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Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Simplify:

(a) $\frac{4x+4}{x+1}$

(b) $\frac{2x-1}{6x-3}$

(c) $\frac{x+4}{x+2}$

(d) $\frac{x+\frac{1}{2}}{4x+2}$

(e) $\frac{4x+2y}{6x+3y}$

(f) $\frac{a+3}{a+6}$

(g) $\frac{5p-5q}{10p-10q}$

(h) $\frac{\frac{1}{2}a+b}{2a+4b}$

(i) $\frac{x^2}{x^2+3x}$

(j) $\frac{x^2-3x}{x^2-9}$

(k) $\frac{x^2+5x+4}{x^2+8x+16}$

(l) $\frac{x^3 - 2x^2}{x^2 - 4}$

(m) $\frac{x^2 - 4}{x^2 + 4}$

(n) $\frac{x + 2}{x^2 + 5x + 6}$

(o) $\frac{2x^2 - 5x - 3}{2x^2 - 7x - 4}$

(p) $\frac{\frac{1}{2}x^2 + x - 4}{\frac{1}{4}x^2 + \frac{3}{2}x + 2}$

(q) $\frac{3x^2 - x - 2}{\frac{1}{2}x + \frac{1}{3}}$

(r) $\frac{x^2 - 5x - 6}{\frac{1}{3}x - 2}$

Solution:

(a)

$$\frac{4x+4}{x+1} = \frac{4x \times (x+1)}{1x \times (x+1)} = \frac{4}{1} = 4$$

(b)

$$\frac{2x-1}{6x-3} = \frac{1 \times (2x-1)}{3 \times (2x-1)} = \frac{1}{3}$$

(c) $\frac{x + 4}{x + 2}$ will not simplify any further

(d)

$$\frac{x+12\times2}{4x+2\times2} = \frac{2x+1}{8x+4} = \frac{1 \times (2x+1)}{4 \times (2x+1)} = \frac{1}{4}$$

(e)

$$\frac{4x+2y}{6x+3y}=\frac{2\cancel{(2x+y)}}{3\cancel{(2x+y)}}=\frac{2}{3}$$

(f) $\frac{a+3}{a+6}$ will not simplify any further

(g)

$$\frac{5p-5q}{10p-10q}=\frac{5\cancel{(p-q)}}{10\cancel{(p-q)}}=\frac{5^1}{10_2}=\frac{1}{2}$$

(h)

$$\frac{12a+b\times 2}{2a+4b\times 2}=\frac{a+2b}{4a+8b}=\frac{1\times\cancel{(a+2b)}}{4\times\cancel{(a+2b)}}=\frac{1}{4}$$

(i)

$$\frac{x^2}{x^2+3x}=\frac{\cancel{x}\times x}{\cancel{x}(x+3)}=\frac{x}{x+3}$$

(j)

$$\frac{x^2-3x}{x^2-9}=\frac{x\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}=\frac{x}{x+3}$$

(k)

$$\frac{x^2+5x+4}{x^2+8x+16}=\frac{(x+1)\cancel{(x+4)}}{(x+4)\cancel{(x+4)}}=\frac{x+1}{x+4}$$

(l)

$$\frac{x^3-2x^2}{x^2-4}=\frac{x^2\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}=\frac{x^2}{x+2}$$

(m) $\frac{x^2-4}{x^2+4}$ will not simplify any further. The denominator doesn't factorise.

(n)

$$\frac{x+2}{x^2+5x+6}=\frac{1\times\cancel{(x+2)}}{(x+3)\cancel{(x+2)}}=\frac{1}{x+3}$$

(o)

$$\frac{2x^2-5x-3}{2x^2-7x-4}=\frac{\cancel{(2x+1)}(x-3)}{\cancel{(2x+1)}(x-4)}=\frac{x-3}{x-4}$$

(p)

$$\frac{12x^2+x-4}{14x^2-32x+2}\begin{matrix}\times 4\\ \times 4\end{matrix}=\frac{2x^2+4x-16}{x^2+6x+8}=\frac{2(x-2)\cancel{(x+4)}}{(x+2)\cancel{(x+4)}}=\frac{2(x-2)}{x+2}$$

(q)

$$\frac{3x^2-x-2}{12x+13}\begin{matrix}\times 6\\ \times 6\end{matrix}=\frac{6(3x^2-x-2)}{3x+2}=\frac{6\cancel{(3x+2)}(x-1)}{1\times\cancel{(3x+2)}}=6(x-1)$$

(r)

$$\frac{x^2-5x-6}{13x-2}\begin{matrix}\times 3\\ \times 3\end{matrix}=\frac{3(x^2-5x-6)}{x-6}=\frac{3(x+1)\cancel{(x-6)}}{1\times\cancel{(x-6)}}=3(x+1)$$

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Exercise B, Question 1

Question:

Simplify:

(a) $\frac{a}{d} \times \frac{a}{c}$

(b) $\frac{a^2}{c} \times \frac{c}{a}$

(c) $\frac{2}{x} \times \frac{x}{4}$

(d) $\frac{3}{x} \div \frac{6}{x}$

(e) $\frac{4}{\text{xy}} \div \frac{x}{y}$

(f) $\frac{2r^2}{5} \div \frac{4}{r^3}$

(g) $\left(x + 2 \right) \times \frac{1}{x^2 - 4}$

(h) $\frac{1}{a^2 + 6a + 9} \times \frac{a^2 - 9}{2}$

(i) $\frac{x^2 - 3x}{y^2 + y} \times \frac{y + 1}{x}$

(j) $\frac{y}{y + 3} \div \frac{y^2}{y^2 + 4y + 3}$

(k) $\frac{x^2}{3} \div \frac{2x^3 - 6x^2}{x^2 - 3x}$

(l) $\frac{4x^2 - 25}{4x - 10} \div \frac{2x + 5}{8}$

(m) $\frac{x+3}{x^2+10x+25} \times \frac{x^2+5x}{x^2+3x}$

(n) $\frac{3y^2+4y-4}{10} \div \frac{3y+6}{15}$

(o) $\frac{x^2+2xy+y^2}{2} \times \frac{4}{(x-y)^2}$

Solution:

(a) $\frac{a}{d} \times \frac{a}{c} = \frac{a \times a}{d \times c} = \frac{a^2}{cd}$

(b)

$$\frac{a^{\cancel{2}}}{\cancel{c}_1} \times \frac{\cancel{c}^1}{\cancel{a}_1} = \frac{a\times 1}{1\times 1} = a$$

(c)

$$\frac{\cancel{2}^1}{\cancel{x}_1} \times \frac{\cancel{x}^1}{\cancel{4}_2} = \frac{1\times 1}{1\times 2} = \frac{1}{2}$$

(d)

$$\frac{3}{x} \div \frac{6}{x} = \frac{\cancel{3}^1}{\cancel{x}_1} \times \frac{\cancel{x}^1}{\cancel{6}_2} = \frac{1\times 1}{1\times 2} = \frac{1}{2}$$

(e)

$$\frac{4}{xy} \div \frac{x}{y} = \frac{4}{x\cancel{y}_1} \times \frac{\cancel{y}^1}{x} = \frac{4\times 1}{x\times x} = \frac{4}{x^2}$$

(f)

$$\frac{2r^2}{5} \div \frac{4}{r^3} = \frac{{}^1\cancel{2}r^2}{5} \times \frac{r^3}{\cancel{4}_2} = \frac{r^5}{10}$$

(g)

$$(x+2) \times \frac{1}{x^2-4} = \frac{x+2^1}{1} = \frac{1}{\cancel{(x+2)}_1(x-2)} = \frac{1\times 1}{1\times(x-2)} = \frac{1}{x-2}$$

(h)

$$\frac{1}{a^2+6a+9} \times \frac{a^2-9}{2} = \frac{1}{\cancel{(a+3)}(a+3)} \times \frac{\cancel{(a+3)}(a-3)}{2} = \frac{a-3}{2(a+3)}$$

(i)

$$\frac{x^2-3x}{y^2+y}\times\frac{y+1}{x}=\frac{\cancel{x}^1(x-3)}{y\cancel{(y+1)}_1}\times\frac{\cancel{(y+1)}^1}{\cancel{x}_1}=\frac{x-3}{y}$$

(j)

$$\frac{y}{y+3}\div\frac{y^2}{y^2+4y+3}=\frac{y}{y+3}\times\frac{y^2+4y+3}{y^2}=\frac{\cancel{y}}{\cancel{y+3}}\times\frac{(y+1)\cancel{(y+3)}}{y^{\cancel{2}}}=\frac{y+1}{y}$$

(k)

$$\frac{x^2}{3}\div\frac{2x^3-6x^2}{x^2-3x}=\frac{x^2}{3}\times\frac{x^2-3x}{2x^3-6x^2}=\frac{\cancel{x}^{\cancel{2}^1}}{3}\times\frac{x\cancel{(x-3)}^1}{2\cancel{x^2}\cancel{(x-3)}_1}=\frac{1\times x}{3\times 2}=\frac{x}{6}$$

(l)

$$\frac{4x^2-25}{4x-10}\div\frac{2x+5}{8}=\frac{4x^2-25}{4x-10}\times\frac{8}{(2x+5)}=\frac{\cancel{(2x+5)}^1\cancel{(2x-5)}^1}{2\cancel{(2x-5)}_1}\times\frac{8}{\cancel{(2x+5)}_1}=\frac{1\times 8}{2\times 1}=4$$

(m)

$$\frac{x+3}{x^2+10x+25}\times\frac{x^2+5x}{x^2+3x}=\frac{\cancel{x+3}^1}{\cancel{(x+5)}_1(x+5)}\times\frac{\cancel{x}^1\cancel{(x+5)}^1}{\cancel{x}_1\cancel{(x+3)}_1}=\frac{1}{x+5}$$

(n)

$$\frac{3y^2+4y-4}{10}\div\frac{3y+6}{15}=\frac{3y^2+4y-4}{10}\times\frac{15}{3y+6}=\frac{(3y-2)\cancel{(y+2)}^1}{\cancel{10}_2}\times\frac{\cancel{15}^3}{\cancel{3}\cancel{(y+2)}_1}=\frac{3y-2}{2}$$

(o)

$$\frac{x^2+2xy+y^2}{2}\times\frac{4}{(x-y)^2}=\frac{(x+y)^2}{\cancel{2}_1}\times\frac{\cancel{4}^2}{(x-y)^2}=\frac{2(x+y)^2}{(x-y)^2}$$

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Exercise C, Question 1

Question:

Simplify:

(a) $\frac{1}{p}+\frac{1}{q}$

(b) $\frac{a}{b}-1$

(c) $\frac{1}{2x}+\frac{1}{x}$

(d) $\frac{3}{x^2}-\frac{1}{x}$

(e) $\frac{3}{4x}+\frac{1}{8x}$

(f) $\frac{x}{y}+\frac{y}{x}$

(g) $\frac{1}{x+2}-\frac{1}{x+1}$

(h) $\frac{2}{x+3}-\frac{1}{x-2}$

(i) $\frac{1}{3}(x+2)-\frac{1}{2}(x+3)$

(j) $\frac{3x}{(x+4)^2}-\frac{1}{(x+4)}$

(k) $\frac{1}{2(x+3)}+\frac{1}{3(x-1)}$

(l) $\frac{2}{x^2+2x+1}+\frac{1}{x+1}$

(m) $\frac{3}{x^2 + 3x + 2} - \frac{2}{x^2 + 4x + 4}$

(n) $\frac{2}{a^2 + 6a + 9} - \frac{3}{a^2 + 4a + 3}$

(o) $\frac{2}{y^2 - x^2} + \frac{3}{y - x}$

(p) $\frac{x + 2}{x^2 - x - 12} - \frac{x + 1}{x^2 + 5x + 6}$

(q) $\frac{3x + 1}{(x + 2)^3} - \frac{2}{(x + 2)^2} + \frac{4}{(x + 2)}$

Solution:

(a) $\frac{1}{p} + \frac{1}{q} = \frac{q}{\text{pq}} + \frac{p}{\text{pq}} = \frac{q + p}{\text{pq}}$

(b) $\frac{a}{b} - 1 = \frac{a}{b} - \frac{1}{1} = \frac{a}{b} - \frac{b}{b} = \frac{a - b}{b}$

(c) $\frac{1}{2x} + \frac{1}{x} = \frac{1}{2x} + \frac{2}{2x} = \frac{1 + 2}{2x} = \frac{3}{2x}$

(d) $\frac{3}{x^2} - \frac{1}{x} = \frac{3}{x^2} - \frac{x}{x^2} = \frac{3 - x}{x^2}$

(e) $\frac{3}{4x} + \frac{1}{8x} = \frac{6}{8x} + \frac{1}{8x} = \frac{7}{8x}$

(f) $\frac{x}{y} + \frac{y}{x} = \frac{x^2}{\text{xy}} + \frac{y^2}{\text{xy}} = \frac{x^2 + y^2}{\text{xy}}$

(g) $\frac{1}{(x + 2)} - \frac{1}{(x + 1)} = \frac{x + 1}{(x + 2)(x + 1)} - \frac{x + 2}{(x + 2)(x + 1)} =$

$\frac{(x + 1) - (x + 2)}{(x + 2)(x + 1)} = \frac{-1}{(x + 2)(x + 1)}$

(h) $\frac{2}{(x+3)} - \frac{1}{(x-2)} = \frac{2(x-2)}{(x+3)(x-2)} - \frac{(x+3)}{(x+3)(x-2)} =$

$= \frac{x-7}{(x+3)(x-2)}$

(i) $\frac{1}{3}(x+2) - \frac{1}{2}(x+3) = \frac{x+2}{3} - \frac{x+3}{2} = \frac{2(x+2)}{6} - \frac{3(x+3)}{6} =$

$\frac{-x-5}{6}$

(j) $\frac{3x}{(x+4)^2} - \frac{1}{(x+4)} = \frac{3x}{(x+4)^2} - \frac{x+4}{(x+4)^2} = \frac{3x-(x+4)}{(x+4)^2} = \frac{2x-4}{(x+4)^2}$

(k) $\frac{1}{2(x+3)} + \frac{1}{3(x-1)}$

$= \frac{3(x-1)}{6(x+3)(x-1)} + \frac{2(x+3)}{6(x+3)(x-1)}$

$= \frac{3(x-1)+2(x+3)}{6(x+3)(x-1)}$

$= \frac{5x+3}{6(x+3)(x-1)}$

(l) $\frac{2}{x^2+2x+1} + \frac{1}{x+1}$

$= \frac{2}{(x+1)^2} + \frac{1}{(x+1)}$

$= \frac{2}{(x+1)^2} + \frac{x+1}{(x+1)^2}$

$= \frac{2+x+1}{(x+1)^2}$

$= \frac{x+3}{(x+1)^2}$

(m) $\frac{3}{x^2+3x+2} - \frac{2}{x^2+4x+4}$

$= \frac{3}{(x+1)(x+2)} - \frac{2}{(x+2)^2}$

$= \frac{3(x+2)}{(x+1)(x+2)^2} - \frac{2(x+1)}{(x+1)(x+2)^2}$

$$= \frac{3(x+2) - 2(x+1)}{(x+1)(x+2)^2}$$

$$= \frac{x+4}{(x+1)(x+2)^2}$$

(n) $\frac{2}{a^2+6a+9} - \frac{3}{a^2+4a+3}$

$$= \frac{2}{(a+3)^2} - \frac{3}{(a+1)(a+3)}$$

$$= \frac{2(a+1)}{(a+1)(a+3)^2} - \frac{3(a+3)}{(a+1)(a+3)^2}$$

$$= \frac{2(a+1) - 3(a+3)}{(a+1)(a+3)^2}$$

$$= \frac{-a-7}{(a+1)(a+3)^2}$$

(o) $\frac{2}{y^2-x^2} + \frac{3}{y-x}$

$$= \frac{2}{(y+x)(y-x)} + \frac{3}{(y-x)}$$

$$= \frac{2}{(y+x)(y-x)} + \frac{3(y+x)}{(y+x)(y-x)}$$

$$= \frac{2+3(y+x)}{(y+x)(y-x)}$$

$$= \frac{2+3y+3x}{(y+x)(y-x)}$$

(p) $\frac{x+2}{x^2-x-12} - \frac{x+1}{x^2+5x+6}$

$$= \frac{x+2}{(x-4)(x+3)} - \frac{x+1}{(x+3)(x+2)}$$

$$= \frac{(x+2)(x+2)}{(x+2)(x+3)(x-4)} - \frac{(x+1)(x-4)}{(x+2)(x+3)(x-4)}$$

$$= \frac{(x^2+4x+4) - (x^2-3x-4)}{(x+2)(x+3)(x-4)}$$

$$= \frac{7x+8}{(x+2)(x+3)(x-4)}$$

(q) $\frac{3x+1}{(x+2)^3} - \frac{2}{(x+2)^2} + \frac{4}{(x+2)}$

$$= \frac{3x+1}{(x+2)^3} - \frac{2(x+2)}{(x+2)^3} + \frac{4(x+2)^2}{(x+2)^3}$$

$$= \frac{(3x+1) - (2x+4) + 4(x^2+4x+4)}{(x+2)^3}$$

$$= \frac{4x^2+17x+13}{(x+2)^3}$$

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Exercise D, Question 1

Question:

Express the following improper fractions in 'mixed' number form by: (i) using long division, (ii) using the remainder theorem

(a) $\frac{x^3 + 2x^2 + 3x - 4}{x - 1}$

(b) $\frac{2x^3 + 3x^2 - 4x + 5}{x + 3}$

(c) $\frac{x^3 - 8}{x - 2}$

(d) $\frac{2x^2 + 4x + 5}{x^2 - 1}$

(e) $\frac{8x^3 + 2x^2 + 5}{2x^2 + 2}$

(f) $\frac{4x^3 - 5x^2 + 3x - 14}{x^2 + 2x - 1}$

(g) $\frac{x^4 + 3x^2 - 4}{x^2 + 1}$

(h) $\frac{x^4 - 1}{x + 1}$

(i) $\frac{2x^4 + 3x^3 - 2x^2 + 4x - 6}{x^2 + x - 2}$

Solution:

(a) (i)

$$\begin{array}{r} x^2+3x+6 \\ x-1\overline{)x^3+2x^2+3x-4} \\ \underline{x^3-x^2} \\ 3x^2+3x \\ \underline{3x^2-3x} \\ 6x-4 \\ \underline{6x-6} \\ 2 \end{array}$$

(ii) Let $x^3 + 2x^2 + 3x - 4 \equiv (Ax^2 + \text{B}x + C)(x - 1) + R$

Let $x = 1$

$1 + 2 + 3 - 4 = (A + B + C) \times 0 + R$

$\Rightarrow \quad 2 = R$

Equate terms in $x^3 \quad \Rightarrow \quad 1 = A$

Equate terms in x^2

$\Rightarrow \quad 2 = -A + B$ (substitute $A = 1$)

$\Rightarrow \quad 2 = -1 + B$

$\Rightarrow \quad B = 3$

Equate constant terms

$\Rightarrow \quad -4 = -C + R$ (substitute $R = 2$)

$\Rightarrow \quad -4 = -C + 2$

$\Rightarrow \quad C = 6$

Hence $\dfrac{x^3 + 2x^2 + 3x - 4}{x - 1} \equiv x^2 + 3x + 6 + \dfrac{2}{x - 1}$

(b) (i)

$$\begin{array}{r} 2x^2-3x+5 \\ x+3\overline{)2x^3+3x^2-4x+5} \\ \underline{2x^3+6x^2} \\ -3x^2-4x \\ \underline{-3x^2-9x} \\ 5x+5 \\ \underline{5x+15} \\ -10 \end{array}$$

(ii) Let $2x^3 + 3x^2 - 4x + 5 \equiv (Ax^2 + \text{B}x + C)(x + 3) + R$

Let $x = -3$

$2 \times -27 + 3 \times 9 + 12 + 5 = (9A - 3B + C) \times 0 + R$

$\Rightarrow \quad -10 = R$

Equate terms in $x^3 \quad \Rightarrow \quad 2 = A$

Equate terms in x^2

$\Rightarrow \quad 3 = B + 3A \quad$ (substitute $A = 2$)

$\Rightarrow \quad 3 = B + 6$

$\Rightarrow \quad B = -3$

Equate constant terms

$\Rightarrow \quad 5 = 3C + R \quad$ (substitute $R = -10$)

$\Rightarrow \quad 5 = 3C - 10$

$\Rightarrow \quad 3C = 15$

$\Rightarrow \quad C = 5$

Hence $\quad \dfrac{2x^3 + 3x^2 - 4x + 5}{x + 3} \equiv 2x^2 - 3x + 5 - \dfrac{10}{x + 3}$

(c) (i)

$$\begin{array}{r}
x^2 + 2x + 4 \\
x - 2 \overline{\big) x^3 + 0x^2 + 0x - 8} \\
\underline{x^3 - 2x^2} \qquad\qquad \\
2x^2 + 0x \qquad \\
\underline{2x^2 - 4x} \qquad \\
4x - 8 \\
\underline{4x - 8} \\
0
\end{array}$$

(ii) Let $x^3 - 8 \equiv (Ax^2 + Bx + C)(x - 2) + R$

Let $x = 2$

$8 - 8 = (4A + 2B + C) \times 0 + R$

$\Rightarrow \quad 0 = R$

Equate terms in $x^3 \quad \Rightarrow \quad 1 = A$

Equate terms in x^2

$\Rightarrow \quad 0 = B - 2A \quad$ (substitute $A = 1$)

$\Rightarrow \quad 0 = B - 2$

$\Rightarrow \quad B = 2$

Equate constant terms

$\Rightarrow \quad -8 = -2C + R \quad$ (substitute $R = 0$)

$\Rightarrow \quad -2C = -8$

$\Rightarrow \quad C = 4$

Hence $\quad \dfrac{x^3 - 8}{x - 2} \equiv x^2 + 2x + 4$

There is no remainder. So $(x - 2)$ is a factor.

(d) (i)

$$\begin{array}{r} 2 \\ x^2+0x-1\overline{)2x^2+4x+5} \\ \underline{2x^2+0x-2} \\ 4x+7 \end{array}$$

$4x + 7$ is 'less than' $(x^2 - 1)$ so it is the remainder.

(ii) Let

$$2x^2+4x+5 \equiv A(x^2-1)+\underset{\uparrow}{\underline{Bx+C}}$$

If the divisor is quadratic then the remainder can be linear.

Equate terms in x^2 $\Rightarrow$ $2 = A$

Equate terms in x $\Rightarrow$ $4 = B$

Equate constant terms

$\Rightarrow$ $5 = -A + C$ (substitute $A = 2$)

$\Rightarrow$ $5 = -2 + C$

$\Rightarrow$ $C = 7$

Hence $\dfrac{2x^2 + 4x + 5}{x^2 - 1} \equiv 2 + \dfrac{4x + 7}{x^2 - 1}$

(e) (i)

$$\begin{array}{r} 4x+1 \\ 2x^2+0x+2\overline{)8x^3+2x^2+0x+5} \\ \underline{8x^3+0x^2+8x} \\ 2x^2-8x+5 \\ \underline{2x^2+0x+2} \\ -8x+3 \end{array}$$

(ii) Let $8x^3 + 2x^2 + 5 \equiv (Ax + B) (2x^2 + 2) + Cx + D$

Equate terms in x^3

$\Rightarrow$ $8 = 2A$

$\Rightarrow$ $A = 4$

Equate terms in x^2

$\Rightarrow$ $2 = 2B$

$\Rightarrow$ $B = 1$

Equate terms in x

$\Rightarrow$ $0 = 2A + C$ (substitute $A = 4$)

$\Rightarrow$ $0 = 8 + C$

$\Rightarrow \quad C = -8$

Equate constant terms

$\Rightarrow \quad 5 = 2B + D \quad$ (substitute $B = 1$)

$\Rightarrow \quad 5 = 2 + D$

$\Rightarrow \quad D = 3$

Hence $\quad \frac{8x^3 + 2x^2 + 5}{2x^2 + 2} \equiv 4x + 1 + \frac{-8x + 3}{2x^2 + 2}$

(f) (i)

$$\begin{array}{r l}
 & 4x - 13 \\
x^2 + 2x - 1 \,\big) & 4x^3 - 5x^2 + 3x - 14 \\
 & \underline{4x^3 + 8x^2 - 4x} \\
 & \quad -13x^2 + 7x - 14 \\
 & \quad \underline{-13x^2 - 26x + 13} \\
 & \qquad\qquad 33x - 27
\end{array}$$

(ii) Let $4x^3 - 5x^2 + 3x - 14 \equiv (Ax + B)(x^2 + 2x - 1) + Cx + D$

Equate terms in $x^3 \quad \Rightarrow \quad 4 = A$

Equate terms in x^2

$\Rightarrow \quad -5 = B + 2A \quad$ (substitute $A = 4$)

$\Rightarrow \quad -5 = B + 8$

$\Rightarrow \quad B = -13$

Equate terms in x

$\Rightarrow \quad 3 = -A + 2B + C \quad$ (substitute $A = 4$, $B = -13$)

$\Rightarrow \quad 3 = -4 + (-26) + C$

$\Rightarrow \quad C = 33$

Equate constant terms

$\Rightarrow \quad -14 = -B + D \quad$ (substitute $B = -13$)

$\Rightarrow \quad -14 = 13 + D$

$\Rightarrow \quad D = -27$

Hence $\quad \frac{4x^3 - 5x^2 + 3x - 14}{x^2 + 2x - 1} \equiv 4x - 13 + \frac{33x - 27}{x^2 + 2x - 1}$

(g) (i)

$$\begin{array}{r}
x^2+2 \\
x^2+0x+1\overline{)x^4+0x^3+3x^2+0x-4} \\
\underline{x^4+0x^3+x^2} \\
2x^2+0x-4 \\
\underline{2x^2+0x+2} \\
-6
\end{array}$$

(ii) Let $x^4 + 3x^2 - 4 \equiv (Ax^2 + Bx + C)(x^2 + 1) + Dx + E$

Equate terms in $x^4 \Rightarrow 1 = A$

Equate terms in $x^3 \Rightarrow 0 = B$

Equate terms in x^2

$\Rightarrow 3 = A + C$ (substitute $A = 1$)

$\Rightarrow 3 = 1 + C$

$\Rightarrow C = 2$

Equate terms in x

$\Rightarrow 0 = B + D$ (substitute $B = 0$)

$\Rightarrow 0 = 0 + D$

$\Rightarrow D = 0$

Equate constant terms

$\Rightarrow -4 = C + E$ (substitute $C = 2$)

$\Rightarrow -4 = 2 + E$

$\Rightarrow E = -6$

Hence $\dfrac{x^4 + 3x^2 - 4}{x^2 + 1} \equiv x^2 + 2 - \dfrac{6}{x^2 + 1}$

(h) (i)

$$\begin{array}{r}
x^3-x^2+x-1 \\
x+1\overline{)x^4+0x^3+0x^2+0x-1} \\
\underline{x^4+x^3} \\
-x^3+0x^2 \\
\underline{-x^3-x^2} \\
x^2+0x \\
\underline{x^2+x} \\
-x-1 \\
\underline{-x-1} \\
0
\end{array}$$

There is no remainder so $(x + 1)$ is a factor of $x^4 - 1$.

(ii) Let $x^4 - 1 \equiv (Ax^3 + Bx^2 + Cx + D)(x + 1) + E$

Let $x = -1$

$(-1)^4 - 1 = (-A + B - C + D) \times 0 + E$

$\Rightarrow \quad E = 0$

Equate terms in $x^4 \quad \Rightarrow \quad 1 = A$

Equate terms in x^3

$\Rightarrow \quad 0 = A + B \quad$ (substitute $A = 1$)

$\Rightarrow \quad 0 = 1 + B$

$\Rightarrow \quad B = -1$

Equate terms in x^2

$\Rightarrow \quad 0 = B + C \quad$ (substitute $B = -1$)

$\Rightarrow \quad 0 = -1 + C$

$\Rightarrow \quad C = 1$

Equate terms in x

$\Rightarrow \quad 0 = D + C \quad$ (substitute $C = 1$)

$\Rightarrow \quad 0 = D + 1$

$\Rightarrow \quad D = -1$

Hence $\quad \frac{x^4 - 1}{x + 1} \equiv x^3 - x^2 + x - 1$

(i) (i)

$$\begin{array}{r l}
 & 2x^2 + x + 1 \\
x^2 + x - 2 \,\big) & 2x^4 + 3x^3 - 2x^2 + 4x - 6 \\
 & \underline{2x^4 + 2x^3 - 4x^2} \\
 & x^3 + 2x^2 + 4x \\
 & \underline{x^3 + x^2 - 2x} \\
 & x^2 + 6x - 6 \\
 & \underline{x^2 + x - 2} \\
 & 5x - 4
\end{array}$$

(ii) Let $2x^4 + 3x^3 - 2x^2 + 4x - 6 \equiv (Ax^2 + \mathrm{B}x + C) (x^2 + x - 2) + \mathrm{D}x + E$

Equate terms in $x^4 \quad \Rightarrow \quad 2 = A$

Equate terms in x^3

$\Rightarrow \quad 3 = A + B \quad$ (substitute $A = 2$)

$\Rightarrow \quad 3 = 2 + B$

$\Rightarrow \quad B = 1$

Equate terms in x^2

$\Rightarrow \quad -2 = -2A + B + C \quad$ (substitute $A = 2$, $B = 1$)

$\Rightarrow \quad -2 = -4 + 1 + C$

$\Rightarrow \quad C = 1$

Equate terms in x

$\Rightarrow \quad 4 = C - 2B + D \quad$ (substitute $C = 1$, $B = 1$)

$\Rightarrow \quad 4 = 1 - 2 + D$

$\Rightarrow \quad D = 5$

Equate constant terms

$\Rightarrow \quad -6 = -2C + E \quad$ (substitute $C = 1$)

$\Rightarrow \quad -6 = -2 + E$

$\Rightarrow \quad E = -4$

Hence $\quad \dfrac{2x^4 + 3x^3 - 2x^2 + 4x - 6}{x^2 + x - 2} \equiv 2x^2 + x + 1 + \dfrac{5x - 4}{x^2 + x - 2}$

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Exercise D, Question 2

Question:

Find the value of the constants A, B, C, D and E in the following identity:
$3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (Ax^2 + \mathrm{B}x + C)(x^2 - 3) + \mathrm{D}x + E$

Solution:

$3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (Ax^2 + \mathrm{B}x + C)(x^2 - 3) + \mathrm{D}x + E$

Equate terms in x^4 $\Rightarrow$ $3 = A$

Equate terms in x^3 $\Rightarrow$ $-4 = B$

Equate terms in x^2

$\Rightarrow$ $-8 = -3A + C$ (substitute $A = 3$)

$\Rightarrow$ $-8 = -9 + C$

$\Rightarrow$ $C = 1$

Equate terms in x

$\Rightarrow$ $16 = -3B + D$ (substitute $B = -4$)

$\Rightarrow$ $16 = 12 + D$

$\Rightarrow$ $D = 4$

Equate constant terms

$\Rightarrow$ $-2 = -3C + E$ (substitute $C = 1$)

$\Rightarrow$ $-2 = -3 + E$

$\Rightarrow$ $E = 1$

Hence $3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (3x^2 - 4x + 1)(x^2 - 3) + 4x + 1$

A good idea in equalities is to check with an easy value of x because it should be true for all values of x.

Substitute $x = 1$ into LHS $\Rightarrow$ $3 - 4 - 8 + 16 - 2 = 5$

Substitute $x = 1$ into RHS $\Rightarrow$ $\left(3 - 4 + 1\right) \times \left(1 - 3\right)$

$+ 4 + 1 = 0 \times -2 + 4 + 1 = 5$ ✓

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Exercise E, Question 1

Question:

Simplify the following fractions:

(a) $\frac{ab}{c} \times \frac{c^2}{a^2}$

(b) $\frac{x^2 + 2x + 1}{4x + 4}$

(c) $\frac{x^2 + x}{2} \div \frac{x + 1}{4}$

(d) $\frac{x + \frac{1}{x} - 2}{x - 1}$

(e) $\frac{a + 4}{a + 8}$

(f) $\frac{b^2 + 4b - 5}{b^2 + 2b - 3}$

Solution:

(a)

$$\frac{ab}{c} \times \frac{c^2}{a^2} = \frac{b \times c}{a} = \frac{bc}{a}$$

(b)

$$\frac{x^2 + 2x + 1}{4x + 4} = \frac{(x+1)\cancel{(x+1)}}{4\cancel{(x+1)}} = \frac{x+1}{4}$$

(c)

$$\frac{x^2 + x}{2} \div \frac{x+1}{4} = \frac{x^2 + x}{2} \times \frac{4}{x+1} = \frac{x\cancel{(x+1)}^1}{\cancel{2}_1} \times \frac{\cancel{4}^2}{\cancel{(x+1)}_1} = 2x$$

(d)

$$\frac{x+1x-2}{x-1}\frac{\times x}{\times x}=\frac{x^2+1-2x}{x(x-1)}=\frac{(x-1)\cancel{(x-1)}^1}{x\cancel{(x-1)}_1}=\frac{x-1}{x}$$

(e) $\frac{a+4}{a+8}$ doesn't simplify as there are no common factors.

(f)

$$\frac{b^2+4b-5}{b^2+2b-3}=\frac{(b+5)\cancel{(b-1)}^1}{(b+3)\cancel{(b-1)}_1}=\frac{b+5}{b+3}$$

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Exercise E, Question 2

Question:

Simplify:

(a) $\frac{x}{4} + \frac{x}{3}$

(b) $\frac{4}{y} - \frac{3}{2y}$

(c) $\frac{x+1}{2} - \frac{x-2}{3}$

(d) $\frac{x^2-5x-6}{x-1}$

(e) $\frac{x^3+7x-1}{x+2}$

(f) $\frac{x^4+3}{x^2+1}$

Solution:

(a) $\frac{x}{4} + \frac{x}{3} = \frac{3x}{12} + \frac{4x}{12} = \frac{7x}{12}$

(b) $\frac{4}{y} - \frac{3}{2y} = \frac{8}{2y} - \frac{3}{2y} = \frac{5}{2y}$

(c) $\frac{x+1}{2} - \frac{x-2}{3} = \frac{3(x+1)}{6} - \frac{2(x-2)}{6} = \frac{3(x+1)-2(x-2)}{6} = \frac{x+7}{6}$

(d) $\frac{x^2-5x-6}{x-1} = \frac{(x-6)(x+1)}{(x-1)}$ No common factors so divide.

$$
\begin{array}{r}
x-4 \\
x-1\overline{)x^2-5x-6} \\
\underline{x^2-1x} \\
-4x-6 \\
\underline{-4x+4} \\
-10
\end{array}
$$

Hence $\dfrac{x^2 - 5x - 6}{x - 1} = x - 4 - \dfrac{10}{x - 1}$

(e) $\dfrac{x^3 + 7x - 1}{x + 2}$

$$\begin{array}{r} x^2 - 2x + 11 \\ x+2 \overline{\smash{)} x^3 + 0x^2 + 7x - 1} \\ \underline{x^3 + 2x^2} \quad\quad\quad\quad \\ -2x^2 + 7x \quad\;\; \\ \underline{-2x^2 - 4x} \quad\;\; \\ 11x - 1 \\ \underline{11x + 22} \\ -23 \end{array}$$

Hence $\dfrac{x^3 + 7x - 1}{x + 2} = x^2 - 2x + 11 - \dfrac{23}{x + 2}$

(f) $\dfrac{x^4 + 3}{x^2 + 1}$

$$\begin{array}{r} x^2 - 1 \quad\quad\quad\quad\quad\quad \\ x^2 + 0x + 1 \overline{\smash{)} x^4 + 0x^3 + 0x^2 + 0x + 3} \\ \underline{x^4 + 0x^3 + 1x^2} \quad\quad\quad\quad \\ -1x^2 + 0x + 3 \\ \underline{-1x^2 + 0x - 1} \\ 4 \end{array}$$

Hence $\dfrac{x^4 + 3}{x^2 + 1} = x^2 - 1 + \dfrac{4}{x^2 + 1}$

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Exercise E, Question 3

Question:

Find the value of the constants A, B, C and D in the following identity:

$x^3 - 6x^2 + 11x + 2 \equiv (x - 2)(Ax^2 + Bx + C) + D$

Solution:

$x^3 - 6x^2 + 11x + 2 \equiv (x - 2)(Ax^2 + Bx + C) + D$

Let $x = 2$

$8 - 24 + 22 + 2 = 0 \times (4A + 2B + C) + D$

$\Rightarrow \quad D = 8$

Equate coefficients in $x^3 \quad \Rightarrow \quad 1 = A$

Equate coefficients in x^2

$\Rightarrow \quad -6 = -2A + B \quad$ (substitute $A = 1$)

$\Rightarrow \quad -6 = -2 + B$

$\Rightarrow \quad B = -4$

Equate coefficients in x

$\Rightarrow \quad 11 = C - 2B \quad$ (substitute $B = -4$)

$\Rightarrow \quad 11 = C + 8$

$\Rightarrow \quad C = 3$

Hence $\quad x^3 - 6x^2 + 11x + 2 = (x - 2)(x^2 - 4x + 3) + 8$

Check. Equate constant terms: $2 = -2 \times 3 + 8$✓

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Exercise E, Question 4

Question:

$$f(x) = x + \frac{3}{x-1} - \frac{12}{x^2+2x-3} \quad \{ x \in \mathbb{R}, \quad x > 1 \{$$

Show that $f(x) = \dfrac{x^2+3x+3}{x+3}$

[E]

Solution:

$$f(x) = x + \frac{3}{x-1} - \frac{12}{x^2+2x-3}$$

$$= \frac{x}{1} + \frac{3}{x-1} - \frac{12}{(x+3)(x-1)}$$

$$= \frac{x(x+3)(x-1)}{(x+3)(x-1)} + \frac{3(x+3)}{(x+3)(x-1)} - \frac{12}{(x+3)(x-1)}$$

$$= \frac{x(x+3)(x-1) + 3(x+3) - 12}{(x+3)(x-1)}$$

$$= \frac{x(x^2+2x-3) + 3x + 9 - 12}{(x+3)(x-1)}$$

$$= \frac{x^3+2x^2-3x+3x+9-12}{(x+3)(x-1)}$$

$$= \frac{x^3+2x^2-3}{(x+3)(x-1)}$$

[Factorise numerator. $(x-1)$ is a factor as $f(1) = 0$.]

$$= \frac{(x-1)(x^2+3x+3)}{(x+3)(x-1)}$$

(cancel common factors)

$$= \frac{x^2+3x+3}{x+3}$$

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Exercise E, Question 5

Question:

Show that $\frac{x^4+2}{x^2-1} \equiv x^2 + B + \frac{C}{x^2-1}$ for constants B and C, which should be found.

Solution:

We need to find B and C such that

$$\frac{x^4+2}{x^2-1} \equiv x^2 + B + \frac{C}{x^2-1}$$

Multiply both sides by (x^2-1) :

$$x^4 + 2 \equiv (x^2 + B)(x^2 - 1) + C$$

$$x^4 + 2 \equiv x^4 + Bx^2 - x^2 - B + C$$

Compare terms in x^2

$\Rightarrow \quad 0 = B - 1$

$\Rightarrow \quad B = 1$

Compare constant terms

$\Rightarrow \quad 2 = -B + C \quad$ (substitute $B = 1$)

$\Rightarrow \quad 2 = -1 + C$

$\Rightarrow \quad C = 3$

Hence $\quad x^4 + 2 \equiv (x^2 + 1)(x^2 - 1) + 3$

So $\quad \frac{x^4+2}{x^2-1} = x^2 + 1 + \frac{3}{x^2-1}$

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Exercise E, Question 6

Question:

Show that $\frac{4x^3 - 6x^2 + 8x - 5}{2x + 1}$ can be put in the form $Ax^2 + \text{B}x + C + \frac{D}{2x + 1}$. Find the values of the constants A, B, C and D.

Solution:

$$\begin{array}{r|l} & 2x^2 - 4x + 6 \\ \hline 2x+1 & 4x^3 - 6x^2 + 8x - 5 \\ & \underline{4x^3 + 2x^2} \\ & \quad -8x^2 + 8x \\ & \quad \underline{-8x^2 - 4x} \\ & \qquad\quad 12x - 5 \\ & \qquad\quad \underline{12x + 6} \\ & \qquad\qquad -11 \end{array}$$

Hence $\frac{4x^3 - 6x^2 + 8x - 5}{2x + 1} = 2x^2 - 4x + 6 \text{ R } -11 = 2x^2 - 4x + 6 - \frac{11}{2x + 1}$

So $A = 2$, $B = -4$, $C = 6$ and $D = -11$