sqrt{1} - \frac{2}{1} = -1$

$f(2) = \sqrt{2} - \frac{2}{2} = 0.414$

$f(1) < 0$ and $f(2) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 1$ and $x = 2$.

(d) $\sqrt{x} = \frac{2}{x}$

$x^{\frac{1}{2}} = \frac{2}{x}$

$x^{\frac{1}{2}} \times x = 2$

$x^{\frac{1}{2} + 1} = 2$

$x^{\frac{3}{2}} = 2$

$\left(x^{\frac{3}{2}}\right)^2 = 2^2$

$x^3 = 4$

So $p = 3$ and $q = 4$

(e) $x^{\frac{3}{2}} = 2$

$\Rightarrow$ $x = 2^{\frac{2}{3}}$ $\left[= \left(2^2\right)^{\frac{1}{3}} = 4^{\frac{1}{3}} \right]$

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Exercise A, Question 10

Question:

(a) On the same axes, sketch the graphs of $y = \frac{1}{x}$ and $y = x + 3$.

(b) Write down the number of roots of the equation $\frac{1}{x} = x + 3$.

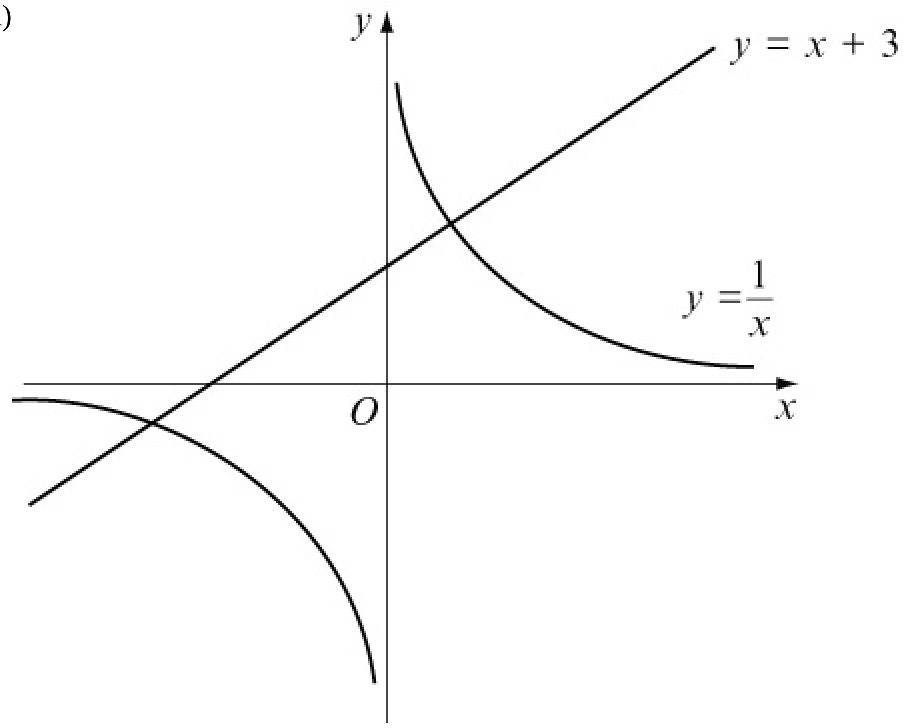
(c) Show that the positive root of the equation $\frac{1}{x} = x + 3$ lies in the interval (0.30, 0.31).

(d) Show that the equation $\frac{1}{x} = x + 3$ may be written in the form $x^2 + 3x - 1 = 0$.

(e) Use the quadratic formula to find the positive root of the equation $x^2 + 3x - 1 = 0$ to 3 decimal places.

Solution:

(a)



(b) The line meets the curve at two points, so there are two values of x that satisfy the equation $\frac{1}{x} = x + 3$.

So $\frac{1}{x} = x + 3$ has **two** roots.

(c) Let $f(x) = \frac{1}{x} - x - 3$

$f(0.30) = \frac{1}{0.30} - (0.30) - 3 = 0.0333$

$f(0.31) = \frac{1}{0.31} - (0.31) - 3 = -0.0842$

$f(0.30) > 0$ and $f(0.31) < 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 0.30$ and $x = 0.31$.

(d) $\frac{1}{x} = x + 3$

$\frac{1}{x} \times x = x \times x + 3 \times x \quad (\times x)$

$1 = x^2 + 3x$

So $x^2 + 3x - 1 = 0$

(e) Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 1$, $b = 3$, $c = -1$

$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-1)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 4}}{2} = \frac{-3 \pm \sqrt{13}}{2}$

So $x = \frac{-3 + \sqrt{13}}{2} = 0.303$

and $x = \frac{-3 - \sqrt{13}}{2} = -3.303$

The positive root is 0.303 to 3 decimal places.

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 1

Question:

Show that $x^2 - 6x + 2 = 0$ can be written in the form:

(a) $x = \frac{x^2 + 2}{6}$

(b) $x = \sqrt{6x - 2}$

(c) $x = 6 - \frac{2}{x}$

Solution:

(a) $x^2 - 6x + 2 = 0$

$6x = x^2 + 2$ Add $6x$ to each side

$x = \frac{x^2 + 2}{6}$ Divide each side by 6

(b) $x^2 - 6x + 2 = 0$

$x^2 + 2 = 6x$ Add $6x$ to each side

$x^2 = 6x - 2$ Subtract 2 from each side

$x = \sqrt{6x - 2}$ Take the square root of each side

(c) $x^2 - 6x + 2 = 0$

$x^2 + 2 = 6x$ Add $6x$ to each side

$x^2 = 6x - 2$ Subtract 2 from each side

$\frac{x^2}{x} = \frac{6x}{x} - \frac{2}{x}$ Divide each term by x

$x = 6 - \frac{2}{x}$ Simplify

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 2

Question:

Show that $x^3 + 5x^2 - 2 = 0$ can be written in the form:

(a) $x = \sqrt[3]{2 - 5x^2}$

(b) $x = \frac{2}{x^2} - 5$

(c) $x = \sqrt{\frac{2 - x^3}{5}}$

Solution:

(a) $x^3 + 5x^2 - 2 = 0$

$x^3 + 5x^2 = 2$ Add 2 to each side

$x^3 = 2 - 5x^2$ Subtract $5x^2$ from each side

$x = \sqrt[3]{2 - 5x^2}$ Take the cube root of each side

(b) $x^3 + 5x^2 - 2 = 0$

$x^3 + 5x^2 = 2$ Add 2 to each side

$x^3 = 2 - 5x^2$ Subtract $5x^2$ from each side

$\frac{x^3}{x^2} = \frac{2}{x^2} - \frac{5x^2}{x^2}$ Divide each term by x^2

$x = \frac{2}{x^2} - 5$ Simplify

(c) $x^3 + 5x^2 - 2 = 0$

$x^3 + 5x^2 = 2$ Add 2 to each side

$5x^2 = 2 - x^3$ Subtract x^3 from each side

$x^2 = \frac{2 - x^3}{5}$ Divide each side by 5

$x = \sqrt{\frac{2 - x^3}{5}}$ Take the square root of each side

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 3

Question:

Rearrange $x^3 - 3x + 4 = 0$ into the form $x = \frac{x^3}{3} + a$, where the value of a is to be found.

Solution:

$x^3 - 3x + 4 = 0$

$3x = x^3 + 4$ Add $3x$ to each side

$\frac{3x}{3} = \frac{x^3}{3} + \frac{4}{3}$ Divide each term by 3

$x = \frac{x^3}{3} + \frac{4}{3}$ Simplify

So $a = \frac{4}{3}$

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Exercise B, Question 4

Question:

Rearrange $x^4 - 3x^3 - 6 = 0$ into the form $x = \sqrt[3]{px^4 - 2}$, where the value of p is to be found.

Solution:

$x^4 - 3x^3 - 6 = 0$

$3x^3 = x^4 - 6$ Add $3x^3$ to each side

$\frac{3x^3}{3} = \frac{x^4}{3} - \frac{6}{3}$ Divide each term by 3

$x^3 = \frac{x^4}{3} - 2$ Simplify

$x = \sqrt[3]{\frac{x^4}{3} - 2}$ Take the cube root of each side

So $p = \frac{1}{3}$

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 5

Question:

(a) Show that the equation $x^3 - x^2 + 7 = 0$ can be written in the form $x = \sqrt[3]{x^2 - 7}$.

(b) Use the iteration formula $x_{n+1} = \sqrt[3]{x_n^2 - 7}$, starting with $x_0 = 1$, to find x_2 to 1 decimal place.

Solution:

(a) $x^3 - x^2 + 7 = 0$

$x^3 + 7 = x^2$ Add x^2 to each side

$x^3 = x^2 - 7$ Subtract 7 from each side

$x = \sqrt[3]{x^2 - 7}$ Take the cube root of each side

(b) $x_0 = 1$

$x_1 = \sqrt[3]{(1)^2 - 7} = -1.817...$

$x_2 = \sqrt[3]{(-1.817...)^2 - 7} = -1.546...$

So $x_2 = -1.5$ (1 d.p.)

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 6

Question:

(a) Show that the equation $x^3 + 3x^2 - 5 = 0$ can be written in the form $x = \sqrt{\frac{5}{x + 3}}$.

(b) Use the iteration formula $x_{n+1} = \sqrt{\frac{5}{x_n + 3}}$, starting with $x_0 = 1$, to find x_4 to 3 decimal places.

Solution:

(a) $x^3 + 3x^2 - 5 = 0$
$x^2 (x + 3) - 5 = 0$ Factorise x^2
$x^2 (x + 3) = 5$ Add 5 to each side
$x^2 = \frac{5}{x + 3}$ Divide each side by $(x + 3)$
$x = \sqrt{\frac{5}{x + 3}}$ Take the square root of each side

(b) $x_0 = 1$
$x_1 = \sqrt{\frac{5}{(1) + 3}} = 1.118...$
$x_2 = \sqrt{\frac{5}{(1.118...) + 3}} = 1.101...$
$x_3 = \sqrt{\frac{5}{(1.101...) + 3}} = 1.104...$
$x_4 = \sqrt{\frac{5}{(1.104...) + 3}} = 1.103768...$
So $x_4 = 1.104$ (3 d.p.)

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 7

Question:

(a) Show that the equation $x^6 - 5x + 3 = 0$ has a root between $x = 1$ and $x = 1.5$.

(b) Use the iteration formula $x_{n+1} = \sqrt[5]{5 - \frac{3}{x_n}}$ to find an approximation for the root of the equation $x^6 - 5x + 3 = 0$, giving your answer to 2 decimal places.

Solution:

(a) Let $f(x) = x^6 - 5x + 3$

$f(1) = (1)^6 - 5(1) + 3 = -1$

$f(1.5) = (1.5)^6 - 5(1.5) + 3 = 6.89$

$f(1) < 0$ and $f(1.5) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 1$ and $x = 1.5$.

(b) $x_0 = 1$

$x_1 = \sqrt[5]{5 - \frac{3}{1}} = 1.148...$

Similarly,

$x_2 = 1.190...$

$x_3 = 1.199...$

$x_4 = 1.200...$

$x_5 = 1.201...$

So the root is 1.20 (2 d.p.)

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 8

Question:

(a) Rearrange the equation $x^2 - 6x + 1 = 0$ into the form $x = p - \frac{1}{x}$, where p is a constant to be found.

(b) Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = p - \frac{1}{x_n}$ with your value of p, to find x_3 to 2 decimal places.

Solution:

(a) $x^2 - 6x + 1 = 0$

$x^2 + 1 = 6x$ Add $6x$ to each side

$x^2 = 6x - 1$ Subtract 1 from each side

$\frac{x^2}{x} = \frac{6x}{x} - \frac{1}{x}$ Divide each term by x

$x = 6 - \frac{1}{x}$ Simplify

So $p = 6$

(b) $x_0 = 3$

$x_1 = 6 - \frac{1}{3} = 5.666...$

$x_2 = 6 - \frac{1}{5.666...} = 5.823...$

$x_3 = 6 - \frac{1}{5.823...} = 5.828...$

So $x_3 = 5.83$ (2 d.p.)

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 9

Question:

(a) Show that the equation $x^3 - x^2 + 8 = 0$ has a root in the interval $(-2, -1)$.

(b) Use a suitable iteration formula to find an approximation to 2 decimal places for the negative root of the equation $x^3 - x^2 + 8 = 0$.

Solution:

(a) Let $f(x) = x^3 - x^2 + 8$

$f(-2) = (-2)^3 - (-2)^2 + 8 = -8 - 4 + 8 = -4$

$f(-1) = (-1)^3 - (-1)^2 + 8 = -1 - 1 + 8 = 6$

$f(-2) < 0$ and $f(-1) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = -2$ and $x = -1$.

(b) $x^3 - x^2 + 8 = 0$

$x^3 + 8 = x^2$ Add x^2 to each side

$x^3 = x^2 - 8$ Subtract 8 from each side

$x = \sqrt[3]{x^2 - 8}$ Take the cube root of each side

Using $x_{n+1} = \sqrt[3]{x_{n}^2 - 8}$ and any value for x_0, the root is -1.72 (2 d.p.).

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Exercise B, Question 10

Question:

(a) Show that $x^7 - 5x^2 - 20 = 0$ has a root in the interval (1.6, 1.7).

(b) Use a suitable iteration formula to find an approximation to 3 decimal places for the root of $x^7 - 5x^2 - 20 = 0$ in the interval (1.6, 1.7).

Solution:

(a) Let $f(x) = x^7 - 5x^2 - 20$

$f(1.6) = (1.6)^7 - 5(1.6)^2 - 20 = -5.96$

$f(1.7) = (1.7)^7 - 5(1.7)^2 - 20 = 6.58$

$f(1.6) < 0$ and $f(1.7) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 1.6$ and $x = 1.7$.

(b) $x^7 - 5x^2 - 20 = 0$

$x^7 - 20 = 5x^2$ Add $5x^2$ to each side

$x^7 = 5x^2 + 20$ Add 20 to each side

$x = \sqrt[7]{5x^2 + 20}$ Take the seventh root of each side

So let $x_{n+1} = \sqrt[7]{5x_{\ n}^{2} + 20}$ and $x_0 = 1.6$, then

$x_1 = \sqrt[7]{5(1.6)^2 + 20} = 1.6464...$

Similarly,

$x_2 = 1.6518...$

$x_3 = 1.6524...$

$x_4 = 1.6525...$

$x_5 = 1.6525...$

So the root is 1.653 (3 d.p.)

Solutionbank
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

(a) Rearrange the cubic equation $x^3 - 6x - 2 = 0$ into the form $x = \pm\sqrt{a + \frac{b}{x}}$. State the values of the constants a and b.

(b) Use the iterative formula $x_{n+1} = \sqrt{a + \frac{b}{x_n}}$ with $x_0 = 2$ and your values of a and b to find the approximate positive solution x_4 of the equation, to an appropriate degree of accuracy. Show all your intermediate answers.

[E]

Solution:

(a) $x^3 - 6x - 2 = 0$

$x^3 - 2 = 6x$ Add $6x$ to each side

$x^3 = 6x + 2$ Add 2 to each side

$\frac{x^3}{x} = \frac{6x}{x} + \frac{2}{x}$ Divide each term by x

$x^2 = 6 + \frac{2}{x}$ Simplify

$x = \sqrt{6 + \frac{2}{x}}$ Take the square root of each side

So $a = 6$ and $b = 2$

(b) $x_0 = 2$

$x_1 = \sqrt{6 + \frac{2}{2}} = \sqrt{7} = 2.64575...$

$x_2 = \sqrt{6 + \frac{2}{2.64575...}} = 2.59921...$

$x_3 = \sqrt{6 + \frac{2}{2.59921...}} = 2.60181...$

$x_4 = \sqrt{6 + \frac{2}{2.60181...}} = 2.60167...$

So $x_4 = 2.602$ (3 d.p.)

Solutionbank
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Exercise C, Question 2

Question:

(a) By sketching the curves with equations $y = 4 - x^2$ and $y = e^x$, show that the equation $x^2 + e^x - 4 = 0$ has one negative root and one positive root.

(b) Use the iteration formula $x_{n+1} = -\left(4 - e^{x_n}\right)^{\frac{1}{2}}$ with $x_0 = -2$ to find in turn x_1, x_2, x_3 and x_4 and hence write down an approximation to the negative root of the equation, giving your answer to 4 decimal places.
An attempt to evaluate the positive root of the equation is made using the iteration formula

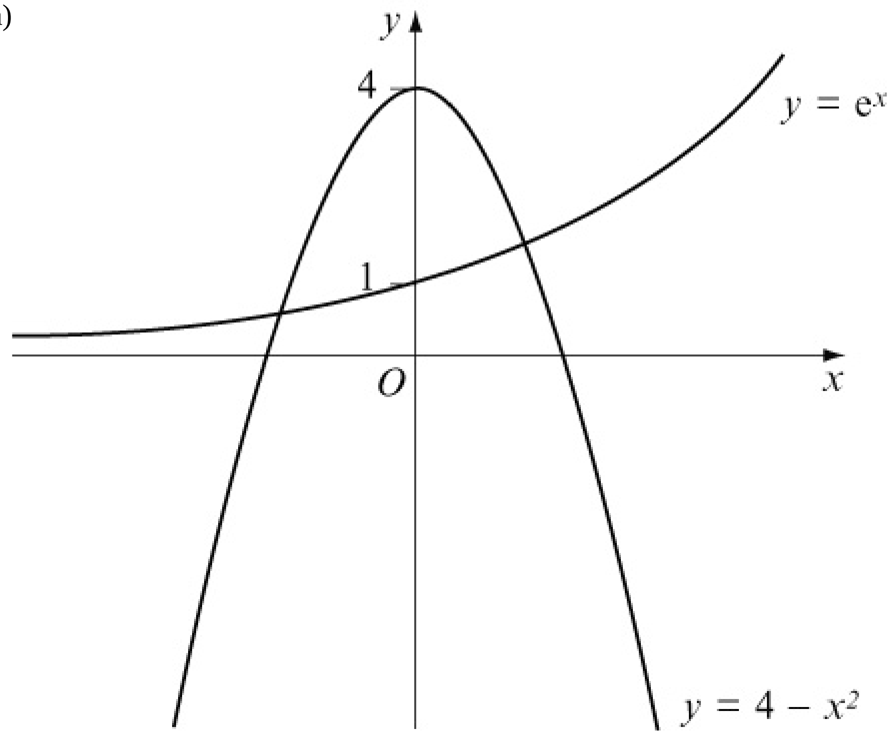
$x_{n+1} = \left(4 - e^{x_n}\right)^{\frac{1}{2}}$ with $x_0 = 1.3$.

(c) Describe the result of such an attempt.

[E]

Solution:

(a)



The curves meet when $x < 0$ and $x > 0$, so the equation $e^x = 4 - x^2$ has one negative and one positive root.

(Note that $e^x = 4 - x^2$ is the same as $x^2 + e^x - 4 = 0$).

(b) $x_0 = -2$

$x_1 = -(4 - e^{-2})^{\frac{1}{2}} = -1.965875051$

$x_2 = -(4 - e^{-1.965875051})^{\frac{1}{2}} = -1.964679797$

$x_3 = -(4 - e^{-1.964679797})^{\frac{1}{2}} = -1.964637175$

$x_4 = -(4 - e^{-1.964637175})^{\frac{1}{2}} = -1.964635654$

So $x_4 = -1.9646$ (4 d.p.)

(c) $x_0 = 1.3$

$x_1 = (4 - e^{1.3})^{\frac{1}{2}} = 0.575...$

$x_2 = (4 - e^{0.575...})^{\frac{1}{2}} = 1.490...$

$x_3 = (4 - e^{1.490...})^{\frac{1}{2}}$ **No solution**

The value of $4 - e^{1.490...}$ is **negative**.

You can not take the square root of a negative number.

Solutionbank

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Exercise C, Question 3

Question:

(a) Show that the equation $x^5 - 5x - 6 = 0$ has a root in the interval (1, 2).

(b) Stating the values of the constants p, q and r, use an iteration of the form $x_{n+1} = \left(px_n + q \right)^{\frac{1}{r}}$ an appropriate number of times to calculate this root of the equation $x^5 - 5x - 6 = 0$ correct to 3 decimal places. Show sufficient working to justify your final answer.

[E]

Solution:

(a) Let $f(x) = x^5 - 5x - 6$

$f(1) = (1)^5 - 5(1) - 6 = 1 - 5 - 6 = -10$

$f(2) = (2)^5 - 5(2) - 6 = 32 - 10 - 6 = 16$

$f(1) < 0$ and $f(2) > 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = 1$ and $x = 2$.

(b) $x^5 - 5x - 6 = 0$

$x^5 - 6 = 5x$ Add $5x$ to each side

$x^5 = 5x + 6$ Add 6 to each side

$x = (5x + 6)^{\frac{1}{5}}$ Take the fifth root of each side

So $p = 5$, $q = 6$ and $r = 5$

Let $x_0 = 1$ then

$x_1 = [5(1) + 6]^{\frac{1}{5}} = 1.6153...$

$x_2 = [5(1.6153...) + 6]^{\frac{1}{5}} = 1.6970...$

$x_3 = 1.7068...$

$x_4 = 1.7079...$

$x_5 = 1.7080...$

$x_6 = 1.7081...$

$x_7 = 1.7081...$

So the root is 1.708 (3 d.p.)

Solutionbank

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Exercise C, Question 4

Question:

$f(x) \equiv 5x - 4\sin x - 2$, where x is in radians.

(a) Evaluate, to 2 significant figures, f(1.1) and f(1.15).

(b) State why the equation $f(x) = 0$ has a root in the interval (1.1, 1.15).
An iteration formula of the form $x_{n+1} = p\sin x_n + q$ is applied to find an approximation to the root of the equation $f(x) = 0$ in the interval (1.1, 1.15).

(c) Stating the values of p and q, use this iteration formula with $x_0 = 1.1$ to find x_4 to 3 decimal places. Show the intermediate results in your working.

[E]

Solution:

(a) $f(1.1) = 5(1.1) - 4\sin(1.1) - 2 = -0.0648...$
$f(1.15) = 5(1.15) - 4\sin(1.15) - 2 = 0.0989...$

(b) $f(1.1) < 0$ and $f(1.15) > 0$ so there is a change of sign.
$\Rightarrow$ There is a root between $x = 1.1$ and $x = 1.15$.

(c) $5x - 4\sin x - 2 = 0$
$5x - 2 = 4\sin x$ Add $4\sin x$ to each side
$5x = 4\sin x + 2$ Add 2 to each side
$\frac{5x}{5} = \frac{4\sin x}{5} + \frac{2}{5}$ Divide each term by 5
$x = 0.8\sin x + 0.4$ Simplify
So $p = 0.8$ and $q = 0.4$
$x_0 = 1.1$
$x_1 = 0.8\sin(1.1) + 0.4 = 1.112965888$
$x_2 = 0.8\sin(1.112965888) + 0.4 = 1.117610848$
$x_3 = 0.8\sin(1.117610848) + 0.4 = 1.11924557$
$x_4 = 0.8\sin(1.11924557) + 0.4 = 1.119817195$
So $x_4 = 1.120$ (3 d.p.)

Solutionbank
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Exercise C, Question 5

Question:

$f(x) \equiv 2\sec x + 2x - 3$, where x is in radians.

(a) Evaluate f(0.4) and f(0.5) and deduce the equation $f(x) = 0$ has a solution in the interval $0.4 < x < 0.5$.

(b) Show that the equation $f(x) = 0$ can be arranged in the form $x = p + \frac{q}{\cos x}$, where p and q are constants, and state the value of p and the value of q.

(c) Using the iteration formula $x_{n+1} = p + \frac{q}{\cos x_n}$, $x_0 = 0.4$, with the values of p and q found in part (b), calculate x_1, x_2, x_3 and x_4, giving your final answer to 4 decimal places.

[E]

Solution:

(a) $f(0.4) = 2\sec(0.4) + 2(0.4) - 3 = -0.0286$
$f(0.5) = 2\sec(0.5) + 2(0.5) - 3 = 0.279$
$f(0.4) < 0$ and $f(0.5) > 0$ so there is a change of sign.
$\Rightarrow$ There is a root between $x = 0.4$ and $x = 0.5$.

(b) $2\sec x + 2x - 3 = 0$
$2\sec x + 2x = 3$ Add 3 to each side
$2x = 3 - 2\sec x$ Subtract $2\sec x$ from each side
$\frac{2x}{2} = \frac{3}{2} - \frac{2\sec x}{2}$ Divide each term by 2
$x = 1.5 - \sec x$ Simplify
$x = 1.5 - \frac{1}{\cos x}$ Use $\sec x = \frac{1}{\cos x}$
So $p = 1.5$ and $q = -1$

(c) $x_0 = 0.4$

$x_1 = 1.5 - \frac{1}{\cos(0.4)} = 0.4142955716$

$$x_2 = 1.5 - \frac{1}{\cos(0.4142955716)} = 0.4075815187$$

$$x_3 = 1.5 - \frac{1}{\cos(0.4075815187)} = 0.4107728765$$

$$x_4 = 1.5 - \frac{1}{\cos(0.4107728765)} = 0.4092644032$$

So $x_4 = 0.4093$ (4 d.p.)

Solutionbank
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Exercise C, Question 6

Question:

$f(x) \equiv e^{0.8x} - \frac{1}{3-2x}, x \neq \frac{3}{2}$

(a) Show that the equation $f(x) = 0$ can be written as $x = 1.5 - 0.5e^{-0.8x}$.

(b) Use the iteration formula $x_{n+1} = 1.5 - 0.5e^{-0.8x_n}$ with $x_0 = 1.3$ to obtain x_1, x_2 and x_3. Give the value of x_3, an approximation to a root of $f(x) = 0$, to 3 decimal places.

(c) Show that the equation $f(x) = 0$ can be written in the form $x = p \ln(3 - 2x)$, stating the value of p.

(d) Use the iteration formula $x_{n+1} = p \ln(3 - 2x_n)$ with $x_0 = -2.6$ and the value of p found in part (c) to obtain x_1, x_2 and x_3. Give the value of x_3, an approximation to the second root of $f(x) = 0$, to 3 decimal places.

[E]

Solution:

(a) $e^{0.8x} - \frac{1}{3-2x} = 0$

$e^{0.8x} = \frac{1}{3-2x}$ Add $\frac{1}{3-2x}$ to each side

$(3-2x)e^{0.8x} = \frac{1}{3-2x} \times (3-2x)$ Multiply each side by $(3-2x)$

$(3-2x)e^{0.8x} = 1$ Simplify

$\frac{(3-2x)e^{0.8x}}{e^{0.8x}} = \frac{1}{e^{0.8x}}$ Divide each side by $e^{0.8x}$

$3 - 2x = e^{-0.8x}$ Simplify (remember $\frac{1}{e^a} = e^{-a}$)

$3 = e^{-0.8x} + 2x$ Add $2x$ to each side

$2x = 3 - e^{-0.8x}$ Subtract $e^{-0.8x}$ from each side

$\frac{2x}{2} = \frac{3}{2} - \frac{e^{-0.8x}}{2}$ Divide each term by 2

$x = 1.5 - 0.5e^{-0.8x}$ Simplify

(b) $x_0 = 1.3$
$x_1 = 1.5 - 0.5e^{-0.8(1.3)} = 1.323272659$
$x_2 = 1.5 - 0.5e^{-0.8(1.323272659)} = 1.32653255$
$x_3 = 1.5 - 0.5e^{-0.8(1.32653255)} = 1.326984349$
So $x_3 = 1.327$ (3 d.p.)

(c) $e^{0.8x} - \frac{1}{3-2x} = 0$

$e^{0.8x} = \frac{1}{3-2x}$ Add $\frac{1}{3-2x}$ to each side

$0.8x = \ln\left(\frac{1}{3-2x}\right)$ Taking logs

$0.8x = -\ln(3-2x)$ Simplify using $\ln\left(\frac{1}{c}\right) = -\ln c$

$\frac{0.8x}{0.8} = -\frac{\ln(3-2x)}{0.8}$ Divide each side by 0.8

$x = -1.25\ln(3-2x)$ Simplify $\left(\frac{1}{0.8} = 1.25\right)$

So $p = -1.25$

(d) $x_0 = -2.6$
$x_1 = -1.25\ln[3-2(-2.6)] = -2.630167693$
$x_2 = -1.25\ln[3-2(-2.630167693)] = -2.639331488$
$x_3 = -1.25\ln[3-2(-2.639331488)] = -2.642101849$
So $x_3 = -2.642$ (3 d.p.)

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Exercise C, Question 7

Question:

(a) Use the iteration $x_{n+1} = (3x_n + 3)^{\frac{1}{3}}$ with $x_0 = 2$ to find, to 3 significant figures, x_4.

The only real root of the equation $x^3 - 3x - 3 = 0$ is α. It is given that, to 3 significant figures, $\alpha = x_4$.

(b) Use the substitution $y = 3^x$ to express $27^x - 3^{x+1} - 3 = 0$ as a cubic equation.

(c) Hence, or otherwise, find an approximate solution to the equation $27^x - 3^{x+1} - 3 = 0$, giving your answer to 2 significant figures.

[E]

Solution:

(a) $x_0 = 2$

$x_1 = [3(2) + 3]^{\frac{1}{3}} = 2.080083823$

$x_2 = [3(2.080083823) + 3]^{\frac{1}{3}} = 2.098430533$

$x_3 = [3(2.098430533) + 3]^{\frac{1}{3}} = 2.102588765$

$x_4 = [3(2.102588765) + 3]^{\frac{1}{3}} = 2.103528934$

So $x_4 = 2.10$ (3 s.f.)

(b) $27^x - 3^{x+1} - 3 = 0$

$(3^3)^x - 3(3^x) - 3 = 0$

$3^{3x} - 3(3^x) - 3 = 0$

$(3^x)^3 - 3(3^x) - 3 = 0$

Let $y = 3^x$

then $y^3 - 3y - 3 = 0$

(c) The root of the equation $y^3 - 3y - 3 = 0$ is x_4

so $y = 2.10$ (3 s.f.)

but $y = 3^x$

so $3^x = 2.10$

$\ln 3^x = \ln 2.10$ Take logs of each side

$x \ln 3 = \ln 2.10$ Simplify using $\ln a^b = b \ln a$

$\frac{x \ln 3}{\ln 3} = \frac{\ln 2.10}{\ln 3}$ Divide each side by ln 3

$x = \frac{\ln 2.10}{\ln 3}$ Simplify

$x = 0.6753...$

So $x = 0.68$ (2 s.f.)

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Exercise C, Question 8

Question:

The equation $x^x = 2$ has a solution near $x = 1.5$.

(a) Use the iteration formula $x_{n+1} = 2^{\frac{1}{x_n}}$ with $x_0 = 1.5$ to find the approximate solution x_5 of the equation. Show the intermediate iterations and give your final answer to 4 decimal places.

(b) Use the iteration formula $x_{n+1} = 2x_n^{(1 - x_n)}$ with $x_0 = 1.5$ to find x_1, x_2, x_3, x_4. Comment briefly on this sequence.

[E]

Solution:

(a) $x_0 = 1.5$

$x_1 = 2^{\frac{1}{1.5}} = 1.587401052$

$x_2 = 2^{\frac{1}{1.587401052}} = 1.54752265$

$x_3 = 2^{\frac{1}{1.54752265}} = 1.565034105$

$x_4 = 2^{\frac{1}{1.565034105}} = 1.557210213$

$x_5 = 2^{\frac{1}{1.557210213}} = 1.560679241$

So $x_5 = 1.5607$ (4 d.p.)

(b) $x_0 = 1.5$

$x_1 = 2 \times (1.5)^{1 - (1.5)} = 1.632993162$

$x_2 = 2 \times (1.632993162)^{1 - (1.632993162)} = 1.466264596$

$x_3 = 2 \times (1.466264596)^{1 - (1.466264596)} = 1.673135301$

$x_4 = 2 \times (1.673135301)^{1 - (1.673135301)} = 1.414371012$

The sequence x_0, x_1, x_2, x_3, x_4 gets further from the root. It is a divergent sequence.

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Exercise C, Question 9

Question:

(a) Show that the equation $2^{1-x} = 4x + 1$ can be arranged in the form $x = \frac{1}{2}\left(2^{-x}\right) + q$, stating the value of the constant q.

(b) Using the iteration formula $x_{n+1} = \frac{1}{2}\left(2^{-x_n}\right) + q$ with $x_0 = 0.2$ and the value of q found in part (a), find x_1, x_2, x_3 and x_4. Give the value of x_4, to 4 decimal places.

[E]

Solution:

(a) $2^{1-x} = 4x + 1$

$4x = 2^{1-x} - 1$ Subtract 1 from each side

$4x = 2\left(2^{-x}\right) - 1$ Use $2^{a+b} = 2^a \times 2^b$ and $2^1 = 2$

$\frac{4x}{4} = \frac{2}{4}\left(2^{-x}\right) - \frac{1}{4}$ Divide each term by 4

$x = \frac{1}{2}\left(2^{-x}\right) - \frac{1}{4}$ Simplify

So $q = -\frac{1}{4}$

(b) $x_0 = 0.2$

$x_1 = \frac{1}{2}\left(2^{-0.2}\right) - \frac{1}{4} = 0.1852752816$

$x_2 = \frac{1}{2}\left(2^{-0.1852752816}\right) - \frac{1}{4} = 0.1897406227$

$x_3 = \frac{1}{2}\left(2^{-0.1897406227}\right) - \frac{1}{4} = 0.1883816687$

$x_4 = \frac{1}{2}\left(2^{-0.1883816687}\right) - \frac{1}{4} = 0.1887947991$

So $x_4 = 0.1888$ (4 d.p.)

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Exercise C, Question 10

Question:

The curve with equation $y = \ln\ (3x)$ crosses the x-axis at the point P $(p, 0)$.

(a) Sketch the graph of $y = \ln\ (3x)$, showing the exact value of p.
The normal to the curve at the point Q, with x-coordinate q, passes through the origin.

(b) Show that $x = q$ is a solution of the equation $x^2 + \ln 3x = 0$.

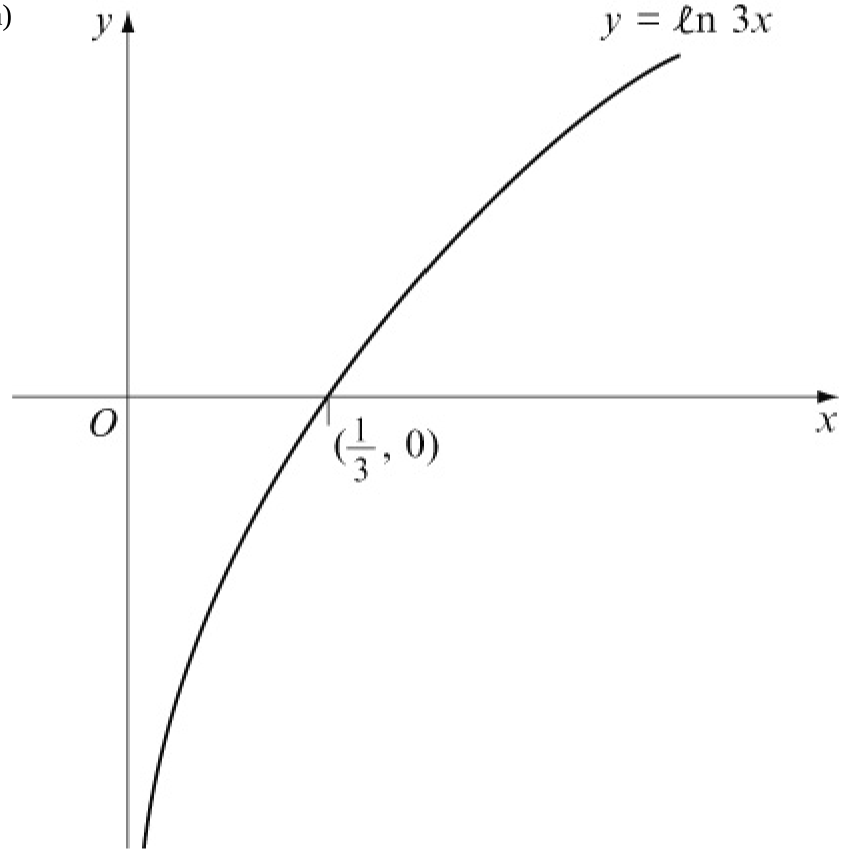
(c) Show that the equation in part (b) can be rearranged in the form $x = \frac{1}{3}e^{-x^2}$.

(d) Use the iteration formula $x_{n+1} = \frac{1}{3}e^{-x_n^2}$, with $x_0 = \frac{1}{3}$, to find x_1, x_2, x_3 and x_4. Hence write down, to 3 decimal places, an approximation for q.

[E]

Solution:

(a)



So $p = \frac{1}{3}$

(b)① $\frac{d}{dx} \ln 3x = \frac{1}{x}$

So the gradient of the tangent at Q is $\frac{1}{q}$.

The gradient of the normal is $-q$ (because the product of the gradients of perpendicular lines is -1).

The equation of the line with gradient $-q$ that passes through (0, 0) is

$y - y_1 = m (x - x_1)$

$y - 0 = -q (x - 0)$

$y = -qx$

② The line $y = -qx$ meets the curve $y = \ln 3x$ when

$\ln 3x = -qx$

We know they meet at Q.

So, substitute $x = q$ into $\ln 3x = -qx$:

$\ln 3q = -q (q)$

$\ln 3q = -q^2$

$q^2 + \ln 3q = 0$ Add q^2 to each side

This is $x^2 + \ln 3x = 0$ with $x = q$

So $x = q$ is a solution of the equation $x^2 + \ln 3x = 0$

(c) $x^2 + \ln 3x = 0$

$\ln 3x = -x^2$ Subtract x^2 from each side

$3x = e^{-x^2}$ Use $\ln a = b \Rightarrow a = e^b$

$x = \frac{1}{3}e^{-x^2}$ Divide each term by 3

(d) $x_0 = \frac{1}{3}$

$x_1 = \frac{1}{3}e^{-\left(\frac{1}{3}\right)^2} = 0.2982797723$

$x_2 = \frac{1}{3}e^{-(0.2982797723)^2} = 0.3049574223$

$x_3 = \frac{1}{3}e^{-(0.3049574223)^2} = 0.3037314616$

$x_4 = \frac{1}{3}e^{-(0.3037314616)^2} = 0.3039581993$

So $x_4 = 0.304$ (3 d.p.)

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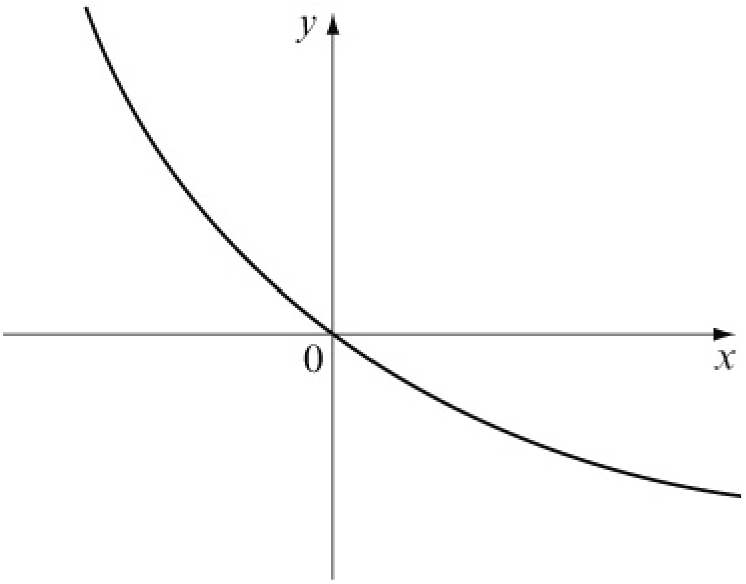
Exercise C, Question 11

Question:

(a) Copy this sketch of the curve with equation $y = e^{-x} - 1$. On the same axes sketch the graph of $y = \frac{1}{2}\left(x - 1\right)$, for $x \geq 1$, and $y = -\frac{1}{2}\left(x - 1\right)$, for $x < 1$. Show the coordinates of the points where the graph meets the axes.



The x-coordinate of the point of intersection of the graphs is α.

(b) Show that $x = \alpha$ is a root of the equation $x + 2e^{-x} - 3 = 0$.

(c) Show that $-1 < \alpha < 0$.

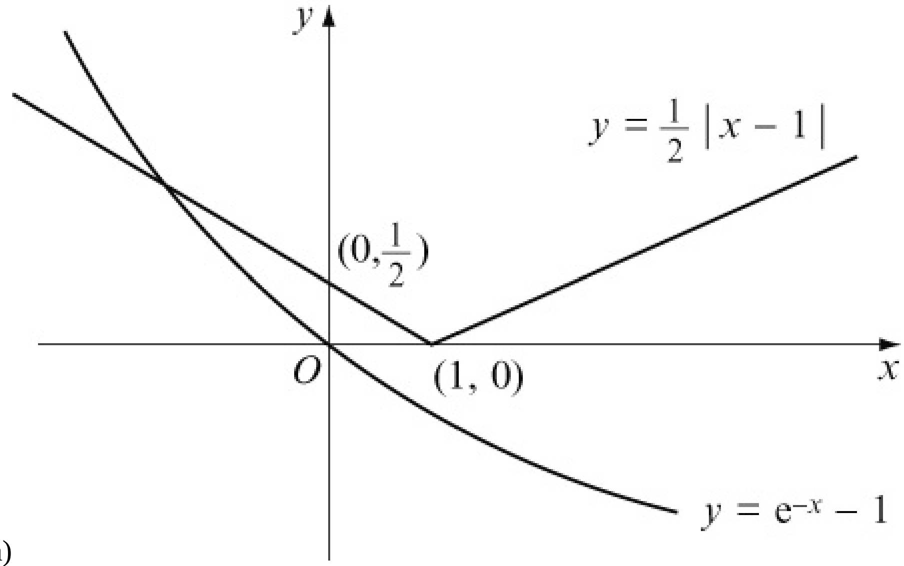
The iterative formula $x_{n+1} = -\ln\left[\frac{1}{2}\left(3 - x_n\right)\right]$ is used to solve the equation $x + 2e^{-x} - 3 = 0$.

(d) Starting with $x_0 = -1$, find the values of x_1 and x_2.

(e) Show that, to 2 decimal places, $\alpha = -0.58$.

[E]

Solution:



(a)

① Substitute $x = 0$ into $y = \frac{1}{2}\left| x - 1 \right|$:

$y = \frac{1}{2}\left| -1 \right| = \frac{1}{2}$

So $y = \frac{1}{2}\left| x - 1 \right|$ meets the y-axis at $\left(0 , \frac{1}{2} \right)$

② Substitute $y = 0$ into $y = \frac{1}{2}\left| x - 1 \right|$:

$\frac{1}{2}\left| x - 1 \right| = 0$

$x = 1$

So $y = \frac{1}{2}\left| x - 1 \right|$ meets the x-axis at (1, 0)

(b) The equation of the branch of the curve for $x < 1$ is $y = \frac{1}{2}\left(1 - x \right)$.

This line meets the curve $y = e^{-x} - 1$ when

$\frac{1}{2}\left(1 - x \right) = e^{-x} - 1$

$(1 - x) = 2 (e^{-x} - 1)$ Multiply each side by 2

$1 - x = 2e^{-x} - 2$ Simplify

$-x = 2e^{-x} - 3$ Subtract 1 from each side

$0 = x + 2e^{-x} - 3$ Add x to each side

or $x + 2e^{-x} - 3 = 0$

The line meets the curve when $x = \alpha$, so $x = \alpha$ is a root of the equation

$x + 2e^{-x} - 3 = 0$

(c) Let $f(x) = x + 2e^{-x} - 3$

$f(-1) = (-1) + 2e^{-(-1)} - 3 = 1.44$

$f(0) = (0) + 2e^{-(0)} - 3 = -1$

$f(-1) > 0$ and $f(0) < 0$ so there is a change of sign.

$\Rightarrow$ There is a root between $x = -1$ and $x = 0$,

i.e. $-1 < \alpha < 0$

(d) $x_0 = -1$

$$x_1 = -\ln\left\{\frac{1}{2}\left[3 - \left(-1\right)\right]\right\} = -0.6931471806$$

$$x_2 = -\ln\left\{\frac{1}{2}\left[3 - \left(-0.6931471806\right)\right]\right\} = -0.6133318084$$

(e) $x_3 = -\ln\left\{\frac{1}{2}\left[3 - \left(-0.6133318084\right)\right]\right\} = -0.5914831048$

$$x_4 = -\ln\left\{\frac{1}{2}\left[3 - \left(-0.5914831048\right)\right]\right\} = -0.5854180577$$

$$x_5 = -\ln\left\{\frac{1}{2}\left[3 - \left(-0.5854180577\right)\right]\right\} = -0.5837278997$$

$$x_6 = -\ln\left\{\frac{1}{2}\left[3 - \left(-0.5837278997\right)\right]\right\} = -0.5832563908$$

So $\alpha = -0.58$ (2 d.p.)