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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise A, Question 1

Question:

Simplify $\frac{x^2 - 2x - 3}{x^2 - 7x + 12}$.

Solution:

$$\frac{x^2 - 2x - 3}{x^2 - 7x + 12}$$

$$= \frac{(x - 3)(x + 1)}{(x - 3)(x - 4)}$$

Factorise $x^2 - 2x - 3$:
$(-3) \times (+1) = -3$
$(-3) + (+1) = -2$
so $x^2 - 2x - 3 = (x - 3)(x + 1)$
Factorise $x^2 - 7x + 12$:
$(-3) \times (-4) = +12$
$(-3) + (-4) = -7$
so $x^2 - 7x + 12 = (x - 3)(x - 4)$

$$= \frac{x + 1}{x - 4}$$

Divide top and bottom by $(x - 3)$

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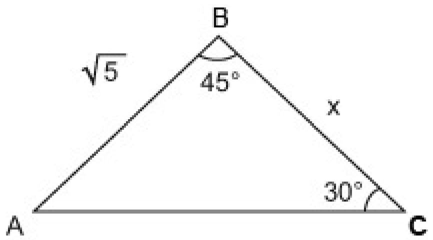
Algebra and functions

Exercise A, Question 2

Question:

In ΔABC, AB = $\sqrt{5}$cm, $\angle ABC = 45°$, $\angle BCA = 30°$. Find the length of BC.

Solution:



Draw a diagram to show the given information

$$\frac{x}{\sin A} = \frac{\sqrt{5}}{\sin 30°}$$

Use the sine rule $\frac{a}{\sin A} = \frac{c}{\sin C}$, where $a = x$, $c = \sqrt{5}$ and $C = 30°$

$$A + 30 + 45 = 180°$$
$$A = 105°$$

Find angle A. The angles in a triangle add to 180°.

so $\frac{x}{\sin 105°} = \frac{\sqrt{5}}{\sin 30°}$

$$x = \frac{\sqrt{5}\sin 105°}{\sin 30°}$$

Multiply throughout by sin105°

$$= 4.32$$

Give answer to 3 significant figures

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Algebra and functions
Exercise A, Question 3

Question:

(a) Write down the value of $\log_3 81$

(b) Express $2\log_a 4 + \log_a 5$ as a single logarithm to base a.

Solution:

(a)

$\log_3 81 = \log_3 (3^4)$ — Write 81 as a power of 3, $81 = 3 \times 3 \times 3 \times 3 = 3^4$.

$= 4\log_3 3$ — Use the power law: $\log_a (x^k) = k\log_a x$, so that $\log_3 (3^4) = 4\log_3 3$

$= 4 \times 1$ — Use $\log_a a = 1$, so that $\log_3 3 = 1$.

$= 4$

(b)

$2\log_a 4 + \log_a 5$

$= \log_a 4^2 + \log_a 5$ — Use the power law: $\log_a (x^k) = k\log_a x$, so that $2\log_a 4 = \log_a 4^2$

$= \log_a (4^2 \times 5)$ — Use the, multiplication law: $\log_a xy = \log_a x + \log_a y$ so that $\log_a 4^2 + \log_a 5 = \log_a (4^2 \times 5)$

$= \log_a 80$

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Algebra and functions

Exercise A, Question 4

Question:

P is the centre of the circle $(x-1)^2 + (y+4)^2 = 81$.

Q is the centre of the circle $(x+3)^2 + y^2 = 36$.

Find the exact distance between the points P and Q.

Solution:

$(x-1)^2 + (y+4)^2 = 81$

The Coordinates of P are $(1, -4)$.

Compare $(x-1)^2 + (y+4)^2 = 8$ to $(x-a)^2 + (y-b)^2 = r^2$, where (a, b) is the centre.

$(x+3)^2 + y^2 = 36$

The Coordinates of Q are $(-3, 0)$.

Compare $(x+3)^2 + y^2 = 36$ to $(x-a)^2 + (y-b)^2 = r^2$ where (a, b) is the centre.

$PQ = \sqrt{(-3-1)^2 + (0-(-4))^2}$

use $d = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$, where $(x_1, y_1) = (1, -4)$ and $(x_2, y_2) = (-3, 0)$

$= \sqrt{(-4)^2 + (4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

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Algebra and functions
Exercise A, Question 5

Question:

Divide $2x^3 + 9x^2 + 4x - 15$ by $(x + 3)$.

Solution:

$$\begin{array}{r|l} & 2x^2 \\ \hline x+3 & 2x^3 + 9x^2 + 4x - 15 \\ & 2x^3 + 6x^2 \\ & \qquad 3x^2 + 4x \end{array}$$

Start by dividing the first term of the polynomial by x, so that $2x^3 \div x = 2x^2$. Next multiply $(x + 3)$ by $2x^2$, so that $2x^2 \times (x + 3) = 2x^3 + 6x^2$. Now subtract, so that $(2x^3 + 9x^2) - (2x^3 + 6x^2) = 3x^2$. Copy $+ 4x$.

$$\begin{array}{r|l} & 2x^2 + 3x \\ \hline x+3 & 2x^3 + 9x^2 + 4x - 15 \\ & 2x^3 + 6x^2 \\ & \qquad 3x^2 + 4x \\ & \qquad 3x^2 + 9x \\ & \qquad\qquad -5x - 15 \end{array}$$

Repeat the method. Divide $3x^2$ by x, so that $3x^2 \div x = 3x$. Multiply $(x + 3)$ by $3x$, so that $3x \times (x + 3) = 3x^2 + 9x$. Subtract, so that $(3x^2 + 4x) - (3x^2 + 9x) = -5x$. Copy -15

$$\begin{array}{r|l} & 2x^2 + 3x - 5 \\ \hline x+3 & 2x^3 + 9x^2 + 4x - 15 \\ & 2x^3 + 6x^2 \\ & \qquad 3x^2 + 4x \\ & \qquad 3x^2 + 9x \\ & \qquad\qquad -5x - 15 \\ & \qquad\qquad -5x - 15 \\ 0 & \end{array}$$

Repeat the method. Divide $-5x$ by x, so that $-5x \div x = -5$. Multiply $(x + 3)$ by -5, so that $-5 \times (x + 3) = -5x - 15$. Subtract, so that $(-5x - 15) - (-5x - 15) = 0$.

So $2x^3 + 9x^2 + 4x - 15 \div (x + 3) = 2x^2 + 3x - 5$.

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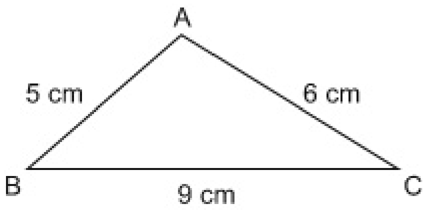
Algebra and functions
Exercise A, Question 6

Question:

In ΔABC, AB = 5cm, BC = 9cm and CA = 6cm. Show that $\cos \angle TRS = -\frac{1}{3}$.

Solution:



Draw a diagram using the given data.

$$\cos \angle \text{BAC} = \frac{5^2 + 6^2 - 9^2}{2 \times 5 \times 6}$$

$$= \frac{25 + 36 - 81}{60}$$

$$= \frac{-20}{60}$$

$$= \frac{-1}{3}$$

Use the Cosine rule $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, where $A = \angle BAC$, $a = 9\ (\text{cm})$, $b = 6\ (\text{cm})$, $c = 5\ (\text{cm})$

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Algebra and functions

Exercise A, Question 7

Question:

(a) Find, to 3 significant figures, the value of x for which $5^x = 0.75$

(b) Solve the equation $2\log_5 x - \log_5 3x = 1$

Solution:

(a)

$5^x = 0.75$

$\log_{10}(5^x) = \log_{10} 0.75$ Take logs to base 10 of each side.

$x\log_{10}5 = \log_{10} 0.75$ Use the Power law: $\log_a(x^k) = k\log_a x$ so that $\log_{10}(5^x) = x\log_{10}5$

$x = \frac{\log_{10} 0.75}{\log_{10}5}$ Divide both sides by $\log_{10}5$

$= -0.179$ Give answer to 3 significant figures

(b)

$2\log_5 x - \log_5 3x = 1$

$\log_5(x^2) - \log_5 3x = 1$ Use the Power law: $\log_a(x^k) = k\log_a x$ so that $2\log_5 x = \log_5(x^2)$

$\log_5\left(\frac{x^2}{3x}\right) = 1$ Use the division law: $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_b y$ so that $\log_5(x^2) - \log_5(3x) = \log_5\left(\frac{x^2}{3x}\right)$.

$\log_5\left(\frac{x}{3}\right) = 1$ Simplify. Divide top and bottom by x, so that $\frac{x^2}{3x} = \frac{x}{3}$.

$\log_5\left(\frac{x}{3}\right) = \log_5 5$ Use $\log_a a = 1$, so that $1 = \log_5 5$

so $\frac{x}{3} = 5$ Compare the logarithms, they each have the same base, so $\frac{x}{3} = 5$.

$x = 15$.

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Algebra and functions
Exercise A, Question 8

Question:

The circle C has equation $(x+4)^2+(y-1)^2=25$.

The point P has coordinates $(-1,5)$.

(a) Show that the point P lies on the circumference of C.

(b) Show that the centre of C lies on the line $x-2y+6=0$.

Solution:

(a)

Substitute $(-1,5)$ into $(x+4)^2+(y-1)^2=25$.

$$(-1+4)^2+(5-1)^2=3^2+4^2$$
$$=9+16$$
$$=25 \text{ as required}$$

so P lies on the circumference of the circle.

Any point (x,y) on the circumference of a circle satisfies the equation of the circle.

(b)

The Centre of C is $(-4,1)$

Compare $(x+4)^2+(y-1)^2=25$ to $(x-a)^2+(y-b)^2=r^2$ where (a,b) is the centre.

Substitute $(-4,1)$ into $x-2y+6=0$

$(-4)-2(1)+6=-4-2+6=0$ As required

so the centre of C lies on the line $x-2y+6=0$.

Any point (x,y) on a line satisfies the equation of the line.

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Algebra and functions
Exercise A, Question 9

Question:

(a) Show that $(2x-1)$ is a factor of $2x^3-7x^2-17x+10$.

(b) Factorise $2x^3-7x^2-17x+10$ completely.

Solution:

(a)

$f(x)=2x^3-7x^2-17x+10$

$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-7\left(\frac{1}{2}\right)^2-17\left(\frac{1}{2}\right)+10$

$=2\times\frac{1}{8}-7\times\frac{1}{4}-17\times\frac{1}{2}+10$

$=\frac{1}{4}-\frac{7}{4}-\frac{17}{2}+10$

$=0$

so, $(2x-1)$ is a factor of $2x^3-7x^2-17x+10$.

Use the remainder theorem: if $f(x)$ is divided by $(ax-b)$, then the remainder is $g\left(\frac{b}{a}\right)$. Compare $(2x-1)$ to $(ax-b)$, so $a=2$, $b=1$ and the remainder is $f\left(\frac{1}{2}\right)$.

The remainder $=0$, so $(2x-1)$ is a factor of $2x^3-7x^2-17x+10$.

(b)

$$
\begin{array}{r l}
 & x^2-3x-10 \\
2x-1 \,\big) & 2x^3-7x^2-17x+10 \\
 & 2x^3-x^2 \\
 & -6x^2-17x \\
 & -6x^2+3x \\
 & -20x+10 \\
 & -20x+10 \\
 & 0
\end{array}
$$

First divide $2x^3-7x^2-17x+10$ by $(2x-1)$.

so $2x^3-7x^2-17x+10=(2x-1)(x^2-3x-10)$

$=(2x-1)(x-5)(x+2)$

Now factorise $x^2-3x-10$:

$(-5)\times(+2)=-10$

$(-5)+(+2)=-3$

so $x^2-3x-10=(x-5)(x+2)$.

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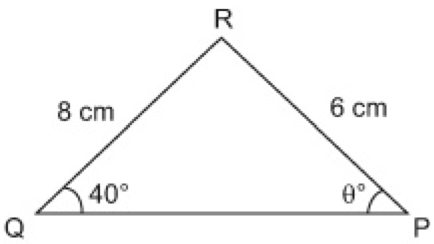
Algebra and functions
Exercise A, Question 10

Question:

In ΔPQR, $QR = 8$ cm, $PR = 6$ cm and $\angle PQR = 40°$.

Calculate the two possible values of $\angle QPR$.

Solution:



Draw a diagram using the given data.

Let $\angle QPR = \theta°$

$$\frac{\sin \theta}{8} = \frac{\sin 40°}{6}$$

Use $\frac{\sin P}{p} = \frac{\sin Q}{q}$, where $P = \theta°$, $p = 8$ (cm), $Q = 40°$, $q = 6$ (cm).

$\theta = 59.0°$ and $121.0°$

As $\sin(180 - \theta)° = \sin \theta°$, $\theta = 180° - 59.0° = 121.0°$ is the other possible answer.

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Algebra and functions
Exercise A, Question 11

Question:

(a) Express $\log_2 \left(\frac{4a}{b^2} \right)$ in terms of $\log_2 a$ and $\log_2 b$.

(b) Find the value of $\log_{27} \frac{1}{9}$.

Solution:

(a) $\log_2 \left(\frac{4a}{b^2} \right)$

$= \log_2 4a - \log_2 (b^2)$

Use the division law: $\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$, so that $\log_2 \left(\frac{4a}{b^2} \right) = \log_2 4a - \log_2 b^2$.

$= \log_2 4 + \log_2 a - \log_2 (b^2)$

Use the multiplication law: $\log_a (xy) = \log_a x + \log_b y$, so that $\log_2 4a = \log_2 4 + \log_2 a$

$= 2 + \log_2 a - 2 \log_2 b$

Simplify $\log_2 4$

$$\begin{aligned} \log_2 4 &= \log_2 (2^2) \\ &= 2 \log_2 2 \\ &= 2 \times 1 \\ &= 2 \end{aligned}$$

Use the power law: $\log_a (x^K) = K \log_a x$, so that $\log_2 (b^2) = 2 \log_2 b$.

(b)

$$\log_{27} \left(\frac{1}{9} \right) = \frac{\log_{10} \left(\frac{1}{9} \right)}{\log_{10} (27)} = -\frac{2}{3}$$

Change the base of the logarithm. Use $\log_a x = \frac{\log_b x}{\log_b a}$, so that $\log_{27} \left(\frac{1}{9} \right) = \frac{\log_{10} \left(\frac{1}{9} \right)}{\log_{10} (27)}$.

Alternative method:

$$\begin{aligned} \log_{27} \left(\frac{1}{9} \right) &= \log_{27} (9^{-1}) \\ &= -\log_{27} (9) \end{aligned}$$

Use index rules: $x^{-1} = \frac{1}{x}$, so that $\frac{1}{9} = 9^{-1}$

Use the power law $\log_a (x^K) = K \log_a x$.

$= - \log_{27}(3^2)$	
$= -2\log_{27}(3)$	Use the power law $\log_a(x^K) = K\log_a x$.
$= -2\log_{27}(27^{\frac{1}{3}})$	$27 = 3 \times 3 \times 3$, so $3 = \sqrt[3]{27} = 27^{\frac{1}{3}}$
$= \frac{-2}{3}\log_{27}27$	Use the power law $\log_a(x^K) = K\log_a x$.
$= \frac{-2}{3} \times 1$	Use $\log_a a = 1$, so that $\log_{27}27 = 1$
$= \frac{-2}{3}$	

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Algebra and functions
Exercise A, Question 12

Question:

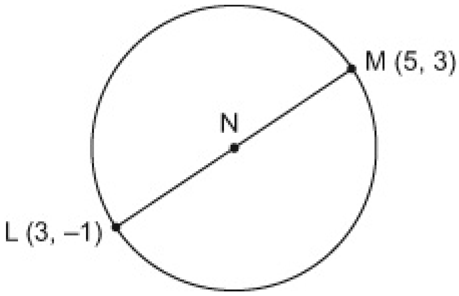
The points $L(3, -1)$ and $M(5, 3)$ are the end points of a diameter of a circle, centre N.

(a) Find the exact length of LM.

(b) Find the coordinates of the point N.

(c) Find an equation for the circle.

Solution:



Draw a diagram using the given information

(a)

$$\begin{aligned} LM &= \sqrt{(5-3)^2 + 3 - (-1)^2} \\ &= \sqrt{(2)^2 + (4)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \end{aligned}$$

Use $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ with $(x_1, y_1) = (3, -1)$ and $(x_2, y_2) = (5, 3)$

(b)

The Coordinates of N are $\left(\frac{3+5}{2}, \frac{-1+3}{2}\right) = (4, 1)$.

Use $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ with $(x_1, y_1) = (3, -1)$ and $(x_2, y_2) = (5, 3)$.

(c)

The equation of the Circle is

$$(x-4)^2 + (y-1)^2 = \left(\frac{\sqrt{20}}{2}\right)^2$$

$$(x-4)^2 + (y-1)^2 = 5$$

Use $(x-a)^2 + (y-b)^2 = r^2$ where (a, b) is the centre and r is the radius. Here $(a, b) = (4, 1)$ and $r = \frac{\sqrt{20}}{2}$.

$$\left(\frac{\sqrt{20}}{2}\right)^2 = \frac{\sqrt{20}}{2} \times \frac{\sqrt{20}}{2} = \frac{20}{4} = 5$$

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Algebra and functions
Exercise A, Question 13

Question:

$f(x) = 3x^3 + x^2 - 38x + c$

Given that $f(3) = 0$,

(a) find the value of c,

(b) factorise $f(x)$ completely,

(c) find the remainder when $f(x)$ is divided by $(2x - 1)$.

Solution:

$f(x) = 3x^3 + x^2 - 38x + c$

(a)

$3(3)^3 + (3)^2 - 38(3) + c = 0$

$3 \times 27 + 9 - 114 + c = 0$

$c = 24$

so $f(x) = 3x^3 + x^2 - 38x + 24$.

Substitute $x = 3$ into the polynomial.

(b)

$f(3) = 0$, so $(x - 3)$ is a factor of $3x^3 + x^2 - 38x + 24$

Use the factor theorem: If $f(p) = 0$, then $(x - p)$ is a factor of $f(x)$. Here $p = 3$

$$
\begin{array}{r|l}
 & 3x^2 - 10x \quad - \quad 8 \\
\hline
x - 3 & 3x^3 + \quad x^2 - 38x + 24 \\
 & 3x^3 - 9x^2 \\
 & \qquad 10x^2 - 38x \\
 & \qquad 10x^2 - 30x \\
 & \qquad\qquad - 8x - 24 \\
 & \qquad\qquad - 8x + 24 \\
 & 0
\end{array}
$$

First divide $3x^3 + x^2 - 38x + 24$ by $(x - 3)$.

so $3x^3 + x^2 - 38x + 24 = (x - 3)(3x^2 + 10x - 8) = (x - 3)(3x - 2)(x + 4)$.

Now factorise $3x^2 + 10x - 8$. $ac = -24$ and $(-2) + (+12) = +10 (= b)$ so
$3x^2 + 10x - 8 = 3x^2 - 2x + 12x - 8$.
$= x(3x - 2) + 4(3x - 2)$
$= (3x - 2)(x + 4)$

(c)

The remainder when $f(x)$ is divided by $(2x - 1)$ is $f(\frac{1}{2})$

Use the rule that if $f(x)$ is divided by $(ax - b)$ then the remainder is $f\left(\frac{a}{b}\right)$.

$f(\tfrac{1}{2}) = 3(\tfrac{1}{2})^3 + (\tfrac{1}{2})^2 - 38(\tfrac{1}{2}) + 24$

$= \frac{3}{8} + \frac{1}{4} - 19 + 24$

$= 5\frac{5}{8}$

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Algebra and functions
Exercise A, Question 14

Question:

In ΔABC, $AB = 5$cm, $BC = (2x - 3)$ cm, $CA = (x + 1)$ cm and $\angle ABC = 60°$.

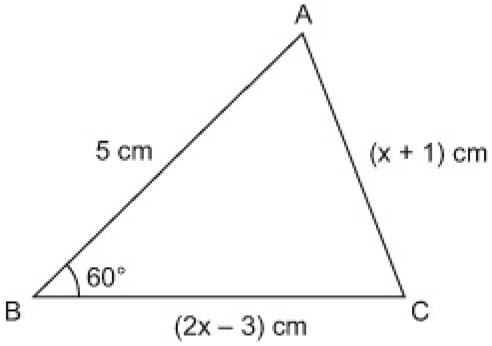
(a) Show that x satisfies the equation $x^2 - 8x + 16 = 0$.

(b) Find the value of x.

(c) Calculate the area of the triangle, giving your answer to 3 significant figures.

Solution:

(a)



Draw a diagram using the given data.

$(x+1)^2 = (2x-3)^2 + 5^2 - 2(2x-3) \times 5 \times \cos 60°$

Use the cosine rule:
$b^2 = a^2 + c^2 - 2ac \cos B$, where $a = (2x-3)$ cm, $b = (x+1)$ cm, $c = 5$cm, B $= 60°$.

$(x+1)^2 = (2x-3)^2 + 5^2 - 5(2x-3)$

$\cos 60° = \frac{1}{2}$, so $2(2x-3) \times 5 \times \cos 60°$
$= 2(2x-3) \times 5 \times \frac{1}{2}$
$= 5(2x-3)$

$x^2 + 2x + 1 = 4x^2 - 12x + 9 + 5^2 - 10x + 15$
$3x^2 - 24x + 48 = 0$
$x^2 - 8x + 16 = 0$

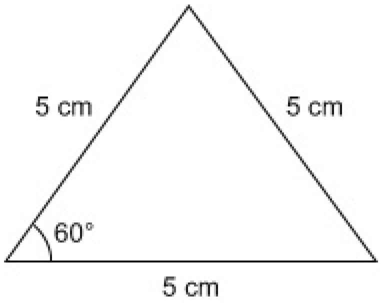
(b)

$x^2 - 8x + 16 = 0$
$(x-4)(x-4) = 0$

$x = 4$

Factorize $x^2 - 8x + 16 = 0$
$(-4) \times (-4) = +16$
$(-4) + (-4) = -8$
so $x^2 - 8x + 16 = (x-4)(x-4)$

(c)



Draw the diagram using $x = 4$

Area $= \frac{1}{2} \times 5 \times 5 \sin 60°$

$= 10.8\text{cm}^2$

Use Area $= \frac{1}{2}ac \sin B$, where

$a = 5\text{cm}$, $c = 5\text{cm}$, $B = 60°$

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Algebra and functions
Exercise A, Question 15

Question:

(a) Solve $0.6^{2x} = 0.8$, giving your answer to 3 significant figures.

(b) Find the value of x in $\log_x 243 = 2.5$

Solution:

(a) $0.6^{2x} = 0.8$

$\log_{10} 0.6^{2x} = \log_{10} 0.8$ Take logs to base 10 of each side.

$2x \log_{10} 0.6 = \log_{10} 0.8$ Use the power law: $\log_a(x^K) = K \log_a x$, so that $\log_{10} 0.6^{2x} = 2x \log_{10} 0.6$.

$2x = \dfrac{\log_{10} 0.8}{\log_{10} 0.6}$ Divide throughout by $\log_{10} 0.6$

$x = \dfrac{1}{2}\left(\dfrac{\log_{10} 0.8}{\log_{10} 0.6}\right)$

$= 0.218$

(b)

$\log_x 243 = 2.5$

$\dfrac{\log_{10} 243}{\log_{10} x} = 2.5$ Change the base of the logarithm. Use $\log_a x = \dfrac{\log_b x}{\log_b a}$, so that $\log_x 243 = \dfrac{\log_{10} 243}{\log_{10} x}$.

$\log_{10} x = \dfrac{\log_{10} 243}{2.5}$ Rearrange the equation for x.

so $x = 10^{\left(\frac{\log_{10} 243}{2.5}\right)}$ $\log_a n = x$ means that $a^x = n$, so $\log_{10} x = C$ means $x = 10^c$, where $c = \dfrac{\log_{10} 243}{2.5}$.

$= 9$

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Algebra and functions

Exercise A, Question 16

Question:

Show that part of the line $3x + y = 14$ forms a chord to the circle $(x-2)^2 + (y-3)^2 = 5$ and find the length of this chord.

Solution:

$(x-2)^2 + (y-3)^2 = 5$
$3x + y = 14$

Solve the equations simultaneously.

$y = 14 - 3x$

$(x-2)^2 + (14 - 3x - 3)^2 = 5$

Rearrange $3x + y = 14$ for y and substitute into $(x-2)^2 + (y-3)^2 = 5$.

$(x-2)^2 + (11-3x)^2 = 5$

Expand and simplify.
$(x-2)^2 = x^2 - 4x + 4$
$(11-3x)^2 = 121 - 66x + 9x^2$

$x^2 - 4x + 4 + 121 - 66x + 9x^2 = 5$

$10x^2 - 70x + 120 = 0$

Divide throughout by 10

$x^2 - 7x + 12 = 0$

Factorize $x^2 - 7x + 12 = 0$
$(-4) \times (-3) = +12$
$(-4) + (-3) = -7$
so $x^2 - 7x + 12 = (x-3)(x-4)$

$(x-3)(x-4) = 0$

so $x = 3$, $x = 4$

Two values of x, so two points of intersection.

So part of the line forms a chord to the Circle .

When $x = 3$, $y = 14 - 3(3)$
$= 14 - 9$
$= 5$

Find the coordinates of the points where the line meets the circle. Substitute $x = 3$ into $y = 14 - 3x$. Substitute $x = 4$ into $y = 14 - 3x$

When $x = 4$, $y = 14 - 3(4)$
$= 14 - 12$
$= 2$

So the line meets the chord at the points (3,5) and (4,2).

The distance between these points is

$\sqrt{(4-3)^2 + (2-5)^2} = \sqrt{1^2 + (-3)^2}$
$= \sqrt{1+9}$
$= \sqrt{10}$

Find the distance between the points (3,5) and (4,2) use $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$ with $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (4, 2)$.

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Algebra and functions
Exercise A, Question 17

Question:

$g(x) = x^3 - 13x + 12$

(a) Find the remainder when $g(x)$ is divided by $(x - 2)$.

(b) Use the factor theorem to show that $(x - 3)$ is a factor of $g(x)$.

(c) Factorise $g(x)$ completely.

Solution:

(a) $g(x) = x^3 - 13x + 12$

$$\begin{aligned} g(2) &= (2)^3 - 13(2) + 12 \\ &= 8 - 26 + 12 \\ &= -6. \end{aligned}$$

Use the remainder theorem: If $g(x)$ is divided by $(ax - b)$, then the remainder is $g\left(\frac{b}{a}\right)$. Compare $(x - 2)$ to $(ax - b)$, so $a = 1$, $b = 2$ and the remainder is $g\left(\frac{2}{1}\right)$, ie $g(2)$.

(b)

$$\begin{aligned} g(3) &= (3)^3 - 13(3) + 12 \\ &= 27 - 29 + 12 \\ &= 0 \end{aligned}$$

so $(x - 3)$ is a factor of $x^3 - 13x + 12$.

Use the factor theorem: If $g(p) = 0$, then $(x - p)$ is a factor of $g(x)$. Here $p = 3$

(c)

$$\begin{array}{r} x^2 + 3x - 4 \\ x - 3 \enclose{longdiv}{x^3 + 0x^2 - 13x + 12} \\ x^3 - 3x^2 \qquad\qquad\quad \\ 3x^2 - 13x \qquad\quad \\ 3x^2 - 9x \qquad\quad \\ -4x + 12 \\ -4x + 12 \\ 0 \end{array}$$

First divide $x^3 - 13x + 12$ by $(x - 3)$. Use $0x^2$ so that the sum is laid out correctly

so $x^3 - 13x + 12 = (x - 3)(x^2 + 3x - 4)$

$= (x - 3)(x + 4)(x - 1)$.

Factorize $x^2 + 3x - 4$:
$(+4) \times (-1) = -4$
$(+4) + (-1) = +3$
so $x^2 + 3x - 4 = (x + 4)(x - 1)$.

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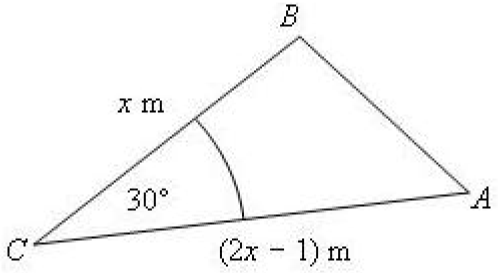
Algebra and functions
Exercise A, Question 18

Question:

The diagram shows ΔABC, with $BC = x$ m, $CA = (2x - 1)$ m and $\angle BCA = 30°$.



Given that the area of the triangle is 2.5 m^2,

(a) find the value of x,

(b) calculate the length of the line AB, giving your answer to 3 significant figures.

Solution:

(a)

$\frac{1}{2}x(2x-1)\sin 30° = 2.5$ — Here $a = x$ (m), $b = (2x - 1)$ (m) and angle $C = 30°$, so use area $= \frac{1}{2}ab \sin C$.

$\frac{1}{2}x(2x-1) \times \frac{1}{2} = 2.5$ — $\sin 30° = \frac{1}{2}$

$x(2x-1) = 10$ — Multiply both side by 4

$2x^2 - x - 10 = 0$ — Expand the brackets and rearrange into the form $ax^2 + bx + c = 0$

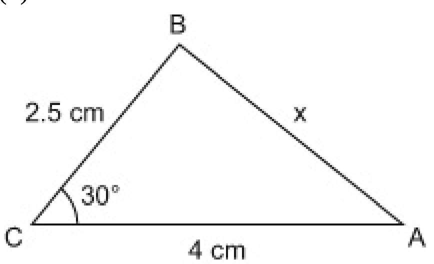
$(x+2)(2x-5) = 0$ — Factorize $2x^2 - x - 10 = 0$: $ac = -20$ and $(+4) + (-5) = -1$ so

$$2x^2 - x - 10 = 2x^2 + 4x - 5x - 10$$
$$= 2x(x+2) - 5(x+2)$$
$$= (x+2)(2x-5)$$

$x = -2$ and $x = \frac{5}{2}$

so $x = 2.5$ m — $x = -2$ is not feasible for this problem as BC would have a negative length.

(b)



Draw the diagram using $x = 2.5$ m

$x^2 = 2.5^2 + 4^2 - 2 \times 2.5 \times 4 \times \cos 30°$ — Use the cosine rule $c^2 = a^2 + b^2 - 2ab \cos C$, where

$x = 2.22$ m

$c = x$ (m) , $a = 2.5$ (m) , $b = 4$ (m) , C = 30 °

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Algebra and functions

Exercise A, Question 19

Question:

(a) Solve $3^{2x-1} = 10$, giving your answer to 3 significant figures.

(b) Solve $\log_2 x + \log_2 (9 - 2x) = 2$

Solution:

(a)

$3^{2x-1} = 10$	
$\log_{10}(3^{2x-1}) = \log_{10} 10$	Take logs to base 10 of each side.
$(2x-1)\log_{10} 3 = 1$	Use the power law: $\log_a (x^K) = K \log_a x$, so that $\log_{10}(3^{2x-1}) = (2x-1)\log_{10} 3$. Use $\log_a a = 1$ so that $\log_{10} 10 = 1$
$2x - 1 = \frac{1}{\log_{10} 3}$	Rearrange the expression, divide both sides by $\log_{10} 3$.
$2x = \frac{1}{\log_{10} 3} + 1$	Add 1 to both sides.
$x = \frac{\frac{1}{\log_{10} 3} + 1}{2}$	Divide both sides by 2
$x = 1.55$	

(b)

$\log_2 x + \log_2 (9 - 2x) = 2$	
$\log_2 x(9 - 2x) = 2$	Use the multiplication law: $\log_a (xy) = \log_a x + \log_a y$ so that $\log_2 x + \log_2 (9 - 2x) = \log_2 x(9 - 2x)$.
so $x(9 - 2x) = 2^2$	$\log_a n = x$ means $a^x = n$ so $\log_2 x(9 - 2x) = 2$ means $2^2 = x(9 - 2x)$
$x(9 - 2x) = 4$	
$9x - 2x^2 = 4$	
$2x^2 - 9x + 4 = 0$	
$(x-4)(2x-1) = 0$	Factorise $2x^2 - 9x + 4 = 0$ $ac = 8$, and $(-8) + (-1) = -9$ so
$x = 4, x = \frac{1}{2}$	$2x^2 - 9x + 4$ $= 2x^2 - 8x - x + 4$ $= 2x(x-4) - 1(x-4)$ $= (x-4)(2x-1)$

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Algebra and functions
Exercise A, Question 20

Question:

Prove that the circle $(x+4)^2 + (y-5)^2 = 8^2$ lies completely inside the circle $x^2 + y^2 + 8x - 10y = 59$.

Solution:

(a)

$x^2 + y^2 + 8x - 10y = 59$ — Write this circle in the form $(x-a)^2 + (y-b)^2 = r^2$

$x^2 + 8x + y^2 - 10y = 59$ — Rearrange the equation to bring the x terms together and the y terms together.

$(x+4)^2 - 16 + (y-5)^2 - 25 = 59$ — Complete the square, use $x^2 + 2ax = (x+a)^2 - a^2$ where $a = 4$, so that $x^2 + 8x = (x+4)^2 - 4^2$, and where $a = -5$, so that $x^2 - 10x = (x-5)^2 - 5^2$.

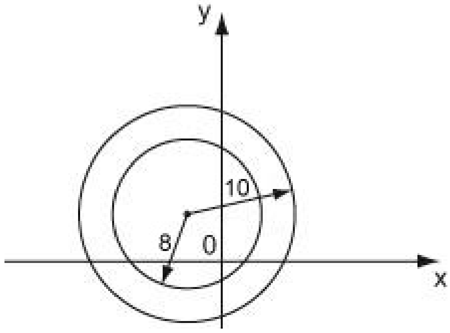
$(x+4)^2 + (y-5)^2 = 100$

$(x+4)^2 + (y-5)^2 = 10^2$

The centre and radius of $x^2 + y^2 + 8x - 10y = 59$ are $(-4, 5)$ and 10. — Compare $(x+4)^2 + (y-5)^2 = 100$ to $(x-a)^2 + (y-b)^2 = r^2$, where (a,b) is the centre and r is the radius. Here $(a, b) = (-4, 5)$ and $r = 10$.

The centre and radius of $(x+4)^2 + (y-5)^2 = 8^2$ are $(-4, 5)$ and 8. — Compare $(x+4)^2 + (y-5)^2 = 8^2$ to $(x-a)^2 + (y-b)^2 = r^2$, where (a,b) is the centre and r is the radius. Here $(a, b) = (-4, 5)$ and $r = 8$.

Both circles have the same centre, but each has a different radius. So, $(x+4)^2 + (y-5)^2 = 8^2$ lies completely inside $x^2 + y^2 + 8x - 10y = 59$.



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Algebra and functions
Exercise A, Question 21

Question:

$f(x) = x^3 + ax + b$, where a and b are constants.

When $f(x)$ is divided by $(x - 4)$ the remainder is 32.

When $f(x)$ is divided by $(x + 2)$ the remainder is -10.

(a) Find the value of a and the value of b.

(b) Show that $(x - 2)$ is a factor of $f(x)$.

Solution:

(a)

$f(4) = 32$

so, $(4)^3 + 4a + b = 32$

$4a + b = -32$

Use the remainder theorem: If $f(x)$ is divided by $(ax - b)$, then the remainder is $f\left(\frac{b}{a}\right)$. Compare $(x - 4)$ to $(ax - b)$, so $a = 1$, $b = 4$ and the remainder is $f(4)$.

$f(-2) = -10$,

so $(-2)^3 + a(-2) + b = 32$

$-8 - 2a + b = 32$

$-2a + b = 40$

Use the remainder theorem: Compare $(x + 2)$ to $(ax - b)$, so $a = 1$, $b = -2$ and the remainder is $f(-2)$.

Solve simultaneously

$4a + b = -32$

$-2a + b = 40$

$6a = -72$

so $a = -12$

Eliminate b: Subtract the equations, so $(4a + b) - (-2a + b) = 6a$ and $(-32) - (40) = -72$

Substitute $a = -12$ into $4a + b = -32$

$4(-12) + b = -32$

$-48 + b = -32$

$b = 16$

Substitute $a = -12$ into one of the equations. Here we use $4a + b = -32$

Substitute the values of a and b into the other equation to check the answer. Here we use $-2a + b = 40$

Check $-2a + b = 40$

$-2(-12) + 16 = 24 + 16 = 40$ (correct)

so $a = -12$, $b = 16$.

so $f(x) = x^3 - 12x + 16$

(b)

$f(2) = (2)^3 - 12(2) + 16$

Use the factor theorem : If $f(p) = 0$, then $(x - p)$ is a factor of $f(x)$. Here $p = 2$.

$= 8 - 24 + 16$
$= 0$

so $(x - 2)$ is a factor of $x^3 - 12x + 16$

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Algebra and functions
Exercise A, Question 22

Question:

Ship *B* is 8km, on a bearing of 30 ° , from ship *A*.

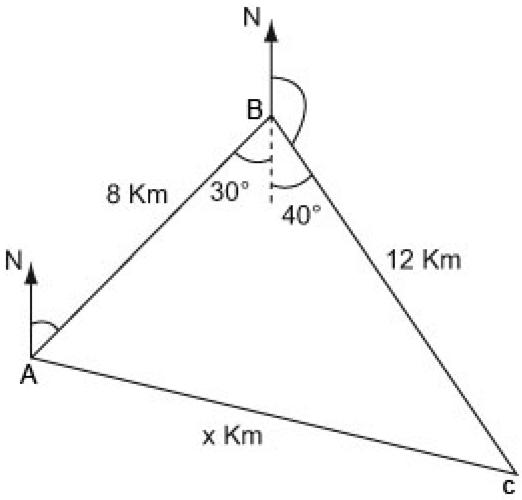
Ship *C* is 12 km, on a bearing of 140 ° , from ship *B*.

(a) Calculate the distance of ship *C* from ship *A*.

(b) Calculate the bearing of ship *C* from ship *A*.

Solution:

(a)



Draw a diagram using the given data.

Find the angle ABC: Angles on a straight line add to 180 ° , so 140 ° + 40 ° = 180 ° . Alternate angles are equal (= 30 °) so ∠ ABC = 30 ° + 40 ° = 70 °

$x^2 = 8^2 + 12^2 - 2 \times 8 \times 12 \times \cos 70°$

You have $a = 12$ (km) , $c = 8$ (km) , $b = x$ (km) , B = 70 ° . Use the cosine rule $b^2 = a^2 + c^2 - 2ac \cos B$

$x = 11.93$ km

The distance of ship C from ship A is 11.93 km.

(b)

$\frac{\sin 70°}{11.93} = \frac{\sin A}{12}$

Find the bearing of C from A. First calculate the angle BAC (= A) . Use $\frac{\sin B}{b} = \frac{\sin A}{12}$, where B = 70 ° , $b = x = 11.93$ (km) , $a = 12$ (km)

$A = 70.9°$

The Bearing of ship C from Ship A is 100.9 °

30 ° + 70.9 ° = 100.9 °

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Algebra and functions
Exercise A, Question 23

Question:

(a) Express $\log_p 12 - \left(\frac{1}{2} \log_p 9 + \frac{1}{3} \log_p 8 \right)$ as a single logarithm to base p.

(b) Find the value of x in $\log_4 x = -1.5$

Solution:

(a) $\log_p 12 - \frac{1}{2} \left(\log_p 9 + \frac{2}{3} \log_p 8 \right)$

$= \log_p 12 - \frac{1}{2} \left(\log_p 9 + \log_p \left(8^{2/3} \right) \right)$

Use the power low: $\log_a \left(x^K \right) = K \log_a x$, so that $\frac{2}{3} \log_p 8 = \log_p \left(8^{2/3} \right)$.

$= \log_p 12 - \frac{1}{2} \left(\log_p 9 + \log_p 4 \right)$

$8^{2/3} = \left(8^{1/3} \right)^2 = 2^2 = 4$

$= \log_p 12 - \frac{1}{2} \log_p 36$

Use the multiplication law: $\log_a \left(xy \right) = \log_a x + \log_a y$, so that $\log_p 9 + \log_p 4 = \log_p \left(9 \times 4 \right) = \log_p 36$

$= \log_p 12 - \log_p \left(36^{1/2} \right)$

Use the power law: $\log_a \left(x^k \right) = k \log_a x$, so that $\frac{1}{2} \log_p 36 = \log_p \left(36^{1/2} \right) = \log_p 6$

$= \log_p 12 - \log_p 6$

$= \log_p \left(\frac{12}{6} \right)$

$= \log_p 2$

Use the division law: $\log_a \left(\frac{x}{y} \right) = \log_a x - \log_b y$, so that $\log_p 12 - \log_p 6 = \log_p \left(\frac{12}{6} \right) = \log_p 2$

(b) $\log_4 x = -1.5$

$\frac{\log_{10} x}{\log_{10} 4} = -1.5$

Change the base of the logarithm. Use $\log_a x = \frac{\log_{10} x}{\log_{10} a}$, so that $\log_4 x = \frac{\log_{10} x}{\log_{10} 4}$.

$\log_{10} x = -1.5 \log_{10} 4$

Multiply throughout by $\log_{10} 4$

$x = 10^{-1.5 \log_{10} 4}$

$= 0.125$

$\log_a n = x$ means $a^x = n$, so $\log_{10} x = c$ means $x = 10^c$, where $c = -1.5 \log_{10} 4$.

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Algebra and functions
Exercise A, Question 24

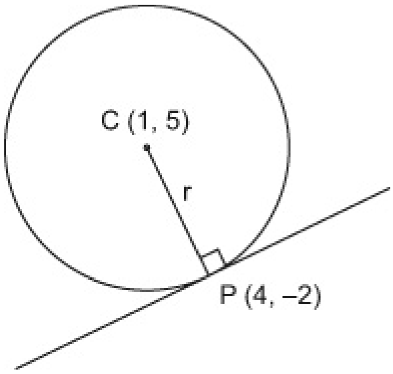
Question:

The point $P(4, -2)$ lies on a circle, centre $C(1, 5)$.

(a) Find an equation for the circle.

(b) Find an equation for the tangent to the circle at P.

Solution:



Draw a diagram using the given information

Let $CP = r$

(a)

$(x-1)^2 + (y-5)^2 = r^2$

Use $(x-a)^2 + (y-b)^2 = r^2$ where (a, b) is the centre of the circle. Here $(a, b) = (1, 5)$.

$r = \sqrt{(4-1)^2 + (-2-5)^2}$
$= \sqrt{3^2 + (-7)^2}$
$= \sqrt{9 + 49}$
$= \sqrt{58}$

Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ where $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (4, -2)$.

The equation of the circle is
$(x-1)^2 + (y-5)^2 = (\sqrt{58})^2$
$(x-1)^2 + (y-5)^2 = 58$

(b)

The gradient of CP is $\frac{-2-5}{4-1} = \frac{-7}{3}$

Use $\frac{y_2 - y_1}{x_2 - x_1}$, where $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (4, -2)$.

So the gradient of the tangent is $\frac{3}{7}$

The tangent at P is perpendicular to the gradient at P. Use $\frac{-1}{m}$. Here $m = -\frac{7}{3}$ so $\frac{-1}{\left(\frac{-7}{3}\right)} = \frac{3}{7}$

The equation of the tangent at P is

Use $y - y_1 = m(x - x_1)$, where $(x_1, y,) = (4, -$

$y + 2 = \frac{3}{7}(x - 4)$ 2) and $m = \frac{3}{7}$.

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Algebra and functions

Exercise A, Question 25

Question:

The remainder when $x^3 - 2x + a$ is divided by $(x - 1)$ is equal to the remainder when $2x^3 + x - a$ is divided by $(2x + 1)$. Find the value of a.

Solution:

$f(x) = x^3 - 2x + a$

$g(x) = 2x^3 + x - a$

$f(1) = g(-\frac{1}{2})$

Use the remainder theorem: If $f(x)$ is divided by $ax - b$, then the remainder is $f(\frac{b}{a})$. Compare $(x - 1)$ to $ax - b$, so $a = 1$, $b = 1$ and the remainder is $f(1)$.

Use the remainder theorem: If $g(x)$ is divided by $ax - b$, then the remainder is $g(\frac{b}{a})$. Compare $(2x + 1)$ to $ax - b$, so $a = 2$, $b = -1$ and the remainder is $g(-\frac{1}{2})$

The remainders are equal so $f(1) = g(-1/2)$.

$(1)^3 - 2(1) + a = 2(-\frac{1}{2})$

$^3 + (\frac{-1}{2}) - a$

$1 - 2 + a = -\frac{1}{4} - \frac{1}{2} - a$

$(\frac{-1}{2})^3 = \frac{-1}{8}$

$2 \times -\frac{1}{8} = -\frac{1}{4}$

$2a = \frac{1}{4}$

so $a = \frac{1}{8}$.

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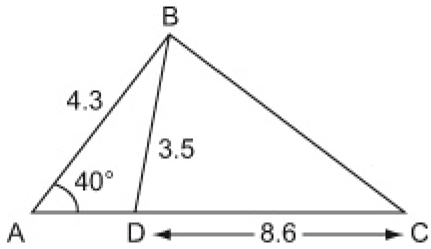
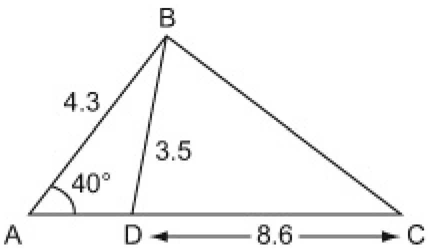
Algebra and functions
Exercise A, Question 26

Question:

The diagram shows ΔABC.
Calculate the area of ΔABC.



Solution:



$$\frac{\sin \angle BDA}{4.3} = \frac{\sin 40^\circ}{3.5}$$

In ΔABD, use $\frac{\sin D}{d} = \frac{\sin A}{a}$, where D = $\angle$ BDA, d = 4.3, A = 40°, a = 3.5.

$$\sin \angle BDA = \frac{4.3 \sin 40^\circ}{3.5}$$

$$\angle BDA = 52.16^\circ$$

$$\angle ABD = 180^\circ - (52.16^\circ + 40^\circ)$$

Angles in a triangle sum to 180°.

$$= 87.84^\circ$$

$$\frac{AD}{\sin 87.84} = \frac{3.5}{\sin 40^\circ}$$

In ΔABD, use $\frac{b}{\sin B} = \frac{a}{\sin A}$, where b = AD, B = 87.84°, a = 3.5, A = 40°.

$$AD = \frac{3.5 \sin 87.84}{\sin 40}$$

$$= 5.44 \text{ cm}$$

$$AC = AD + DC = 5.44 + 8.6$$

$$= 14.04$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 4.3 \times 14.04 \times \sin 40^\circ$$

In ΔABC, use Area = $\frac{1}{2} bc \sin A$ where b = 14.04, c = 4.3, A = 40°.

$$= 19.4 \text{ cm}^2$$

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Algebra and functions

Exercise A, Question 27

Question:

Solve $3^{2x+1} + 5 = 16(3^x)$.

Solution:

$$3^{2x+1} + 5 = 16(3^x)$$

$$3(3^{2x}) + 5 = 16(3^x)$$

$$3(3^x)^2 + 5 = 16(3^x)$$

Use the rules for indices: $a^m \times a^n = a^{m+n}$, so that

$3^{2x+1} = 3^{2x} \times 3^1$

$= 3(3^{2x})$.

Also, $(a^m)^n = a^{mn}$, so that $3^{2x} = (3^x)^2$.

let $y = 3^x$

so $3y^2 + 5 = 16y$

$$3y^2 - 16y + 5 = 0$$

$$(3y - 1)(y - 5) = 0$$

Factorise $3y^2 - 16y + 5 = 0$. $ac = 15$ and $(-15) + (-1) = -16$, so that

$3y^2 - 16y + 5 = 3y^2 - 15y - y + 5$

$= 3y(y - 5) - 1(y - 5)$

$= (y - 5)(3y - 1)$

$y = \frac{1}{3}$, $y = 5$

Now $3^x = \frac{1}{3}$, so $x = -1$.

and $3^x = 5$,

$\log_{10}(3^x) = \log_{10}5$

Take logarithm to base 10 of each side.

$x \log_{10}3 = \log_{10}5$

Use the power law: $\log_a(x^K) = K \log_a x$, so that

$\log_{10}(3^x) = x \log_{10}3$

Divide throughout by $\log_{10}3$

$$x = \frac{\log_{10}5}{\log_{10}3}$$

$$= 1.46$$

so $x = -1$ and $x = 1.46$

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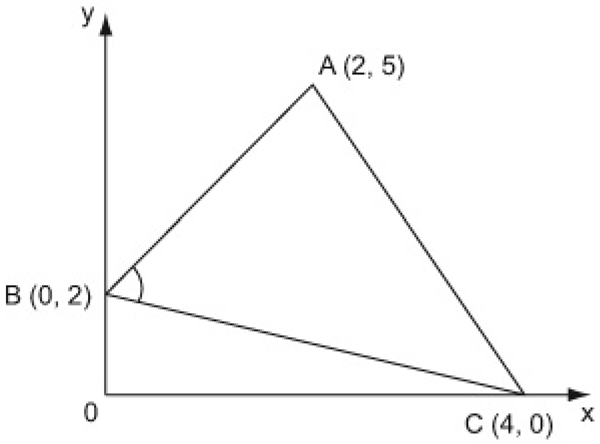
Algebra and functions
Exercise A, Question 28

Question:

The coordinates of the vertices of ΔABC are $A(2,5)$, $B(0,2)$ and $C(4,0)$.

Find the value of $\cos \angle ABC$.

Solution:



Draw a diagram using the given information.

$$AB^2 = (2-0)^2 + (5-2)^2$$
$$= 2^2 + 3^2$$
$$= 4 + 9$$
$$= 13$$

Use $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, with $(x_1, y_1) = (0,2)$ and $(x_2, y_2) = (2,5)$.

$$BC^2 = (0-4)^2 + (2-0)^2$$
$$= (-4)^2 + (2)^2$$
$$= 16 + 4$$
$$= 20$$

Use $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ with $(x_1, y_1) = (4,0)$ and $(x_2, y_2) = (0,2)$.

$$CA^2 = (4-2)^2 + (0-5)^2$$
$$= 2^2 + (-5)^2$$
$$= 4 + 25$$
$$= 29$$

Use $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ with $(x_1, y_1) = (2,5)$ and $(x_2, y_2) = (4,0)$.

$$\cos \angle ABC = \frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC}$$
$$= \frac{13 + 20 - 29}{2\sqrt{13}\sqrt{20}}$$

Use $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$, where $B = \angle ABC$, $a = BC$, $c = AB$, $b = AC$

$$\angle ABC = 82.9^\circ$$

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Algebra and functions

Exercise A, Question 29

Question:

Solve the simultaneous equations

$4\log_9 x + 4\log_3 y = 9$

$6\log_3 x + 6\log_{27} y = 7$

Solution:

$4\log_9 x + 4\log_3 y = 9$

Change the base of the logarithm, use $\log_a x = \frac{\log_b x}{\log_b a}$, so that $\log_9 x = \frac{\log_3 x}{\log_3 9}$.

$4\frac{\log_3 x}{\log_3 9} + 4\log_3 y = 9$

$\log_3 9 = \log_3 (3^2)$
$= 2\log_3 3 = 2 \times 1 = 2$

$\frac{4\log_3 x}{\log_3 9} = \frac{4\log_3 x}{2} = 2\log_3 x$

$2\log_3 x + 4\log_3 y = 9$ ①

$6\log_3 x + 6\log_{27} y = 7$

Change the base of the logarithm, use $\log_a x = \frac{\log_b x}{\log_b a}$, so that $\log_{27} y = \frac{\log_3 y}{\log_3 27}$

$6\log_3 x + \frac{6\log_3 y}{\log_3 27} = 7$

$\log_3 27 = \log_3 (3^3)$
$= 3\log_3 3$
$= 3 \times 1 = 3$

so $\frac{6\log_3 y}{\log_3 27} = \frac{6\log_3 y}{3}$
$= 2\log_3 y$

$6\log_3 x + 2\log_3 y = 7$ ②

Solve ① & ② simultaneously.

Let $\log_3 x = X$ and $\log_3 y = Y$

so $2X + 4Y = 9$
$6X + 2Y = 7$

Multiply ① throughout by 3

$6X + 12Y = 27$
$-\ 6X + 2Y = 7$
$10Y = 20$

$Y = 2$

Sub $Y = 2$ into $2X + 4Y = 9$

$2X + 4(2) = 9$
$2X + 8 = 9$
$2X = 1$
$X = \frac{1}{2}$

Check sub $X = \frac{1}{2}$ and $Y = 2$ into $6x + 2y = 7$

$6\left(\frac{1}{2}\right) + 2(2)$
$= 3 + 4 = 7$ ✓ ✓ (correct)

so $(X =)\ \log_3 x = \frac{1}{2}$

i.e. $x = 3^{1/2}$

$\log_a n = x$ means $a^x = n$, so $\log_3 x = \frac{1}{2}$ means $x = 3^{1/2}$.

and $(Y =)\ \log_3 y = 2$

i.e. $y = 3^2 = 9$

$\log_a n = x$ means $a^x = n$, so $\log_3 y = 2$ means $y = 3^2$

so $(x, y) = (3^{1/2}, 9)$

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise A, Question 30

Question:

The line $y = 5x - 13$ meets the circle $(x-2)^2 + (y+3)^2 = 26$ at the points A and B.

(a) Find the coordinates of the points A and B.

M is the midpoint of the line AB.

(b) Find the equation of the line which passes through M and is perpendicular to the line AB. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

(a)

$y = 5x - 13$

$(x-2)^2 + (y+3)^2 = 26$

$(x-2)^2 + (5x-13+3)^2 = 26$

Solve the equations simultaneously. Substitute $y = 5x - 13$ into $(x-2)^2 + (y+3)^2 = 26$.

$(x-2)^2 + (5x-10)^2 = 26$

Expand and Simplify

$x^2 - 4x + 4 + 25x^2 - 100x + 100 = 26$

$26x^2 - 104x + 78 = 0$

Divide throughout by 26

$x^2 - 4x + 3 = 0$

Factorise $x^2 - 4x + 3$.

$(x-3)(x-1) = 0$

$(-3) \times (-1) = +3$

$(-3) + (-1) = -4$

$x = 3, x = 1$

so $x^2 - 4x + 3 = (x-3)(x-1)$

When $x = 1$, $y = 5(1) - 13$
$= 5 - 13$
$= -8$

Find the Corresponding y coordinates. Substitute $x = 1$ into $y = 5x - 13$.

When $x = 3$, $y = 5(3) - 13$
$= 15 - 13$
$= 2$

Substitute $x = 3$ into $y = 5x - 13$

So the coordinates of the points of intersection are $(1, -8)$ and $(3, 2)$.

(b)

The Midpoint of AB is $\left(\frac{1+3}{2}, \frac{-8+2}{2}\right) = (2, -3)$.

Use $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ with $(x_1, y_1) = (1, -8)$ and $(x_2, y_2) = (3, 2)$

The gradient of the line perpendicular to $y = 5x - 13$ is $-\frac{1}{5}$	The gradient of the line perpendicular to $y = mx + c$ is $-\frac{1}{m}$. Here $m = 5$.
so, $y + 3 = \frac{-1}{5}(x - 2)$	Use $y - y_1 = m(x - x_1)$ with $m = \frac{-1}{5}$ and $(x_1, y_1) = (2, -3)$
$5y + 15 = -1(x - 2)$ $5y + 15 = -x + 2$ $x + 5y + 13 = 0$	Clear the fraction. Multiply each side by 5.

Solutionbank C1
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise A, Question 31

Question:

The circle C has equation $x^2 + y^2 - 10x + 4y + 20 = 0$.
Find the length of the tangent to C from the point $(-4, 4)$.

Solution:

The angle between a tangent and a radius is a right-angle, so form a right-angled triangle with the tangent, the radius and the distance between the centre of the circle and the point $(-4, 4)$.

$x^2 + y^2 - 10x + 4y + 20 = 0$

$(x-5)^2 - 25 + (y+2)^2 - 4 = -20$

$(x-5)^2 + (y+2)^2 = 9$

Find the equation of the tangent in the form $(x-a)^2 + (y-b)^2 = r^2$

So circle has centre $(5, -2)$ and radius 3

$\sqrt{(5 - -4)^2 + (-2-4)^2}$

$= \sqrt{81 + 36} = \sqrt{117}$

Calculate the distance between the centre of the circle and $(-4, 4)$

Therefore $117 = 3^2 + x^2$

Using Pythagoras

$x^2 = 108$

$x = \sqrt{108}$