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Revision Exercises 2

Exercise A, Question 1

Question:

Expand and simplify $(1 - x)^5$.

Solution:

$(1 - x)^5 = 1 + 5(-x) + 10(-x)^2 + 10(-x)^3 + 5(-x)^4 + (-x)^5$

$= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$

Compare $(1 + x)^n$ with $(1 - x)^n$. Replace n by 5 and 'x' by $-x$.

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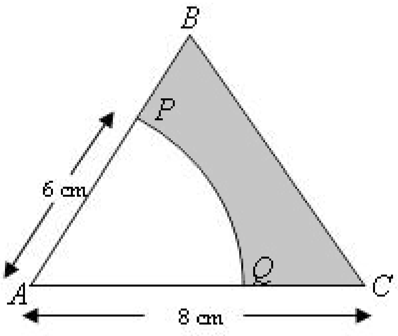
Revision Exercises 2
Exercise A, Question 2

Question:

In the diagram, ABC is an equilateral triangle with side 8 cm. PQ is an arc of a circle centre A, radius 6 cm. Find the perimeter of the shaded region in the diagram.

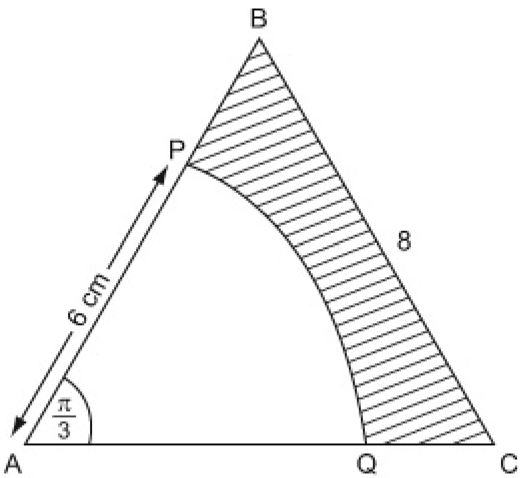


Solution:

Remember: The length of an arc of a circle is $L = r\theta$.

The area of a sector is $A = \frac{1}{2}r^2\theta$.

Draw a diagram. Remember: $60° = \frac{\pi}{3}$ radius



Length of arc $PQ = r\theta$

$= 6 \left(\frac{\pi}{3} \right)$

$= 2\pi$ cm

Perimeter of shaded region

$= 2 + 8 + 2 + 2\pi$

$= 12 + 2\pi$

$= 18.28$ cm

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Revision Exercises 2
Exercise A, Question 3

Question:

The sum to infinity of a geometric series is 15. Given that the first term is 5,

(a) find the common ratio,

(b) find the third term.

Solution:

(a)

$\frac{a}{1-r} = 15 , \quad a = 5$

$\frac{5}{1-r} = 15$

$1 - r = \frac{1}{3}$

$r = \frac{2}{3}$

Remember: $s_{\infty} = \frac{a}{1-r}$, where $|r| < 1$. Here $s_{\infty} = 15$ and $a = 5$ so that $15 = \frac{5}{1-r}$.

(b)

$ar^2 = 5 \left(\frac{2}{3} \right)^2$

$= 5 \times \frac{4}{9}$

$= \frac{20}{9}$

Remember: nth term $= ar^{n-1}$. Here $a = 5$, $r = \frac{2}{3}$ and $n = 3$, so that

$ar^{n-1} = 5 \left(\frac{2}{3} \right)^{3-1}$

$= 5 \left(\frac{2}{3} \right)^2$

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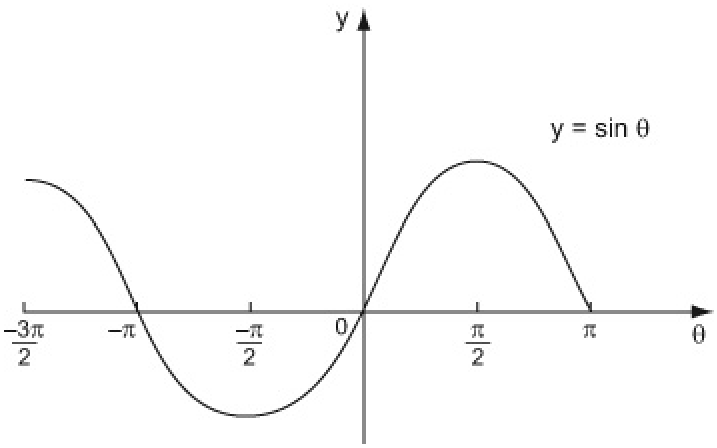
Revision Exercises 2
Exercise A, Question 4

Question:

Sketch the graph of $y = \sin\ \theta°$ in the interval $-\frac{3\pi}{2} \le \theta < \pi$.

Solution:



Remember: $180° = \pi$ radians

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Revision Exercises 2

Exercise A, Question 5

Question:

Find the first three terms, in descending powers of b, of the binomial expansion of $(2a+3b)^6$, giving each term in its simplest form.

Solution:

$$(2a+3b)^6 = (2a)^6 + \binom{6}{1}(2a)^5(3b) + \binom{6}{2}(2a)^4(3b)^2 + \cdots$$

$$= 2^6a^6 + 6 \times 2^5 \times 3 \times a^5b + 15 \times 2^4 \times 3^2 \times a^4b^2 + \cdots$$

$$= 64a^6 + 576a^5b + 2160a^4b^2 + \cdots$$

Compare $(2a+3b)^n$ with $(a+b)^n$. Replace n by 6, 'a' by $2a$ and 'b' by $3b$.

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Revision Exercises 2
Exercise A, Question 6

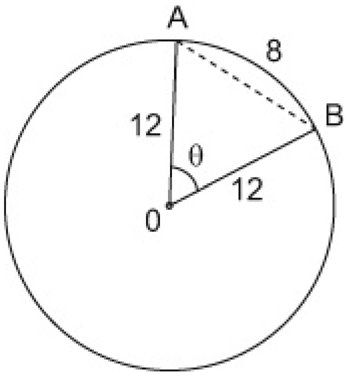
Question:

AB is an arc of a circle centre O. Arc $AB = 8$ cm and $OA = OB = 12$ cm.

(a) Find, in radians, $\angle AOB$.

(b) Calculate the length of the chord AB, giving your answer to 3 significant figures.

Solution:



Draw a diagram. Let $\angle \text{AOB} = \theta$.

(a)

$\angle \text{AOB} = \theta$

$12\theta = 8$

so $\theta = \frac{2}{3}$

Use $l = r\theta$. Here $l = 8$ and $r = 12$.

(b)

$AB^2 = 12^2 + 12^2 - 2(12)(12)\cos\left(\frac{2}{3}\right)$

$= 61.66 \ldots$

$AB = 7.85 \text{ cm } (3 \text{ s.f.})$

Use the cosine formula $c^2 = a^2 + b^2 - 2ab \cos C$. Here $c =$ AB, $a = 12$, $b = 12$ and $C = \theta = \frac{2}{3}$. Remember to change your to radians.

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Revision Exercises 2
Exercise A, Question 7

Question:

A geometric series has first term 4 and common ratio r. The sum of the first three terms of the series is 7.

(a) Show that $4r^2 + 4r - 3 = 0$.

(b) Find the two possible values of r.

Given that r is positive,

(c) find the sum to infinity of the series.

Solution:

(a)

$4 , \quad 4r , \quad 4r^2 , \quad \ldots$

$4 + 4r + 4r^2 = 7$

$4r^2 + 4r - 3 = 0$ (as required)

Use ar^{n-1} to write down expressions for the first 3 terms. Here $a = 4$ and $n = 1 , \quad 2 , \quad 3$.

(b)

$4r^2 + 4r - 3 = 0$

$(2r - 1) (2r + 3) = 0$

$r = \frac{1}{2} , \quad r = \frac{-3}{2}$

Factorize $4r^2 + 4r - 3$. $ac = -12$. $(-2) + (+6) = +4$, so

$$4r^2 - 2r + 6r - 3 = 2r (2r - 1) + 3 (2r - 1) = (2r - 1) (2r + 3) .$$

(c)

$r = \frac{1}{2}$

$\frac{a}{1-r} = \frac{4}{1 - \frac{1}{2}} = 8$

Use $S_\infty = \frac{a}{1-r}$. Here $a = 4$ and $r = \frac{1}{2}$, so that

$$\frac{a}{1-r} = \frac{4}{1 - \frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8.$$

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Revision Exercises 2
Exercise A, Question 8

Question:

(a) Write down the number of cycles of the graph $y = \sin\ nx$ in the interval $0 \le x \le 360°$.

(b) Hence write down the period of the graph $y = \sin\ nx$.

Solution:

(a)

n	Consider the graphs of $y = \sin\ x$, $y = \sin\ 2x$, $y = \sin\ 3x$ … $y = \sin\ x$ has 1 cycle in the interval $0 \le x \le 360°$. $y = \sin\ 2x$ has 2 cycles in the interval $0 \le x \le 360°$. $y = \sin\ 3x$ has 3 cycles in the interval $0 \le x \le 360°$. etc. So $y = \sin\ nx$ has n cycles in the internal $0 \le x \le 360°$.

(b)

$\frac{360°}{n}$ (or $\frac{2\pi}{n}$)	Period = length of cycle. If there are n cycles in the interval $0 \le x \le 360°$, the length of each cycle will be $\frac{360°}{n}$.

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Revision Exercises 2
Exercise A, Question 9

Question:

(a) Find the first four terms, in ascending powers of x, of the binomial expansion of $\left(1 + px\right)^7$, where p is a non-zero constant.

Given that, in this expansion, the coefficients of x and x^2 are equal,

(b) find the value of p,

(c) find the coefficient of x^3.

Solution:

(a)

$$\left(1 + px\right)^7 = 1 + nx + \frac{n\left(n-1\right)}{2!}x^2 + \frac{n\left(n-1\right)\left(n-2\right)}{3!}x^3 + \ldots$$

$$= 1 + 7\left(px\right) + \frac{7\left(6\right)}{2!}\left(px\right)^2 + \frac{7\left(6\right)\left(5\right)}{3!}\left(px\right)^3 + \ldots$$

$$= 1 + 7px + 21p^2x^2 + 35p^3x^3 + \ldots$$

Compare $\left(1 + x\right)^n$ with $\left(1 + px\right)^n$. Replace n by 7 and 'x' by px.

(b)

$7p = 21p^2$

$p \neq 0$, so $7 = 21p$

$p = \frac{1}{3}$

The coefficients of x and x^2 are equal, so $7p = 21p^2$.

(c)

$35p^3 = 35\left(\frac{1}{3}\right)^3 = \frac{35}{27}$

The coefficient of x^3 is $35p^3$. Here $p = \frac{1}{3}$, so that $35p^3 = 35\left(\frac{1}{3}\right)^3$.

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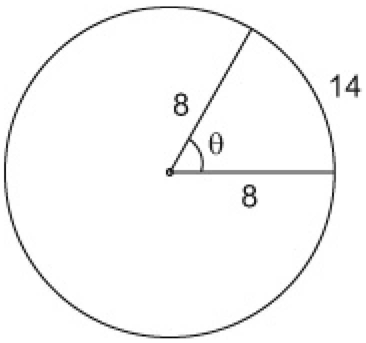
Revision Exercises 2
Exercise A, Question 10

Question:

A sector of a circle of radius 8 cm contains an angle of θ radians. Given that the perimeter of the sector is 30 cm, find the area of the sector.

Solution:



Draw a diagram. Perimeter of sector $= 30$cm, so arc length $= 14$ cm.

$8\theta = 14$

$\theta = \frac{14}{8}$

Find the value of θ. Use $L = r\theta$. Here $L = 14$ and $r = 8$ so that $8\theta = 14$.

Area of sector $= \frac{1}{2}(8)^2\theta$

$= \frac{1}{2}(8)^2\left(\frac{14}{8}\right)$

$= 56 \text{ cm}^2$

Use $A = \frac{1}{2}r^2\theta$. Here $r = 8$ and $\theta = \frac{14}{8}$, so that $A = \frac{1}{2}(8)^2\left(\frac{14}{8}\right)$.

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Revision Exercises 2

Exercise A, Question 11

Question:

A pendulum is set swinging. Its first oscillation is through 30 ° . Each succeeding oscillation is $\frac{9}{10}$ of the one before it. What is the total angle described by the pendulum before it stops?

Solution:

$30 , \quad 30 \left(\frac{9}{10} \right) , \quad 30 \left(\frac{9}{10} \right)^2 , \ldots$

Write down the first 3 term. Use ar^{n-1}. Here $a = 30$, $r = \frac{9}{10}$ and $n = 1 , 2 , 3$.

$\frac{a}{1-r} = \frac{30}{1 - \frac{9}{10}}$

$= \frac{30}{\left(\frac{1}{10} \right)}$

$= 300°$

Use $S_\infty = \frac{a}{1-r}$. Here $a = 30$ and $r = \frac{9}{10}$ so that $S_\infty = \frac{30}{1 - \frac{9}{10}}$.

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Revision Exercises 2
Exercise A, Question 12

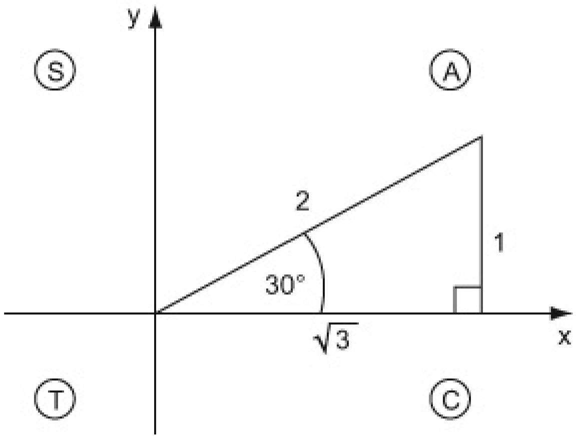
Question:

Write down the exact value

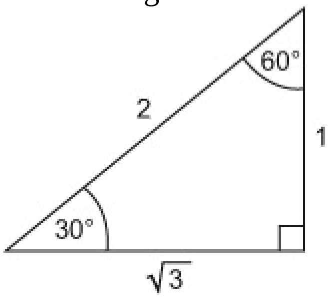
(a) $\sin 30°$, (b) $\cos 330°$, (c) $\tan(-60°)$.

Solution:

(a)

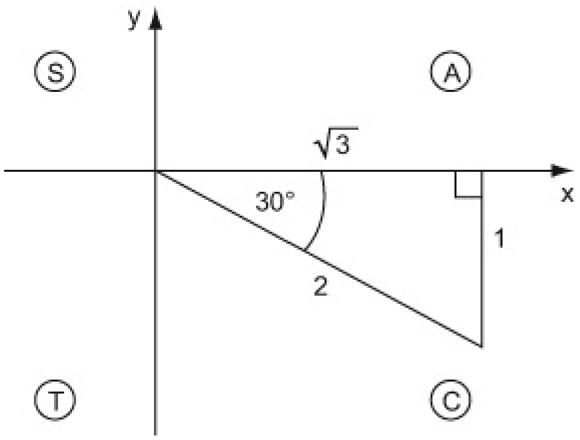


Draw a diagram for each part. Remember



$\sin 30° = \frac{1}{2}$

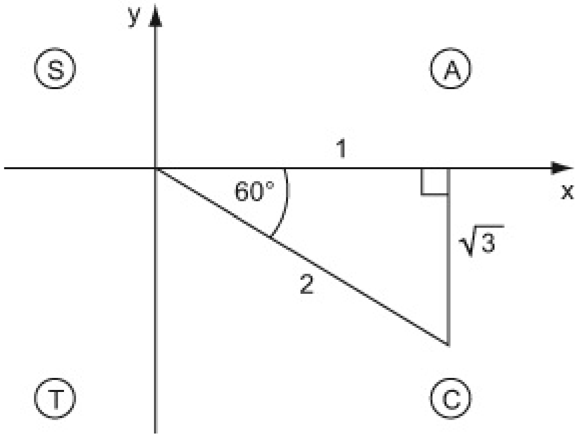
(b)



$\cos 330° = \cos 30° = \frac{\sqrt{3}}{2}$

330° is in the fourth quadrant.

(c)



$\tan(-60°) = -\tan 60°$ — $-60°$ is in the fourth quadrant.

$= -\frac{\sqrt{3}}{1}$

$= -\sqrt{3}$

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Revision Exercises 2
Exercise A, Question 13

Question:

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of $(1 - ax)^8$, where a is a non-zero integer.

The first three terms are 1, $-24x$ and bx^2, where b is a constant.

(b) Find the value of a and the value of b.

Solution:

(a)

$(1 - ax)^8 = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots$

$= 1 + 8(-ax) + \frac{8(8-1)}{2!}(-ax)^2 + \dots$

$= 1 - 8ax + 28a^2x^2 + \dots$

Compare $(1 + x)^n$ with $(1 - ax)^n$ Replace n by 8 and 'x' by $-ax$.

(b)

$-8a = -24$

$a = 3$

Compare coefficients of x, so that $-8a = -24$.

$b = 28a^2$

$= 28(3)^2$

$= 252$

so $a = 3$ and $b = 252$

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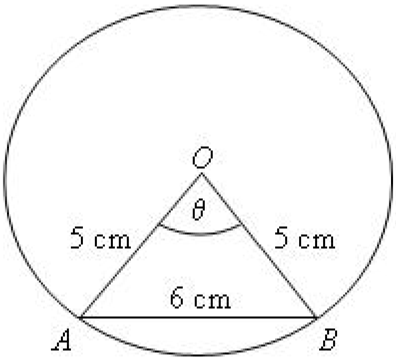
Revision Exercises 2
Exercise A, Question 14

Question:

In the diagram, A and B are points on the circumference of a circle centre O and radius 5 cm.
$\angle AOB = \theta$ radians.
$AB = 6$ cm.

(a) Find the value of θ.

(b) Calculate the length of the minor arc AB.



Solution:

(a)

$$\cos\theta = \frac{5^2 + 5^2 - 6^2}{2(5)(5)}$$
$$= \frac{7}{25}$$
$$\theta = 1.287 \text{ radians}$$

Use the cosine formula $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$. Here $c = \theta$, $a = 5$, $b = 5$ and $c = 6$.

(b)

$$\text{arc } AB = 5\theta$$
$$= 5 \times 1.287$$
$$= 6.44 \text{ cm}$$

Use $C = r\theta$. Here C = arc AB, $r = 5$ and $\theta = 1.287$ radians.

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Revision Exercises 2
Exercise A, Question 15

Question:

The fifth and sixth terms of a geometric series are 4.5 and 6.75 respectively.

(a) Find the common ratio.

(b) Find the first term.

(c) Find the sum of the first 20 terms, giving your answer to 3 decimal places.

Solution:

(a)

$ar^4 = 4.5,\ ar^5 = 6.75$

$\frac{ar^5}{ar^4} = \frac{6.75}{4.5}$

$r = \frac{3}{2}$

Find r. Divide ar^5 by ar^4

so that $\frac{ar^5}{ar^4} = \frac{ar^{5-4}}{a}$

$= r$

and $\frac{6.75}{4.5} = 1.5.$

(b)

$a(1.5)^4 = 4.5$

$a = \frac{4.5}{(1.5)^4}$

$= \frac{8}{9}$

(c)

$S_{20} = \frac{\frac{8}{9}\left((1.5)^{20} - 1\right)}{1.5 - 1}$

$= 5909.790 \quad (3 \text{ d.p.})$

Use $S_n = \frac{a(r^n - 1)}{r - 1}$. Here $a = \frac{8}{9}$, $r = 1.5$ and $n = 20$.

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Revision Exercises 2

Exercise A, Question 16

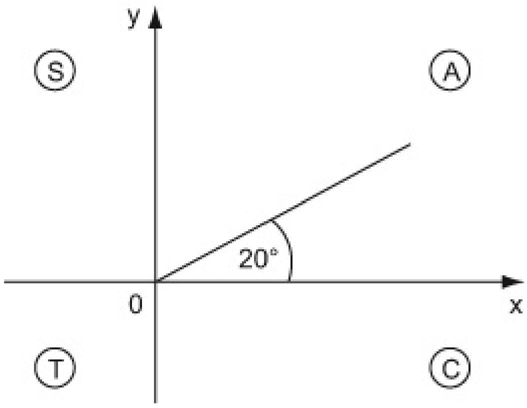
Question:

Given that θ is an acute angle measured in degrees, express in term of $\cos 2\theta$

(a) $\cos(360° + 2\theta)$, (b) $\cos(-2\theta)$, (c) $\cos(180° - 2\theta)$

Solution:

(a)

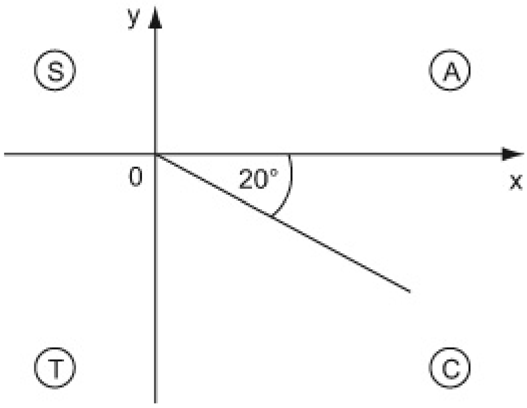


Draw a diagram for each part.

$\cos(360° + 2\theta) = \cos 2\theta$

$360° + 2\theta$ is in the first quadrant.

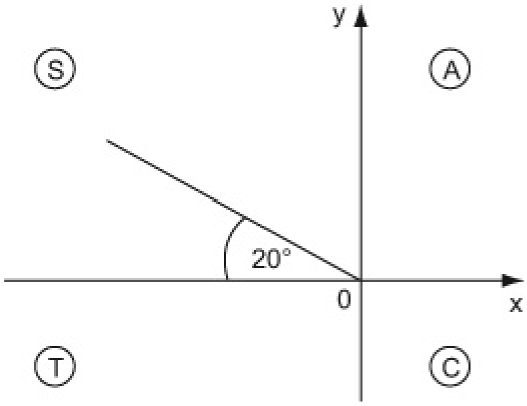
(b)



$\cos(-2\theta) = \cos 2\theta$

-2θ is in the fourth quadrant.

(c)



$\cos\ (180° - 2\theta) = -\cos\ 2\theta$ $180° - 2\theta$ is in the second quadrant.

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Revision Exercises 2

Exercise A, Question 17

Question:

(a) Expand $(1-2x)^{10}$ in ascending powers of x up to and including the term in x^3.

(b) Use your answer to part (a) to evaluate $(0.98)^{10}$ correct to 3 decimal places.

Solution:

(a)

$$(1-2x)^{10} = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

$$= 1 + 10(-2x) + \frac{10(a)}{2}(-2x)^2 + \frac{10(9)(8)}{6}(-2x)^3 + \cdots$$

$$= 1 - 20x + 180x^2 - 960x^3 + \cdots$$

Compare $(1+x)^n$ with $(1-2x)^n$. Replace n by 10 and 'x' by $-2x$.

(b)

$$(1-2(0.01))^{10} = 1 - 20(0.01) + 180(0.01)^2 - 960(0.01)^3 + \cdots$$

$$0.98^{10} \simeq 0.817 \quad (3 \text{ d.p.})$$

Find the value of x.

$$0.98 = 1 - 0.02$$

$$= 1 - 2(0.01)$$

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Revision Exercises 2
Exercise A, Question 18

Question:

In the diagram,

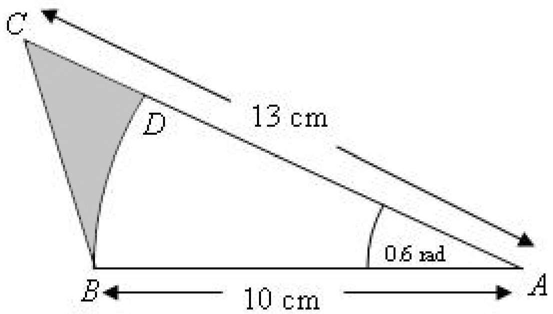
$AB = 10$ cm, $AC = 13$ cm.

$\angle CAB = 0.6$ radians.

BD is an arc of a circle centre A and radius 10 cm.

(a) Calculate the length of the arc BD.

(b) Calculate the shaded area in the diagram.



Solution:

(a)

$\text{arc BD} = 10 \times 0.6$
$= 6$ cm

Use $L = r\theta$. Here L = arc BD, $r = 10$ and $\theta = 0.6$ radians.

(b)

Shaded area

$= \frac{1}{2}(10)(13)\sin(0.6) - \frac{1}{2}(10)^2(0.6)$

$= 6.70 \text{ cm}^2$ (3 s.f.)

Use area of triangle $= \frac{1}{2}bc \sin A$ and area of sector $= \frac{1}{2}r^2\theta$. Here $b = 13$, $c = 10$ and A = (θ =) 0.6 ; $r = 10$ and $\theta = 0.6$.

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Revision Exercises 2
Exercise A, Question 19

Question:

The value of a gold coin in 2000 was £180. The value of the coin increases by 5% per annum.

(a) Write down an expression for the value of the coin after *n* years.

(b) Find the year in which the value of the coin exceeds £360.

Solution:

$180, 180(1.05), 180(1.05)^2, \ldots$

Write down the first 3 terms. Use ar^{n-1}. Here $a = 180$, $r = 1.05$ and $n = 1, 2, 3$.

(a)

Value after n years $= 180(1.05)^n$

(b)

$180(1.05)^n > 360$

$180(1.05)^{14} = 356.39$

$180(1.05)^{15} = 374.21$

Substitute values of n. The value of the coin after 14 years is £ 356.39, and after 15 years is £ 374.21. So the value of the coin will exceed £ 360 in the 15th year.

The value of the coin will exceed £ 360 in 2014.

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Revision Exercises 2
Exercise A, Question 20

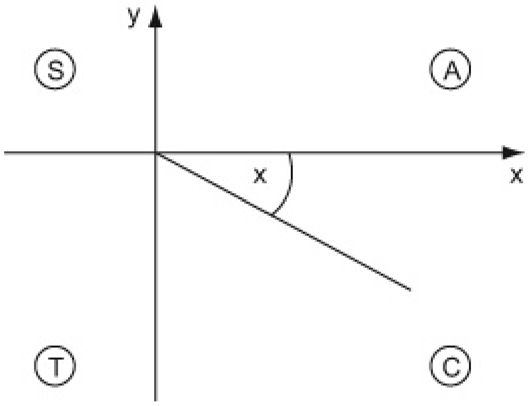
Question:

Given that x is an acute angle measured in radians, express in terms of $\sin x$

(a) $\sin(2\pi - x)$, (b) $\sin(\pi + x)$, (c) $\cos\left(\frac{\pi}{2} - x\right)$.

Solution:

(a)



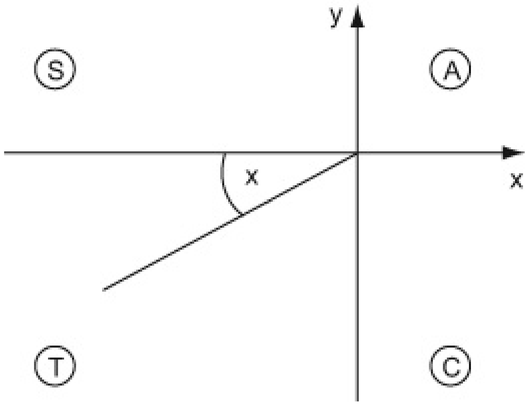
Draw a diagram for each part.

Remember: π Radians $= 180°$

$\sin(2\pi - x) = -\sin x$

$2\pi - x$ is in the fourth quadrant.

(b)

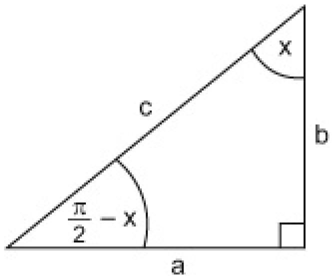


$\sin(\pi + x) = -\sin x$

$\pi + x$ is in the third quadrant.

(c)

$180° = \pi$ radians, so $90° = \frac{\pi}{2}$ radians.



$\cos\left(\frac{\pi}{2} - x\right) = \sin x \left(= \frac{a}{c}\right)$.

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Revision Exercises 2
Exercise A, Question 21

Question:

Expand and simplify $\left(x - \frac{1}{x} \right)^6$

Solution:

$$\left(x - \frac{1}{x} \right)^6 = x^6 + \binom{6}{1} x^5 \left(\frac{-1}{x} \right) + \binom{6}{2} x^4 \left(\frac{-1}{x} \right)^2 + \binom{6}{3} x^3 \left(\frac{-1}{x} \right)^3 + \binom{6}{4} x^2 \left(\frac{-1}{x} \right)^4 + \binom{6}{5} x \left(\frac{-1}{x} \right)^5 + \left(\frac{-1}{x} \right)^6$$

Compare $\left(x - \frac{1}{x} \right)^n$ with $(a + b)^n$. Replace n by 6, 'a' with x and 'b' with $\frac{-1}{x}$.

$$= x^6 + 6x^5 \left(\frac{-1}{x} \right) + 15x^4 \left(\frac{1}{x^2} \right) + 20x^3 \left(\frac{-1}{x^3} \right) + 15x^2 \left(\frac{1}{x^4} \right) + 6x \left(\frac{-1}{x^5} \right) + \frac{1}{x^6}$$

$$\left(\frac{-1}{x} \right)^2 = \frac{-1}{x} \times \frac{-1}{x} = \frac{1}{x^2}$$

$$\left(\frac{-1}{x} \right)^3 = \frac{-1}{x} \times \frac{-1}{x} \times \frac{-1}{x} = \frac{-1}{x^3}$$

$$\left(\frac{-1}{x} \right)^4 = \frac{-1}{x} \times \frac{-1}{x} \times \frac{-1}{x} \times \frac{-1}{x} = \frac{1}{x^4}$$

$$= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}$$

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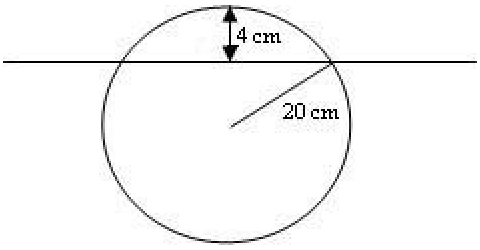
Revision Exercises 2

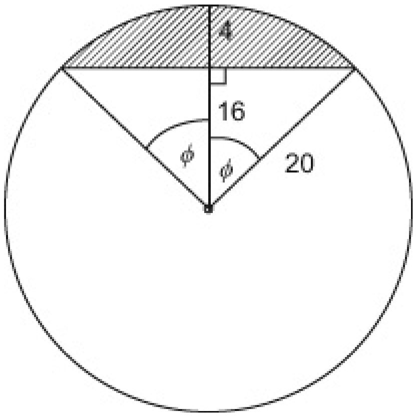
Exercise A, Question 22

Question:

A cylindrical log, length 2m, radius 20 cm, floats with its axis horizontal and with its highest point 4 cm above the water level. Find the volume of the log in the water.



Solution:



Draw a diagram. Let sector angle = 2φ.

$\cos\ \varphi = \frac{16}{20}\ (\ = 0.8\ )$

Area above water level $= \frac{1}{2}r^2\ (\ 2\varphi\ )\ -\ \frac{1}{2}r^2\sin\ (\ 2\varphi\ )$

$= \frac{1}{2}\ (\ 20\ )^2\ (\ 2\varphi\ )\ -\ \frac{1}{2}\ (\ 20\ )^2\sin\ (\ 2\varphi\ )$

$= 65.40\ \text{cm}^2$

Use area of segment $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\ \theta$. Here $r = 20$ cm and $\theta = 2 \times \cos^{-1}(\ 0.8\ )$

Area below water level $= \pi\ (\ 20\ )^2 - 65.40$

$= 1191.24\ \text{cm}^2$

Volume below water level $= 1191.24 \times 200$

$= 238248\ \text{cm}^3$

$(\ = 0.238\ \text{cm}^3\ )$

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Revision Exercises 2
Exercise A, Question 23

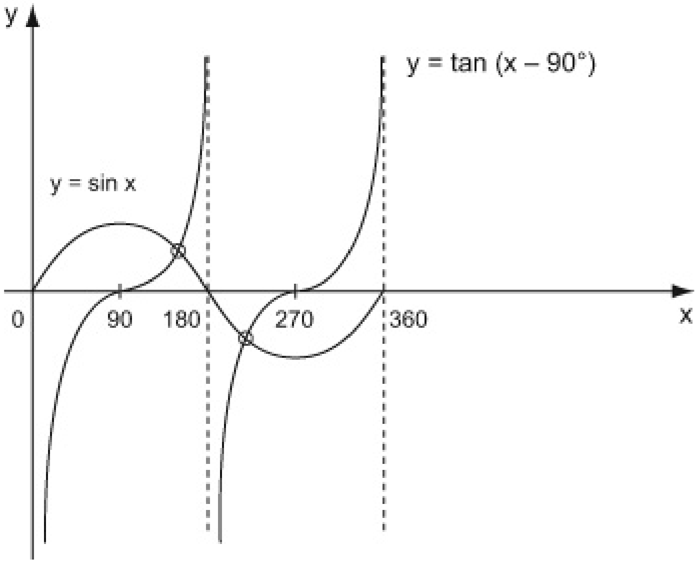
Question:

(a) On the same axes, in the interval $0 \le x \le 360°$, sketch the graphs of $y = \tan(x - 90°)$ and $y = \sin x$.

(b) Hence write down the number of solutions of the equation $\tan(x - 90°) = \sin x$ in the interval $0 \le x \le 360°$.

Solution:

(a)



$y = \tan(x - 90°)$ is a translation of $y = \tan x$ by $+90°$ in the x-direction.

(b)

2 solutions in the interval $0 \le x \le 360$.

From the sketch, the graphs of $y = \tan(x - 90°)$ and $y = \sin x$ meet at two points. So there are 2 solutions in the internal $0 \le x \le 360°$.

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Revision Exercises 2
Exercise A, Question 24

Question:

A geometric series has first term 4 and common ratio $\frac{4}{3}$. Find the greatest number of terms the series can have without its sum exceeding 100.

Solution:

$a = 4 \quad , \quad r = \frac{4}{3}$

$$S_n = \frac{4\left(\left(\frac{4}{3}\right)^n - 1\right)}{\frac{4}{3} - 1}$$

Use $S_n = \frac{a(r^n - 1)}{r - 1}$. Here $a = 4$ and $r = \frac{4}{3}$.

$$= \frac{4\left(\left(\frac{4}{3}\right)^n - 1\right)}{\frac{1}{3}}$$

$$= 12\left(\left(\frac{4}{3}\right)^n - 1\right)$$

Now, $12\left(\left(\frac{4}{3}\right)^n - 1\right) < 100$

$$\left(\frac{4}{3}\right)^n - 1 < \frac{100}{12}$$

$$\left(\frac{4}{3}\right)^n < \frac{100}{12} + 1$$

$$\left(\frac{4}{3}\right)^n < 9\frac{1}{3}$$

$$\left(\frac{4}{3}\right)^7 = 7.492$$

$$\left(\frac{4}{3}\right)^8 = 9.990$$

so $\quad n = 7$

Substitute values of n. The largest value of n for which $\left(\frac{4}{3}\right)^n < 9^{1/3}$ is 7.

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Revision Exercises 2
Exercise A, Question 25

Question:

Describe geometrically the transformation which maps the graph of

(a) $y = \tan x$ onto the graph of $y = \tan(x - 45°)$,

(b) $y = \sin x$ onto the graph of $y = 3\sin x$,

(c) $y = \cos x$ onto the graph of $y = \cos \frac{x}{2}$,

(d) $y = \sin x$ onto the graph of $y = \sin x - 3$.

Solution:

(a) A translation of $+45°$ in the x direction
(b) A stretch of scale factor 3 in the y direction
(c) A stretch of scale factor 2 in the x direction
(d) A translation of -3 in the y direction

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Revision Exercises 2
Exercise A, Question 26

Question:

If x is so small that terms of x^3 and higher can be ignored, and $(2-x)(1+2x)^5 \approx a+bx+cx^2$, find the values of the constants a, b and c.

Solution:

$(1+2x)^5 = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$

Compare $(1+x)^n$ with $(1+2x)^n$. Replace n by 5 and 'x' by $2x$.

$= 1 + 5(2x) + \frac{5(4)}{2}(2x)^2 + \ldots$

$= 1 + 10x + 40x^2 + \ldots$

$(2-x)(1+10x+40x^2+\ldots)$

$= 2 + 20x + 80x^2 + \ldots$

$\quad - x - 10x^2 + \ldots$

$2 + 19x + 70x^2 + \ldots$

Expand $(2-x)(1+10x+40x^2+\ldots)$ ignoring terms in x^3, so that

$2 \times (1+10x+40x^2+\ldots)$

$= 2 + 20x + 80x^2 + \ldots$

and

$-x \times (1+10x+40x^2+\ldots)$

$= -x - 10x^2 - 40x^3$

simplify so that

$2 + 20x - x + 80x^2 - 10x^2 + \ldots$

$= 2 + 19x + 70x^2 + \ldots$

$(2-x)(1+2x)^5 \simeq 2 + 19x + 70x^2$

so $a = 2$, $b = 19$, $c = 70$

Compare $2 + 19x + 70x^2 + \ldots$ with $a + bx + cx^2 + \ldots$ so that $a = 2$, $b = 19$ and $c = 70$.

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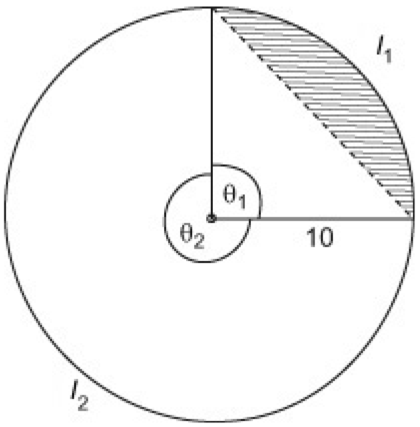
Revision Exercises 2
Exercise A, Question 27

Question:

A chord of a circle, radius 20 cm, divides the circumference in the ratio 1:3.

Find the ratio of the areas of the segments into which the circle is divided by the chord.

Solution:



Draw a diagram. Let the minor are l_1 have angle θ_1 and the major are l_2 have angle θ_2.

$l_1 : l_2 = 1 : 3$

$10\theta_1 : 10\theta_2 = 1 : 3$

$\theta_1 : \theta_2 = 1 : 3$

Use $l = r\theta$ so that $l_1 = 10\theta_1$ and $l_2 = 10\theta_2$.

so $\theta_1 = \frac{1}{4} \times 2\pi = \frac{\pi}{2}$

Shaded area $= \frac{1}{2}(10)^2\theta_1 - \frac{1}{2}(10)^2\sin\theta_1$

$= 25\pi - 50$

Use area of segment $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$. Here $r = 10$ and $\theta = \theta_1 = \frac{\pi}{2}$

Area of large segment $= \pi(10)^2 - (25\pi - 50)$

$= 100\pi - 25\pi + 50$

$= 75\pi + 50$

Ratio of small segment to large segment is

$25\pi - 50 : 75\pi + 50$

Divide throughout by 25.

$\pi - 2 : 3\pi + 2$

or $1 : \frac{3\pi + 2}{\pi - 2}$

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Revision Exercises 2
Exercise A, Question 28

Question:

x, 3 and $x + 8$ are the fourth, fifth and sixth terms of geometric series.

(a) Find the two possible values of x and the corresponding values of the common ratio.

Given that the sum to infinity of the series exists,

(b) find the first term,

(c) the sum to infinity of the series.

Solution:

$ar^3 = x$

$ar^4 = 3$

$ar^5 = x + 8$

$\frac{ar^5}{ar^4} = \frac{ar^4}{ar^3}$

$\frac{ar^5}{ar} = r$ and $\frac{ar^4}{ar^3} = r$ so $\frac{ar^5}{ar^4} = \frac{ar^4}{ar^3}$.

so $\frac{x+8}{3} = \frac{3}{x}$

Clear the fractions. Multiply each side by $3x$ so that $3x \times \frac{x+8}{3} = x(x+8)$ and $3x \times \frac{3}{x} = 9$.

$x(x+8) = 9$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x = 1, x = -9$

$r = \frac{ar^4}{ar^3} = \frac{x}{3}$

When $x = 1$, $r = \frac{1}{3}$

When $x = -9$, $r = -3$

Find r. Substitute $x = 1$, then $x = -9$, into $\frac{ar^4}{ar^3} = \frac{x}{3}$, so that $r = \frac{1}{3}$ and $r = \frac{-9}{3} = -3$.

(b)

$r = \frac{1}{3}$

Remember $S_\infty = \frac{a}{1-r}$ for $|r| < 1$, so $r = \frac{1}{3}$.

$ar^4 = 3$

$a\left(\frac{1}{3}\right)^4 = 3$

$a = 243$

(c)

$$\frac{a}{1-r} = \frac{243}{1-\frac{1}{3}} = 364\frac{1}{2}$$

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Revision Exercises 2
Exercise A, Question 29

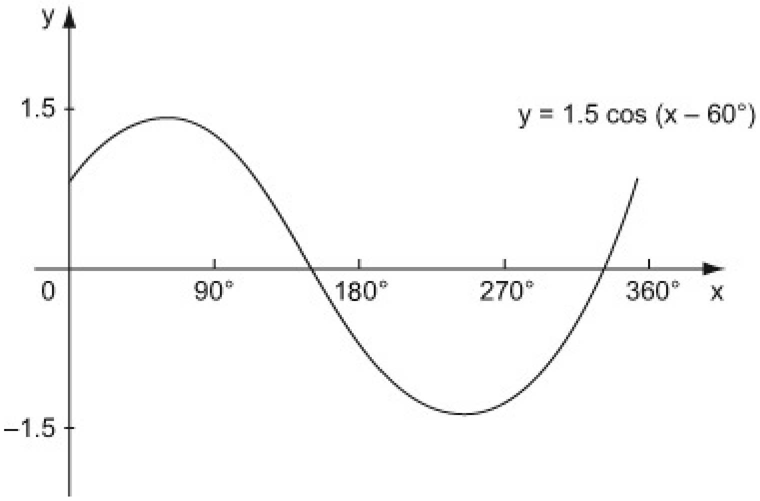
Question:

(a) Sketch the graph of $y = 1.5 \cos (x - 60°)$ in the interval $0 \le x < 360°$

(b) Write down the coordinates of the points where your graph meets the coordinate axes.

Solution:

(a)



(b)

When $x = 0$,

$y = 1.5 \cos (-60°)$
$= 0.75$
so $(0, 0.75)$

The graph of $y = 1.5 \cos (x - 60°)$ meets the y axis when $x = 0$. Substitute $x = 0$ into $y = 1.5 \cos (x - 60°)$ so that $y = 1.5 \cos (-60°)$. $\cos (-60°) = \cos 60° = \frac{1}{2}$ so $y = 1.5 \times \frac{1}{2} = 0.75$

$y = 1.5 \cos (x - 60°)$

$y = 0$,

when $x = 90° + 60°$
$= 150°$

and $x = 270° + 60°$
$= 330°$

The graph of $y = 1.5 \cos (x - 60°)$ meets the x-axis when $y = 0$. $\cos (x - 60°)$ represents a translation of $\cos x$ by $+60°$ in the x-direction $\cos x$ meets the x-axis at $90°$ and $270°$, so $y = 1.5 (\cos x - 60°)$ meets the x-axis at $90° + 60° = 150°$ and $270° + 60° = 330°$.

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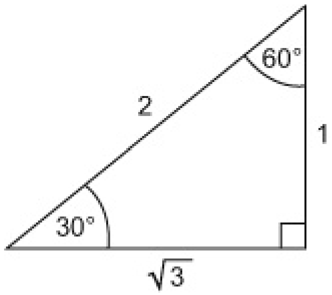
Revision Exercises 2
Exercise A, Question 30

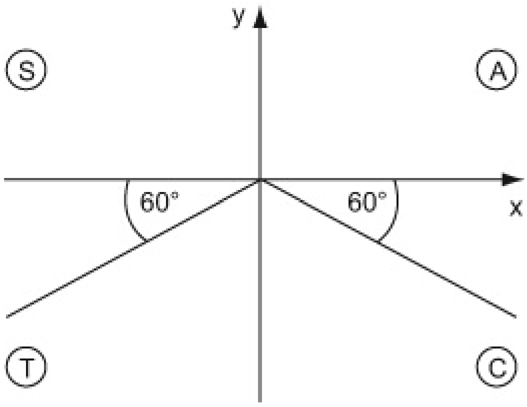
Question:

Without using a calculator, solve $\sin\left(x-20^{\circ}\right)=-\frac{\sqrt{3}}{2}$ in the interval $0 \le x \le 360^{\circ}$.

Solution:



$\sin 60^{\circ}=\frac{\sqrt{3}}{2}$



$\sin x=-\frac{\sqrt{3}}{2}$. $\sin x$ is negative in the 3rd and 4th quadrants.

$\sin\left(240^{\circ}\right)=-\frac{\sqrt{3}}{2}$

$\sin\left(300^{\circ}\right)=-\frac{\sqrt{3}}{2}$

so $x-20^{\circ}=240^{\circ}$

$x=260^{\circ}$

and $x-20^{\circ}=300^{\circ}$

$x=320^{\circ}$

$\sin 240^{\circ}=-\frac{\sqrt{3}}{2}$ but $\sin\left(x-20^{\circ}\right)=-\frac{\sqrt{3}}{2}$ so $x-20^{\circ}=240^{\circ}$, i.e. $x=260^{\circ}$. Similarly $\sin 300^{\circ}=-\frac{\sqrt{3}}{2}$ but $\sin\left(x-20^{\circ}\right)=-\frac{\sqrt{3}}{2}$ so $x-20^{\circ}=300^{\circ}$, i.e. $x=320^{\circ}$.