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Edexcel Modular Mathematics for AS and A-Level

Radian measure and its applications

Exercise A, Question 1

Question:

Convert the following angles in radians to degrees:

(a) $\frac{\pi}{20}$

(b) $\frac{\pi}{15}$

(c) $\frac{5\pi}{12}$

(d) $\frac{\pi}{2}$

(e) $\frac{7\pi}{9}$

(f) $\frac{7\pi}{6}$

(g) $\frac{5\pi}{4}$

(h) $\frac{3\pi}{2}$

(i) 3π

Solution:

(a) $\frac{\pi}{20}$ rad = $\frac{180\ ^\circ}{20}$ = 9 °

(b) $\frac{\pi}{15}$ rad = $\frac{180\ ^\circ}{15}$ = 12 °

(c) $\frac{5\pi}{12}$ rad = $\frac{5 \times \overset{15^\circ}{\cancel{180^\circ}}}{\cancel{12}}$ = 75 °

(d) $\frac{\pi}{2}$ rad = $\frac{180\ ^\circ}{2}$ = 90 °

(e) $\frac{7\pi}{9}$ rad = $\frac{7 \times \overset{20^\circ}{\cancel{180^\circ}}}{\cancel{9}}$ = 140 °

(f) $\frac{7\pi}{6}$ rad = $\frac{7 \times \overset{30^\circ}{\cancel{180^\circ}}}{\cancel{6}}$ = 210 °

(g) $\frac{5\pi}{4}$ rad = $\frac{5 \times \overset{45^\circ}{\cancel{180^\circ}}}{\cancel{4}}$ = 225 °

(h) $\frac{3\pi}{2}$ rad = 3 × 90 ° = 270 °

(i) 3π rad = 3 × 180 ° = 540 °

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Radian measure and its applications

Exercise A, Question 2

Question:

Use your calculator to convert the following angles to degrees, giving your answer to the nearest 0.1 ° :

(a) 0.46^c

(b) 1^c

(c) 1.135^c

(d) $\sqrt{3}^c$

(e) 2.5^c

(f) 3.14^c

(g) 3.49^c

Solution:

(a) $0.46^c = 26.356 \ldots ° = 26.4 °$ (nearest 0.1 °)

(b) $1^c = 57.295 \ldots ° = 57.3 °$ (nearest 0.1 °)

(c) $1.135^c = 65.030 \ldots ° = 65.0 °$ (nearest 0.1 °)

(d) $\sqrt{3}^c = 99.239 \ldots ° = 99.2 °$ (nearest 0.1 °)

(e) $2.5^c = 143.239 \ldots ° = 143.2 °$ (nearest 0.1 °)

(f) $3.14^c = 179.908 \ldots ° = 179.9 °$ (nearest 0.1 °)

(g) $3.49^c = 199.96 \ldots ° = 200.0 °$ (nearest 0.1 °)

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Radian measure and its applications
Exercise A, Question 3

Question:

Use your calculator to write down the value, to 3 significant figures, of the following trigonometric functions.

(a) $\sin\ 0.5^c$

(b) $\cos\ \sqrt{2}^c$

(c) $\tan\ 1.05^c$

(d) $\sin\ 2^c$

(e) $\cos\ 3.6^c$

Solution:

(a) $\sin\ 0.5^c = 0.47942 \ldots = 0.479$ (3 s.f.)

(b) $\cos\ \sqrt{2}^c = 0.1559 \ldots = 0.156$ (3 s.f.)

(c) $\tan\ 1.05^c = 1.7433 \ldots = 1.74$ (3 s.f.)

(d) $\sin\ 2^c = 0.90929 \ldots = 0.909$ (3 s.f.)

(e) $\cos\ 3.6^c = -0.8967 \ldots = -0.897$ (3 s.f.)

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Radian measure and its applications
Exercise A, Question 4

Question:

Convert the following angles to radians, giving your answers as multiples of π.

(a) 8°

(b) 10°

(c) 22.5°

(d) 30°

(e) 45°

(f) 60°

(g) 75°

(h) 80°

(i) 112.5°

(j) 120°

(k) 135°

(l) 200°

(m) 240°

(n) 270°

(o) 315°

(p) 330°

Solution:

(a) $8° = \overset{2}{\cancel{8}} \times \dfrac{\pi}{\underset{45}{\cancel{180}}}$ rad $= \dfrac{2\pi}{45}$ rad

(b) $10° = 10 \times \dfrac{\pi}{180}$ rad $= \dfrac{\pi}{18}$ rad

(c) $22.5° = \cancel{22.5} \times \dfrac{\pi}{\underset{8}{\cancel{180}}}$ rad $= \dfrac{\pi}{8}$ rad

(d) $30° = 30 \times \frac{\pi}{180}$ rad $= \frac{\pi}{6}$ rad

(e) $45° = 45 \times \frac{\pi}{180}$ rad $= \frac{\pi}{4}$ rad

(f) $60° = 2 \times$ answer to (d) $= \frac{\pi}{3}$ rad

(g) $75° = \overset{5}{\cancel{75}} \times \frac{\pi}{\underset{12}{\cancel{180}}}$ rad $= \frac{5\pi}{12}$ rad

(h) $80° = 8\cancel{0} \times \frac{\pi}{18\cancel{0}}$ rad $= \frac{4\pi}{9}$ rad

(i) $112.5° = 5 \times$ answer to (c) $= \frac{5\pi}{8}$ rad

(j) $120° = 2 \times$ answer to (f) $= \frac{2\pi}{3}$ rad

(k) $135° = 3 \times$ answer to (e) $= \frac{3\pi}{4}$ rad

(l) $200° = 20\cancel{0} \times \frac{\pi}{18\cancel{0}}$ rad $= \frac{10\pi}{9}$ rad

(m) $240° = 2 \times$ answer to (j) $= \frac{4\pi}{3}$ rad

(n) $270° = 3 \times 90° = \frac{3\pi}{2}$ rad

(o) $315° = 180° + 135° = \pi + \frac{3\pi}{4} = \frac{7\pi}{4}$ rad

(p) $330° = 11 \times 30° = \frac{11\pi}{6}$ rad

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Radian measure and its applications
Exercise A, Question 5

Question:

Use your calculator to convert the following angles to radians, giving your answers to 3 significant figures:

(a) $50°$

(b) $75°$

(c) $100°$

(d) $160°$

(e) $230°$

(f) $320°$

Solution:

(a) $50° = 0.8726 \ldots^c = 0.873^c$ (3 s.f.)

(b) $75° = 1.3089 \ldots^c = 1.31^c$ (3 s.f.)

(c) $100° = 1.7453 \ldots^c = 1.75^c$ (3 s.f.)

(d) $160° = 2.7925 \ldots^c = 2.79^c$ (3 s.f.)

(e) $230° = 4.01425 \ldots^c = 4.01^c$ (3 s.f.)

(f) $320° = 5.585 \ldots^c = 5.59^c$ (3 s.f.)

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Radian measure and its applications

Exercise B, Question 1

Question:

An arc AB of a circle, centre O and radius r cm, subtends an angle θ radians at O. The length of AB is l cm.

(a) Find l when
(i) $r = 6$, $\theta = 0.45$
(ii) $r = 4.5$, $\theta = 0.45$
(iii) $r = 20$, $\theta = \frac{3}{8}\pi$

(b) Find r when
(i) $l = 10$, $\theta = 0.6$
(ii) $l = 1.26$, $\theta = 0.7$
(iii) $l = 1.5\pi$, $\theta = \frac{5}{12}\pi$

(c) Find θ when
(i) $l = 10$, $r = 7.5$
(ii) $l = 4.5$, $r = 5.625$
(iii) $l = \sqrt{12}$, $r = \sqrt{3}$

Solution:

(a) Using $l = r\theta$
(i) $l = 6 \times 0.45 = 2.7$
(ii) $l = 4.5 \times 0.45 = 2.025$
(iii) $l = 20 \times \frac{3}{8}\pi = 7.5\pi$ (23.6 3 s.f.)

(b) Using $r = \frac{l}{\theta}$

(i) $r = \frac{10}{0.6} = 16\frac{2}{3}$

(ii) $r = \frac{1.26}{0.7} = 1.8$

(iii) $r = \frac{1.5\pi}{\frac{5}{12}\pi} = 1.5 \times \frac{12}{5} = \frac{18}{5} = 3\frac{3}{5}$

(c) Using $\theta = \frac{l}{r}$

(i) $\theta = \frac{10}{7.5} = 1\frac{1}{3}$

(ii) $\theta = \frac{4.5}{5.625} = 0.8$

(iii) $\theta = \frac{\sqrt{12}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$

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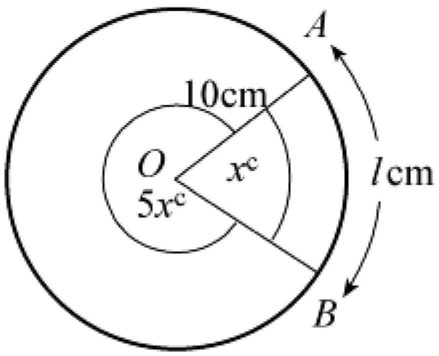
Radian measure and its applications
Exercise B, Question 2

Question:

A minor arc *AB* of a circle, centre *O* and radius 10 cm, subtends an angle *x* at *O*. The major arc *AB* subtends an angle 5*x* at *O*. Find, in terms of π, the length of the minor arc *AB*.

Solution:



The total angle at the centre is $6x^c$ so

$6x = 2\pi$

$x = \frac{\pi}{3}$

Using $l = r\theta$ to find minor arc *AB*

$l = 10 \times \frac{\pi}{3} = \frac{10\pi}{3}$ cm

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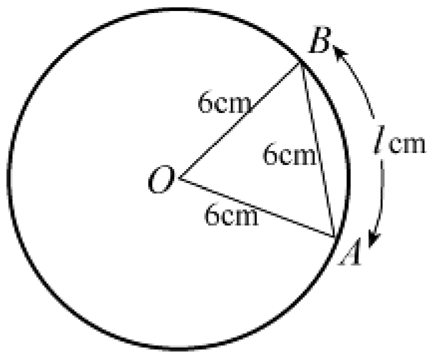
Radian measure and its applications
Exercise B, Question 3

Question:

An arc *AB* of a circle, centre *O* and radius 6 cm, has length *l* cm. Given that the chord *AB* has length 6 cm, find the value of *l*, giving your answer in terms of π.

Solution:



$\triangle$OAB is equilateral, so $\angle$ AOB = $\frac{\pi}{3}$ rad.

Using $l = r\theta$

$l = 6 \times \frac{\pi}{3} = 2\pi$

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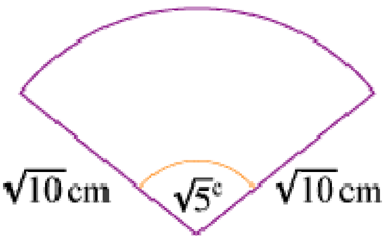
Radian measure and its applications

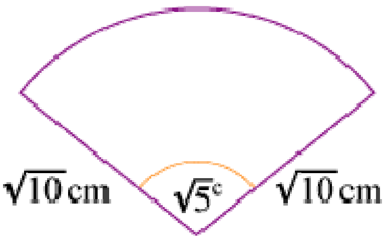
Exercise B, Question 4

Question:

The sector of a circle of radius $\sqrt{10}$ cm contains an angle of $\sqrt{5}$ radians, as shown in the diagram. Find the length of the arc, giving your answer in the form $p\sqrt{q}$ cm, where p and q are integers.



Solution:



Using $l = r\theta$ with $r = \sqrt{10}$ cm and $\theta = \sqrt{5}^c$
$l = \sqrt{10} \times \sqrt{5} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$ cm

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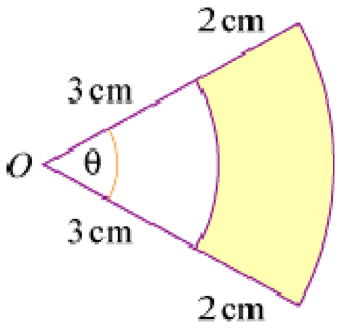
Radian measure and its applications
Exercise B, Question 5

Question:

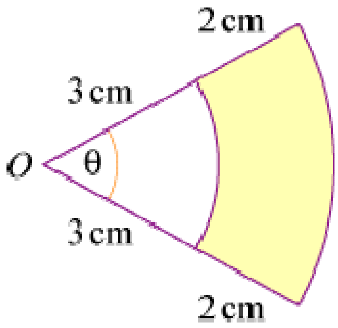
Referring to the diagram, find:



(a) The perimeter of the shaded region when $\theta = 0.8$ radians.

(b) The value of θ when the perimeter of the shaded region is 14 cm.

Solution:



(a) Using $l = r\theta$,
the smaller arc $= 3 \times 0.8 = 2.4$ cm
the larger arc $= (3 + 2) \times 0.8 = 4$ cm
Perimeter $= 2.4$ cm $+ 2$ cm $+ 4$ cm $+ 2$ cm $= 10.4$ cm

(b) The smaller arc $= 3\theta$ cm, the larger arc $= 5\theta$ cm.
So perimeter $= (3\theta + 5\theta + 2 + 2)$ cm .
As perimeter is 14 cm,
$8\theta + 4 = 14$
$8\theta = 10$
$\theta = \frac{10}{8} = 1\frac{1}{4}$

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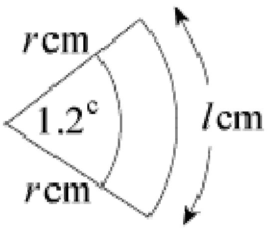
Radian measure and its applications
Exercise B, Question 6

Question:

A sector of a circle of radius r cm contains an angle of 1.2 radians. Given that the sector has the same perimeter as a square of area 36 cm^2, find the value of r.

Solution:

Using $l = r\theta$, the arc length $= 1.2r$ cm.
The area of the square $= 36$ cm^2, so each side $= 6$ cm and the perimeter is, therefore, 24 cm.
The perimeter of the sector = arc length $+ 2r$ cm $= (1.2r + 2r)$ cm $= 3.2r$ cm.
The perimeter of square = perimeter of sector so
$24 = 3.2r$
$r = \frac{24}{3.2} = 7.5$



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Radian measure and its applications
Exercise B, Question 7

Question:

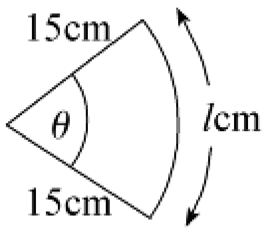
A sector of a circle of radius 15 cm contains an angle of θ radians. Given that the perimeter of the sector is 42 cm, find the value of θ.

Solution:

Using $l = r\theta$, the arc length of the sector $= 15\theta$ cm.
So the perimeter $= (15\theta + 30)$ cm .
As the perimeter $= 42$ cm
$15\theta + 30 = 42$

$\Rightarrow \quad 15\theta = 12$

$\Rightarrow \quad \theta = \frac{12}{15} = \frac{4}{5}$



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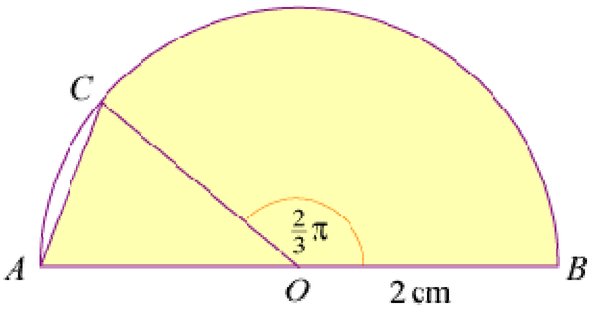
Radian measure and its applications
Exercise B, Question 8

Question:

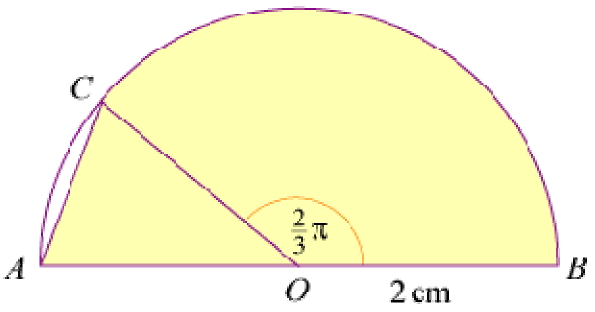
In the diagram *AB* is the diameter of a circle, centre *O* and radius 2 cm. The point *C* is on the circumference such that $\angle$ COB = $\frac{2}{3}\pi$ radians.



(a) State the value, in radians, of $\angle$ COA.
The shaded region enclosed by the chord *AC*, arc *CB* and *AB* is the template for a brooch.

(b) Find the exact value of the perimeter of the brooch.

Solution:



(a) $\angle$ COA $= \pi - \frac{2}{3}\pi = \frac{\pi}{3}$ rad

(b) The perimeter of the brooch = AB + arc BC + chord AC.
AB = 4 cm

arc BC = $r\theta$ with $r = 2$ cm and $\theta = \frac{2}{3}\pi$ so

arc BC $= 2 \times \frac{2}{3}\pi = \frac{4}{3}\pi$ cm

As $\angle$ COA $= \frac{\pi}{3}$ (60 °), △COA is equilateral, so

chord AC = 2 cm

The perimeter $= 4$ cm $+ \frac{4}{3}\pi$ cm $+ 2$ cm $= \left(6 + \frac{4}{3}\pi \right)$ cm

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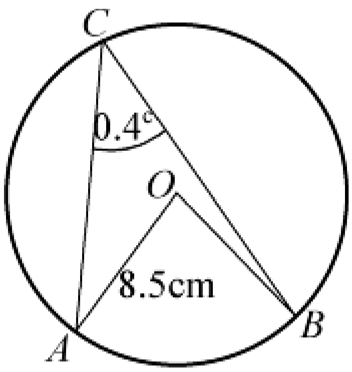
Radian measure and its applications
Exercise B, Question 9

Question:

The points *A* and *B* lie on the circumference of a circle with centre *O* and radius 8.5 cm. The point *C* lies on the major arc *AB*. Given that $\angle$ ACB = 0.4 radians, calculate the length of the minor arc *AB*.

Solution:



Using the circle theorem:
Angle subtended at the centre of the circle = 2 × angle subtended at the circumference
$\angle$ AOB = 2 $\angle$ ACB = 0.8^c
Using $l = r\theta$
length of minor arc AB = 8.5 × 0.8 cm = 6.8 cm

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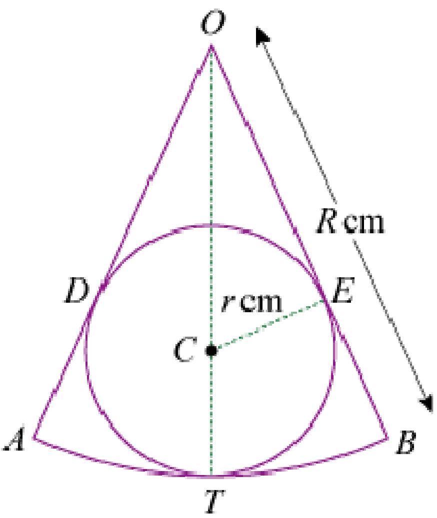
Radian measure and its applications
Exercise B, Question 10

Question:

In the diagram *OAB* is a sector of a circle, centre *O* and radius *R* cm, and $\angle AOB = 2\theta$ radians. A circle, centre *C* and radius *r* cm, touches the arc *AB* at *T*, and touches *OA* and *OB* at *D* and *E* respectively, as shown.

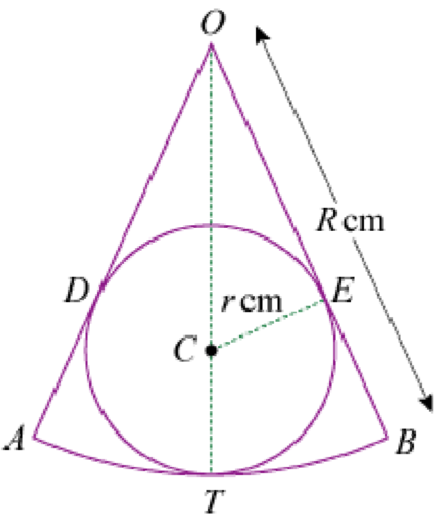


(a) Write down, in terms of *R* and *r*, the length of *OC*.

(b) Using $\triangle OCE$, show that $R\sin\theta = r(1 + \sin\theta)$.

(c) Given that $\sin\theta = \frac{3}{4}$ and that the perimeter of the sector *OAB* is 21 cm, find *r*, giving your answer to 3 significant figures.

Solution:



(a) $OC = OT - CT = R \text{ cm} - r \text{ cm} = (R - r) \text{ cm}$

(b) In $\triangle$OCE, $\angle$ CEO = 90 ° (radius perpendicular to tangent)

and $\angle$ COE = θ (*OT* bisects $\angle$ AOB)

Using $\sin \angle \text{COE} = \frac{\text{CE}}{\text{OC}}$

$\sin \theta = \frac{r}{R - r}$

$(R - r) \sin \theta = r$
$R \sin \theta - r \sin \theta = r$
$R \sin \theta = r + r \sin \theta$
$R \sin \theta = r (1 + \sin \theta)$

(c) As $\sin \theta = \frac{3}{4}$, $\frac{3}{4}R = \frac{7}{4}r \quad \Rightarrow \quad R = \frac{7}{3}r$

and $\theta = \sin^{-1} \frac{3}{4} = 0.84806 \ldots^{c}$

The perimeter of the sector $= 2R + 2R\theta = 2R \left(1 + \theta \right) = \frac{14}{3}r \left(1.84806 \ldots \right)$

So $21 = \frac{14}{3}r \left(1.84806 \ldots \right)$

$\Rightarrow \quad r = \frac{21 \times 3}{14 (1.84806 \ldots)} = \frac{9}{2 (1.84806 \ldots)} = 2.43$ (3 s.f.)

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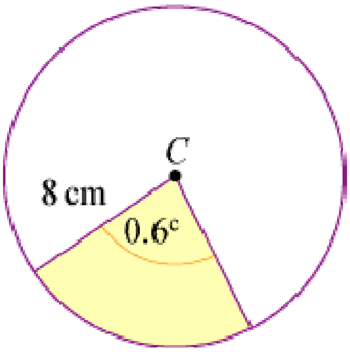
Radian measure and its applications
Exercise C, Question 1

Question:

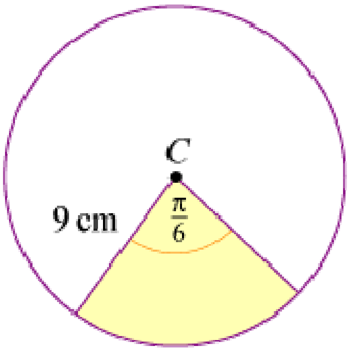
(*Note:* give non-exact answers to 3 significant figures.)

Find the area of the shaded sector in each of the following circles with centre *C*. Leave your answer in terms of π, where appropriate.

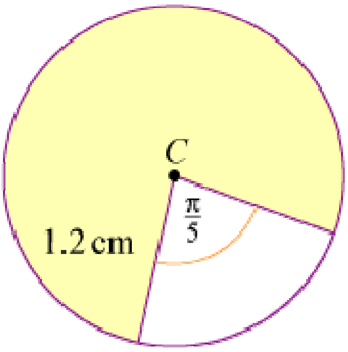
(a)



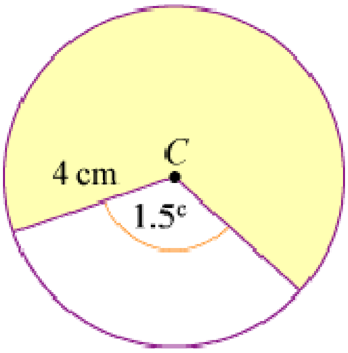
(b)



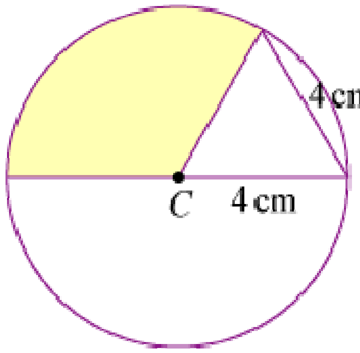
(c)



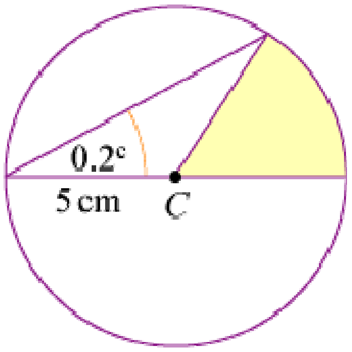
(d)



(e)

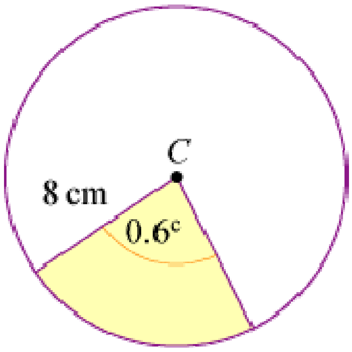


(f)



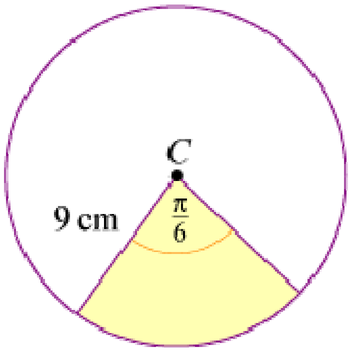
Solution:

(a)



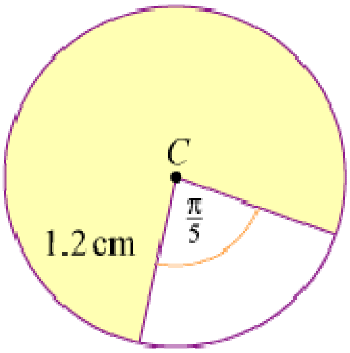
Area of shaded sector $= \frac{1}{2} \times 8^2 \times 0.6 = 19.2 \text{ cm}^2$

(b)



Area of shaded sector $= \frac{1}{2} \times 9^2 \times \frac{\pi}{6} = \frac{27\pi}{4} \text{ cm}^2 = 6.75\pi \text{ cm}^2$

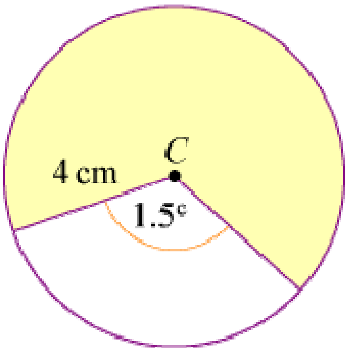
(c)



Angle subtended at C by major arc $= 2\pi - \frac{\pi}{5} = \frac{9\pi}{5}$ rad

Area of shaded sector $= \frac{1}{2} \times 1.2^2 \times \frac{9\pi}{5} = 1.296\pi \text{ cm}^2$

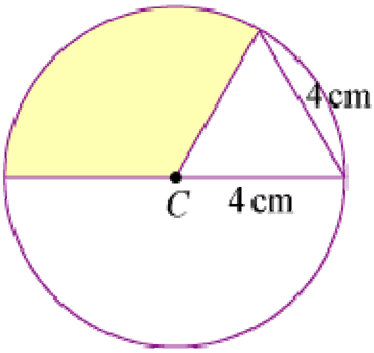
(d)



Angle subtended at C by major arc $= (2\pi - 1.5)$ rad

Area of shaded sector $= \frac{1}{2} \times 4^2 \times \left(2\pi - 1.5\right) = 38.3 \text{ cm}^2$ (3 s.f.)

(e)

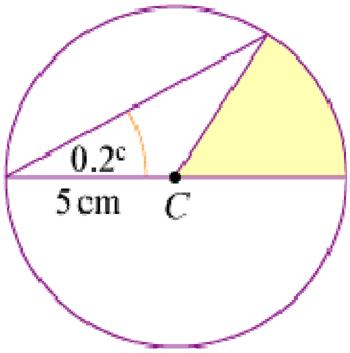


The triangle is equilateral so angle at C in the triangle is $\frac{\pi}{3}$ rad.

Angle subtended at C by shaded sector $= \pi - \frac{\pi}{3}$ rad $= \frac{2\pi}{3}$ rad

Area of shaded sector $= \frac{1}{2} \times 4^2 \times \frac{2\pi}{3} = \frac{16}{3}\pi \text{ cm}^2$

(f)



As triangle is isosceles, angle at C in shaded sector is 0.4^c.

Area of shaded sector $= \frac{1}{2} \times 5^2 \times 0.4 = 5 \text{ cm}^2$

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Radian measure and its applications

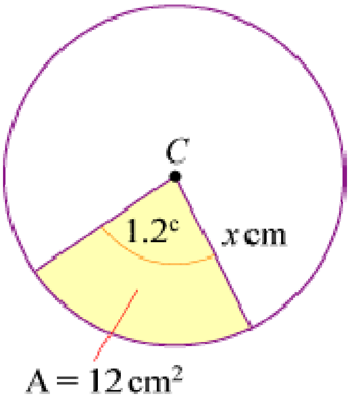
Exercise C, Question 2

Question:

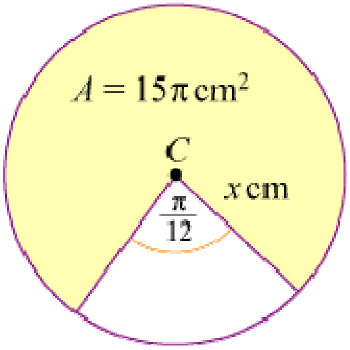
(*Note:* give non-exact answers to 3 significant figures.)

For the following circles with centre C, the area A of the shaded sector is given. Find the value of x in each case.

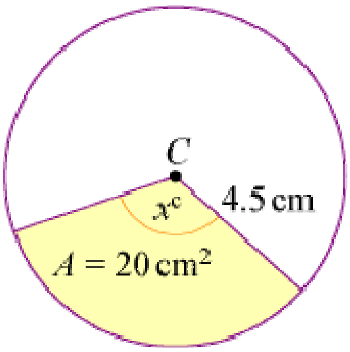
(a)



(b)

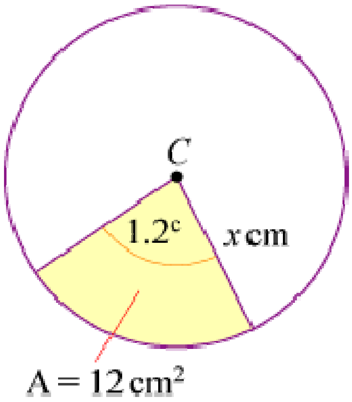


(c)



Solution:

(a)



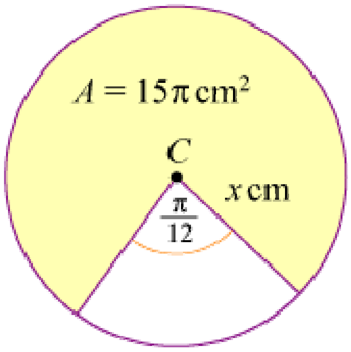
Area of shaded sector $= \frac{1}{2} \times x^2 \times 1.2 = 0.6x^2$ cm^2

So $0.6x^2 = 12$

$\Rightarrow x^2 = 20$

$\Rightarrow x = 4.47$ (3 s.f.)

(b)



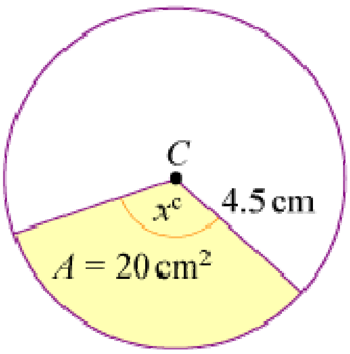
Area of shaded sector $= \frac{1}{2} \times x^2 \times \left(2\pi - \frac{\pi}{12} \right) = \frac{1}{2} x^2 \times \frac{23\pi}{12}$ cm^2

So $15\pi = \frac{23}{24} \pi x^2$

$\Rightarrow x^2 = \frac{24 \times 15}{23}$

$\Rightarrow x = 3.96$ (3 s.f.)

(c)



Area of shaded sector $= \frac{1}{2} \times 4.5^2 \times x$ cm^2

So $20 = \frac{1}{2} \times 4.5^2 x$

$\Rightarrow \quad x = \frac{40}{4.5^2} = 1.98$ (3 s.f.)

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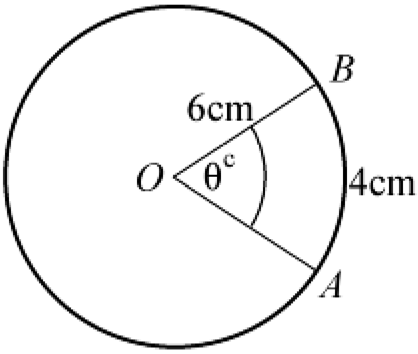
Radian measure and its applications
Exercise C, Question 3

Question:

(*Note:* give non-exact answers to 3 significant figures.)

The arc *AB* of a circle, centre *O* and radius 6 cm, has length 4 cm.
Find the area of the minor sector *AOB*.

Solution:



Using $l = r\theta$
$4 = 6\theta$
$\theta = \frac{2}{3}$

So area of sector $= \frac{1}{2} \times 6^2 \times \frac{2}{3} = 12 \text{ cm}^2$

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Radian measure and its applications
Exercise C, Question 4

Question:

(*Note:* give non-exact answers to 3 significant figures.)

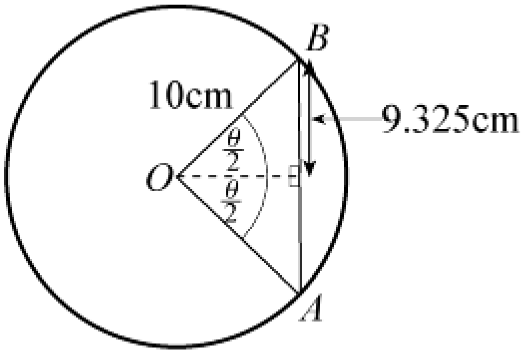
The chord *AB* of a circle, centre *O* and radius 10 cm, has length 18.65 cm and subtends an angle of θ radians at *O*.

(a) Show that $\theta = 2.40$ (to 3 significant figures).

(b) Find the area of the minor sector *AOB*.

Solution:

(a)



Using the line of symmetry in the isosceles triangle *OAB*

$\sin \frac{\theta}{2} = \frac{9.325}{10}$

$\frac{\theta}{2} = \sin^{-1}\left(\frac{9.325}{10}\right)$ (Use radian mode)

$\theta = 2\sin^{-1}\left(\frac{9.325}{10}\right) = 2.4025 \quad \ldots \quad = 2.40$ (3 s.f.)

(b) Area of minor sector *AOB* = $\frac{1}{2} \times 10^2 \times \theta = 120 \text{ cm}^2$ (3 s.f.)

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Radian measure and its applications
Exercise C, Question 5

Question:

(*Note:* give non-exact answers to 3 significant figures.)

The area of a sector of a circle of radius 12 cm is 100 cm^2.
Find the perimeter of the sector.

Solution:

Using area of sector $= \frac{1}{2} r^2 \theta$

$100 = \frac{1}{2} \times 12^2 \theta$

$\Rightarrow \quad \theta = \frac{100}{72} = \frac{25}{18}^{c}$

The perimeter of the sector $= 12 + 12 + 12\theta = 12 \left(2 + \theta \right) = 12 \times \frac{61}{18} = \frac{122}{3} = 40 \frac{2}{3}$ cm

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Radian measure and its applications
Exercise C, Question 6

Question:

(*Note:* give non-exact answers to 3 significant figures.)

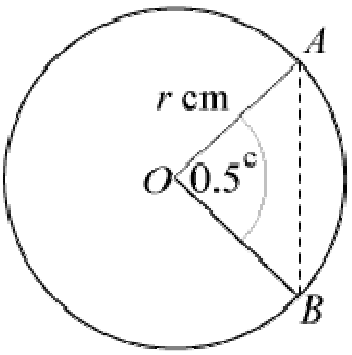
The arc *AB* of a circle, centre *O* and radius *r* cm, is such that $\angle$ AOB = 0.5 radians. Given that the perimeter of the minor sector *AOB* is 30 cm:

(a) Calculate the value of *r*.

(b) Show that the area of the minor sector *AOB* is 36 cm^2.

(c) Calculate the area of the segment enclosed by the chord *AB* and the minor arc *AB*.

Solution:



(a) The perimeter of minor sector AOB = $r + r + 0.5r = 2.5r$ cm
So $30 = 2.5r$

$\Rightarrow \quad r = \frac{30}{2.5} = 12$

(b) Area of minor sector $= \frac{1}{2} \times r^2 \times \theta = \frac{1}{2} \times 12^2 \times 0.5 = 36$ cm^2

(c) Area of segment

$= \frac{1}{2}r^2 \left(\theta - \sin \theta \right)$

$= \frac{1}{2} \times 12^2 \left(0.5 - \sin 0.5 \right)$

$= 72 \left(0.5 - \sin 0.5 \right)$
$= 1.48$ cm^2 (3 s.f.)

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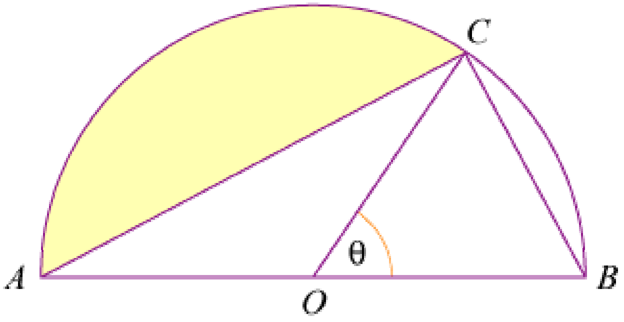
Radian measure and its applications
Exercise C, Question 7

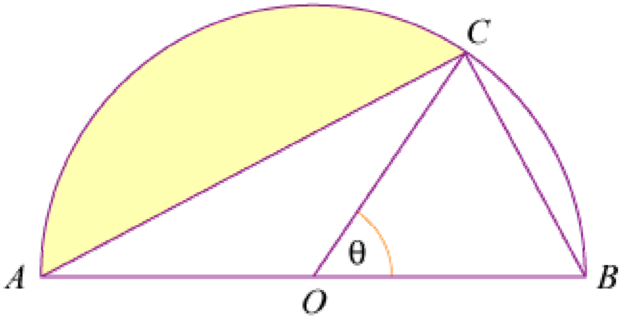
Question:

(*Note:* give non-exact answers to 3 significant figures.)

In the diagram, *AB* is the diameter of a circle of radius *r* cm and $\angle$ BOC = θ radians. Given that the area of △COB is equal to that of the shaded segment, show that $\theta + 2\sin\theta = \pi$.



Solution:



Using the formula

area of a triangle $= \frac{1}{2}ab\sin C$

area of △COB $= \frac{1}{2}r^2\sin\theta$ ①

$\angle$ AOC $= (\pi - \theta)$ rad

Area of shaded segment $= \frac{1}{2}r^2\left[(\pi-\theta) - \sin(\pi-\theta)\right]$ ②

As ① and ② are equal

$\frac{1}{2}r^2\sin\theta = \frac{1}{2}r^2\left[\pi - \theta - \sin(\pi-\theta)\right]$

$\sin\theta = \pi - \theta - \sin(\pi - \theta)$
and as $\sin(\pi-\theta) = \sin\theta$
$\sin\theta = \pi - \theta - \sin\theta$
So $\theta + 2\sin\theta = \pi$

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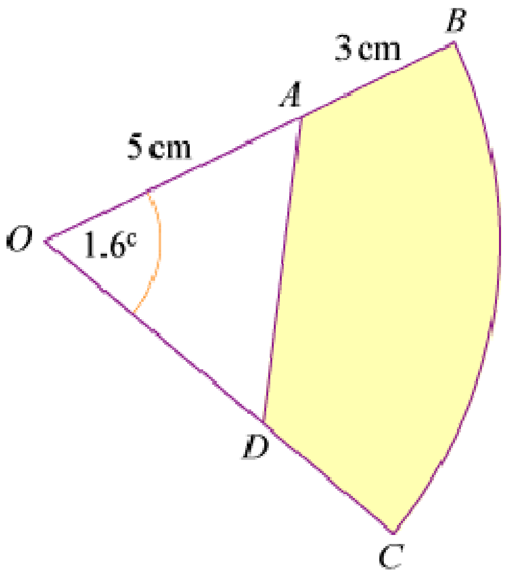
Radian measure and its applications
Exercise C, Question 8

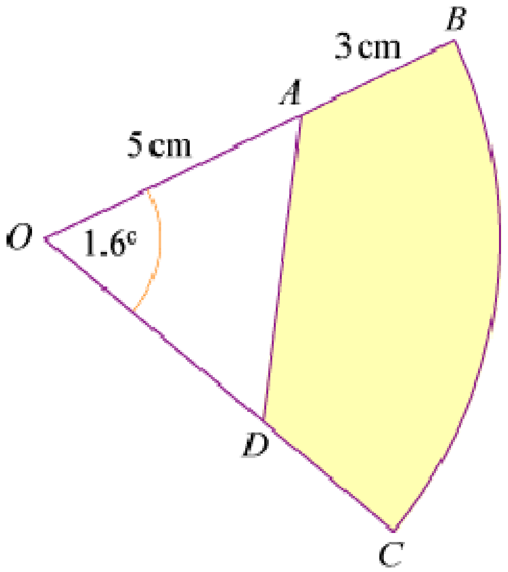
Question:

(*Note:* give non-exact answers to 3 significant figures.)

In the diagram, *BC* is the arc of a circle, centre *O* and radius 8 cm. The points *A* and *D* are such that OA = OD = 5 cm. Given that ∠ BOC = 1.6 radians, calculate the area of the shaded region.



Solution:



Area of sector OBC = $\frac{1}{2}r^2\theta$ with $r = 8$ cm and $\theta = 1.6^c$

Area of sector OBC = $\frac{1}{2} \times 8^2 \times 1.6 = 51.2$ cm^2

Using area of triangle formula

Area of $\triangle$OAD = $\frac{1}{2} \times 5 \times 5 \times \sin 1.6^c = 12.495 \text{ cm}^2$

Area of shaded region $= 51.2 - 12.495 = 38.7 \text{ cm}^2$ (3 s.f.)

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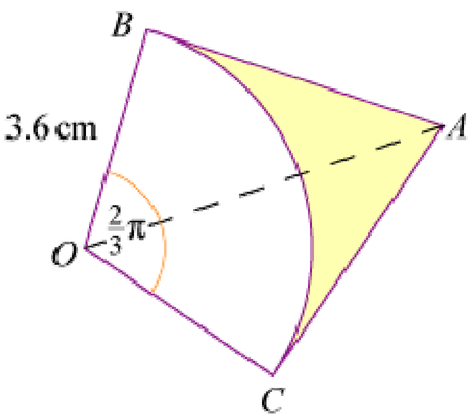
Radian measure and its applications
Exercise C, Question 9

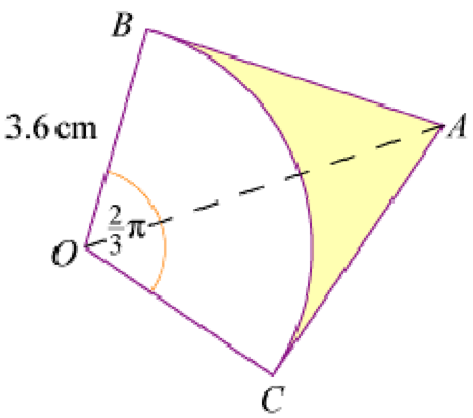
Question:

(*Note:* give non-exact answers to 3 significant figures.)

In the diagram, *AB* and *AC* are tangents to a circle, centre *O* and radius 3.6 cm. Calculate the area of the shaded region, given that $\angle BOC = \frac{2}{3}\pi$ radians.



Solution:



In right-angled $\triangle OBA$: $\tan \frac{\pi}{3} = \frac{AB}{3.6}$

$\Rightarrow AB = 3.6 \tan \frac{\pi}{3}$

Area of $\triangle OBA = \frac{1}{2} \times 3.6 \times 3.6 \times \tan \frac{\pi}{3}$

So area of quadrilateral $OBAC = 3.6^2 \times \tan \frac{\pi}{3} = 22.447 \ldots \text{ cm}^2$

Area of sector $= \frac{1}{2} \times 3.6^2 \times \frac{2}{3}\pi = 13.57 \ldots \text{ cm}^2$

Area of shaded region

= area of quadrilateral *OBAC* − area of sector *OBC*
= 8.88 cm^2 (3 s.f.)

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Radian measure and its applications
Exercise C, Question 10

Question:

(*Note:* give non-exact answers to 3 significant figures.)

A chord *AB* subtends an angle of θ radians at the centre *O* of a circle of radius 6.5 cm. Find the area of the segment enclosed by the chord *AB* and the minor arc *AB*, when:

(a) $\theta = 0.8$

(b) $\theta = \frac{2}{3}\pi$

(c) $\theta = \frac{4}{3}\pi$

Solution:

(a) Area of sector OAB = $\frac{1}{2} \times 6.5^2 \times 0.8$

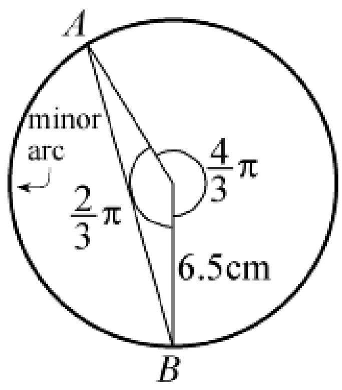
Area of $\triangle$OAB = $\frac{1}{2} \times 6.5^2 \times \sin 0.8$

Area of segment $= \frac{1}{2} \times 6.5^2 \times 0.8 - \frac{1}{2} \times 6.5^2 \times \sin 0.8 = 1.75 \text{ cm}^2$ (3 s.f.)

(b) Area of segment $= \frac{1}{2} \times 6.5^2 \left(\frac{2}{3}\pi - \sin \frac{2}{3}\pi \right) = 25.9 \text{ cm}^2$ (3 s.f.)

(c) Area of segment $= \frac{1}{2} \times 6.5^2 \left(\frac{2}{3}\pi - \sin \frac{2}{3}\pi \right) = 25.9 \text{ cm}^2$ (3 s.f.)

Diagram shows why $\frac{2}{3}\pi$ is required.



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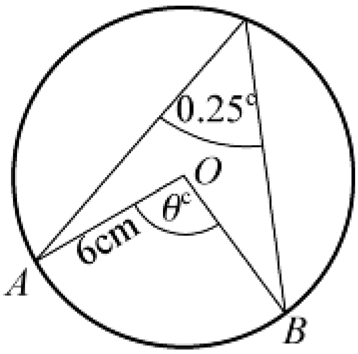
Radian measure and its applications
Exercise C, Question 11

Question:

(*Note:* give non-exact answers to 3 significant figures.)

An arc *AB* subtends an angle of 0.25 radians at the *circumference* of a circle, centre *O* and radius 6 cm. Calculate the area of the minor sector *OAB*.

Solution:



Using the circle theorem: angle at the centre $= 2 \times$ angle at circumference
$\angle AOB = 0.5^c$

Area of minor sector AOB $= \frac{1}{2} \times 6^2 \times 0.5 = 9\ \text{cm}^2$

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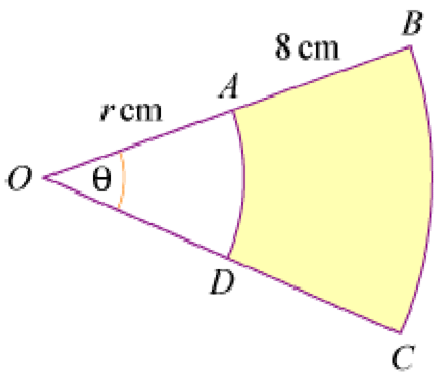
Radian measure and its applications

Exercise C, Question 12

Question:

(*Note:* give non-exact answers to 3 significant figures.)

In the diagram, AD and BC are arcs of circles with centre O, such that $OA = OD = r$ cm, $AB = DC = 8$ cm and $\angle BOC = \theta$ radians.



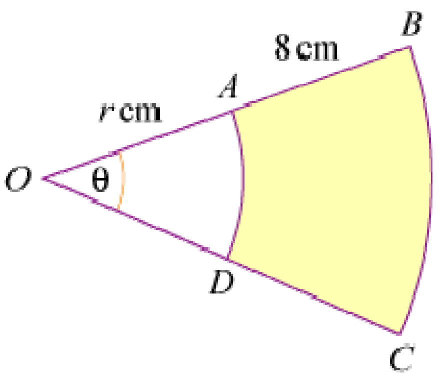
(a) Given that the area of the shaded region is 48 cm^2, show that

$r = \frac{6}{\theta} - 4$.

(b) Given also that $r = 10\theta$, calculate the perimeter of the shaded region.

Solution:

(a)



Area of larger sector $= \frac{1}{2}(r + 8)^2\theta$ cm^2

Area of smaller sector $= \frac{1}{2}r^2\theta$ cm^2

Area of shaded region

$= \frac{1}{2}(r + 8)^2\theta - \frac{1}{2}r^2\theta$ cm^2

$= \frac{1}{2}\theta\left[\left(r^2 + 16r + 64\right) - r^2\right]$ cm^2

$= \frac{1}{2}\theta \left(16r + 64 \right) \text{ cm}^2$

$= 8\theta (r + 4) \text{ cm}^2$

So $48 = 8\theta (r + 4)$

$\Rightarrow \quad 6 = r\theta + 4\theta \quad *$

$\Rightarrow \quad r\theta = 6 - 4\theta$

$\Rightarrow \quad r = \frac{6}{\theta} - 4$

(b) As $r = 10\theta$, using *

$10\theta^2 + 4\theta - 6 = 0$

$5\theta^2 + 2\theta - 3 = 0$

$(5\theta - 3) (\theta + 1) = 0$

So $\theta = \frac{3}{5}$ and $r = 10\theta = 6$

Perimeter of shaded region $= [r\theta + 8 + (r + 8) \theta + 8]$ cm

So perimeter $= \frac{18}{5} + 8 + \frac{42}{5} + 8 = 28$ cm

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Radian measure and its applications
Exercise C, Question 13

Question:

(*Note:* give non-exact answers to 3 significant figures.)

A sector of a circle of radius 28 cm has perimeter P cm and area A cm^2.
Given that $A = 4P$, find the value of P.

Solution:

The area of the sector $= \frac{1}{2} \times 28^2 \times \theta = 392\theta \text{ cm}^2 = A \text{ cm}^2$

The perimeter of the sector $= (28\theta + 56) \text{ cm} = P \text{ cm}$

As $A = 4P$

$392\theta = 4(28\theta + 56)$

$98\theta = 28\theta + 56$

$70\theta = 56$

$\theta = \frac{56}{70} = 0.8$

$P = 28\theta + 56 = 28(0.8) + 56 = 78.4$

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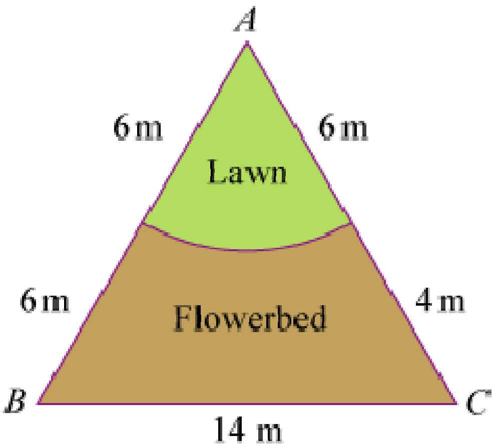
Radian measure and its applications
Exercise C, Question 14

Question:

(*Note:* give non-exact answers to 3 significant figures.)

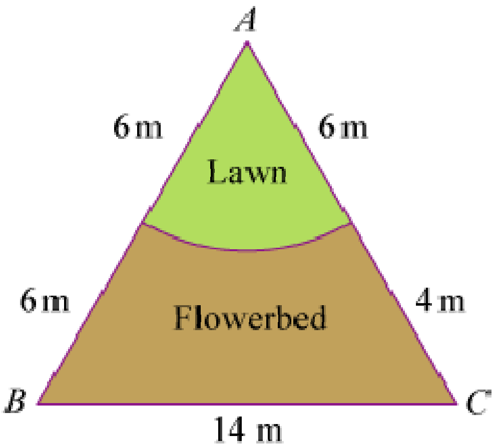
The diagram shows a triangular plot of land. The sides *AB*, *BC* and *CA* have lengths 12 m, 14 m and 10 m respectively. The lawn is a sector of a circle, centre *A* and radius 6 m.



(a) Show that ∠BAC = 1.37 radians, correct to 3 significant figures.

(b) Calculate the area of the flowerbed.

Solution:



(a) Using cosine rule

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{10^2 + 12^2 - 14^2}{2 \times 10 \times 12} = 0.2$

$A = \cos^{-1}(0.2)$ (use in radian mode)
$A = 1.369 \ldots = 1.37$ (3 s.f.)

(b) Area of $\triangle ABC = \frac{1}{2} \times 12 \times 10 \times \sin A = 58.787 \ldots \text{ m}^2$

Area of sector (lawn) $= \frac{1}{2} \times 6^2 \times A = 24.649 \ldots \text{ m}^2$

Area of flowerbed = area of $\triangle ABC$ − area of sector = 34.1m^2 (3 s.f.)

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Radian measure and its applications
Exercise D, Question 1

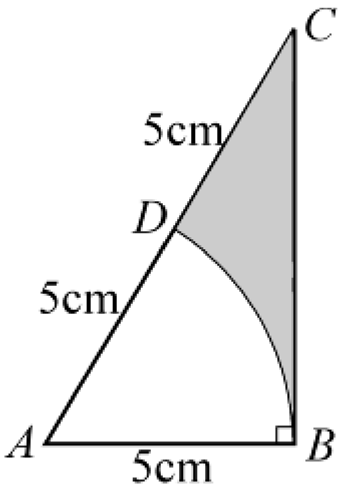
Question:

Triangle *ABC* is such that AB = 5 cm, AC = 10 cm and $\angle$ ABC = 90 ° . An arc of a circle, centre *A* and radius 5 cm, cuts *AC* at *D*.

(a) State, in radians, the value of $\angle$ BAC.

(b) Calculate the area of the region enclosed by *BC*, *DC* and the arc *BD*.

Solution:



(a) In the right-angled △ABC

$\cos \angle \text{BAC} = \frac{5}{10} = \frac{1}{2}$

$\angle \text{BAC} = \frac{\pi}{3}$

(b) Area of △ABC = $\frac{1}{2} \times 5 \times 10 \times \sin \frac{\pi}{3} = 21.650 \ldots \text{ cm}^2$

Area of sector DAB = $\frac{1}{2} \times 5^2 \times \frac{\pi}{3} = 13.089 \ldots \text{ cm}^2$

Area of shaded region = area of △ABC − area of sector *DAB* = 8.56 cm^2 (3 s.f.)

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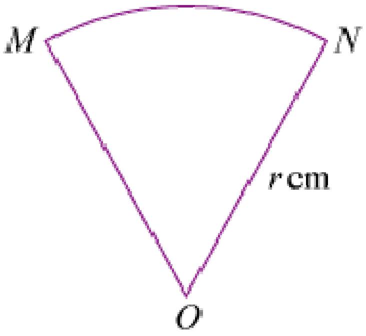
Radian measure and its applications
Exercise D, Question 2

Question:

The diagram shows a minor sector *OMN* of a circle centre *O* and radius *r* cm. The perimeter of the sector is 100 cm and the area of the sector is *A* cm^2.

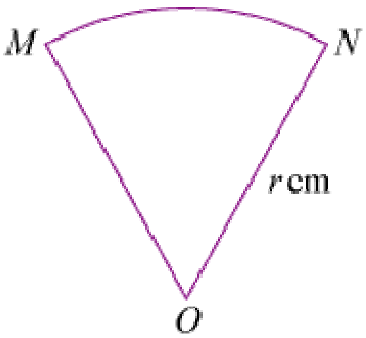


(a) Show that $A = 50r - r^2$.

(b) Given that *r* varies, find:
(i) The value of *r* for which *A* is a maximum and show that *A* is a maximum.
(ii) The value of ∠ MON for this maximum area.
(iii) The maximum area of the sector *OMN*.

[E]

Solution:



(a) Let $\angle MON = \theta^c$
Perimeter of sector = $(2r + r\theta)$ cm
So $100 = 2r + r\theta$

$\Rightarrow \quad r\theta = 100 - 2r$

$\Rightarrow \quad \theta = \frac{100}{r} - 2$ *

The area of the sector $= A \text{ cm}^2 = \frac{1}{2}r^2\theta \text{ cm}^2$

So $A = \frac{1}{2}r^2\left(\frac{100}{r} - 2\right)$

$\Rightarrow \quad A = 50r - r^2$

(b) (i) $A = -(r^2 - 50r) = -[(r-25)^2 - 625] = 625 - (r-25)^2$
The maximum value occurs when $r = 25$, as for all other values of r something is subtracted from 625.

(ii) Using *, when $r = 25$, $\theta = \frac{100}{25} - 2 = 2^c$

(iii) Maximum area = 625 cm^2

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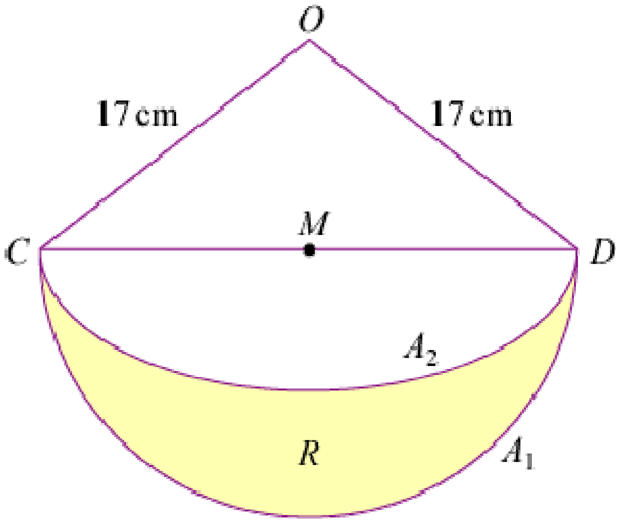
Radian measure and its applications
Exercise D, Question 3

Question:

The diagram shows the triangle *OCD* with OC = OD = 17 cm and CD = 30 cm. The mid-point of *CD* is *M*. With centre *M*, a semicircular arc A_1 is drawn on *CD* as diameter. With centre *O* and radius 17 cm, a circular arc A_2 is drawn from *C* to *D*. The shaded region *R* is bounded by the arcs A_1 and A_2. Calculate, giving answers to 2 decimal places:



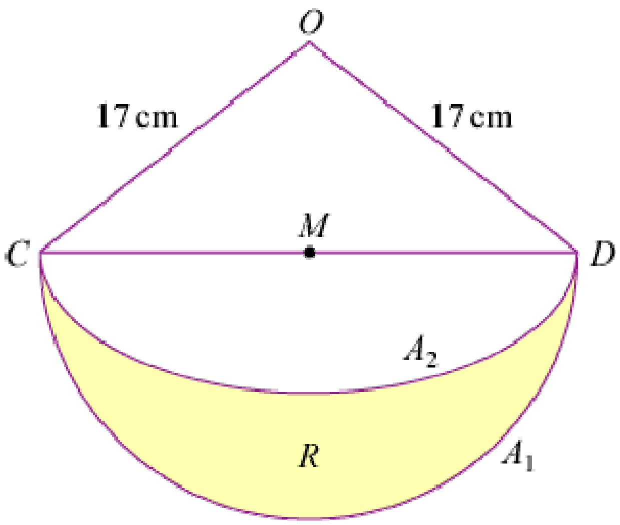
(a) The area of the triangle *OCD*.

(b) The angle *COD* in radians.

(c) The area of the shaded region *R*.

[E]

Solution:



(a) Using Pythagoras' theorem to find *OM*:
$OM^2 = 17^2 - 15^2 = 64$

$\Rightarrow \quad OM = 8$ cm

Area of $\triangle OCD = \frac{1}{2} CD \times OM = \frac{1}{2} \times 30 \times 8 = 120$ cm^2

(b) In $\triangle OCM$: $\sin \angle COM = \frac{15}{17} \Rightarrow \angle COM = 1.0808 \ldots^c$

So $\angle COD = 2 \times \angle COM = 2.16^c$ (2 d.p.)

(c) Area of shaded region R = area of semicircle $-$ area of segment CDA_2

Area of segment = area of sector OCD – area of sector $\triangle$ OCD

$= \frac{1}{2} \times 17^2 \left(\angle COD - \sin \angle COD \right)$ (angles in radians)

$= 192.362 \ldots$ cm^2 (use at least 3 d.p.)

Area of semicircle $= \frac{1}{2} \times \pi \times 15^2 = 353.429 \ldots$ cm^2

So area of shaded region $R = 353.429 \ldots - 192.362 \ldots = 161.07$ cm^2 (2 d.p.)

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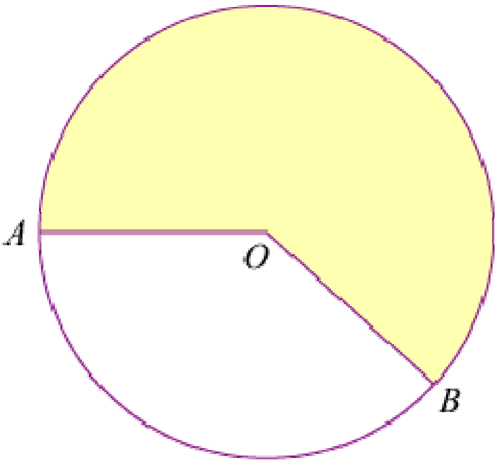
Radian measure and its applications
Exercise D, Question 4

Question:

The diagram shows a circle, centre O, of radius 6 cm. The points A and B are on the circumference of the circle. The area of the shaded major sector is 80 cm^2. Given that $\angle AOB = \theta$ radians, where $0 < \theta < \pi$, calculate:

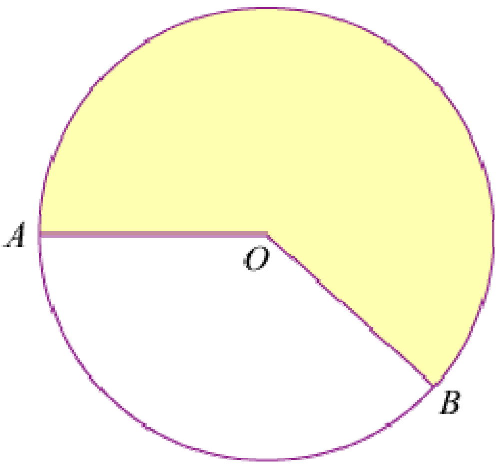


(a) The value, to 3 decimal places, of θ.

(b) The length in cm, to 2 decimal places, of the minor arc AB.

[E]

Solution:



(a) Reflex angle AOB = $(2\pi - \theta)$ rad

Area of shaded sector $= \frac{1}{2} \times 6^2 \times \left(2\pi - \theta \right) = 36\pi - 18\theta$ cm^2

So $80 = 36\pi - 18\theta$

$\Rightarrow \quad 18\theta = 36\pi - 80$

$\Rightarrow \quad \theta = \frac{36\pi - 80}{18} = 1.839$ (3 d.p.)

(b) Length of minor arc AB = 6θ = 11.03 cm (2 d.p.)

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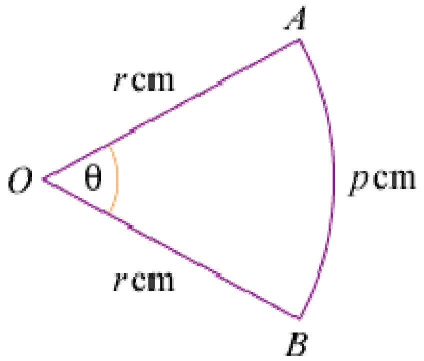
Radian measure and its applications
Exercise D, Question 5

Question:

The diagram shows a sector *OAB* of a circle, centre *O* and radius *r* cm. The length of the arc *AB* is *p* cm and ∠ AOB is θ radians.



(a) Find θ in terms of *p* and *r*.

(b) Deduce that the area of the sector is $\frac{1}{2}$pr cm^2.

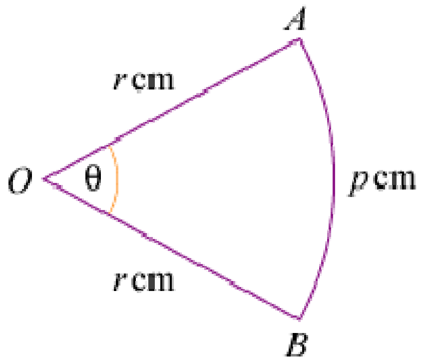
Given that $r = 4.7$ and $p = 5.3$, where each has been measured to 1 decimal place, find, giving your answer to 3 decimal places:

(c) The least possible value of the area of the sector.

(d) The range of possible values of θ.

[E]

Solution:



(a) Using $l = r\theta \quad \Rightarrow \quad p = r\theta$

So $\theta = \frac{p}{r}$

(b) Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times \frac{p}{r} = \frac{1}{2}$pr cm^2

(c) $4.65 \leq r < 4.75$, $5.25 \leq p < 5.35$

Least value for area of sector $= \frac{1}{2} \times 5.25 \times 4.65 = 12.207 \text{ cm}^2$ (3 d.p.)

(**Note**: Lowest is 12.20625, so 12.207 should be given.)

(d) Max value of $\theta = \frac{\max p}{\min r} = \frac{5.35}{4.65} = 1.1505 \ldots$

So give 1.150 (3 d.p.)

Min value of $\theta = \frac{\min p}{\max r} = \frac{5.25}{4.75} = 1.10526 \ldots$

So give 1.106 (3 d.p.)

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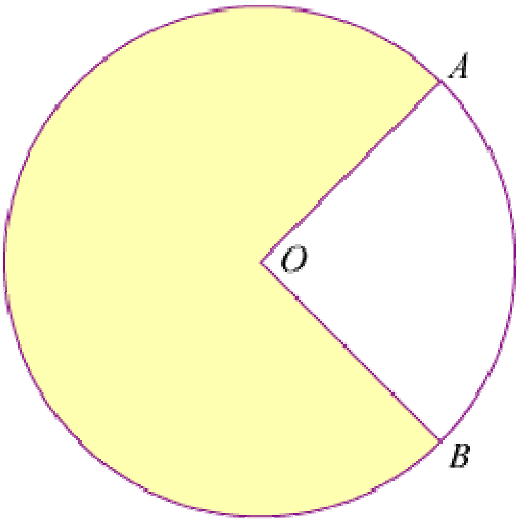
Radian measure and its applications
Exercise D, Question 6

Question:

The diagram shows a circle centre *O* and radius 5 cm. The length of the minor arc *AB* is 6.4 cm.



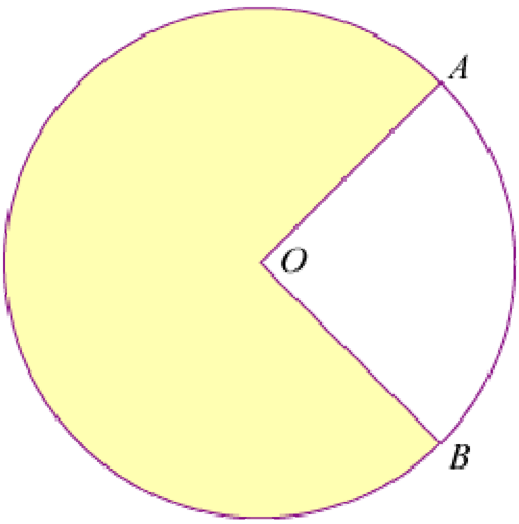
(a) Calculate, in radians, the size of the acute angle *AOB*.
The area of the minor sector *AOB* is R_1 cm^2 and the area of the shaded major sector *AOB* is R_2 cm^2.

(b) Calculate the value of R_1.

(c) Calculate R_1: R_2 in the form 1: *p*, giving the value of *p* to 3 significant figures.

[E]

Solution:



(a) Using $l = r\theta$, $6.4 = 5\theta$

$\Rightarrow \quad \theta = \frac{6.4}{5} = 1.28^c$

(b) Using area of sector $= \frac{1}{2}r^2\theta$

$R_1 = \frac{1}{2} \times 5^2 \times 1.28 = 16$

(c) $R_2 =$ area of circle $- R_1 = \pi 5^2 - 16 = 62.5398 \ldots$

So $\frac{R_1}{R_2} = \frac{16}{62.5398 \ldots} = \frac{1}{3.908 \ldots} = \frac{1}{p}$

$\Rightarrow \quad p = 3.91$ (3 s.f.)

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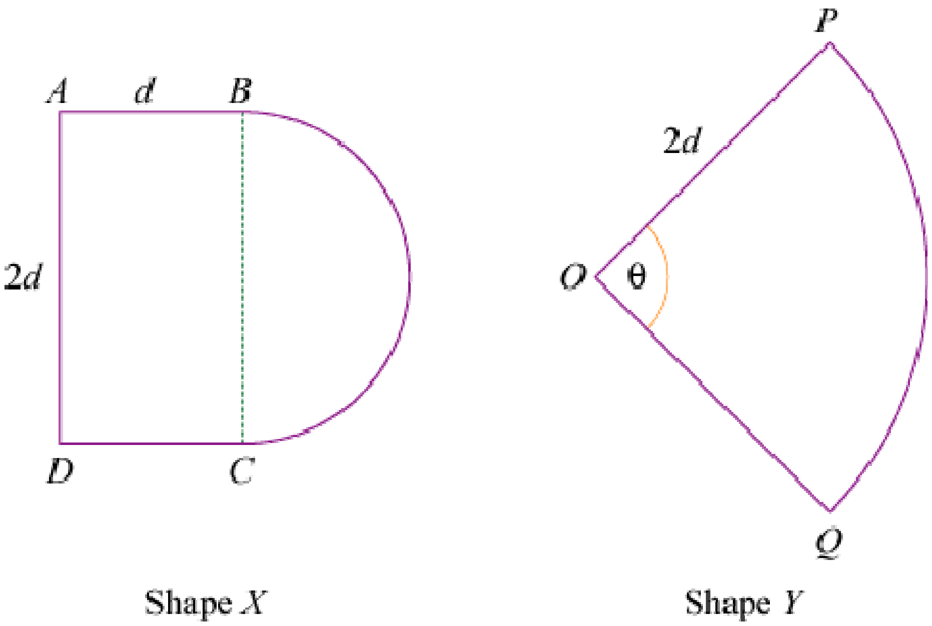
Radian measure and its applications
Exercise D, Question 7

Question:



The diagrams show the cross-sections of two drawer handles.
Shape *X* is a rectangle *ABCD* joined to a semicircle with *BC* as diameter. The length AB = *d* cm and BC = 2*d* cm. Shape *Y* is a sector *OPQ* of a circle with centre *O* and radius 2*d* cm. Angle *POQ* is θ radians.
Given that the areas of shapes *X* and *Y* are equal:

(a) Prove that $\theta = 1 + \frac{1}{4}\pi$.

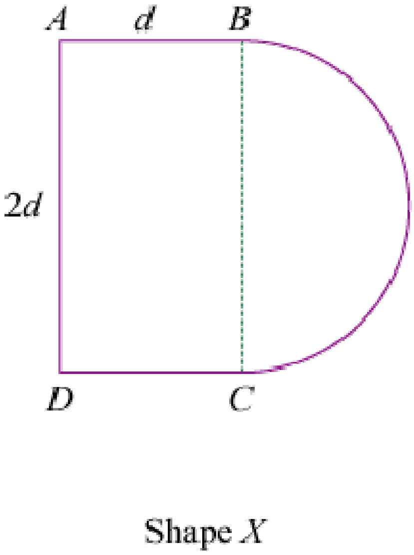
Using this value of θ, and given that $d = 3$, find in terms of π:

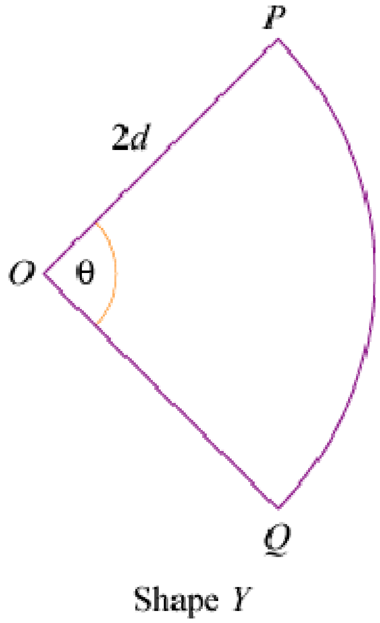
(b) The perimeter of shape *X*.

(c) The perimeter of shape *Y*.

(d) Hence find the difference, in mm, between the perimeters of shapes *X* and *Y*. **[E]**

Solution:





(a) Area of shape X
= area of rectangle + area of semicircle

$= 2d^2 + \frac{1}{2}\pi d^2 \text{ cm}^2$

Area of shape $Y = \frac{1}{2}(2d)^2\theta = 2d^2\theta \text{ cm}^2$

As $X = Y$: $2d^2 + \frac{1}{2}\pi d^2 = 2d^2\theta$

Divide by $2d^2$: $1 + \frac{\pi}{4} = \theta$

(b) Perimeter of X
$= (d + 2d + d + \pi d)$ cm with $d = 3$
$= (3\pi + 12)$ cm

(c) Perimeter of Y

$= (2d + 2d + 2d\theta)$ cm with $d = 3$ and $\theta = 1 + \frac{\pi}{4}$

$= 12 + 6\left(1 + \frac{\pi}{4}\right)$

$= \left(18 + \frac{3\pi}{2}\right)$ cm

(d) Difference (in mm)

$= \left[\left(18 + \frac{3\pi}{2}\right) - \left(3\pi + 12\right)\right] \times 10$

$= 10\left(6 - \frac{3\pi}{2}\right)$

$= 12.87 \ldots$
$= 12.9$ (3 s.f.)

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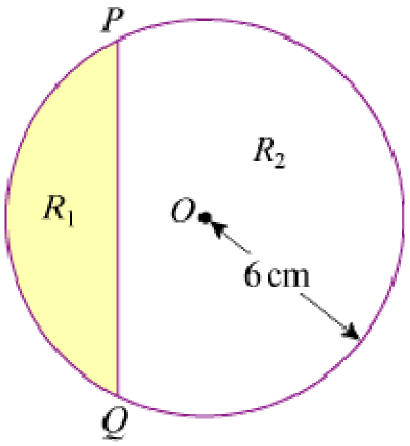
Radian measure and its applications
Exercise D, Question 8

Question:

The diagram shows a circle with centre *O* and radius 6 cm. The chord *PQ* divides the circle into a minor segment R_1 of area A_1 cm^2 and a major segment R_2 of area A_2 cm^2. The chord *PQ* subtends an angle θ radians at *O*.

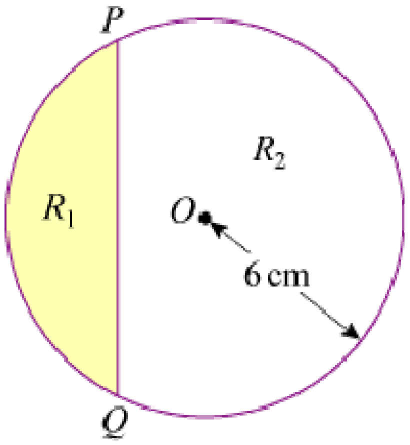


(a) Show that $A_1 = 18 (\theta - \sin \theta)$.
Given that $A_2 = 3A_1$ and $f (\theta) = 2\theta - 2 \sin \theta - \pi$:

(b) Prove that $f (\theta) = 0$.

(c) Evaluate f(2.3) and f(2.32) and deduce that $2.3 < \theta < 2.32$. **[E]**

Solution:



(a) Area of segment R_1 = area of sector *OPQ* − area of triangle *OPQ*

$\Rightarrow A_1 = \frac{1}{2} \times 6^2 \times \theta - \frac{1}{2} \times 6^2 \times \sin \theta$

$\Rightarrow A_1 = 18 (\theta - \sin \theta)$

(b) Area of segment R_2 = area of circle − area of segment R_1

$\Rightarrow A_2 = \pi 6^2 - 18 (\theta - \sin \theta)$

$\Rightarrow \quad A_2 = 36\pi - 18\theta + 18 \sin \theta$

As $A_2 = 3A_1$

$36\pi - 18\theta + 18 \sin \theta = 3 (18\theta - 18 \sin \theta) = 54\theta - 54 \sin \theta$

So $72\theta - 72 \sin \theta - 36\pi = 0$

$\Rightarrow \quad 36 (2\theta - 2 \sin \theta - \pi) = 0$

$\Rightarrow \quad 2\theta - 2 \sin \theta - \pi = 0$

So $f (\theta) = 0$

(c) $f (2.3) = - 0.0330 \ldots$

$f (2.32) = + 0.0339 \ldots$

As there is a change of sign θ lies between 2.3 and 2.32.

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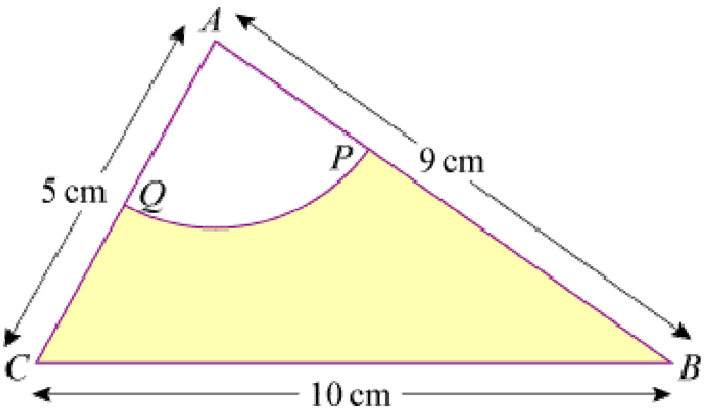
Radian measure and its applications

Exercise D, Question 9

Question:

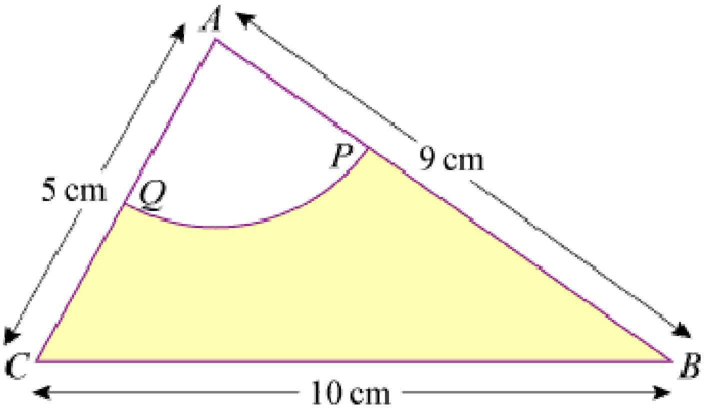
Triangle ABC has AB = 9 cm, BC = 10 cm and CA = 5 cm. A circle, centre A and radius 3 cm, intersects AB and AC at P and Q respectively, as shown in the diagram.



(a) Show that, to 3 decimal places, $\angle BAC = 1.504$ radians.

(b) Calculate:
(i) The area, in cm^2, of the sector APQ.
(ii) The area, in cm^2, of the shaded region $BPQC$.
(iii) The perimeter, in cm, of the shaded region $BPQC$. **[E]**

Solution:



(a) In $\triangle ABC$ using the cosine rule:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \quad \cos \angle BAC = \frac{5^2 + 9^2 - 10^2}{2 \times 5 \times 9} = 0.06$$

$$\Rightarrow \quad \angle BAC = 1.50408 \ \ldots \ \text{radians} = 1.504^c \text{ (3 d.p.)}$$

(b) (i) Using the sector area formula: area of sector $= \frac{1}{2}r^2\theta$

$\Rightarrow$ area of sector APQ $= \frac{1}{2} \times 3^2 \times 1.504 = 6.77$ cm^2 (3 s.f.)

(ii) Area of shaded region *BPQC*
= area of $\triangle ABC -$ area of sector *APQ*
$= \frac{1}{2} \times 5 \times 9 \times \sin 1.504^c - \frac{1}{2} \times 3^2 \times 1.504$ cm^2
= 15.681 … cm^2
= 15.7 cm^2 (3 s.f.)
(iii) Perimeter of shaded region *BPQC*
= QC + CB + BP + arc *PQ*
= 2 + 10 + 6 + (3 × 1.504) cm
= 22.51 … cm
= 22.5 cm (3 s.f.)

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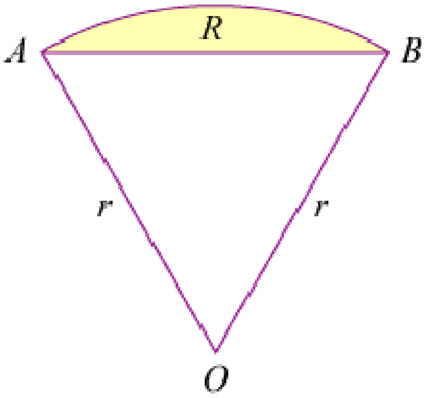
Radian measure and its applications
Exercise D, Question 10

Question:

The diagram shows the sector *OAB* of a circle of radius *r* cm. The area of the sector is 15 cm^2 and ∠ AOB = 1.5 radians.



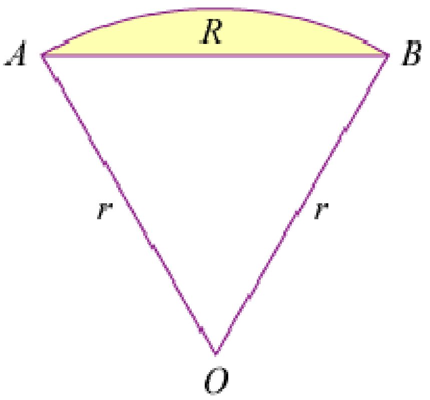
(a) Prove that $r = 2\sqrt{5}$.

(b) Find, in cm, the perimeter of the sector *OAB*.
The segment *R*, shaded in the diagram, is enclosed by the arc *AB* and the straight line *AB*.

(c) Calculate, to 3 decimal places, the area of *R*.

[E]

Solution:



(a) Area of sector $= \frac{1}{2}r^2 \left(1.5 \right) cm^2$

So $\frac{3}{4}r^2 = 15$

$\Rightarrow \quad r^2 = \frac{60}{3} = 20$

$\Rightarrow \quad r = \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

(b) Arc length AB = $r(1.5) = 3\sqrt{5}$ cm
Perimeter of sector
= AO + OB + arc *AB*
= $(2\sqrt{5} + 2\sqrt{5} + 3\sqrt{5})$ cm
= $7\sqrt{5}$ cm
= 15.7 cm (3 s.f.)

(c) Area of segment *R*
= area of sector − area of triangle
= $15 - \frac{1}{2}r^2 \sin 1.5^c$ cm^2
= $(15 - 10 \sin 1.5^c)$ cm^2
= 5.025 cm^2 (3 d.p.)

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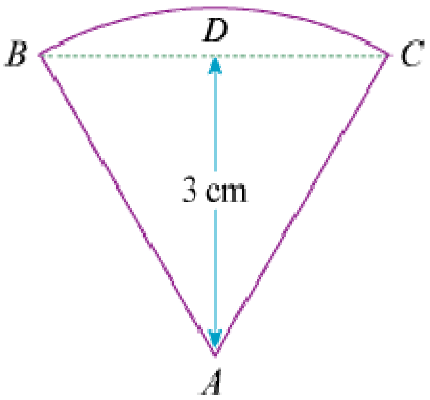
Radian measure and its applications
Exercise D, Question 11

Question:

The shape of a badge is a sector *ABC* of a circle with centre *A* and radius *AB*, as shown in the diagram. The triangle *ABC* is equilateral and has perpendicular height 3 cm.



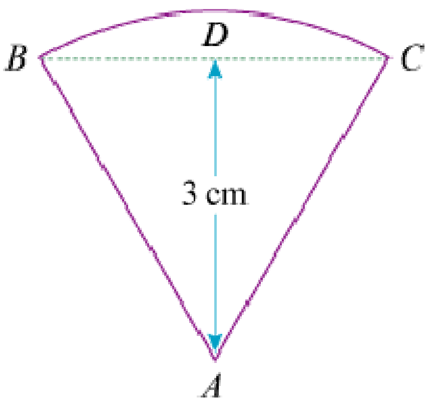
(a) Find, in surd form, the length of *AB*.

(b) Find, in terms of π, the area of the badge.

(c) Prove that the perimeter of the badge is $\frac{2\sqrt{3}}{3}\left(\pi + 6\right)$ cm.

[E]

Solution:



(a) Using the right-angled $\triangle ABD$, with $\angle ABD = 60°$,

$\sin 60° = \frac{3}{AB}$

$$\Rightarrow \quad AB = \frac{3}{\sin 60°} = \frac{3}{\frac{\sqrt{3}}{2}} = 3 \times \frac{2}{\sqrt{3}} = 2\sqrt{3} \text{ cm}$$

(b) Area of badge
= area of sector

$= \frac{1}{2} \times (2\sqrt{3})^2 \theta$ where $\theta = \frac{\pi}{3}$

$= \frac{1}{2} \times 12 \times \frac{\pi}{3}$

$= 2\pi$ cm^2

(c) Perimeter of badge
= AB + AC + arc BC

$= \left(2\sqrt{3} + 2\sqrt{3} + 2\sqrt{3}\frac{\pi}{3} \right)$ cm

$= 2\sqrt{3}\left(2 + \frac{\pi}{3} \right)$ cm

$= \frac{2\sqrt{3}}{3}\left(6 + \pi \right)$ cm

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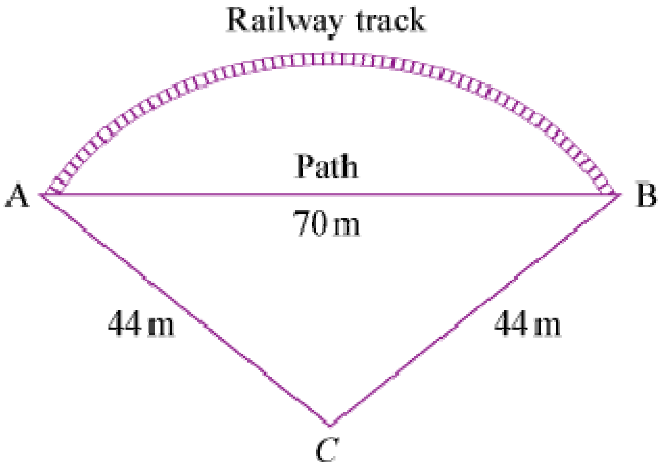
Radian measure and its applications

Exercise D, Question 12

Question:

There is a straight path of length 70 m from the point *A* to the point *B*. The points are joined also by a railway track in the form of an arc of the circle whose centre is *C* and whose radius is 44 m, as shown in the diagram.

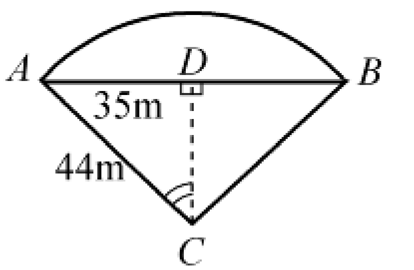


(a) Show that the size, to 2 decimal places, of ∠ ACB is 1.84 radians.

(b) Calculate:
(i) The length of the railway track.
(ii) The shortest distance from *C* to the path.
(iii) The area of the region bounded by the railway track and the path.

[E]

Solution:



(a) Using right-angled △ADC

sin ∠ ACD = $\frac{35}{44}$

So ∠ ACD = $\sin^{-1}\left(\frac{35}{44}\right)$

and ∠ ACB = $2\sin^{-1}\left(\frac{35}{44}\right)$ (work in radian mode)

⇒ ∠ ACB = 1.8395 … = 1.84^c (2 d.p.)

(b) (i) Length of railway track = length of arc AB = $44 \times 1.8395 \ldots = 80.9$ m (3 s.f.)
(ii) Shortest distance from C to AB is DC.
Using Pythagoras' theorem:
$DC^2 = 44^2 - 35^2$
$DC = \sqrt{44^2 - 35^2} = 26.7$ m (3 s.f.)
(iii) Area of region = area of segment
= area of sector ABC − area of $\triangle ABC$

$= \frac{1}{2} \times 44^2 \times 1.8395 \ldots - \frac{1}{2} \times 70 \times DC \quad (\text{or } \frac{1}{2} \times 44^2 \times \sin 1.8395 \ldots^c)$

$= 847$ m^2 (3 s.f.)

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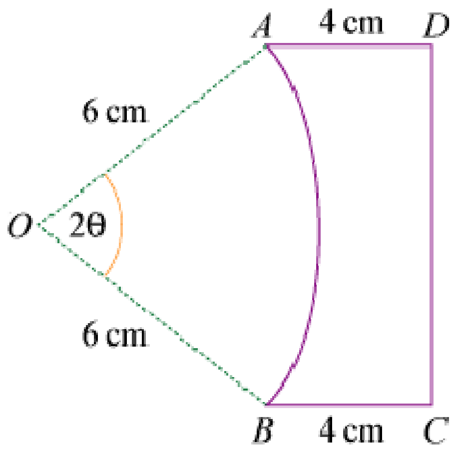
Radian measure and its applications
Exercise D, Question 13

Question:



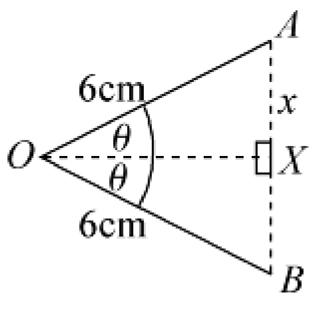
The diagram shows the cross-section *ABCD* of a glass prism. AD = BC = 4 cm and both are at right angles to *DC*. *AB* is the arc of a circle, centre *O* and radius 6 cm. Given that $\angle$ AOB = 2θ radians, and that the perimeter of the cross-section is $2(7+\pi)$ cm:

(a) Show that $\left(2\theta + 2\sin\theta - 1\right) = \frac{\pi}{3}$.

(b) Verify that $\theta = \frac{\pi}{6}$.

(c) Find the area of the cross-section.

Solution:



(a) In △OAX (see diagram)

$\frac{x}{6} = \sin\theta$

$\Rightarrow \quad x = 6\sin\theta$

So AB = $2x = 12\sin\theta$ (AB = DC)

The perimeter of cross-section

= arc AB + AD + DC + BC

= $[6(2\theta) + 4 + 12\sin\theta + 4]$ cm

= $(8 + 12\theta + 12\sin\theta)$ cm

So $2(7+\pi) = 8 + 12\theta + 12\sin\theta$

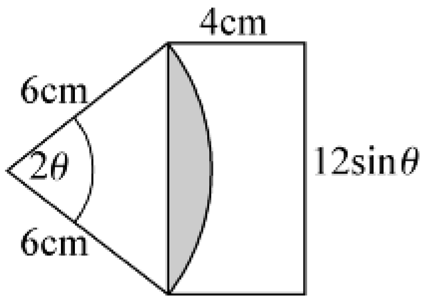
$\Rightarrow \quad 14 + 2\pi = 8 + 12\theta + 12\sin\theta$

$\Rightarrow \quad 12\theta + 12\sin\theta - 6 = 2\pi$

Divide by 6: $2\theta + 2\sin\theta - 1 = \frac{\pi}{3}$

(b) When $\theta = \frac{\pi}{6}$, $2\theta + 2\sin\theta - 1 = \frac{\pi}{3} + \left(2 \times \frac{1}{2}\right) - 1 = \frac{\pi}{3}$ ✓

(c)



The area of cross-section = area of rectangle *ABCD* − area of shaded segment

Area of rectangle $= 4 \times \left(12\sin\frac{\pi}{6}\right) = 24\text{ cm}^2$

Area of shaded segment
= area of sector − area of triangle

$= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} - \frac{1}{2} \times 6^2 \sin\frac{\pi}{3}$

$= 3.261 \ldots \text{ cm}^2$

So area of cross-section = 20.7 cm^2 (3 s.f.)

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Radian measure and its applications

Exercise D, Question 14

Question:

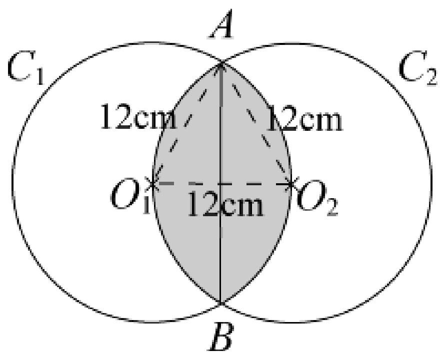
Two circles C_1 and C_2, both of radius 12 cm, have centres O_1 and O_2 respectively. O_1 lies on the circumference of C_2; O_2 lies on the circumference of C_1. The circles intersect at A and B, and enclose the region R.

(a) Show that $\angle AO_1B = \frac{2}{3}\pi$ radians.

(b) Hence write down, in terms of π, the perimeter of R.

(c) Find the area of R, giving your answer to 3 significant figures.

Solution:



(a) $\triangle AO_1O_2$ is equilateral.

So $\angle AO_1O_2 = \frac{\pi}{3}$ radians

$\angle AO_1B = 2\angle AO_1O_2 = \frac{2\pi}{3}$ radians

(b) Consider arc AO_2B in circle C_1.
Using arc length $= r\theta$

arc $AO_2B = 12 \times \frac{2\pi}{3} = 8\pi$ cm

Perimeter of R = arc AO_2B + arc $AO_1B = 2 \times 8\pi = 16\pi$ cm

(c) Consider the segment AO_2B in circle C_1.
Area of segment AO_2B
= area of sector O_1AB − area of $\triangle O_1$AB

$= \frac{1}{2} \times 12^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 12^2 \times \sin\frac{2\pi}{3}$

= 88.442 … cm^2
Area of region R
= area of segment AO_2B + area of segment AO_1B
= 2 × 88.442 … cm^2
= 177 cm^2 (3 s.f.)