

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Practice paper

Exercise 1, Question 1

Question:

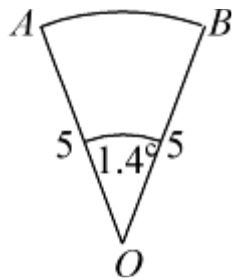
The sector AOB is removed from a circle of radius 5 cm.

The $\angle AOB$ is 1.4 radians and $OA = OB$.

(a) Find the perimeter of the sector AOB . (3)

(b) Find the area of sector AOB . (2)

Solution:



(a)

$$\text{Arc length} = r\theta = 5 \times 1.4 = 7 \text{ cm}$$

$$\text{Perimeter} = 10 + \text{Arc} = 17 \text{ cm}$$

$$(b) \text{ Sector area} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times 1.4 = 17.5 \text{ cm}^2$$

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Practice paper Exercise 1, Question 2

Question:

Given that $\log_2 x = p$:

- (a) Find $\log_2 (8x^2)$ in terms of p . (4)
- (b) Given also that $p = 5$, find the value of x . (2)

Solution:

(a) $\log_2 x = p$

$$\log_2 (8x^2) = \log_2 8 + \log_2 x^2 = 3 + 2 \log_2 x = 3 + 2p$$

(b) $\log_2 x = 5 \Rightarrow x = 2^5 \Rightarrow x = 32$

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Practice paper Exercise 1, Question 3

Question:

- (a) Find the value of the constant a so that $(x - 3)$ is a factor of $x^3 - ax - 6$. (3)
- (b) Using this value of a , factorise $x^3 - ax - 6$ completely. (4)

Solution:

(a) Let $f(x) = x^3 - ax - 6$
If $(x - 3)$ is a factor then $f(3) = 0$
i.e. $0 = 27 - 3a - 6$
So $3a = 21 \Rightarrow a = 7$

(b) $x^3 - 7x - 6$ has $(x - 3)$ as a factor, so
 $x^3 - 7x - 6 = (x - 3)(x^2 + 3x + 2) = (x - 3)(x + 2)(x + 1)$

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Practice paper Exercise 1, Question 4

Question:

(a) Find the coefficient of x^{11} and the coefficient of x^{12} in the binomial expansion of $(2 + x)^{15}$. (4)

The coefficient of x^{11} and the coefficient of x^{12} in the binomial expansion of $(2 + kx)^{15}$ are equal.

(b) Find the value of the constant k . (3)

Solution:

$$(a) (2 + x)^{15} = \dots \binom{15}{11} 2^4 x^{11} + \binom{15}{12} 2^3 x^{12} + \dots$$

$$\text{Coefficient of } x^{11} = \binom{15}{11} \times 16 = 1365 \times 16 = 21\,840$$

$$\text{Coefficient of } x^{12} = \binom{15}{12} \times 8 = 455 \times 8 = 3640$$

$$(b) 21\,840k^{11} = 3640k^{12}$$

$$\text{So } k = \frac{21\,840}{3640}$$

$$\text{i.e. } k = 6$$

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Practice paper Exercise 1, Question 5

Question:

(a) Prove that:

$$\frac{\cos^2 \theta}{\sin \theta + \sin^2 \theta} \equiv \frac{1 - \sin \theta}{\sin \theta}, 0 < \theta < 180^\circ. \quad (4)$$

(b) Hence, or otherwise, solve the following equation for $0 < \theta < 180^\circ$:

$$\frac{\cos^2 \theta}{\sin \theta + \sin^2 \theta} = 2$$

Give your answers to the nearest degree. (4)

Solution:

$$\begin{aligned} \text{(a) LHS} &= \frac{\cos^2 \theta}{\sin \theta + \sin^2 \theta} \\ &= \frac{1 - \sin^2 \theta}{\sin \theta + \sin^2 \theta} \text{ (using } \sin^2 \theta + \cos^2 \theta \equiv 1) \\ &= \frac{(1 - \sin \theta)(1 + \sin \theta)}{\sin \theta(1 + \sin \theta)} \text{ (factorising)} \\ &= \frac{1 - \sin \theta}{\sin \theta} \text{ (cancelling } [1 + \sin \theta] \text{)} \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(b) } 2 &= \frac{\cos^2 \theta}{\sin \theta + \sin^2 \theta} \\ \Rightarrow 2 &= \frac{1 - \sin \theta}{\sin \theta} \\ \Rightarrow 2 \sin \theta &= 1 - \sin \theta \text{ (can multiply by } \sin \theta \text{ } \because 0 < \theta < 180) \\ \Rightarrow 3 \sin \theta &= 1 \\ \Rightarrow \sin \theta &= \frac{1}{3} \end{aligned}$$

So $\theta = 19.47 \dots^\circ, 160.5 \dots^\circ = 19^\circ, 161^\circ$ (to nearest degree)

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Exercise 1, Question 6

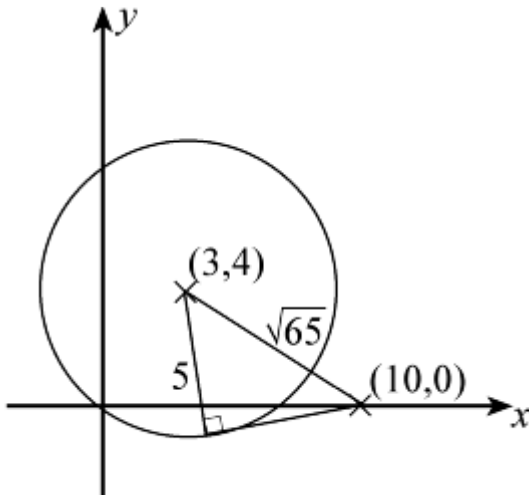
Question:

- (a) Show that the centre of the circle with equation $x^2 + y^2 = 6x + 8y$ is (3, 4) and find the radius of the circle. (5)
- (b) Find the exact length of the tangents from the point (10, 0) to the circle. (4)

Solution:

$$\begin{aligned} \text{(a)} \quad x^2 - 6x + y^2 - 8y &= 0 \\ &\Rightarrow (x - 3)^2 + (y - 4)^2 = 9 + 16 \\ \text{i.e. } (x - 3)^2 + (y - 4)^2 &= 5^2 \\ \text{Centre } (3, 4), \text{ radius } &5 \end{aligned}$$

$$\text{(b) Distance from } (3, 4) \text{ to } (10, 0) = \sqrt{7^2 + 4^2} = \sqrt{65}$$



$$\text{Length of tangent} = \sqrt{\sqrt{65^2} - 5^2} = \sqrt{40} = 2\sqrt{10}$$

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Practice paper Exercise 1, Question 7

Question:

A father promises his daughter an eternal gift on her birthday. On day 1 she receives £75 and each following day she receives $\frac{2}{3}$ of the amount given to her the day before. The father promises that this will go on for ever.

- (a) Show that after 2 days the daughter will have received £125. (2)
- (b) Find how much money the father should set aside to ensure that he can cover the cost of the gift. (3)
After k days the total amount of money that the daughter will have received exceeds £200.
- (c) Find the smallest value of k . (5)

Solution:

(a) Day 1 = £ 75, day 2 = £ 50, total = £ 125

(b) $a = 75$, $r = \frac{2}{3}$ —geometric series

$$S_{\infty} = \frac{a}{1-r} = \frac{75}{1-\frac{2}{3}}$$

Amount required = £ 225

$$(c) S_k = \frac{a(1-r^k)}{1-r}$$

$$\text{Require } \frac{75 \left[1 - \left(\frac{2}{3} \right)^k \right]}{1 - \frac{2}{3}} > 200$$

$$\text{i.e. } 225 \left[1 - \left(\frac{2}{3} \right)^k \right] > 200$$

$$\Rightarrow 1 - \left(\frac{2}{3} \right)^k > \frac{8}{9}$$

$$\Rightarrow \frac{1}{9} > \left(\frac{2}{3} \right)^k$$

$$\text{Take logs: } \log \left(\frac{1}{9} \right) > k \log \left(\frac{2}{3} \right)$$

Since $\log \left(\frac{2}{3} \right)$ is negative, when we divide by this the inequality will change around.

$$\text{So } k > \frac{\log \left(\frac{1}{9} \right)}{\log \left(\frac{2}{3} \right)}$$

i.e. $k > 5.419 \dots$
So need $k = 6$

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Practice paper Exercise 1, Question 8

Question:

Given $I = \int_1^3 \left(\frac{1}{x^2} + 3\sqrt{x} \right) dx$:

(a) Use the trapezium rule with the table below to estimate I to 3 significant figures. (4)

x	1	1.5	2	2.5	3
y	4	4.119	4.493	4.903	5.307

(b) Find the exact value of I . (4)

(c) Calculate, to 1 significant figure, the percentage error incurred by using the trapezium rule as in part (a) to estimate I . (2)

Solution:

(a) $h = 0.5$

$$\begin{aligned}
 I &\approx \frac{0.5}{2} \left[4 + 2 \left(4.119 + 4.493 + 4.903 \right) + 5.307 \right] \\
 &= \frac{1}{4} \left[36.337 \right] \\
 &= 9.08425
 \end{aligned}$$

(b) $I = \int_1^3 \left(x^{-2} + 3x^{\frac{1}{2}} \right) dx$

$$\begin{aligned}
 &= \left[\frac{x^{-1}}{-1} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^3 \\
 &= \left[-\frac{1}{x} + 2x^{\frac{3}{2}} \right]_1^3 \\
 &= \left(-\frac{1}{3} + 2 \times 3\sqrt{3} \right) - \left(-1 + 2 \right) \\
 &= 6\sqrt{3} - \frac{4}{3}
 \end{aligned}$$

(c) Percentage error = $\frac{|6\sqrt{3} - \frac{4}{3} - 9.08425|}{6\sqrt{3} - \frac{4}{3}} \times 100 = 0.279 \dots \% = 0.3 \%$

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Practice paper Exercise 1, Question 9

Question:

The curve C has equation $y = 6x^{\frac{7}{3}} - 7x^2 + 4$.

(a) Find $\frac{dy}{dx}$. (2)

(b) Find $\frac{d^2y}{dx^2}$. (2)

(c) Use your answers to parts (a) and (b) to find the coordinates of the stationary points on C and determine their nature. (9)

Solution:

(a) $y = 6x^{\frac{7}{3}} - 7x^2 + 4$

$$\frac{dy}{dx} = 6 \times \frac{7}{3} x^{\frac{4}{3}} - 14x$$

$$\frac{dy}{dx} = 14x^{\frac{4}{3}} - 14x$$

(b) $\frac{d^2y}{dx^2} = \frac{56}{3} x^{\frac{1}{3}} - 14$

(c) $\frac{dy}{dx} = 0 \Rightarrow x^{\frac{4}{3}} - x = 0 \Rightarrow x \left(x^{\frac{1}{3}} - 1 \right) = 0$

So $x = 0$ or 1

$$x = 0 \Rightarrow \frac{d^2y}{dx^2} = -14 < 0 \therefore (0, 4) \text{ is a maximum}$$

$$x = 1 \Rightarrow \frac{d^2y}{dx^2} = \frac{56}{3} - 14 > 0 \therefore (1, 3) \text{ is a minimum}$$