Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

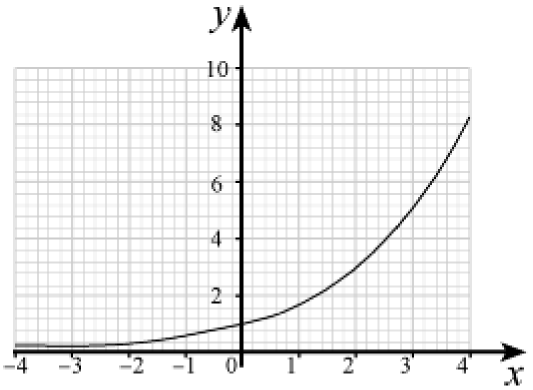
Exponentials and logarithms
Exercise A, Question 1

Question:

(a) Draw an accurate graph of $y = (1.7)^x$, for $-4 \leq x \leq 4$.

(b) Use your graph to solve the equation $(1.7)^x = 4$.

Solution:



(a)

(b) Where $y = 4$, $x \simeq 2.6$

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Exponentials and logarithms
Exercise A, Question 2

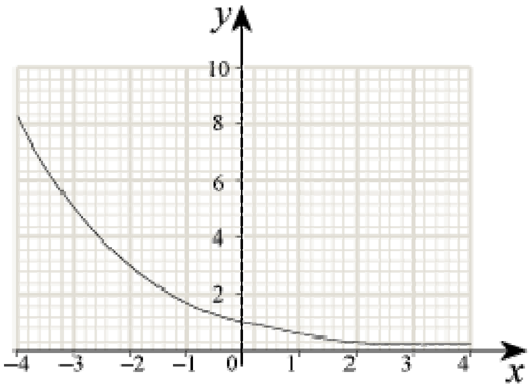
Question:

(a) Draw an accurate graph of $y = (0.6) ^{x}$, for $-4 \leq x \leq 4$.

(b) Use your graph to solve the equation $(0.6) ^{x} = 2$.

Solution:

(a)



(b) Where $y = 2$, $x \simeq -1.4$

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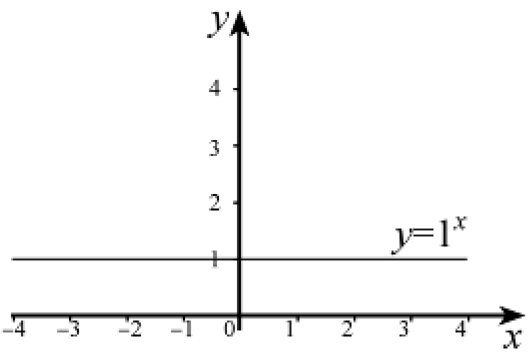
Exponentials and logarithms
Exercise A, Question 3

Question:

Sketch the graph of $y = 1^x$.

Solution:



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Exponentials and logarithms
Exercise B, Question 1

Question:

Rewrite as a logarithm:

(a) $4^4 = 256$

(b) $3^{-2} = \frac{1}{9}$

(c) $10^6 = 1\ 000\ 000$

(d) $11^1 = 11$

(e) $(0.2)^3 = 0.008$

Solution:

(a) $\log_4 256 = 4$

(b) $\log_3 \left(\frac{1}{9} \right) = -2$

(c) $\log_{10} 1\ 000\ 000 = 6$

(d) $\log_{11} 11 = 1$

(e) $\log_{0.2} 0.008 = 3$

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Exponentials and logarithms
Exercise B, Question 2

Question:

Rewrite using a power:

(a) $\log_2 16 = 4$

(b) $\log_5 25 = 2$

(c) $\log_9 3 = \frac{1}{2}$

(d) $\log_5 0.2 = -1$

(e) $\log_{10} 100\ 000 = 5$

Solution:

(a) $2^4 = 16$

(b) $5^2 = 25$

(c) $9^{\frac{1}{2}} = 3$

(d) $5^{-1} = 0.2$

(e) $10^5 = 100\ 000$

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Exponentials and logarithms

Exercise B, Question 3

Question:

Find the value of:

(a) $\log_2 8$

(b) $\log_5 25$

(c) $\log_{10} 10\ 000\ 000$

(d) $\log_{12} 12$

(e) $\log_3 729$

(f) $\log_{10} \sqrt{10}$

(g) $\log_4 (0.25)$

(h) $\log_{0.25} 16$

(i) $\log_a (a^{10})$

(j) $\log_{\left(\frac{2}{3}\right)} \left(\frac{9}{4}\right)$

Solution:

(a) If $\log_2 8 = x$ then $2^x = 8$, so $x = 3$

(b) If $\log_5 25 = x$ then $5^x = 25$, so $x = 2$

(c) If $\log_{10} 10\ 000\ 000 = x$ then $10^x = 10\ 000\ 000$, so $x = 7$

(d) If $\log_{12} 12 = x$ then $12^x = 12$, so $x = 1$

(e) If $\log_3 729 = x$ then $3^x = 729$, so $x = 6$

(f) If $\log_{10} \sqrt{10} = x$ then $10^x = \sqrt{10}$, so $x = \frac{1}{2}$

(Power $\frac{1}{2}$ means 'square root'.)

(g) If $\log_4 (0.25) = x$ then $4^x = 0.25 = \frac{1}{4}$, so $x = -1$

(Negative power means ‘reciprocal’.)

(h) $\log_{0.25} 16 = x$

$\Rightarrow \quad 0.25^x = 16$

$\Rightarrow \quad \left(\frac{1}{4}\right)^x = 16$, so $x = -2$

$$\left[\left(\frac{1}{4}\right)^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2} = \frac{1}{\left(\frac{1}{16}\right)} = 16 \right]$$

(i) $\log_a \left(a^{10}\right) = x$

$\Rightarrow \quad a^x = a^{10}$, so $x = 10$

(j) $\log_{\left(\frac{2}{3}\right)} \left(\frac{9}{4}\right) = x$

$\Rightarrow \quad \left(\frac{2}{3}\right)^x = \frac{9}{4}$, so $x = -2$

$$\left[\left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{1}{\left(\frac{4}{9}\right)} = \frac{9}{4} \right]$$

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Exponentials and logarithms
Exercise B, Question 4

Question:

Find the value of x for which:

(a) $\log_5 x = 4$

(b) $\log_x 81 = 2$

(c) $\log_7 x = 1$

(d) $\log_x (2x) = 2$

Solution:

(a) Using a power, $5^4 = x$
So $x = 625$

(b) Using a power, $x^2 = 81$
So $x = 9$
(The base of a logarithm cannot be negative, so $x = -9$ is not possible.)

(c) Using a power, $7^1 = x$
So $x = 7$

(d) Using a power,
$x^2 = 2x$
$x^2 - 2x = 0$
$x(x - 2) = 0$
$x = 2$
(The base of a logarithm cannot be zero, so $x = 0$ is not possible.)

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Exponentials and logarithms
Exercise C, Question 1

Question:

Find from your calculator the value to 3 s.f. of:

$\log_{10} 20$

Solution:

$\log_{10} 20 = 1.3010 \quad \ldots \quad = 1.30$ (3 s.f.)

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Exponentials and logarithms
Exercise C, Question 2

Question:

Find from your calculator the value to 3 s.f. of:
$\log_{10} 4$

Solution:

$\log_{10} 4 = 0.6020 \quad \ldots \quad = 0.602$ (3 s.f.)

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Exponentials and logarithms
Exercise C, Question 3

Question:

Find from your calculator the value to 3 s.f. of:
$\log_{10} 7000$

Solution:

$\log_{10} 7000 = 3.8450 \ldots = 3.85$ (3 s.f.)

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Exponentials and logarithms
Exercise C, Question 4

Question:

Find from your calculator the value to 3 s.f. of:
$\log_{10} 0.786$

Solution:

$\log_{10} 0.786 = -0.1045 \ldots = -0.105$ (3 s.f.)

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Exponentials and logarithms
Exercise C, Question 5

Question:

Find from your calculator the value to 3 s.f. of:
$\log_{10} 11$

Solution:

$\log_{10} 11 = 1.0413 \quad \ldots \quad = 1.04$ (3 s.f.)

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Exponentials and logarithms
Exercise C, Question 6

Question:

Find from your calculator the value to 3 s.f. of:
$\log_{10} 35.3$

Solution:

$\log_{10} 35.3 = 1.5477 \quad \ldots \quad = 1.55$ (3 s.f.)

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Exponentials and logarithms

Exercise C, Question 7

Question:

Find from your calculator the value to 3 s.f. of:

$\log_{10} 0.3$

Solution:

$\log_{10} 0.3 = -0.5228 \ldots = -0.523$ (3 s.f.)

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Exponentials and logarithms
Exercise C, Question 8

Question:

Find from your calculator the value to 3 s.f. of:
$\log_{10} 999$

Solution:

$\log_{10} 999 = 2.9995 \quad \ldots \quad = 3.00$ (3 s.f.)

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Exponentials and logarithms
Exercise D, Question 1

Question:

Write as a single logarithm:

(a) $\log_2 7 + \log_2 3$

(b) $\log_2 36 - \log_2 4$

(c) $3 \log_5 2 + \log_5 10$

(d) $2 \log_6 8 - 4 \log_6 3$

(e) $\log_{10} 5 + \log_{10} 6 - \log_{10} \left(\frac{1}{4} \right)$

Solution:

(a) $\log_2 (7 \times 3) = \log_2 21$

(b) $\log_2 \left(\frac{36}{4} \right) = \log_2 9$

(c) $3 \log_5 2 = \log_5 2^3 = \log_5 8$
$\log_5 8 + \log_5 10 = \log_5 (8 \times 10) = \log_5 80$

(d) $2 \log_6 8 = \log_6 8^2 = \log_6 64$
$4 \log_6 3 = \log_6 3^4 = \log_6 81$
$\log_6 64 - \log_6 81 = \log_6 \left(\frac{64}{81} \right)$

(e) $\log_{10} 5 + \log_{10} 6 = \log_{10} (5 \times 6) = \log_{10} 30$
$\log_{10} 30 - \log_{10} \left(\frac{1}{4} \right) = \log_{10} \left[\frac{30}{\left(\frac{1}{4} \right)} \right] = \log_{10} 120$

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Exponentials and logarithms
Exercise D, Question 2

Question:

Write as a single logarithm, then simplify your answer:

(a) $\log_2 40 - \log_2 5$

(b) $\log_6 4 + \log_6 9$

(c) $2 \log_{12} 3 + 4 \log_{12} 2$

(d) $\log_8 25 + \log_8 10 - 3 \log_8 5$

(e) $2 \log_{10} 20 - (\log_{10} 5 + \log_{10} 8)$

Solution:

(a) $\log_2 \left(\frac{40}{5} \right) = \log_2 8 = 3 \quad \left(2^3 = 8 \right)$

(b) $\log_6 (4 \times 9) = \log_6 36 = 2 \quad (6^2 = 36)$

(c) $\log_{12} (3^2) + \log_{12} (2^4)$
$= \log_{12} 9 + \log_{12} 16$
$= \log_{12} (9 \times 16)$
$= \log_{12} 144$
$= 2 \quad (12^2 = 144)$

(d) $\log_8 (25 \times 10) - \log_8 (5^3)$
$= \log_8 250 - \log_8 125$
$= \log_8 \left(\frac{250}{125} \right)$
$= \log_8 2$
$= \frac{1}{3} \quad \left(8^{\frac{1}{3}} = 2 \right)$

(e) $\log_{10} (20^2) - \log_{10} (5 \times 8)$
$= \log_{10} 400 - \log_{10} 40$
$= \log_{10} \left(\frac{400}{40} \right)$
$= \log_{10} 10$
$= 1 \quad (10^1 = 10)$

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Exponentials and logarithms

Exercise D, Question 3

Question:

Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$:

(a) $\log_a (x^3 y^4 z)$

(b) $\log_a \left(\frac{x^5}{y^2} \right)$

(c) $\log_a (a^2 x^2)$

(d) $\log_a \left(\frac{x \sqrt{y}}{z} \right)$

(e) $\log_a \sqrt{ax}$

Solution:

(a) $\log_a x^3 + \log_a y^4 + \log_a z$
$= 3 \log_a x + 4 \log_a y + \log_a z$

(b) $\log_a x^5 - \log_a y^2$
$= 5 \log_a x - 2 \log_a y$

(c) $\log_a a^2 + \log_a x^2$
$= 2 \log_a a + 2 \log_a x$
$= 2 + 2 \log_a x \quad (\log_a a = 1)$

(d) $\log_a x + \log_a y^{\frac{1}{2}} - \log_a z$

$= \log_a x + \frac{1}{2} \log_a y - \log_a z$

(e) $\log_a (ax)^{\frac{1}{2}}$

$= \frac{1}{2} \log_a (ax)$

$= \frac{1}{2} \log_a a + \frac{1}{2} \log_a x$

$= \frac{1}{2} + \frac{1}{2} \log_a x$

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Exponentials and logarithms
Exercise E, Question 1

Question:

Solve, giving your answer to 3 significant figures:

(a) $2^x = 75$

(b) $3^x = 10$

(c) $5^x = 2$

(d) $4^{2x} = 100$

(e) $9^{x+5} = 50$

(f) $7^{2x-1} = 23$

(g) $3^{x-1} = 8^{x+1}$

(h) $2^{2x+3} = 3^{3x+2}$

(i) $8^{3-x} = 10^x$

(j) $3^{4-3x} = 4^{x+5}$

Solution:

(a) $2^x = 75$
$\log 2^x = \log 75$
$x \log 2 = \log 75$
$x = \frac{\log 75}{\log 2}$
$x = 6.23$ (3 s.f.)

(b) $3^x = 10$
$\log 3^x = \log 10$
$x \log 3 = \log 10$
$x = \frac{\log 10}{\log 3}$
$x = 2.10$ (3 s.f.)

(c) $5^x = 2$
$\log 5^x = \log 2$
$x \log 5 = \log 2$
$x = \frac{\log 2}{\log 5}$
$x = 0.431$ (3 s.f.)

(d) $4^{2x} = 100$
$\log 4^{2x} = \log 100$
$2x \log 4 = \log 100$

$x = \frac{\log 100}{2 \log 4}$

$x = 1.66$ (3 s.f.)

(e) $9^{x+5} = 50$

$\log 9^{x+5} = \log 50$

$(x+5) \log 9 = \log 50$

$x \log 9 + 5 \log 9 = \log 50$

$x \log 9 = \log 50 - 5 \log 9$

$x = \frac{\log 50 - 5 \log 9}{\log 9}$

$x = -3.22$ (3 s.f.)

(f) $7^{2x-1} = 23$

$\log 7^{2x-1} = \log 23$

$(2x-1) \log 7 = \log 23$

$2x \log 7 - \log 7 = \log 23$

$2x \log 7 = \log 23 + \log 7$

$x = \frac{\log 23 + \log 7}{2 \log 7}$

$x = 1.31$ (3 s.f.)

(g) $3^{x-1} = 8^{x+1}$

$\log 3^{x-1} = \log 8^{x+1}$

$(x-1) \log 3 = (x+1) \log 8$

$x \log 3 - \log 3 = x \log 8 + \log 8$

$x(\log 3 - \log 8) = \log 3 + \log 8$

$x = \frac{\log 3 + \log 8}{\log 3 - \log 8}$

$x = -3.24$ (3 s.f.)

(h) $2^{2x+3} = 3^{3x+2}$

$\log 2^{2x+3} = \log 3^{3x+2}$

$(2x+3) \log 2 = (3x+2) \log 3$

$2x \log 2 + 3 \log 2 = 3x \log 3 + 2 \log 3$

$2x \log 2 - 3x \log 3 = 2 \log 3 - 3 \log 2$

$x(2 \log 2 - 3 \log 3) = 2 \log 3 - 3 \log 2$

$x = \frac{2 \log 3 - 3 \log 2}{2 \log 2 - 3 \log 3}$

$x = -0.0617$ (3 s.f.)

(i) $8^{3-x} = 10^x$

$\log 8^{3-x} = \log 10^x$

$(3-x) \log 8 = x \log 10$

$3 \log 8 - x \log 8 = x \log 10$

$3 \log 8 = x(\log 10 + \log 8)$

$x = \frac{3 \log 8}{\log 10 + \log 8}$

$x = 1.42$ (3 s.f.)

(j) $3^{4-3x} = 4^{x+5}$

$\log 3^{4-3x} = \log 4^{x+5}$

$(4-3x) \log 3 = (x+5) \log 4$

$4 \log 3 - 3x \log 3 = x \log 4 + 5 \log 4$

$4 \log 3 - 5 \log 4 = x \log 4 + 3x \log 3$

$4 \log 3 - 5 \log 4 = x(\log 4 + 3 \log 3)$

$x = \frac{4 \log 3 - 5 \log 4}{\log 4 + 3 \log 3}$

$x = -0.542$ (3 s.f.)

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Exponentials and logarithms
Exercise E, Question 2

Question:

Solve, giving your answer to 3 significant figures:

(a) $2^{2x} - 6(2^x) + 5 = 0$

(b) $3^{2x} - 15(3^x) + 44 = 0$

(c) $5^{2x} - 6(5^x) - 7 = 0$

(d) $3^{2x} + 3^{x+1} - 10 = 0$

(e) $7^{2x} + 12 = 7^{x+1}$

(f) $2^{2x} + 3(2^x) - 4 = 0$

(g) $3^{2x+1} - 26(3^x) - 9 = 0$

(h) $4(3^{2x+1}) + 17(3^x) - 7 = 0$

Solution:

(a) Let $y = 2^x$
$y^2 - 6y + 5 = 0$
$(y - 1)(y - 5) = 0$
So $y = 1$ or $y = 5$
If $y = 1$, $2^x = 1$, $x = 0$
If $y = 5$, $2^x = 5$
$\log 2^x = \log 5$
$x \log 2 = \log 5$
$x = \frac{\log 5}{\log 2}$
$x = 2.32$ (3 s.f.)
So $x = 0$ or $x = 2.32$

(b) Let $y = 3^x$
$y^2 - 15y + 44 = 0$
$(y - 4)(y - 11) = 0$
So $y = 4$ or $y = 11$
If $y = 4$, $3^x = 4$
$\log 3^x = \log 4$
$x \log 3 = \log 4$
$x = \frac{\log 4}{\log 3}$
$x = 1.26$ (3 s.f.)
If $y = 11$, $3^x = 11$
$\log 3^x = \log 11$
$x \log 3 = \log 11$
$x = \frac{\log 11}{\log 3}$
$x = 2.18$ (3 s.f.)

So $x = 1.26$ or $x = 2.18$

(c) Let $y = 5^x$
$y^2 - 6y - 7 = 0$
$(y + 1)(y - 7) = 0$
So $y = -1$ or $y = 7$
If $y = -1$, $5^x = -1$. No solution.
If $y = 7$, $5^x = 7$
$\log 5^x = \log 7$
$x \log 5 = \log 7$
$x = \frac{\log 7}{\log 5}$
$x = 1.21$ (3 s.f.)

(d) Let $y = 3^x$
$(3^x)^2 + (3^x \times 3) - 10 = 0$
$y^2 + 3y - 10 = 0$
$(y + 5)(y - 2) = 0$
So $y = -5$ or $y = 2$
If $y = -5$, $3^x = -5$. No solution.
If $y = 2$, $3^x = 2$
$\log 3^x = \log 2$
$x \log 3 = \log 2$
$x = \frac{\log 2}{\log 3}$
$x = 0.631$ (3 s.f.)

(e) Let $y = 7^x$
$(7^x)^2 + 12 = 7^x \times 7$
$y^2 + 12 = 7y$
$y^2 - 7y + 12 = 0$
$(y - 3)(y - 4) = 0$
So $y = 3$ or $y = 4$
If $y = 3$, $7^x = 3$
$x \log 7 = \log 3$
$x = \frac{\log 3}{\log 7}$
$x = 0.565$ (3 s.f.)
If $y = 4$, $7^x = 4$
$x \log 7 = \log 4$
$x = \frac{\log 4}{\log 7}$
$x = 0.712$ (3 s.f.)
So $x = 0.565$ or $x = 0.712$

(f) $2^{2x} + 3(2^x) - 4 = 0$
Let $y = 2^x$
Then $y^2 + 3y - 4 = 0$
So $(y + 4)(y - 1) = 0$
So $y = -4$ or $y = 1$
$2^x = -4$ has no solution
Therefore $2^x = 1$
So $x = 0$ is the only solution

(g) $3^{2x + 1} - 26(3^x) - 9 = 0$
Let $y = 3^x$
Then $3y^2 - 26y - 9 = 0$
So $(3y + 1)(y - 9) = 0$

So $y = -\frac{1}{3}$ or $y = 9$

$3^x = -\frac{1}{3}$ has no solution

Therefore $3^x = 9$
So $x = 2$ is the only solution

(h) $4(3^{2x+1}) + 17(3^x) - 7 = 0$
$12(3^{2x}) + 17(3^x) - 7 = 0$
Let $y = 3^x$
So $12y^2 + 17y - 7 = 0$
So $(3y - 1)(4y + 7) = 0$
So $y = \frac{1}{3}$ or $y = -\frac{7}{4}$

$3^x = -\frac{7}{4}$ has no solution

Therefore $3^x = \frac{1}{3}$

So $x = -1$ is the only solution

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Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms
Exercise F, Question 1

Question:

Find, to 3 decimal places:

(a) $\log_7 120$

(b) $\log_3 45$

(c) $\log_2 19$

(d) $\log_{11} 3$

(e) $\log_6 4$

Solution:

(a) $\log_7 120 = \frac{\log_{10} 120}{\log_{10} 7} = 2.460$ (3 d.p.)

(b) $\log_3 45 = \frac{\log_{10} 45}{\log_{10} 3} = 3.465$ (3 d.p.)

(c) $\log_2 19 = \frac{\log_{10} 19}{\log_{10} 2} = 4.248$ (3 d.p.)

(d) $\log_{11} 3 = \frac{\log_{10} 3}{\log_{10} 11} = 0.458$ (3 d.p.)

(e) $\log_6 4 = \frac{\log_{10} 4}{\log_{10} 6} = 0.774$ (3 d.p.)

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Exponentials and logarithms
Exercise F, Question 2

Question:

Solve, giving your answer to 3 significant figures:

(a) $8^x = 14$

(b) $9^x = 99$

(c) $12^x = 6$

Solution:

(a) $\log 8^x = \log 14$
$x \log 8 = \log 14$
$x = \frac{\log_{10} 14}{\log_{10} 8}$
$x = 1.27$ (3 s.f.)

(b) $\log 9^x = \log 99$
$x \log 9 = \log 99$
$x = \frac{\log_{10} 99}{\log_{10} 9}$
$x = 2.09$ (3 s.f.)

(c) $\log 12^x = \log 6$
$x \log 12 = \log 6$
$x = \frac{\log_{10} 6}{\log_{10} 12}$
$x = 0.721$ (3 s.f.)

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Exponentials and logarithms
Exercise F, Question 3

Question:

Solve, giving your answer to 3 significant figures:

(a) $\log_2 x = 8 + 9 \log_x 2$

(b) $\log_4 x + 2 \log_x 4 + 3 = 0$

(c) $\log_2 x + \log_4 x = 2$

Solution:

(a) $\log_2 x = 8 + 9 \log_x 2$

$\log_2 x = 8 + \frac{9}{\log_2 x}$

Let $\log_2 x = y$

$y = 8 + \frac{9}{y}$

$y^2 = 8y + 9$
$y^2 - 8y - 9 = 0$
$(y + 1)(y - 9) = 0$
So $y = -1$ or $y = 9$
If $y = -1$, $\log_2 x = -1$

$\Rightarrow \quad x = 2^{-1} = \frac{1}{2}$

If $y = 9$, $\log_2 x = 9$
$\Rightarrow \quad x = 2^9 = 512$

So $x = \frac{1}{2}$ or $x = 512$

(b) $\log_4 x + 2 \log_x 4 + 3 = 0$

$\log_4 x + \frac{2}{\log_4 x} + 3 = 0$

Let $\log_4 x = y$

$y + \frac{2}{y} + 3 = 0$

$y^2 + 2 + 3y = 0$
$y^2 + 3y + 2 = 0$
$(y + 1)(y + 2) = 0$
So $y = -1$ or $y = -2$
If $y = -1$, $\log_4 x = -1$

$\Rightarrow \quad x = 4^{-1} = \frac{1}{4}$

If $y = -2$, $\log_4 x = -2$

$\Rightarrow \quad x = 4^{-2} = \frac{1}{16}$

So $x = \frac{1}{4}$ or $x = \frac{1}{16}$

(c) $\log_2 x + \log_4 x = 2$

$\log_2 x + \frac{\log_2 x}{\log_2 4} = 2$

But $\log_2 4 = 2$ (because $2^2 = 4$) , so

$\log_2 x + \frac{\log_2 x}{2} = 2$

$\frac{3}{2}\log_2 x = 2$

$\log_2 x = \frac{4}{3}$

$x = 2^{\frac{4}{3}}$

$x = 2.52$ (3 s.f.)

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Exponentials and logarithms

Exercise G, Question 1

Question:

Find the possible values of x for which $2^{2x+1} = 3(2^x) - 1$. **[E]**

Solution:

$2^{2x+1} = 3(2^x) - 1$
$2^{2x} \times 2^1 = 3(2^x) - 1$
Let $2^x = y$
$2y^2 = 3y - 1$
$2y^2 - 3y + 1 = 0$
$(2y - 1)(y - 1) = 0$
So $y = \frac{1}{2}$ or $y = 1$

If $y = \frac{1}{2}$, $2^x = \frac{1}{2}$, $x = -1$

If $y = 1$, $2^x = 1$, $x = 0$
So $x = 0$ or $x = -1$

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Exponentials and logarithms
Exercise G, Question 2

Question:

(a) Express $\log_a (p^2 q)$ in terms of $\log_a p$ and $\log_a q$.

(b) Given that $\log_a (pq) = 5$ and $\log_a (p^2 q) = 9$, find the values of $\log_a p$ and $\log_a q$. **[E]**

Solution:

(a) $\log_a (p^2 q) = \log_a (p^2) + \log_a q = 2 \log_a p + \log_a q$

(b) $\log_a (pq) = \log_a p + \log_a q$
So
$\log_a p + \log_a q = 5$ ①
$2 \log_a p + \log_a q = 9$ ②
Subtracting equation ① from equation ②:
$\log_a p = 4$
So $\log_a q = 1$

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Exponentials and logarithms

Exercise G, Question 3

Question:

Given that $p = \log_q 16$, express in terms of p,

(a) $\log_q 2$,

(b) $\log_q (8q)$. **[E]**

Solution:

(a) $p = \log_q 16$

$p = \log_q (2^4)$

$p = 4 \log_q 2$

$\log_q 2 = \frac{p}{4}$

(b) $\log_q (8q) = \log_q 8 + \log_q q$

$= \log_q (2^3) + \log_q q$

$= 3 \log_q 2 + \log_q q$

$= \frac{3p}{4} + 1$

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Exponentials and logarithms
Exercise G, Question 4

Question:

(a) Given that $\log_3 x = 2$, determine the value of x.

(b) Calculate the value of y for which $2 \log_3 y - \log_3 (y + 4) = 2$.

(c) Calculate the values of z for which $\log_3 z = 4 \log_z 3$.

[E]

Solution:

(a) $\log_3 x = 2$
$x = 3^2 = 9$

(b) $2 \log_3 y - \log_3 (y + 4) = 2$
$\log_3 (y^2) - \log_3 (y + 4) = 2$

$\log_3 \left(\frac{y^2}{y+4} \right) = 2$

$\frac{y^2}{y+4} = 9$

$y^2 = 9y + 36$
$y^2 - 9y - 36 = 0$
$(y + 3)(y - 12) = 0$
$y = -3$ or $y = 12$
But $\log_3 (-3)$ is not defined,
So $y = 12$

(c) $\log_3 z = 4 \log_z 3$

$\log_3 z = \frac{4}{\log_3 z}$

$(\log_3 z)^2 = 4$
Either $\log_3 z = 2$ or $\log_3 z = -2$
$z = 3^2$ or $z = 3^{-2}$
$z = 9$ or $z = \frac{1}{9}$

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Exponentials and logarithms
Exercise G, Question 5

Question:

(a) Using the substitution $u = 2^x$, show that the equation $4^x - 2^{(x+1)} - 15 = 0$ can be written in the form $u^2 - 2u - 15 = 0$.

(b) Hence solve the equation $4^x - 2^{(x+1)} - 15 = 0$, giving your answer to 2 decimal places. **[E]**

Solution:

(a) $4^x - 2^{(x+1)} - 15 = 0$
$4^x = (2^2)^x = (2^x)^2$
$2^{x+1} = 2^x \times 2^1$
Let $u = 2^x$
$u^2 - 2u - 15 = 0$

(b) $(u+3)(u-5) = 0$
So $u = -3$ or $u = 5$
If $u = -3$, $2^x = -3$. No solution.
If $u = 5$, $2^x = 5$
$\log 2^x = \log 5$
$x \log 2 = \log 5$
$x = \frac{\log 5}{\log 2}$
$x = 2.32$ (2 d.p.)

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Exponentials and logarithms
Exercise G, Question 6

Question:

Solve, giving your answers as exact fractions, the simultaneous equations:

$8^y = 4^{2x+3}$
$\log_2 y = \log_2 x + 4.$ **[E]**

Solution:

$8^y = 4^{2x+3}$
$(2^3)^y = (2^2)^{2x+3}$
$2^{3y} = 2^{2(2x+3)}$
$3y = 4x + 6$ ①

$\log_2 y - \log_2 x = 4$

$\log_2 \left(\frac{y}{x} \right) = 4$

$\frac{y}{x} = 2^4 = 16$

$y = 16x$ ①

Substitute ② into ①:
$48x = 4x + 6$
$44x = 6$
$x = \frac{3}{22}$

$y = 16x = \frac{48}{22} = 2\frac{2}{11}$

So $x = \frac{3}{22}, y = 2\frac{2}{11}$

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Exponentials and logarithms
Exercise G, Question 7

Question:

Find the values of x for which $\log_3 x - 2 \log_x 3 = 1$. **[E]**

Solution:

$\log_3 x - 2 \log_x 3 = 1$

$\log_3 x - \frac{2}{\log_3 x} = 1$

Let $\log_3 x = y$

$y - \frac{2}{y} = 1$

$y^2 - 2 = y$
$y^2 - y - 2 = 0$
$(y + 1) (y - 2) = 0$
So $y = -1$ or $y = 2$
If $y = -1$, $\log_3 x = -1$

$\Rightarrow \quad x = 3^{-1} = \frac{1}{3}$

If $y = 2$, $\log_3 x = 2$
$\Rightarrow \quad x = 3^2 = 9$

So $x = \frac{1}{3}$ or $x = 9$

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Exponentials and logarithms
Exercise G, Question 8

Question:

Solve the equation
$\log_3 (2-3x) = \log_9 (6x^2 - 19x + 2)$. **[E]**

Solution:

$\log_3 (2-3x) = \log_9 (6x^2 - 19x + 2)$

$$\log_9 \left(6x^2 - 19x + 2 \right) = \frac{\log_3 (6x^2 - 19x + 2)}{\log_3 9} = \frac{\log_3 (6x^2 - 19x + 2)}{2}$$

So

$2 \log_3 (2-3x) = \log_3 (6x^2 - 19x + 2)$

$\log_3 (2-3x)^2 = \log_3 (6x^2 - 19x + 2)$

$(2-3x)^2 = 6x^2 - 19x + 2$

$4 - 12x + 9x^2 = 6x^2 - 19x + 2$

$3x^2 + 7x + 2 = 0$

$(3x+1)(x+2) = 0$

$x = -\frac{1}{3}$ or $x = -2$

(Both solutions are valid, since they give logs of positive numbers in the original equation.)

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Exponentials and logarithms
Exercise G, Question 9

Question:

If xy = 64 and $\log_x y + \log_y x = \frac{5}{2}$, find x and y. **[E]**

Solution:

$\log_x y + \log_y x = \frac{5}{2}$

$\log_x y + \frac{1}{\log_x y} = \frac{5}{2}$

Let $\log_x y = u$

$u + \frac{1}{u} = \frac{5}{2}$

$2u^2 + 2 = 5u$
$2u^2 - 5u + 2 = 0$
$(2u - 1)(u - 2) = 0$

$u = \frac{1}{2}$ or $u = 2$

If $u = \frac{1}{2}$, $\log_x y = \frac{1}{2}$

$\Rightarrow \quad y = x^{\frac{1}{2}} = \sqrt{x}$

Since xy = 64,

$x\sqrt{x} = 64 \quad \left(x^{\frac{3}{2}} = 64 \right)$

$x = 16$
$y = \sqrt{x} = 4$
If $u = 2$, $\log_x y = 2$

$\Rightarrow \quad y = x^2$

Since xy = 64,
$x^3 = 64$
$x = 4$
$y = x^2 = 16$
So $x = 16$, $y = 4$ or $x = 4$, $y = 16$

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Exponentials and logarithms
Exercise G, Question 10

Question:

Prove that if $a^x = b^y = (ab)^{xy}$, then $x + y = 1$. **[E]**

Solution:

Given that $a^x = b^y = (ab)^{xy}$
Take logs to base a for $a^x = b^y$:
$\log_a (a^x) = \log_a (b^y)$
$x \log_a a = y \log_a b$
$x = y \log_a b$ ①

Take logs to base a for $a^x = (ab)^{xy}$
$x = \log_a (ab)^{xy}$
$x = xy \log_a (ab)$
$x = xy (\log_a a + \log_a b)$
$x = xy (1 + \log_a b)$
$1 = y (1 + \log_a b)$ ②

But, from ①, $\log_a b = \frac{x}{y}$
Substitute into ②:
$1 = y \left(1 + \frac{x}{y} \right)$
$1 = y + x$
$x + y = 1$

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Exponentials and logarithms
Exercise G, Question 11

Question:

(a) Show that $\log_4 3 = \log_2 \sqrt{3}$.

(b) Hence or otherwise solve the simultaneous equations:
$2 \log_2 y = \log_4 3 + \log_2 x$,
$3^y = 9^x$,
given that x and y are positive. **[E]**

Solution:

(a) $\log_4 3 = \frac{\log_2 3}{\log_2 4} = \frac{\log_2 3}{2}$

$\log_4 3 = \frac{1}{2}\log_2 3 = \log_2 3^{\frac{1}{2}} = \log_2 \sqrt{3}$

(b) $3^y = 9^x$
$3^y = (3^2)^x = 3^{2x}$
So $y = 2x$
$2 \log_2 y = \log_4 3 + \log_2 x$
$\log_2 (y^2) = \log_2 \sqrt{3} + \log_2 x = \log_2 (x\sqrt{3})$
So $y^2 = x\sqrt{3}$
Since $y = 2x$, $(2x)^2 = x\sqrt{3}$
$\Rightarrow \quad 4x^2 = x\sqrt{3}$

x is positive, so $x \neq 0$, $x = \frac{\sqrt{3}}{4}$

$\Rightarrow \quad y = 2x = \frac{\sqrt{3}}{2}$

So $x = \frac{\sqrt{3}}{4}$, $y = \frac{\sqrt{3}}{2}$

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Exponentials and logarithms
Exercise G, Question 12

Question:

(a) Given that $3 + 2 \log_2 x = \log_2 y$, show that $y = 8x^2$.

(b) Hence, or otherwise, find the roots α and β, where $\alpha < \beta$, of the equation $3 + 2 \log_2 x = \log_2 (14x - 3)$.

(c) Show that $\log_2 \alpha = - 2$.

(d) Calculate $\log_2 \beta$, giving your answer to 3 significant figures. **[E]**

Solution:

(a) $3 + 2 \log_2 x = \log_2 y$
$\log_2 y - 2 \log_2 x = 3$
$\log_2 y - \log_2 x^2 = 3$

$\log_2 \left(\frac{y}{x^2} \right) = 3$

$\frac{y}{x^2} = 2^3 = 8$
$y = 8x^2$

(b) Comparing equations,
$y = 14x - 3$
$8x^2 = 14x - 3$
$8x^2 - 14x + 3 = 0$
$(4x - 1) (2x - 3) = 0$
$x = \frac{1}{4}$ or $x = \frac{3}{2}$

$\alpha = \frac{1}{4}, \beta = \frac{3}{2}$

(c) $\log_2 \alpha = \log_2 \left(\frac{1}{4} \right) = - 2$,

since $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

(d) $\log_2 \beta = \log_2 \left(\frac{3}{2} \right)$

$\log_2 1.5 = \frac{\log_{10} 1.5}{\log_{10} 2} = 0.585$ (3 s.f.)